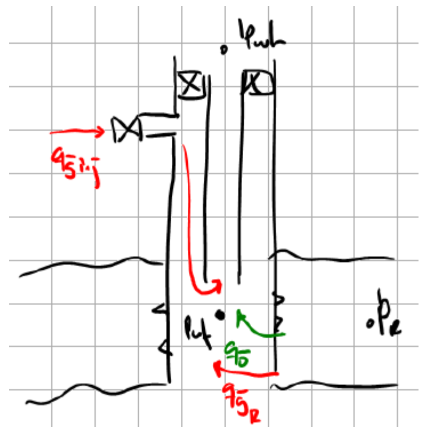
PROBLEM 1 (20 POINTS).

Consider a gas-lifted oil well. The injection point is very close to the bottom of the tubing, so it is reasonable to assume that the lift gas is injected at the end of the tubing (see the figure below). The end of the tubing is very close to the perforations.

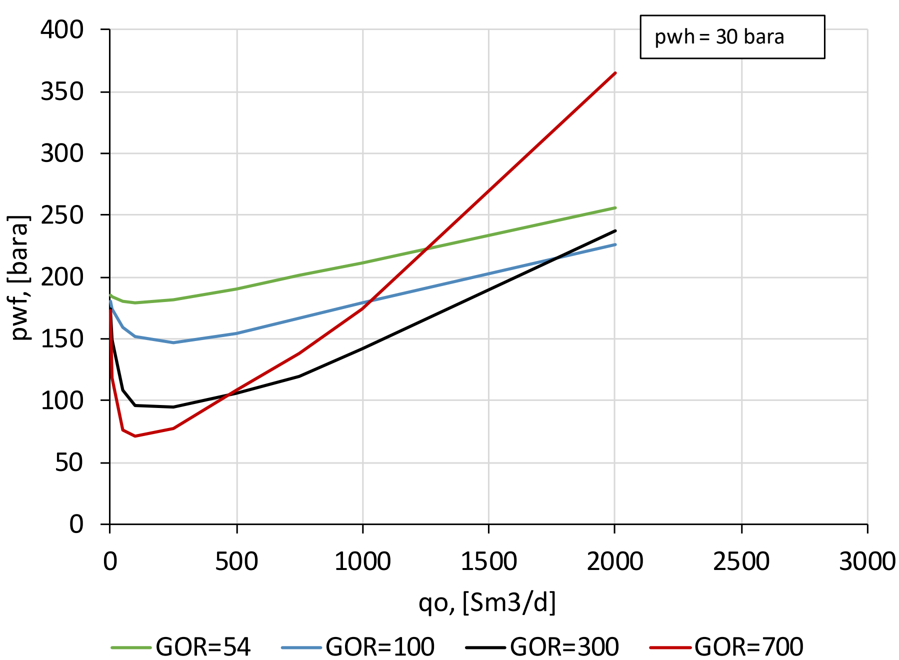


The reservoir GOR is equal to 54. The reservoir IPR can be modeled with Vogel equation (programmed in VBA, available with the name “IPR_Vogel_qo”:

$$q_{\bar{o}} = q_{\bar{o},max}\left[1 - 0.2\cdot\frac{p_{wf}}{p_R} - 0.8\cdot\left(\frac{p_{wf}}{p_R}\right)^2\right]$$

using a p_R = 200 bara, and a q_{omax} = 3000 Sm3/d.

The figure below shows the curves of Tubing performance relationship at a constant wellhead pressure of 30 bara, for different values of GOR in the tubing.

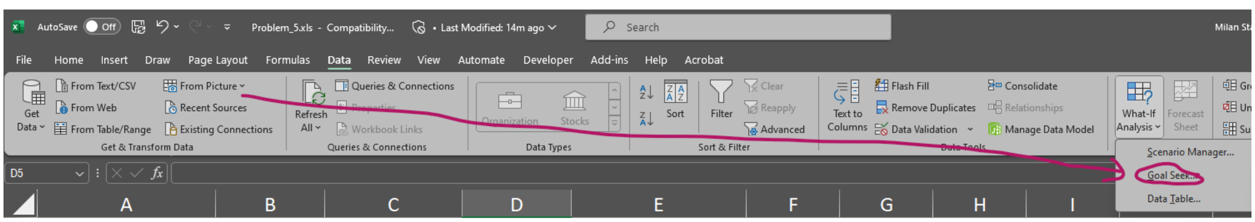


Task. Using the curves given, determine the gas lift performance curve (oil reservoir rate versus gas lift rate injected). Find at least 4 points of the gas lift performance curve.

Use the Excel file provided (it has user defined VBA functions to perform calculations in inflow, all the data you need to perform your calculations and a recommended solving structure). **Write an explanation in the excel file about how you have solved the problem**.

Additional information

- To solve this problem, you need to find intersection between curves. You can find the intersection by reading the values from the chart (naked eye reading), or using the goal seek.
    - In case you decide to use the goalseek, a VBA function for interpolation in the tubing tables is provided. The function is called TwoDimInterpol(x1 As Double, x2 As Double, col As Integer, Matrix As Range, OrderX As Integer, OrderY As Integer). It takes as arguments:
        - Value of rate
        - Value of GOR
        - Column where the data to interpolate is (in this case 3)
        - Matrix: the complete tubing table (without headers)
        - OrderX: 1
        - OrderY: 1
    - The goal seek can be found here:



PROBLEM 2 (10 POINTS). Explain what is gas lift and how does it work

Solution:

Gas lift is an artificial lift method that allows to increase the well rate higher than the equilibrium rate. It consists of injecting gas at the surface, through the annulus and further into the tubing through a port installed at a specific depth.

The pressure drop in the tubing has mainly two components: hydrostatic and frictional. When gas is injected into the tubing, it reduces the density of the fluids in the tubing, reducing the hydrostatic pressure drop component, and because of this, if the well is operating with constant wellhead pressure, the flowing bottomhole pressure is reduced. However injecting gas also increases the flow rate and the velocity in the tubing, which increases the frictional pressure drop component also. Usually, the decrease in the hydrostatic component is higher than the increase in the frictional component.

PROBLEM 3 (20 POINTS). Drawing an analogy to the dry gas IPR, the IPR for a CO_2 injector is:

$$q_{\bar{g}} = -\frac{2 \cdot \pi \cdot k \cdot h}{\left[ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \cdot \int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp$$

Using the pure CO_2 PVT data provided in the Excel file, compute the term

$$\int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp$$

for reservoir pressure 200 bara, and flowing bottom-hole pressures ranging from 200 to 500 bara (for example use 200, 300, 400, and 500.

Will it be possible to approximate the IPR of this well with a linear equation? (something like):

$$q_{\bar{g}} = J\left(p_{wf} - p_R\right)$$

Use the Excel file provided to solve this problem.

Additional information.

- The trapezoidal rule for numerical integration is:
$$\int_a^b f(x)\,dx \approx (b-a) \cdot \tfrac{1}{2}(f(a) + f(b))$$
- The gas formation volume factor, B_g ,is volume of gas at local conditions divided by volume of gas at standard conditions.

PROBLEM 4 (10 POINTS).

Consider the IPR equation:

$$q_{\bar{o}} = \frac{k \cdot h}{18.68 \cdot \left[ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \cdot \left(\frac{1}{\mu_o \cdot B_o}\right)_{@p_{av}} \cdot \left(p_R - p_{wf}\right)$$

With μ_o in cP, k in md, h in m, p in bara, and rate in Sm3/d.

1. Briefly explain for which conditions is this equation valid.
2. When deriving IPRs, there is usually an assumption about the type of outer boundary (either no flow or constant pressure). Briefly explain which term in this equation captures the effect of the boundary, and what type of boundary is assumed?
3. The IPR equation above is simplified from solving the integral: $\int_{p_{wf}}^{p_R} \frac{1}{\mu_o \cdot B_o} dp$. What approximation was made to solve it?

Solution:

- This equation is valid for vertical well, pseudo steady state, undersaturated oil
- The effect of the boundary is on the value -0.75. When integrating the IPR, one integrates from wellbore to the location where reservoir pressure is. In pseudo steady state, the location of the boundary is at 0.47 re (external radius). The type of boundary assumed is no flow.
- The approximation used to solve the integral is that the term 1/(visco Bo) is linear (negative slope) with pressure. Therefore the integral is approximated with the trapezoidal rule.

PROBLEM 5 (20 POINTS).

A test has been performed on a gas well and the following pair of dry gas rate and flowing bottom-hole pressure are reported.

Test point	q_g [Sm3/d]	p_{wf} [bara]
1	6.89 E04	80

Reservoir pressure is 120 bara.

You will use this data to find the parameters in the low-pressure dry gas backpressure IPR equation:

$$q_{\bar{g}} = C_R \cdot \left(p_R^2 - p_{wf}^2\right)^n$$

This equation will be later used to perform a flow equilibrium calculation in the well, assuming wellhead pressure is equal to 13 bara.

However, the issue is that there is only one test point available, so you need to make an assumption about the value of n (you can assume Darcy flow, turbulent flow, or anything in between). **Quantify what effect could the exponent "n" have on your equilibrium calculations and determine if it is worthwhile or not to ask management to perform an additional test to estimate the value of n**.

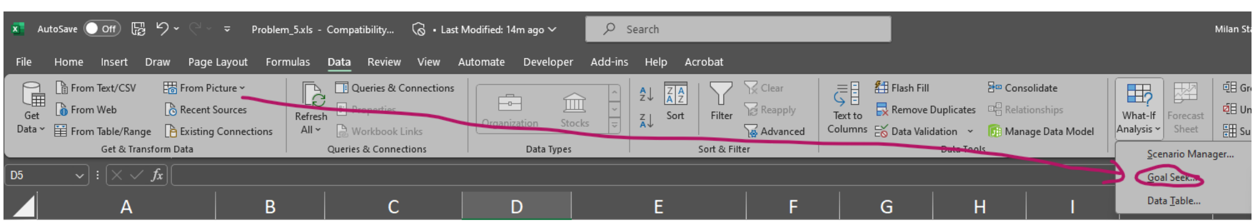
Use the Excel file provided (it has user defined VBA functions to perform calculations in inflow, tubing, and all the data you need to perform your calculations). **Write an explanation in the excel file about how you have solved the problem**.

Additional information:

- Dry gas flow equations programmed in VBA

Inflow equation: $q_{\bar{g}} = C_R \cdot \left(p_R^2 - p_{wf}^2\right)^n$	C_R Inflow backpressure coefficient n, inflow backpressure exponent p_R reservoir pressure , bara p_{wf} flowing bottomhole pressure , bara
Tubing equation: $q_{\bar{g}} = C_T \cdot \left(\frac{p_{wf}^2}{e^S} - p_{wh}^2\right)^{0.5}$	C_T, tubing coefficient, Sm^3/bar S, tubing elevation coefficient, [-] p_{wh}, wellhead pressure , bara

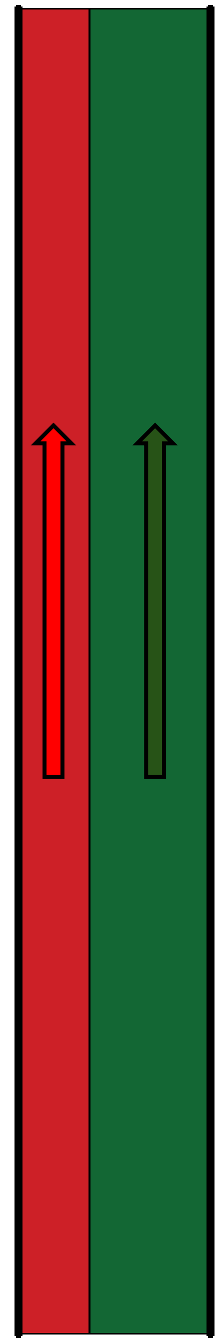
- If you need to use a solver in Excel, you can use the GoalSeek



PROBLEM 6 (20 POINTS).

Consider a vertical tubing, 2000 m long and with an internal diameter of 0.1 m that has gas and oil circulating through it. Assume both are incompressible and that there is no mass transfer between them.
The mass flow of the oil is 8 kg/s while the mass flow of gas is 4 kg/s. The density of the oil is 800 kg/m^3, while the density of the gas is 100 kg/m^3.
Calculate:

- Total amount of liquid (in kg) that is in the tubing
- How long does it take a particle of oil to travel from the bottom of the well to the top
- How long does it take a particle of gas to travel from the bottom of the well to the top.



Additional information:

- The liquid holdup (H_L) can be estimated with the Chisholm correlation:

$$H_L = \frac{1}{\dfrac{\rho_L \cdot x}{\left(\dfrac{\rho_L}{\rho_g}\right)^{0.25} \cdot \rho_g \cdot (1-x)} + 1}$$

Where x is the mass fraction (mg/(ml+mg))

Solution:

We calculate the holdup

The gass mass fraction = 4/12=0.33

$$H_L = \frac{1}{\frac{800 \cdot 0.33}{\left(\frac{800}{100}\right)^{0.25} \cdot 100 \cdot (0.66)} + 1} = 0.29599$$

The total volume the liquid occupies in the tubing is then:

$$H_L \cdot L \cdot \frac{\pi \cdot \emptyset^2}{4} = 4.65\ m^3$$

To find the mass, we multiply the volume by the liquid density = 4.65*800=3719 kg

The velocity of the liquid is $\frac{\dot{m}_L}{\rho_L \cdot A_L} = \frac{\dot{m}}{\rho_L \cdot A \cdot H_L} = \frac{8}{800 \cdot \frac{\pi \cdot \emptyset^2}{4} \cdot 0.296} = 4.3\ m/s$

The velocity of the gas is: is $\frac{\dot{m}_G}{\rho_g \cdot A_G} = \frac{\dot{m}}{\rho_L \cdot A \cdot (1 - H_L)} = \frac{4}{100 \cdot \frac{\pi \cdot \emptyset^2}{4} \cdot (1 - 0.296)} = 7.23\ m/s$

It takes a particle of liquid to travel from bottom up 2000m /4.27 m/s = 465 s

It takes a particle of gas to travel from bottom up 2000 m /7.25 m/s = 276 s