NTNU | Norwegian University of Science and Technology

TPG4245 – Production wells

Autumn Semester 2024

Information

- Lecturer: Assoc. Prof. Milan Stanko (Production Tech) (milan.stanko@ntnu.no). Office 510.
- Teaching assistant: Even Hognestad, evenhog@stud.ntnu.no

- Lecture schedule (starting 19.08)
  - Mondays, 14:15-16:00 (theory and exercises) – P10
  - Fridays, 08:15-10:00 (theory and exercises) – P11
  - Fridays*, 11:15-12:00 (TA) – P10
- Course description

EXCEPT First week

Course scope

- Production performance of wells and gathering systems.
- Addresses the integrated production and injection system, inflow, tubing and pipe flow, and technologies such as artificial lift
- Developing skills for planning, operating, monitoring, troubleshooting and controlling production of oil and gas production systems, CO_2 injection systems, and natural gas storage subsurface systems.

Goals of the course

At the end of the course, the student should be able to:

- Perform common production engineering calculations
- Understand the fundamentals of petroleum production engineering
- Describe the main components of the production system, the most common well completions, artificial lift methods and configurations of production systems
- Describe, understand and explain the functionality of the main components of a production system
- Understand the factors and drivers involved in the planning and operation of oil and gas wells

Goals of the course

At the end of the course, the student should be able to:

- Have a good starting point to apply the knowledge obtained to other areas such as pipe transport and wells of CO_2, hydrogen, natural gas storage and geothermal energy.

Course content

- Introduction (well layout, production engineering domain)
- Flow equilibrium
- PVT properties
- Inflow performance relationship
  - Undersaturated Oil
    - Radial and horizontal wells
    - Water coning
  - Dry Gas
    - High velocity flow
  - Saturated oil
  - Gas condensate
  - Water, CO2 injector
- Choke performance
- Tubing performance
  - Incompressible liquid
  - Dry Gas flow
  - Tubing size considerations
  - Multiphase flow of oil, gas and water
  - CO_2 flow
- Mechanical properties and stress calculations in tubulars
- Artificial lift
  - Gas lift
  - Electric submersible pump
- Temperature calculations in wellbore

Course scope

- Practical SI units only (bar, m3), no field units (psi, bbl, cf).
- Less on the structural and completion part, e.g. design, material selection
  - Completion tools, technology and procedures may vary between different vendors and companies
- Not all models and topics will be covered - Focus on the fundamentals

Students background

- MTPETR. Petroleumsfag - master (5 year master)
- MSGEOS. Geoscience and georesources (2-year International master program).
- MSG1. Petroleum Engineering (2-year International master program).
- MIUVT. Undervannsteknologi (2 year master)
- Mechanical engineering (5 year program)
- Exchange (Erasmus) students
- PhD candidates

Connection with other courses

- (Spring) TPG4145 (Reservoir fluids and flow).
  - 3rd year course Mandatory for Petroleumsfag - master (sivilingeniør) (5-årig) – Hovedprofil: Petroleumsteknologi.
  - 1st year course mandatory for Geoscience and georesources (2-year International master program)
  - Topics:
    - PVT properties.
    - Dry gas flow equations (tubing and IPR).
    - Undersaturated oil IPR equation.
    - Saturated oil Fetkovich IPR equation
- TPG4150 (reservoir recovery techniques). Several IPR models (undersaturated oil, dry gas, saturated oil, gas condensate).
- TPG4175 (Petrophysics, well logging). Reservoir properties and geometry.

Information

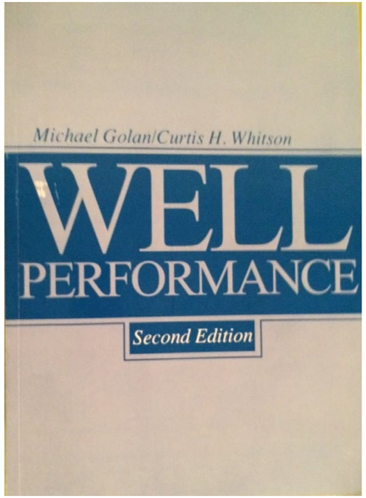
- Lectures until 22 November
- Consultation time: preferably after class. Try to make an email appointment.
- Reference group – **any volunteers?**
- **Use Blackboard to navigate the course**
  - For group deliveries: Join a group before delivering the exercise (even if group consists of only one person!!)

Reference material

- Milan's Compendium
- Book: Well performance (Golan and Whitson)
- Other relevant material, e.g. articles, Excel files, notes, links, will be provided or mentioned in the videos

Other

- Production wells compendium (Asheim)
- Book Nodal analysis of Oil and Gas production Systems, (Jansen)



Evaluation

- 100% «written» school exam
  - Digital exam in Inspera, no written/handwritten material allowed (equations will be provided in the exam papers)
  - Previous years' exams
  - Make it nice, easy to understand and follow. When provided, use the Excel template

Evaluation

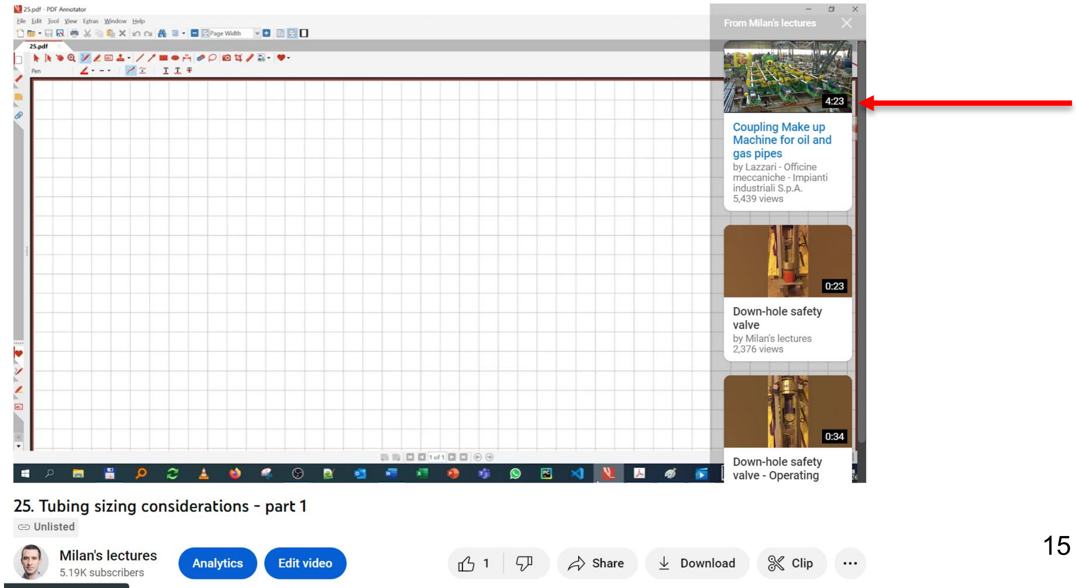
- Mandatory assignments
  - All assignments must be **approved** to get access to the exam
  - All assignments must be delivered in Blackboard by the deadline
  - Some assignments will be discussed in class
  - Groups of up to 3 people may be allowed for some assignments
  - Tentatively 3 assignments
  - **Let me know early if there is a deadline conflict with other courses**

Teaching

- Flipped classroom
  - Participants watch by themselves pre-recorded videos (ca 15-40 min) (on Youtube).
    - There are exercises in the videos
  - Live classes every week
    - Discussing theory, exercises, tutorials on software, Q&A, advanced topics

How to watch the pre-recorded videos

- Watch the entire video (can be watched at 1.5-2x speed)
- At certain time stamps (**or at the end of the video**), the videos might have embedded links to other relevant videos and material

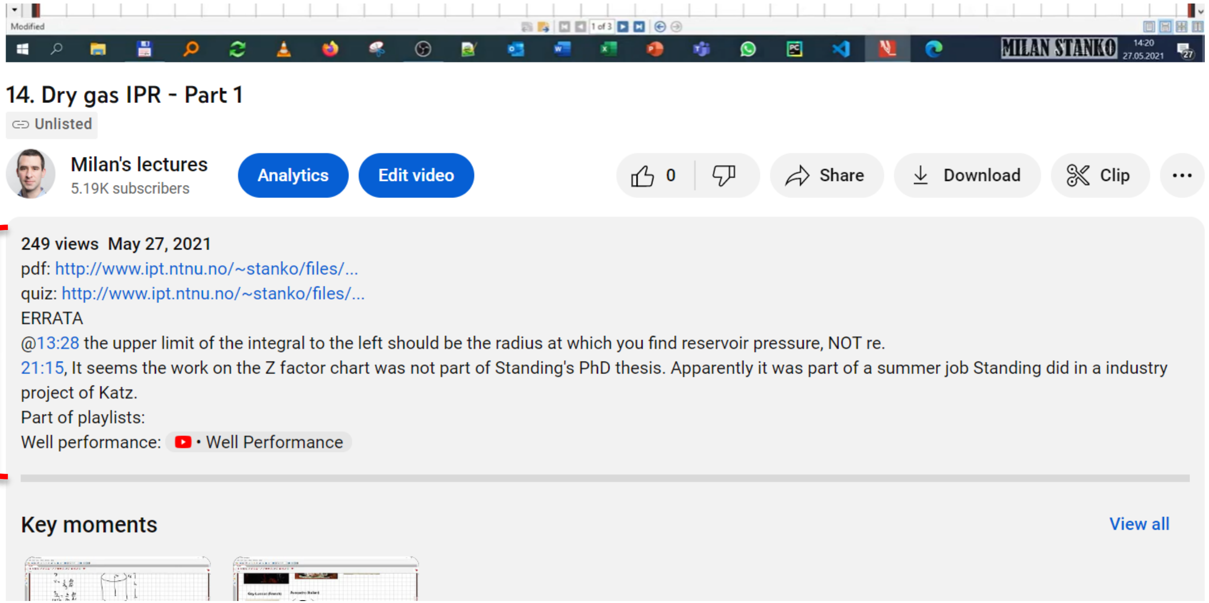


How to watch the videos

- Pause when needed. Try to summarize what was presented with your own words. Take notes.
- DO THE EXERCISES BY YOURSELF

How to watch the videos

- Read the additional material provided, if any



Teaching - streaming

Fysisk tilstedeværelse for studenter ved Fakultet for ingeniørvitenskap fra høsten 2022

Etter en lang periode med koronatiltak er vi nå i en normalfase for undervisning og tilstedeværelse på campus. Fysisk oppmøte har en stor betydning for det psykososiale læringsmiljøet og trivsel blant studentene. Et godt psykososiale læringsmiljø og trivsel bidrar også til mindre frafall.

I løpet av vårsemesteret har vi opplevd manglende oppmøte i forelesninger som gjennomføres fysisk. Manglene oppmøte i undervisningen gjør det vanskeligere å følge opp det pedagogiske opplegget, og diskusjoner og samarbeid blir vanskelig å gjennomføre. I tillegg øker risikoen for faglige hull blant studentene noe som kan gi dårligere gjennomføringsevne.

Fra studiestart høsten 2022 blir forelesninger og undervisning primært gjennomført fysisk på campus for studenter ved Fakultet for ingeniørvitenskap. Studentene skal hovedsakelig være til stede i forelesningssalen eller i klasserom og verksted.

Hovedregelen er at vi ikke strømmer undervisningen.

Teaching – recording?

- Live lectures will be recorded (only the screen of Milan’s computer and voice) and uploaded to Blackboard
- There will be some things that will not be properly captured by the recording (when Milan points to the screen, comments from people in the audience, etc)
- But despite of having the recording available, try to come to class in person, having people in the room allows to have discussions and interactions

Course progress overview – Excel file

Link

Tools

Primary:

- Excel (VBA)
- Pipesim (SLB) or Prosper (PETEX) – Computer lab P2

Secondary:

- Hysys (Aspentech) – farm.ntnu.no
- Python (Jupyter Notebook) – e.g. using Google Colaboratory

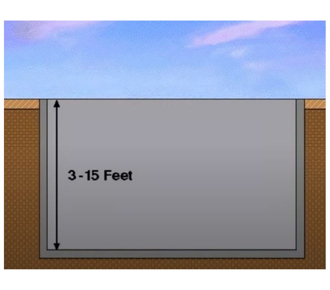
Approx. student time required

Item	hrs
Student time required for a 7.5p course (source)	+200
Live classes (14•4)	-56
TA sessions (13•1)	-13
Final exam (4 exam+40 prep)	-44
Video watching (1X speed)	-19.5
Video exercises	-8
Remaining (e.g. mandatory exercises, etc.)	+59.5

Questions?

ONSHORE WELL DRILLING (easier to visualize and understand)

Digging the cellar:

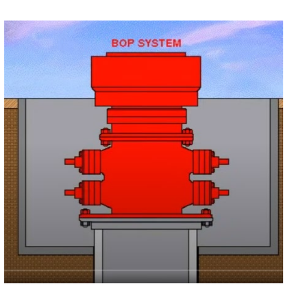


sequence extracted from*

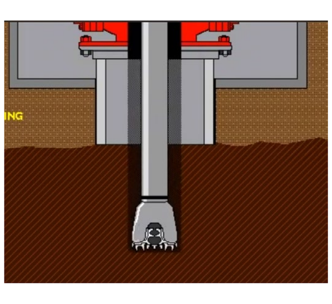
Installing the 36”, 40-120 m conductor (drilled, piled, jetted, etc):

The conductor carries all the well loads (everything inside) and transfers it to the soil

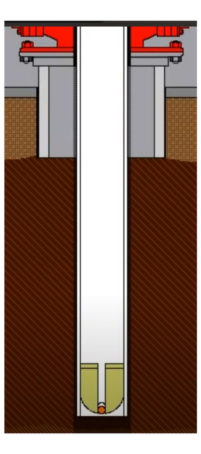
Surface casing (20”):



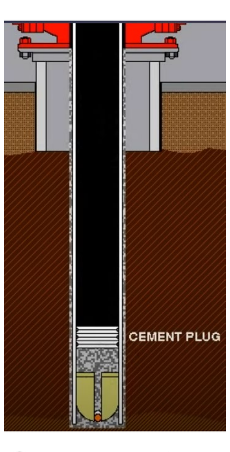
BOP connection



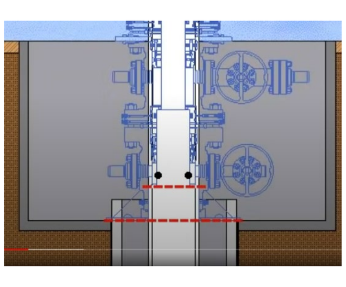
Drilling hole



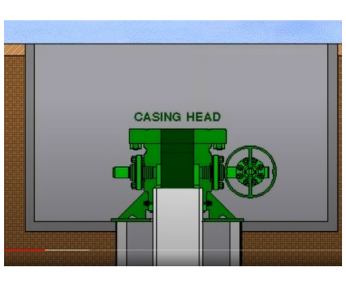
Running casing



Cementing

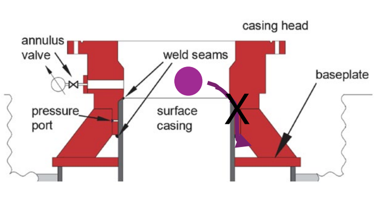


Installing casing head



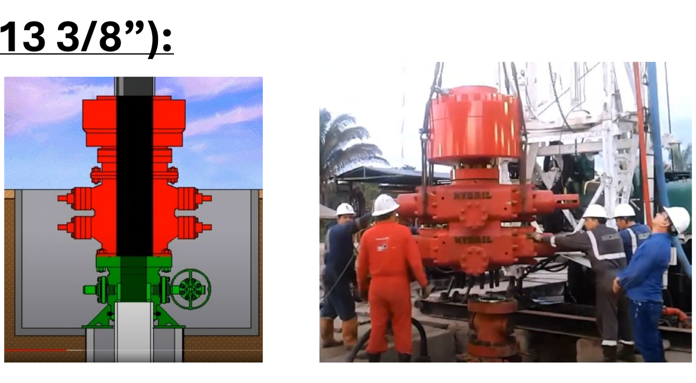
Installing casing head

welding casing head*

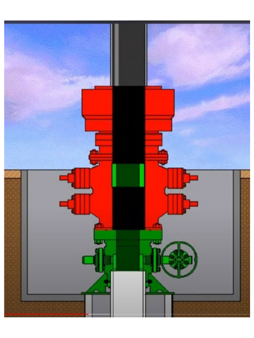


Check weld sealing using the pressure port and a hydraulic pump (the violet dot is high pressure and should not leak out through the welding). **The well is essentially a pressure vessel protecting outside from inside!!**

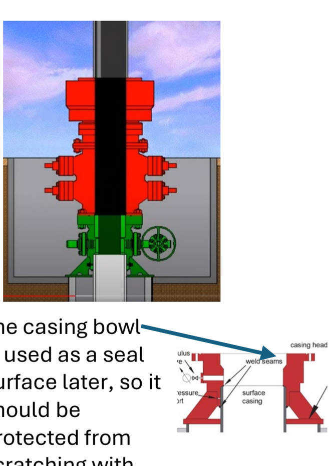
Intermediate casing (13 3/8”):



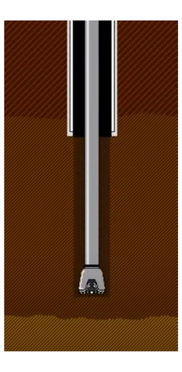
BOP connection (flanged)*



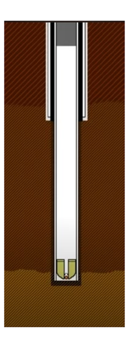
Wear bushing installation



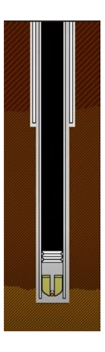
The casing bowl is used as a seal surface later, so it should be protected from scratching with the drill bit and drill string



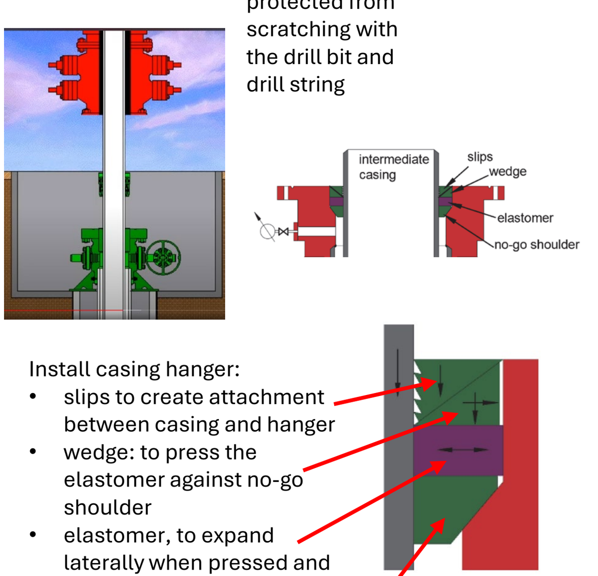
Drilling hole



Running casing (when at the bottom, keep it hanged rig (tension)



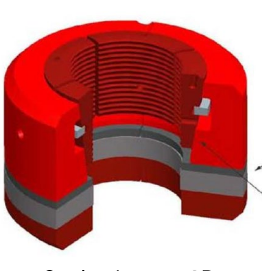
Cementing (either all way to surface or with significant overlap with previous casing)



Install casing hanger:

- slips to create attachment between casing and hanger
- wedge: to press the elastomer against no-go shoulder
- elastomer, to expand laterally when pressed and create seal (high pressure above, low pressure below)
- No-go shoulder, to transfer load to casing bowl

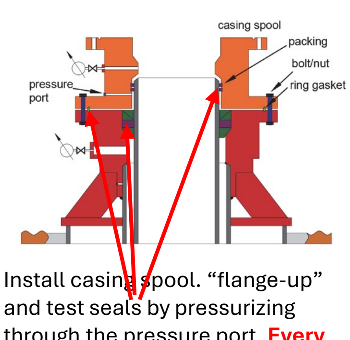
Slip similar to drilling slips*



Casing hanger 3D view

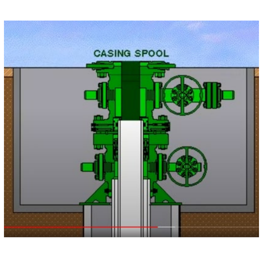
Casing hanger*

Slip marks on casing *

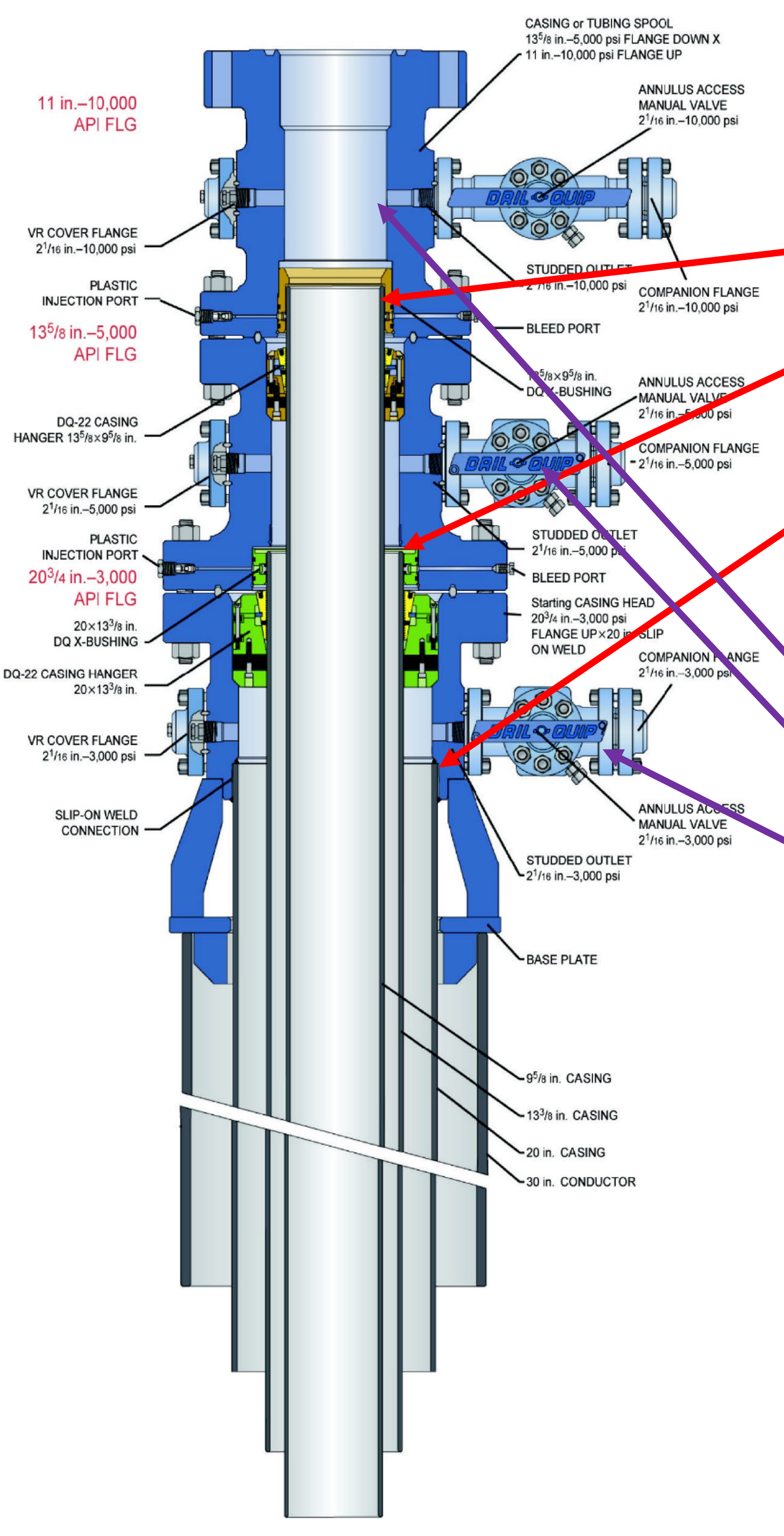


Install casing spool. “flange-up” and test seals by pressurizing through the pressure port. **Every seal should be tested by applying high or low pressure depending where it will have it during normal operation.**

source



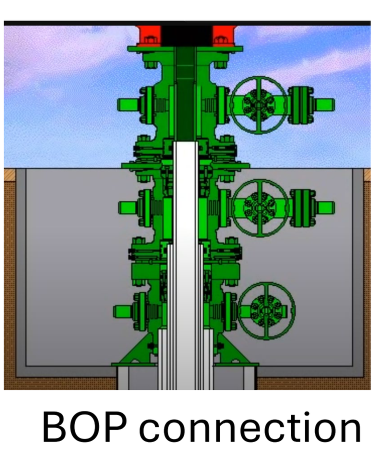
Intermediate casings and production casing (9 5/8”): Repeat process shown above for intermediate casing!



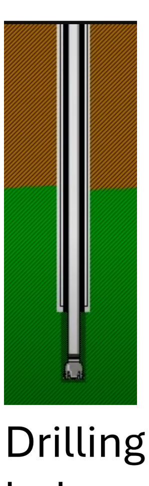
Pressure rating of inner casing higher than outer casing (here production casing 10 000 psi, intermediate casing 5000 psia, surface casing 3000 psia)

The only annulus that might be connected to facilities (for production of injection) is the production casing annulus (A). However, valves and pressure transducers are installed in the others (B,C) to monitor if there are pressure increases (indicates leakages e.g. due to casing hangers seals or cement fails)

Lower completion and tubing:

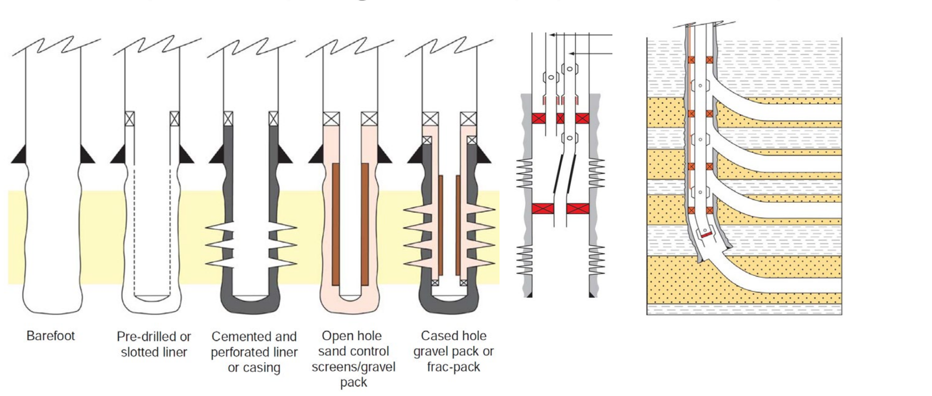


BOP connection (flanged)

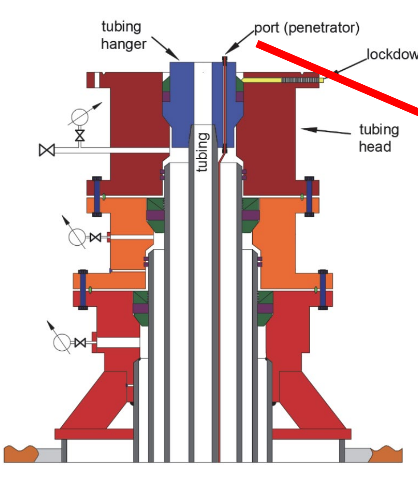


Drilling hole

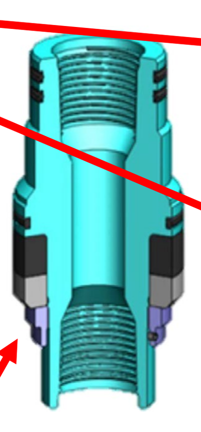
some options (single zone), (multi-zone)



Lower completion. Equipment/materials usually needed: packers (seals), hangers (fix tubulars to casing), drilling (drilling laterals), liners (tubulars that don’t go to surface), perforating gun (establish connection between reservoir and well), cement (isolate reservoir from well).



Run tubing and hang it with tubing hanger

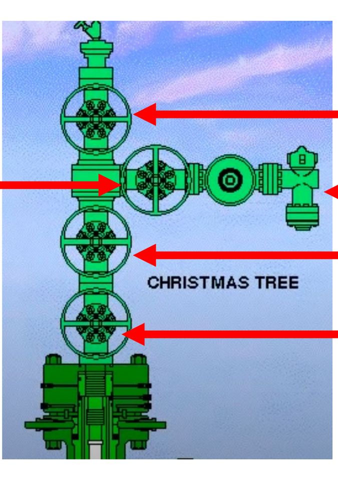


threaded hanger

Lockdown screws are often used to ensure hangers stay in place. When there is thermal expansion of tubing and casing, the hanger could be pushed upwards, releasing the slips and disengaging the seal.

Tubing hanger has “holes” to e.g. chemical injection, pressure gauge, hydraulic lines for subsurface safety valve, etc

Install X-mas tree:



wing valve

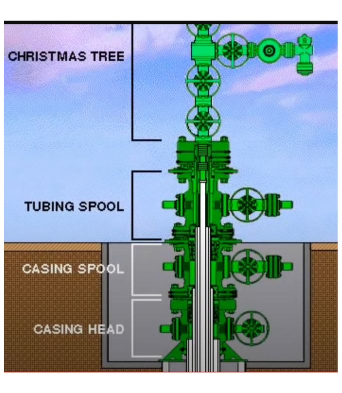
swab valve (to perfom light intervention on well)

Choke (control valve)

upper master valve (remotely operated)

lower master valve (manually operated)

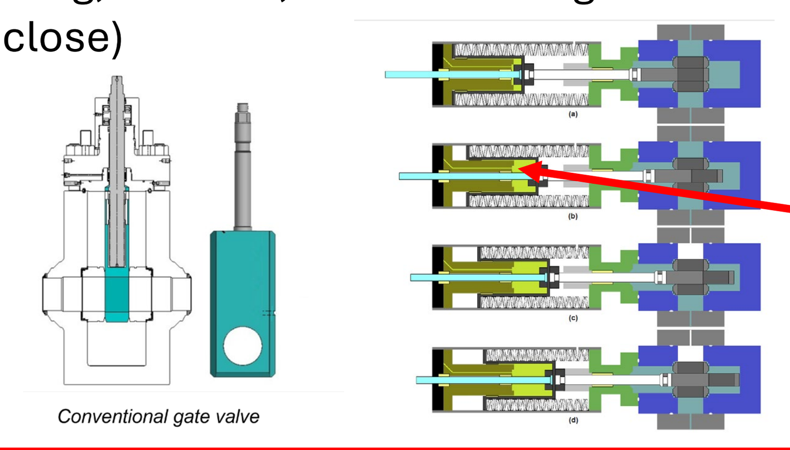
redundancy



X-mas tree

wellhead/ upper completion

wing, masters, and swab are gate valves (only open or close)



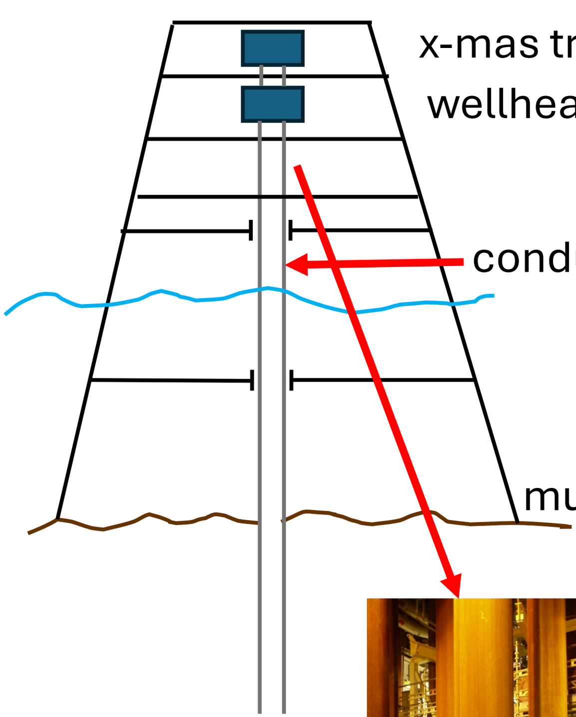
Fail-safe mechanism (closes if hydraulic fluid pressure drops thanks to spring)

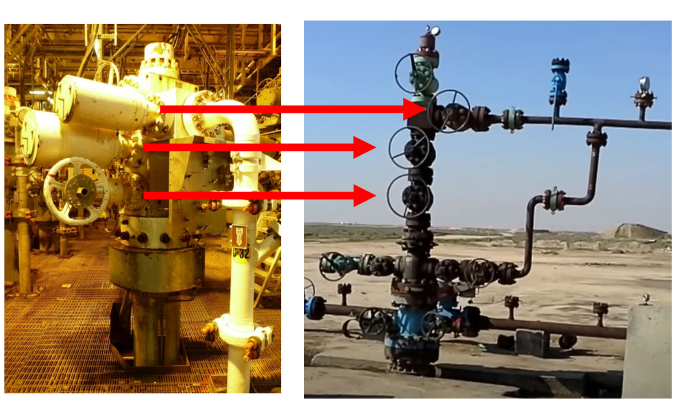
annulus master valve

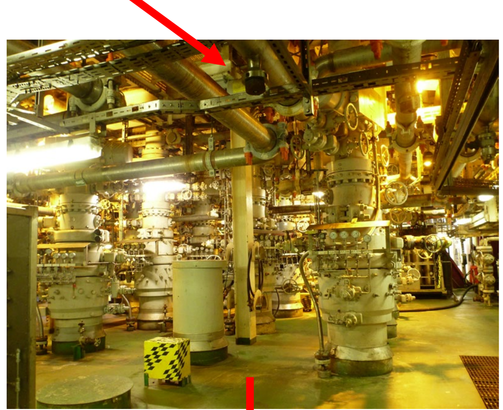
source

PLATFORM WELL DRILLING

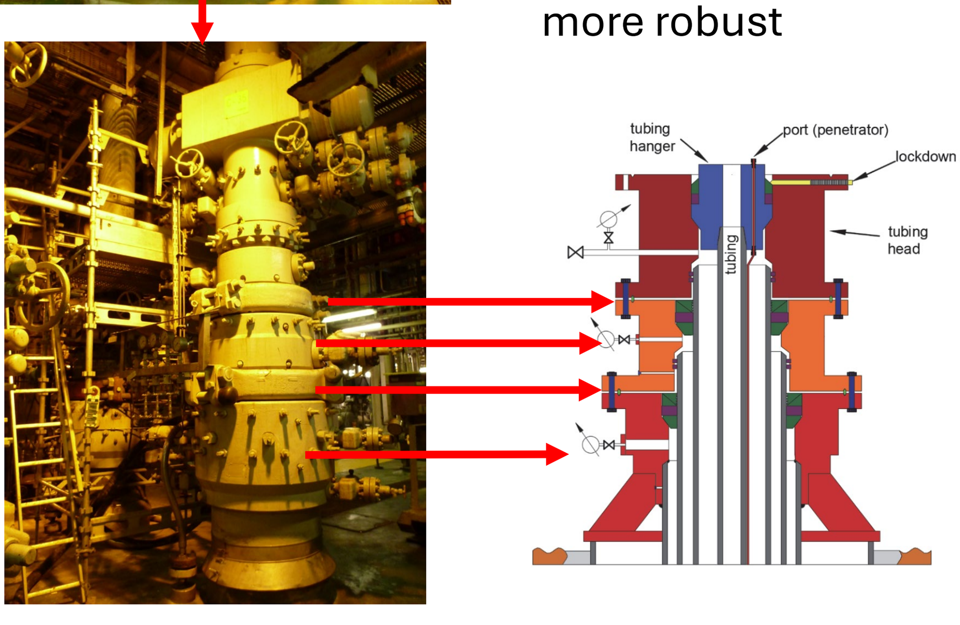
similar to onshore drilling, but the conductor is extended above the mudline and into the platform.



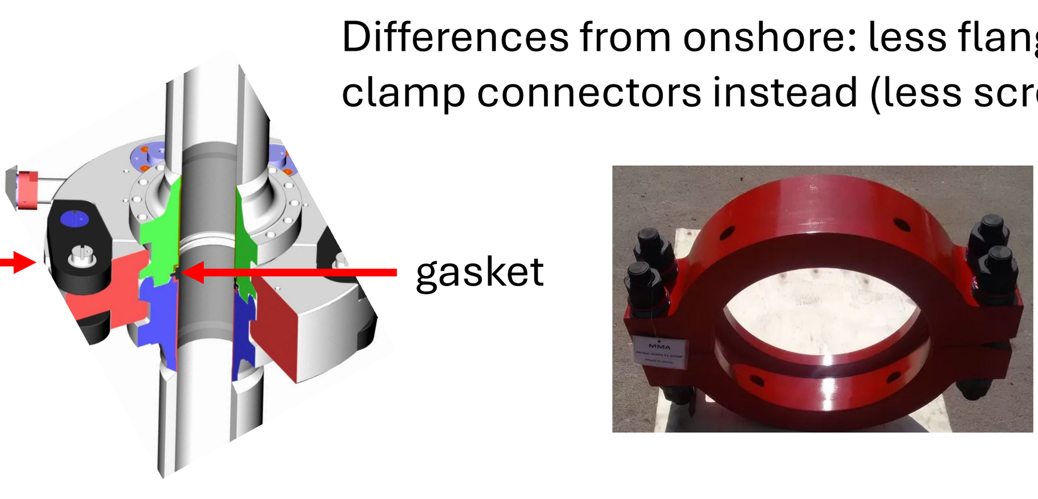




Differences from onshore: x-mas tree is a solid steel block, more robust



Differences from onshore: less flanges, fast clamp connectors instead (less screws)



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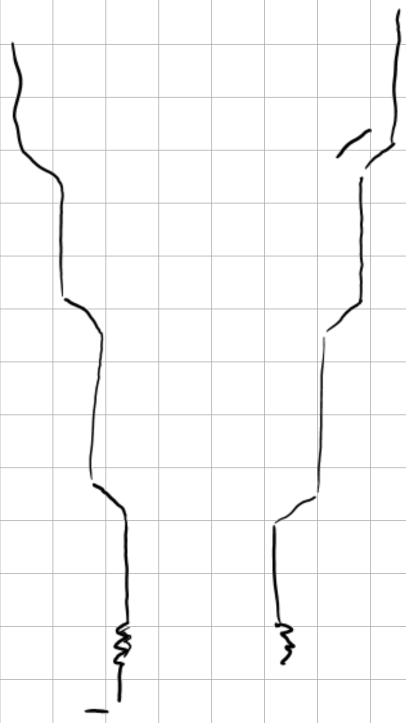
AGENDA:
-Approximate estimation of student time required
-YT video duration per week available in Excel file for course plan
-Re-cap of well construction process onshore using summary poster
-(class activity) Casing hanger puzzle
-well construction process on platform using summary poster
-Comments on well construction process subsea using summary poster.



SUBSEA WELL DRILLING

3D images taken from *

- There is a wellhead housing, usually attached to the top of the surface casing and resting on the conductor. All wellhead items will be inside. It might have some section changes to hang casing and grooves to latch "stuff".

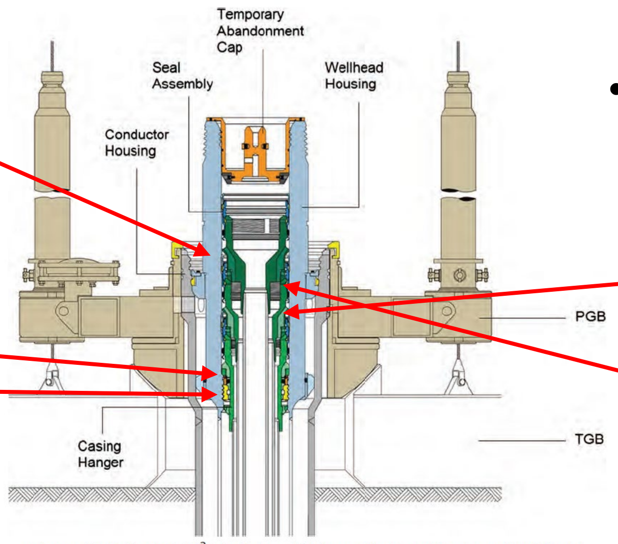
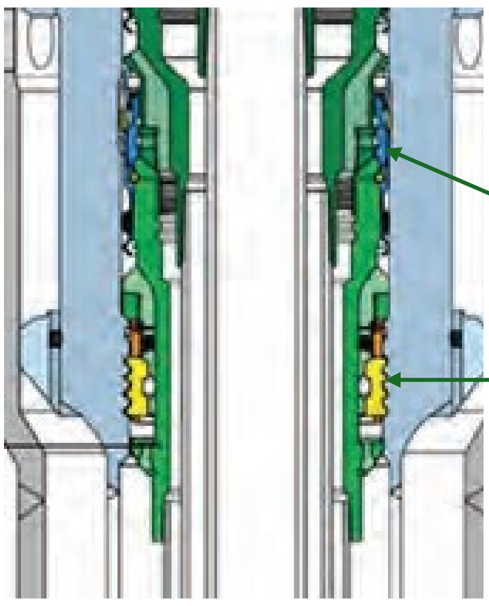
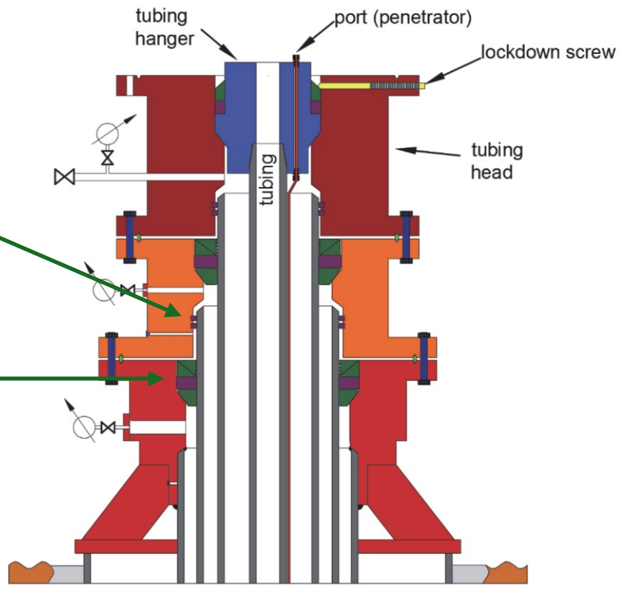
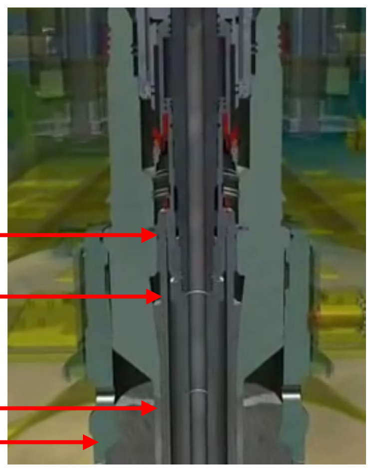


Figure 22-5 Typical 18¾-in. Subsea Wellhead System *(Courtesy of Dril-Quip)*

- Tubulars usually have at the top part the no-go, and above it a "bowl" to hang the next casing.





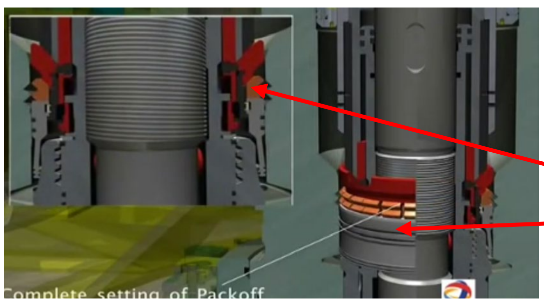


no-go shoulder

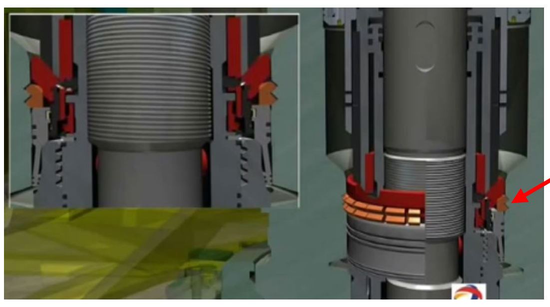
Interm. casing

Surface casing

conductor



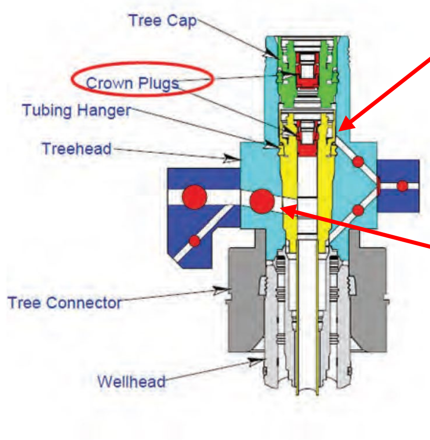
Packoff assembly (equivalent to casing hanger seal)



Lock mechanism (equivalent to lockdown screw)

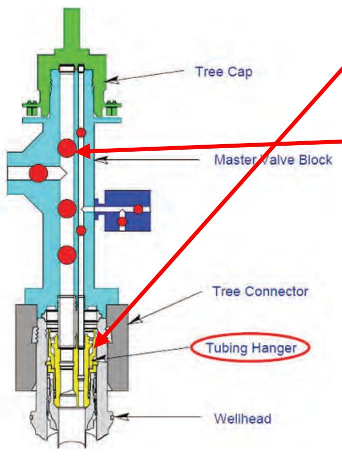
- Two types of wells:

Horizontal tree



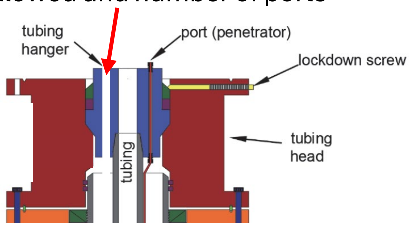
- Tubing hanger is at the top of the x-mas tree. This allows to replace tubing without removing x-mas tree. In the past, SS tubing was expensive, so it was necessary to change it more frequent that X-mas tree. Nowadays, SS tubing price is ok.
- Valves are sideways
- Tubing hanger has a hole, needs to be oriented with the hole in the x-mas tree block.

vertical tree

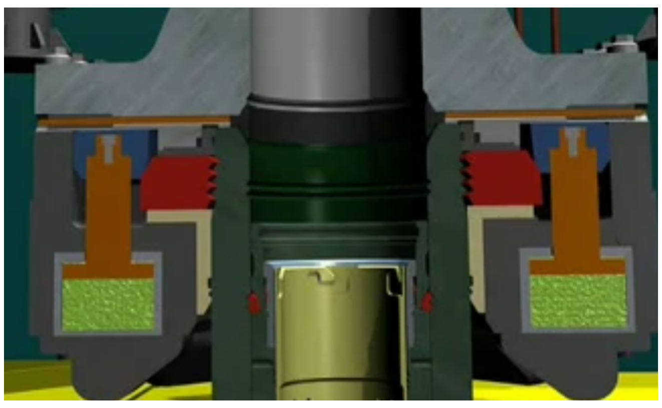


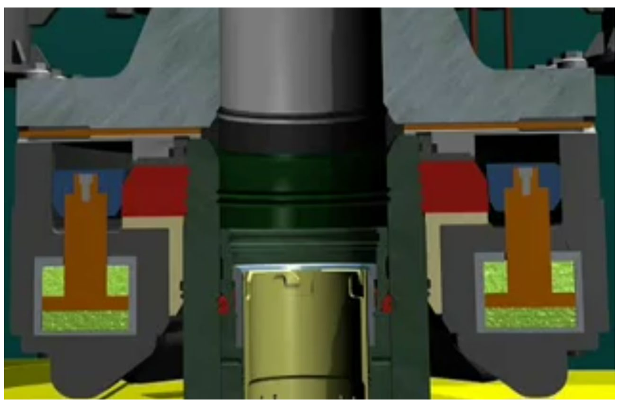
- Tubing hanger is part of the wellhead (tubing lasts longer than x-mas tree)
- Valves are vertical
- Long tree, which can create stresses on the wellhead due to bending while intervention

- The production annulus is accessed through the hanger, which reduces the max tubing size allowed and number of ports

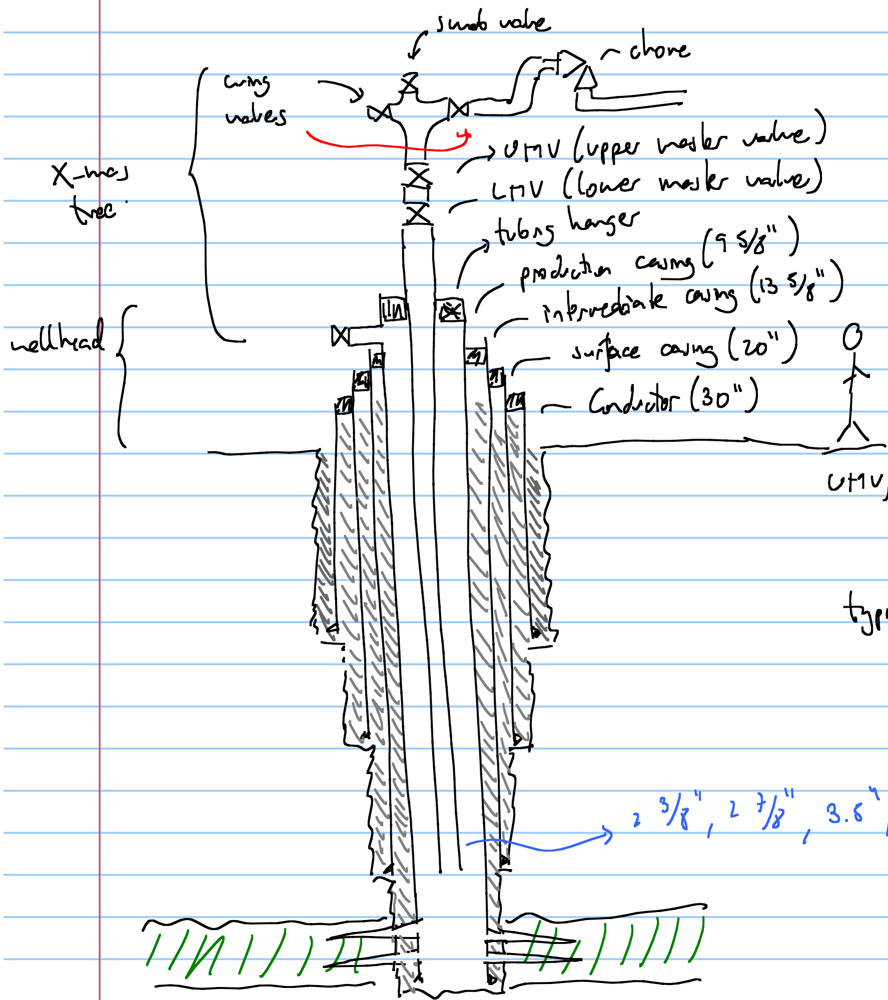


H4 connector, hydraulically driven, for connection to wellhead –BOP, X-mas tree-wellhead, intervention, etc.

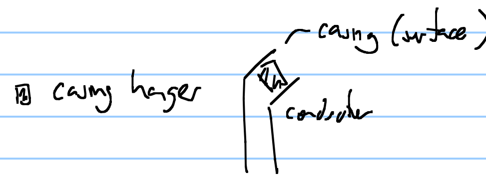




Video 01: Well layout and domain of production engineering

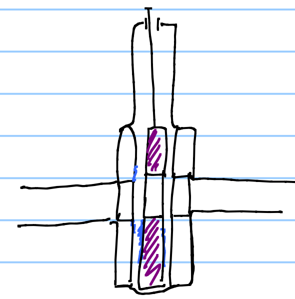


2 3/8", 2 7/8", 3.6", 4", 5", 7", 9"

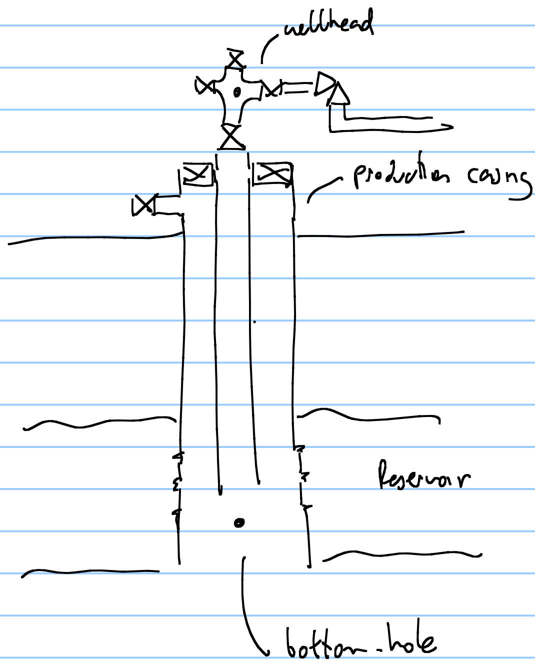


UMV, LMV, wing valves, swab valve on-off (fully open, fully closed)

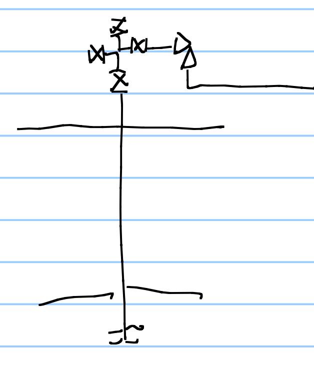
typically gate valves



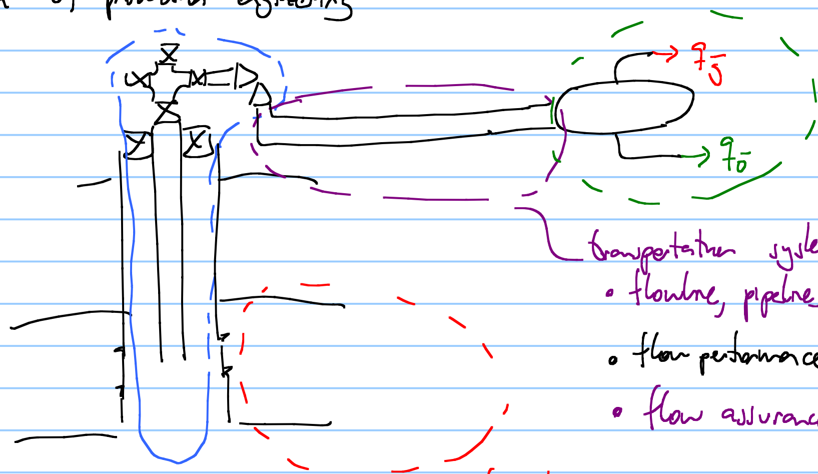
Drawing of a well



Line diagram



Domain of production engineering



processing facilities
- prepare reservoir streams to export (sales)

transportation system
- flowline, pipelines, network
- flow performance
- flow assurance

$\bar{o}$ standard conditions volumetric rate

stb/d

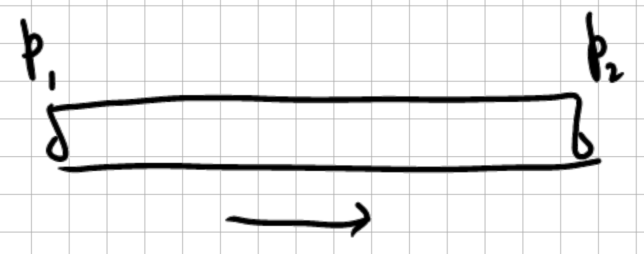
scf/d

Sm^3/d

wellbore
- flow performance
  Δp, ΔT, v
- upper and lower completion
- structural performance
- Artificial lift

near well bore formation
- performance
- fluid distribution
- interface with wellbore
- damage
- stimulation

flow equilibrium

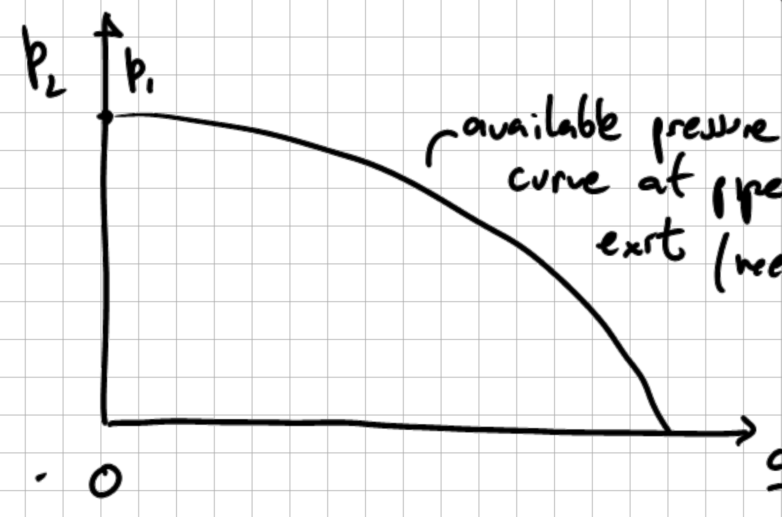


horizontal pipe

q volumetric rate circulating through pipe $[m^3/d]$

1) keep p_1 constant, vary q

$p_2 = p_1 - \Delta p_{pipe}(q)$



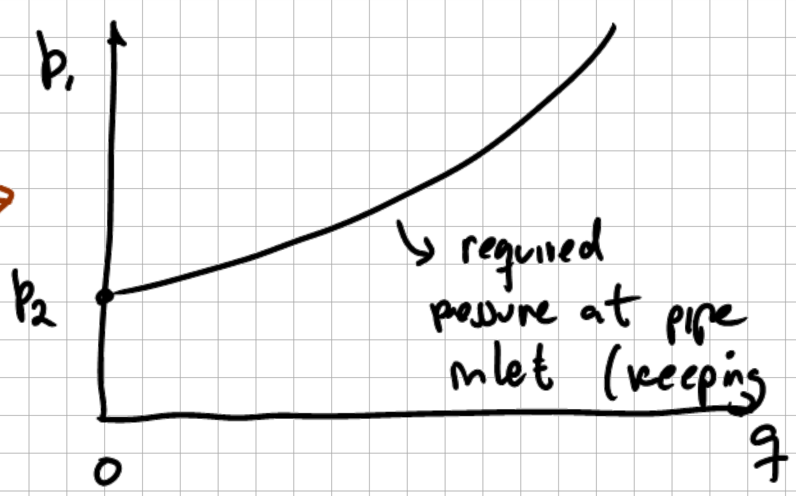
available pressure curve at pipe exit (keeping p_1 constant)

$q=0$
$\Delta p_{pipe}=0$

2) keep p_2 constant, vary q, compute p_1

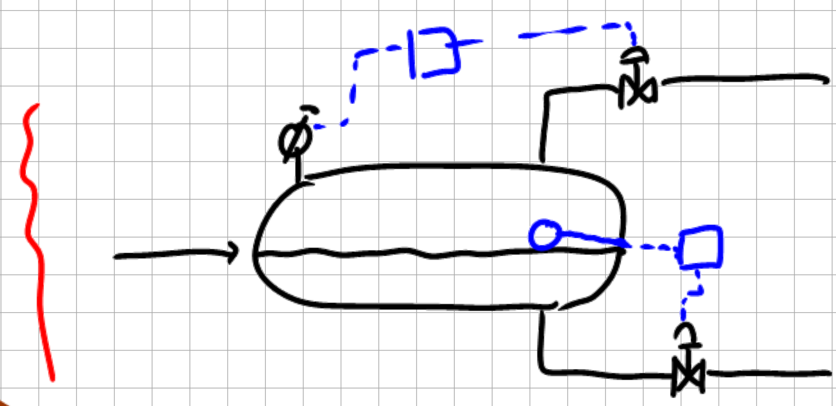
$p_1 = p_2 + \Delta p_{pipe}$

$q=0$
$\Delta p_{pipe}=0$
$q \neq 0$
$\Delta p_{pipe} =$
$\uparrow q \ \uparrow \Delta p_{pipe}$

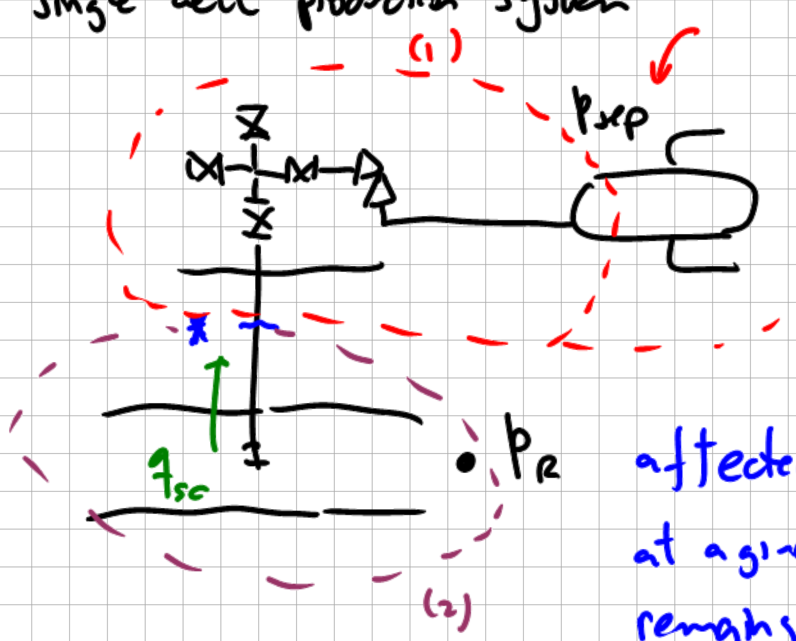


required pressure at pipe inlet (keeping P_2 constant)

3) provide $p_1, p_2 \rightarrow$ calculate q



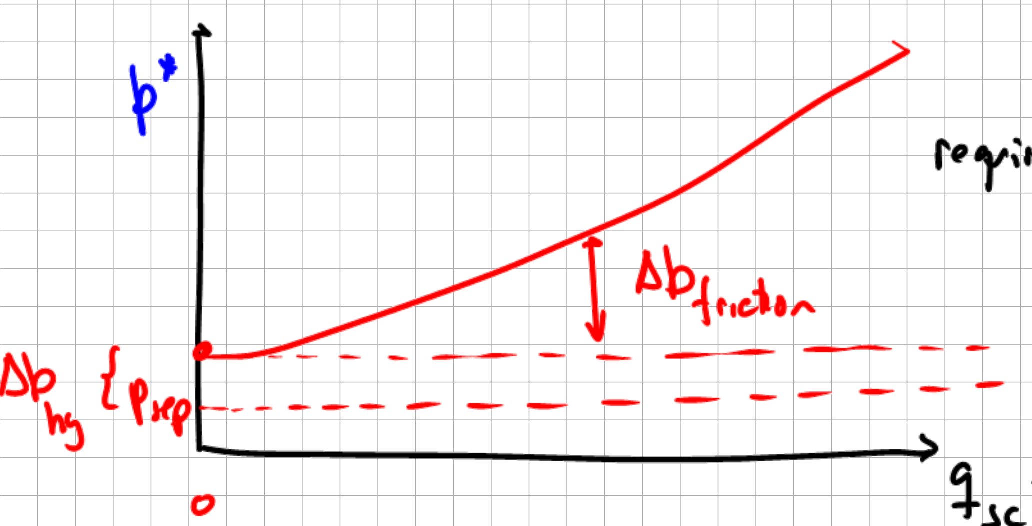
Single well production system



affected by depletion
at a given time "t"
remains fairly "constant"

system (1) resembles case 2

required pressure at p^* (seen from separator)



m^3/d affected by P, T

Sm^3/d unaffected by p, T, $p_{sc} = 1.01325$ bara

$T_{sc} = 15.56°C$

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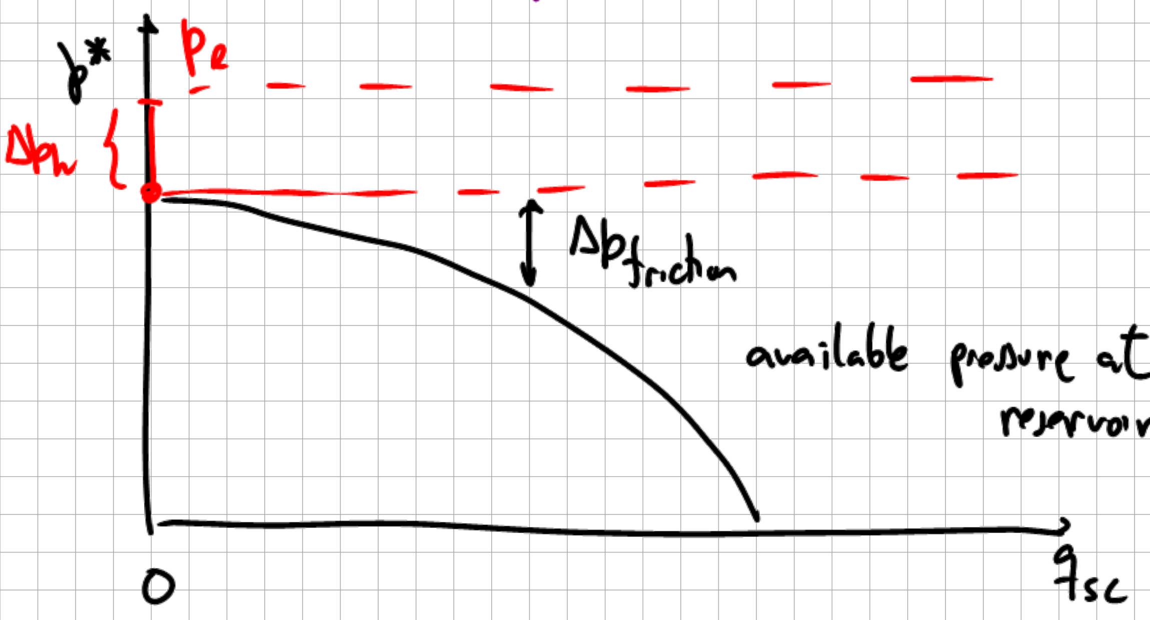
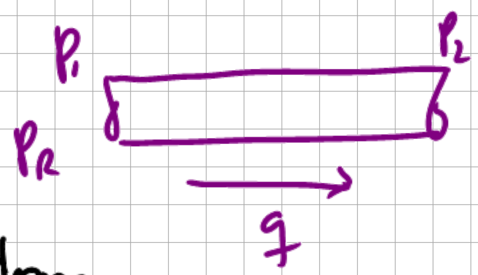
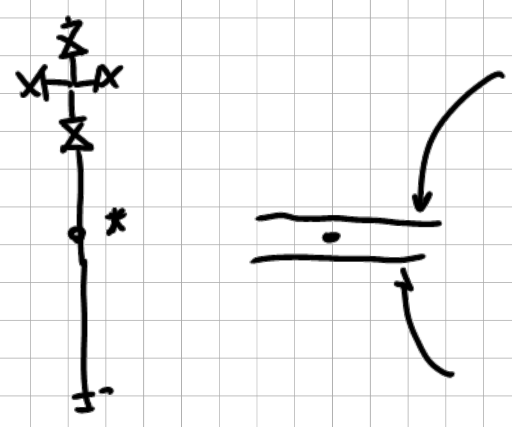
System (2) (formation and part of tubing) resembles case (1)

P_1 fixed, find P_2 by changing q



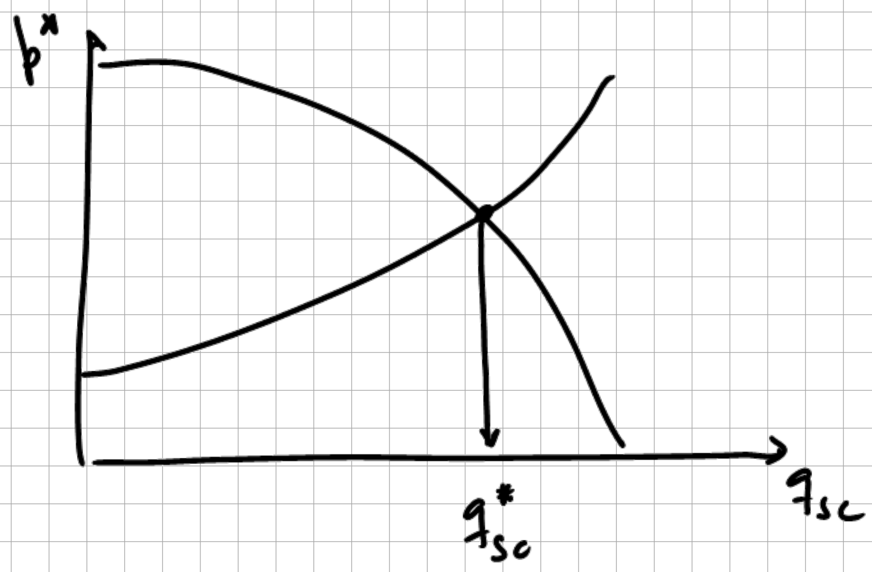


available pressure at * seen from reservoir

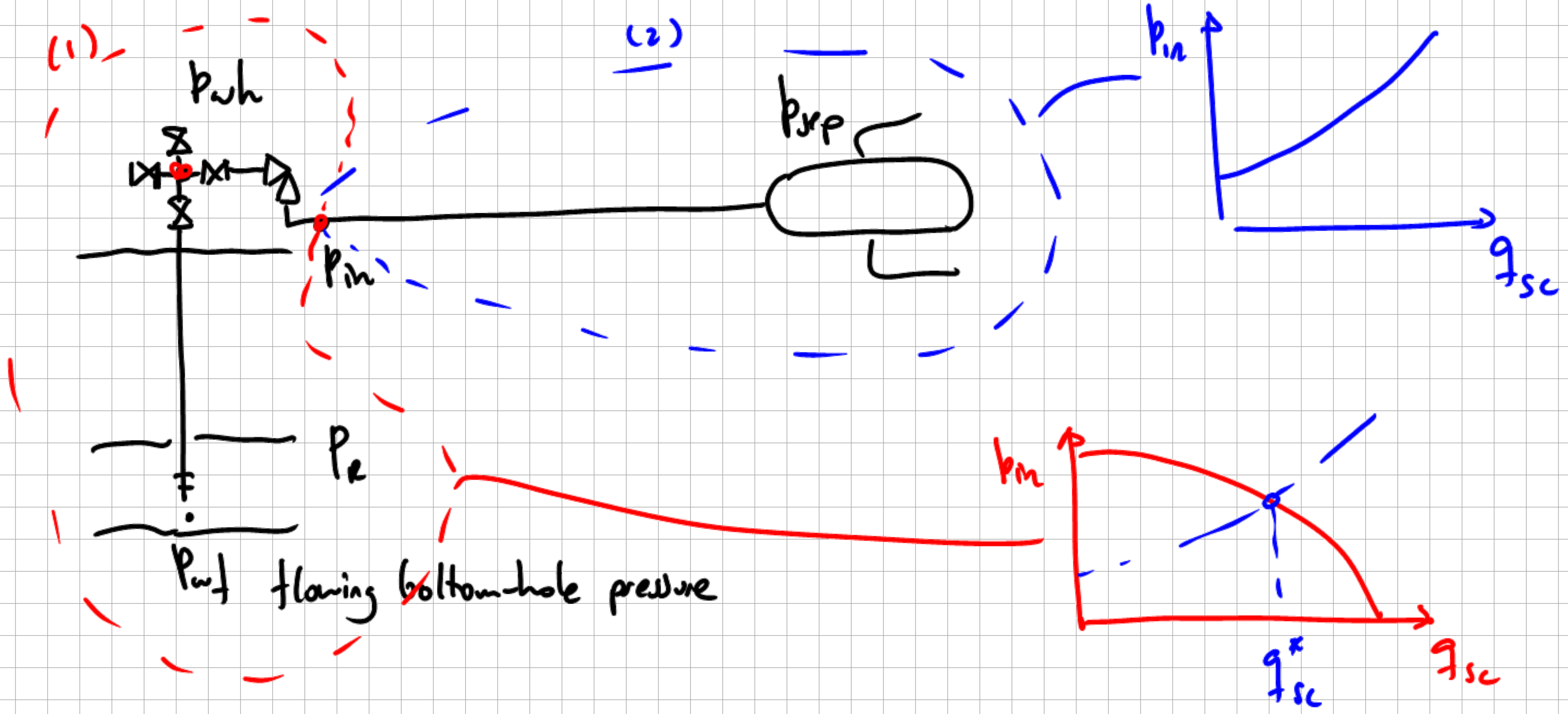


at p^* there should be only one pressure!

overimpose the two curves

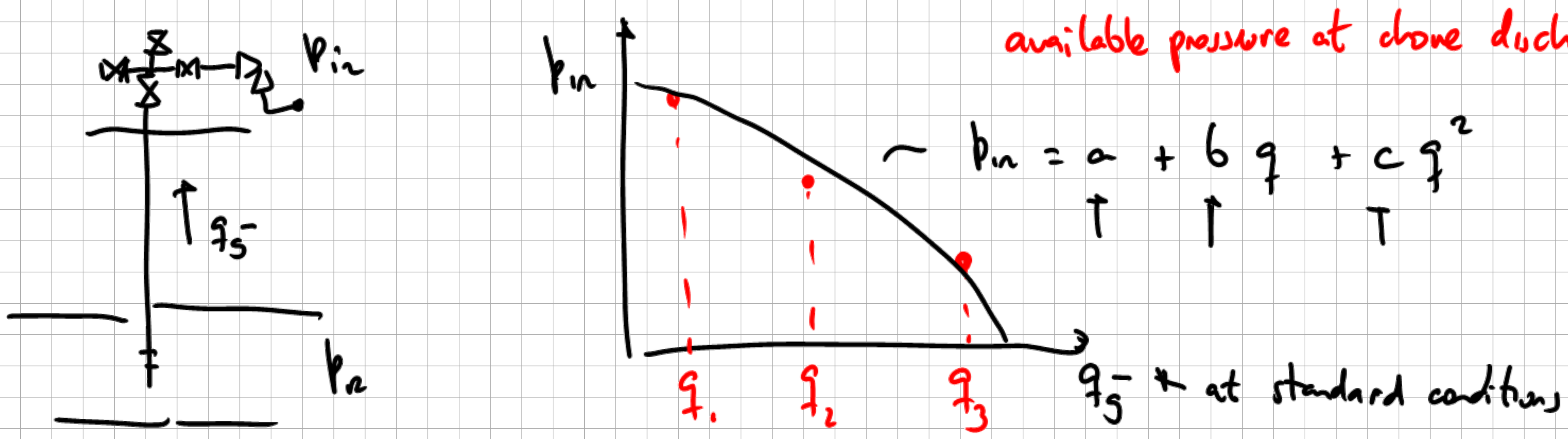


↳ equilibrium rate of the system when put together!



Example

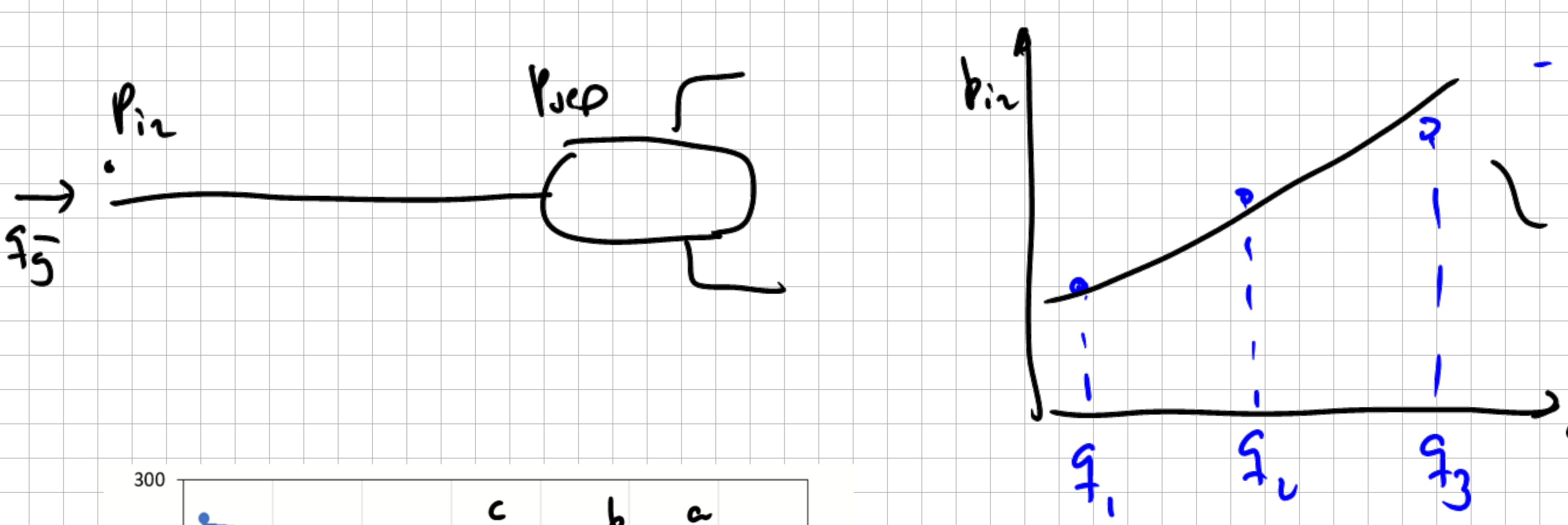
Dry gas production system



available pressure at choke discharge

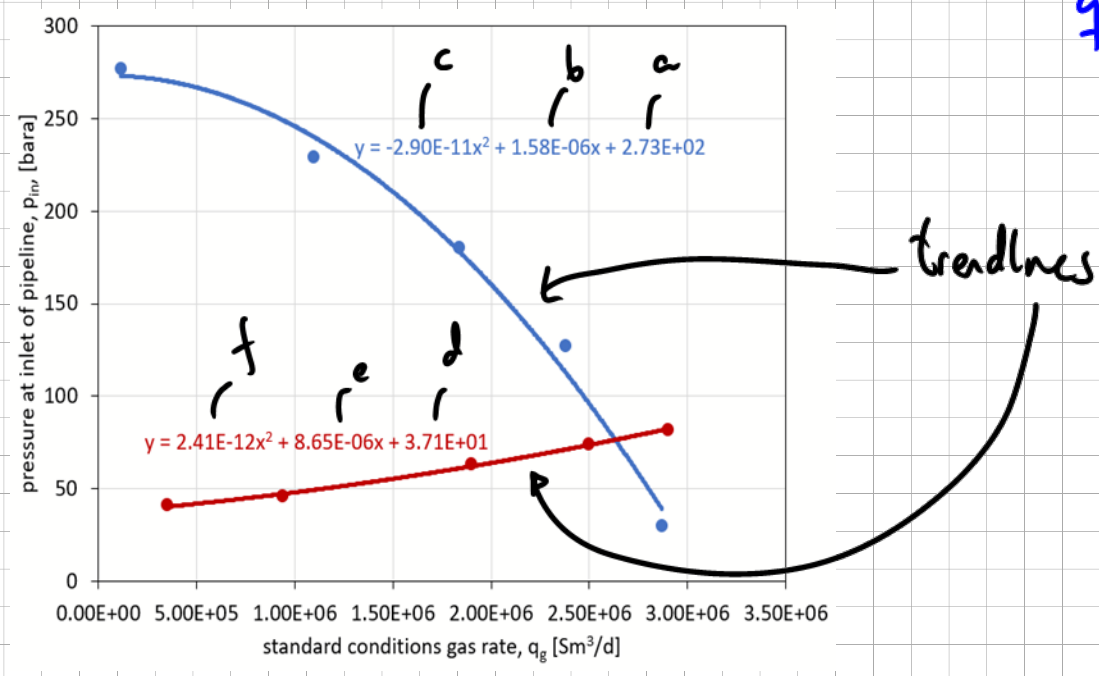
$$\sim \; p_{in} = a + b\,q + c\,q^2$$

q_g * at standard conditions



required pressure at pipeline inlet

$$p_{in} = d + e\,q + f\,q^2$$



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pin (available)								
c	[bara/(Sm3/d)^2]	-2.90E-11						
b	[bara/Sm3/d]	1.58E-06						
a	[bara]	2.73E+02		qg	pin (av)	pin(req)	error	
				[Sm3/d]	[bara]	[bara]	[bara]	
pin(required)				2.63E+06	76.5	76.5	0.0	
f	[bara/(Sm3/d)^2]	2.41E-12						
e	[bara/Sm3/d]	8.65E-06						
d	[bara]	3.71E+01						

$p_{in} = 76.5$ bara

equilibrium gas rate

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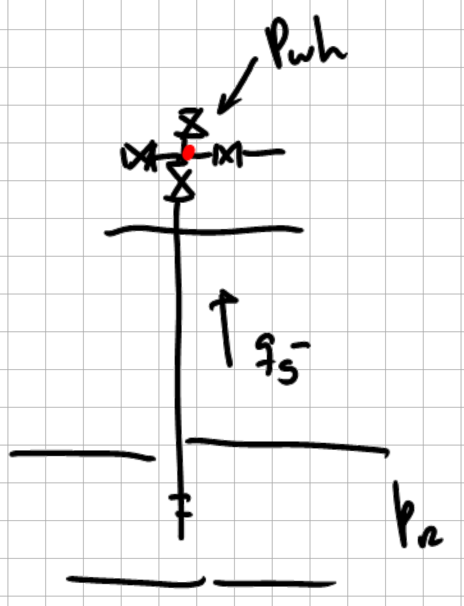
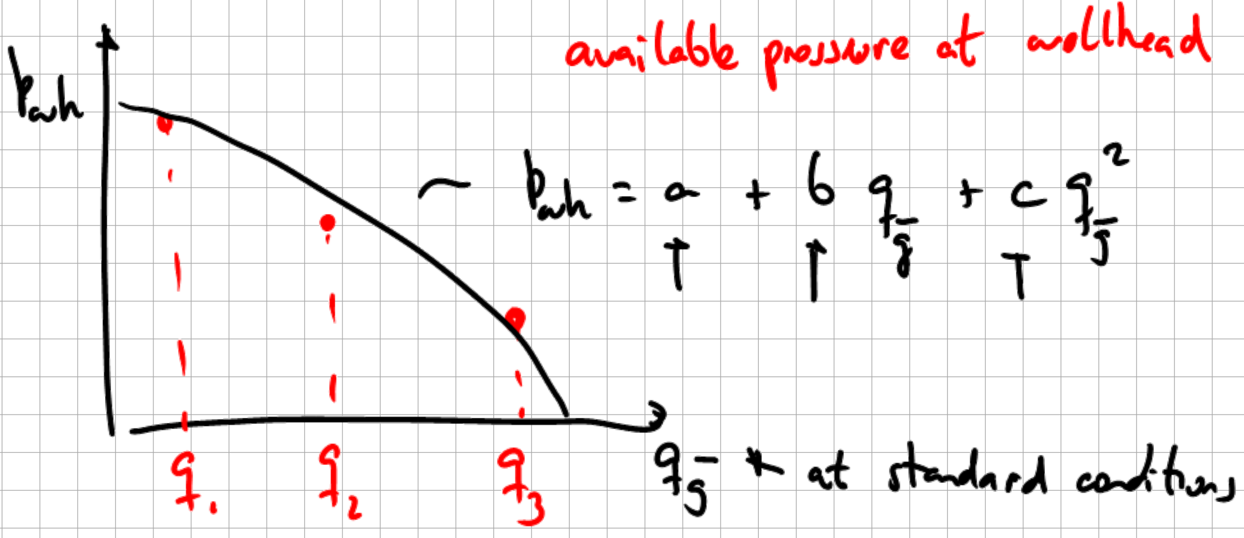
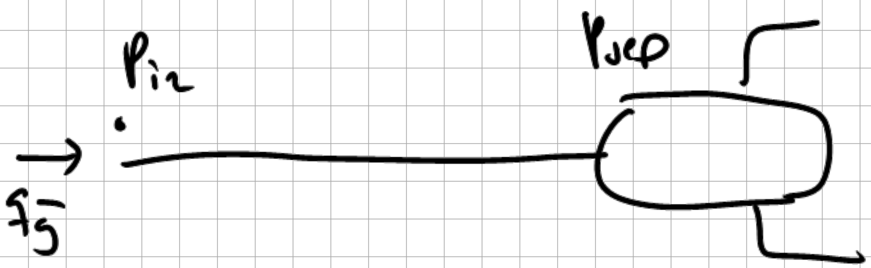
Example

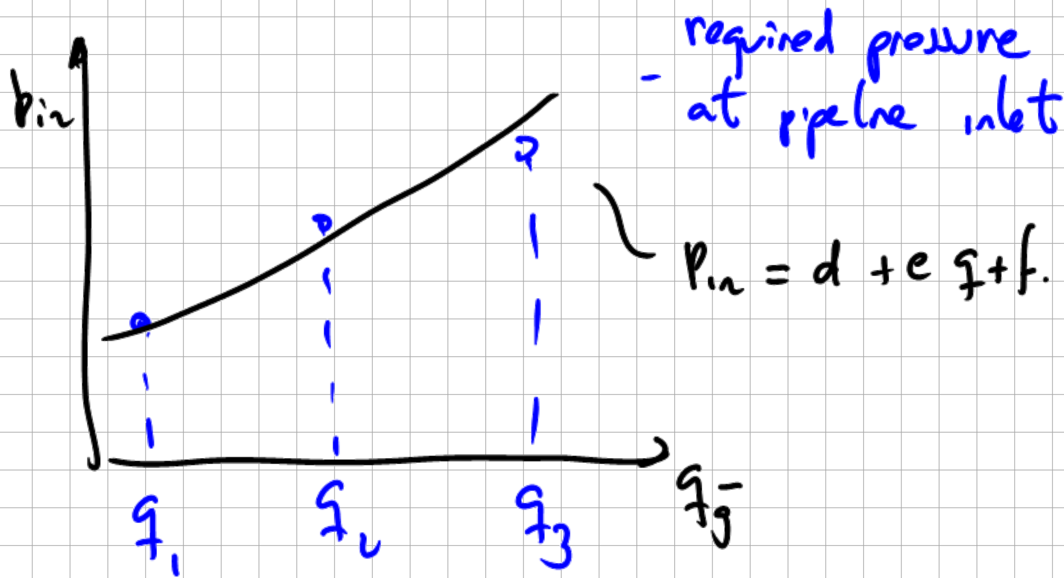
Dry gas production system



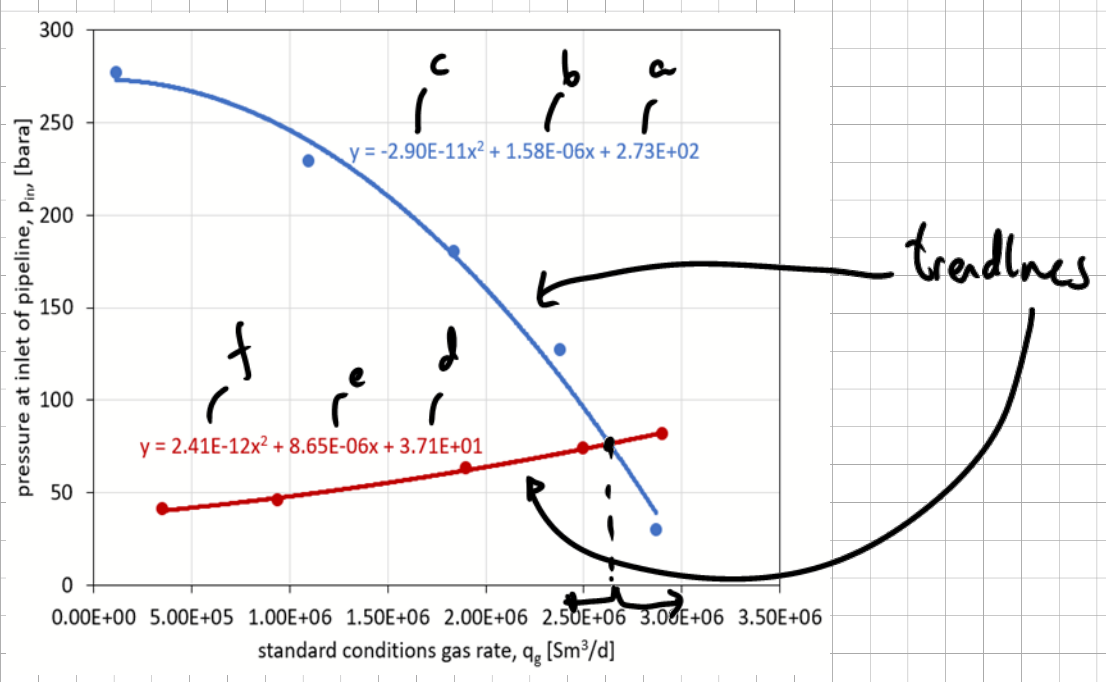


$\sim p_{wh} = a + b\, q_g + c\, q_g^2$





$p_{in} = d + e\, q + f \cdot q^2$



pin (available)						
c	[bara/(Sm3/d)²]	-2.90E-11				
b	[bara/Sm3/d]	1.58E-06				
a	[bara]	2.73E+02	qg	pin (av)	pin(req)	error
			[Sm3/d]	[bara]	[bara]	[bara]
pin(required)			2.63E+06	76.5	76.5	0.0
f	[bara/(Sm3/d)²]	2.41E-12				
e	[bara/Sm3/d]	8.65E-06				
d	[bara]	3.71E+01				

$p_{in} = 76.5$ bara

equilibrium gas rate

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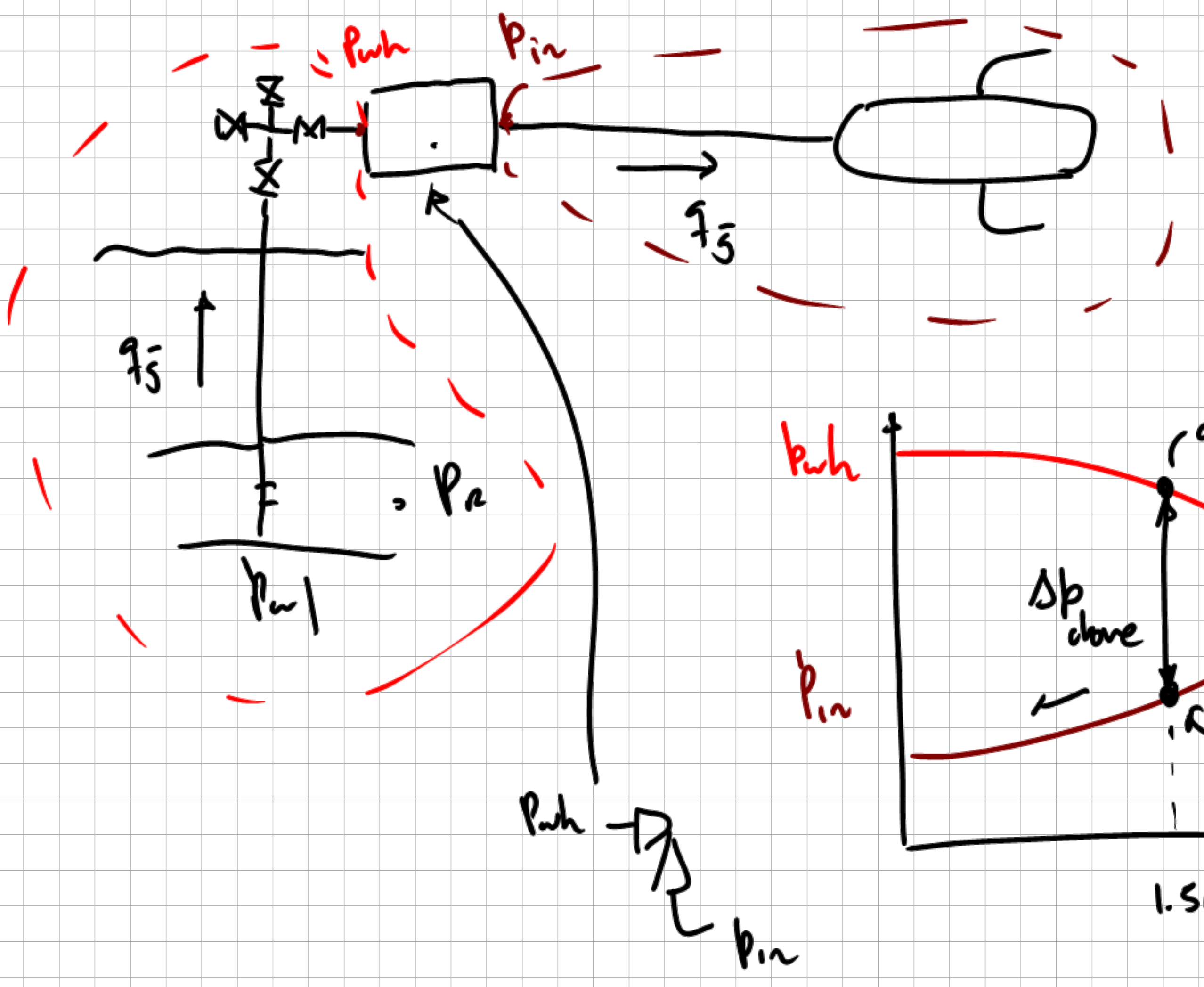
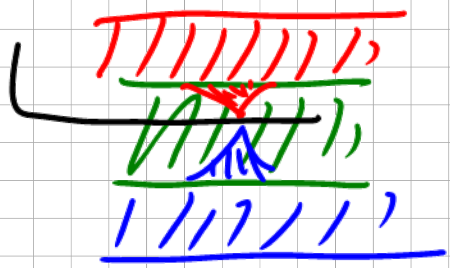
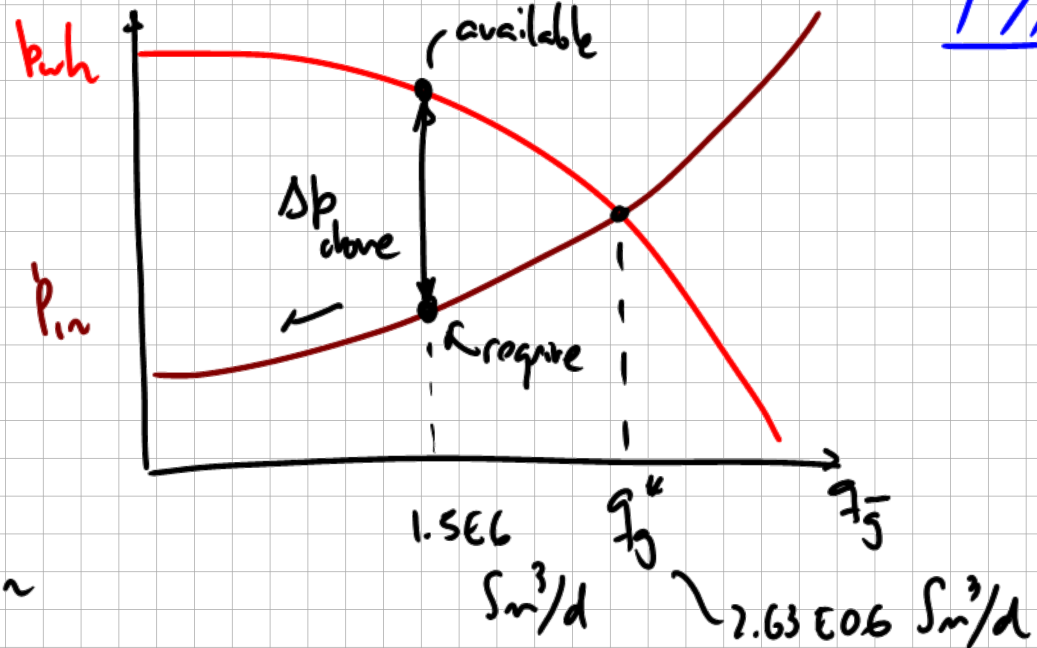
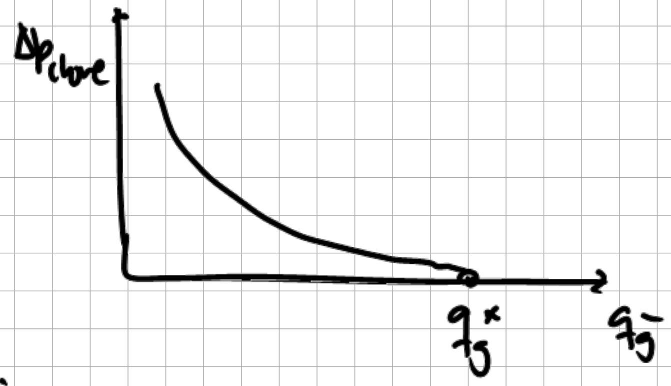


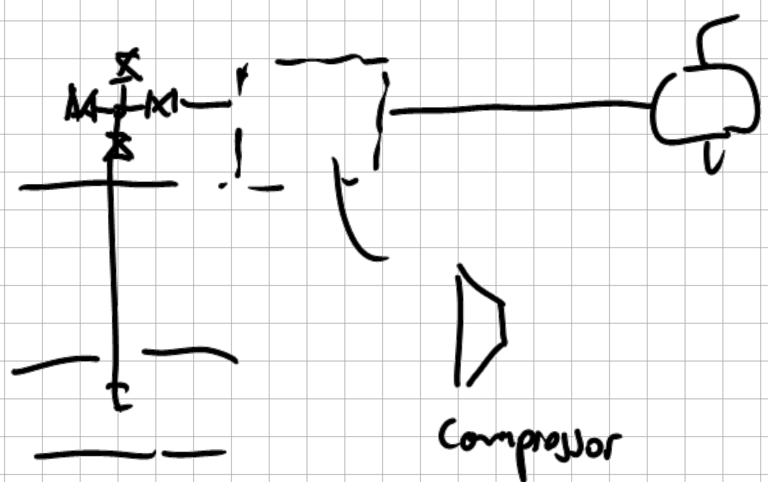


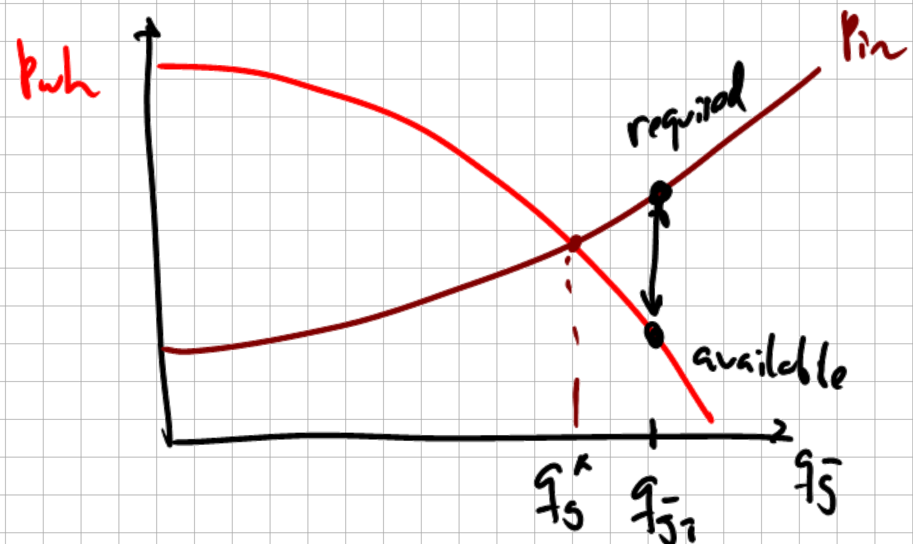


qg	p_{wh} (av)	p_{in} (req)	Deltap
[Sm3/d]	[bara]	[bara]	[bar]
1.50E+06	210.1	55.5	-154.6



what if i need a rate higher than equilibrium





required > avail
energy must be
given to fluid

I desire to produce 2.8 E06 Sm^3/d

qg	p_{wh} (av)	p_{in} (req)	Deltap
[Sm3/d]	[bara]	[bara]	[bar]
2.80E+06	50.1	80.2	30.2

deltap to be provided by wellhead compressor

20240826
OUTLINE:
-Reference group - 2 volunteers required
-video lectures short recap
-Short intro to Excel VBA
-Class exercises

Short intro to Excel VBA

accessing the VBA module :
alt+F11

implicit type declaration versus

```
Function multiply(x, y)
    multiply = x * y
End Function
```

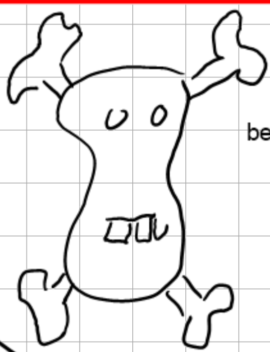
explicit type declaration

```
Function multiply(x As Double, y As Double)
    Dim output As Double
    output = x * y
    multiply = output
End Function
```

$(2.x).x.y = 2x^2.y$

```
Function multiply(x, y)
    double_x = doubling(x)
    multiply = double_x * x * y
End Function

Function doubling(x)
    x = 2 * x
    doubling = x
End Function
```



be aware!!

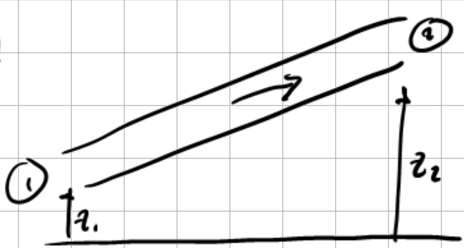
x is changing, and the change affects this !

Naming: descriptive variables   name_name1_name2

NameName1Name

h in m!

Bernoulli eq



$h_1 = h_2 + \Delta h_{friction} + \Delta h_{accessories}$

localized losses:
Section changes
Pipe connections
valves (even if open), elbows

$\left(z_1 + \frac{P_1}{\rho . g} + \frac{V_1^2}{2g}\right)$

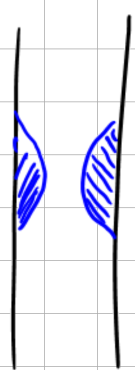
$\Delta h_{friction} = f \frac{L}{\phi}\left(\frac{V^2}{2g}\right)$

$\Delta h_{accessories} =$

$K\left(\frac{V^2}{2g}\right)$

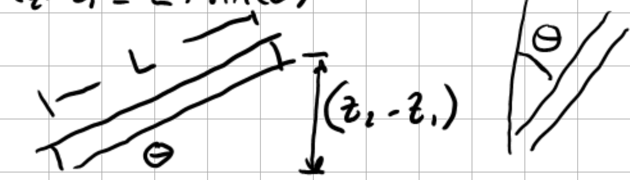
$f \frac{L_{eq}}{\phi}\left(\frac{V^2}{2g}\right)$

since incompressible, $\boxed{V_1 = V_2 = V}$



$$(P_1 - P_2) = \overbrace{\left((z_2 - z_1)\rho . g\right)}^{\text{hydrostatic } \Delta p} + \overbrace{\left(f \frac{L}{\phi}\rho . \frac{V^2}{2}\right)}^{\text{frictional term}}$$

$$z_2 - z_1 = L \cdot \sin(\theta)$$



$$\Phi: (P_1 - P_2) = \left((z_2 - z_1)\rho \cdot g\right) + \underbrace{\left(f \frac{L}{\phi} \rho \cdot \frac{v^2}{2}\right)}$$

$$v = \frac{q}{A} = \frac{q}{\frac{\pi \phi^2}{4}} \qquad v^2 = \frac{q^2\, 16}{\pi^2 \phi^4}$$

$$f L \rho \frac{q^2\, 8}{\pi^2 \phi^5}$$ in Pa

1e5 Pa → 1 bara

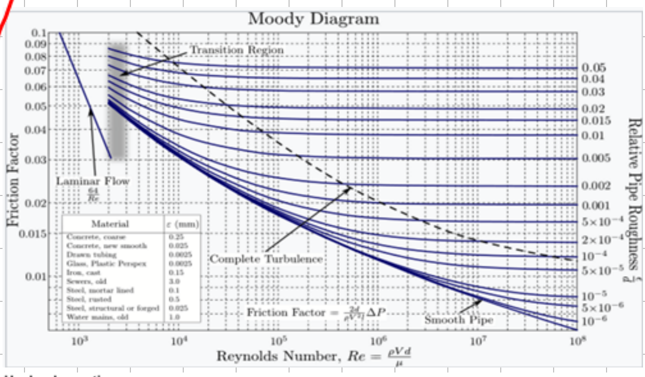
```
Function pressure_drop(q, L, den, visc, teta, roughness, phi)
    'Calculates pressure_drop in bar
    'q, liquid volumetric rate, in m3/d
    'L, pipe length, in m
    'den, liquid density, in kg/m3
    'visc, liquid viscosity, in cP
    'teta, pipe inclination angle from the horizontal in deg
    'roughness, pipe roughness, in m
    'phi, pipe inner diameter, in m
    'defining constants
    Pi = Application.WorksheetFunction.Pi()
    g = 9.81 ' in m/s^2
    'Step 0, convert rate to metric units
    q_ = q / (3600# * 24#) 'changing from m3/d to m3/s
    'Step 1, calculate velocity
    Area = (phi ^ 2) * 0.25 * Pi
    v = q_ / Area
    'Step 2, calculate reynolds number
    Re = Reynolds_f(v, den, visc / 1000, phi) 'visc is required in Pa s, but available in cP
    'Step 3, calculate friction facgtor
    rel_roughness = roughness / phi
    friction_factor = friction_factor_f(Re, rel_roughness)
    'Compute the pressure drop using the Bernoulli equation
    deltap_hydrostatic = 0.00001 * L * Sin(teta * Pi / 180) * den * g
    deltap_frictional = 0.00001 * den * friction_factor * L * 8 * (q_ ^ 2) / ((phi ^ 5) * (Pi ^ 2))
    pressure_drop= deltap_hydrostatic + deltap_frictional
End Function

Function Reynolds_f(v, den, visc, phi)
    'TODO: provide description of function
    'and function input
    Reynolds_f = v * den * phi / visc
End Function

Function friction_factor_f(Re, rel_roughness)
    If Re < 2300 Then
        friction_factor_f = 64 / Re
    Else
        part1 = -1.8 * (Log((rel_roughness / 3.7) ^ 1.11 + 6.9 / Re) / Log(10))
        friction_factor_f = part1 ^ (1 / -0.5)
    End If
End Function
```

in VBA, log is ln

$$\log_{10}(x) = \frac{\ln(x)}{\ln(10)}$$



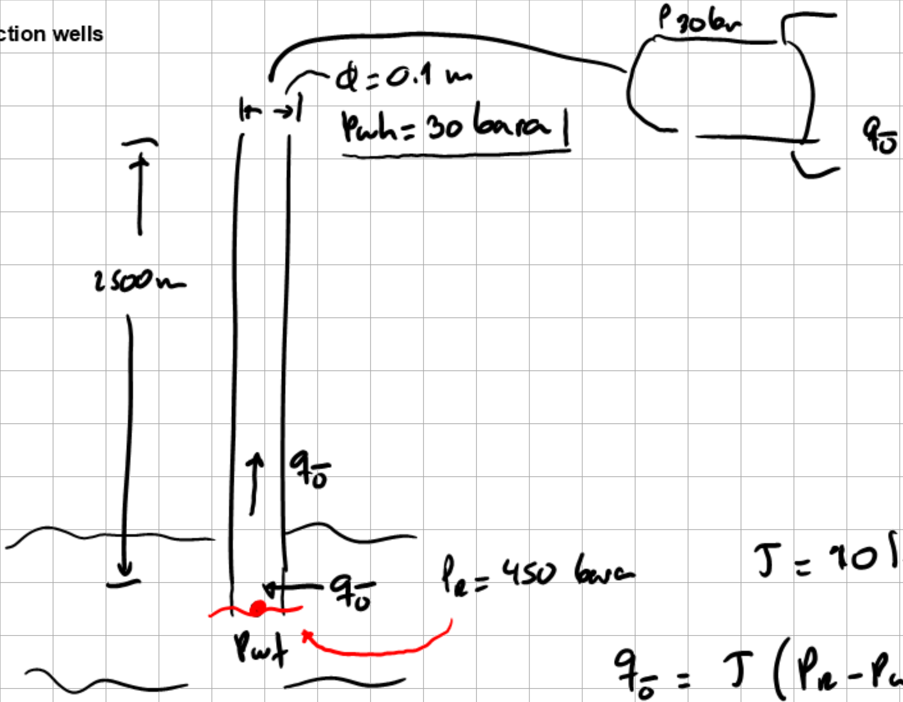
Haaland equation [edit]

The *Haaland equation* was proposed in 1983 by Professor S.E. Haaland of the Norwegian Institute of Technology.[9] It is used to solve directly for the Darcy–Weisbach friction factor *f* for a full-flowing circular pipe. It is an approximation of the implicit Colebrook–White equation, but the discrepancy from experimental data is well within the accuracy of the data.

The Haaland equation[10] is expressed:

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

1. Consider a vertical tubing with an inner diameter of 0.1 m, and length of 2500 m, through which undersaturated oil is flowing. Perform the following tasks:
    a. Calculate the curve of available pressure at the wellhead (in bara versus Sm3/d of oil), if the inlet pressure is 150 bara.
    b. Calculate the curve of required pressure at the bottom-hole (in bara versus Sm3/d of oil) if the outlet pressure is 30 bara.
    c. Consider the flow is not known, and that the bottom-hole pressure and wellhead pressure are measured and equal to 350 bara and 30 bara, respectively. Estimate the liquid rate of oil circulating through the pipe (this calculation is often referred to as virtual metering)
        i. If the pressures have measurement errors of +- 5%, how much will this affect your results.
    d. Assume there is an undersaturated oil well with the following inflow relationship: qo = J * (pR-pwf), with J = 10 Sm3/d/bar and pR = 450 bara. Assume that wellhead pressure is 30 bara. Find the equilibrium flow of the system.
        i. Based on the rate obtained in the previous task. If one wishes to produce 10% more, determine the pump deltap and power to install at the end of the tubing to achieve this.



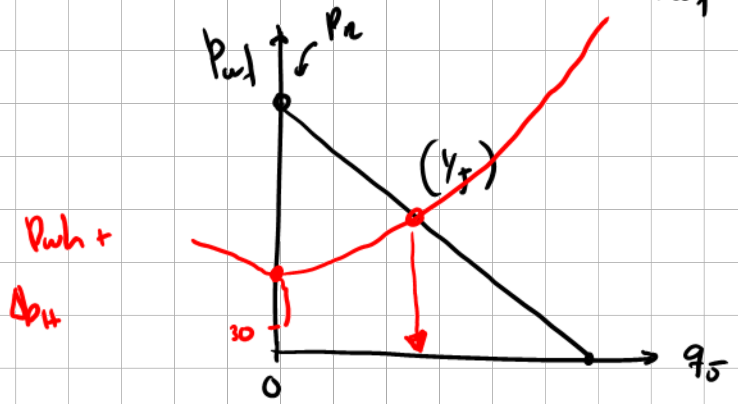
$J = 10 \, m^3/d/bar$

$$q_o = J\,(P_R - P_{wf}) \quad ①$$

① available pressure at bottom-hole from reservoir

$$P_{wf} = P_R - \frac{q_o}{J}$$

$P_R \rightarrow P_{wf}$



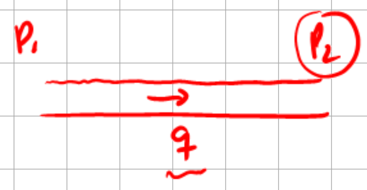
② calculate required pressure to flow against wellhead (counter current)

$$P_{wf} = P_{wh} + \Delta p_{tubing}(q_o) \quad ②$$

↳ VBA eq $\Delta p = \Delta p_H + \Delta p_f$

$1E\text{-}5 \, f \, L \, 8 \, \frac{q^2}{\pi^2 \cdot \phi^5} \rho$

$1E\text{-}5 \cdot L \sin(\theta) \, \rho \cdot g$



Flow equilibrium is equivalent to solve:

$$\overset{?}{P_{wf}} = P_R - \frac{\overset{?}{q_o}}{J}$$

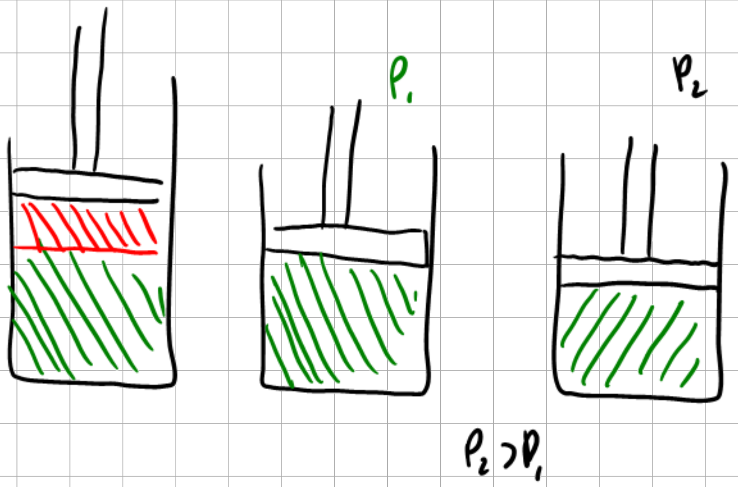
1 eq, 2 unknowns

$$p_{wf} = p_{wh} + \Delta p_{tubing}(q_{\bar{o}})$$

1 Eq, same 2 unknowns

850 kg/m³

$$1E5\ \rho \cdot g \cdot L \cdot \sin(\theta) + f \cdot L \rho \frac{q^2\, 8}{\pi^2 \phi^5} \cdot 1E5$$



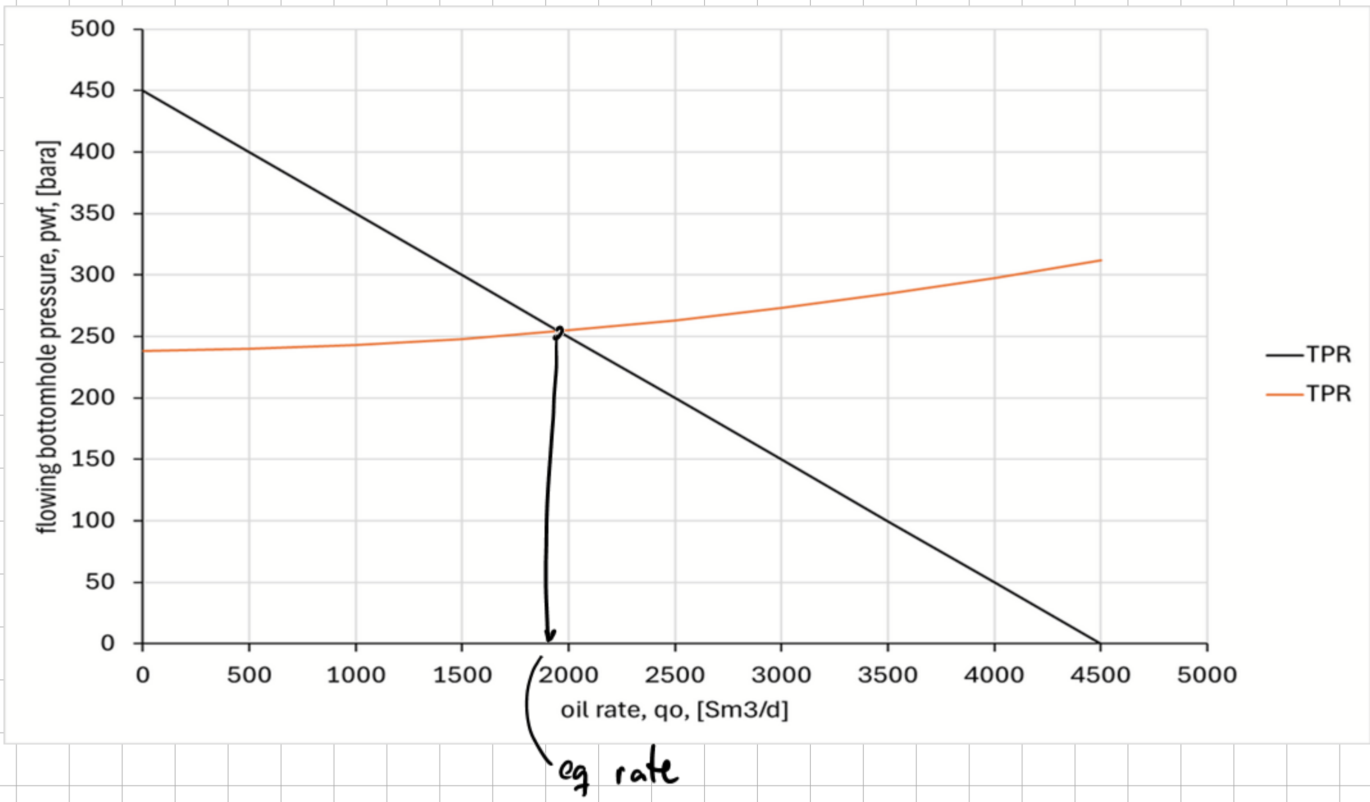
$$q_o \overset{?}{=} q_{\bar{o}}$$

NO! only if the oil is incompressible

$$\mu_o = 2\ cP$$

ubing data		
L	[m]	2500
teta (angle from horizontal)	[deg]	90
roughness	[m]	1.50E-05
nternal diameter	[m]	0.1
wellhead pressure	[bara]	30
Reservoir data		
Reservoir pressure, pR	[bara]	450
Productivity index, J	[Sm3/d]	10
Fluid data		
Density	[kg/m3]	850
viscosity	[cP]	2

IPR (available)		**TPR (req)**	
pwf - avail	**qo**	**DP-tub-req**	**pwf-req (pwh+DP_tub)**
[bara]	**[Sm3/d]**	**[bara]**	**[bara]**
450	0	208	238
400	500	210	240
350	1000	213	243
300	1500	218	248
250	2000	225	255
200	2500	233	263
150	3000	243	273
100	3500	255	285
50	4000	268	298
0	4500	282	312

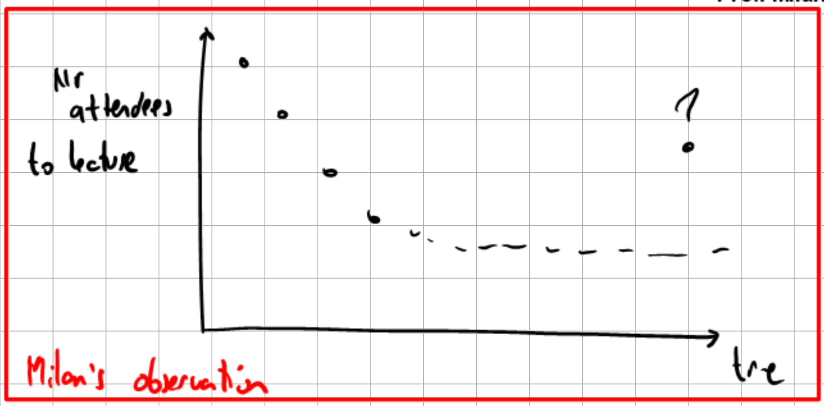


500
450
400
350
300
250
200
150
100
50
0
flowing bottomhole pressure, pwf, [bara]
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
oil rate, qo, [Sm3/d]
TPR
TPR
eq rate

20240830

INFO:
-TA sessions to start today, kl. 11:15.
-Names of members of reference group
-No lecture coming monday 02.09.

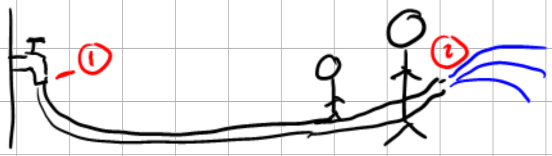
OUTLINE
-Re-cap last lecture
-Solving (partially) problems in mandatory exercise set nr. 1



Typical velocities in water transport pipes:

(v^2) , (1)

$(5)^2 \sim (25)$

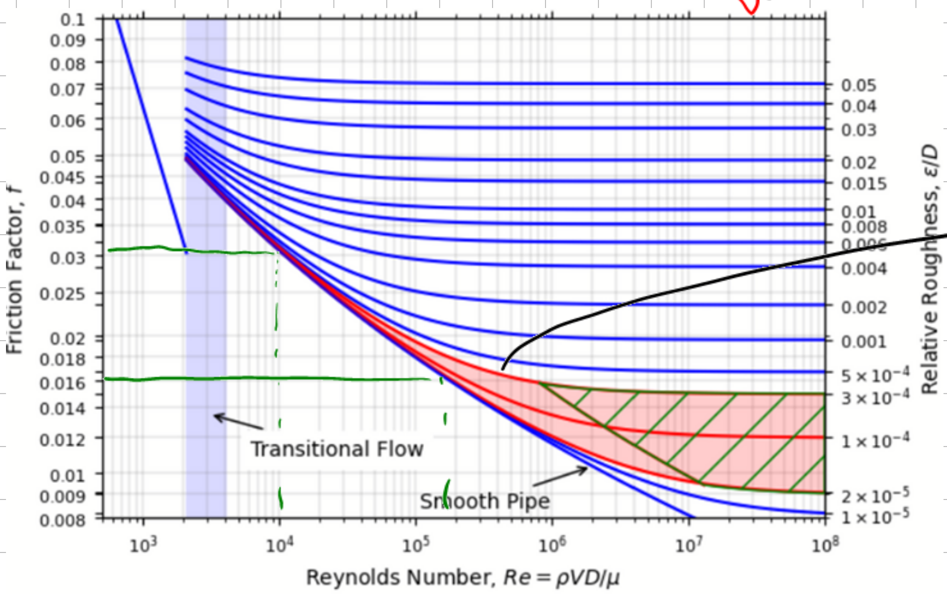


0.5 - 2(5) m/s

gas (5 m/s - 40 m/s)

$f \frac{L}{\phi} \cdot \left(\frac{v^2}{2g}\right)$ $K \frac{v^2}{2g}$

since velocity is squared, it has a huge impact in frictional and accessories losses!



red region: roughnesses typically encountered in peteng / CO_2

quite smooth pipes

Modification required to VBA code originally provided by Milan in the previous lecture to account for cases where V = 0 m/s

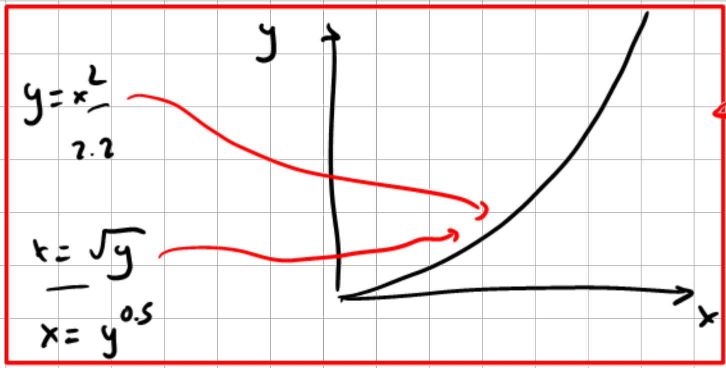
```
Function friction_factor_f(Re, rel_roughness)
    If Re = 0 Then
        friction_factor_f = 0
    ElseIf Re > 0 And Re < 2300 Then 'laminar flow
        friction_factor_f = 64 / Re
    Else 'turbulent flow
        part1 = -1.8 * (Log((rel_roughness / 3.7) ^ 1.11 + 6.9 / Re) / Log(10))
        friction_factor_f = part1 ^ (1 / -0.5)
    End If
End Function
```

oil well typical rates

private-owned onshore	10 stb/d	$\xrightarrow{/6}$	1.66 Sm³/d	10
company owned onshore	200 stb/d	$\xrightarrow{/6}$	33.3 Sm³/d	100
company owned offshore	6000 stb/d	$\xrightarrow{/6}$	1000 Sm³/d	1000
BIG middle east	60000 stb/d	$\xrightarrow{/6}$	10000 Sm³/d	10000

$p_{wf} = p_R - \frac{q_o}{J}$

$- p_{wf}$

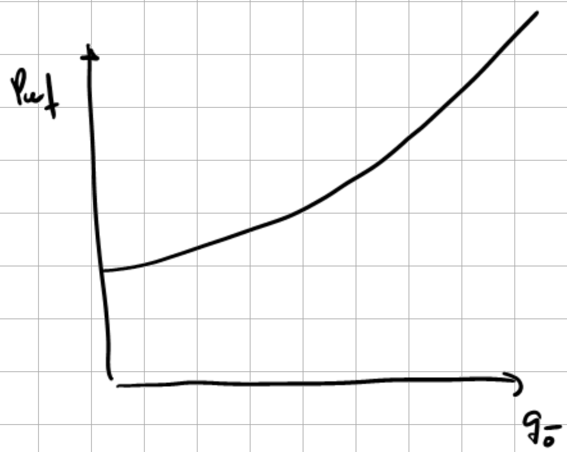


IPR (available)	
pwf - avail [bara]	qo [Sm3/d]
450	0
400	500
350	1000
300	1500
250	2000
200	2500
150	3000
100	3500
50	4000
0	4500

Last lecture, instead of calculating available pwf from pR, I provided a value for it, and calculated the rate, why?
if I assume a value of qo which is too high, I get negative value for qo. It is difficult to know a priori the range of qo. but pwf is always bounded by pR and 0 bara. At the end the only thing I need is the curve, so how I calculate it, is not important. Example:

For convenience, and to avoid having a separate column for DP, we will create two VBA functions

$$P_{wf} = P_{th} + \Delta p(q_o)$$



$$P_{out} = P_{in} - \Delta p(q_o)$$

$$P_{in} = P_{out} + \Delta p(q_o)$$

```
Function pin_pipe(pout, q, L, den, visc, teta, roughness, phi)
    'function to calculate required pressure at pipe inlet to flow with rate q, against pout
    pin_pipe = pout + pressure_drop(q, L, den, visc, teta, roughness, phi)
End Function

Function pout_pipe(pin, q, L, den, visc, teta, roughness, phi)
    'function to calculate available pressure at pipe outlet to flow with rate q, from pin
    pout_pipe = pin - pressure_drop(q, L, den, visc, teta, roughness, phi)
End Function
```

IPR (available)		TPR (req)
pwf - avail [bara]	qo [Sm3/d]	pwf-req [bara]
450	0	238
400	500	240
350	1000	243
300	1500	248
250	2000	255
200	2500	263
150	3000	273
100	3500	285
50	4000	298
0	4500	312

How to find exactly the Equilibrium rate:

IPR (available)		TPR (req)
pwf - avail	qo	pwf-req
[bara]	[Sm3/d]	[bara]
450	0	238
400	500	240
350	1000	243
300	1500	248
250	2000	255
200	2500	263
150	3000	273
100	3500	285
50	4000	298
0	4500	312

when $P_{wf\text{-}avail} = P_{wf\,req}$

P_{wf}-avail (assumed) q_o (calc from IPR) P_{wf}-req (TPR)

$\boxed{P_{wf}} \rightarrow \boxed{q_o} \rightarrow \boxed{P_{wf}}$

change to make equal to

using "Goal Seek" or "Solver"

			diff [bar]
254.3	1957.5	254	0

Data- What if Analysis

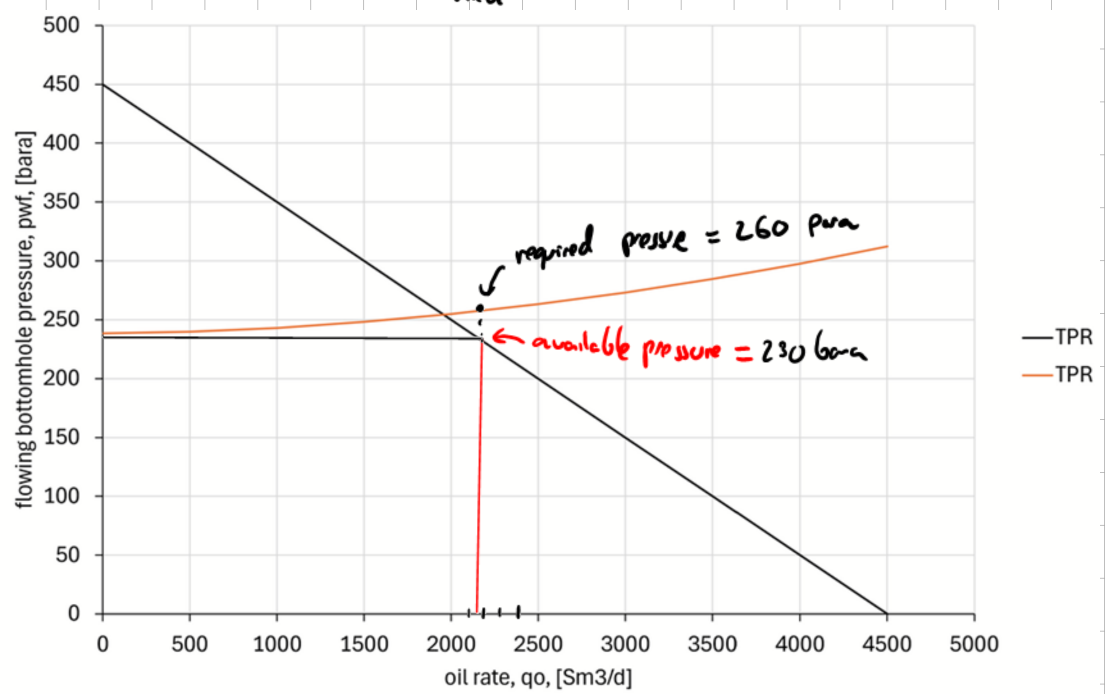
data - towards the end.

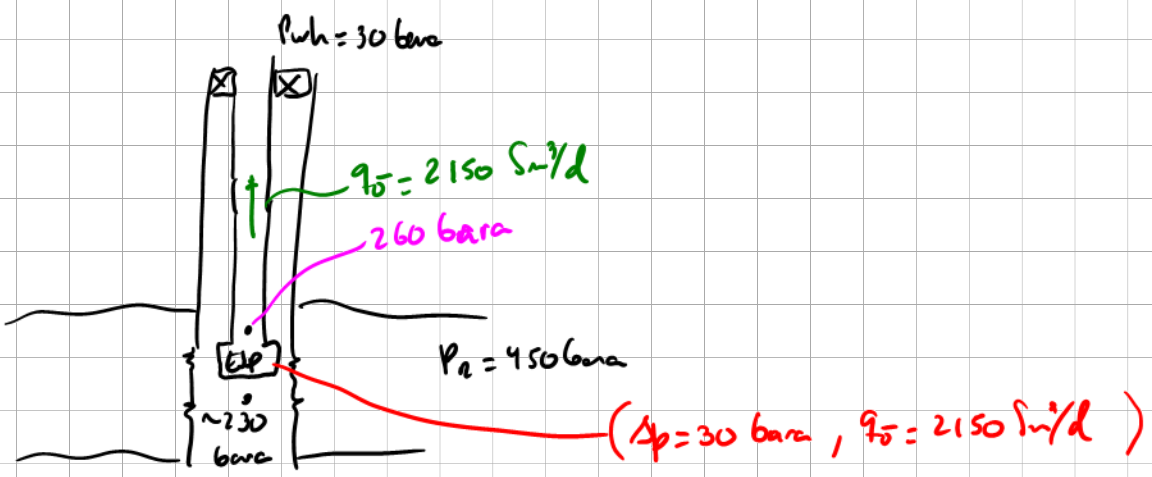
Activation:

Options → Add-ins (side menu). → "go" → check the box for "solver"

i. Based on the rate obtained in the previous task. If one wishes to produce 10% more, determine the pump deltap and power to install at the end of the tubing to achieve this.

$q_{o,wanted} = 1.1 \times 1957.5 = 2150$ Sm3/d

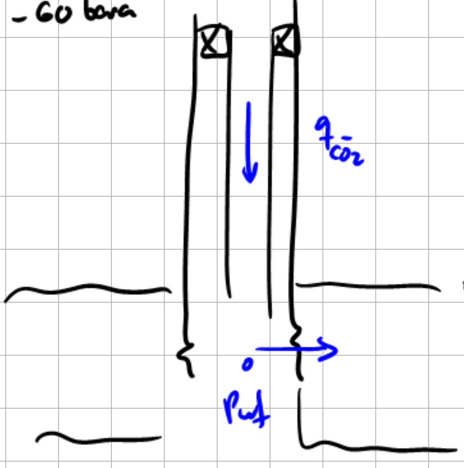




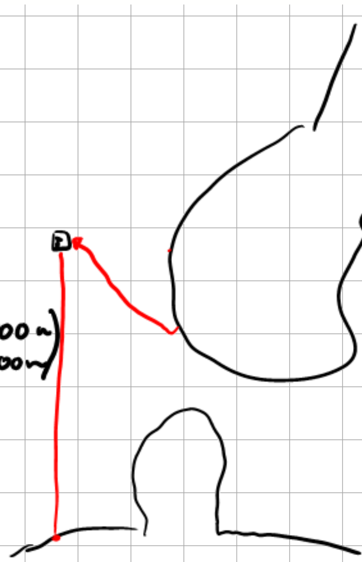
2. Perform a flow equilibrium calculation in a vertical CO_2 injection well with a total tubing length of 1500 m, and 0.168 ID. Use the well bottom-hole as equilibrium point. For the IPR use a linear equation with J = 700 [t/d/bar]. Assume a reservoir pressure equal to 130 bara. Assume the wellhead pressure is kept fixed at a value of 60 bara. Assume an average density and viscosity of 850 kg/m³ and 0.085 cP respectively.
    a. If one wishes to inject 1.5 Mt/y, estimate wellhead pressure required and choke DP.

Pipe transport pressures
200 bara - 60 bara

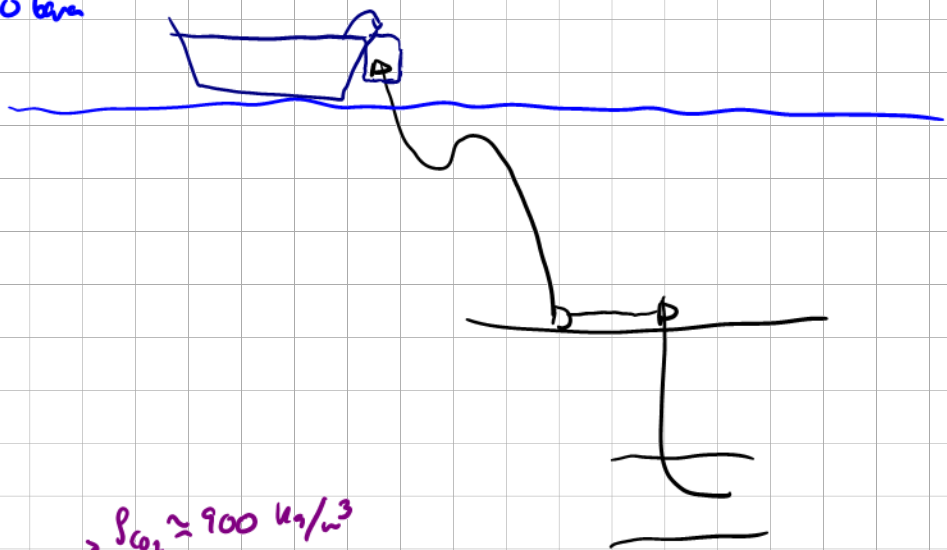
P_{wh} = 60 bara, T_{wh} = 5°C



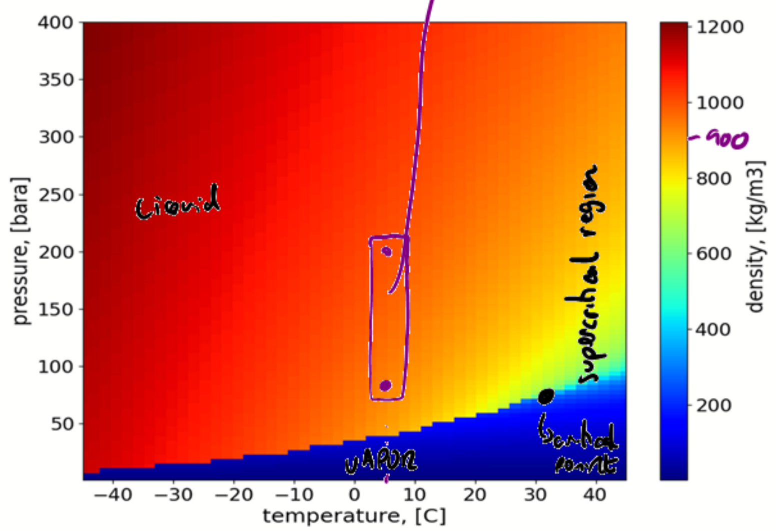
Aquifer (shallow >1000 m - 2000 m)
P_R = 130 bara
T_R = 30-50 °C
3°C/100 m

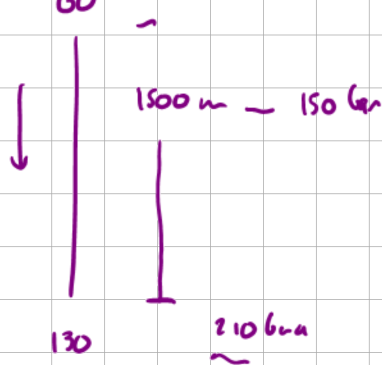


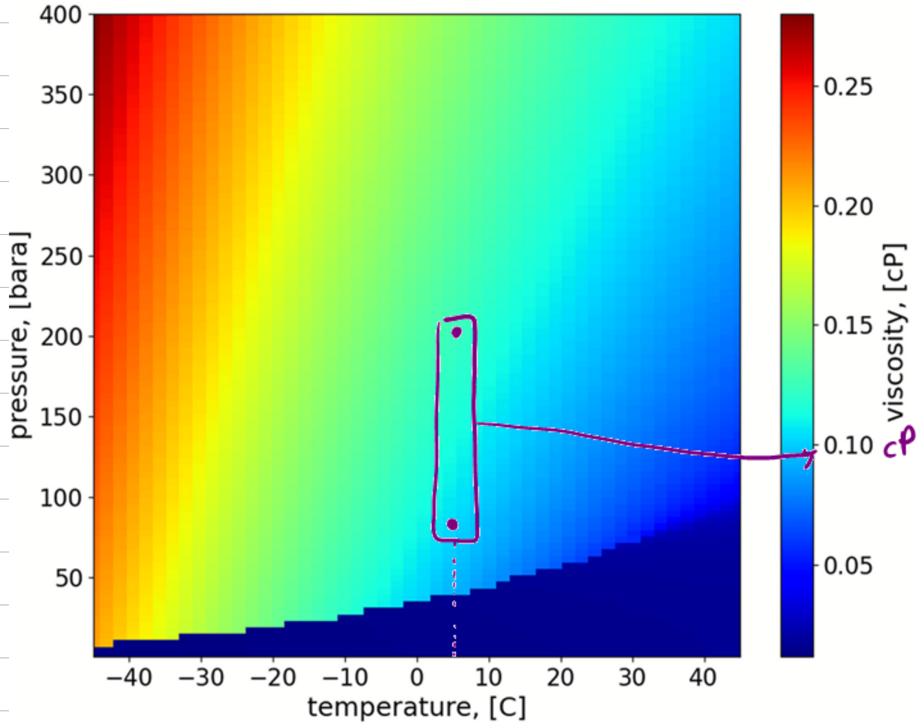
$P_{wf} > P_R$
130 bara



$\rho_{CO_2} \approx 900$ kg/m³







cP

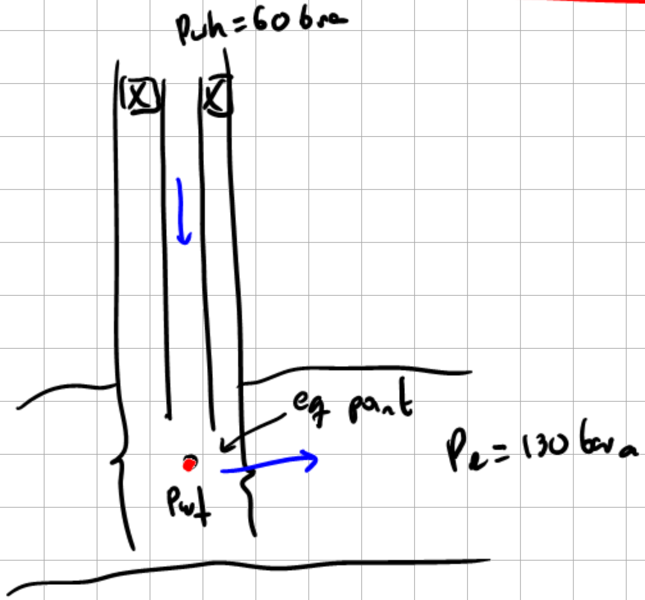
water 1 cP

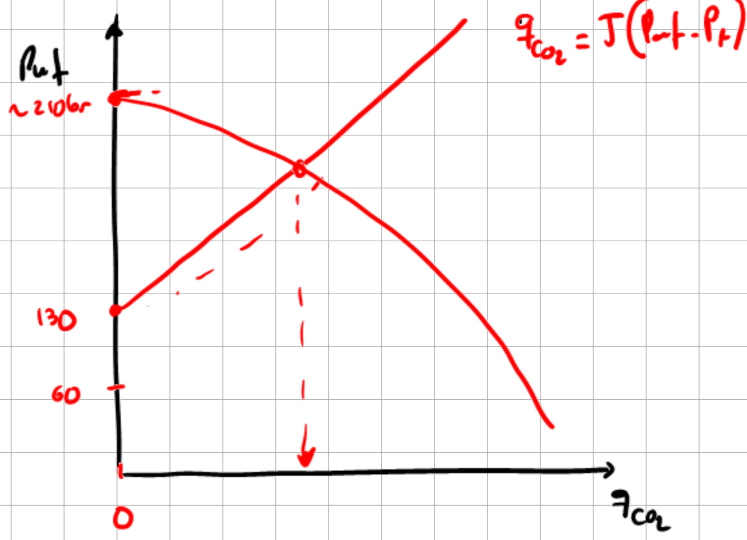
gas ~ 0.01 cP

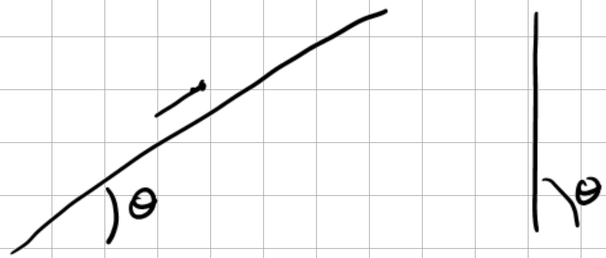
In reality, the temperature of CO2 when flowing in the well increases due to heat up from the formation.

in CO_2 inj wells, $\Delta p_{Hydrostatic} > \Delta p_{friction}$, therefore $P_{wf} > P_{wh}$

This also happens in production wells







for vertical injector $\theta = -90$

$t/d \rightarrow \underbrace{1000\ kg/d}$ $\longrightarrow$ Sm^3/d

In CO2 people work often with t/d, instead of Sm3/d. So there is no need to convert to Sm3/d.

$\rho_{\overline{CO_2}} = 1.8\ kg/Sm^3$

$\rho_{CO_2 @ \text{well condition}} = 850\ kg/m^3$

I calculate IPR in terms of bara versus t/d

IPR (req)	
pwf - req	m-Co2
[bara]	[t/d]
130	0
140	7000
150	14000
160	21000
170	28000

To calculate pressure losses with the DP function, I need rate at local pressure conditions, i.e. at density of 850 kg/m3, I can use this equation

$\dot m_{CO_2}$

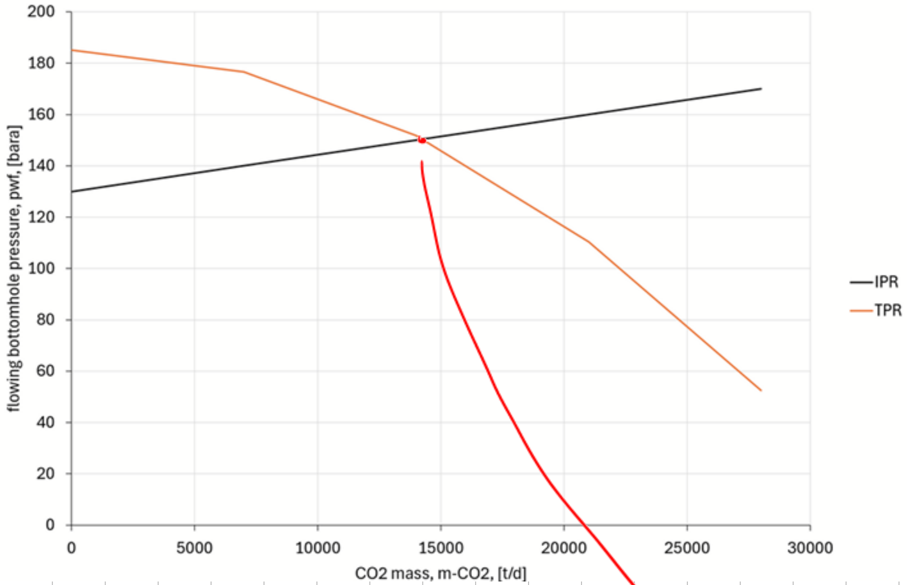
$$q_{CO_2 @ \text{well conditons}} = \frac{\dot m_{CO_2}}{\rho_{CO_2 @ \text{well condtions}}}$$

=F19*1000/C14

	B	C	D	E	F	G	
rof. Milan Stanko (NTNU)							
	[m]	1500					
izontal)	[deg]	-90					
	[m]	1.50E-05					
	[m]	0.168					
	[bara]	60					
pR	[bara]	130					
	[t/d/bar]	700					
	[kg/m3]	850					
	[cP]	0.085					
				IPR (req)			T
				pwf - req	m-Co2	q-Co2	
				[bara]	[t/d]	[m3/d]	
				130	0	1000/	
				140	7000	8235	
				150	14000	16471	
				160	21000	24706	
				170	28000	32941	

=pout_pipe(C8,G21,C4,C14,C15,C5,C6,C7)

A	B	C	D	E	F	G	H
rof. Milan Stanko (NTNU)							
	[m]	1500					
izontal)	[deg]	-90					
	[m]	1.50E-05					
	[m]	0.168					
	[bara]	60					
pR	[bara]	130					
I	[t/d/bar]	700					
	[kg/m3]	850					
	[cP]	0.085					
				IPR (req)			TPR (avail)
				pwf - req	m-Co2	q-Co2	pwf-avail
				[bara]	[t/d]	[m3/d]	[bara]
				130	0	0	185
				140	7000	8235	177
				150	14000	16471	C4,C14,
				160	21000	24706	110
				170	28000	32941	52



Ca 14000 t/d ~ 3.8 E06 t/y

Equinor is planning average CO2 injector rate of 1.5 E06 t/y

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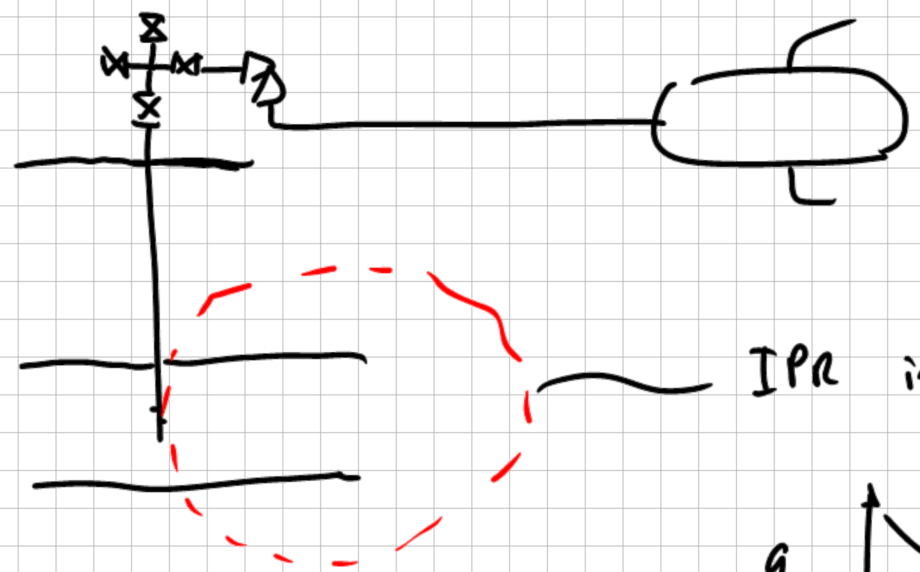
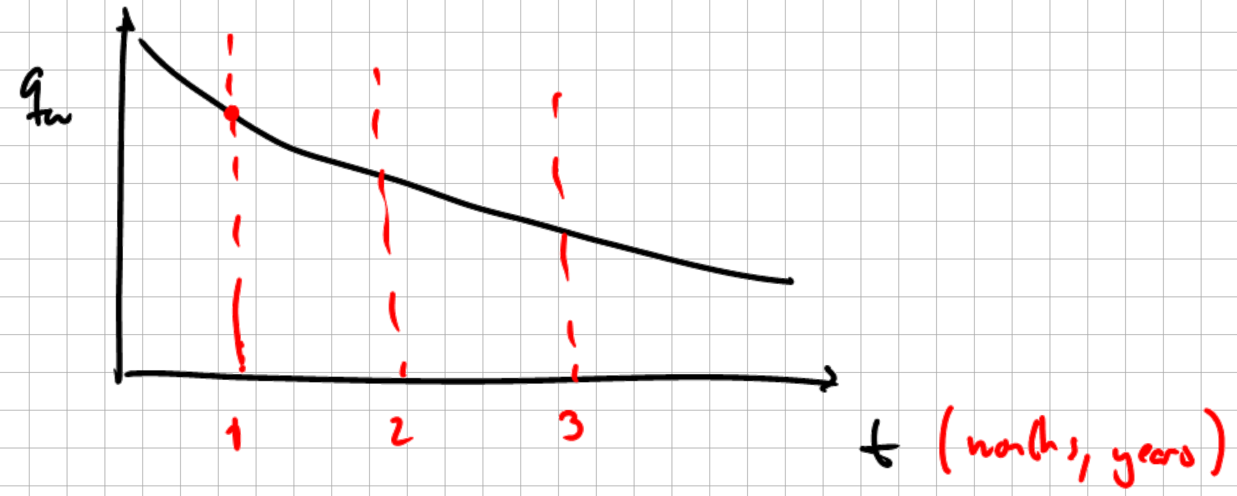
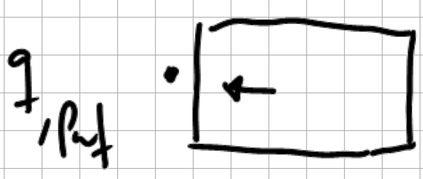


IPR inflow performance relationship

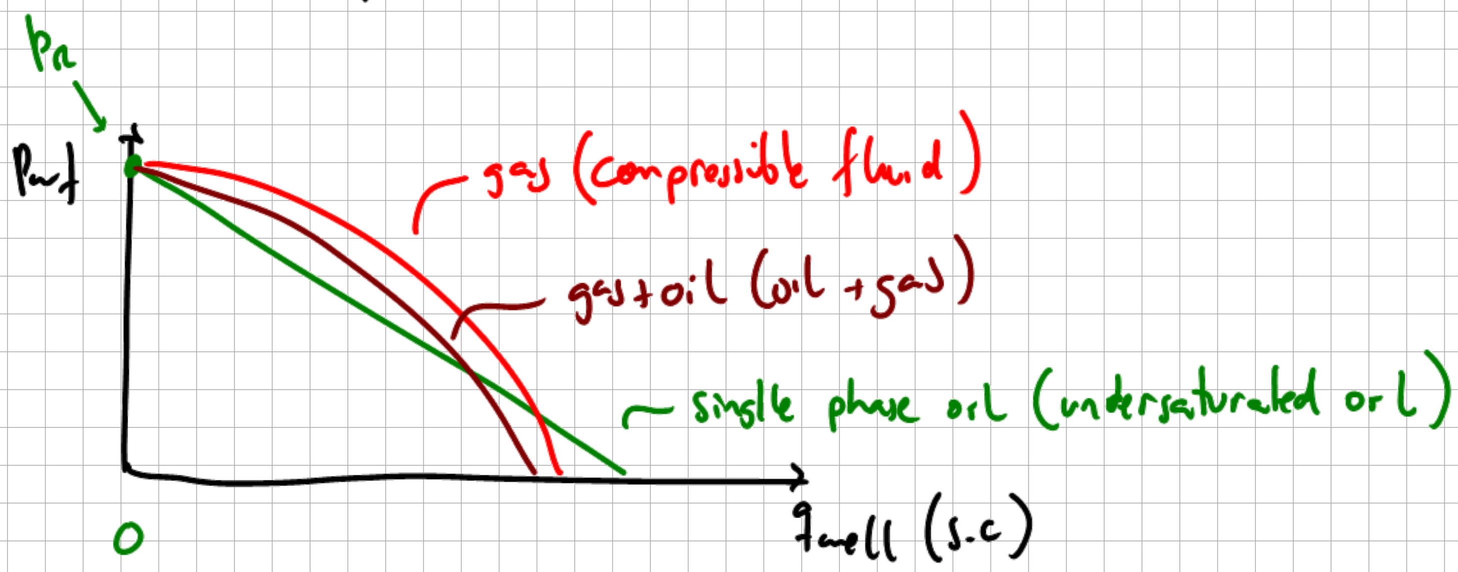




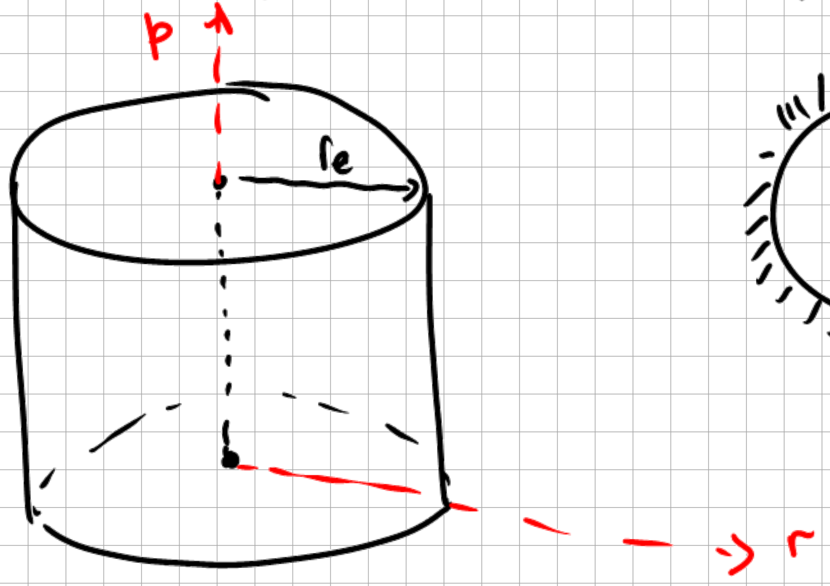
$q = f(P_{wf}, t)$

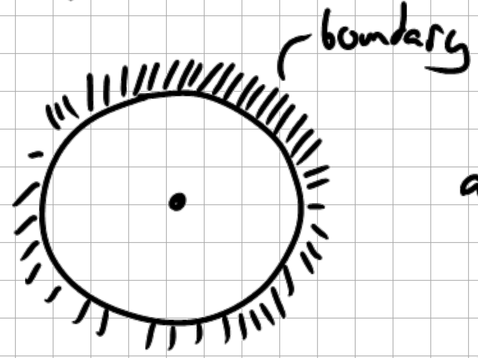
↳ instead of using t, we use P_R

at a given point in time:



vertical well, cylindrical reservoir, single phase fluid





at $t=0$

$q=0$

$P(r_w) = P_R$

at $t>0$

$q \neq 0$

$P(r_w) = P_{wf}^*$

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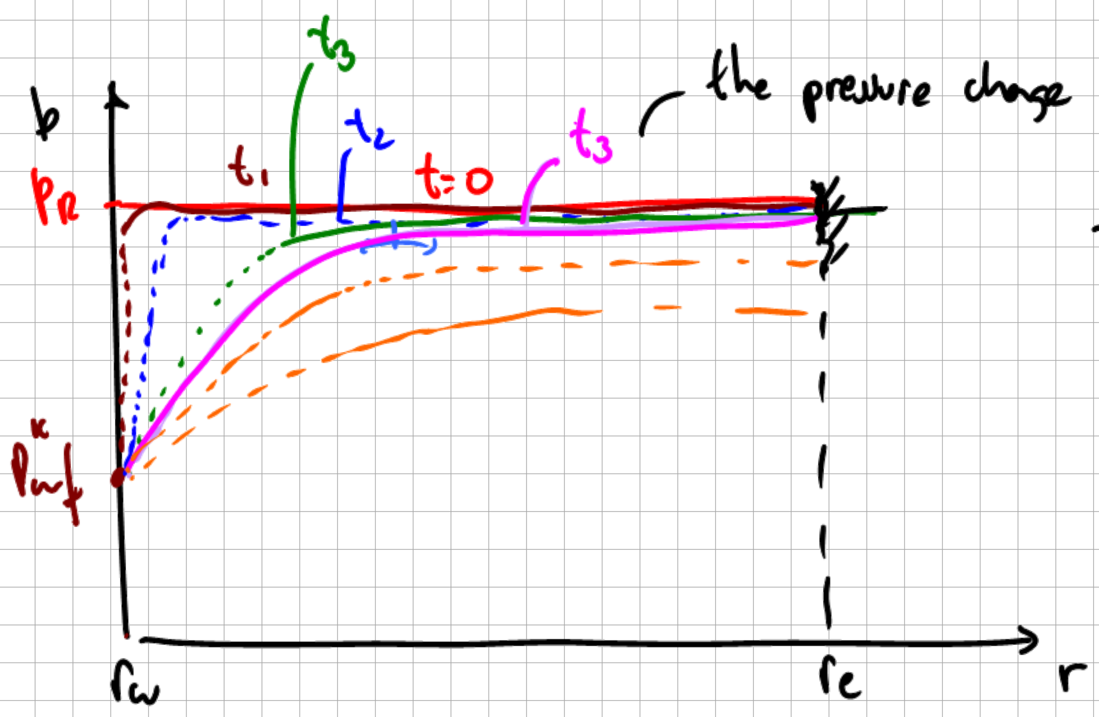

the pressure change reaches the boundary

$t=0 \rightarrow t=t_3$ infinite acting

t is important for IPR

(no boundary effects)

$t_3 \rightarrow$ steady state $p(r_e) = \text{const}$

pseudo steady state $q(r_e)=0$

(pss)

$$t_{pss} = \frac{281.3\ \phi \cdot \mu \cdot C_t \cdot A}{k}$$

ϕ [-]

μ [cp] 1cp = 1E-3 Pa s

C_t [1/bar] $C_t = C_f + C_{fluid}$ → gas, → oil

↳ $S_w C_w + S_o C_o + S_g C_g$

A [m^2]

k [md]

Formation compressibility, c_f	[1/bar]	4.35E-05
Oil compressibility at res conditions, C_o	[1/bar]	2.18E-04
Gas compressibility at res conditions, C_g	[1/bar]	7.25E-03
Total compressibility, assuming oil-filled pore, C_t	[1/bar]	2.61E-04
Total compressibility, assuming gas-filled pore, C_t	[1/bar]	7.30E-03
Oil viscosity at res conditions, μ_o	[cp]	0.5
Gas viscosity at res conditions, μ_g	[cp]	1.20E-02
Porosity, ϕ	[-]	0.3
Outer radius of reservoir, r_e	[m]	600
Top area of reservoir, A	[m2]	1130973.4

FLUID:	Gas	Oil
k [md]	t_{pss} [hr]	t_{pss} [hr]
0.01	835552.28	1245853.30
0.1	83555.23	124585.33
1	8355.52	12458.53
10	835.55	1245.85
1000	8.36	12.46
10000	0.84	1.25

- if t_{pss} is short most of the production will occur in pss (ss) therefore t is not necessary to compute IPR
- if t_{pss} is large, most of the production will occur in IA, therefore it is important to consider time in IPR

IPRs are obtained

- empirical (field data) → "easier to use" → field data is needed
- Derived analytically (semi-analytically) from conservation equations.

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Conservation of mass + Darcy flow in a radial reservoir;
single-phase fluid

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \alpha \frac{\partial p}{\partial t}$$

$$\alpha = \frac{\mu \phi C}{k}$$

- need alot of parameters about the reservoir
- Allow for well design and to estimate future performance
- always need adjustment with field data

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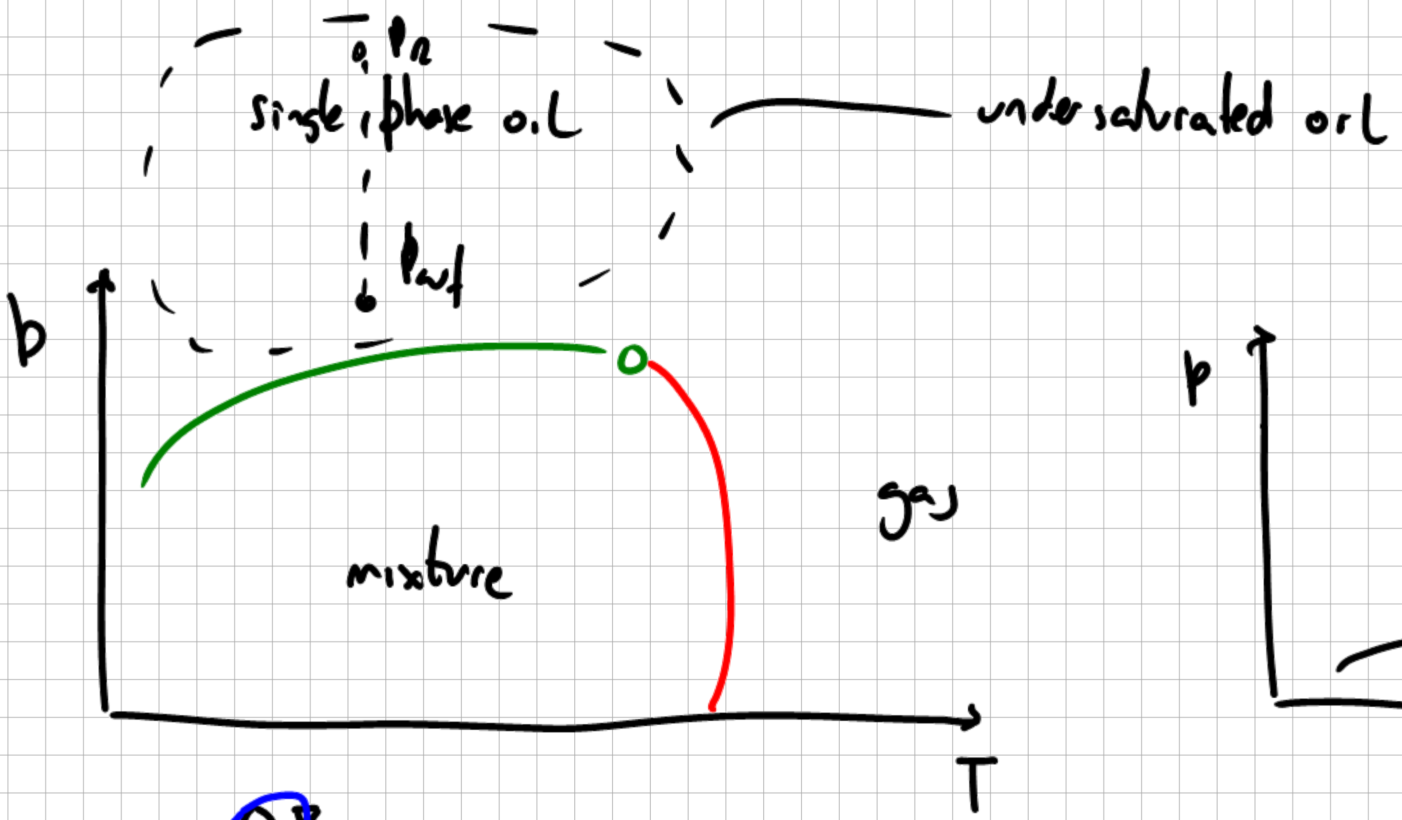
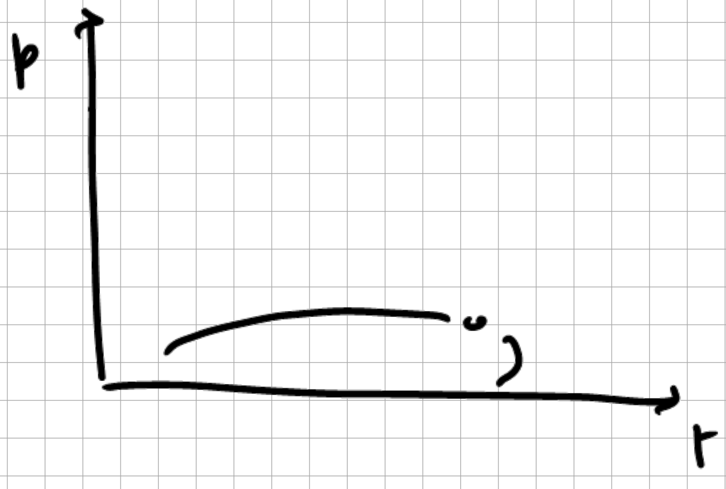
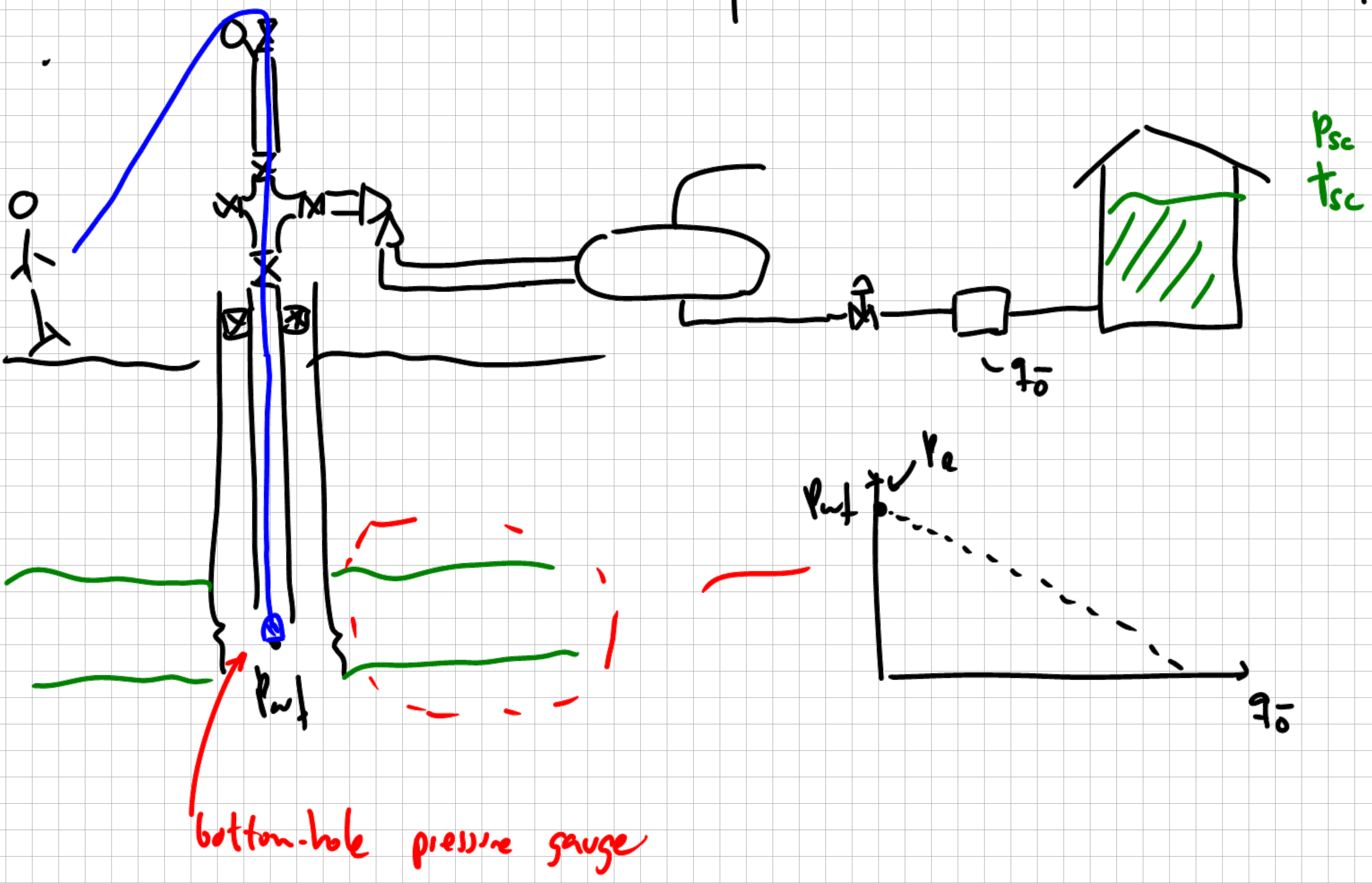
Video 06 - IPR for vertical undersaturated oil well (field test)







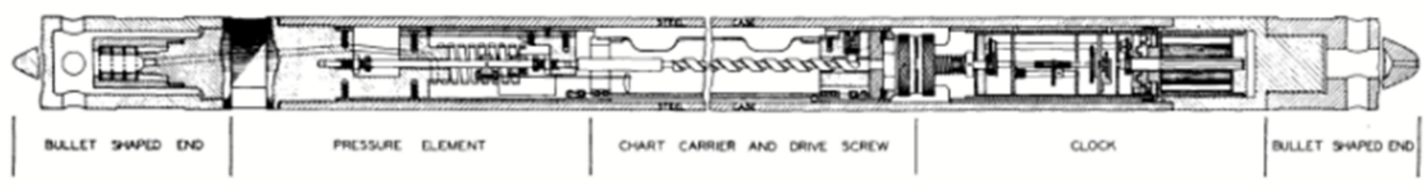


FIG. 1 CROSS-SECTION OF AMERADA PRESSURE GAUGE.

Bottom-hole Pressures in Oil Wells[1]

By Charles V. Millikan,[2] Tulsa, Okla. and Carroll V. Sidwell,[3] Seminole, Okla.

(Tulsa Meeting, October, 1930)

There is nothing more important in petroleum engineering than a definite knowledge of the pressure at the bottom of an oil well at any existing operating condition, and the relation of this pressure to the pressure within the producing formation. A knowledge of bottom-hole pressures is fundamental in determining the most efficient methods of recovery and the most efficient lifting procedure, yet there is less information about these pressures than about any other part of the general problem of producing oil.

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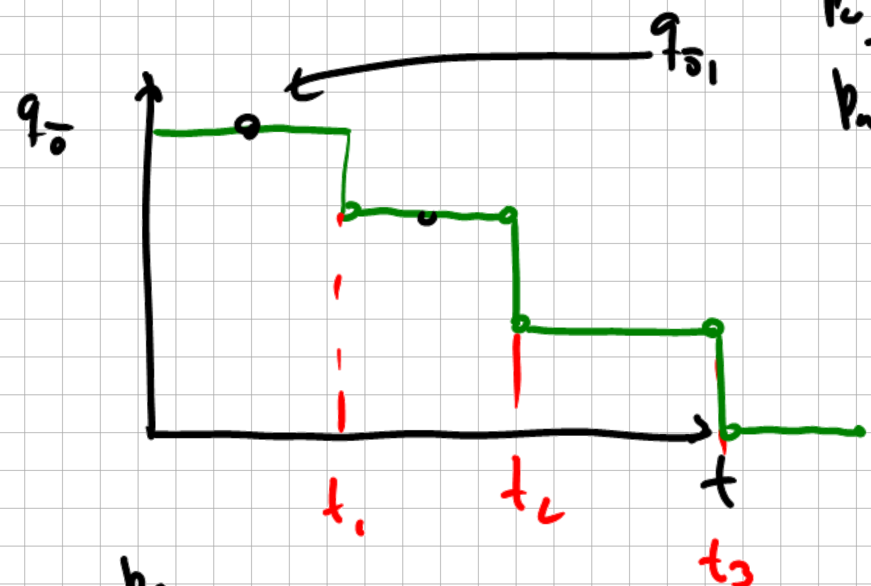
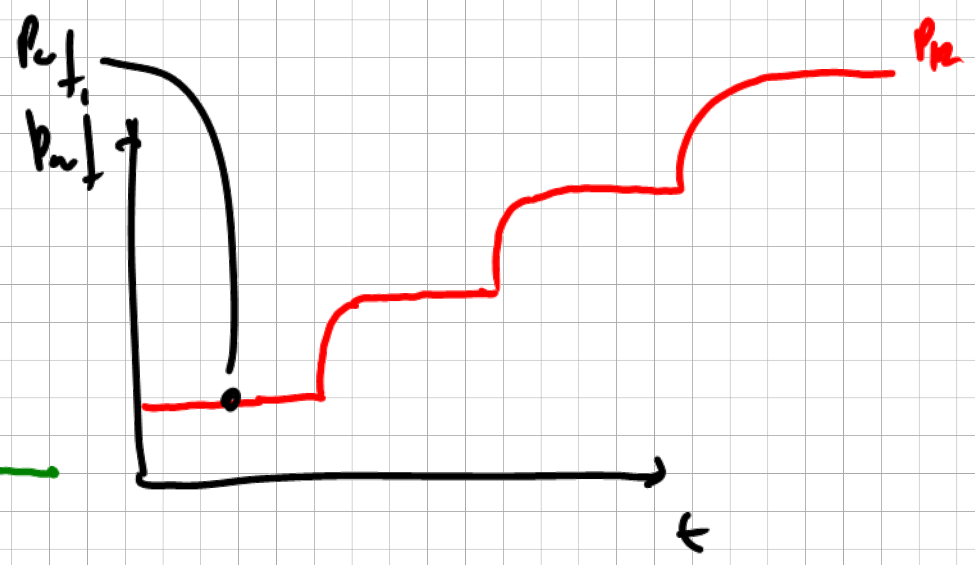
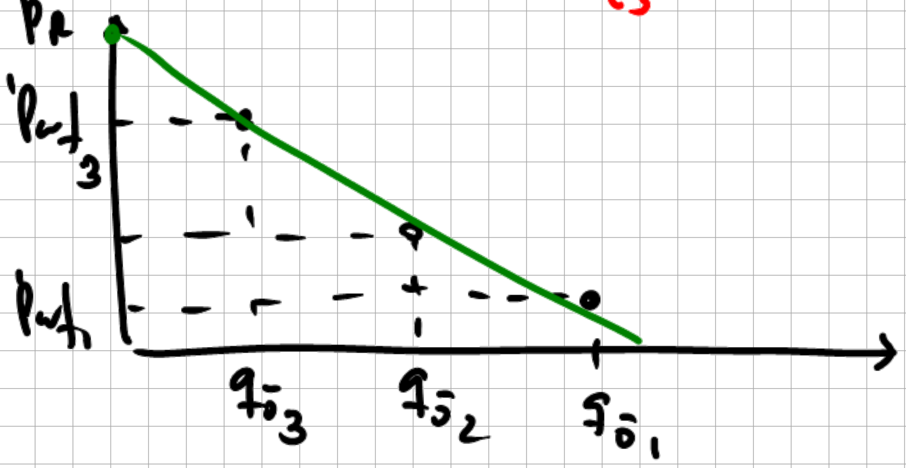


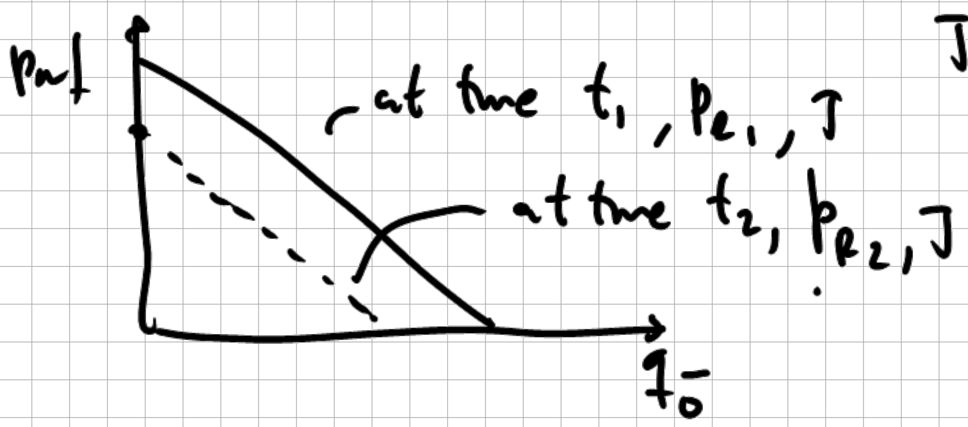




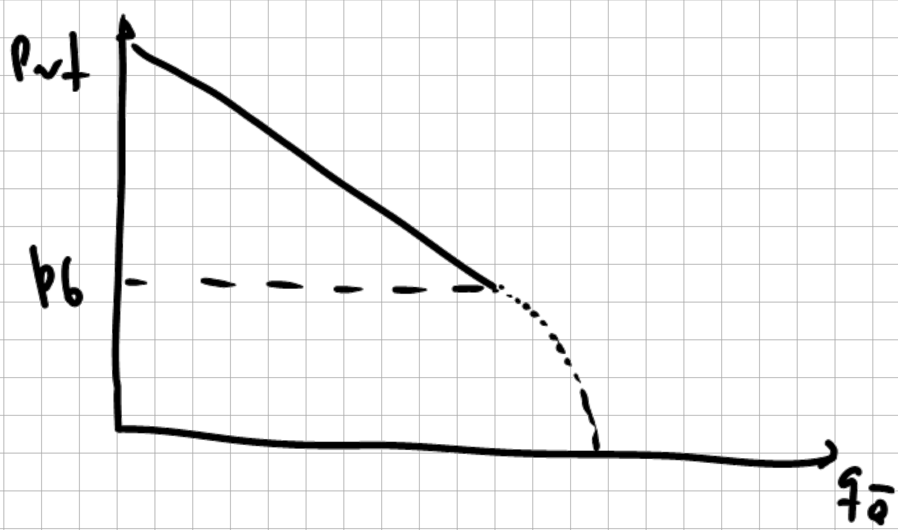
$$\underbrace{q_{\bar{o}}}_{Sm^3/d} = J\,(\overbrace{p_R - p_{wf}}^{bara})$$

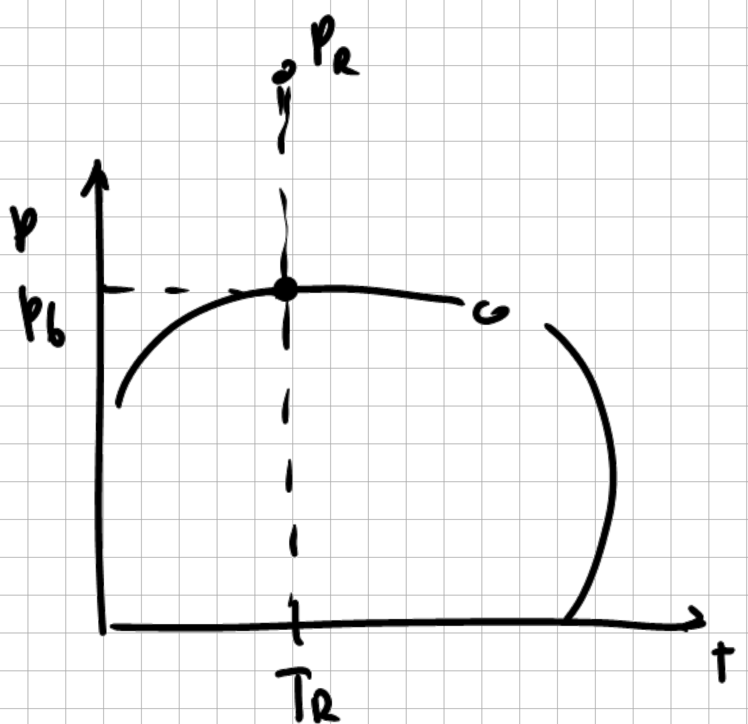
↑ productivity index

$$J = \left[\frac{Sm^3/d}{bara}\right]$$

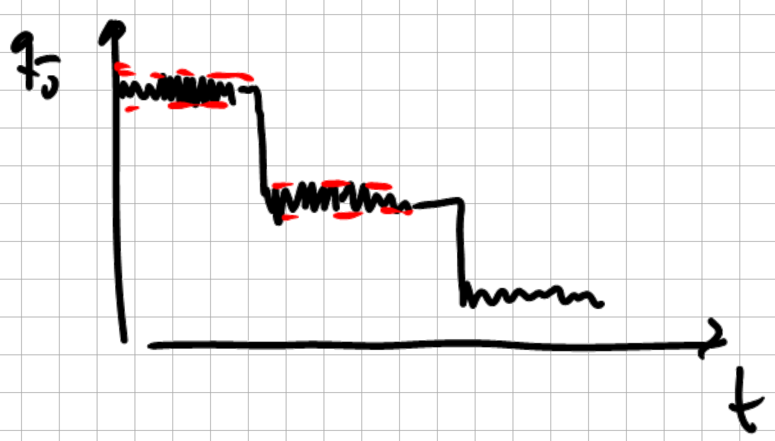


J usually doesn't change with depletion

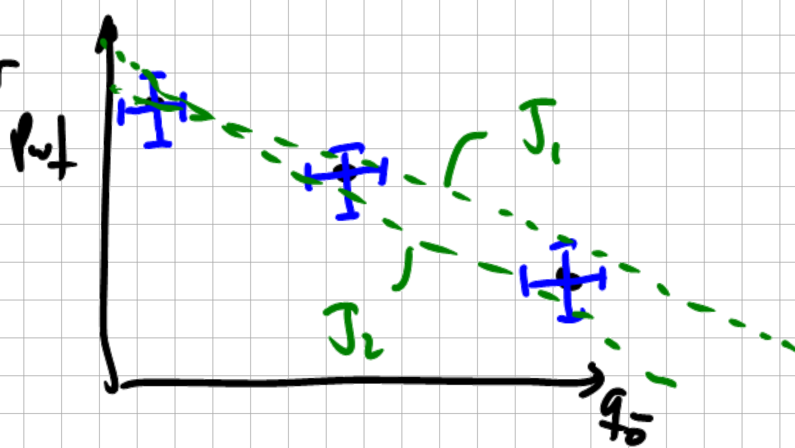




- effect of measurement errors and variability



p = value + error



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if one point is available, and reservoir pressure is known

P_{wf_1} , $q_{\bar{o}_1}$

$$q_{\bar{o}_1} = J\left(P_R - P_{wf_1}\right)$$

$$J = \frac{q_{\bar{o}_1}}{\left(P_R - P_{wf_1}\right)}$$

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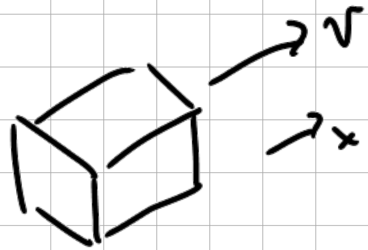
Darcy's law

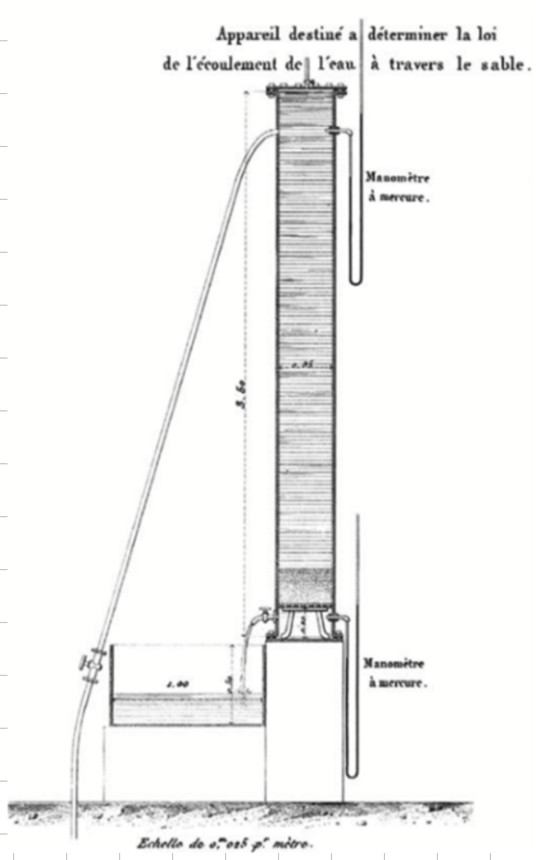
Dijon

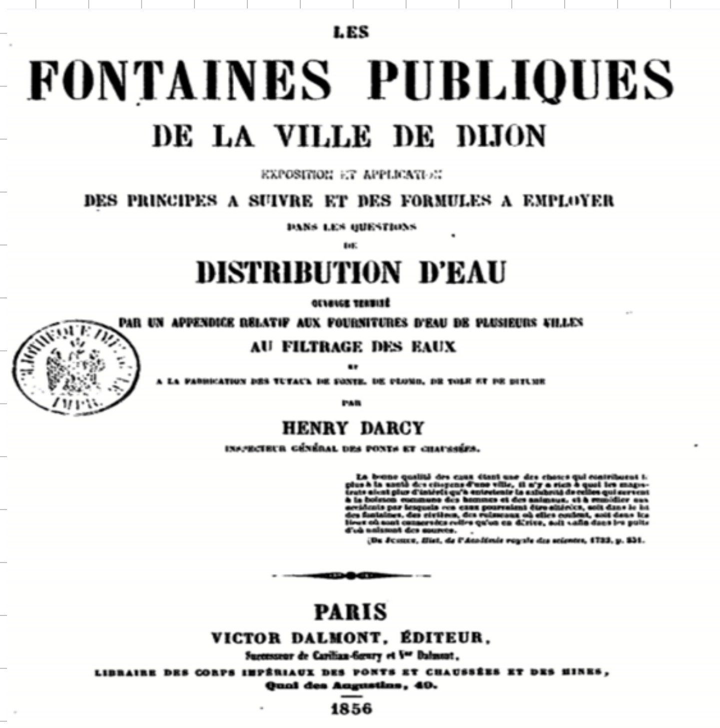
1856 → Retired to Dijon

1858 → died

$$v = \frac{k}{\mu} \cdot \frac{dP}{dx}$$







LES

FONTAINES PUBLIQUES

DE LA VILLE DE DIJON

EXPOSITION ET APPLICATION

DES PRINCIPES A SUIVRE ET DES FORMULES A EMPLOYER

DANS LES QUESTIONS

DE

DISTRIBUTION D'EAU

OUVRAGE TERMINÉ

PAR UN APPENDICE RELATIF AUX FOURNITURES D'EAU DE PLUSIEURS VILLES

AU FILTRAGE DES EAUX

ET

A LA FABRICATION DES TUYAUX DE FONTE, DE PLOMB, DE TOLE ET DE BITUME

PAR

HENRY DARCY

INSPECTEUR GÉNÉRAL DES PONTS ET CHAUSSÉES.

PARIS

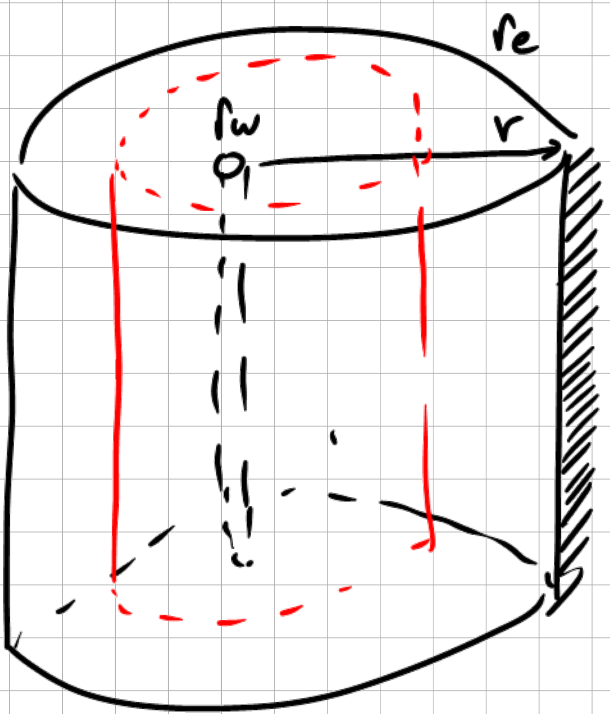
VICTOR DALMONT, ÉDITEUR,

Successeur de Carilian-Gœury et Vve Dalmont,

LIBRAIRE DES CORPS IMPÉRIAUX DES PONTS ET CHAUSSÉES ET DES MINES,

Quai des Augustins, 49.

1856



SS
PSS
$$\left.\begin{matrix} SS \\ PSS \end{matrix}\right\} \frac{\partial}{\partial t} = 0$$

$$v = \frac{k}{\mu} \frac{dP}{dr}$$

$v \rightarrow q_{\bar{o}}$

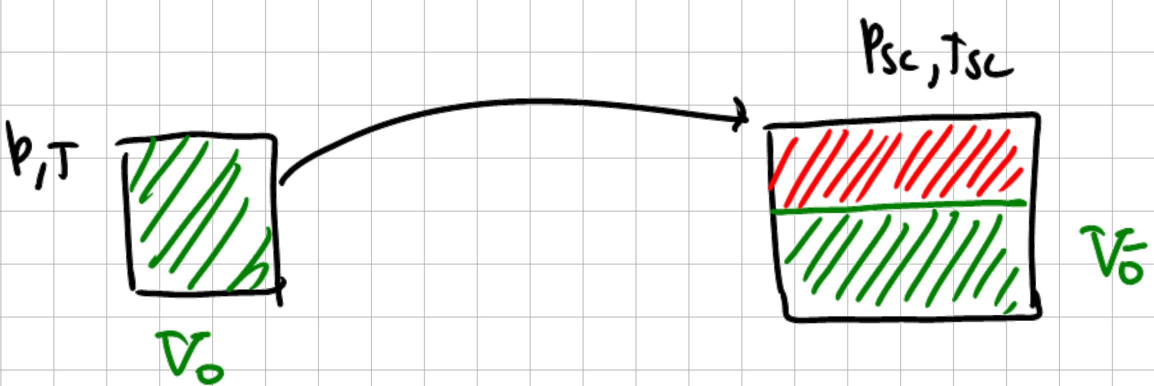
$$v = \frac{q_o}{A} \rightarrow [m^3/d] \quad [m^3/s]$$

$A \rightarrow 2\pi r h$

$q_{\bar{o}}$?

$q_{\bar{o}} \overset{?}{\rightarrow} q_o$

Black oil properties



oil volume factor

$$B_o(P,T) = \frac{V_o(P,T)}{V_{\bar{o}}} > 1$$

$$q_o = B_o(P,T) \cdot q_{\bar{o}}$$

$$B_o = \left[m^3 / Sm^3 \right]$$

$$v = \frac{q_{\bar{o}} \cdot B_o}{2\pi r h} = \underbrace{\frac{k}{\mu_o}}_{f(P,T)} \frac{\partial P}{dr}$$

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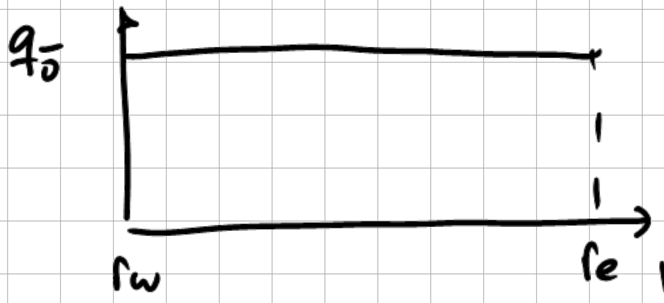
$q_{\bar{o}} \overset{?}{=} f(r)$



$q_{\bar{o}} \neq f(r)$ } SS

$$\int_{r_w}^{r@\bar{p}_R} \frac{dr}{r} = \frac{2\pi K h}{q_{\bar{o}}} \int_{p_{wf}}^{\bar{p}_R} \frac{dp}{B_o \mu_o}$$

(1) under the left integral; (2) under the right integral

$$\bar{p}_R = \frac{\int_V p \, dv}{V}$$

for SS, $r@\bar{p}_R = 0.61\, r_e$

(1) $\ln\left(\frac{0.61\, r_e}{r_w}\right) = \ln(0.61) + \ln\left(\frac{r_e}{r_w}\right) = \ln\left(\frac{r_e}{r_w}\right) - 0.5$

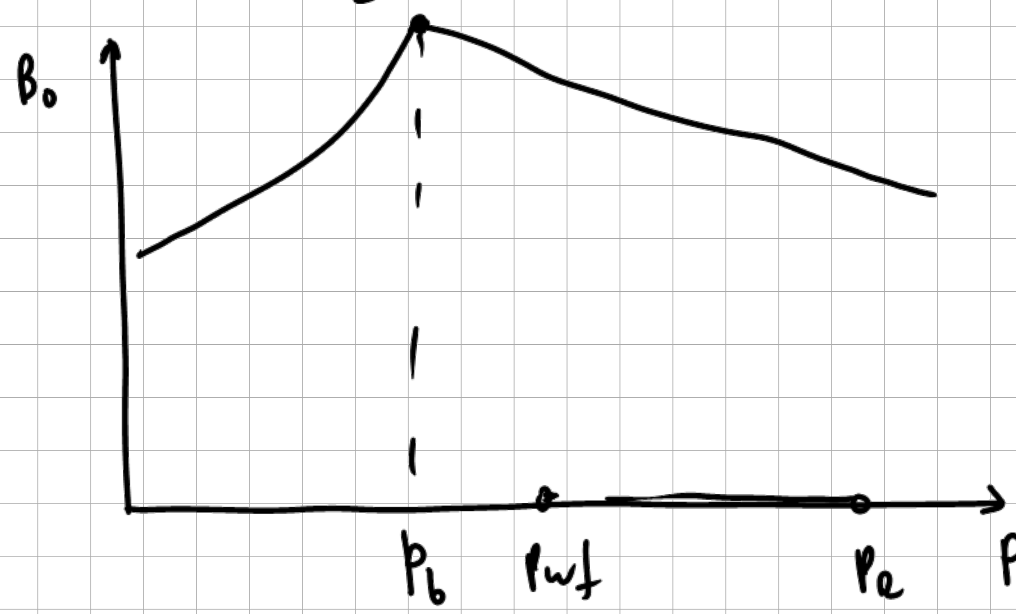
(2) $\int_{p_{wf}}^{\bar{p}_R} \frac{1}{\mu_o B_o} dp$

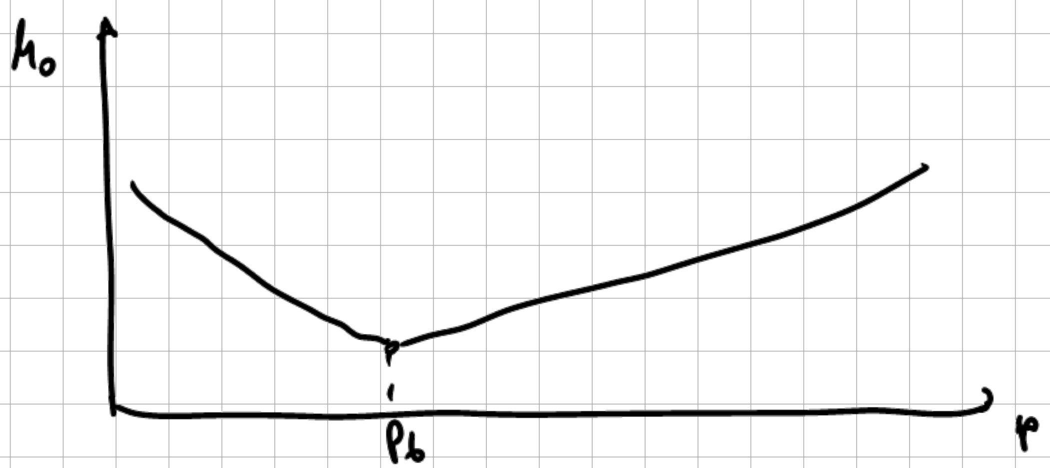
$\mu_o = f_1(p, T)$

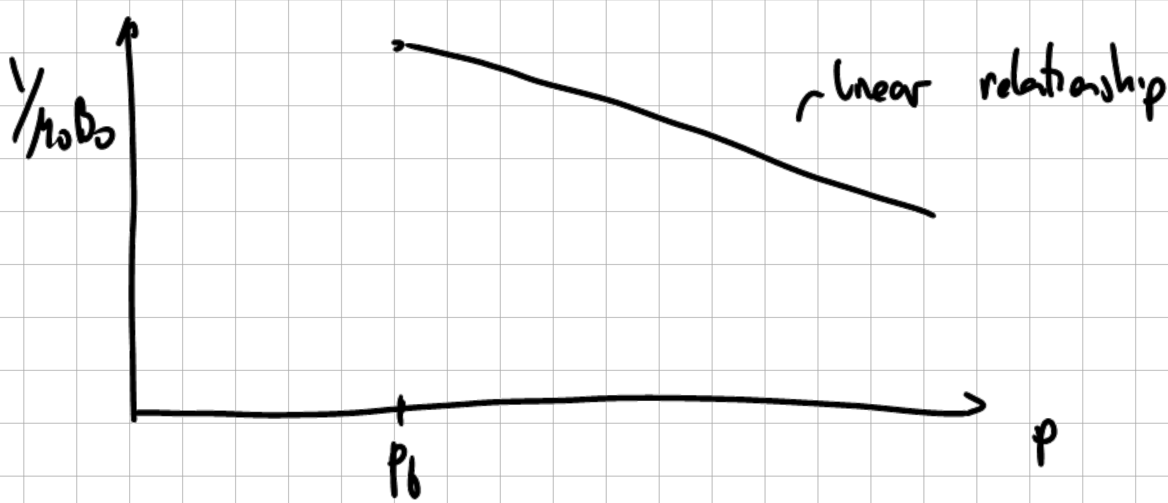
$B_o = f_2(p, T)$

@ reservoir T = constant = T_R

sat ← | → undersaturated







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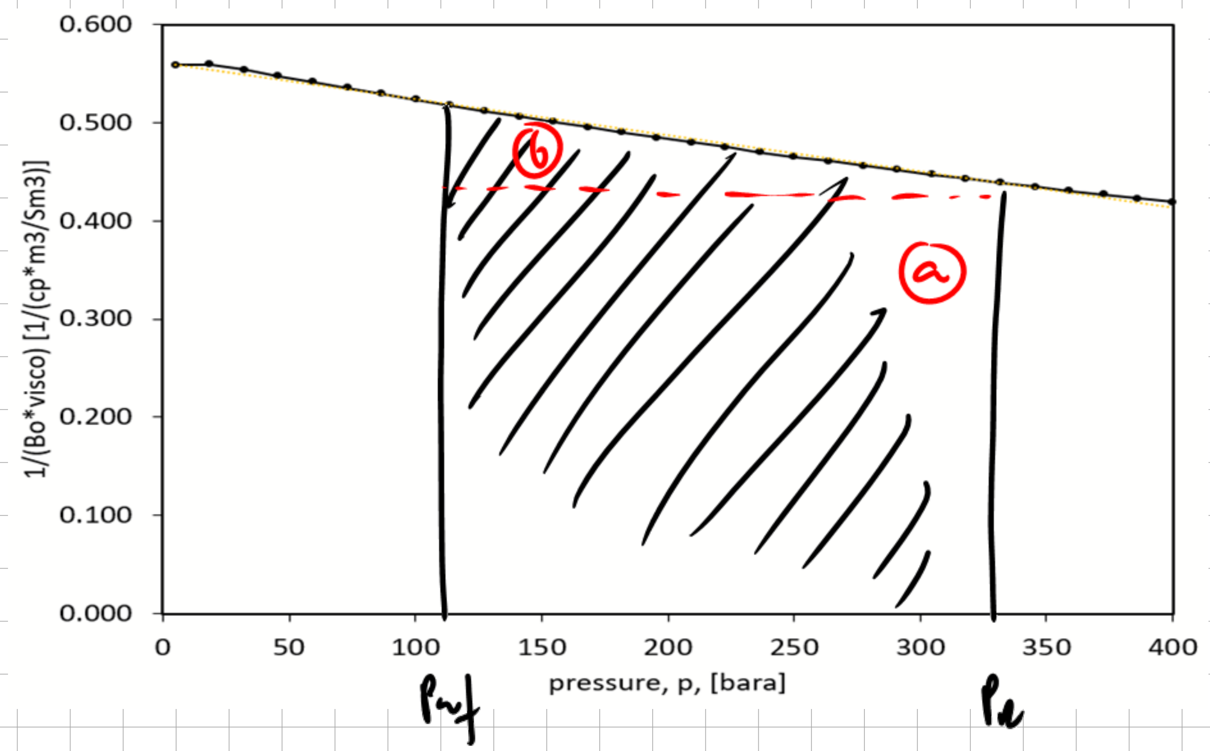

$$② \int_{p_{wf}}^{p_R} \frac{1}{\mu_o B_o}\, dp \approx (p_R - p_{wf}) \cdot \underbrace{\left[\frac{1}{\mu_o B_o}\Big|_{p_{wf}} + \frac{1}{\mu_o B_o}\Big|_{@ p_R} \right] 0.5}_{\frac{1}{\mu_o B_o}\Big|_{@ p_{av} = \frac{(p_R + p_{wf})}{2}}}$$

$$② = \frac{(p_R - p_{wf})}{(\mu_o B_o)_{@ p_{av}}}$$

$$\ln\left(\frac{r_e}{r_w}\right) - 0.5 = \frac{2\pi k h}{q_o} \frac{(p_R - p_{wf})}{(\mu_o B_o)_{@ p_{av}}}$$

$$q_o = \frac{2\pi k h \,(p_R - p_{wf})}{(\mu_o B_o)_{@ p_{av}} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.5 \right]}$$

$J \approx$ constant

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$$q_{\bar{o}} = \frac{2\pi k h (P_R - P_{wf})}{(\mu_o B_o)_{@p_{av}} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.5\right]}$$

- steady-state
- vertical well, fully perforated
- undersaturated oil
- radial drainage volume
- homogeneous permeability

SI system

$$\left[Sm^3/s\right] = \frac{[m^2][m][Pa]}{[Pa\ s]\ [-]}$$

$$\left[Sm^3/d \ \frac{1d}{24h} \ \frac{1h}{3600s}\right] = \frac{\left[1\ md \ \frac{1d}{1000\ md} \ \frac{9.86E-13\ m^2}{1d}\right][m]\left[bara \ \frac{10^5 Pa}{1\ bara}\right]}{\left[1\ cp \ \frac{10^{-3} Pa.s}{1cp}\right]} \ 2\pi$$

$$= \frac{1}{18.68}$$

$$q_{\bar{o}} = \frac{k h}{(\mu_o B_o)|_{@p_{av}} \left(\ln \frac{r_e}{r_w} - 0.5\right)} \frac{1}{18.68} (P_R - P_{wf})$$

J

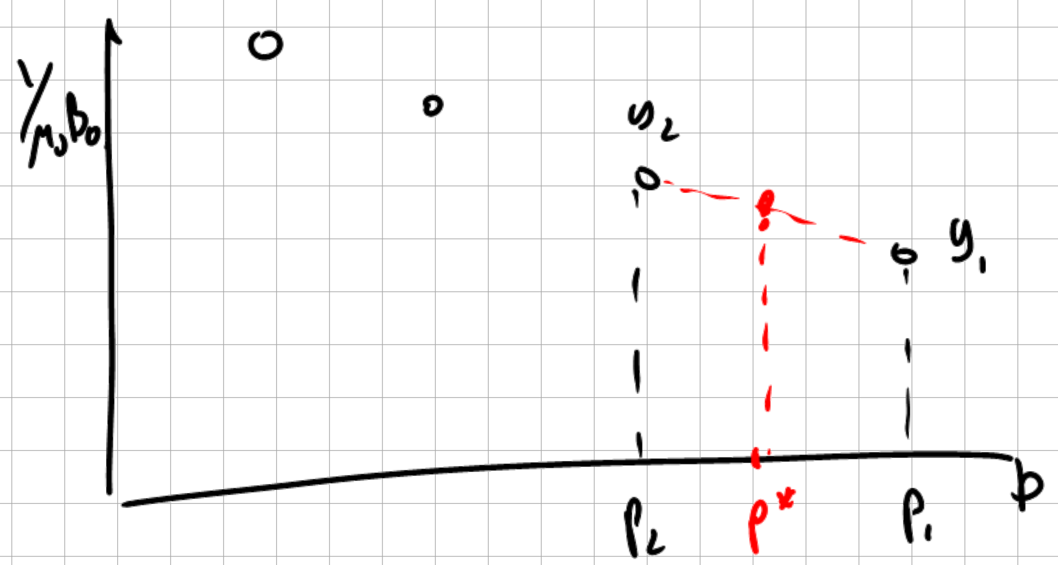
p in bara

k in md

μ_o in cp

$q_{\bar{o}} = Sm^3/d$

$$J = \frac{k h}{(\mu_o B_o)_{av} \left(\ln\left(\frac{r_e}{r_w}\right) - 0.5\right) \cdot 18.68}$$



$$\frac{y_2 - y_1}{P_2 - P_1} = \frac{y_2 - y^*}{P_2 - p^*}$$

$$y = \frac{1}{\mu_o B_o}$$

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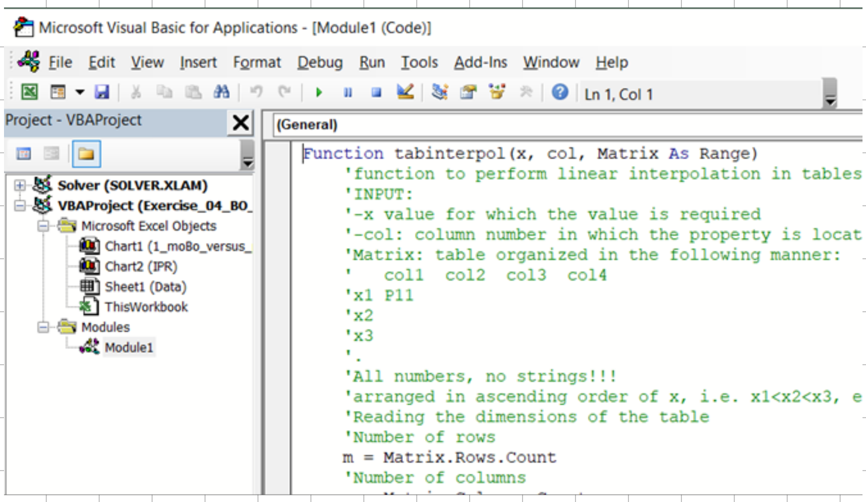
VBA visual basic for application

while in excel press alt + F11



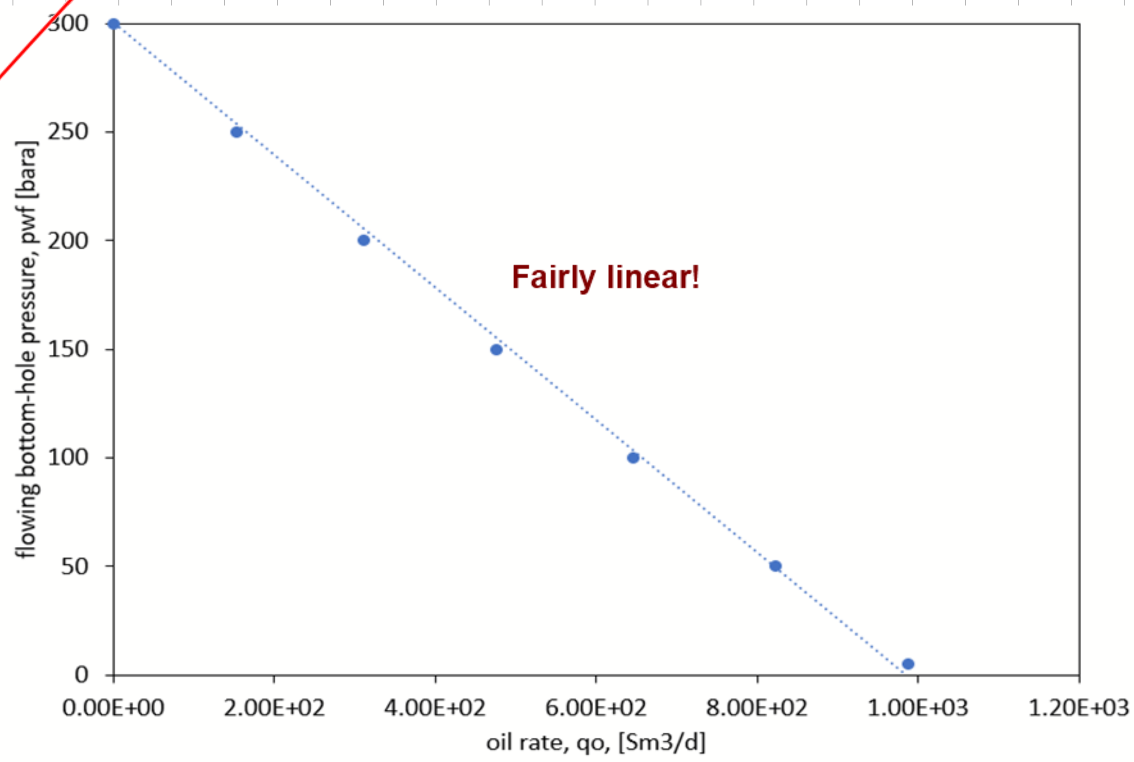

```
Function tabinterpol(x, col, Matrix As Range)
    'function to perform linear interpolation in tables
    'INPUT:
    '-x value for which the value is required
    '-col: column number in which the property is locat
    'Matrix: table organized in the following manner:
    '    col1  col2  col3  col4
    'x1 P11
    'x2
    'x3
    '.
    'All numbers, no strings!!!
    'arranged in ascending order of x, i.e. x1<x2<x3, e
    'Reading the dimensions of the table
    'Number of rows
    m = Matrix.Rows.Count
    'Number of columns
```

Use here the VBA function ProductivityIndex_J

use here the VBA function tabinterpol

r_e	[m]	500
r_w	[m]	0.1
k	[md]	10
h	[m]	100
p_R	[bara]	300

pwf	pav	1/(Bo visco)	J	qo
[bara]	[bara]	[1/(cp*m3/Sm3)]	[Sm3/d/bar]	[Sm3/d]
300	300	0.449	3.0	0.00E+00
250	275	0.457	3.1	1.53E+02
200	250	0.465	3.1	3.11E+02
150	225	0.474	3.2	4.75E+02
100	200	0.483	3.2	6.46E+02
50	175	0.493	3.3	8.23E+02
5	153	0.502	3.3	9.88E+02

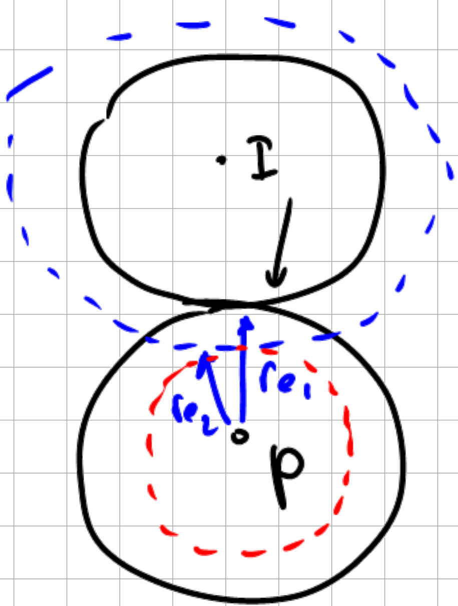


EFFECT OF THE BOUNDARY (re)

EFFECT OF THE REGIME (SS)

r_e	[m]	500
r_w	[m]	0.1
k	[md]	10
h	[m]	100
p_R	[bara]	300

$\ln\left(\frac{r_e}{r_w}\right) =$ 8.52E+00



$$J = \frac{kh}{18.68\,(\mu_o B_o)_{av}\cdot\left(\underbrace{\ln\frac{r_e}{r_w}}_{8.5} - \underbrace{0.5}_{10\%}\right)}$$

SS, boundary effect

r_e	[m]	500
r_w	[m]	0.1
k	[md]	10
h	[m]	100
p_R	[bara]	300

pwf	pav	1/(Bo visco)	J	qo
[bara]	[bara]	[1/(cp*m3/Sm3)]	[Sm3/d/bar]	[Sm3/d]
300	300	0.449	3.0	0.00E+00
250	275	0.457	3.1	1.53E+02
200	250	0.465	3.1	3.11E+02
150	225	0.474	3.2	4.75E+02
100	200	0.483	3.2	6.46E+02
50	175	0.493	3.3	8.23E+02
5	153	0.502	3.3	9.88E+02

r_e	[m]	400
r_w	[m]	0.1
k	[md]	10
h	[m]	100
p_R	[bara]	300

pwf	pav	1/(Bo visco)	J	qo
[bara]	[bara]	[1/(cp*m3/Sm3)]	[Sm3/d/bar]	[Sm3/d]
300	300	0.449	3.1	0.00E+00
250	275	0.457	3.1	1.57E+02
200	250	0.465	3.2	3.20E+02
150	225	0.474	3.3	4.89E+02
100	200	0.483	3.3	6.64E+02
50	175	0.493	3.4	8.46E+02
5	153	0.502	3.4	1.02E+03

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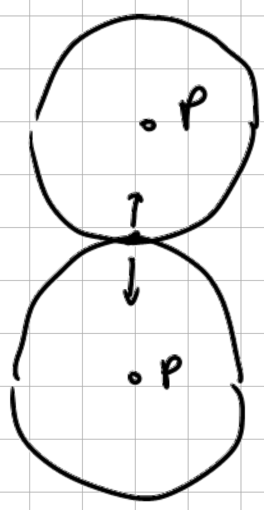
Video 09 - Skin effect for IPR of vertical, undersaturated oil well

$$q_{\bar{o}} = \frac{kh}{(\mu_o B_o)_{av} \left(\underbrace{\ln\left(\frac{r_e}{r_w}\right)}_{8} - 0.5 \right) \cdot 18.68} (p_R - p_{wf})$$

pseudo steady-state pss @re → no-flow boundary



$$\int_{r_w}^{@p_e} \frac{dr}{r} = \frac{kh}{q_{\bar{o}} \cdot 18.68} \int_{p_{wf}}^{p_e} \frac{dp}{B_o \mu_o}$$

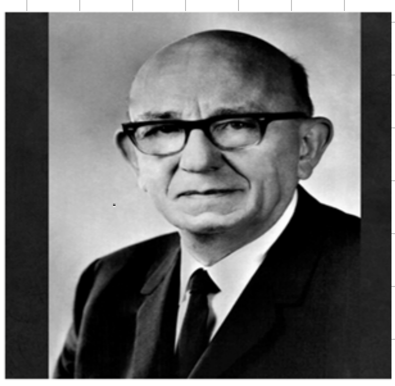
for pss r at which $p = p_e$ $r = 0.47\, r_e$

$$\ln\left(\frac{0.47\, r_e}{r_w}\right) = \ln\left(\frac{r_e}{r_w}\right) - 0.75$$

$$J = \frac{kh}{(\mu_o B_o)_{av} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 \right] 18.68}$$

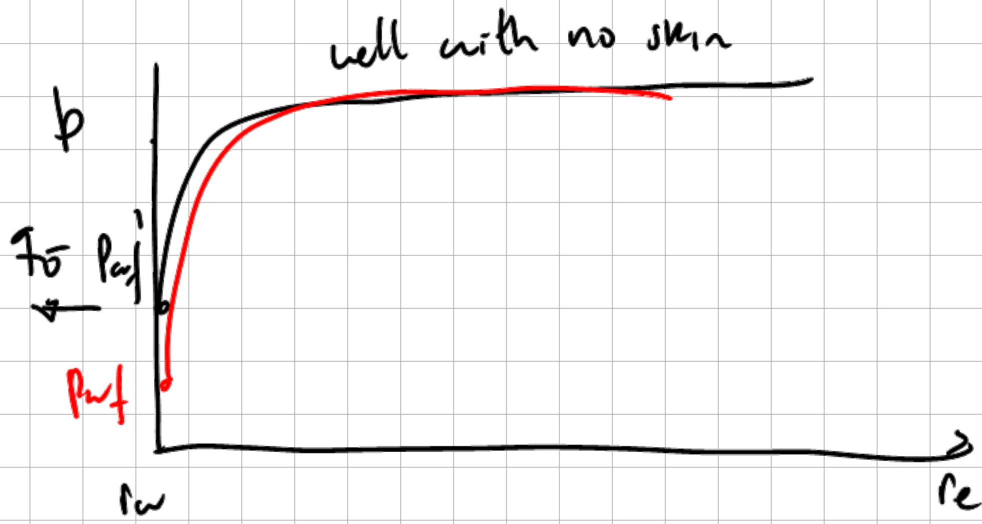
Skin factor 1953 → Hurst

↳ Antonius Van Everdingen (Dutch mining engineer) shell



THE SKIN EFFECT AND ITS INFLUENCE ON THE PRODUCTIVE CAPACITY OF A WELL

A. F. VAN EVERDINGEN, SHELL OIL CO., HOUSTON, TEXAS, MEMBER AIME



$$\left(\underbrace{p_{wf}'}_{\substack{\text{ideal} \\ \downarrow \\ \text{no skin}}} - \underbrace{p_{wf}}_{\substack{\text{actual} \\ \text{with skin}}} \right) = \Delta p_{skin} = S \underbrace{\left[\frac{18.68 \, (\mu_o B_o)_{av} \cdot q_{\bar{o}}}{kh} \right]}_{\text{bar}}$$

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$$\cancel{P_R - P_{wf}'} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{k \cdot h} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 \right]$$

$$\cancel{P_{wf}'} - P_{wf} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{kh} S$$

$$P_R - P_{wf} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{kh} \left[\ln \frac{r_e}{r_w} - 0.75 + S \right]$$

$$J = \frac{kh}{18.68 \, (\mu_o B_o)_{av} \left[\underbrace{\ln \frac{r_e}{r_w}}_{\text{convergence}} \underbrace{- 0.75}_{\text{boundary}} + \underbrace{S}_{\text{skin (near wellbore non-idealities)}} \right]}$$

8

other definitions "equivalent" to skin

- flow efficiency

$$E_F = \frac{P_R - P_{wf}'}{P_R - P_{wf}} = \frac{\frac{\cancel{q_o}}{J'}}{\frac{\cancel{q_o}}{J}} = \frac{\ln\left(\frac{r_e}{r_w}\right) - 0.75}{\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S} \approx \frac{7}{7 + S}$$

- apparent wellbore radius

$$\ln\left(\frac{r_e}{r_w}\right) + S = \ln\left(\frac{r_e}{r_{wa}}\right) \qquad r_{wa} = r_w \, e^{-S}$$

If $S > 0$ ⇝ damage
(less than ideal)
$(P_{wf}' - P_{wf}) > 0$

if $S = 0$ ⇝ ideal

if $S < 0$ ⇝ "stimulation" ⇝ productivity more than ideal

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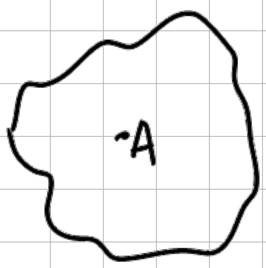
• Drainage area shape → 1965 Daniel Dietz

Determination of Average Reservoir Pressure From Build-Up Surveys

D. N. DIETZ
MEMBER AIME

KONINKLIJKE/SHELL LABORATORIUM
RIJSWIJK, THE NETHERLANDS

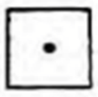
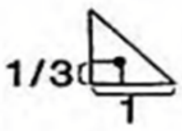
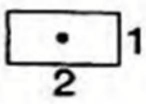
$$\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + \underbrace{S_A}_{\text{shape-area skin}}$$



$$A = \pi \cdot r_e^2 \qquad r_e = \sqrt{\frac{A}{\pi}}$$

well performance (2nd edition)
Golan, Whitson

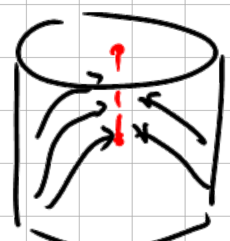
Table 2.4 Shape Factors for Nonradial Outer Boundary Geometries

Geometry	C_A	s_A	t_{DAeia}[a]	t_{DApss}[b]
	31.62	0.000	0.1	0.1
	30.88	0.012	0.09	0.1
	31.60	0.000	0.1	0.1
	27.6	0.068	0.09	0.2
	27.1	0.077	0.09	0.2
	21.9	0.184	0.08	0.4
	21.84	0.185	0.025	0.3

• other cases than can be modeled (represented) with skin

• formation damage (due to drilling) $S > 0$

• partial penetration



$k_v << k_H$ $S > 0$

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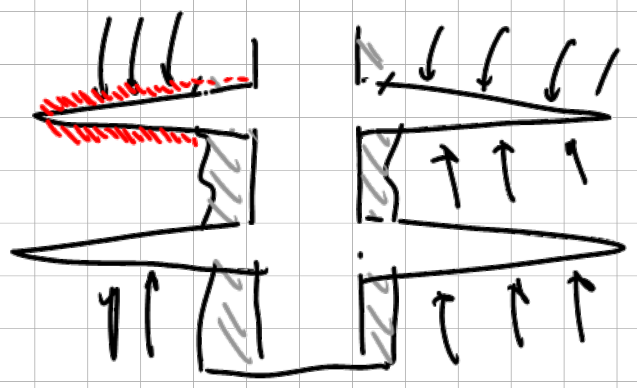
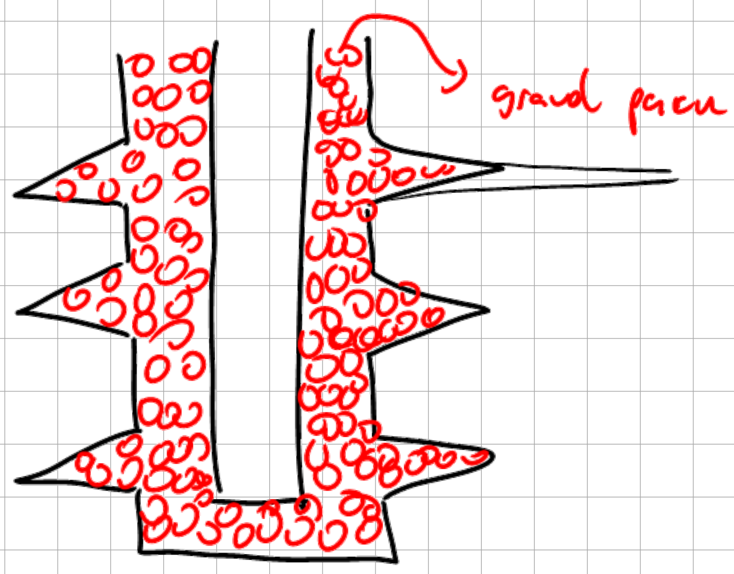
- perforation



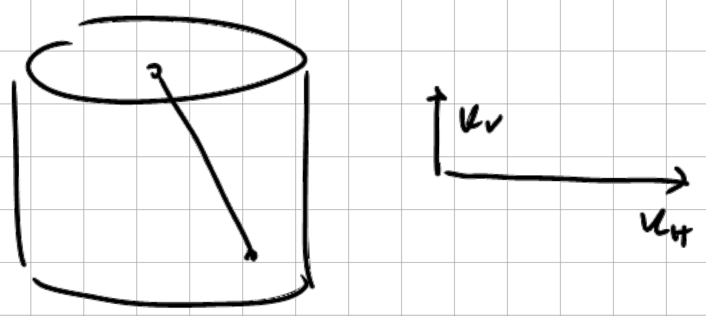
$S > 0$

- Sand control (gravel pack)



$S > 0$

- wellbore inclination
  (permeability anisotropy)



- acidizing (stimulation)
- fracking

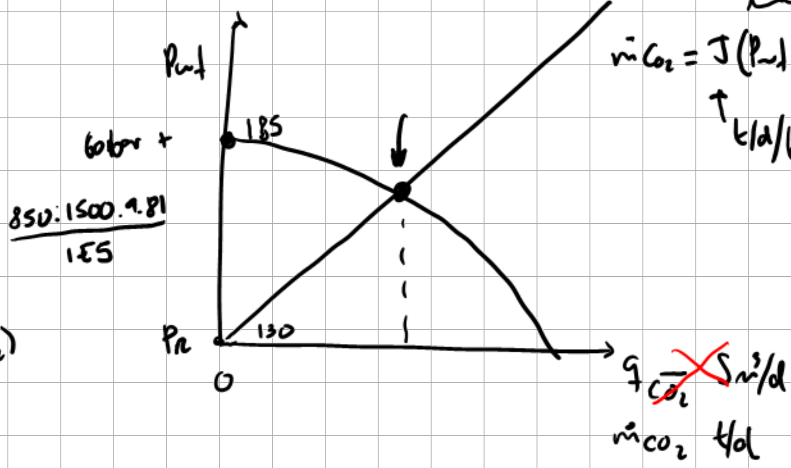
$S < 0$

20240906

OUTLINE:
-Continuation of exercise on flow equilibrium in CO2 injection well
-Re-cap of last week's video lectures (IPR)
-IPR for water/CO2 injector.

$y = x^2$

$x = \sqrt{y}$



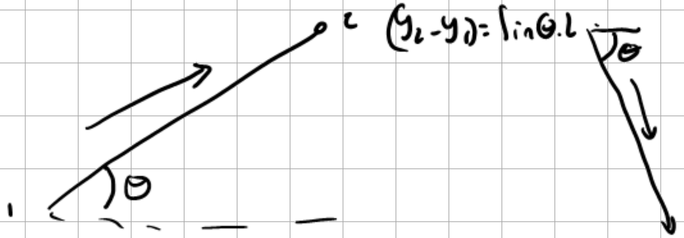
$\dot m_{CO_2} = J(\underbrace{P_{wf} - P_R}_{bar})$

t/d/bar

for tubing 60bara

$p_{wf} = p_{wh} + \Delta p_H - \Delta p_f(\dot m_{CO_2})$

$P_{wf} = 60 + 125 \text{ bar} -$



a) a referere for $\dot m_{CO_2}$, Equinor wants their CO_2 wells to inject 1.5 Mt/y

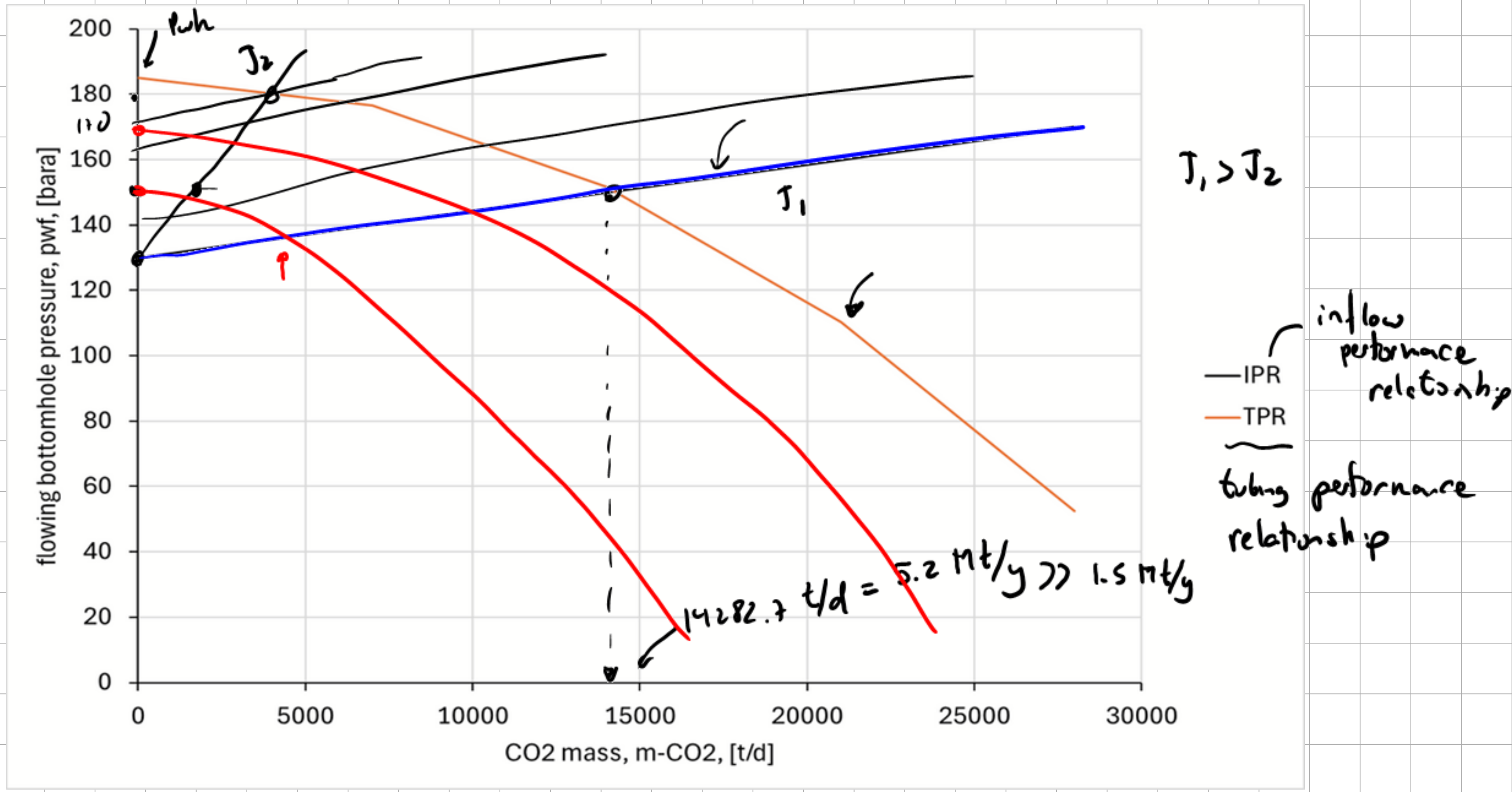
$1.5 \text{ Mt/y} \quad 1.5E6 \frac{t}{1y} \cdot \frac{1y}{365} = 4109.6 \text{ t/d}$

in CO_2 i cannot assume $q_{local} = q_{standard\ conditions}$

$\rho_{local} = 850 \text{ kg/m}^3$ 1.8 kg/m^3

pressure drop in pipe must use ρ_{local}!

$$q_{CO_2} = \frac{\dot m_{CO_2}}{\rho_{CO_2(local)}}$$

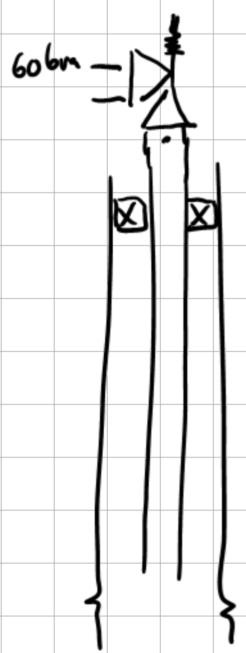


$$\overset{x}{\dot m_{CO_2}} = J\,(\overset{y}{P_{wf}} - P_R)$$

$$y = mx + c$$

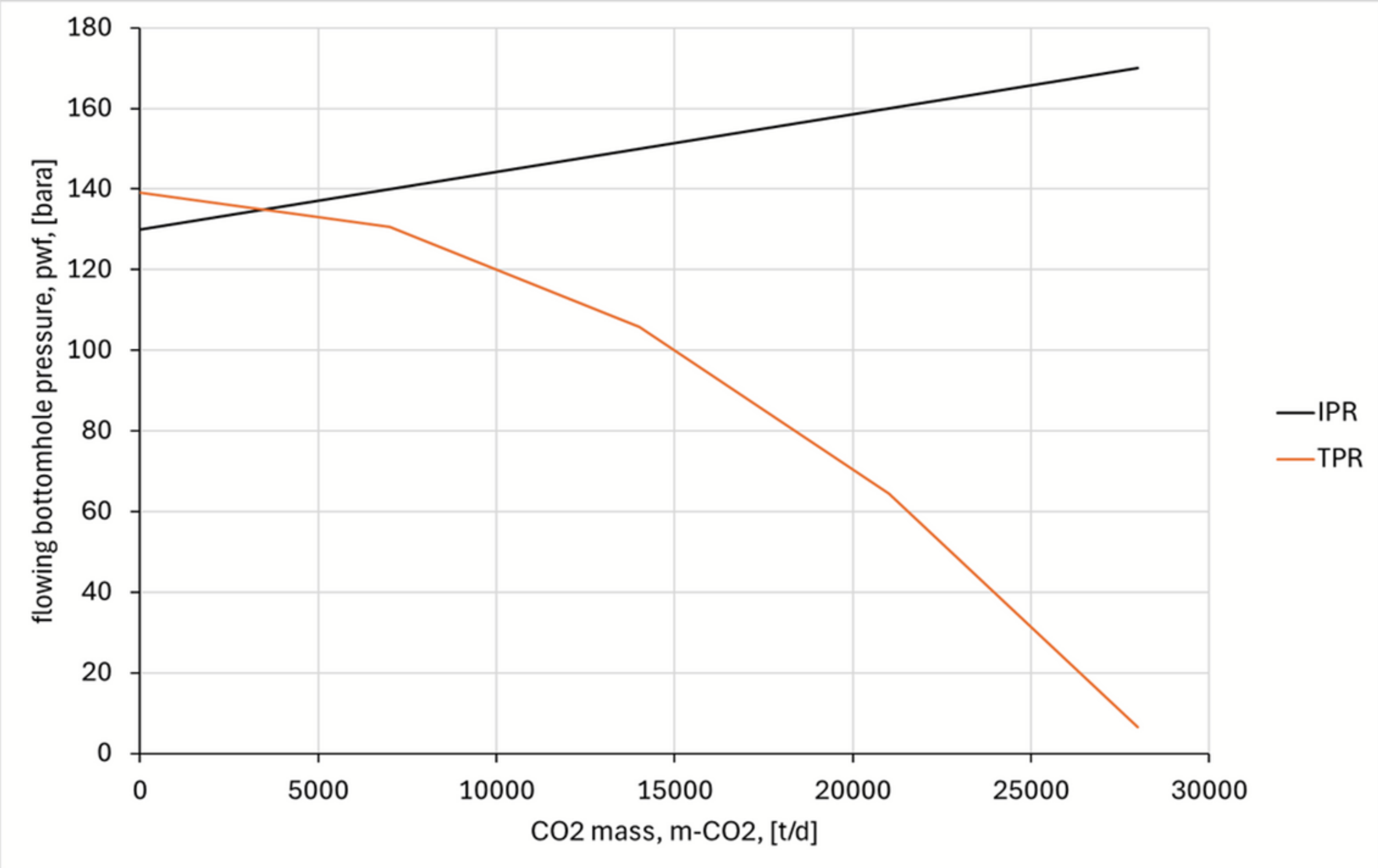
$$\overset{y}{p_{wf}} = \overset{c}{P_R} + \frac{\overset{x}{\dot m_{CO_2}}}{J}$$

$$\frac{1}{J} = m$$



I am injecting too much (5.2 Mt/y)!! how to reduce my injection rate. Options to change IPR and TPR curves:
-Increase reservoir pressure: but pR is usually related with storage capacity, I don't want to fill up the reservoir, and it is also impractical
-reduce the J* (damaging the formation). Difficult and impractical.
-change tubing diameter. Works! but at a later time when pR increases, I will need to install a bigger tubing, which is costly
-Change pwh. Works! but it is not ideal in terms of energy usage (I compress CO2 onshore and then drop the pressure in the choke). But it is the most practical choice. When pR increases, the choke will open gradually.

*reducing the J makes the curve steeper! because the slope m = 1/J



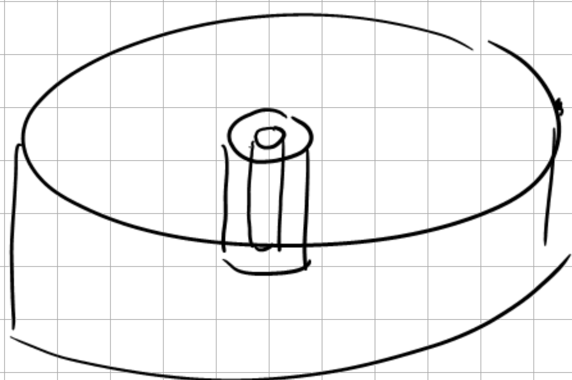
at initial time, pR= 130 bara:

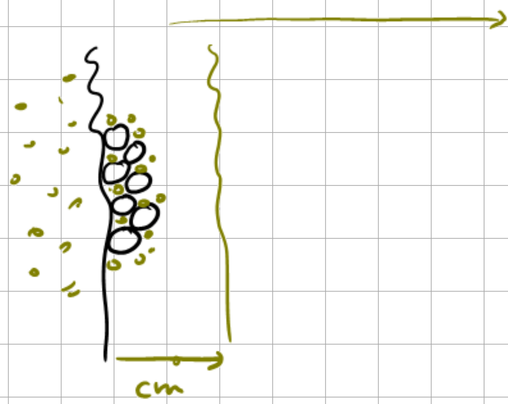
puc [bara]	60			
Dp choke [bar]	46.3			
pwh [bara]	13.7			
IPR (req)			TPR (avail)	
pwf - req [bara]	m-Co2 [t/d]	q-Co2 [m3/d]	pwf-avail [bara]	
130	0	0	139	
140	7000	8235	130	
150	14000	16471	105	
160	21000	24706	64	
170	28000	32941	6	diff
135.87	4109	4834	136	0

AT A LATER TIME, pR=150 bara:

puc [bara]	60			
Dp choke [bar]	26.3			
pwh [bara]	33.7			
IPR (req)			TPR (avail)	
pwf - req [bara]	m-Co2 [t/d]	q-Co2 [m3/d]	pwf-avail [bara]	
150	0	0	159	
160	7000	8235	150	
170	14000	16471	125	
180	21000	24706	84	
190	28000	32941	26	diff
155.87	4109	4834	156	0

Recap about YT lectures on IPR for undersaturated oil well





$$J = \frac{k h}{18.69 (\mu_o B_o)_{av} \left[\ln \frac{r_e}{r_w} - 0.75 + S \right]}$$

$\underbrace{\quad}_{8}$

- $S = 0 \longrightarrow J_1$
- $S = +8 \longrightarrow J_2 \simeq \frac{J_1}{2}$ (a)
- $S = -8 \longrightarrow J_3 = J_1$ (b)

$8 - 0.75 - 8$

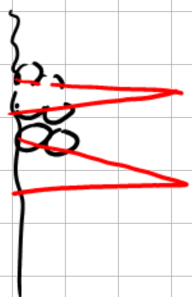
-0.75

$$\frac{J_1}{J_2} = \frac{\left(\ln \frac{r_e}{r_w} - 0.75 + S_2\right)}{\left(\ln \frac{r_e}{r_w} - 0.75 + S_1\right)} =$$

in case (a) $\frac{J_1}{J_2} = \frac{(8 - 0.75 + 8)}{(8 - 0.75)} \simeq 2 \Rightarrow J_2 \simeq \frac{J_1}{2}$

in case (b) $S = -8$ $\frac{J_1}{J_2} = \frac{(8 - 0.75 - 8)}{(8 - 0.75)} \simeq \frac{-0.75}{7.25} \simeq -0.1$

$J_2 = -10 J_1$



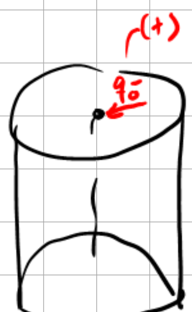
$\frac{8 - 0.75 - 5}{7.25} = \frac{2.25}{7.25} = 0.3 \sim J_2 = 3.\ J_1$

- IPR for vertical water/CO_2 injector

undersaturated oil case

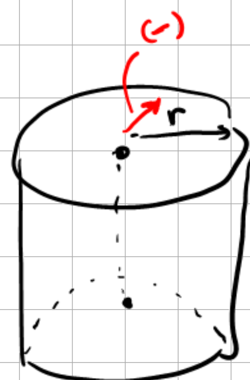
r_w

$$\int_{r_w}^{@r_e} \frac{dr}{r} = \frac{2\pi k h}{q_{\bar{o}}} \int_{p_{wf}}^{p_e} \frac{dp}{B_o \mu_o}$$



water

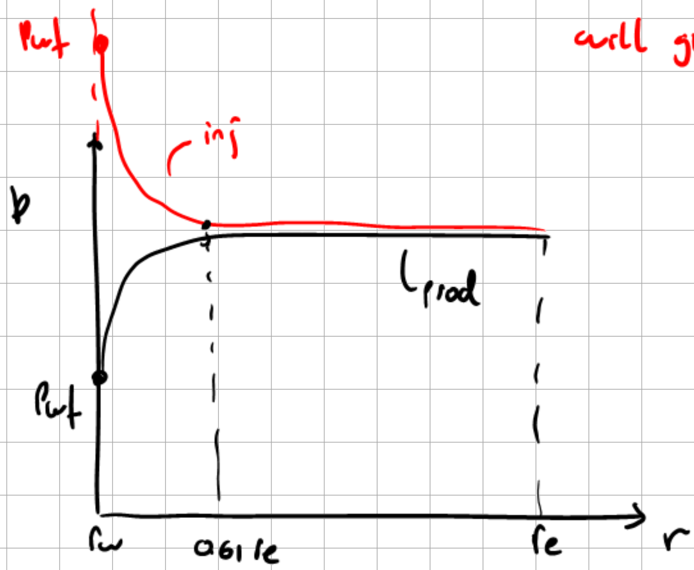
$$\int_{r_w}^{@r_e} \frac{dr}{r} = \frac{2\pi k h}{-q_{\bar{w}}} \int_{p_{wf}}^{p_e} \frac{dp}{B_w \mu_w}$$



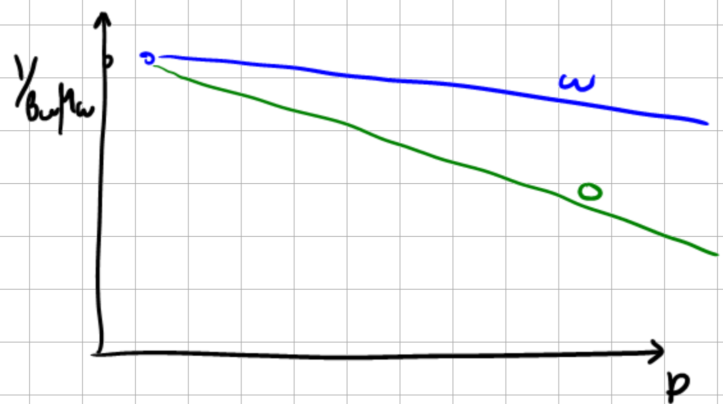
include this sign for $q_{\bar{w}}$ to be positive, otherwise the equation will give a negative number

for oil

@ $p_e = 0.61\, r_e$



for water the pressure distribution is mirrored, and pR is in the same place!



1/Bw uw behaves similar for water

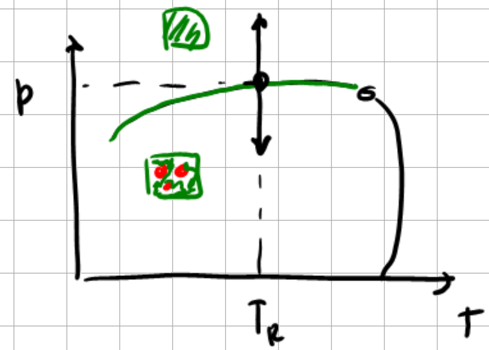
$$q_o = \frac{kh}{(\mu_o B_o)\big|_{@p_{av}}} \frac{(p_e - p_{wf})}{\left(\ln \frac{r_e}{r_w} - 0.5\right)} \frac{1}{18.68}$$

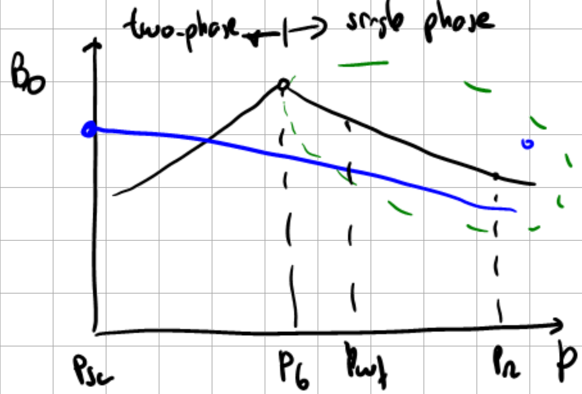
$$q_w = \frac{kh}{(\mu_w B_w)\big|_{@p_{av}}} \frac{\overbrace{(p_{wf} - p_e)}^{\text{due to the negative sign}}}{\left(\ln \frac{r_e}{r_w} - 0.5\right)} \frac{1}{18.68}$$

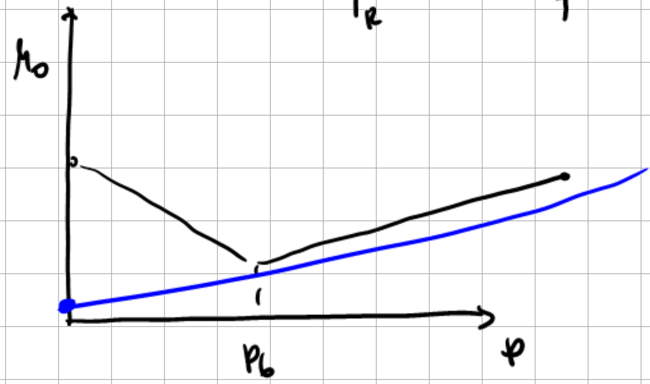
20240909

OUTLINE:

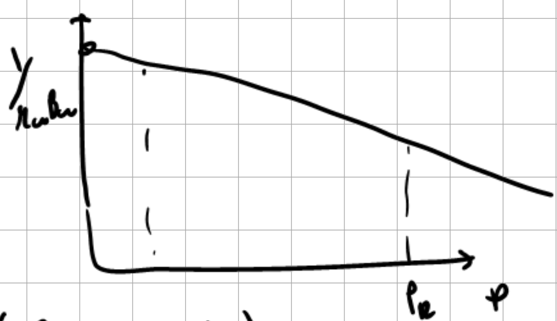
- Water Bw considering the effect of dissolved gas
- Skin for water/CO2 injector
- CO2 injector, IPR in terms of mass flow rate
- Issues with water/CO2 injectors - Example
- Impact of topside facilities on Rp (GOR). gas IPR from undersaturated oil IPR. Calculating Bo from EOS compositional model. Black oil correlations. Tuning BO correlations with EOS model results







$$\int_{P_{wf}}^{P_R} \frac{1}{\mu_w B_w} dP$$



Phase behavior book (SPE Monograph) { Whitson → Brulée

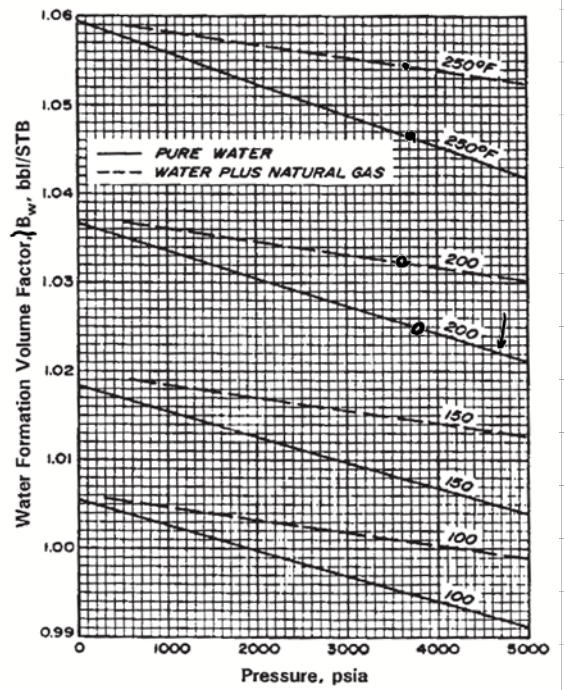
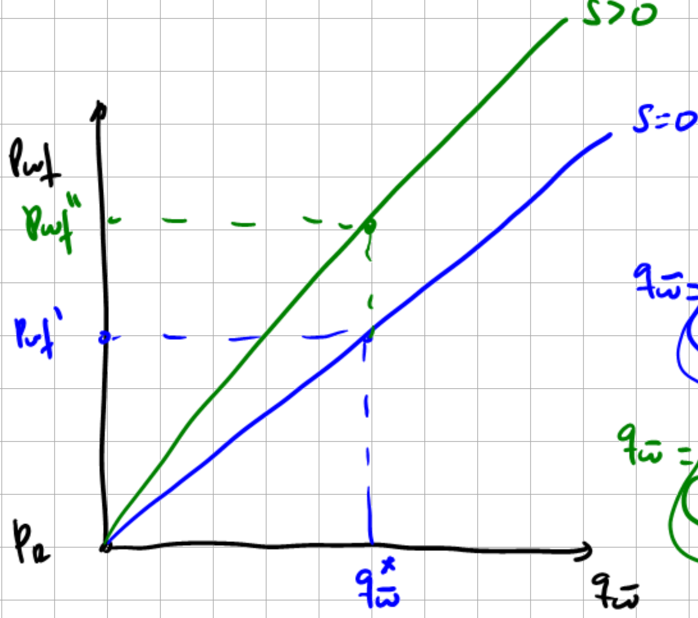


Fig. 9.7—FVF of pure water with and without natural gas (adapted from Ref. 13).

In some cases, water can be saturated with natural gas.

Injector skin



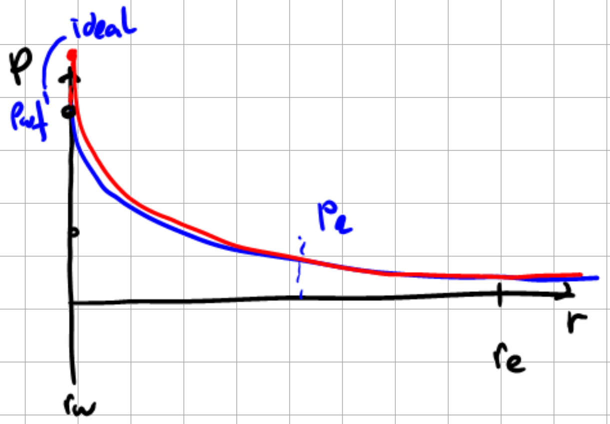
J_{noskin}

$$q_{\bar{w}} = \frac{1}{(\mu_w B_w)_{@p_{av}}} \frac{kh}{18.68} \frac{(p_{wf} - p_R)}{\left(\ln \frac{r_e}{r_w} - 0.5\right)}$$

$$q_{\bar{w}} = \frac{1}{(\mu_w B_w)_{@p_{av}}} \frac{kh}{18.68} \frac{(p_{wf} - p_R)}{\left(\ln \frac{r_e}{r_w} - 0.5 + S\right)}$$

J_{skin}

$J_{skin} < J_{noskin}$



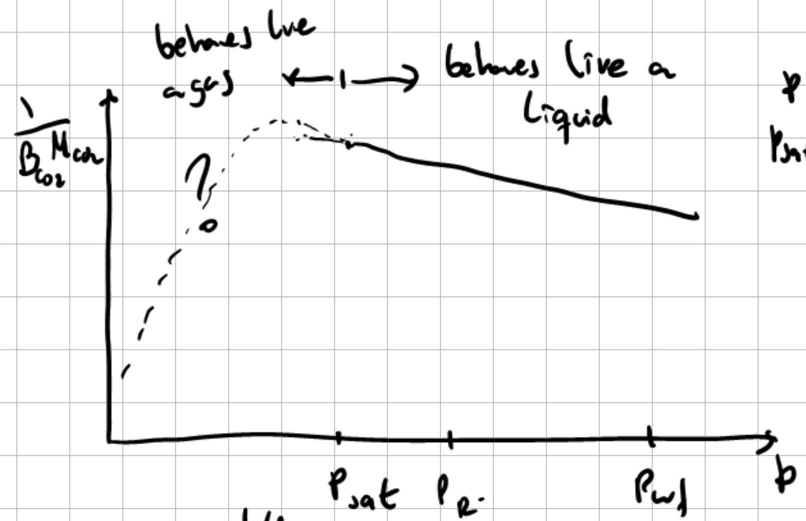
$$p_{wf}' - p_R = q_w \frac{(\mu_w B_w)_{@p_{av}} \cdot \left(\ln\left(\frac{r_e}{r_w}\right) - 0.5\right)}{kh}$$

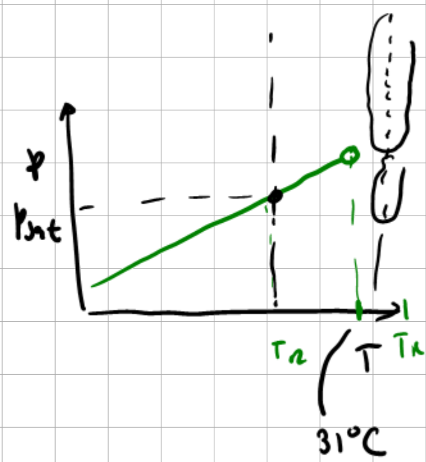
$$p_{wf} - p_{wf}' = S \cdot q_w \left(\frac{(\mu_w B_w)_{@p_{av}}}{k \cdot h}\right)$$

$$\frac{(p_{wf} - p_R) \quad kh}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.5 + S\right) (\mu_w B_w)_{@p_{av}}} = q_{\bar{w}}$$

what about CO_2?

$$\int_{p_{wf}}^{p_R} \frac{1}{B_{CO_2} \mu_{CO_2}} dp$$





in CO_2 injectors we use $\dot{m}_{CO_2}$ (t/d) instead of $q_{\bar{CO_2}}$ (Sm^3/d)

$$q_{\overline{CO_2}} = \frac{k.h}{18.68 \cdot \left(B_{CO_2} \cdot \mu_{CO_2}\right)_{@\,\bar{p}_{av}}} \cdot \frac{\left(p_{wf} - p_R\right)}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.5 + S\right)}$$

$$\dot{m}_{CO_2} = q_{\overline{CO_2}} \cdot \rho_{\overline{CO_2}} \frac{1}{1000}$$

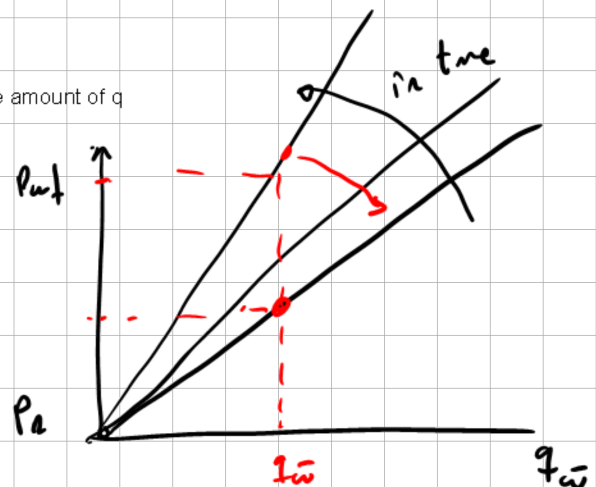
$\left(1.8\ kg/m^3 \right.$

$$\dot{m}_{CO_2} = \frac{\rho_{\overline{CO_2}}}{1000} \cdot \frac{k.h}{18.68 \left(B_{CO_2}\, \mu_{CO_2}\right)_{@\,\bar{p}_{av}}} \cdot \frac{\left(p_{wf} - p_R\right)}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.5 + S\right)}$$

$$1.8E{-3} \cdot \frac{1}{18.68} = \frac{1}{10378}$$

Issues with injectors:

-Decline of J with time: I need a higher pwf to inject the same amount of q

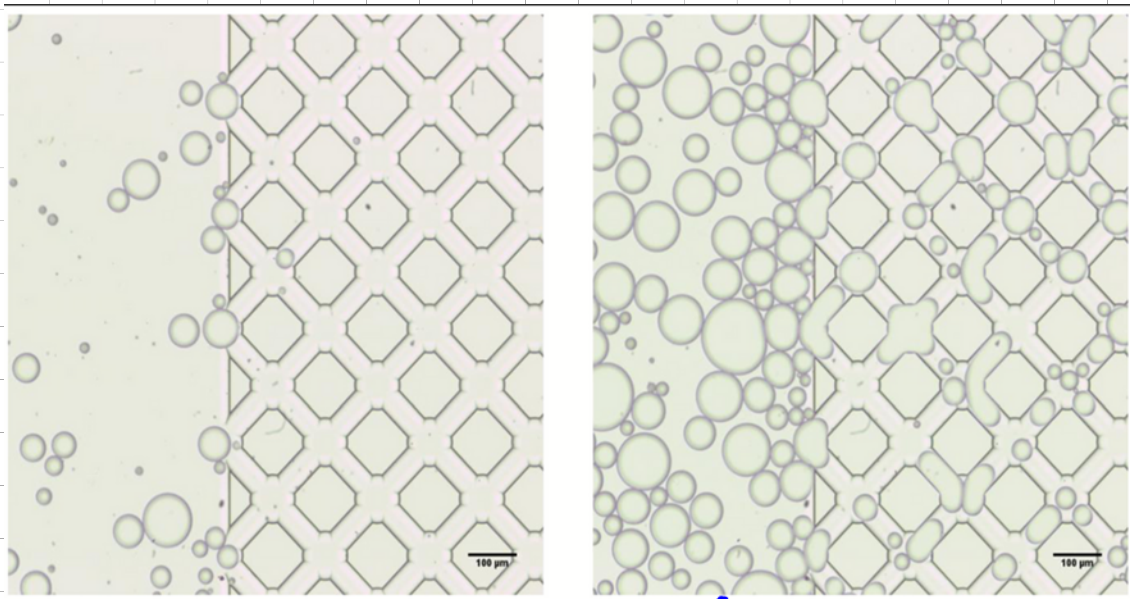


(Peregrino

Injector near formation plugging, due to :

-residual oil concentration (30 mg/l)

-Solids (fines)

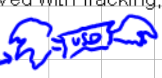


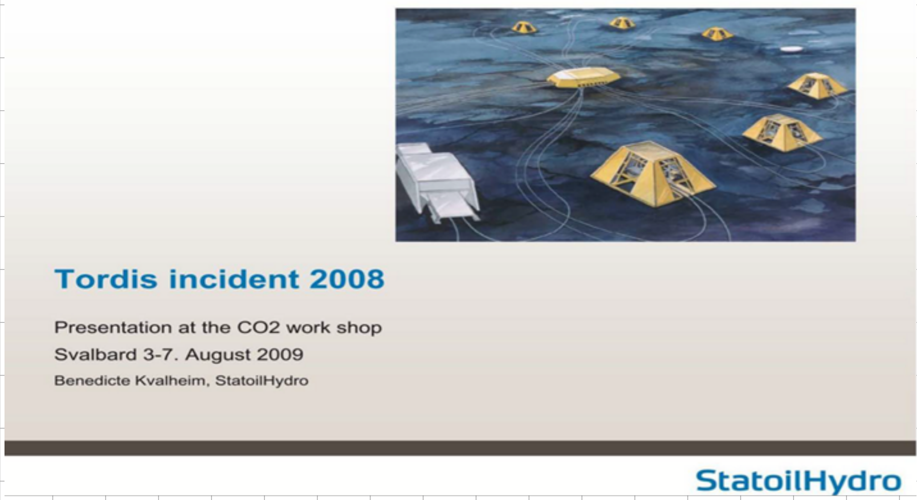
Alternatives:

-Bring back J to original value --> can be achieved with fracking, stimulation

-Drill more injectors

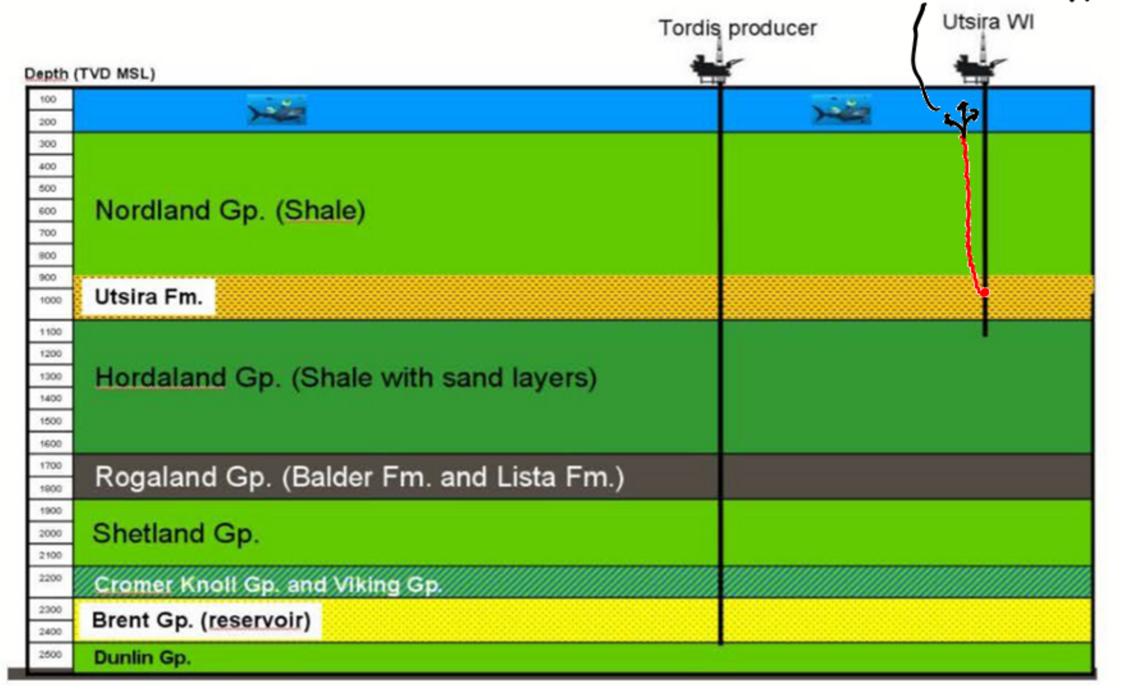
-Choke back production





source

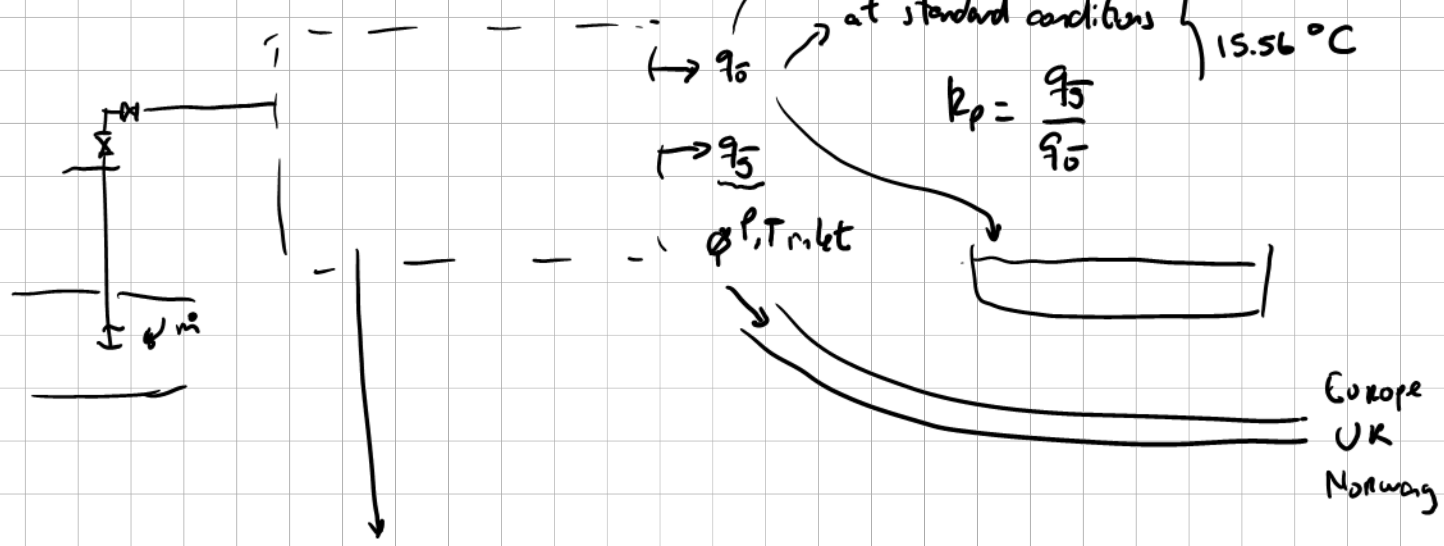
100 - 500 ppm O.W <30 ppm



what went wrong?

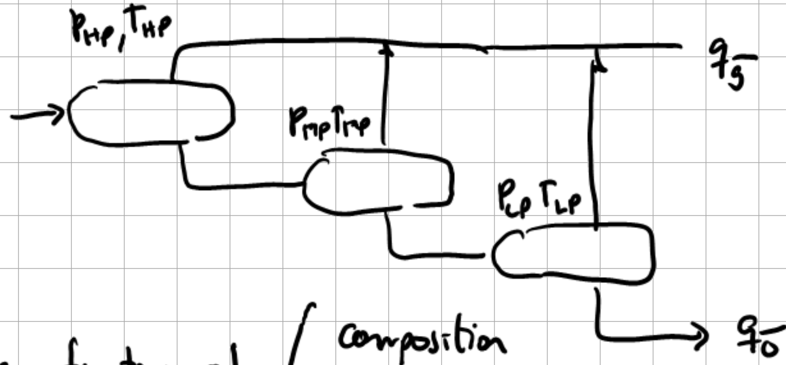
- During the injection phase (December-07 – March -08)
  - Operated the injection at rates and pressures higher than stated in the operation guides (123 bar, well integrity of the producers)
  - Pressure build-up profiles? – vertically vs. horizontally

What is GOR? (Rp)



$$R_p = \frac{q_{\bar{g}}}{q_{\bar{o}}}$$

multi-stage separation process dictates what GOR does the field have



more stages, more gradually GOR ↓

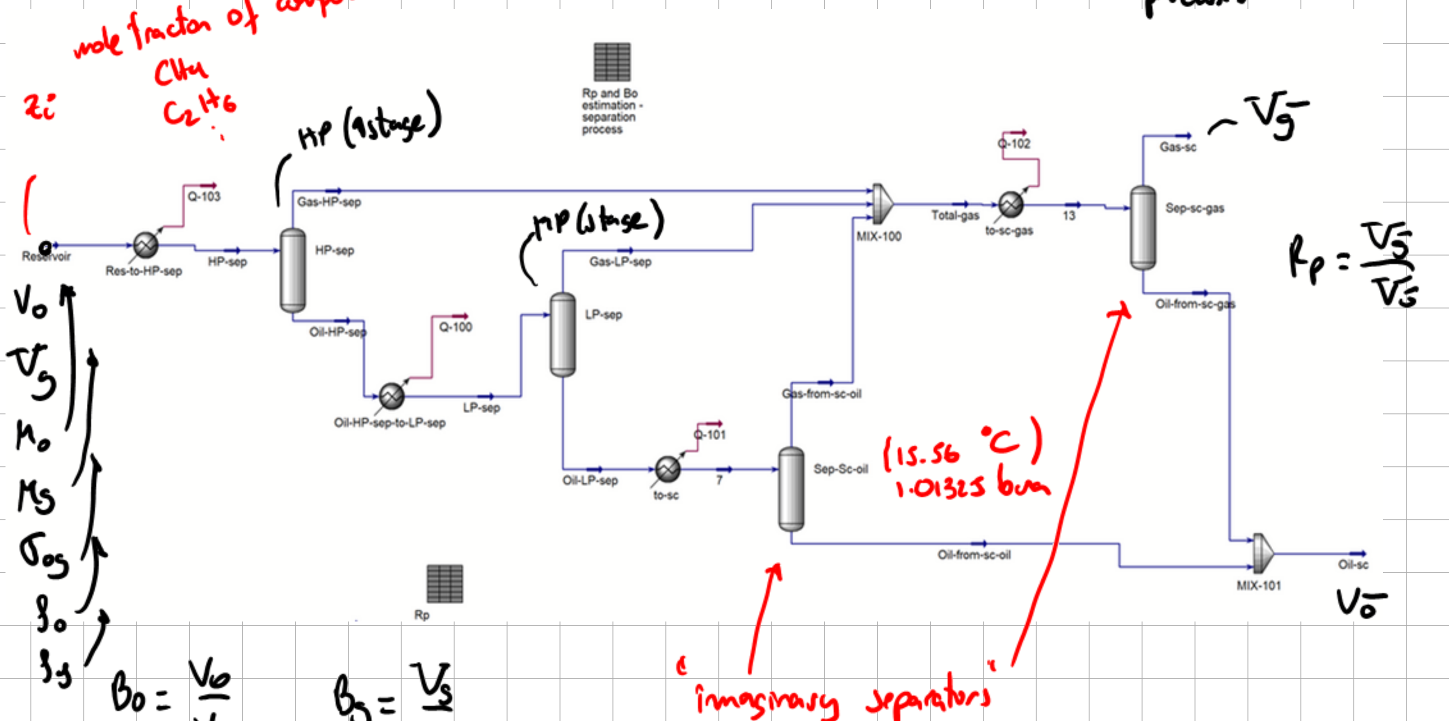
less stages, more directly GOR ↑

GOR is a function of { composition; surface process → multi-stage separation process }

Peng Robinson (PR)
Redlich Kwong Soave (SRK)

How to compute GOR using Hysys, Equation of State calculations (Compositional model):

Hysys / Simulator used to simulate topside processes

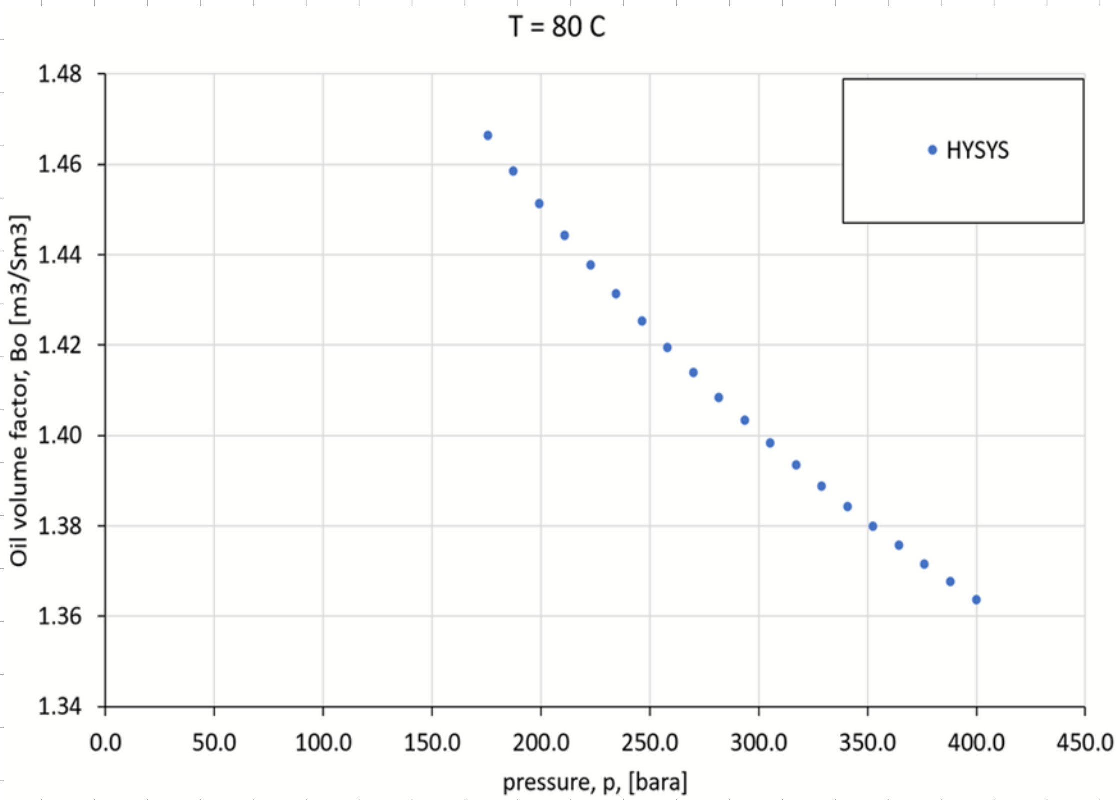


$$B_o = \frac{V_o}{V_{\bar{o}}} \qquad B_g = \frac{V_g}{V_{\bar{g}}}$$

To calculate GOR and BO properties, we can use a Thermodynamic/process simulator that uses EoS
Another approach is to use BO correlations, however, BO correlations are usually developed for a given fluid or field or region, so if our field is not on that region or not similar, it will most likely not work!!!! But one approach that is used, is to generate BO properties using EoS model, and then tune the correlation to the results of the Eo S model

Class exercise:

I have generated Bo versus p at 80 C with Hysys. I want to tune BO correlations to match this data



I have to use three correlations, one for pb, one for Bob, one for Co. I will include a tuning factor for each (multiplier) to match the data of Hysys.
To find Bo versus p, I will use the equation:

Bo_current=Bo_prev*exp(Co_current*(p_prev-p_current)

Taken from Whitson and Brulee Phase behavior monograph

Bubble point pressure

Standing[3,17,40] developed the first accurate bubblepoint cor-elation, which was based on California crude oils.

$$p_b = 18.2(A - 1.4), \quad \text{(3.78)}$$

vhere $A = (R_s/\gamma_g)^{0.83}\, 10^{(0.00091T - 0.0125\gamma_{API})}$, with R_s in scf/STB, T n °F, and p_b in psia.

Saturated oil volume factor

For example, Standing's[3,17,40] correlation for California crude oils is

$$B_{ob} = 0.9759 + (12 \times 10^{-5})A^{1.2}, \quad \text{(3.111)}$$

vhere $A = R_s(\gamma_g/\gamma_o)^{0.5} + 1.25T$.

Undersaturated oil compressibility

Vazquez and Beggs[57] propose the following correlation for nstantaneous undersaturated-oil compressibility.

$$c_o = A/p, \quad \text{(3.107)}$$

vhere
$A = 10^{-5}(5R_{sb} + 17.2T - 1,180\gamma_{gc} + 12.61\gamma_{API} - 1,433)$, with c_o in psi^{-1}, R_{sb} in scf/STB, T in °F, and p in psia.

Vazquez and Beggs correct for the effect of separator conditions using a modified gas specific gravity, γ_{gc}, which is correlated with first-stage-separator pressure and temperature, and stock-tank-oil gravity.

$$\gamma_{gc} = \gamma_g\left[1 + (0.5912 \times 10^{-4})\gamma_{API}T_{sp}\log\left(\frac{p_{sp}}{114.7}\right)\right]. \quad \text{(3.86)}$$

with T_{sp} in °F and p_{sp} in psia.

Undersaturated oil volume factor

$$B_o = B_{ob}\exp[c_o(p_b - p)]$$

Since Co is a function of pressure, this equation is not valid. Workarounds are: or
* to use average pressure to calculate Co
*To use the equation between to consecutive pressures

Standing[3,17,40] developed the first accurate bubblepoint correlation, which was based on California crude oils.

$$p_b = 18.2(A - 1.4) \cdot FO_1 \quad (3.78)$$

where $A = (R_s/\gamma_g)^{0.83}\, 10^{(0.00091T - 0.0125\gamma_{API})}$, with R_s in scf/STB, T in °F, and p_b in psia.

HYSYS DATA (2 stage)

HP sep pressure	[bara]	50
HP sep Temp	[C]	70
GOR	[Sm3/Sm3]	146.5
deno_sc	[kg/m3]	784.6
deng_sc	[kg/m3]	0.840
pb@80 C	[bara]	175.6
Bob@80C	[m3/Sm3]	1.466
gamma o	[-]	0.7846
gamma g	[-]	0.6866
T	[C]	80

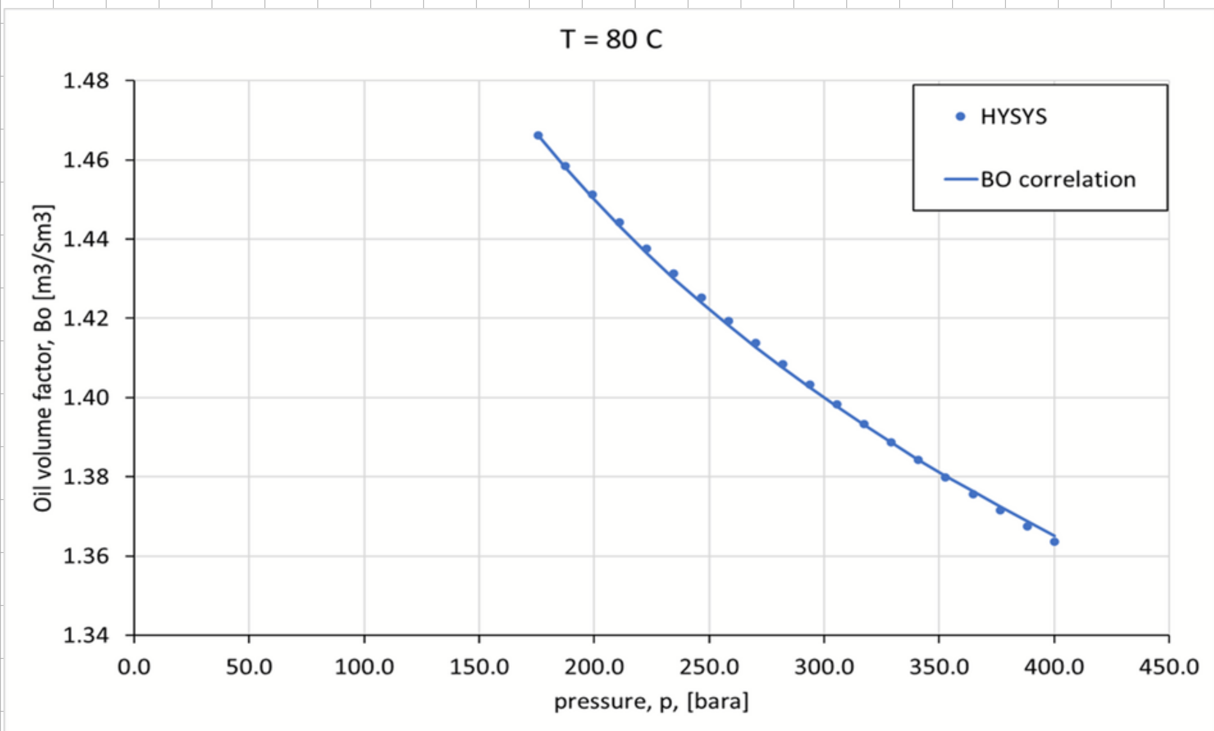
API	[-]	48.9
pb	[bara]	175.60
Bob	[m3/Sm3]	1.4663

Tuning factors	
FO1	1.099
FO2	1.014
FO3 (Co)	1.612

	pb[bara]	Bob[m3/Sm3]
Errors	0.00	0.0000

SUM= 1.50E-05

p [bara]	Co [1/bar]	Bo [m3/Sm3]	Bo (Hysys) [m3/Sm3]	error
175.6	5.06E-04	1.466	1.466	
187.4	4.74E-04	1.458	1.458	1.27E-07
199.2	4.46E-04	1.450	1.451	5.13E-07
211.0	4.21E-04	1.443	1.444	9.23E-07
222.8	3.99E-04	1.437	1.438	1.23E-06
234.7	3.79E-04	1.430	1.431	1.39E-06
246.5	3.60E-04	1.424	1.425	1.41E-06
258.3	3.44E-04	1.418	1.419	1.30E-06
270.1	3.29E-04	1.413	1.414	1.09E-06
281.9	3.15E-04	1.408	1.408	8.37E-07
293.7	3.02E-04	1.402	1.403	5.67E-07
305.5	2.91E-04	1.398	1.398	3.18E-07
317.3	2.80E-04	1.393	1.393	1.25E-07
329.1	2.70E-04	1.389	1.389	1.54E-08
340.9	2.61E-04	1.384	1.384	1.45E-08
352.8	2.52E-04	1.380	1.380	1.43E-07
364.6	2.44E-04	1.376	1.376	4.18E-07
376.4	2.36E-04	1.372	1.372	8.53E-07
388.2	2.29E-04	1.369	1.368	1.46E-06
400.0	2.22E-04	1.365	1.364	2.25E-06



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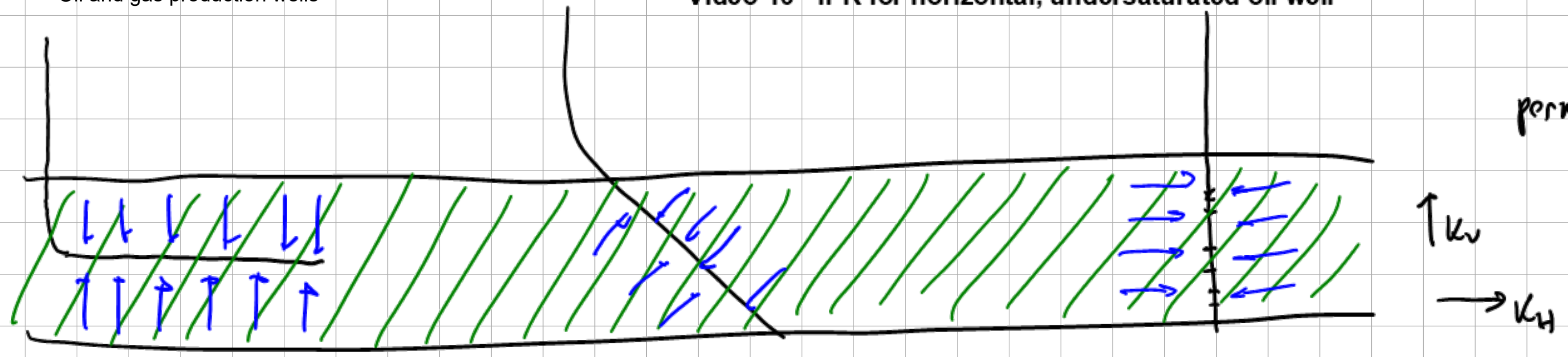
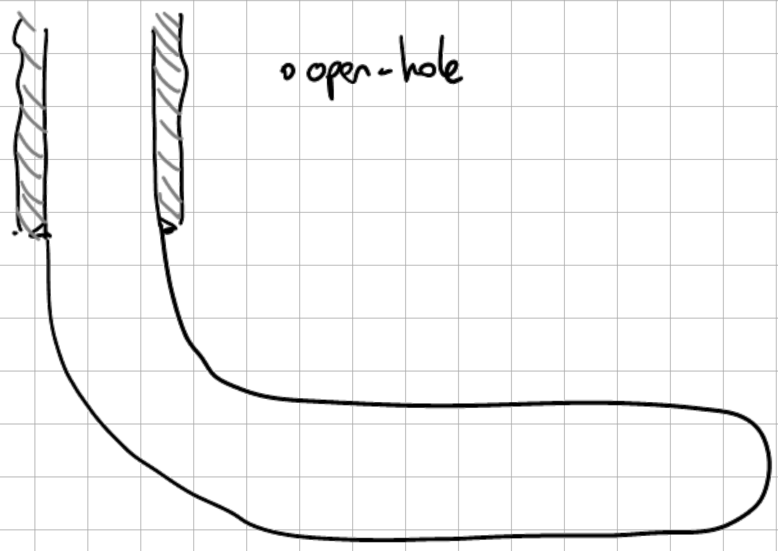
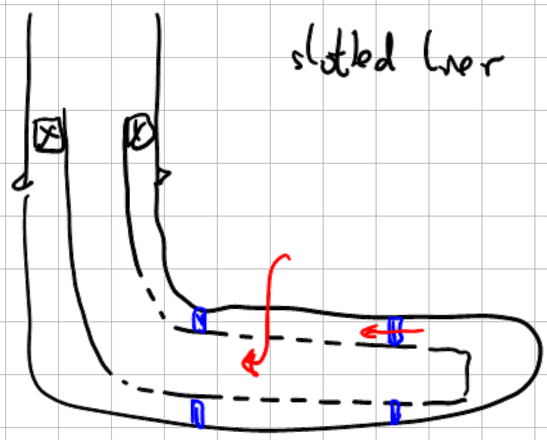


permeability anisotropy

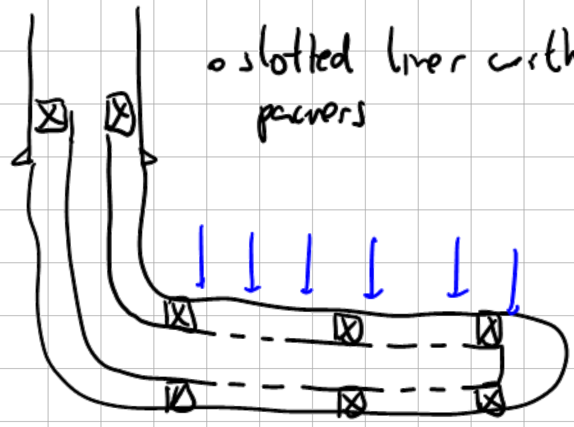
$\frac{k_H}{k_v} >> 1$ e.g. 10



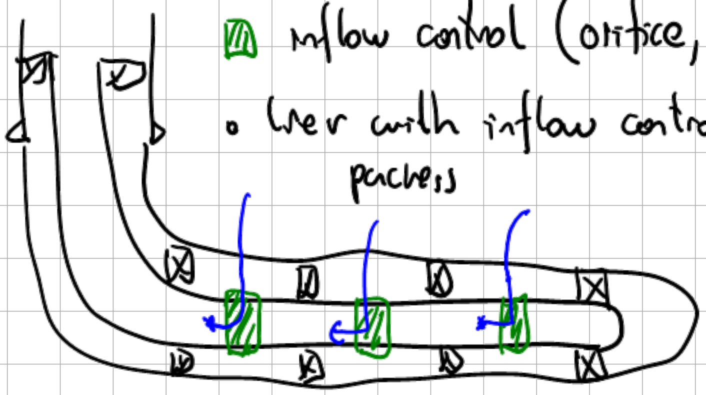
o open-hole



slotted liner



o slotted liner with packers



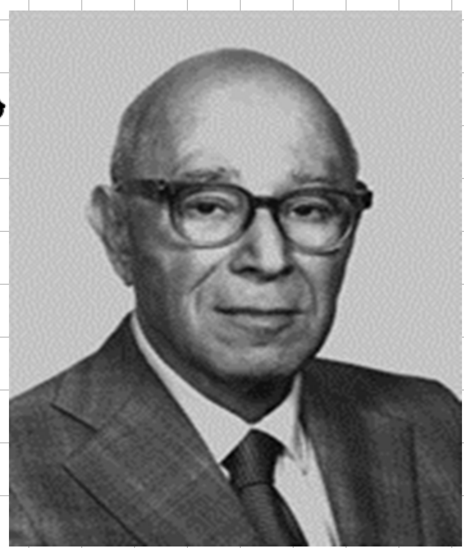
inflow control (orifice, control valve)

o liner with inflow control and packers

cemented and perforated liner

Morris

1937 - Muskat (1937) (Gulf oil)
1958 - Merkulov
1964 - Borisov
1974 - Gringarten (Imperial College), Ramey (Stanford)
1983 - Giger (IFP)
1988 - Joshi
1989 - Babuh, Odeh (Mobil)
1991 - Renard, Dupuy (IFP)
1994 - Buttler
Harald Asheim (NTNU)



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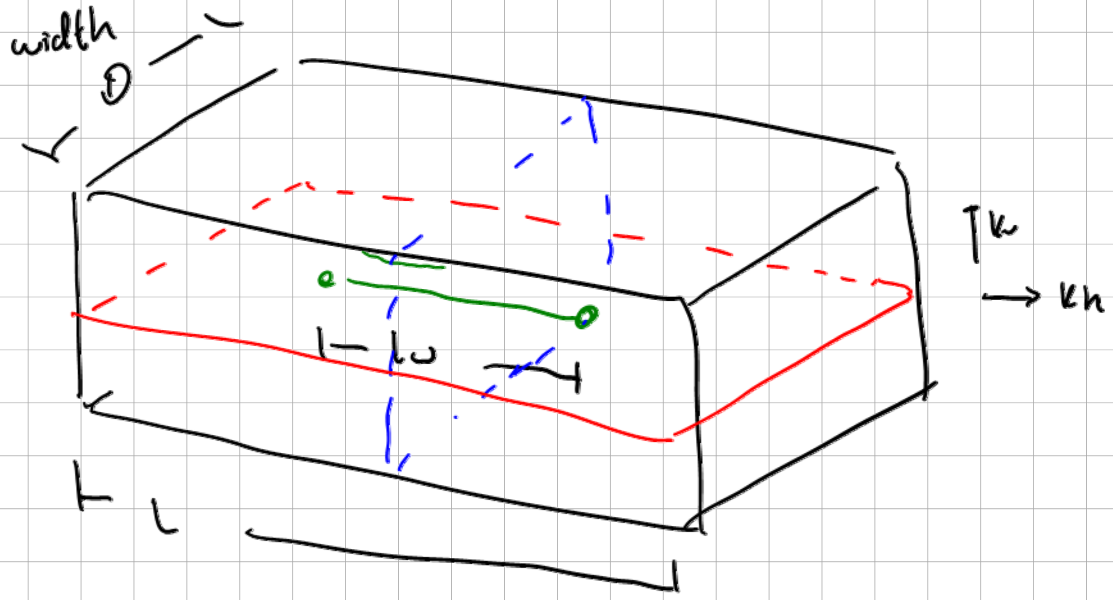
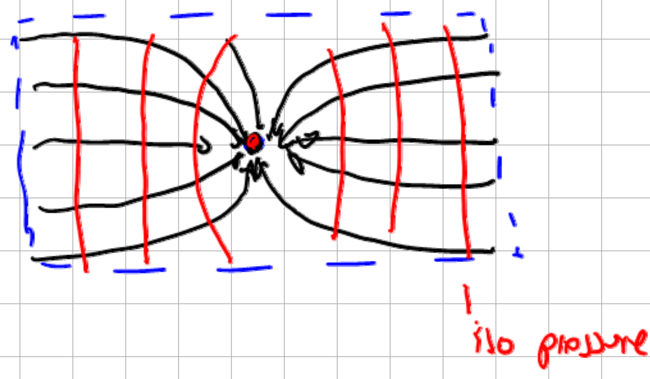
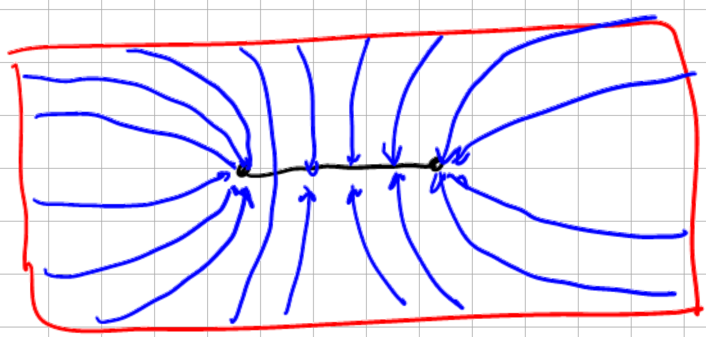
http://www.ipt.ntnu.no/~asheim/prodbr.html





$P = f(x)$

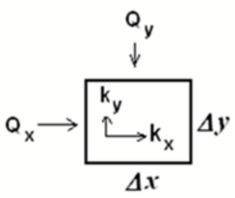


$\sim P = f(x)$

Superposition

Strategy to deal with permeability anisotropy

- Actual reservoir element



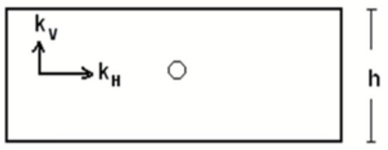
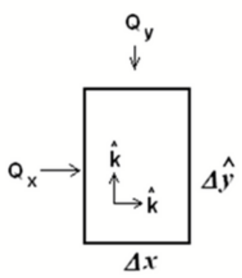


Figure 3.10 Anisotropic reservoir

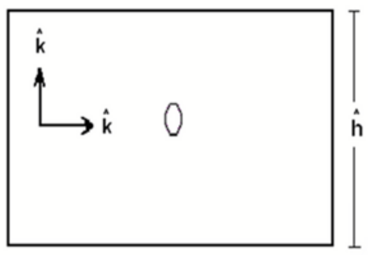
$$k_H \frac{\partial^2 p}{\partial x^2} + k_V \frac{\partial^2 p}{\partial y^2} = \varphi \mu c \frac{\partial p}{\partial t}$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial \left(\sqrt{k_H / k_V}\, y\right)^2} = \frac{\varphi \mu c}{k_H} \frac{\partial p}{\partial t}$$

- Homogenous equivalent:



Permeability scaling: $|\hat{k} = \sqrt{k_x k_y}$



$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial \hat{y}^2} = 0$$

$$\hat{y} = \left(k_H / k_V\right)^{0.5} y$$

Height scaling: $\hat{h} = \left(k_H / k_V\right)^{0.5} h$

geometry related

$$\boxed{q_o = \frac{k_H \cdot h}{6.22 \cdot (\mu_o \cdot B_o)_{@p_{av}} \cdot \left[\frac{\pi \cdot D \cdot \hat{f}_a}{2 \cdot \hat{L}_w} + \frac{3 \cdot h \cdot \beta}{\bar{L}_w} \cdot \left(\ln\left(\frac{h \cdot \beta}{2 \cdot \pi \cdot \hat{r}_w}\right) + \hat{s}_b\right)\right]} [p_R - p_{wf}]}$$

function of PVT, fluid properties

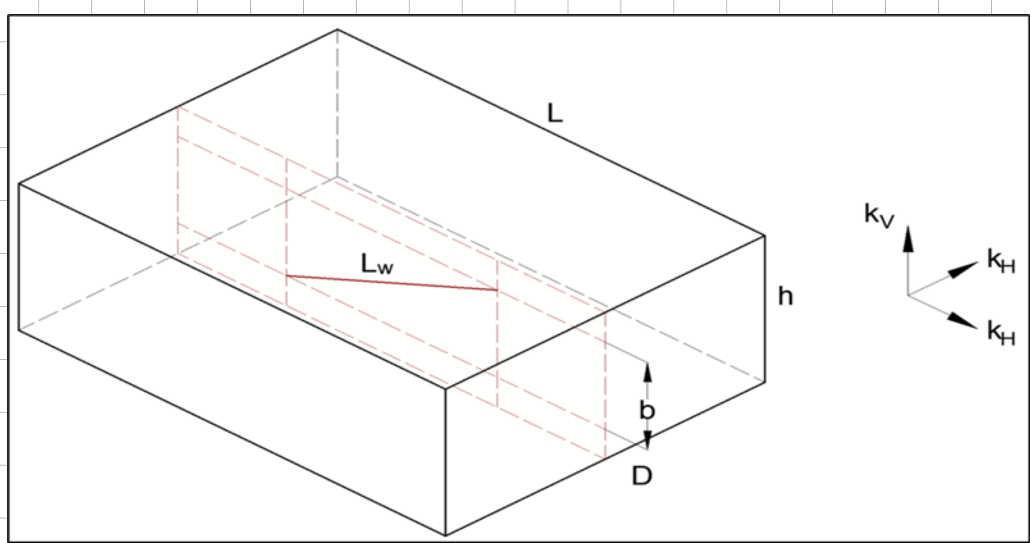
Harald Asheim's IPR equation

- undersaturated oil
- horizontal well
- pss
- scaling due to anisotropy

$$\boxed{\beta = \sqrt{\frac{k_H}{k_V}}}$$

$$\boxed{\hat{L}_w = L_w \cdot \sqrt{1 - \left(\frac{b}{L_w}\right)^2 \cdot (\beta^2 - 1)}}$$

$$\boxed{\bar{L}_w = L_w \cdot \sqrt{1 - \left(\frac{b}{L_w}\right)^2}}$$



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$$\boxed{\hat{r}_w = r_w \cdot \frac{1 + \sqrt{1 + (\beta^2 - 1) \cdot \left(\frac{\bar{L}_w}{L_w}\right)^2}}{2}}$$

well completed partially in the interval

$$\boxed{\hat{f}_a = \frac{\bar{L}_w}{L} \cdot \left[1 + \left(0.53 \cdot \left(\frac{L}{D}\right)^2 + 1.15 \cdot \left(\frac{L}{D}\right) + 0.164\right) \cdot \left(\frac{1 - \left(\frac{\bar{L}_w}{L}\right)}{0.45 + \left(\frac{\bar{L}_w}{L}\right)}\right)\right]}$$

$$\boxed{\hat{s}_b = \begin{Bmatrix} 0.69 \ if \ O(b) = O(h) \\ 0 \ if \ b = 0 \end{Bmatrix}}$$ ~ skin factor, "shielding" effect from top and bottom walls

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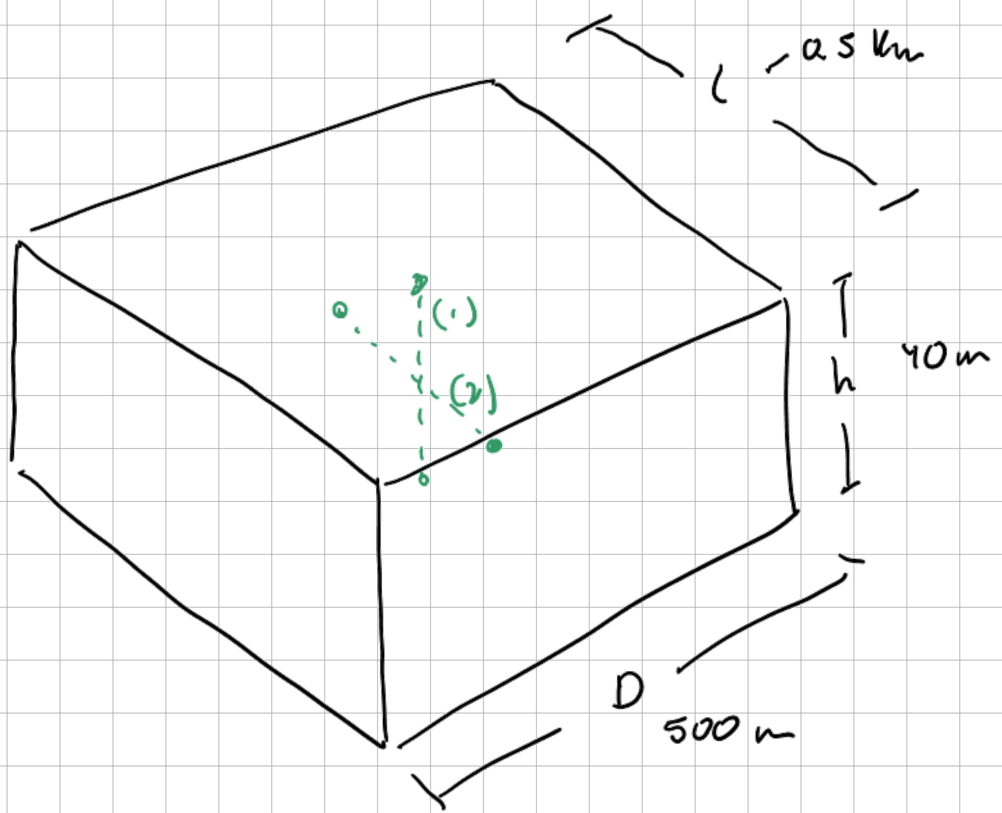

vertical well

$$\pi r_e^2 = A = 500 \cdot 500$$

$$r_e = \sqrt{\frac{500 \cdot 500}{\pi}}$$

Reservoir top area	[m2]	2.50E+05
Reservoir pressure, p_R	[bara]	300
Flowing bottom-hole pressure, p_{wf}	[bara]	200
p_{av}	[bara]	250
Oil viscosity, μ_o at average pressure	[cp]	1.877
Oil volume factor, B_o, at average pressure	[m3/Sm3]	1.144
Wellbore radius, r_w	[m]	0.15
Vertical well located in the center and perforated throughout		
External radius, r_e	[m]	282.1
Skin, s	[-]	0
Shape factor, s_A	[-]	0.012
Productivity Index, J	[Sm3/d/bar]	14.7
Horizontal well		
Wellbore length	[m]	500
Elevation difference between toe and heel, b (sign doesn't matter)	[m]	0
Productivity Index, J	[Sm3/d/bar]	63.2

```
Function J_vertical(k, h, Uo, Bo, re, rw, s, sa)
    'Productivity index for vertical well, undersaturated oil, pss, in Sm3/d/bar
    'k in md
    'h in m
    'Uo in cp
    'Bo in m^3/Sm^3
    'fa is shape factor
    J_vertical = (k * h) / (18.68 * Uo * Bo * (Log(re / rw) - 0.75 + s + sa))
    'Natural Log in Visual Basic is Log, not LN'
End Function
```

```
Function J_horizontal(L, D, h, b, Lw, kh, kv, Bo, viso, rw)
    'Productivity index for horizontal well, undersaturated oil, pss, in Sm3/d/bar
    'L Reservoir length along well direction [m]
    'D reservoir width [m]
    'h reservoir thickness [m]
    'Lw well length [m]
    'kh horizontal permeability [md]
    'kv vertical permeability [md]
    'Bo oil formation volume factor [m^3/Sm^3]
    'viso oil viscosity [cp]
    'rw well radius [m]
    'b, height difference betweeen heel and toe [m]
    Pi = Atn(1) * 4
    b = Abs(b)
    If b / h > 0.1 Then
        s_b = 0.69
    Else
        s_b = 0
    End If
    beta = (kh / kv) ^ 0.5
    Lw_hat = Lw * (1 + ((b / Lw) ^ 2) * (beta ^ 2 - 1)) ^ 0.5
    Lw_bar = Lw * (1 - (b / Lw) ^ 2) ^ 0.5
    rw_hat = 0.5 * rw * (1 + (1 + (beta ^ 2 - 1) * ((Lw_bar / Lw) ^ 2)) ^ 0.5)
    A1 = 0.53 * ((L / D) ^ 2) + 1.15 * (L / D) + 0.164
    A2 = (1 - (Lw_bar / L)) / (0.45 + (Lw_bar / L))
    fa = (Lw_bar / L) * (1 + A1 * A2)
    C1 = 3 * h * beta * (Log(beta * h / (2 * Pi * rw_hat)) + s_b) / Lw_hat
    C2 = (Pi * D * fa / (2 * Lw_bar))
    unit_conversion_constant = (9.869E-13 * 0.001) * 24 * 3600 * 100000 * 6 * Pi / (0.001)
    J_horizontal = unit_conversion_constant * kh * h / (viso * Bo * (C1 + C2))
End Function
```

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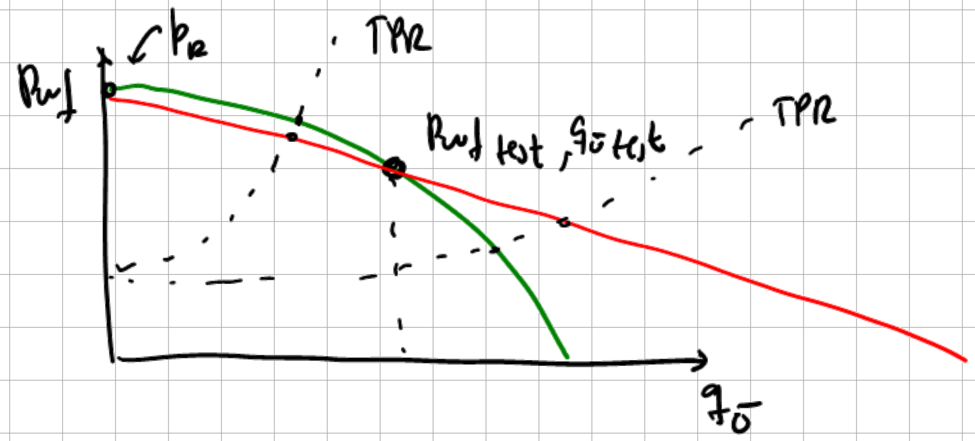
$q_{\bar{o}} = J\,(p_R - p_{wf})$ ⟶ Undersaturated oil

J ~ geometry ~>

fluid-properties ⟶ $(\mu_o B_o)_{@\,p_{av}}$

for gas, saturated oil using a linear IPR is not adequate !

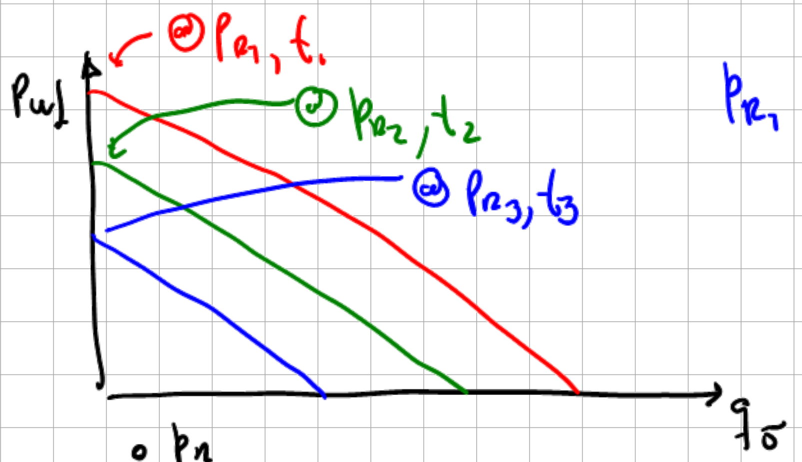


changes to IPR due to depletion

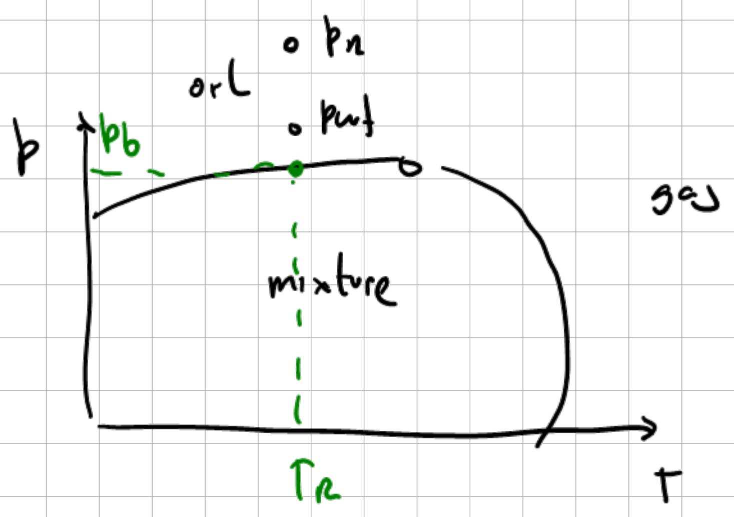
p_R → goes down

J ~> $(\mu_o B_o)_{@p_{av}}$, r_e, p_{ss}, ss, S, otherwise J remains fairly constant with time

$p_{av} = (p_R + p_{wf})\cdot 0.5$



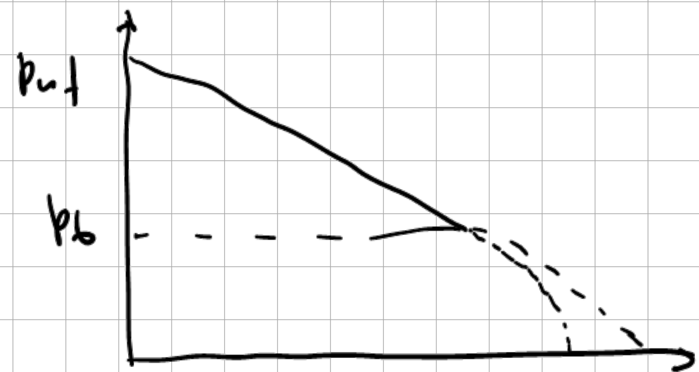
$p_{R_1} > p_{R_2} > p_{R_3}$



linear IPR valid if $p_{wf} > p_b$ $\quad$ $p_R > p_b$

water injection

strong hydrodynamic aquifer



no aquifer, no injection, undersaturated oil reservoir

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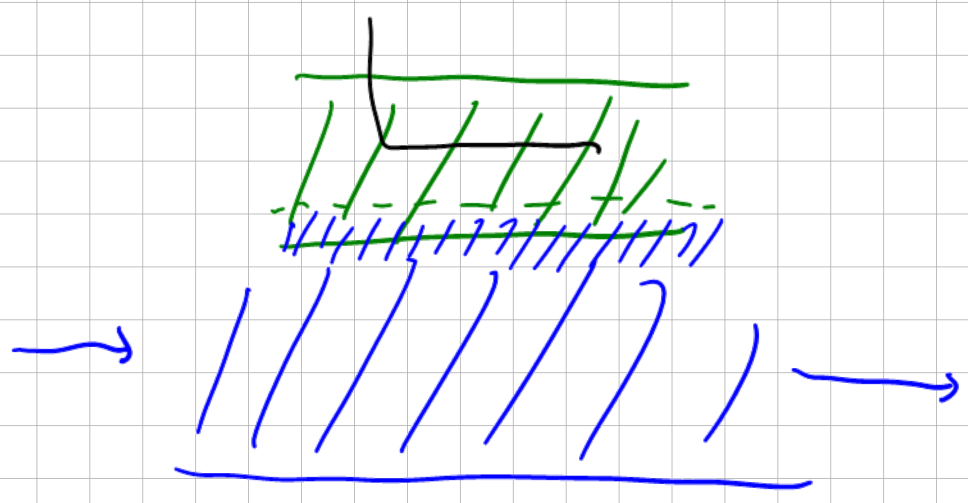


flow of water + oil (undersaturated) → Exhibits a linear IPR

liquid
incompressible

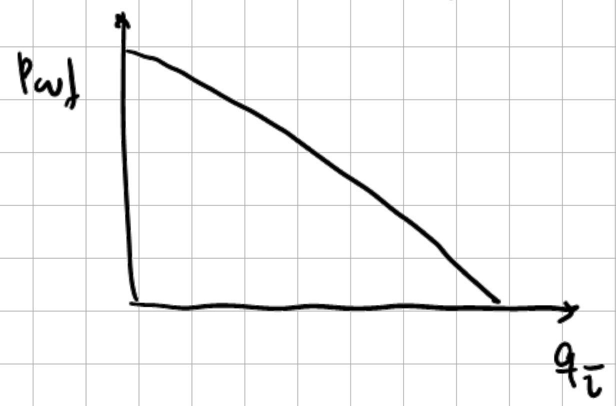
$$J_L = \frac{(q_o + q_w)}{(P_R - P_{wf})}$$

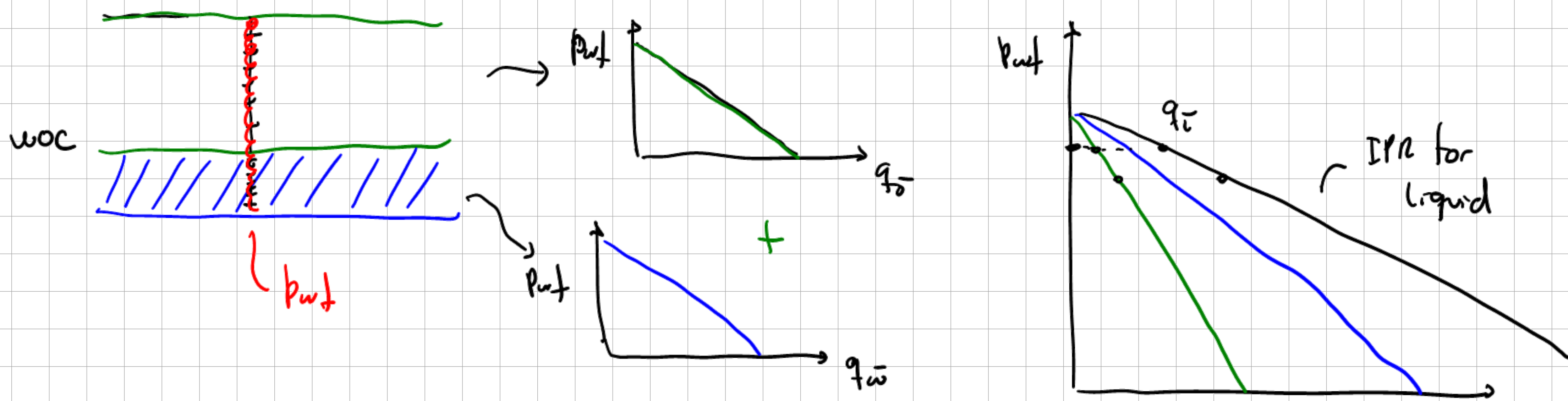
$$q_T = J_L (P_R - P_{wf})$$

$$q_o = q_L (1 - w_c)$$

$$w_c = \frac{q_w}{q_o + q_w}$$

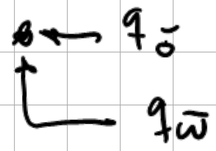
$$q_w = q_T (w_c)$$

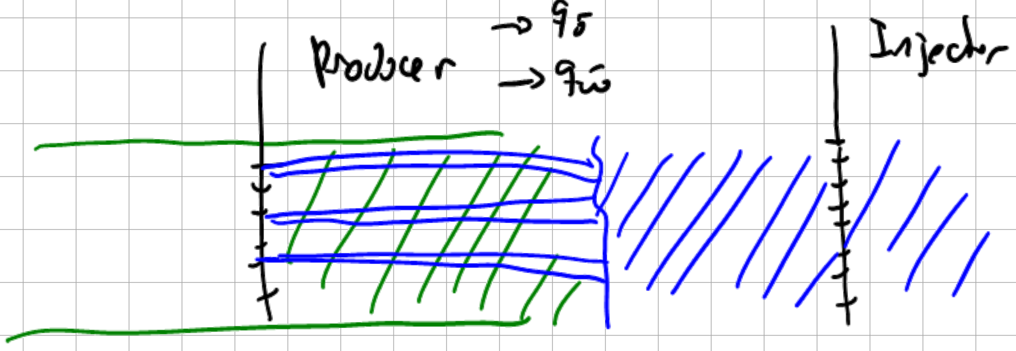


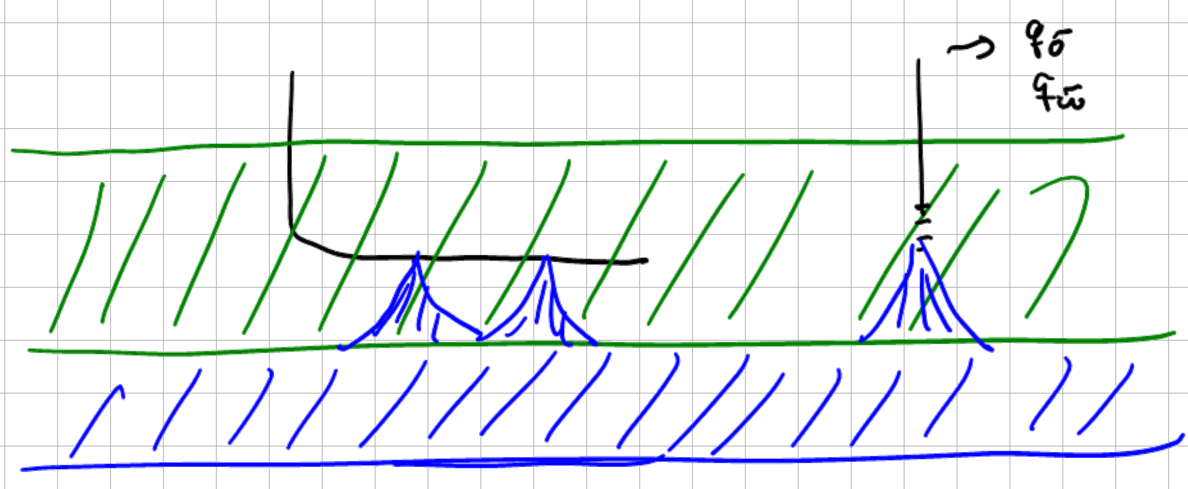


mobility $\lambda_o = \frac{k_{ro}}{\mu_o}$ $\lambda_w = \frac{k_{rw}}{\mu_w}$

$\lambda_o < \lambda_w$







water coning - cusping

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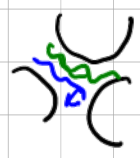
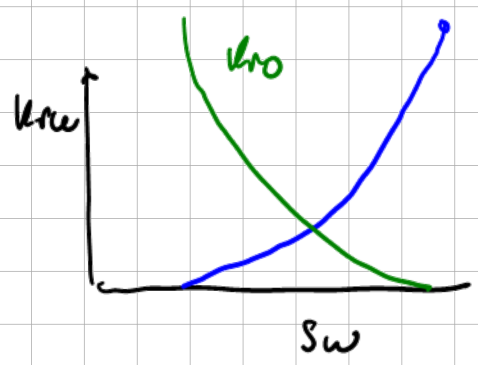
J f(K) when two phases flowing

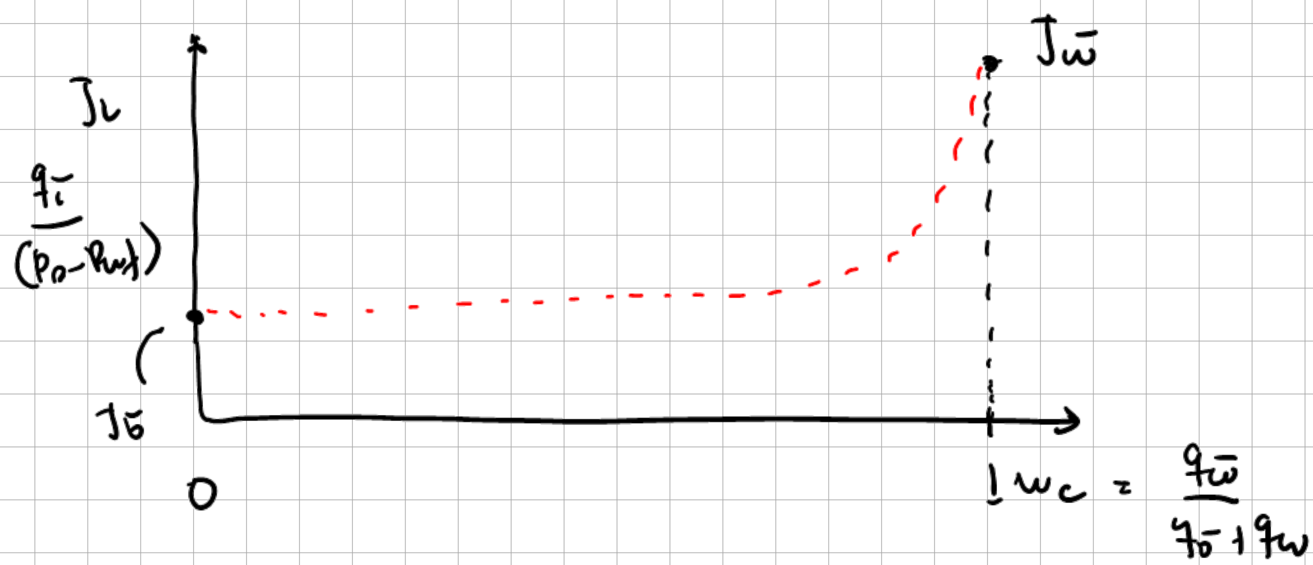
K k_{ro}

k_{rw}





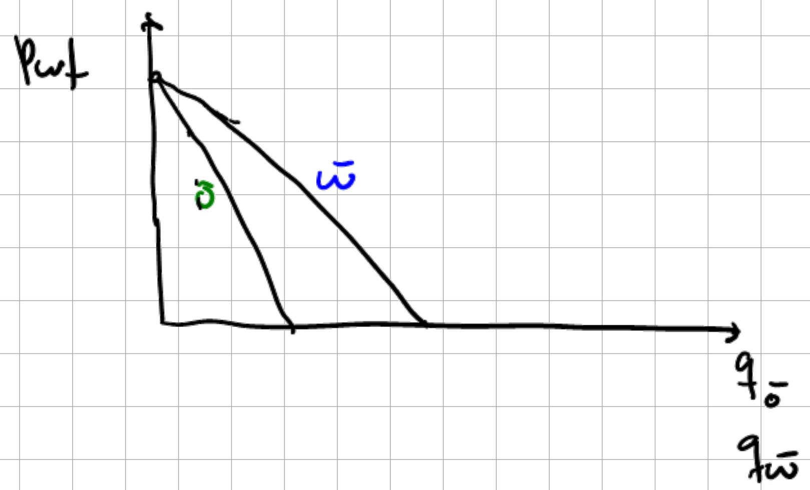
$S_w = \dfrac{V_w}{V_T}$ in the pore



$q_l = J_l\,(p_R - P_{wf})$

$q_o = q_l \cdot (1 - wc)$

$q_w = q_l\,(wc)$



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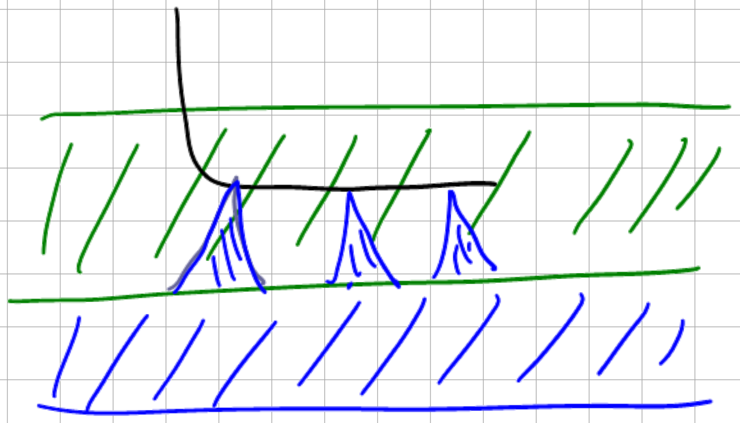
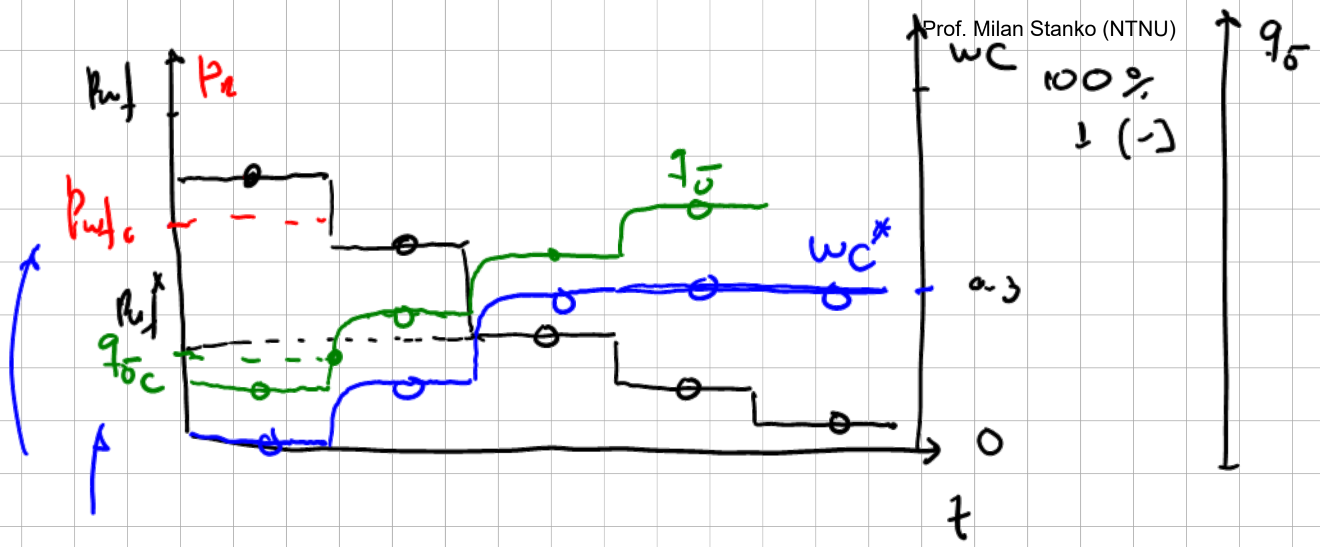
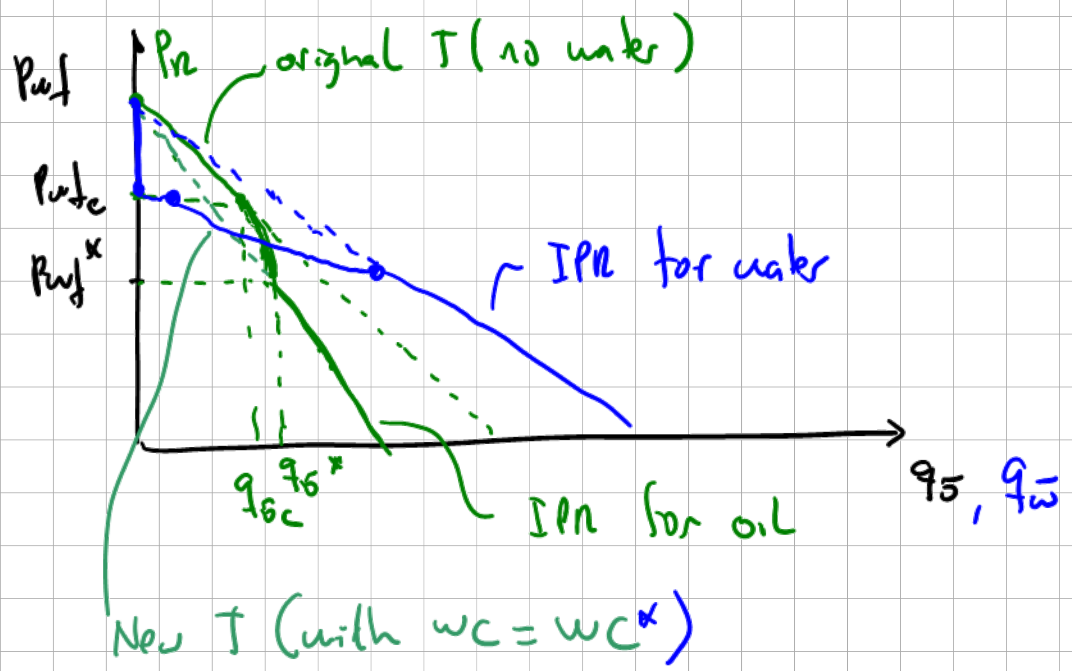




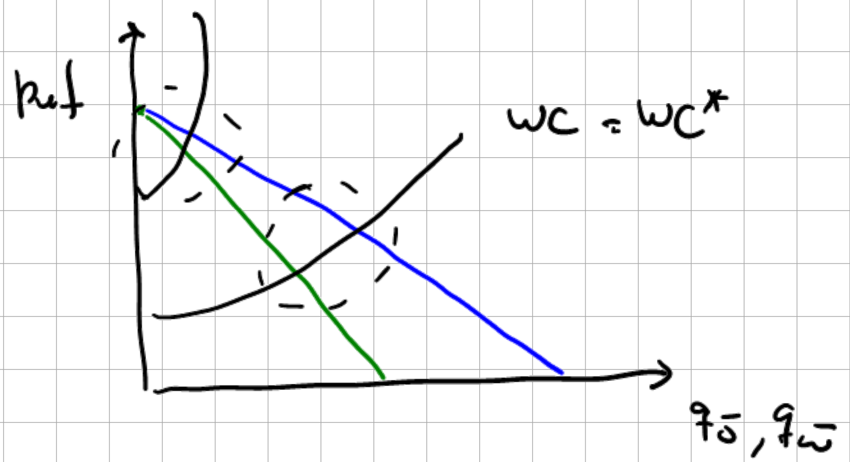
$q_{\bar{o}} < q_{\bar{o}c}$ $P_{wfc} < P_{wf}$ ~> no coning

$q_{\bar{o}} \geq q_{\bar{o}c}$ $P_{wf_c} \geq P_{wf}$ ~> coning



in many cases $q_{\bar{o}c}$ is very small

$q_{\bar{o}}^*$ is very small



Example: one model for water coning from the literature:

Experimental Investigation of Cresting and Critical Flow Rate of Horizontal Wells

Tove Aulie, Evert Grødal, Harald Asheim, Norwegian Inst. of Technology, and Piet Oudeman, Koninklijke/Shell E&P Laboratorium

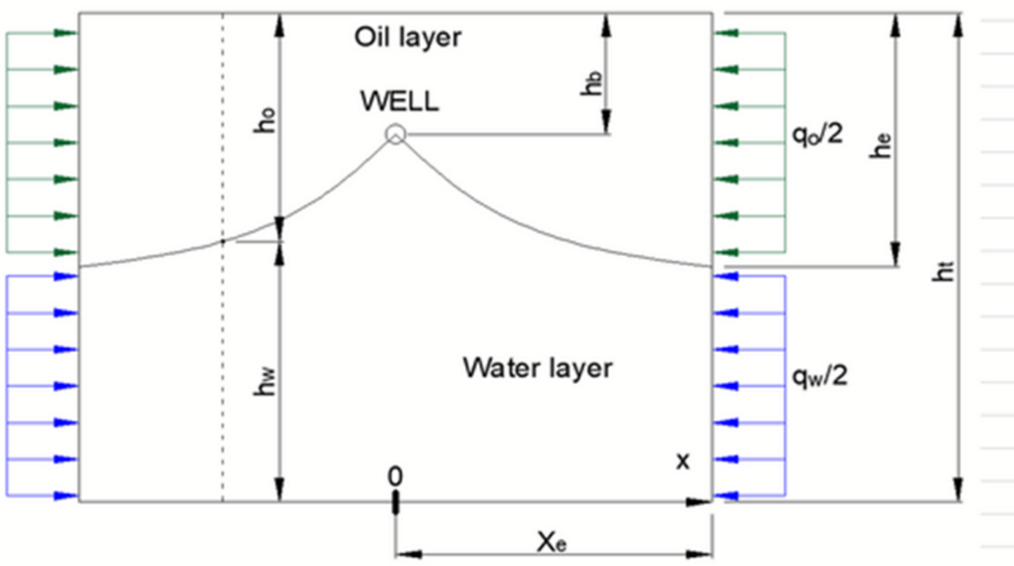
ABSTRACT

Cresting towards horizontal wells with bottom water drive and edge water drive has been experimentally investigated using a

$$h_o(x)^2 - h_e^2 + \frac{q_o}{\lambda_o \Delta\rho g D}\left(\frac{D}{2} - x\right)^2 = 0 \qquad (1)$$

Prof. Milan Stanko (NTNU)		
Total layer height, ht (oil plus water)	[m]	50
Initial water layer height, hw (ht-he)	[m]	10
Initial oil layer height, ho (he)	[m]	40
Horizontal distance from well to outer boundary, xe	[m]	300
Vertical distance between well and top of reservoir, hb	[m]	5
Horizontal permeability, k	[md]	100
Oil viscosity	[cp]	1.0
water viscosity	[cp]	0.6
Oil mobility	[md/cp]	100.0
water mobility	[md/cp]	166.7
Oil density	[kg/m^3]	800
Water density	[kg/m^3]	1024
Oil Bo	[m3/Sm3]	1.0
Water Bw	[m3/Sm3]	1.0
Well length, L, [m]	[m]	500
Critical oil flow to start producing water, qoc (ho=hb@x=0)	[Sm3/d]	49.19
Mobility ratio M (o/w)	[-]	0.6
upper limit of f (qw/qo)	[-]	0.42
upper limit for WC	[%]	29.4
deltaf (qw/qo) - for plotting	[-]	0.014

f (qw/qo)	WC	qo/qoc
[-]	[%]	[-]
0.00	0	1.0
0.01	1	1.0



ASSUMPTIONS:

*Steady state flow, the oil and water volumetric flows in their layers

*Dupuis-Forchheim assumption: the flow towards the well is primari

*Capillary pressure is neglected

$$q_{\bar{o}c} = \frac{(\rho_w - \rho_o) \cdot g \cdot \lambda_o \cdot (h_e^2 - h_b^2) \cdot L}{x_e \cdot B_o}$$

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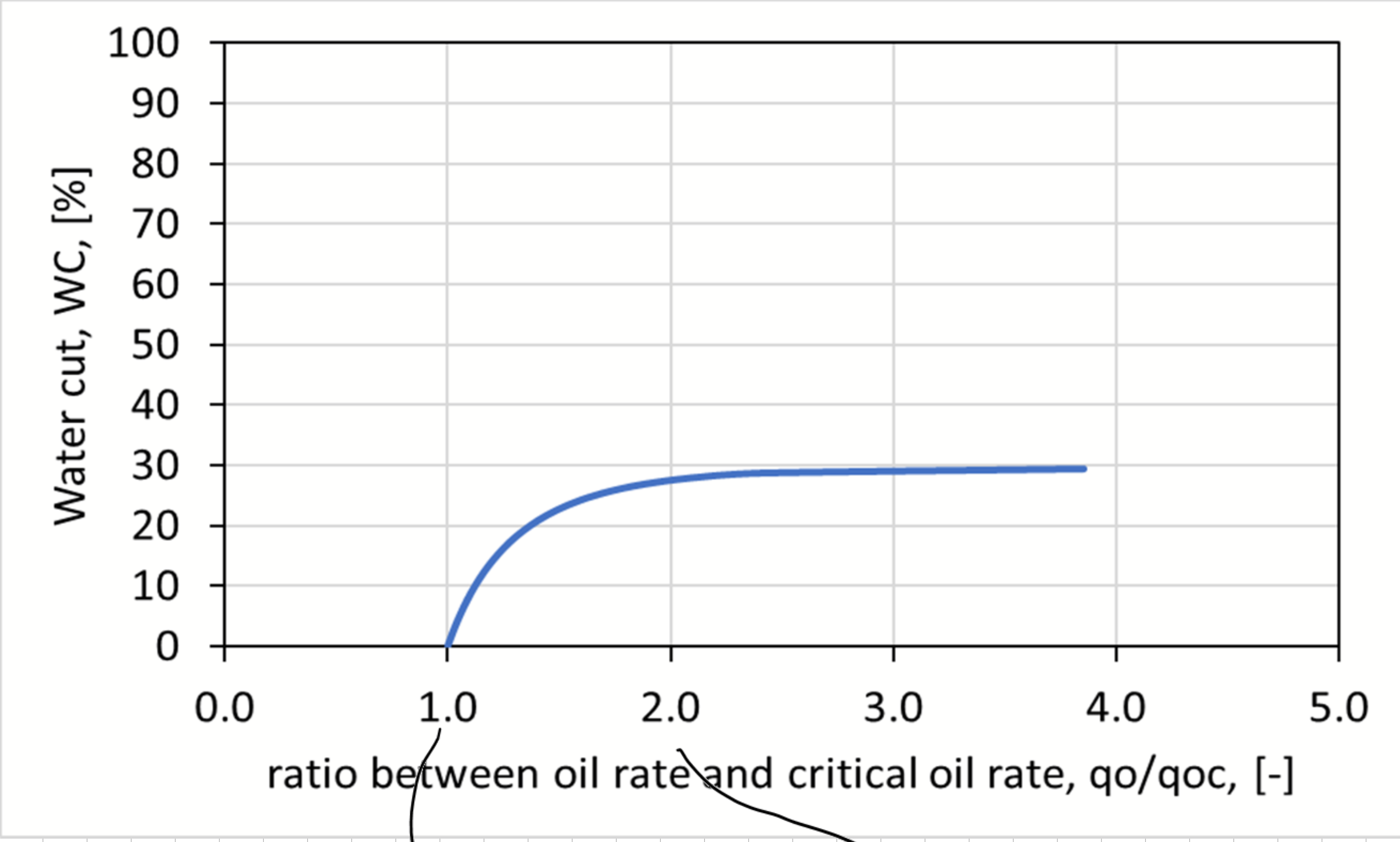
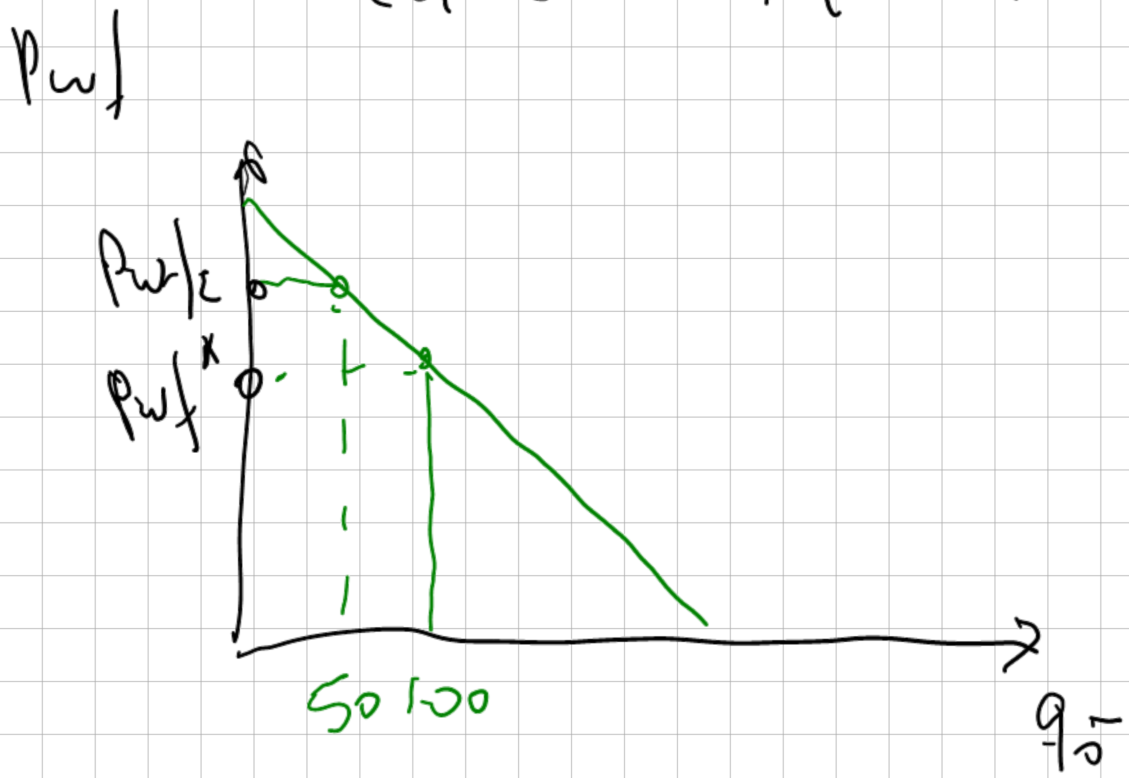


Ca 50 Sm^3/d Ca. 100 Sm^3/d



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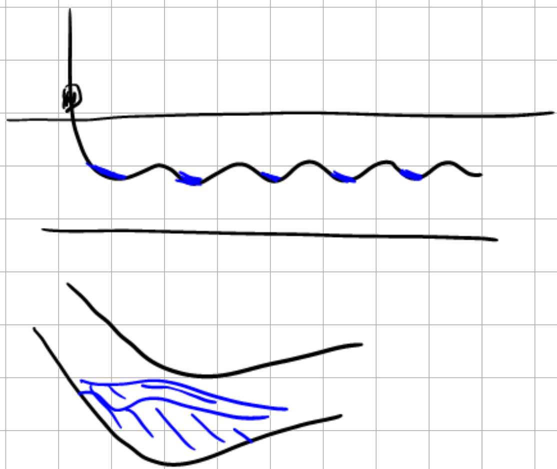
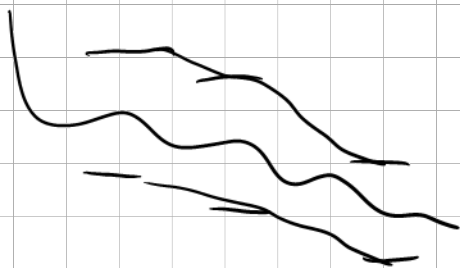
OUTLINE
-Recap - Exercise generation of BO properties with correlations and tuning of correlations with data generated from EoS simulator
-Recap - YT video lecture assignments from last week.
-Class exercises from Mandatory assignment nr. 2

Real horizontal well profile



Liquid slugs



(A vertical undersaturated oil well)

3. **(15 POINTS)** The well exhibits skin damage with s equal to 3. A service company has approached you and offered to perform an acid treatment that would restore the skin to the original undamaged formation (s=0). What increase will this represent in oil rate? Assume that the product $B_o \cdot \mu_o$ remains constant, and that the term $ln\left(\frac{r_e}{r_w}\right)$ is equal to 8.

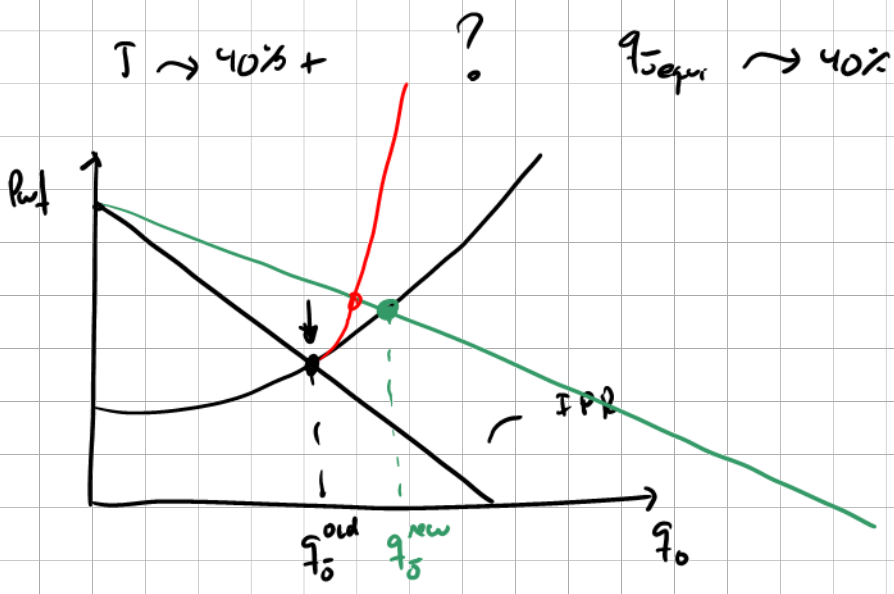
$$J_{original} = \frac{k \cdot h}{(\mu_o B_o)_{@\bar{p}_{av}} \cdot 18.68 \cdot \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S \right]}$$

Assuming pss (no-flow boundary)

$$J_{new} = \frac{k \cdot h}{(\mu_o B_o)_{@\bar{p}_{av}} \cdot 18.68 \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 \right]}$$

$$\frac{J_{new}}{J_{original}} = \frac{\dfrac{\cancel{k \cdot h}}{\cancel{(\mu_o B_o)_{@\bar{p}_{av}}}\ \cancel{18.68} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 \right]}}{\dfrac{\cancel{k h}}{\cancel{(\mu_o B_o)_{@\bar{p}_{av}}}\ \cancel{18.68} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S \right]}} = \frac{\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S \right]}{\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 \right]}$$

$$\frac{J_{new}}{J_{original}} = \frac{(8 - 0.75 + 3)}{(8 - 0.75)} = 1.4$$ ~ 40% increase in J



An increase in J not necessarily gives the same increase in qo, because the TPR is affecting the qo as well!!!

Problem 2. You are considering drilling a well in a undersaturated oil reservoir of thickness 40 m, with a horizontal permeability (k_H) equal to 15 md, and a permeability anisotropy equal to 9 ($\frac{k_H}{k_V}$).

Task 1. Determine how long should you make a horizontal well to have the same productivity index as a vertical well that is completed all through the reservoir thickness.

Additional information

Expressions for productivity index of vertical well and horizontal well (in units of $\frac{Sm3}{d}/bar$) are given below:

- Vertical undersaturated oil well perforated through all the reservoir thickness (h)

$$J_{vertical\ well} = \frac{k_H \cdot h}{18.68 \cdot (\mu_o \cdot B_o)_{@p_{av}} \cdot \left[\ln\left(\frac{r_e}{r_w}\right)\right]}$$

Where permeability is in md, h in m, viscosity in cP and Bo in m3/Sm3

- Horizontal well located in the middle of the layer with thickness h, width D and length L.

$$J_{horizontal\ well} = \frac{k_H \cdot h}{6.22 \cdot (\mu_o \cdot B_o)_{@p_{av}} \cdot \left[\frac{\pi \cdot D}{2 \cdot L_w} + \frac{3 \cdot h \cdot \beta}{L_w} \cdot ln\left(\frac{h \cdot \beta}{\pi \cdot r_w \cdot (1+\beta)}\right)\right]}$$

With

- $\beta = \sqrt{\frac{k_H}{k_V}}$
- L_w is wellbore length
- Assume wellbore radius (r_w) equal to 0.15 m
- Assume $(\mu_o \cdot B_o)_{@p_{av}}$ =2.15
- For the vertical well J expression assume that $r_e = 1500\ m$
- For the horizontal well J expression assume that $D = 1500\ m$ and L = 1500 m

$$J_{vertical} = J_{horizontal}$$

$$\underbrace{\frac{\cancel{k}\cdot h}{\cancel{(\mu_o B_o)}_{@p_{av}} \cdot 18.68 \left[\ln\left(\frac{r_e}{r_w}\right)\right]}}_{\text{neglecting skin, neglecting boundary effect}} = \frac{\cancel{k_H}\cdot \tilde{h}}{6.22\, \cancel{(\mu_o B_o)}_{@p_{av}} \left[\frac{\pi D}{2 L_w} + \frac{3 \cdot h \cdot \beta}{L_w}\left[\ln\left(\frac{h\cdot\beta}{\pi r_w (1+\beta)}\right)\right]\right]}$$

neglecting skin

neglecting boundary effect

$$L_w = \left| \frac{6.22}{18.68} \frac{\left(\frac{\pi \overset{1500}{D}}{2} + 3 \overset{40}{h} \overset{3}{\beta} \ln\left(\frac{\overset{40}{h}\, \overset{3}{\beta}}{\pi \underset{0.15}{r_w} (1+\underset{3}{\beta})}\right)\right)}{\left(\underbrace{\ln\left(\frac{r_e}{r_w}\right)}_{\ln\left(\frac{1500}{0.15}\right)}\right)} \right| = 139 \text{ m}$$

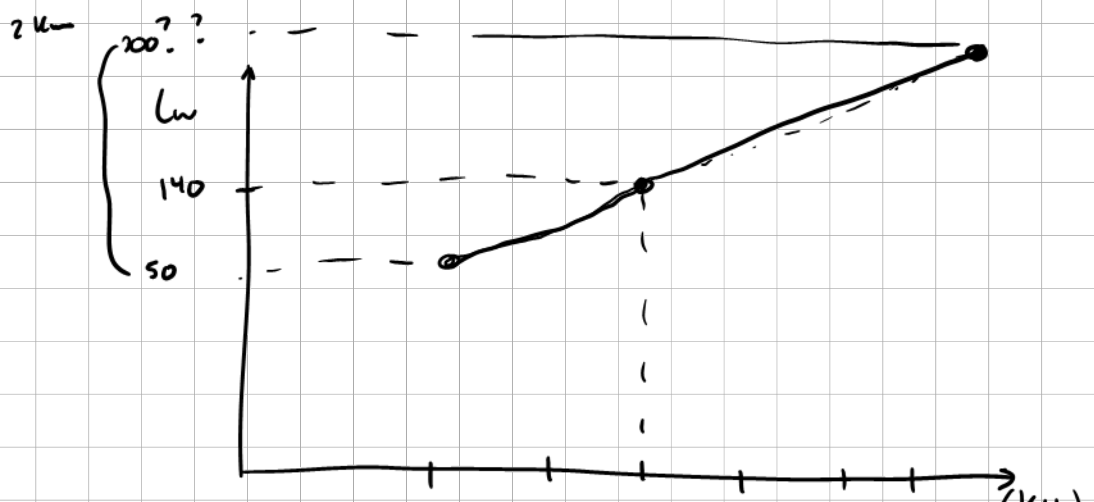
$$\frac{L_w}{h} \simeq \frac{139}{40} = 3.5$$

$$l_w \simeq 1000 \text{ m}$$

$$\frac{1000}{40} = 25$$

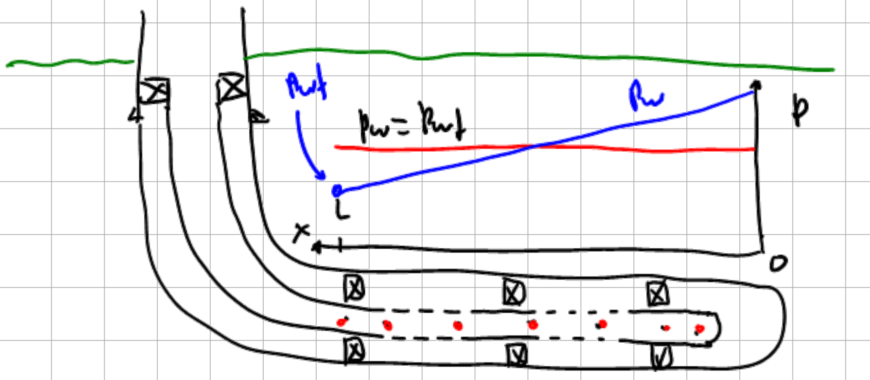
$$\frac{25}{3.5} = 7.$$

a 1000 m long horizontal well will be 7 times more productive as a vertical well completed through the layer (40 m)!!!



K_H/K_v	5	7	9	10	12	15
K_H	15	15	15	15	15	15
K_v	3	2.15	1.6	1.5	1.25	1

Video 13.1 - Effect of wellbore pressure drop on horizontal undersaturated oil well IPR



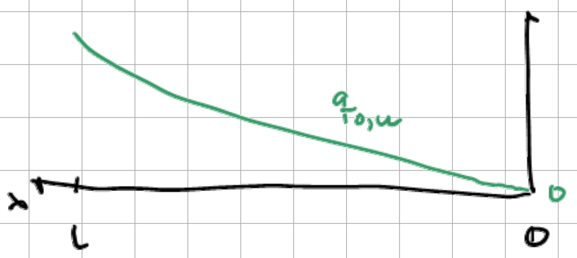
$$q_{\bar{o}} = J\,(p_R - p_{wf})$$

$$\frac{dp_w}{dx} = -\rho_o \,\frac{f}{D}\,\frac{v^2}{2}$$

$$v = \frac{q_{o,w}(x)}{A} \qquad q_{\bar{o},w}\cdot B_{o,w} = q_{o,w}$$

$$A = \frac{\pi D^2}{4}$$

$$\frac{dp_w}{dx} = -\rho_o \,\frac{f}{D^5}\,\frac{8\, q_{\bar{o},w}^2 \cdot B_{o,w}^2}{\pi^2}$$



Pressure Drop in Horizontal Wells and Its Effect on Production Performance

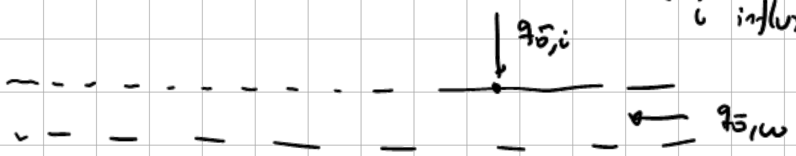
Ben J. Dikken, SPE, Koninklijke/Shell E&P Laboratorium

(1989)

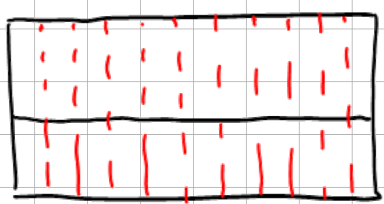
Author

Ben J. Dikken is a reservoir engineer in the Technology Development Dept. of Shell Intl. Petroleum Mij. B.V. in The Hague, with interests in horizontal wells, propagating waterflood-induced fractures, and well testing. He previously worked at the Koninklijke/Shell E&P Laboratory in Rijswijk. He holds MS and PhD degrees in physics, both from the State U. of Utrecht, The Netherlands.

mass conservation in wellbore



$$\frac{d\,q_{\bar{o},w}}{dx} = q_{\bar{o},i}$$



$$q_{\bar{o}} = J\,(p_R - p_{wf})$$

$$q_{\bar{o},i} = j\,(p_R - p_w)$$

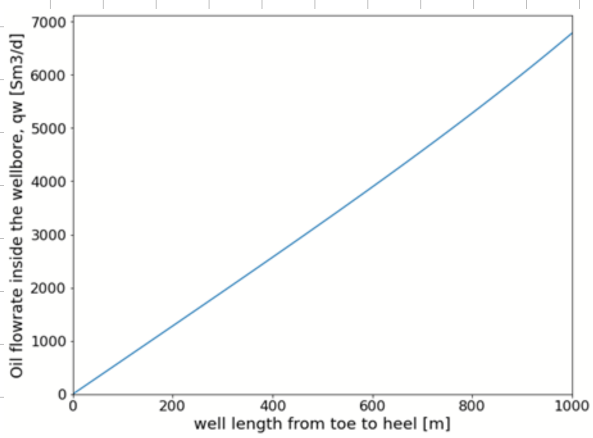
$$j = J/L = \left[\frac{Sm^3/d/bar}{m}\right]$$

$$\left.\begin{aligned} \frac{dq_{\bar{o},w}}{dx} &= j\,(p_R - p_w) \\ \frac{dp_w}{dx} &= -\rho_o \,\frac{f}{D^5}\,\frac{8\, q_{\bar{o},w}^2 \cdot B_{o,w}^2}{\pi^2} \end{aligned}\right\}$$

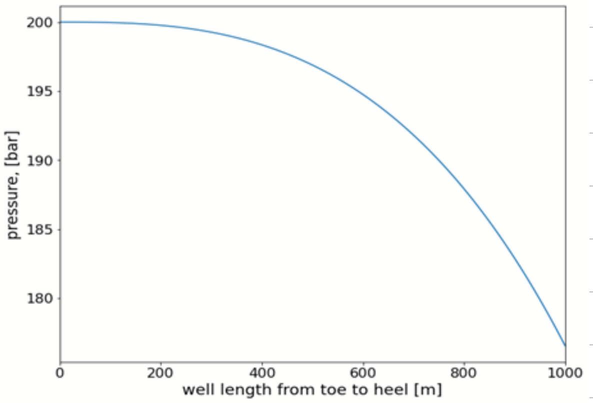
at $x^* = 0$ $\quad p_w$ = assumed value $< p_R$

$q_{\bar{o},w} = 0$

Solution (using Python)

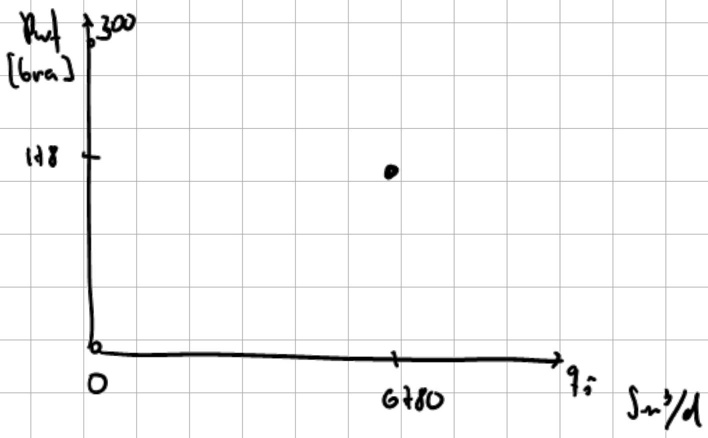


Uniform oil influx along the wellbore

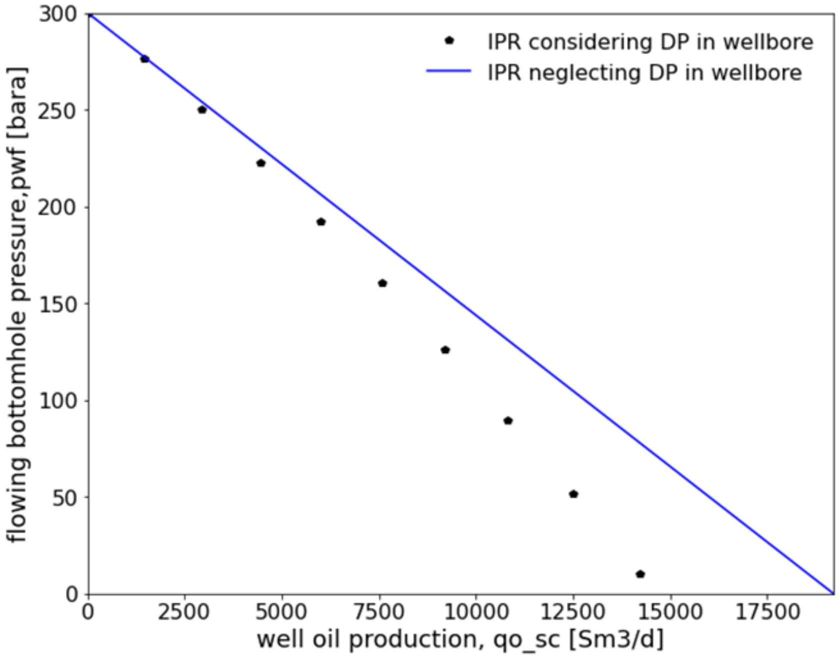


Non-linear distribution of wellbore pressure

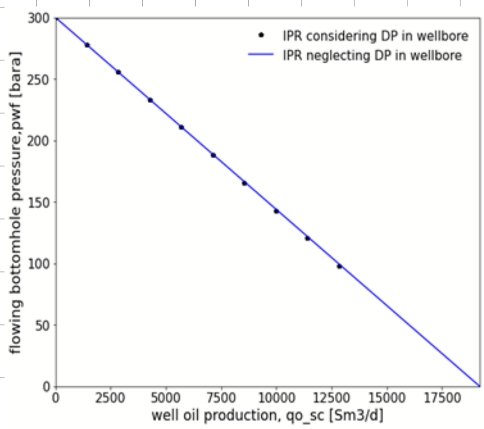
How to obtain the IPR of the well?



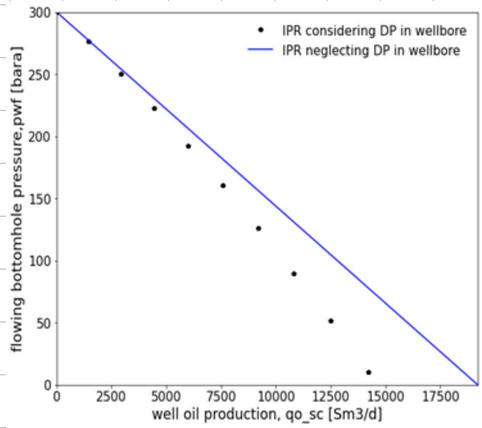
repeat using other values of pw@toe



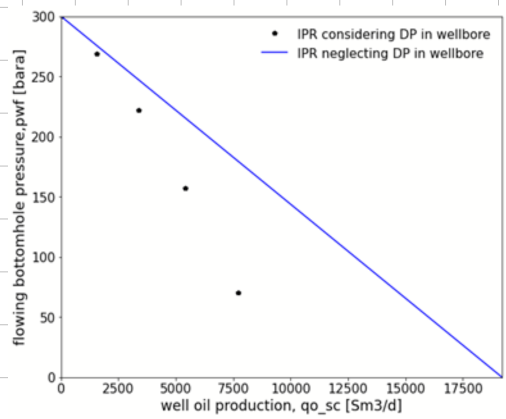
Effect of liner diameter



0.2 m



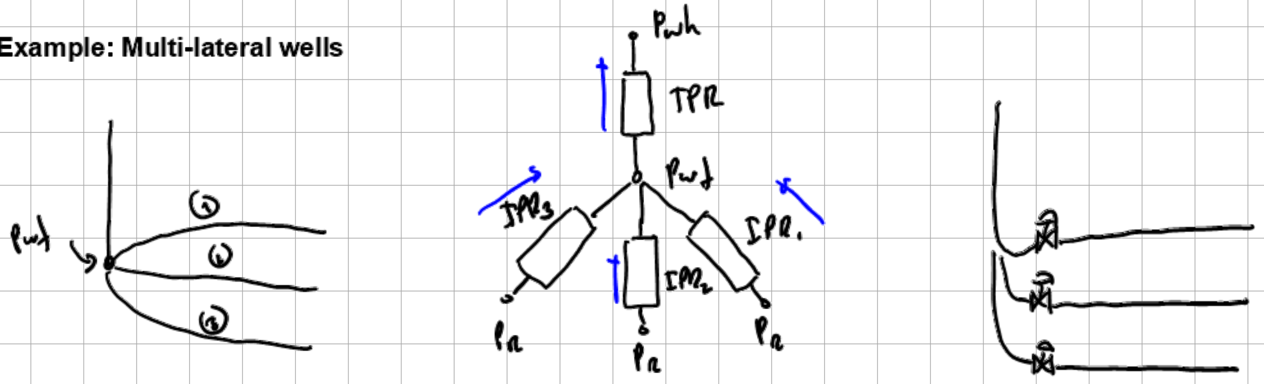
0.1 m



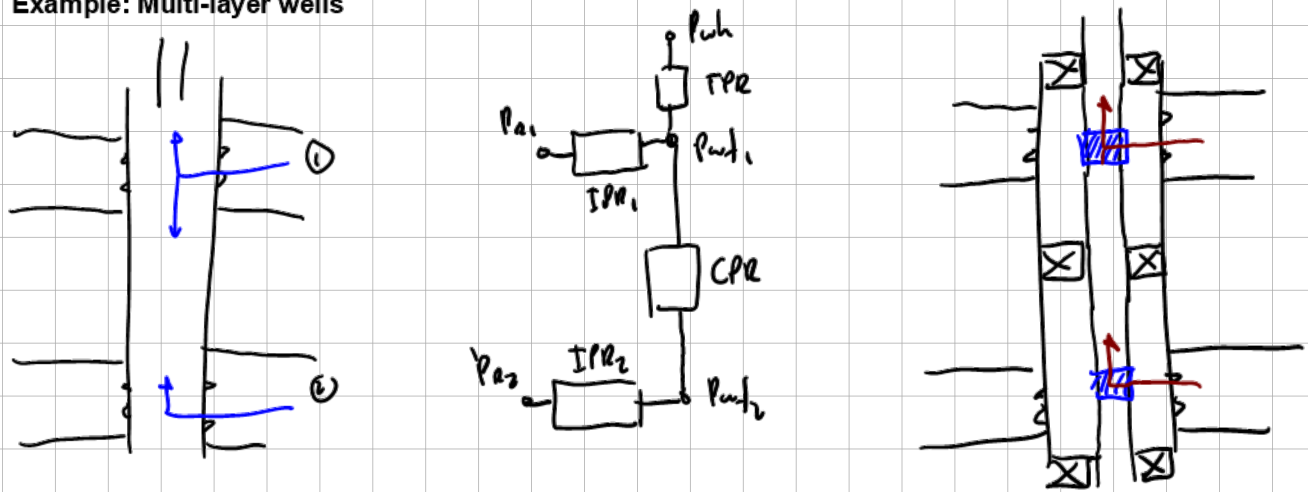
0.07 m

Video 13.2. - Downhole networks and inflow control

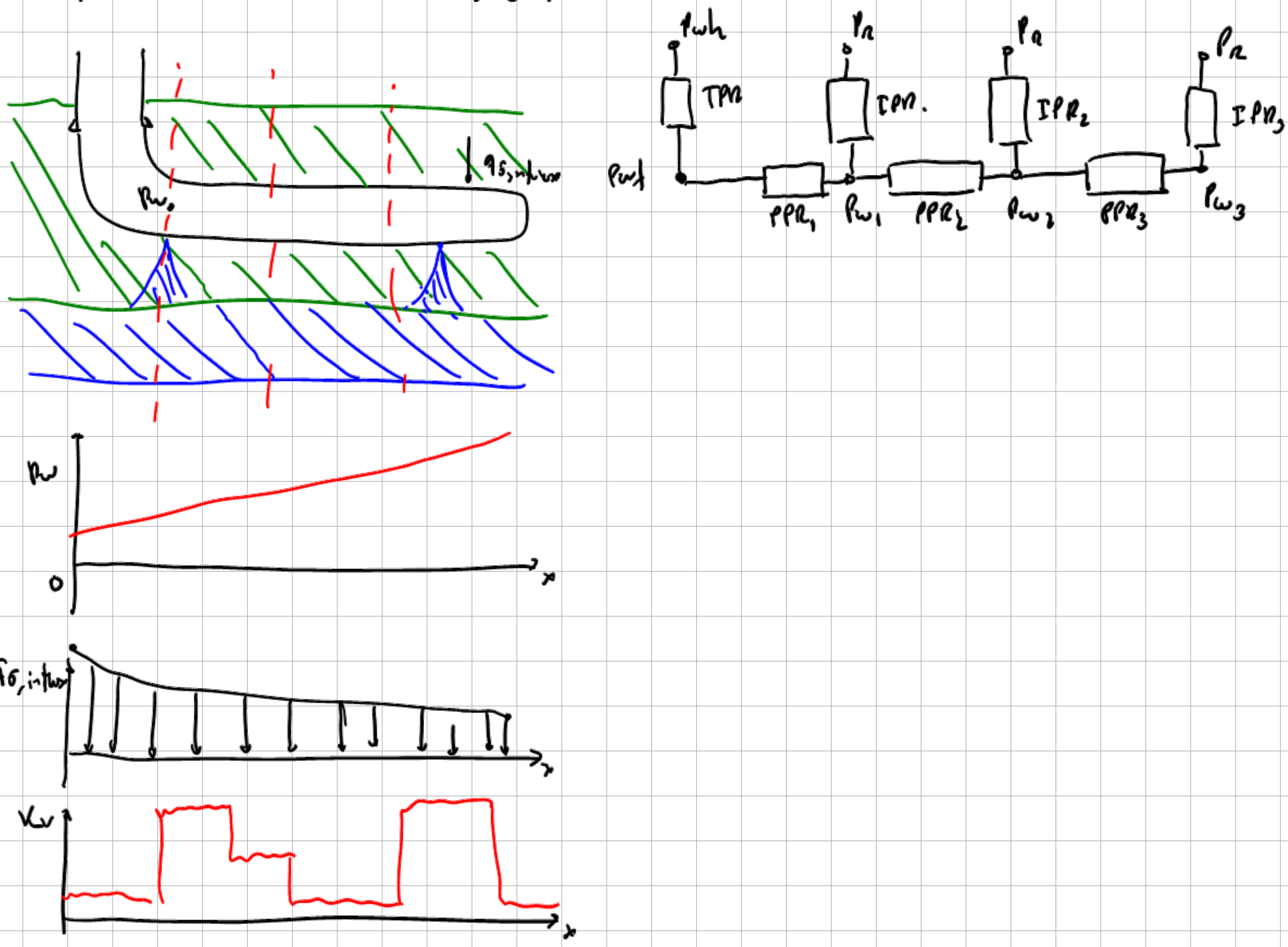
Example: Multi-lateral wells

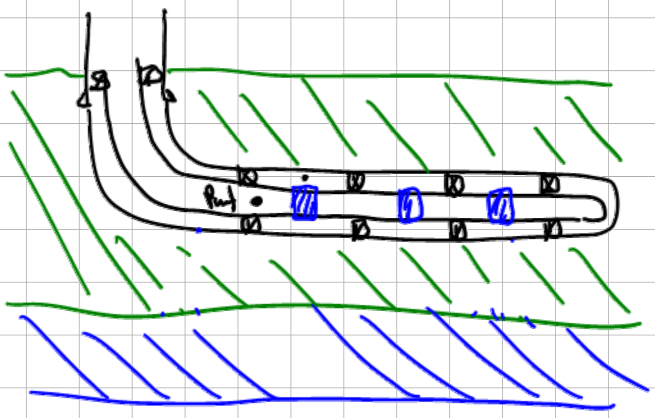


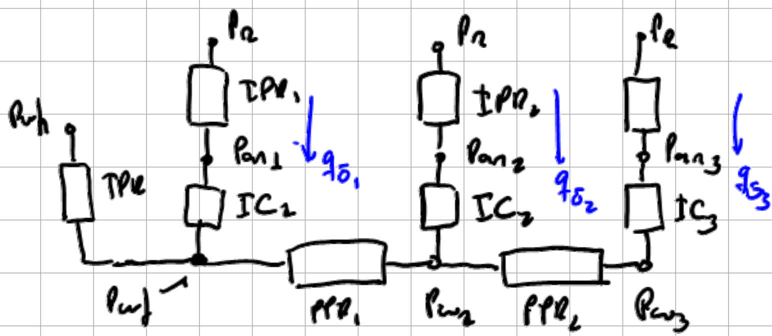
Example: Multi-layer wells

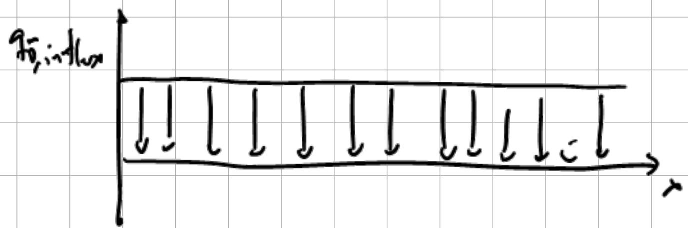


Example: horizontal oil well with an underlying aquifer



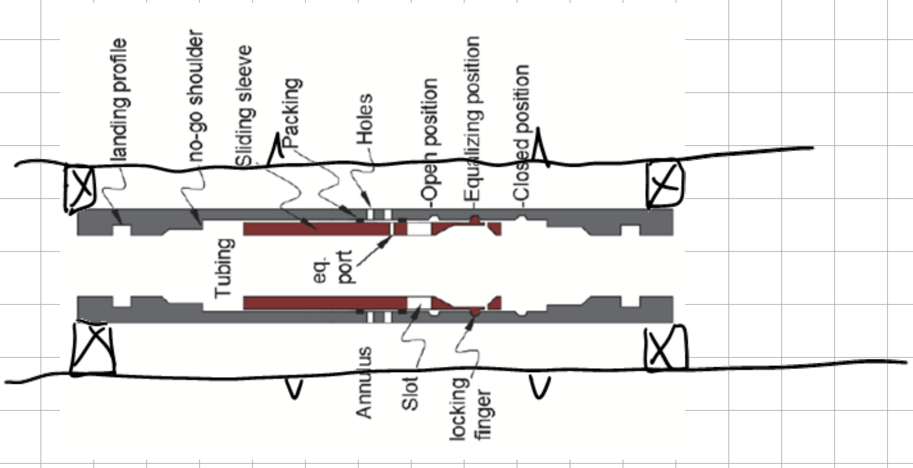




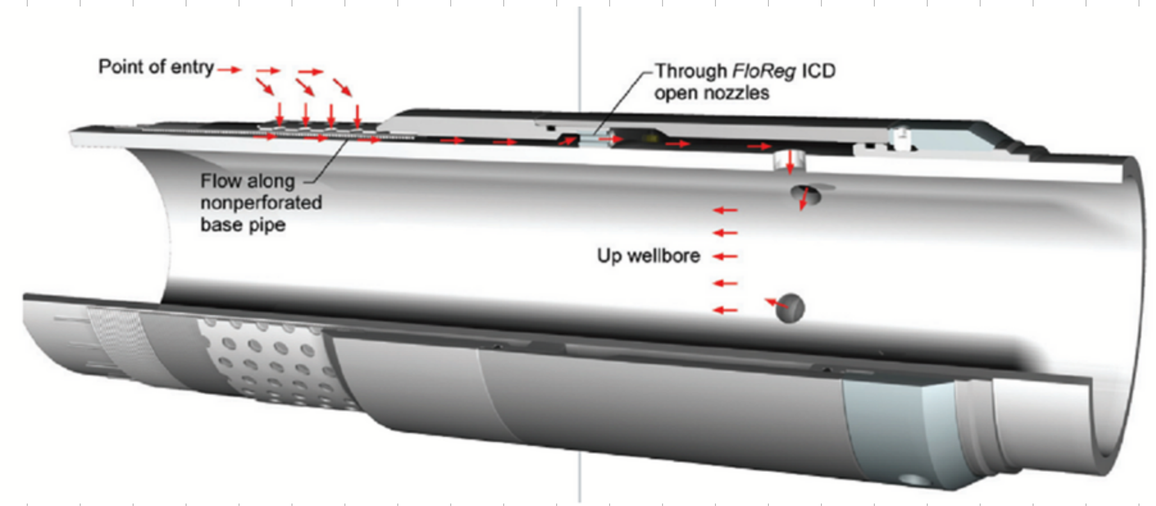


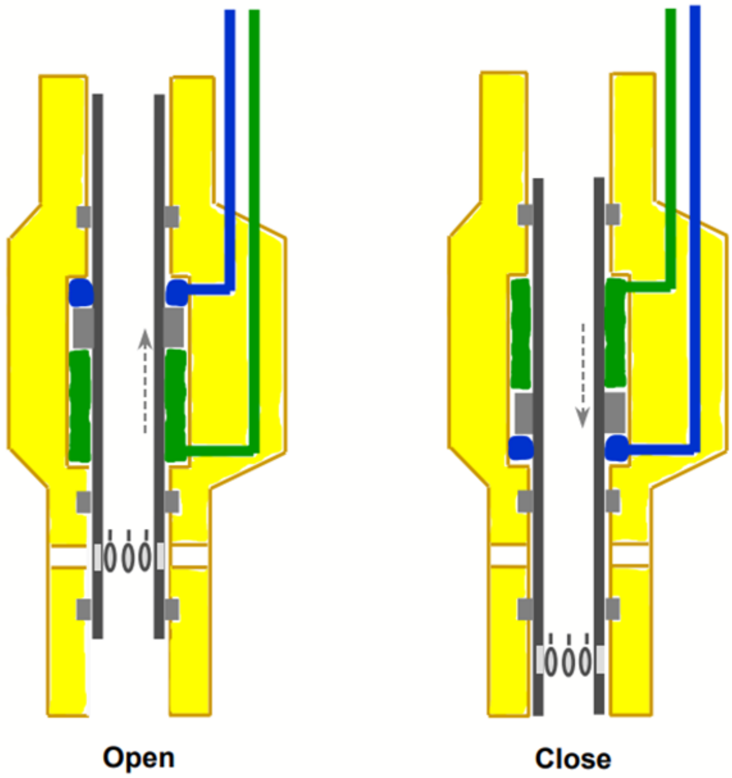
SOME FLOW CONTROL ELEMENTS

Sliding sleeve

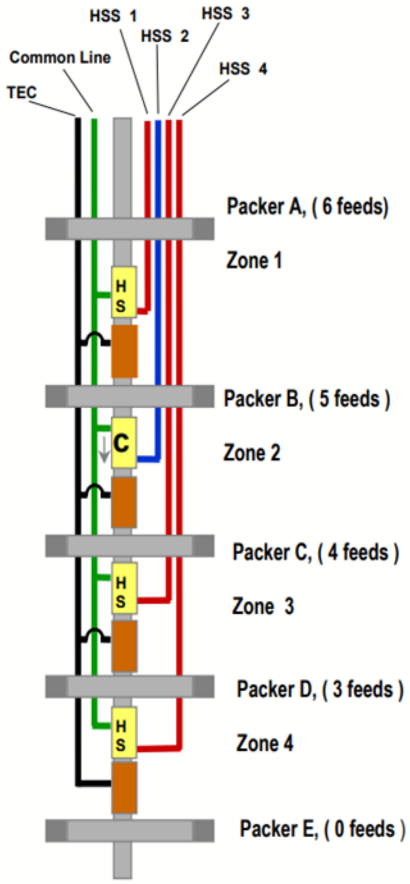


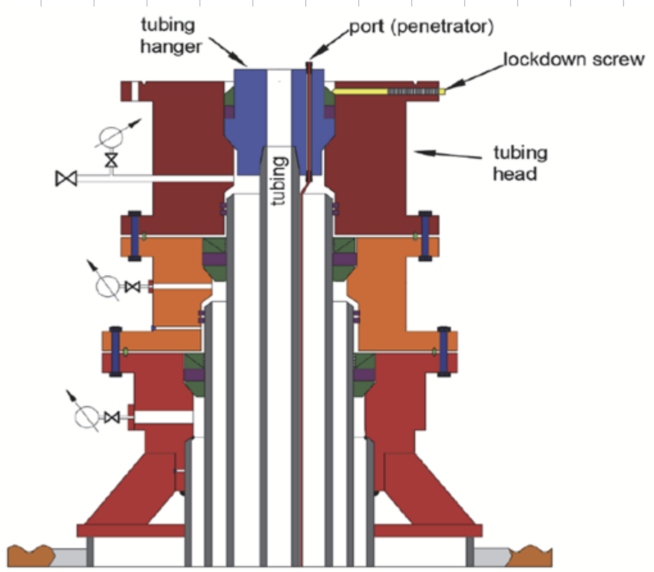
https://www.weatherford.com/documents/brochure/products-and-services/completions/floreg-inflow-control-device-(icd)/



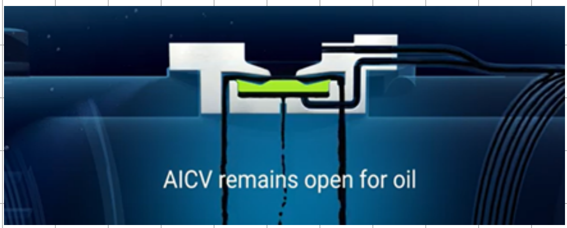


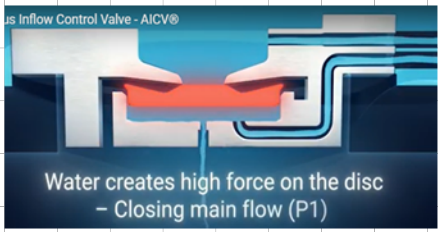
ICV inflow control valve

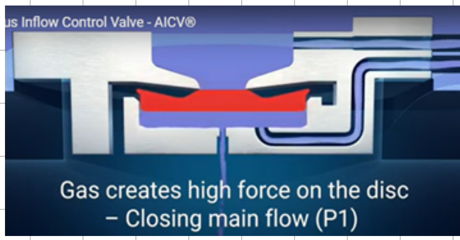




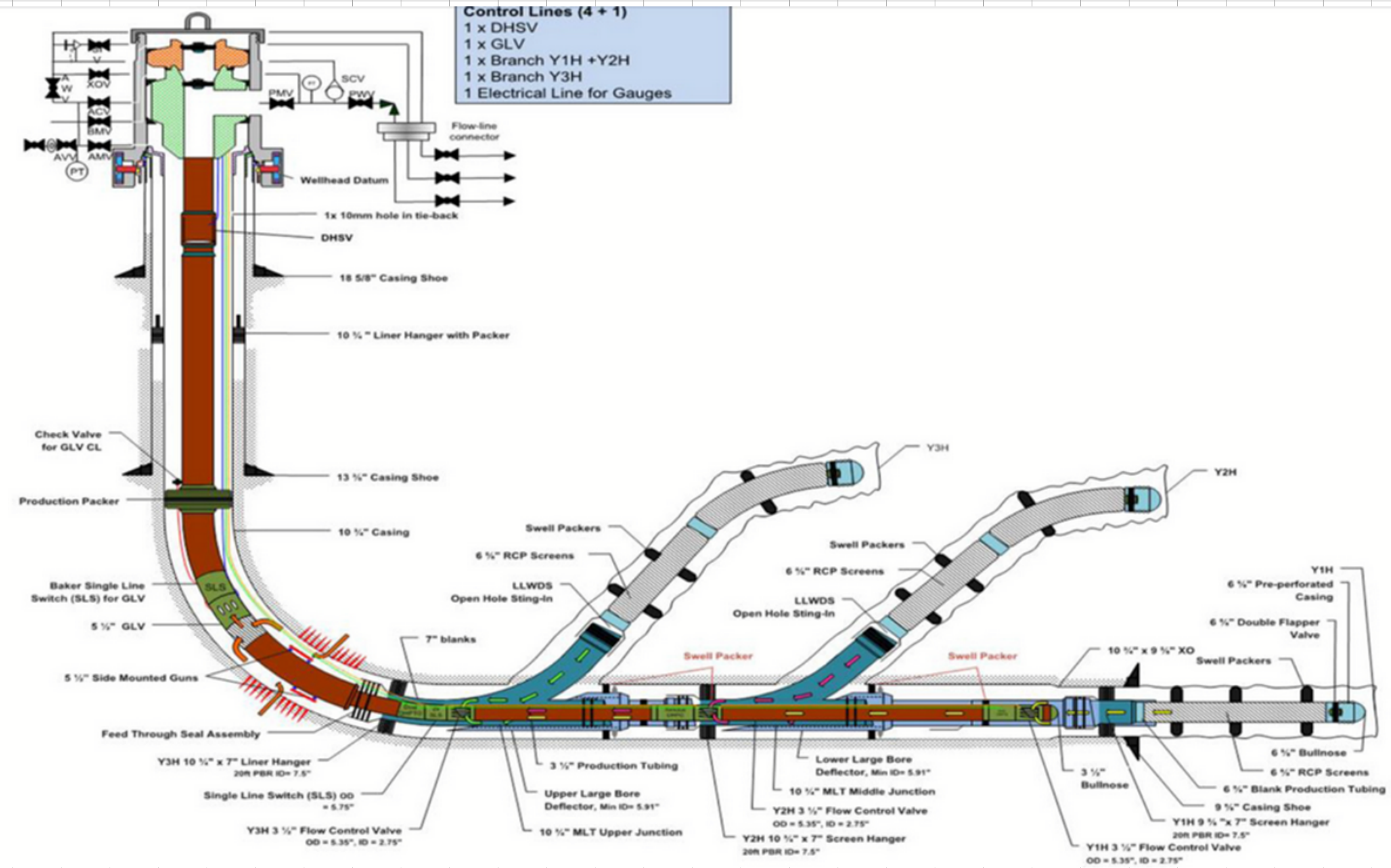
https://www.inflowcontrol.no/aicv-technology







Troll completion (Taken from SPE-180037-MS)



Short on packers:

Inflatable packers (open hole)

https://www.mgs.co.uk/product/borehole-packers/

Swellable packers (open hole)

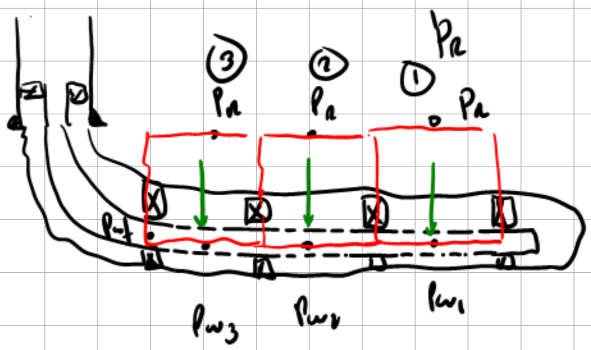
http://poss.com.eg/index.php/portfolio-posts/freecap-i-swellable-packers/

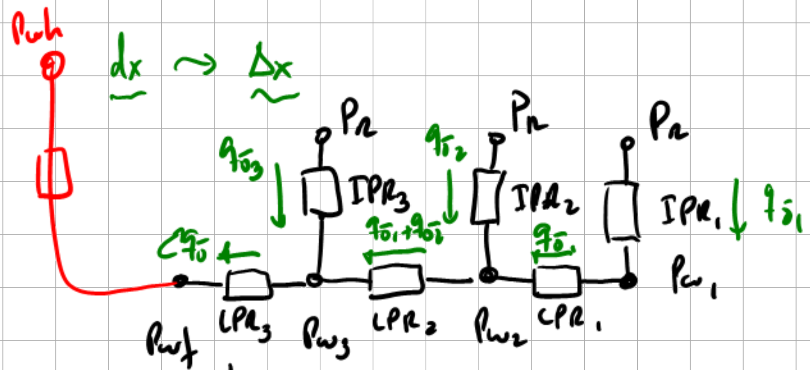
Mechanical/hydraulic packers (cased hole)

https://www.wellcareoiltools.com/packer-system/mechanical-packer/

20240916
OUTLINE
- Recap of video lectures from last week
- Differential versus discrete approximation of long wellbore
- Commercial tool NETool
- Hardware used for inflow control - tour to the lab at IGV

Discrete approximation of distributed flow of undersaturated oil in wellbore





Liner performance relationship

Counting eq. and unknowns:

		nr eq	nr unknowns
IPR_1	$q_{\bar{o}1} = J_1 (P_R - P_{w1})$	1	2
IPR_2	$q_{\bar{o}2} = J_2 (P_R - P_{w2})$	2	4
IPR_3	$q_{\bar{o}3} = J_3 (P_R - P_{w3})$	3	6
LPR_1	$(P_{w1} - P_{w2}) = \rho_{\bar{o}1} g L_1 \sin(\theta_1) + f_1 L_1 \rho_{o1} \dfrac{(q_{\bar{o}1} B_{\bar{o}1})^2 8}{\pi^2 \phi^5}$	4	6
LPR_2	$(P_{w2} - P_{w3}) = \rho_{\bar{o}2} g L_2 \sin(\theta_2) + f_2 L_2 \rho_{o2} \dfrac{((q_{\bar{o}1} + q_{\bar{o}2}) B_{\bar{o}2})^2 8}{\pi^2 \phi_2^5}$	5	6
LPR_3	$(P_{w3} - P_{wf}) = \rho_{o3} g L_3 \sin(\theta_3) + f_3 L_3 \rho_{o3} \left(\dfrac{(q_{\bar{o}1} + q_{\bar{o}2} + q_{\bar{o}3}) B_{o3})^2 8}{\pi^2 \phi_3^5}\right)$	6	7

① i can solve the system of eq, if I provide P_{wf}.

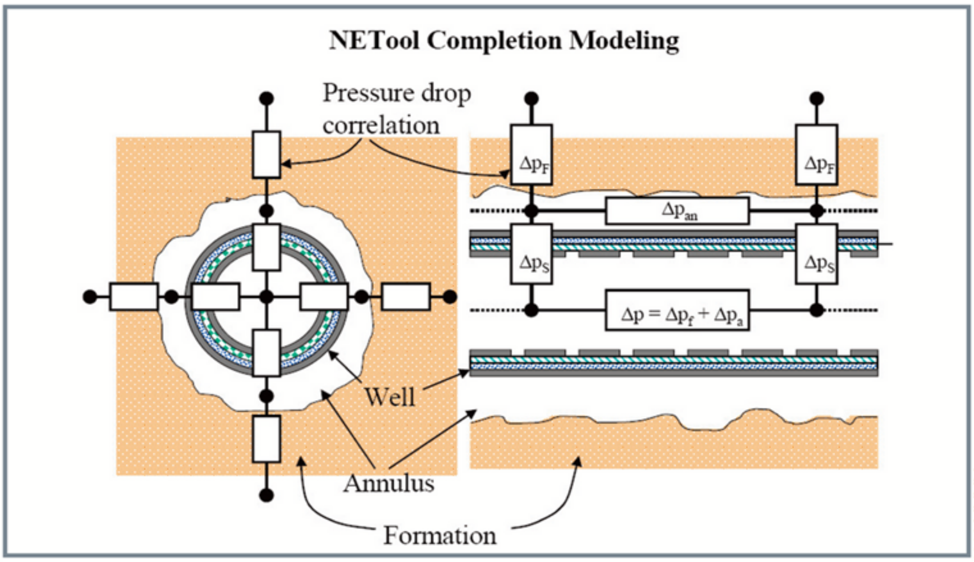
② alternatively, I include the tubing and assume P_{wh}

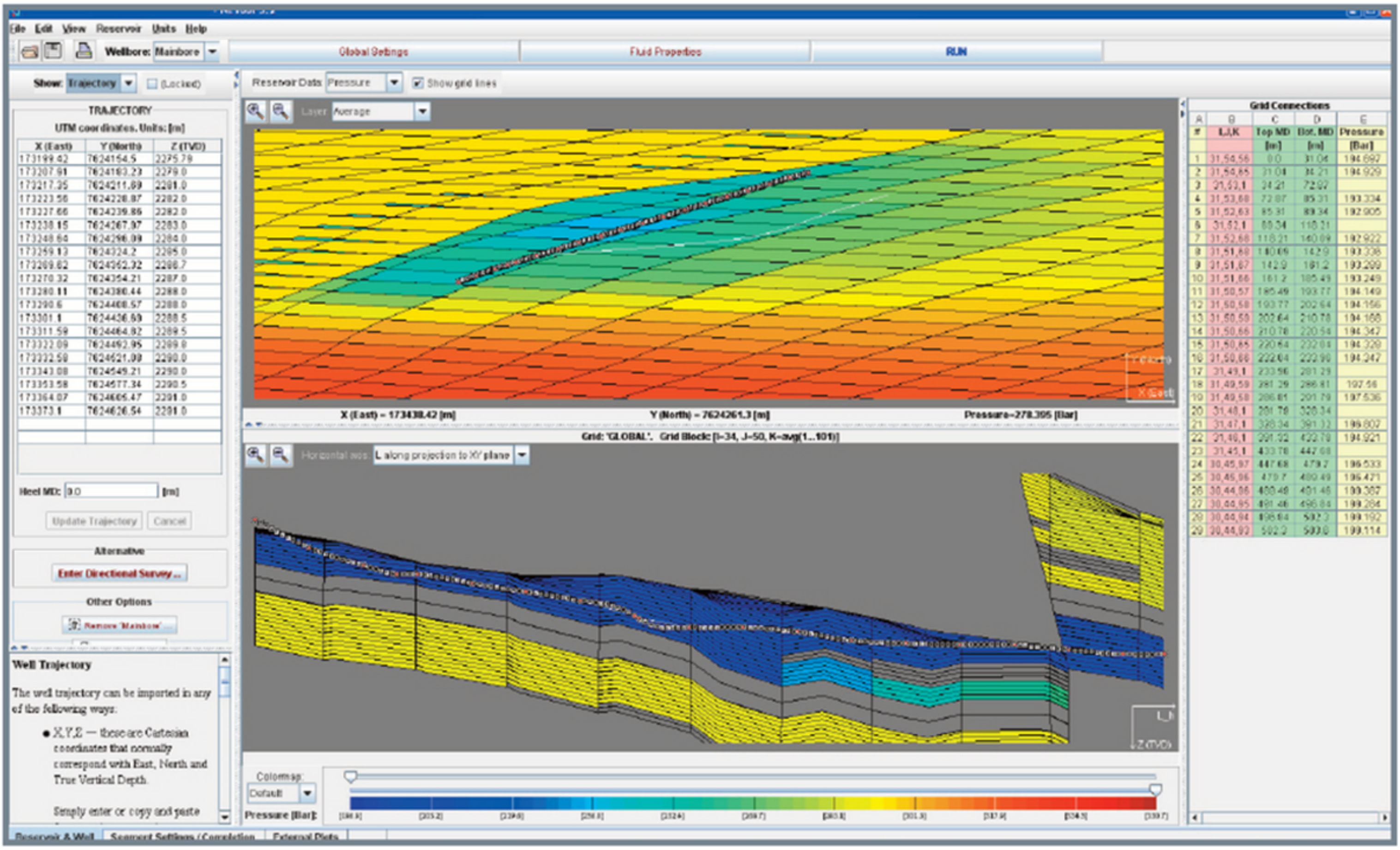
$$(P_{wf} - P_{wh}) = \Delta p_{tubing\ hydros} + \Delta p_{tubing\ friction}(q_{\bar{o}1} + q_{\bar{o}2} + q_{\bar{o}3}) \qquad 7 \qquad 7$$

You solve this system of 7 eq with 7 unknowns using: Newton Raphson (non linear)

The main advantage of this approach is that I can include non-idealities, more realistic conditions, like permeability heterogeneity, different WC per section, etc.

https://www.halliburton.com/en/software/decisionspace-365-enterprise/decisionspace-365-reservoir-and-production/netool-software





This figure shows the main screen of NETool with the reservoir data set. The various values from the reservoir simulator model can be graphically viewed: porosity, permeability, saturations, etc. The visualization reveals the sweet spots for well placement in the reservoir. The well can be entered as a deviation survey and moved with a mouse.

Zonal isolation: sliding sleeve

The sleeve is shifted using slickline and a shifting tool

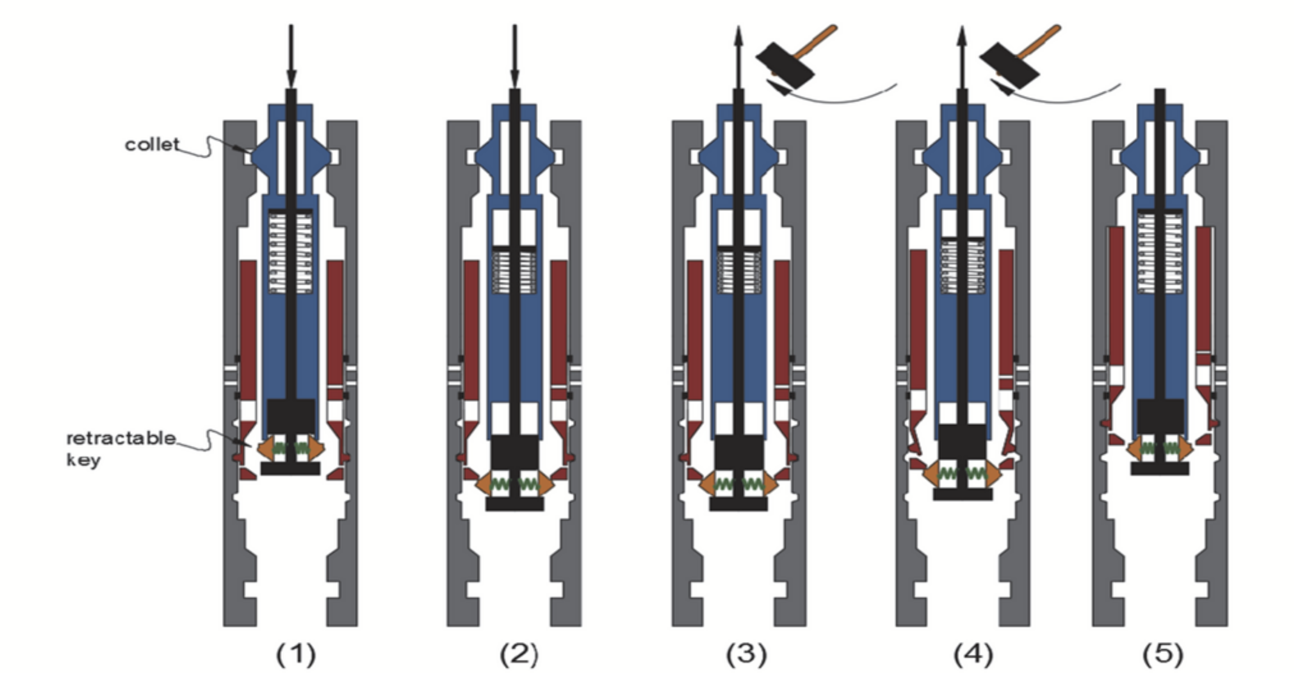
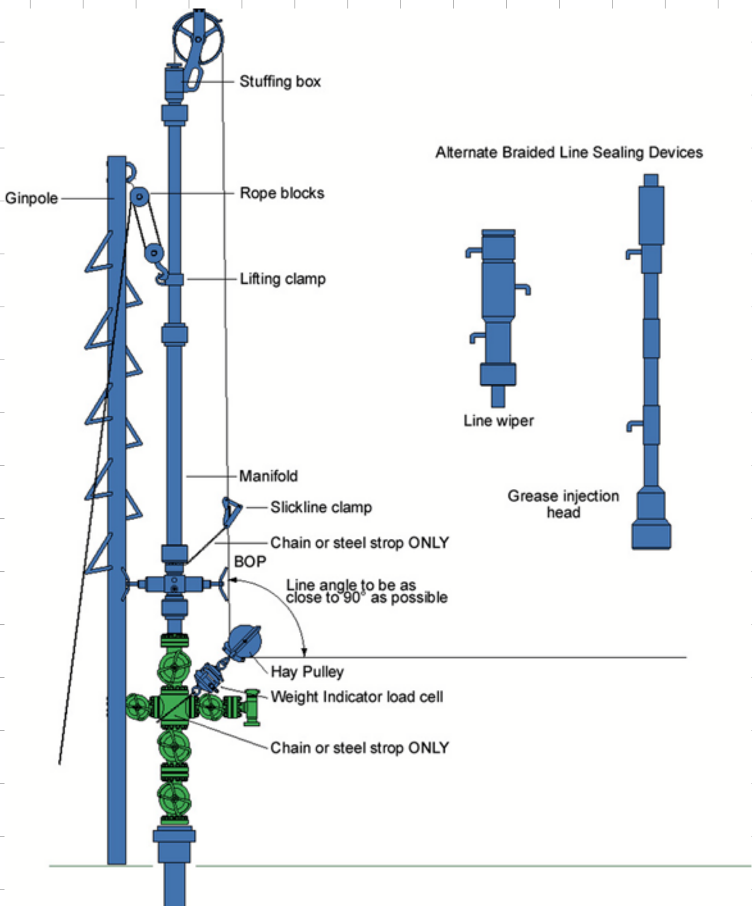
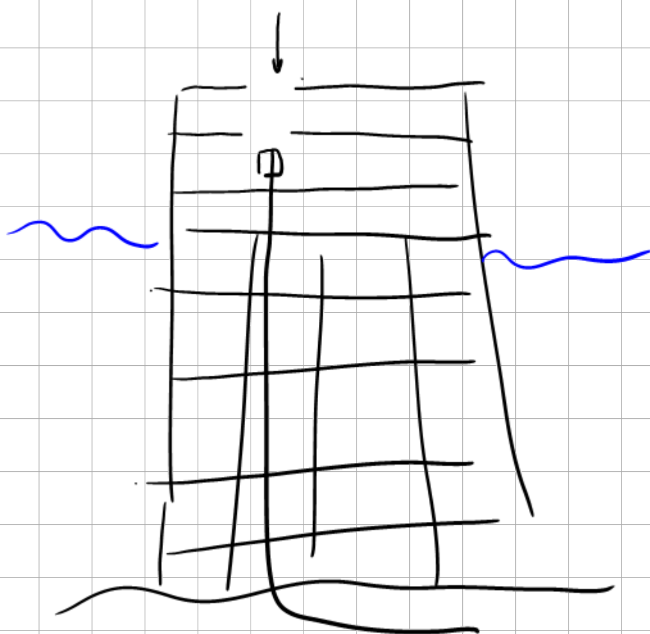


FIGURE 2-58. SHIFTING SEQUENCE OF A SLIDING SLEEVE USING SLICKLINE





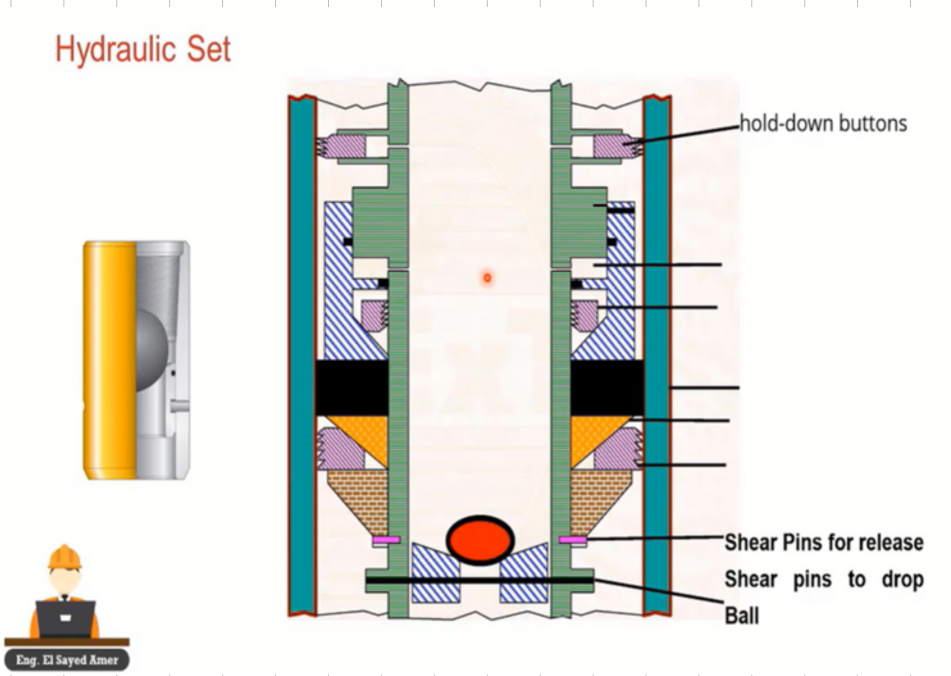
[Slickline video](#)

When the well has significant deviation or it is horizontal, the jarring is not effective anymore, so a solution is to use tractor, that is control with electronics from the surface. The tractor can move up and down and can allow to do rotation.

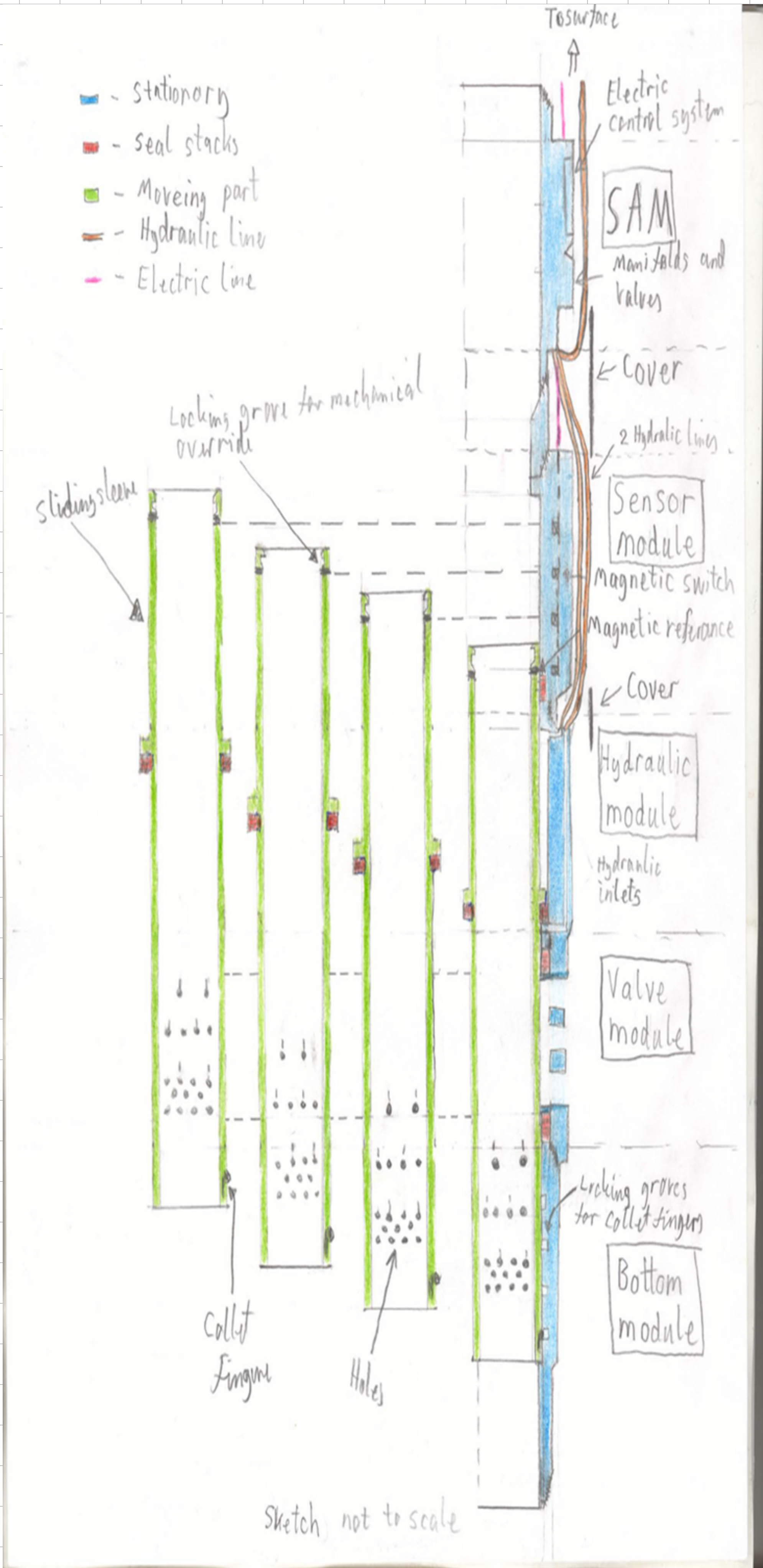
2 Cable Tractor (Aker Solutions/2010 /)

6.23 Cable tractor, equipped for milling (Aker solutions 2010)

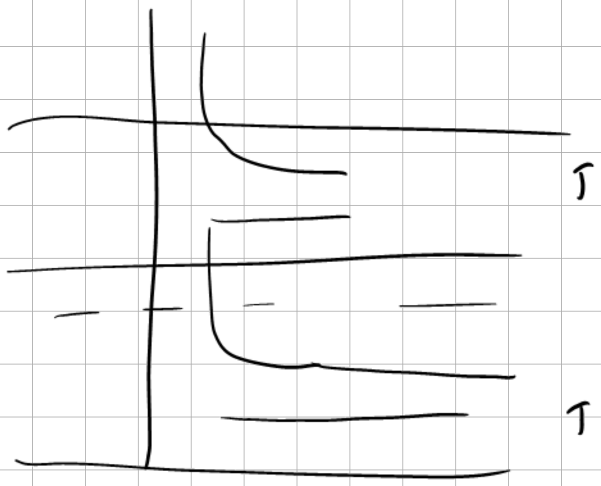
Packer, one example easier to understand



Inflow control valve with 4 positions

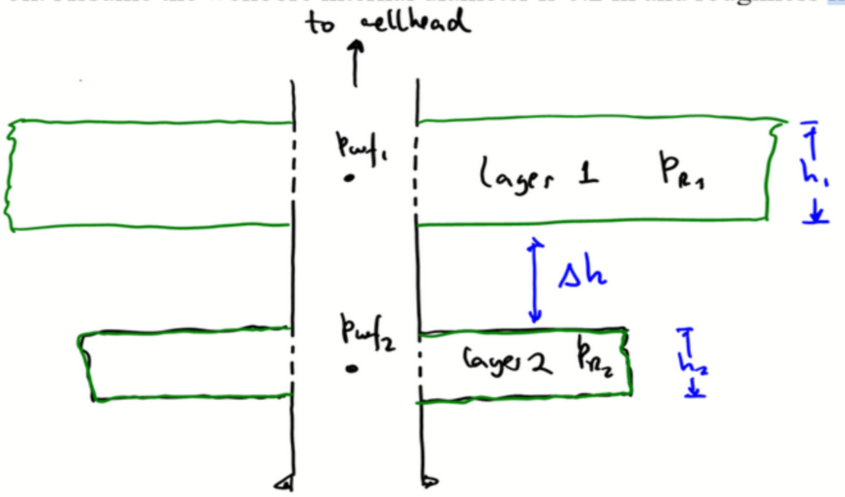


OUTLINE
20240920
-Class exercise - Downhole network in a vertical two-layer undersaturated oil well

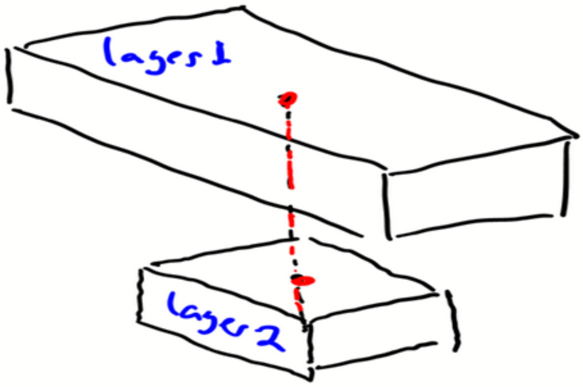


Problem 1: Planning a subsea well in the Alta-Gohta field

You are part of the well planning team in AkerBP that is in charge to design a vertical production well for the Alta-Gohta field development. The plan is to produce two oil layers using the same well. The well will be fully perforated throughout each layer. The two layers contain undersaturated oil. Assume the wellbore internal diameter is 0.2 m and roughness is 1.5 e-5 m.



A reservoir engineer has determined, considering neighboring wells, structural seals, etc. that the drainage volume of the well can be approximated by two rectangular boxes that are vertically stacked one above the other.



The layers have different lengths, widths, and thicknesses (layer 1 has a thickness of 20 m and layer 2 of 25 m). There is a 100 m thick shale layer in between the two oil bearing layers.

Due to the height difference between the layers, the flowing bottom-hole pressure of layer 1 is not the same as layer 2. The pressure difference between the bottom-hole pressures can be calculated using the Bernoulli equation between 1 and 2 (use a density of 715 kg/m3 and a viscosity of 1.6 cP). Assume that the bottom-hole location of each layer is located exactly in the middle of the layer.

Some information about the layers is provided in the following table

	Layer 1	Layer 2
Productivity index of vertical well [Sm3/d/bar]	2.9	7.7
Reservoir pressure [bara]	400	500
Bubble point pressure [bara]	253	185
GOR [Sm3/Sm3]	173	126

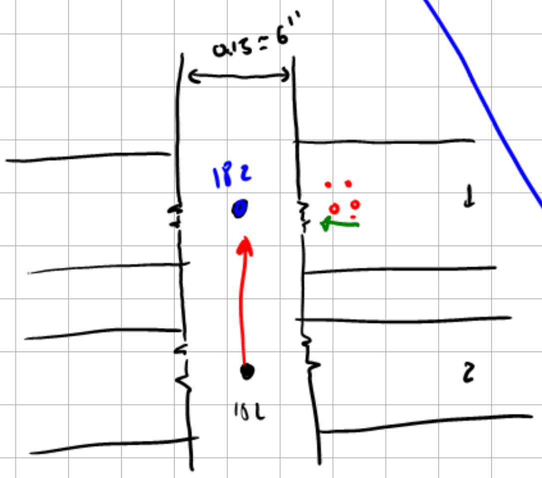
Task 1.

Calculate:

- maximum oil rate that can be produced from the well
- well GOR (R_p)

while ensuring the two following conditions:

- There is no gas is liberated in the reservoir and there is no gas liberated in the wellbore section from layer 2 to 1



$P_{wf_1} \geq P_{b_1} = 253$ bara

$P_{wf_2} \geq P_{b_2} = 182$ bara

$P_{wf_1} \geq P_{b_2} = 182$ bara

max ... is lowest

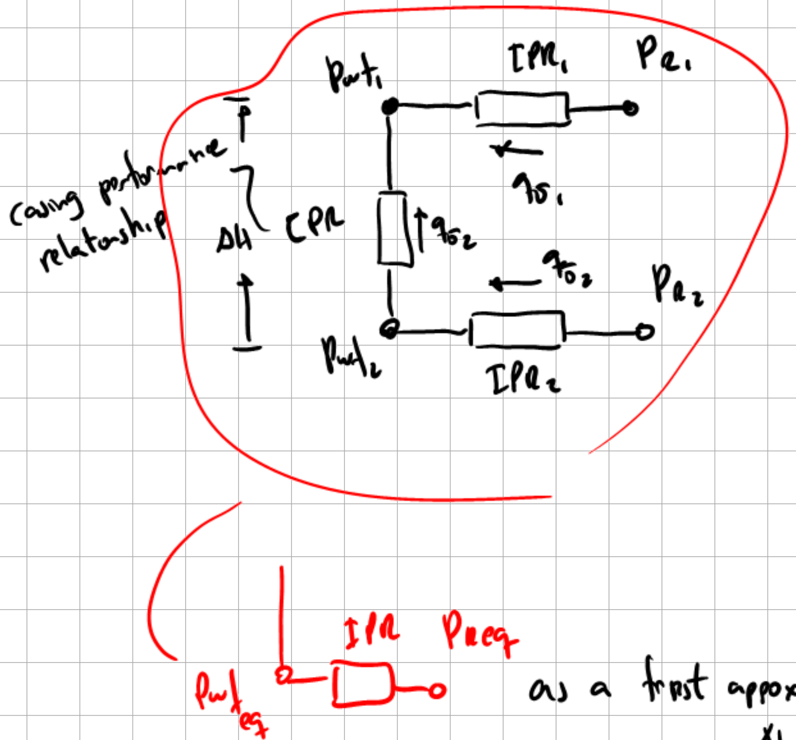
$q_{o_1} = J_1 (P_{R_1} - P_{wf_1})$

$q_{o_2} = J_2 (P_{R_2} - P_{wf_2})$

which P_{wf_1} should I prescribe for layer 1? 182 bara? $\boxed{253 \text{ bara?}}$

$P_{wf_1} = 253$ bara

$P_{wf_2} > P_{wf_1} > 182$ bara



1) set $p_{wf_1} = 253$ bara

$$q_{o_1} = 2.9\cdot(400-253) = 426.3\ Sm^3/d$$

p_{wf_2} ?

$$p_{wf_2} = p_{wf_1} + \Delta p_{casing}(q_{o_2})$$ 1 eq, 2 unknowns

$$q_{o_2} = J_2\,(P_{R_2} - p_{wf_2})$$ 1 eq

as a first approximation, neglect

$$\Delta p_{casing} = \Delta p_{Hydrostatic}(\Delta H, \rho) + \Delta p_{friction}(q_{o_2})$$

$$p_{wf_2} = \underbrace{p_{wf_1}}_{253\,bara} + \frac{\overbrace{9.81\cdot 715\cdot 122.5}^{Pa}}{1E5} = 8.59 \text{ bara}$$

$$p_{wf_2} = 261.6 \text{ bara}$$

$$q_{o_2} = 7.7\cdot(500-261.6) = 1835\ Sm^3/d$$

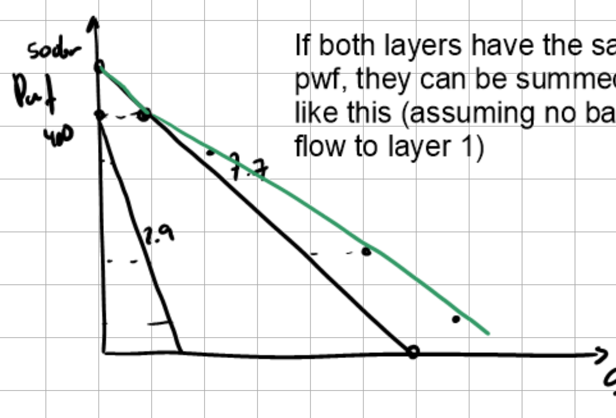
$$q_o = q_{o_1} + q_{o_2} = 1835 + 426 \approx 2262\ Sm^3/d$$

$$GOR = \frac{q_{o_1}\,GOR_1 + q_{o_2}\,GOR_2}{q_{o_T}} =$$

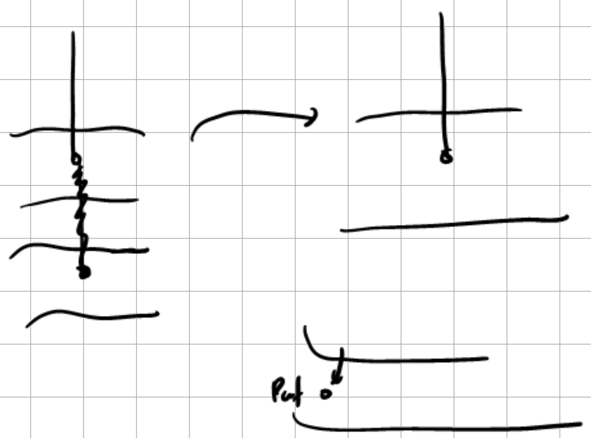
If You are given another value of pwf1, the solving process is exactly the same,

COMPOSITE IPR

Provide an IPR for the two Layers!

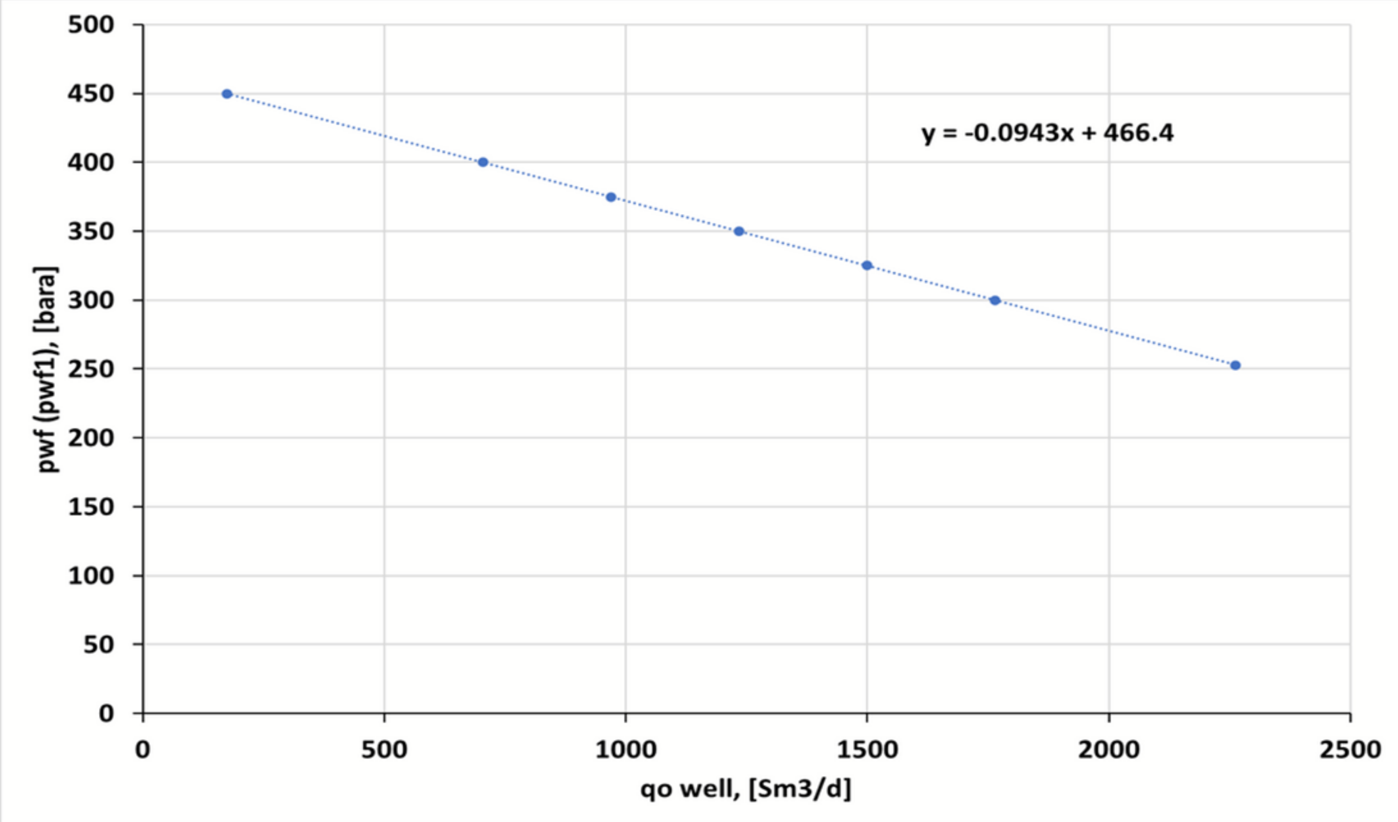


If both layers have the same pwf, they can be summed up like this (assuming no back flow to layer 1)



for composite IPR, $p_{wf} = p_{wf_1}$

q_{o_T} $f(p_{wf_1})$



composite IPR $P_R = 466.4$ bara (where all production from layer 2 is injected in layer 1)

$$y = \overset{J}{-0.0943} \cdot q_o + P_R$$

$$P_{wf} = -\frac{q_o}{J} + P_R$$

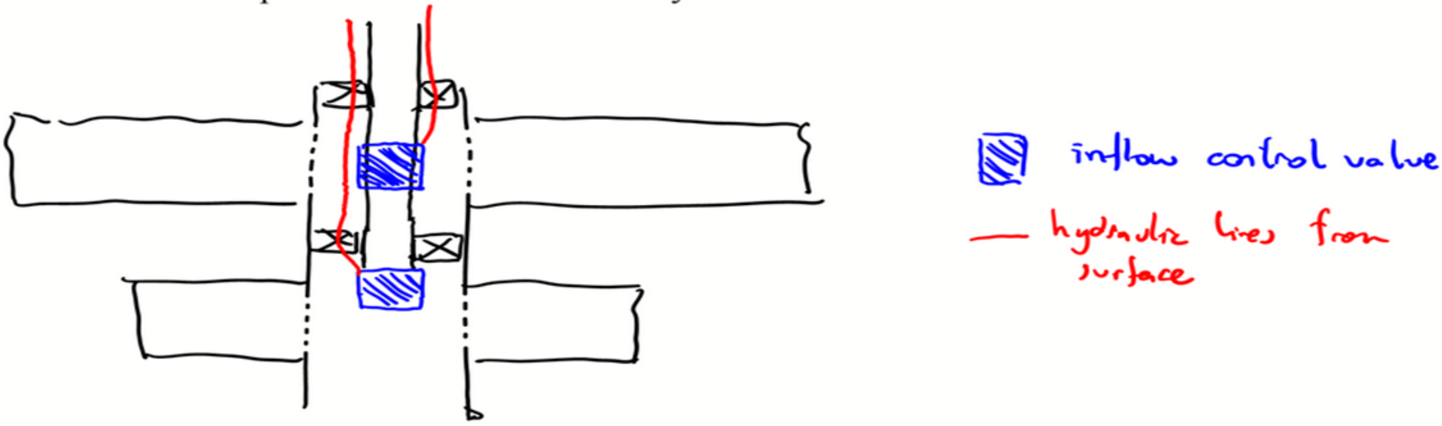
$$J = -\frac{1}{(-0.0943)} = 10.6 \approx J_1 + J_2$$

$$7.7 + 2.9 = 10.6$$

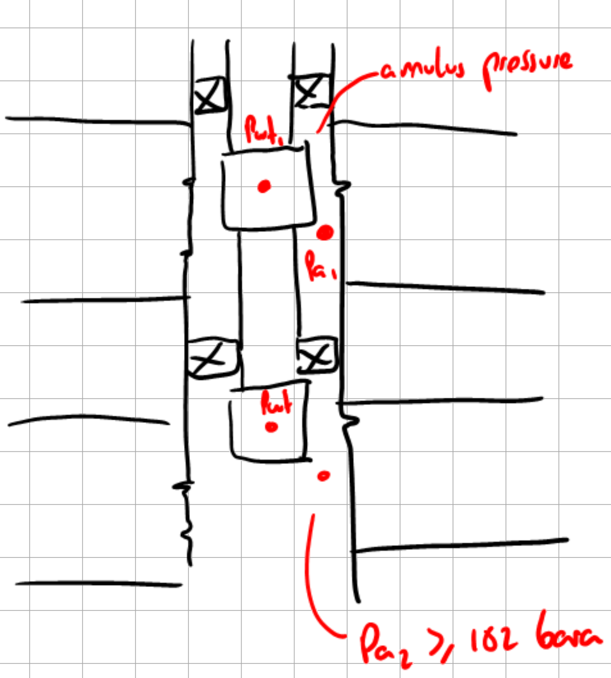
the composite IPR can then be used for flow equilibrium calculation in the wellbore (e..g by intersecting with TPR), conssidering one equilvalent layer, only for pwf>253 bara and pwf<466.5 bara)

Task 2.

Based on the results from task 1, one of your colleagues has suggested to use an inflow control valve to regulate the inflow of each layer, by creating a pressure drop. The colleague claims that this will allow to produce more oil from the system.



Do some calculations to verify the feasibility of this idea.



$$\overset{253}{P_{wt1}} = \overset{182}{P_{wt2}} - \underbrace{\Delta P_{ver}}_{}\ \ 0.(8\ \text{bar})$$

without any inflow control device

$$P_{a1} = P_{wt1}$$

$$P_{a2} = P_{wt2}$$

with an inflow control device

$$\overset{253}{P_{a1}} - \Delta p_{cv_1} = P_{wt1}$$

$$\overset{182}{P_{a2}} - \Delta p_{cv_2} = P_{wt2}$$

$$P_{wt2} - P_{wt1} \simeq 8.6 \text{ bara}$$

		LAYER 1	LAYER 2				
pann	[bara]	253	194	TOTAL	DP 2-->1 (calc)	DP 2-->1 (assumed)	difference
qo	[Sm3/d]	426.3	2359.3	2785.6	[bar]	[bar]	[bar]
pwf	[bara]	185	194		8.6	8.6	0.0
DP-ICV	[bar]	68	0				
GOR-well	[Sm3/Sm3]	133.2					

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U. S. DEPARTMENT OF THE INTERIOR
HAROLD L. ICKES, Secretary

BUREAU OF MINES
John W. Finch, Director

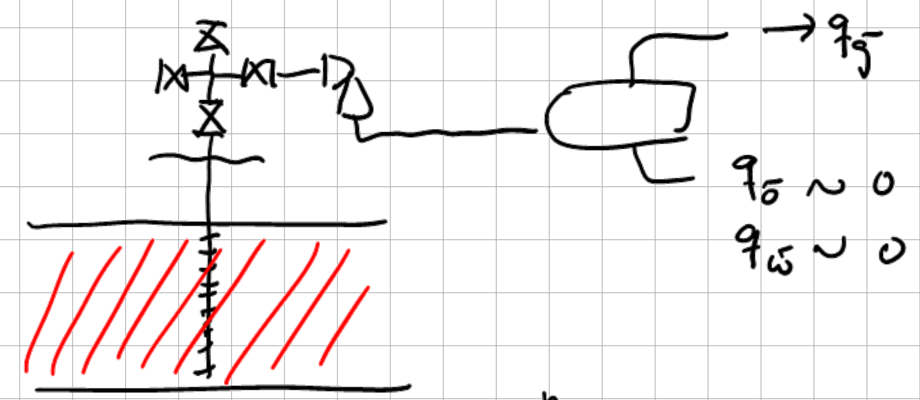
(1935)

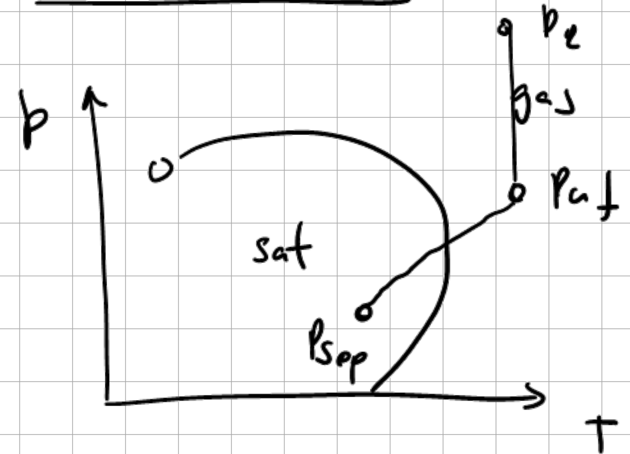
Monograph 7

Back-Pressure Data on Natural-Gas Wells and Their Application to Production Practices

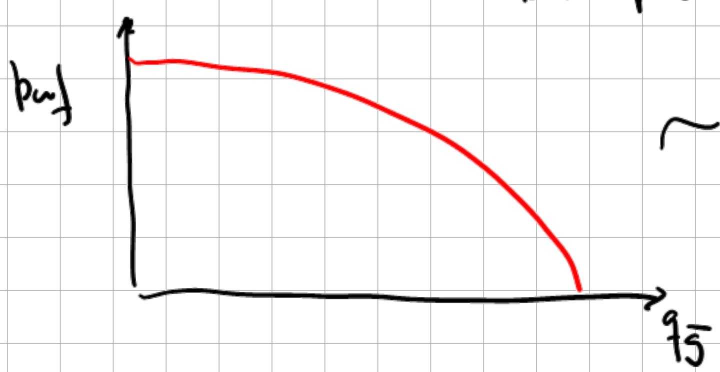
By

E. L. RAWLINS AND M. A. SCHELLHARDT





Back-pressure equation



empirically, measured in the field

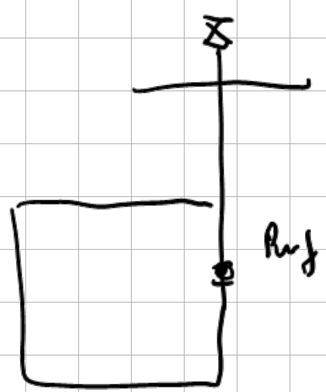
$$\sim \quad q_g = C\left(p_R^2 - p_{wf}^2\right)^n \quad \rightarrow \quad 0 \leq n \leq 1$$

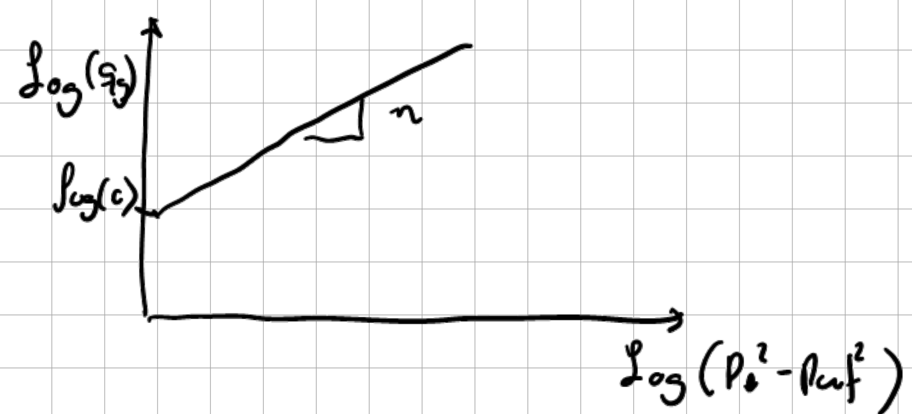
$$\frac{q_g}{C} = \left(p_R^2 - p_{wf}^2\right)^n \qquad \log$$

$$\log\left(\frac{q_g}{C}\right) = \log\left(\left(p_R^2 - p_{wf}^2\right)^n\right)$$

$$\log(q_g) - \log(C) = n \cdot \log\left(p_R^2 - p_{wf}^2\right)$$

$$\log(q_g) = \log(C) + n \log\left(p_R^2 - p_{wf}^2\right)$$





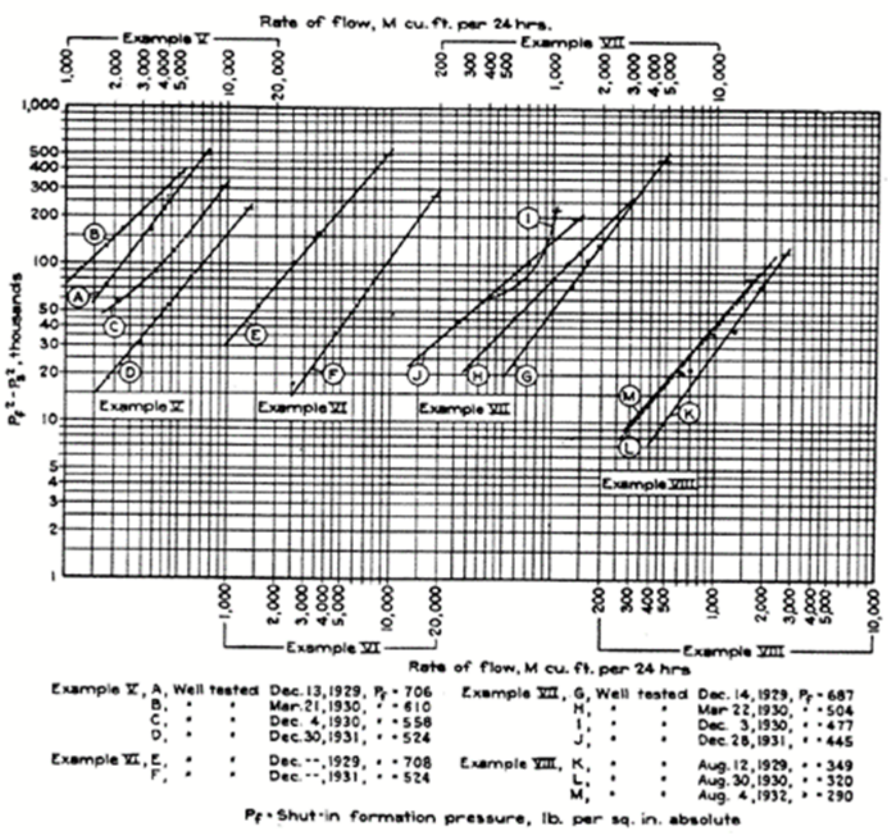
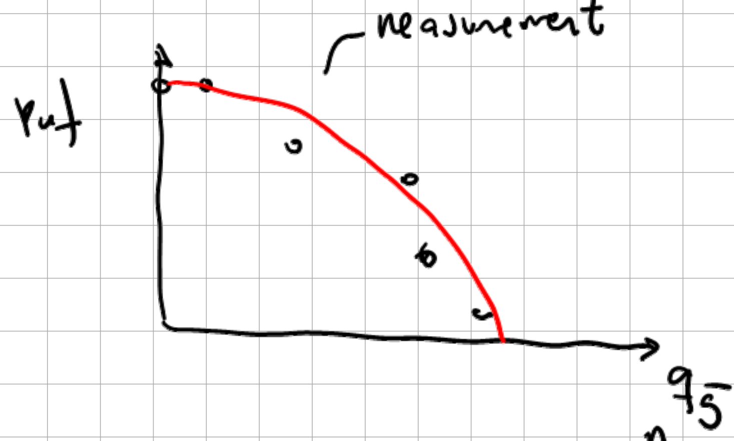


FIGURE 24.—Variation in delivery capacities of gas wells at different times in their productive lives, examples V, VI, VII, and VIII



$$q_g = C\left(p_R^2 - p_{wf}^2\right)^n$$

in Excel change C, n to match data

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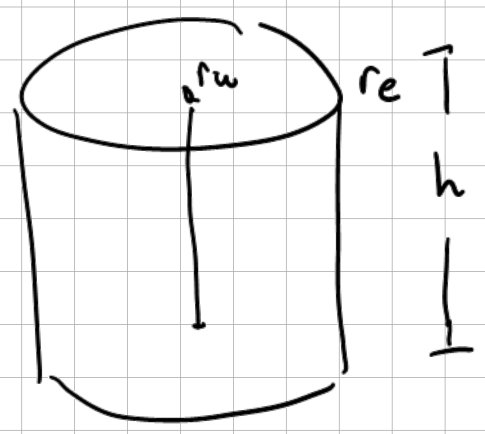
analytical derivation of Dry gas IPR

$q_{\bar{g}}$

$$v = \frac{k}{\mu_g}\frac{dp}{dr}$$

$$\frac{q_g}{2\pi r h} = \frac{k}{\mu_g}\frac{dp}{dr}$$

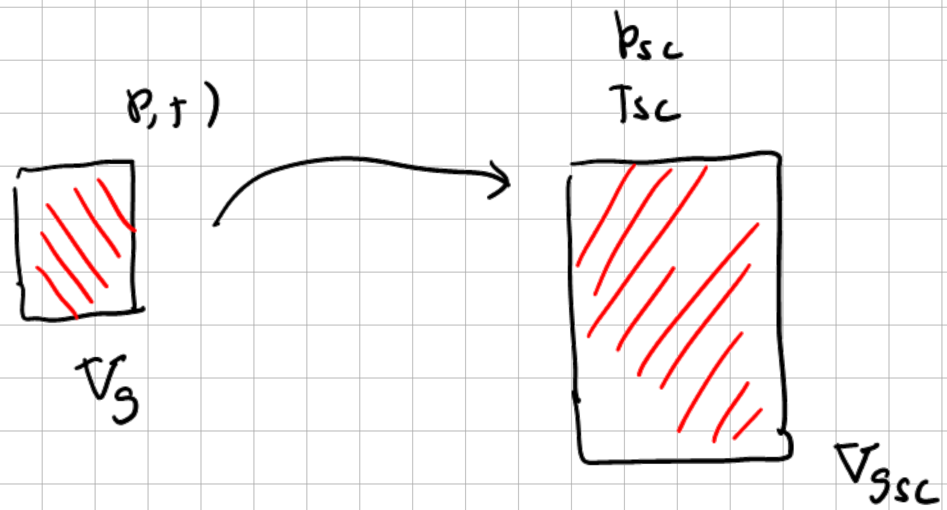


$$q_g = f(q_{\bar{g}})$$

$$B_g = \frac{q_g}{q_{\bar{g}}}$$

$$q_g = q_{\bar{g}} \cdot B_g$$

$$\int_{r_w}^{r_{@pR}} \frac{q_{\bar{g}}}{r} = \int_{p_{wf}}^{p_R} \frac{2\pi k h}{B_g \mu_g}\frac{dp}{dr}$$



$$B_g = \frac{V_g}{V_{\bar{g}}}$$

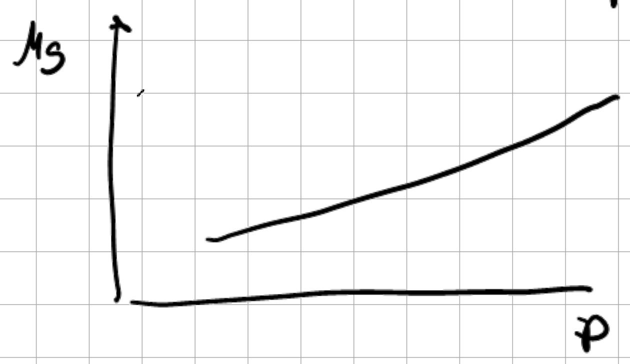
gas volume factor

$$B_g = f(p, T)$$

$$B_g \sim 1E-2 \rightarrow 1E-3$$

for PSS $$q_{\bar{g}} = \frac{2\pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)}$$

$$\int_{p_{wf}}^{p_e} \frac{1}{B_g \mu_g} dp \rightarrow ?$$



mass conservation

$$q_{\bar{g}} \cdot \rho_{\bar{g}} = q_g \rho_g$$

$$B_g = \frac{q_g}{q_{\bar{g}}} = \frac{\rho_{\bar{g}}}{\rho_g}$$

Real gas equation

$$\frac{p}{\rho} = ZRT \qquad R = \frac{R_u}{MW}$$

gas deviation factor

deviation from ideal gas

$$\rho = \frac{p}{ZRT}$$

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Boyle (English)

Hooke (English)

Charles (French)

Gay-Lussac (French)

Avogadro (Italian)

z ~

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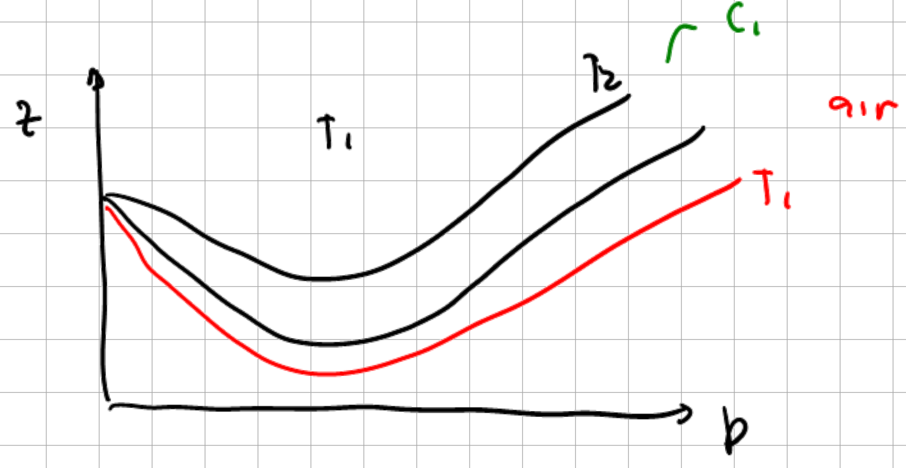
(INCORPORATED)

Density of Natural Gases

BY MARSHALL B. STANDING* AND DONALD L. KATZ,* MEMBER A.I.M.E.

(New York Meeting, February 1941)



$$T_r = \frac{T}{T_c} \qquad p_r = \frac{p}{p_c}$$

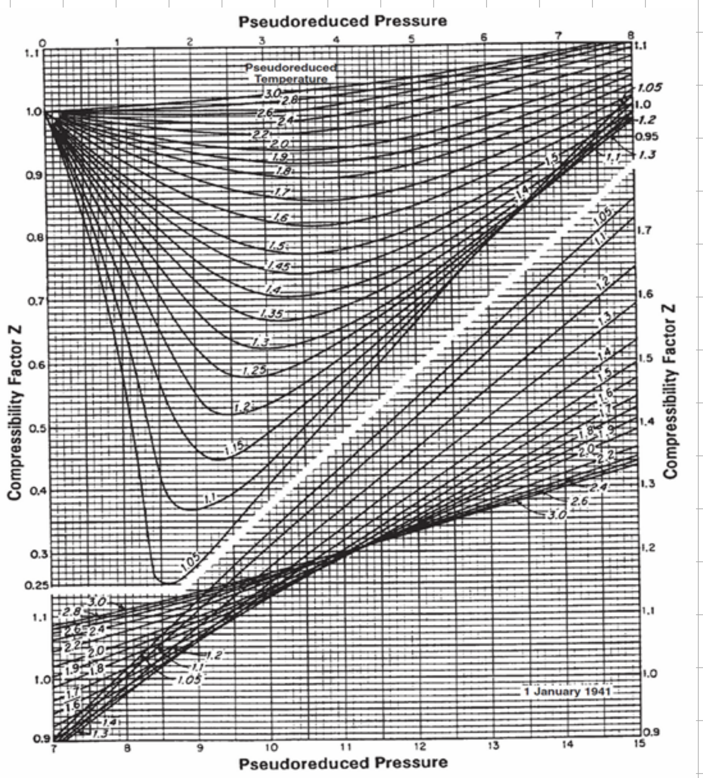
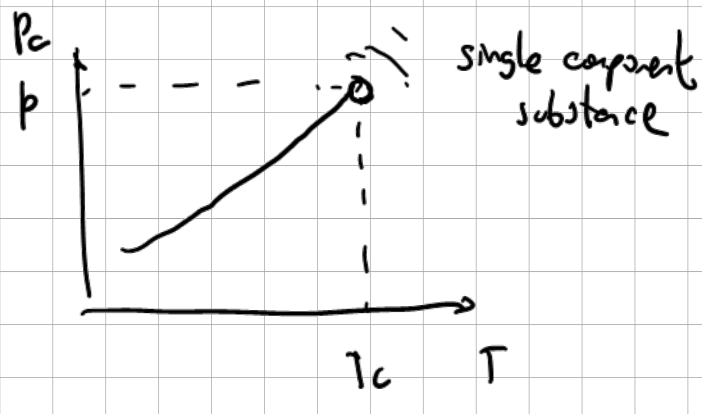


Fig. 3.6—Standing-Katz[4] Z-factor chart.

T_c, p_c

$T_c, p_c = f(MW)$



Gas composition

Component	mole fraction
C_1	z_{c1}
C_2	z_{c2}
C_3	z_{c3}

$$\sum_1^N z_i = 1$$

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$$MW_{mix} = \sum_{1}^{N} Mw_i z_i \qquad T_c, p_c = f(MW_{mix}) \rightsquigarrow \text{Sutton}$$

$$B_g = \frac{\rho_{\bar{g}}}{\rho_g} \qquad \rho_{\bar{g}} = \frac{p_{sc}}{\cancel{z_{sc}}\, R\, T_{sc}} \qquad \rho_g = \frac{p}{z R T}$$

1

$$B_g = \frac{p_{sc}}{T_{sc}} \cdot \frac{z T}{p}$$

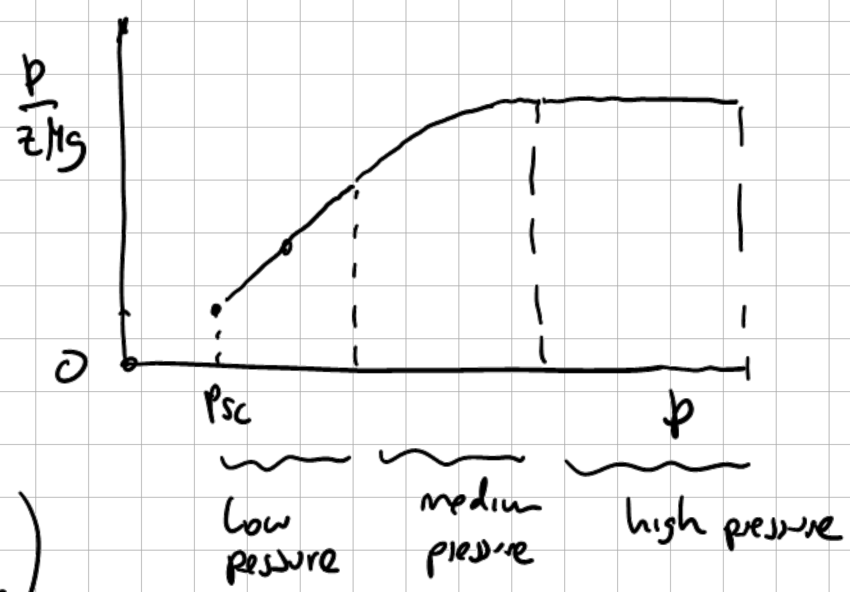
p_{sc} 1.01325 bara

T_{sc} 15.56 °C

$T = T_R$

$$q_{\bar{g}} = \frac{2 \pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)} \int_{p_{wf}}^{p_R} \frac{1}{\mu_g B_g}\, dp$$

$$q_{\bar{g}} = \frac{2 \pi k h}{\ln\left(\frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \int_{p_{wf}}^{p_R} \frac{p}{z \mu_g}\, dp$$



low pressure (p_{wf}, p_R)

$$q_g = \frac{2 \pi k h}{\left(\ln \frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R}\, 0.5 \left[p_R^2 - p_{wf}^2\right] \left(\frac{1}{z \mu_g}\right)_{@ p_R}$$

in the low pressure region $\left(\frac{1}{\mu_g z}\right) = \text{const}$

$$q_{\bar{g}} = C\left(p_R^2 - p_{wf}^2\right)^n$$

if $n = 1$ it's a match!

high pressure

$$q_{\bar{g}} = \frac{2 \pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \underbrace{\left(\frac{p}{\mu B_g}\right)_{@}}_{\text{avg pressure}} \int_{p_{wf}}^{p_R} dp = \underbrace{\frac{2 \pi k h}{\left(\ln \frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \frac{p_R}{(\mu_g z)_{@ p_R}}}_{\text{constant}} (p_R - p_{wf})$$

similar to undersaturated oil IPR

doesn't resemble $q_{\bar{g}} = C\left(p_R^2 - p_{wf}^2\right)^n$

$$q_{\bar{o}} = J\,(p_R - p_{wf})$$

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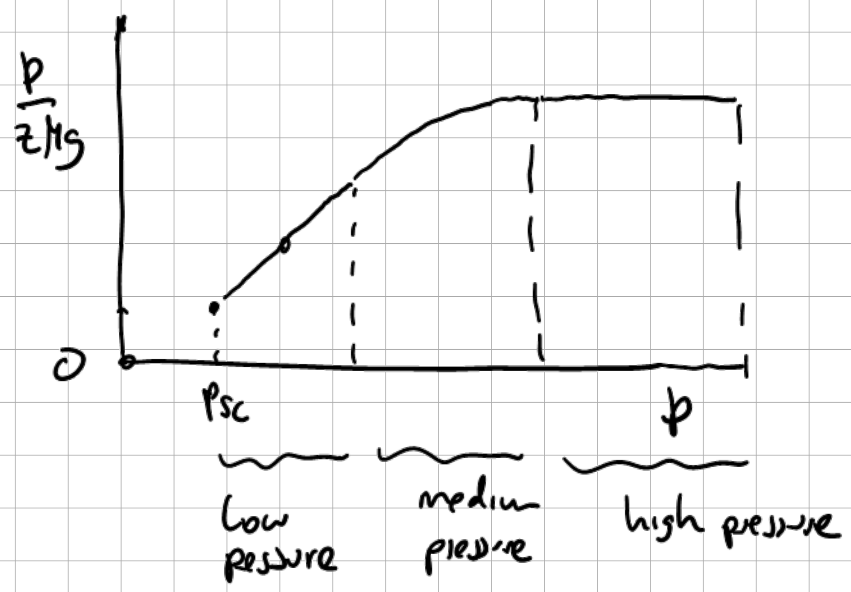
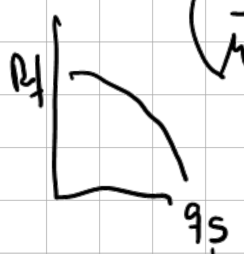
Video 15 - Dry gas IPR (Part 2)

$$q_g = \frac{2\pi k h}{\ln\left(\frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \underbrace{\int_{p_{wf}}^{p_e} \frac{p}{z \mu_g} dp}$$



LP

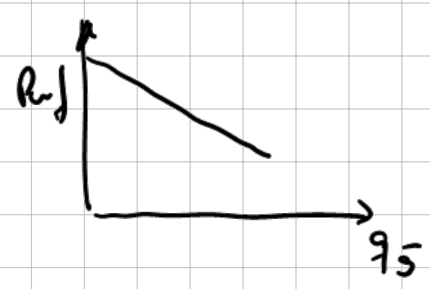
$$\left(\frac{1}{\mu_g z}\right)_{@p_R} \frac{(p_R^2 - p_{wf}^2)}{2}$$



?

HP

$$\frac{p_R}{(\mu_g z)_{@p_R}} (p_R - p_{wf})$$



p [bara] $\quad q_g = [Sm^3/d] \quad p_{sc} = 1.01325$ bara

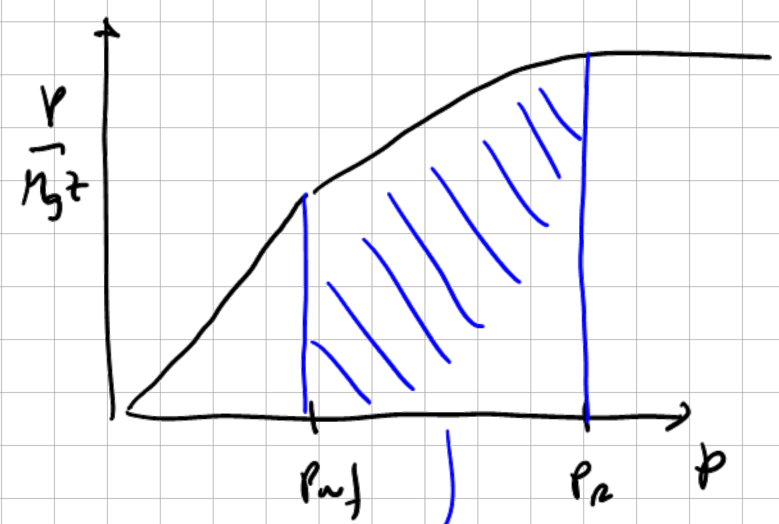
T [K] $\quad \mu_g = [cp] \quad T_{sc} = 15.56$ °C

k [md]

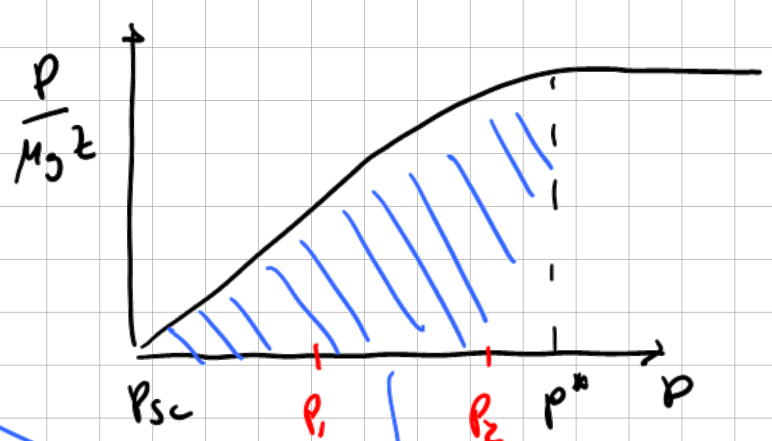
excluding the 2 !

$$q_g = \frac{k h \; 7.63}{\left(\ln \frac{r_e}{r_w} - 0.75\right) T_R} \underbrace{2 \int_{p_{wf}}^{p_R} \frac{p}{\mu_g z} dp}$$

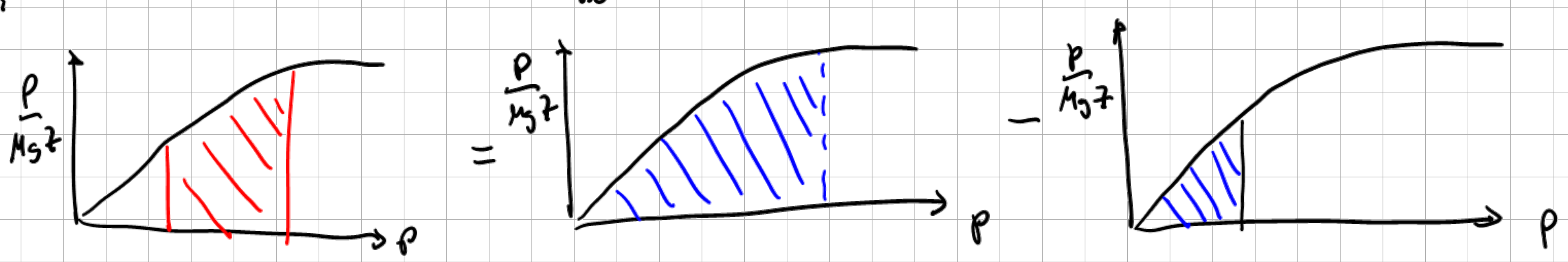
$$(\underbrace{\bigcirc}_{p_R} - \underbrace{\bigcirc}_{p_{wf}})$$



$$m(p^*) = 2 \int_{0 \to p_{sc}}^{p^*} \frac{p}{\mu_g z} dp$$



$$2 \int_{p_1}^{p_2} \frac{p}{\mu_g z} dp = 2 \int_{p_{sc}}^{p_2} \frac{p}{\mu_g z} dp - 2 \int_{p_{sc}}^{p_1} \frac{p}{\mu_g z} dp = m(p_2) - m(p_1)$$



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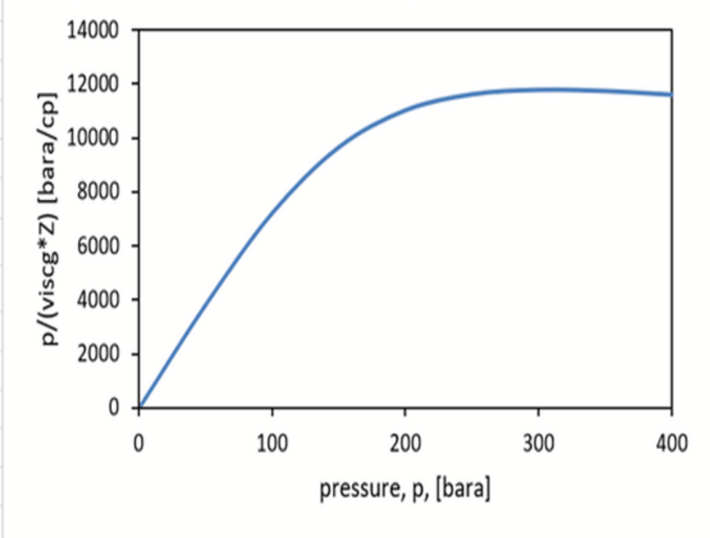
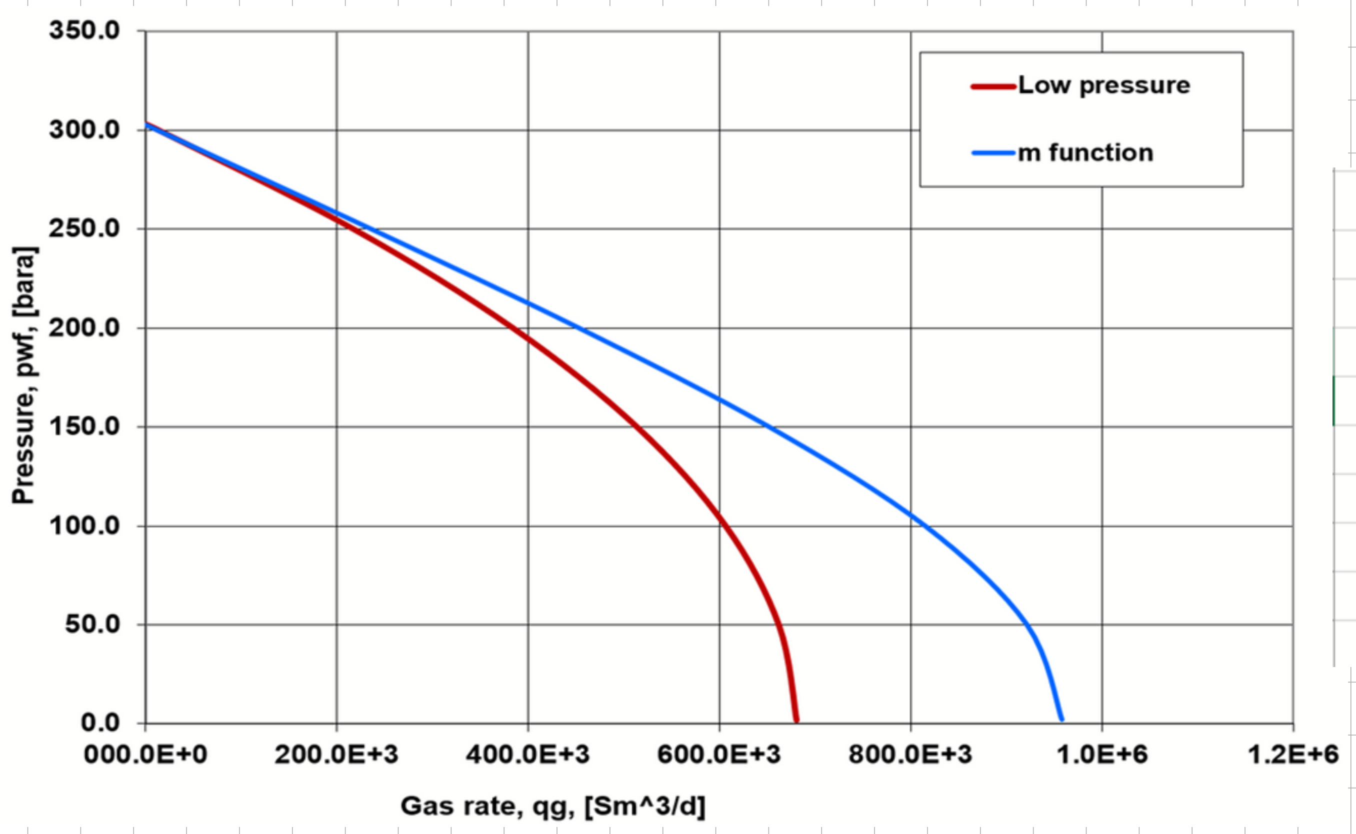
IPR og

$$q_g = \frac{7.63\, k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right)} \frac{1}{T_R} \left[m(P_R) - m(p_{wf}) \right] \quad ?$$

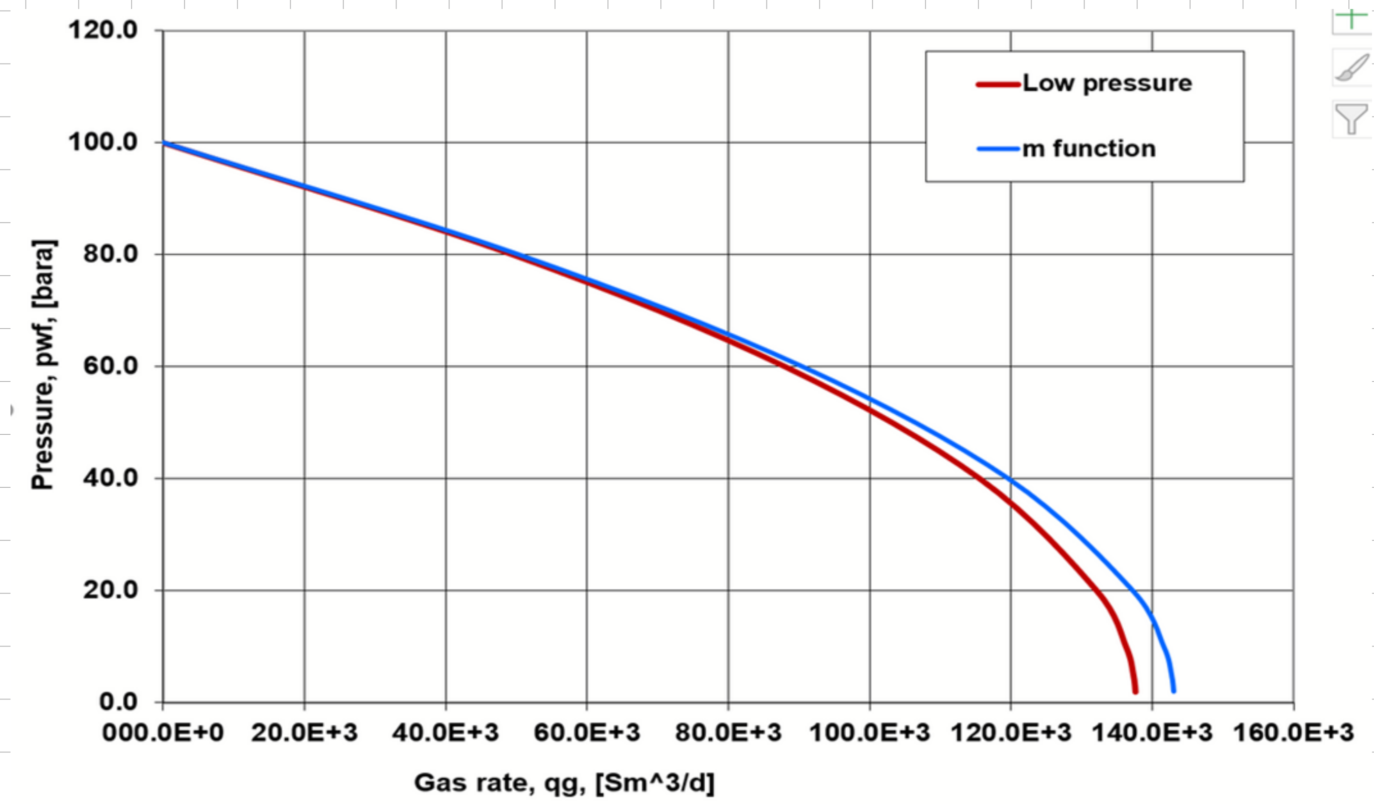
Well Data		
pR	[bara]	400
TR	[C]	105
TR	[K]	378
zR	[-]	1.069
dengR	[kg/m3]	276
ViscosityR	[cp]	0.032

p [bara]	Z [-]	deng [kg/m3]	viscg [cp]	p/viscg*Z [bara/cp]
2	0.997	1.5	0.013	152
50	0.932	39.6	0.014	3825
100	0.882	83.6	0.016	7224
150	0.859	128.8	0.018	9662
200	0.867	170.2	0.021	11029
250	0.899	205.1	0.024	11622
300	0.948	233.6	0.027	11788
350	1.006	256.8	0.030	11745
400	1.069	276.1	0.032	11607





pwf bara	LP - qg [Sm3/d]	m(p) [bara2/cp]	qg [Sm3/d]
303.0	000.0E+0	5026555	000.0E+0
300.0	13.4E+3	4955852	13.5E+3
250.0	217.4E+3	3783006	237.1E+3
200.0	384.3E+3	2645766	453.9E+3
150.0	514.1E+3	1603483	652.6E+3
100.0	606.8E+3	750299.5	815.2E+3
50.0	662.5E+3	191885.9	921.7E+3
2.0	681.0E+3	226.2864	958.2E+3



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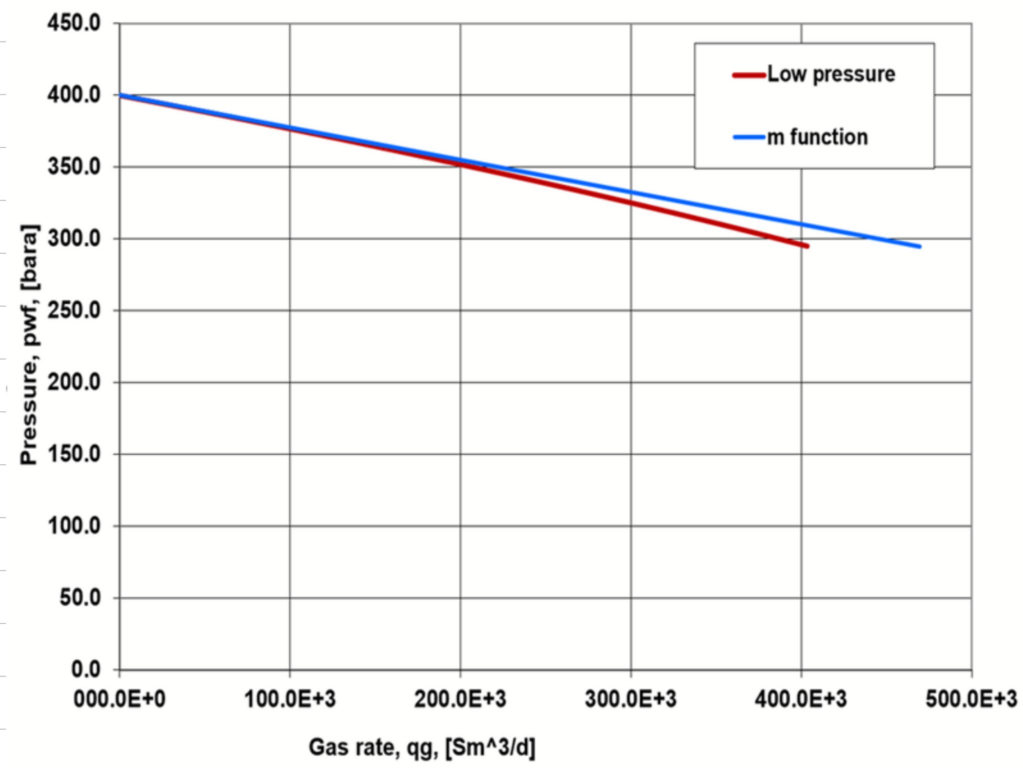

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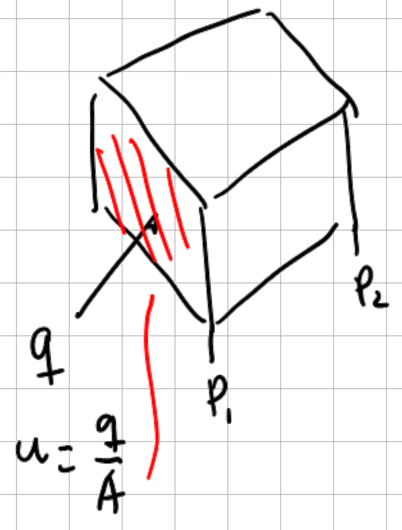
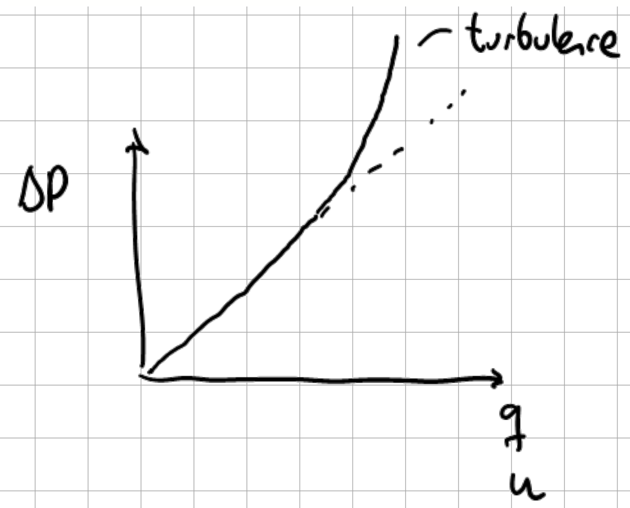
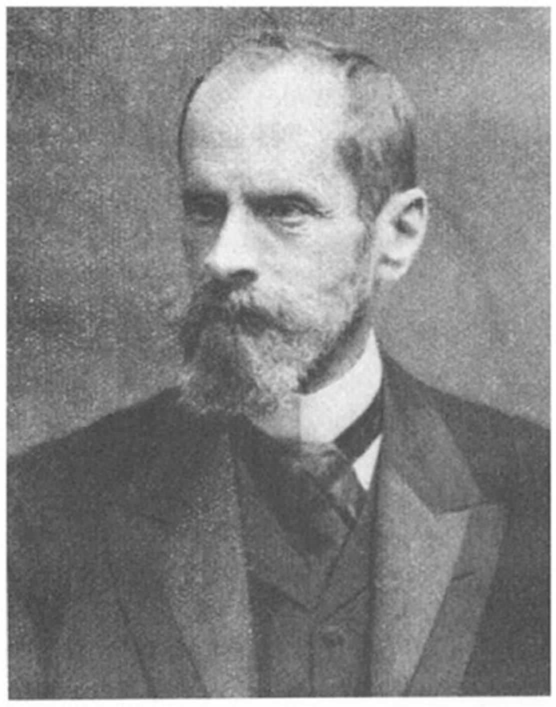
[19] P. Forchheimer. *Wasserbewegung durch Boden.* Zeitschrift des Vereines Deutscher Ingenieuer, 45 edition, 1901.

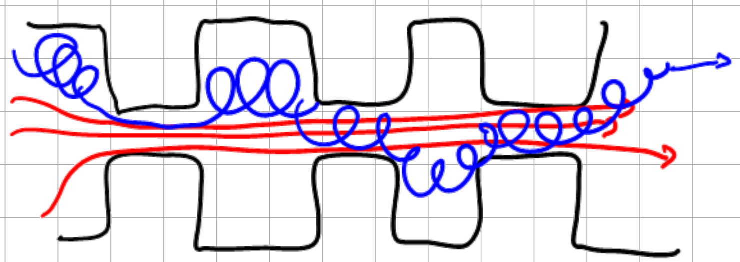






Professor Philipp Forchheimer.

not valid for medium-high rate gas wells
high rate sat oil



$$\Delta P = A \cdot u + B u^2$$

$$\Delta P = A u \left(1 + \frac{B}{A} u\right)$$

Use of Short Term Multiple Rate Flow Tests To Predict Performance of Wells Having Turbulence

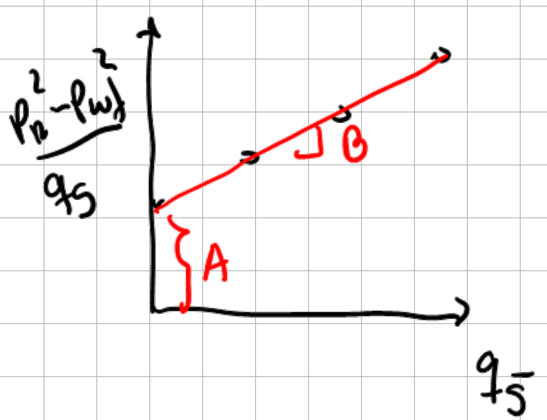
(1976)

Lloyd G. Jones and E. M. Blount, Mobil Research and Development Corp., and O. H. Glaze, Mobil Oil Corp., Members SPE-AIME

LP dry gas wells

$$\left(p_R^2 - p_{wf}^2\right) = A\, q_{\bar{g}} + B\, q_{\bar{g}}^2$$

$$\frac{\left(p_R^2 - p_{wf}^2\right)}{q_{\bar{g}}} = A + B\, q_{\bar{g}}$$



quadratic equation

$$a x^2 + b x + c = 0$$

$$q_{\bar{g}} = \frac{7.63\, k h}{T_R\, M_g\, z} \frac{\left(p_R^2 - p_{wf}^2\right)}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right)}$$

$$\left(p_R^2 - p_{wf}^2\right) = \frac{T_R\, M_g\, z}{7.63\, k h}\left[\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right) q_{\bar{g}}\right] + B\, q_{\bar{g}}^2$$

rate dependent skin

$$B = D\, \frac{z\, M_g\, T_R}{7.63 \cdot k\, h}$$

$$\left(p_R^2 - p_{wf}^2\right)\frac{7.63\, k h}{T_R\, M_g\, z} = \left(\ln \frac{r_e}{r_w} - 0.75 + S\right) q_{\bar{g}} + D\, q_{\bar{g}}^2$$

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$$\frac{(p_R^2 - p_{wf}^2)}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + D\, q_{\bar{g}}\right)} \frac{7.63\ kh}{T_R\, \mu_g\, z} = q_{\bar{g}}$$

rate-dependent skin $\quad D = \left[\frac{d}{Sm^3}\right]$

$\hookrightarrow D = f(?)$

alternative approach

USBM (1935) $\quad q_{\bar{g}} = C\left(p_R^2 - p_{wf}^2\right)^n \quad n\,?$ $\nearrow 1$ laminar (Darcy flow) $\searrow 0.5$ turbulent (HVF)

$$q_{\bar{g}} = \underline{C\ \left(p_R^2 - p_{wf}^2\right)^{n}}$$ — accounts for turbulent flow

$$C = \frac{(7.63\ kh)^n}{(T\mu_g Z)^n D^{1-n}[\ln(r_e/r_w) - 0.75 + s]^{2n-1}}.$$

extends to other IPRs

$$q_{\bar{g}} = C\left[m(p_R) - m(p_{wf})\right]^n$$

$$C = \frac{(7.63\ \ kh)^n}{T^n D^{1-n}[\ln(r_e/r_w) - 0.75 + s]^{2n-1}}.$$

$$q_{\bar g} = \frac{2\pi k h}{\ln\left(\frac{r_e}{r_w}-0.75\right)}\,\frac{7.63}{T_R}\underbrace{\int_{p_{wf}}^{p_R}\frac{p}{z\,\mu_g}\,dp}$$

LP

$$\left(\frac{1}{\mu_g z}\right)_{@p_R}\frac{\left(p_R^2-p_{wf}^2\right)}{2}$$

$$\frac{\left(m(p_R)-m(p_{wf})\right)}{2}$$

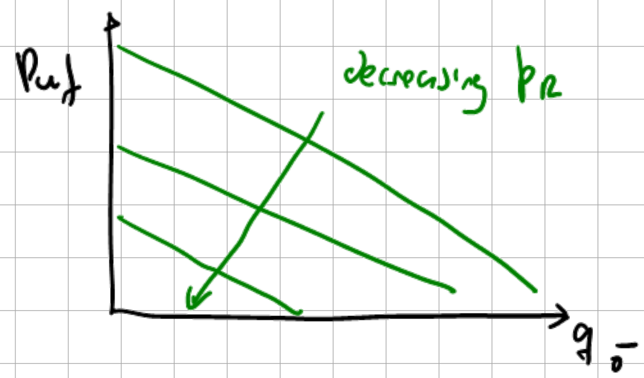
HP

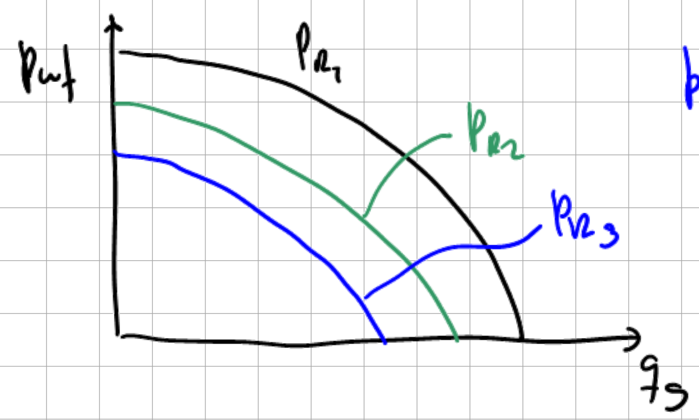
$$\frac{p_R}{(\mu_g z)_{@p_R}}\left(p_R-p_{wf}\right)$$

$$q_{\bar o} = \underset{\sim}{J}\left(p_R - p_{wf}\right)$$

geometry →

fluid properties → $(\mu_o B_o)_{@p_{av}}$



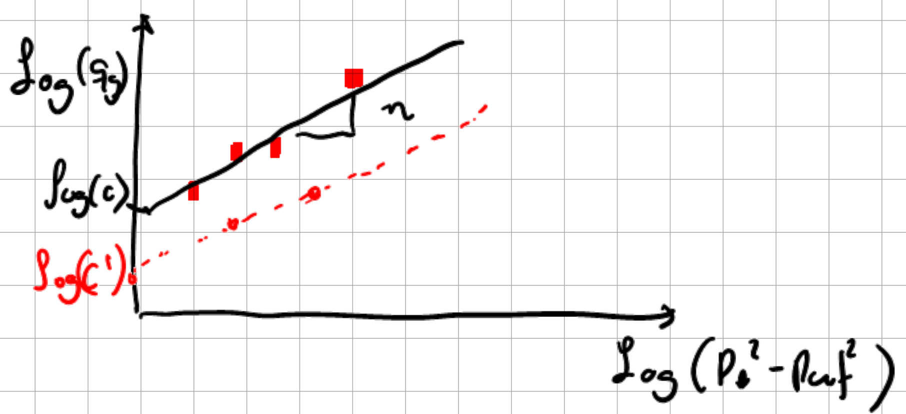


$p_{R_1} > p_{R_2} > p_{R_3}$

Diagnostic of well productivity issues

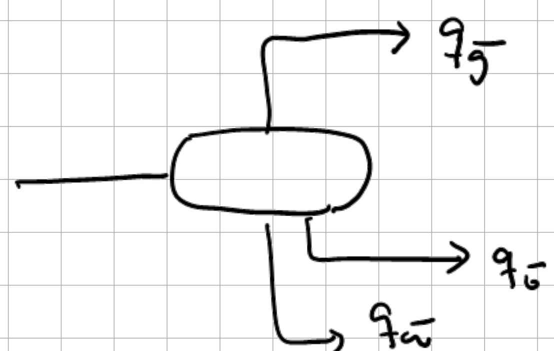
$$q_{\bar g} = C\left(p_R^2 - p_{wf}^2\right)^n$$

$$\log(q_{\bar g}) = \log(C) + n\,\log\left(p_R^2 - p_{wf}^2\right)$$



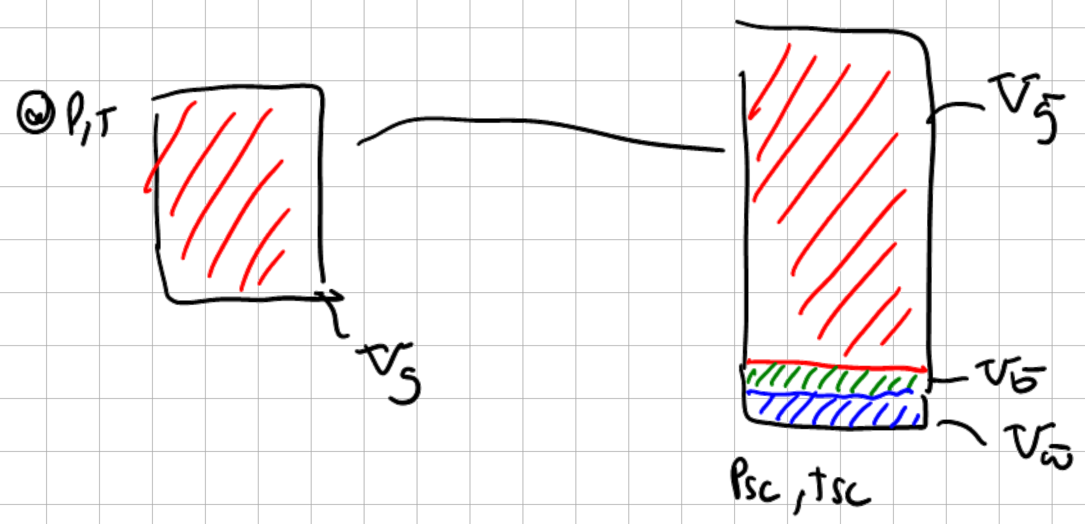
- Gas wells produce condensate $q_{\bar o}$, water $q_{\bar w}$

$$CGR = \frac{q_{\bar o}}{q_{\bar g}} \qquad WGR = \frac{q_{\bar w}}{q_{\bar g}}$$



- the same IPR equations can be used as long as flow $p_R \to p_{wf}$ is single phase

B_g



$$B_{gd} = \frac{V_g}{V_{\bar g}}$$

$$B_{gw} = \frac{V_g}{V_{\bar g} + V_{\bar o} + V_{\bar w}}$$

$\frac{p}{\rho} = zRT$ ⇝ assumes single-phase gas

$\int_{p_{wf}}^{p_R} \frac{p}{z\mu_g} dp$ ⇝ neglects the dropout of condensate / water

If there is dropout of condensate and water ⇝ generate a table of B_{gd} {e.g. EOS}

$\int_{p_{wf}}^{p_R} \frac{1}{\mu_g B_{gd}} dp$ ← integrate B_{gd} directly

---

Expressions are available for horizontal wells:

SPE 99712

Generalized Horizontal Well Inflow Relationships for Liquid, Gas, or Two-Phase Flow

R. Kamkom and D. Zhu, Texas A&M U.

base pressure. The IPR equation for horizontal gas wells in term of the real gas pseudo-pressure is

$$q_g = \frac{kL\left(m(\overline{p}) - m(p_{wf})\right)}{1424T\left(\ln\left[\frac{hI_{ani}}{r_w(I_{ani}+1)}\right] + \frac{\pi y_b}{hI_{ani}} - 1.224 + s\right)} \quad (13)$$

$$I_{ani} = \sqrt{\frac{k_H}{k_V}}$$

IPR:

$$q_g = \underbrace{C}_{\text{geometry}} \int_{p_{wf}}^{p_R} \underbrace{F(p)}_{\text{function of fluid properties}} dp$$

20240923
OUTLINE
-New and final version of Exercise set 2 posted
-Summary of Meeting with Reference group
-New volunteer for member of reference group?
-Recap of YT lectures of last week
-Class exercise (Problems 3 and 4 of exercise set 2)

Problem 3. Drawing an analogy to the dry gas IPR, the IPR for a CO_2 injector is:

$$q_{\bar{g}} = \underbrace{\frac{k \cdot h}{18.68 \cdot \left[ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]}}_{C_R} \int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp$$

Using the pure CO_2 PVT data provided in the Excel file, compute the term

$$\int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp$$

$$\int_{300}^{200} \frac{1}{\mu_g B_g} dP \qquad \int_{400}^{200} \frac{1}{\mu_g B_g} dP$$

p_R (upper limits), p_{wf} (lower limits)

$$\underbrace{\int_{200}^{200} \frac{1}{\mu_g B_g} dP}_{0} \qquad \int_{500}^{200} \frac{1}{\mu_g B_g}$$

for reservoir pressure 200 bara, and flowing bottom-hole pressures ranging from 200 to 500 bara (for example use 200, 300, 400, and 500.

Will it be possible to approximate the IPR of this well with a linear equation? (something like):

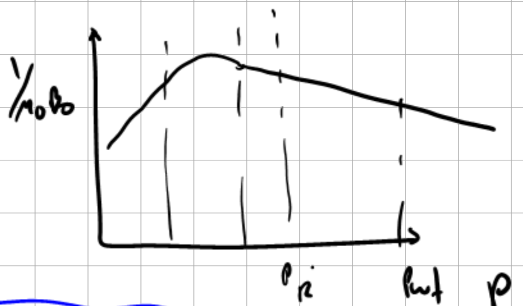
$$q_{\bar{g}} = J(p_{wf} - p_R)$$

$$q_{\bar{g}} = J(P_{wf} - P_R)\ ?$$

Use the Excel file provided to solve this problem.

Additional information.

Stepwise solution:
-Calculate Bg for all pressures
-Calculate 1/Bg*mug for all pressures
-See if the interval pwf and pR fall in the downward linear part
-Calculate (numerically, using the trapezoidal rule):



$$B_g = \frac{\rho_{\bar{g}}}{\rho_g}$$

DANGER! OBS! only valid if the fluid has the same mass at local at standard conditions and there is only one phase at standard conditions!!!

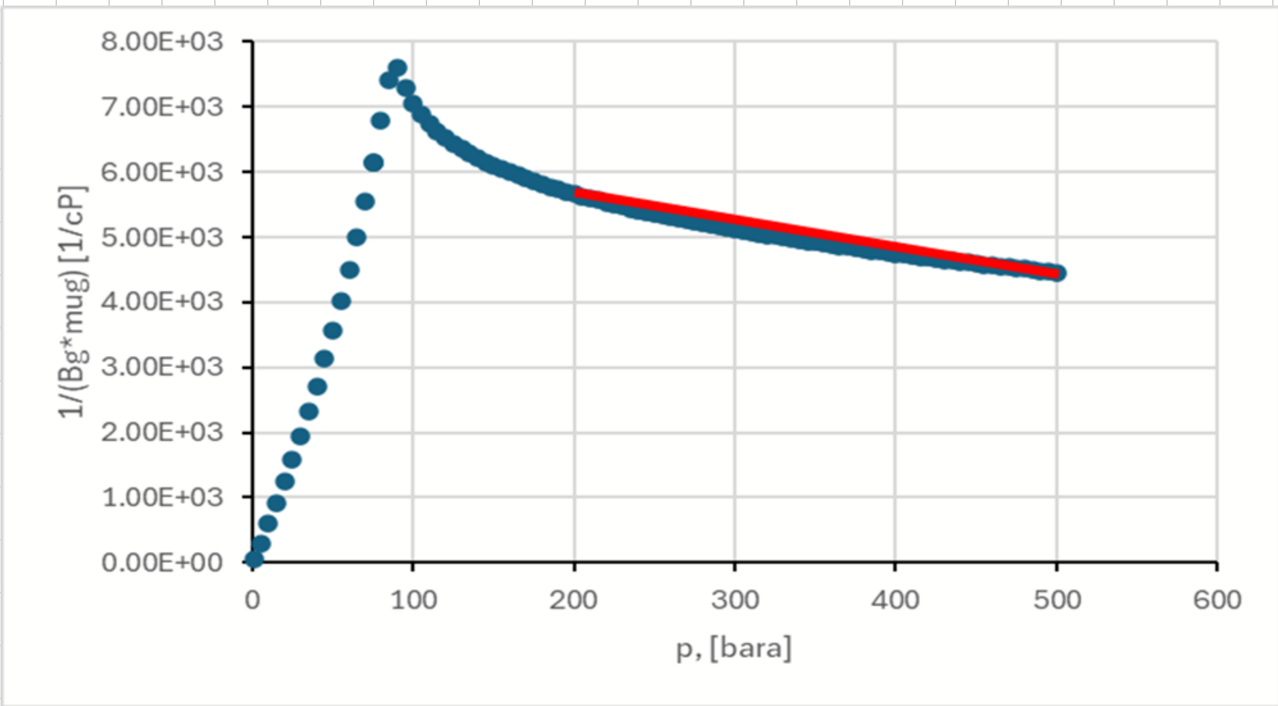
$$V_g = m_g / \rho_g \qquad B_g = \frac{V_g}{V_{\bar{g}}}$$

$$V_{\bar{g}} = m_g / \rho_{\bar{g}}$$

in a OG separator process mass is not conserved

$$int = \int_{P_R}^{P_{wf}} \frac{1}{\mu_g B_g} dP$$

$$int_{P_1}^{P_2} = \int_{P_1}^{P_2} \frac{1}{\mu_g B_g} dP \approx (P_2 - P_1) \cdot 0.5 \cdot \left[\frac{1}{\mu_g B_g}\bigg|_{@P_1} + \frac{1}{\mu_g B_g}\bigg|_{@P_2} \right]$$



Looks somewhat linear above 200 bara, but not quite!

p	deng	viscg	Bg	1/(Bg viscg)	delta_integral
[bara]	[kg/m3]	[cP]	[m3/Sm3]	[1/cP]	[bara/cP]
1.0132	1.7201	0.01563	1.09E+00	5.89E+01	
5	8.6361	0.015676	2.16E-01	2.95E+02	7.05E+02
10	17.669	0.015744	1.06E-01	6.01E+02	2.24E+03
15	27.147	0.015824	6.88E-02	9.18E+02	3.80E+03
20	37.127	0.015919	5.03E-02	1.25E+03	5.42E+03
25	47.68	0.016031	3.92E-02	1.59E+03	7.10E+03
30	58.892	0.016164	3.17E-02	1.95E+03	8.86E+03
35	70.871	0.016321	2.64E-02	2.32E+03	1.07E+04

$$\int_{1.01325}^{5} \frac{1}{\mu_g B_g} dP =$$

$$(5-1.01325)\cdot 0.5\cdot(58.9+295)$$

$$\int_{25}^{30} \frac{1}{\mu_g B_g} dP =$$

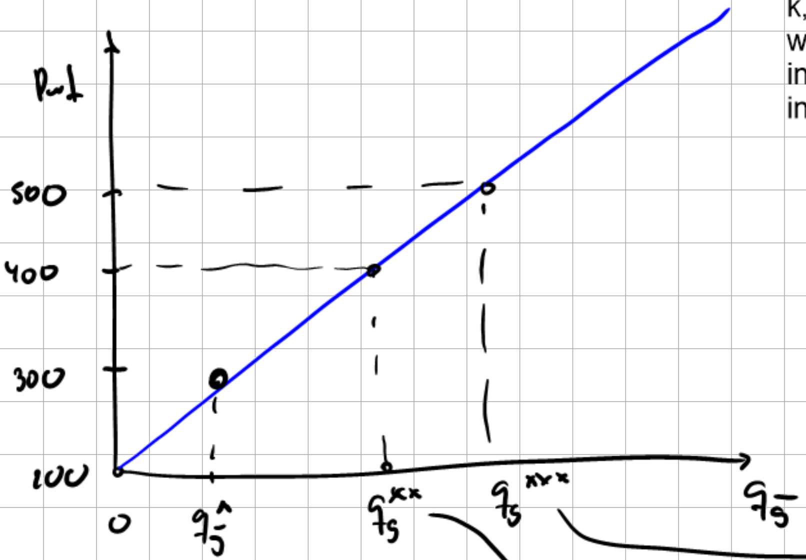
$$(30-25)\ 0.5\cdot(1590+1950)$$

pwf [bara]	Integral [bara/cP]
200	0
300	
400	
500	

$$\int_{200}^{300 \to p_{wf}} \frac{1}{\mu_g B_g} dp = \int_{200}^{205} \frac{1}{\mu_g B_g} dp + \int_{205}^{210} \frac{1}{\mu_g B_g} dp \dots \dots + \int_{295}^{300} \frac{1}{\mu_g B_g} dp$$

$(p_R$

To assess if the IPR is linear or not I need k, h, S, re, rw. But this is a constant, so it will multiply all the points. if pwf versus integral is linear, then pwf versus integral*const will also be linear!

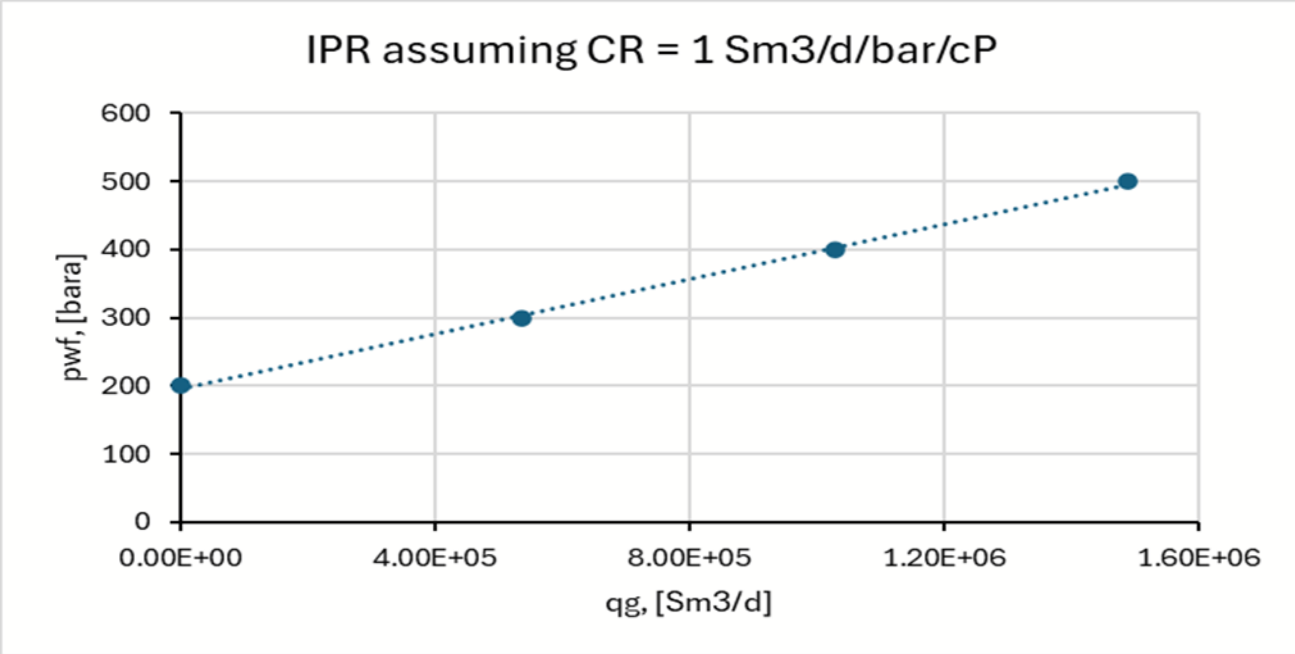


$$q_g = J(p_{wf} - p_R)$$

$$\underbrace{C_R \cdot \int_{200}^{300} \frac{1}{\mu_g B_g}}\qquad \underbrace{C_R \int_{200}^{400} \frac{1}{\mu_g B_g} dp}\qquad \underbrace{C_R \int_{200}^{500} \frac{1}{\mu_g B_g} dp}$$

pwf [bara]	Integral [bara/cP]
200	0
300	5.37E+05
400	1.03E+06
500	1.49E+06

Assume $\underline{C_R} = 1 \quad Sm^3/d \Big/ \frac{bar}{cP}$



the behavior plotted is quite linear, so a linear equation like J*(pwf-pR) should be a good approximation!

Problem 4.

A test has been performed on a gas well and the following values of dry gas rate and flowing bottomhole pressure are reported:

Test point	qg [Sm3/d]	pwf [bara]
1	4.01E+05	493.4
2	1.78E+06	297.0
3	2.53E+06	149.7

The reservoir pressure is 542.5 bara.

Task 1: Propose IPRs for this well and estimate the parameters using the data points. Use the following alternatives:

Darcy flow (laminar):

1. $q_{\bar{g}} = C_R \cdot (p_R^2 - p_{wf}^2)$ (Low pressure)
2. $q_{\bar{g}} = C_R \cdot \int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp$

Forchheimer flow (turbulent):

3. $q_{\bar{g}} = C_R \cdot (p_R^2 - p_{wf}^2)^n$ (Low pressure)
4. $q_{\bar{g}} = C_R \cdot \left(\int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp\right)^n$
5. $q_{\bar{g}} + m \cdot q_{\bar{g}}{}^2 = C_R \cdot \int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp$

Task 2: It is now the future, and a new test has been performed on the well, and the following values of dry gas rate and flowing bottom-hole pressure are reported:

Test point	qg [Sm3/d]	pwf [bara]
1	2.35E+05	272.9
2	8.37E+05	191.4
3	1.35E+06	82.8

Reservoir pressure is 300 bara.

$C_R \cdot (p_R^2 - p_{wf}^2)$

Test point	qg [Sm3/d]	pwf [bara]
1	4.01E+05	493.4
2	1.78E+06	297.0
3	2.53E+06	149.7

$q_{g\,calc}$	ε
x x	$(q_{test} - q_{cal})^2$
x x	
x x	
	Σ

1: assume C_R
2: Calculate $q_{\bar{g}}$ for the p_{wf} given
3: change C_R until $q_{\bar{g}\,test} = q_{\bar{g}\,calc}$

use minimize $\sum_{i=1}^{3} \left(q_{\bar{g}\,test} - q_{\bar{g}\,calc}\right)^2$

$$q_{\bar{g}} = C_R \cdot \left(p_R^2 - p_{wf}^2\right)$$

IPR 1

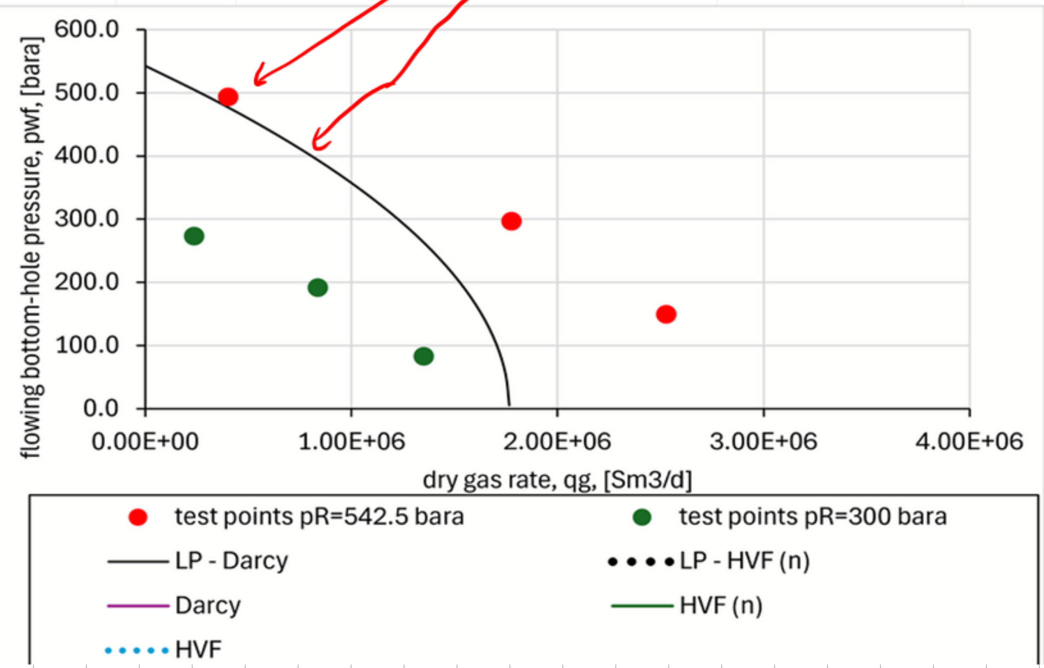
		CR [Sm3/d/bar^2]	6.00
pwf_test [bara]	qg_test [sm3/d]	qg_calc [sm3/d]	(qg_test-qg_calc)^2 [(sm3/d)^2]
493.4	4.01E+05	3.05E+05	9.20E+09
297.0	1.78E+06	1.24E+06	2.92E+11
149.7	2.53E+06	1.63E+06	8.07E+11
			1.11E+12

initial estimate (guess)

do not match, use solver to change to minimize

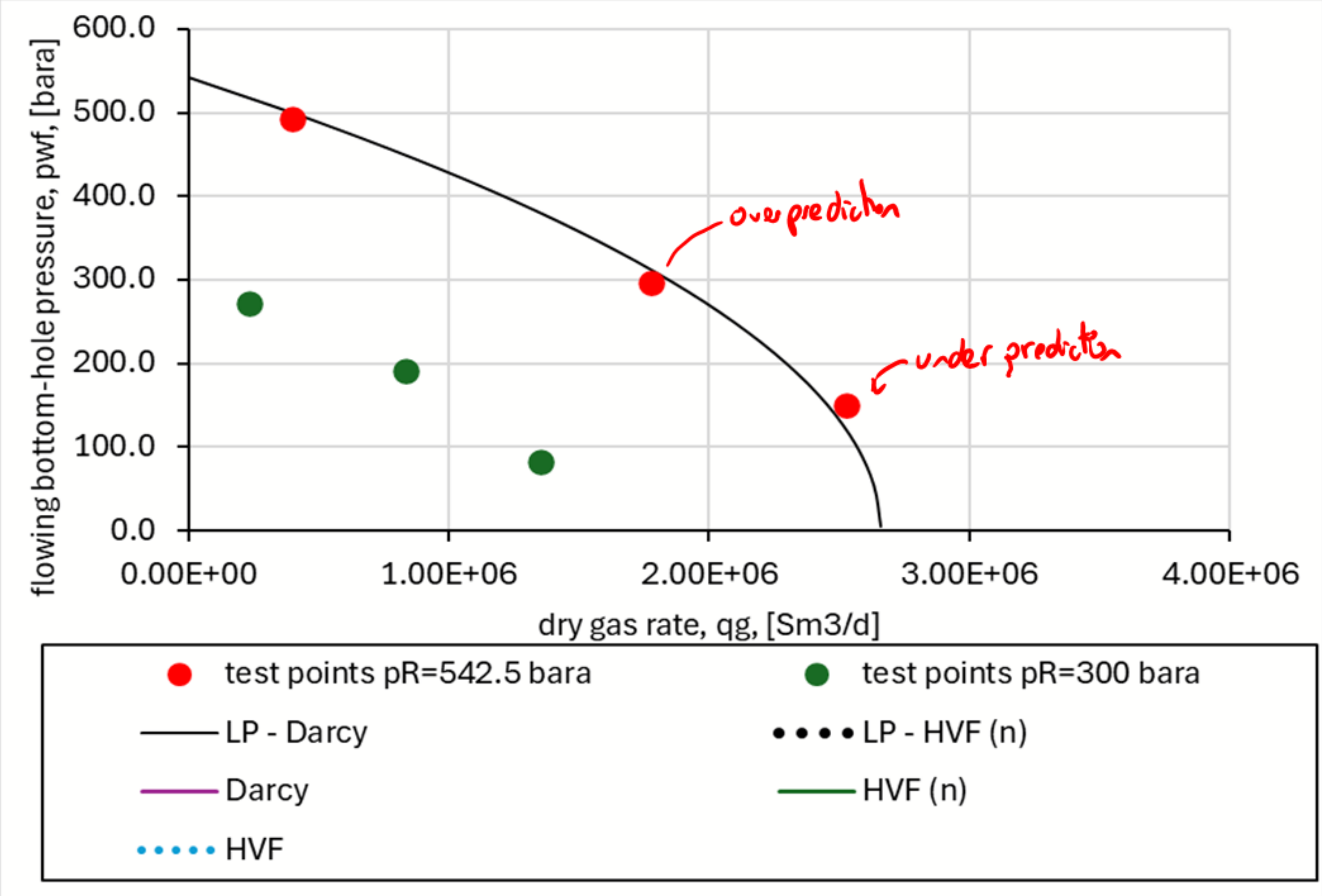
I Plot IPR

82.8 1.35E+06



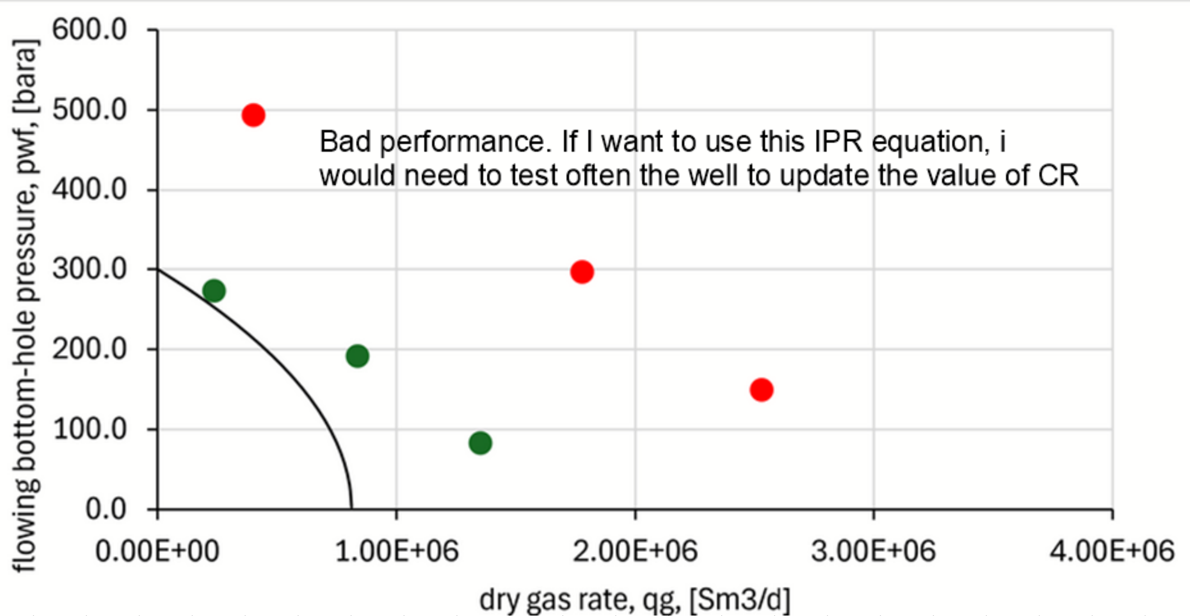
pwf [bara]	IPR 1 LP - Darcy qg [sm3/d]
542.5	0.00E+00
493.7	3.04E+05
444.9	5.78E+05
396.0	8.25E+05
347.2	1.04E+06
298.4	1.23E+06
249.6	1.39E+06
200.7	1.52E+06
151.9	1.63E+06
103.1	1.70E+06
54.3	1.75E+06
5.4	1.77E+06

CR [Sm3/d/bar^2]	9.03



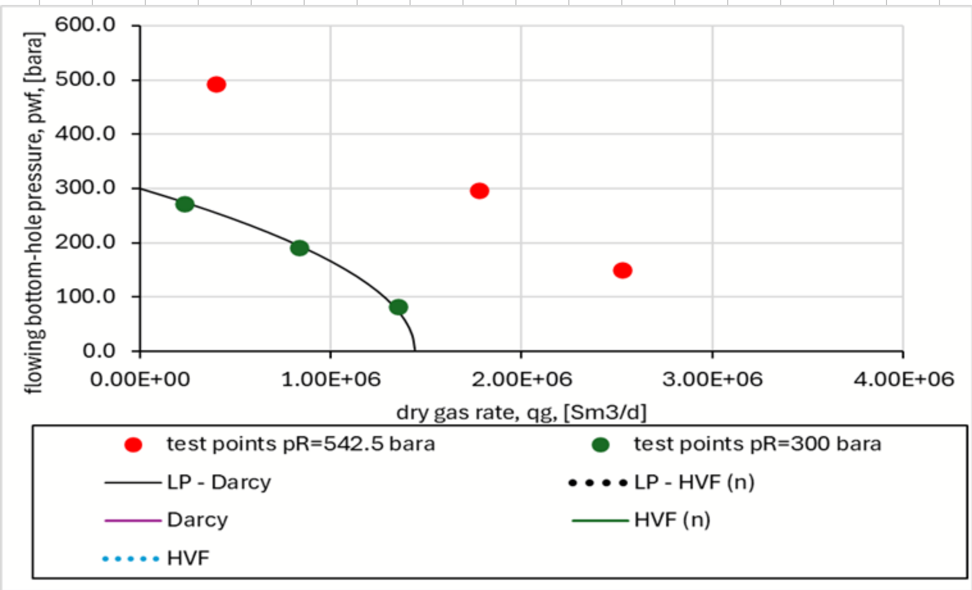
But when I try to use the same equation for pR = 300 bara:

82.8 1.35E+06



	IPR 1
	LP - Darcy
pwf	**qg**
[bara]	**[sm3/d]**
300.0	0.00E+00
273.0	1.40E+05
246.0	2.66E+05
219.0	3.80E+05
192.0	4.80E+05
165.0	5.67E+05
138.0	6.41E+05
111.0	7.01E+05
84.0	7.49E+05
57.0	7.83E+05
30.0	8.05E+05
3.0	8.13E+05

CR [Sm3/d/bar^2]	16.00



20240927
OUTLINE
-Cont of solving problem 4 in exercise set 2
-IPR for gas condensate

To calculate the integral, we are going to follow the same approach we used before when developing the dry gas IPR:

$$\int_{p_{wf}}^{p_R} \frac{1}{\mu_g B_g} dp = \int_{p_{ref}}^{p_R} \frac{1}{\mu_g B_g} dp - \int_{p_{ref}}^{p_{wf}} \frac{1}{\mu_g B_g} dp$$

$$\int_{p_{ref} = p_{min} = 2\ bara}^{p^*} \frac{1}{\mu_g B_g} dp = y(p^*)$$

arbitrary name, to not call it "m function"

p	μ_g	B_g	$1/\mu_g B_g$	integral from 2bara

this is not exactly the m function!!!!(see below).., but it is the same idea, calculating the integral by subtracting two integrals from the same initial pressure

$$m(p) = 2 \cdot \int_{p_{sc}}^{p} \frac{p}{\mu_g \cdot Z} dp \qquad \frac{\left[m(p_R) - m\left(p_{wf}\right)\right]}{2} \cdot \frac{T_{sc}}{T_R \cdot p_{sc}} = \int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp = y(p_R) - y(p_{wf})$$

Why do I need interpolation?: I don't find on the table my exact pR and pwf values

p [bara]	1/viscg*Bg [1/cP]	Integral from 2 bara [bar/cP]
2	114.9	0
50	2885.2	7.20E+04
100	5448.4	2.80E+05
150	7287.3	5.99E+05
200	8318.8	9.89E+05
250	8766.0	1.42E+06
300	8890.8	1.86E+06
350	8858.9	2.30E+06
400	8754.7	2.74E+06
x_1 450	8618.9	3.18E+06 y_1
x_2 x_1 500	8471.3	3.60E+06 y_1 y_2
x_2 550	8321.6	4.02E+06 y_2
600	8174.6	4.44E+06

$$\int_{2\ bara}^{542.5\ bara} \frac{1}{\mu_g B_g} dp$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_{542.5}}{x_2 - 542.5}$$

$$\int_{2\ bara}^{493.4} \frac{1}{\mu_g B_g} dp$$

$$q_{\bar{g}} = C_R \cdot \int_{p_{wf}}^{p_R} \frac{1}{\mu_g \cdot B_g} dp$$

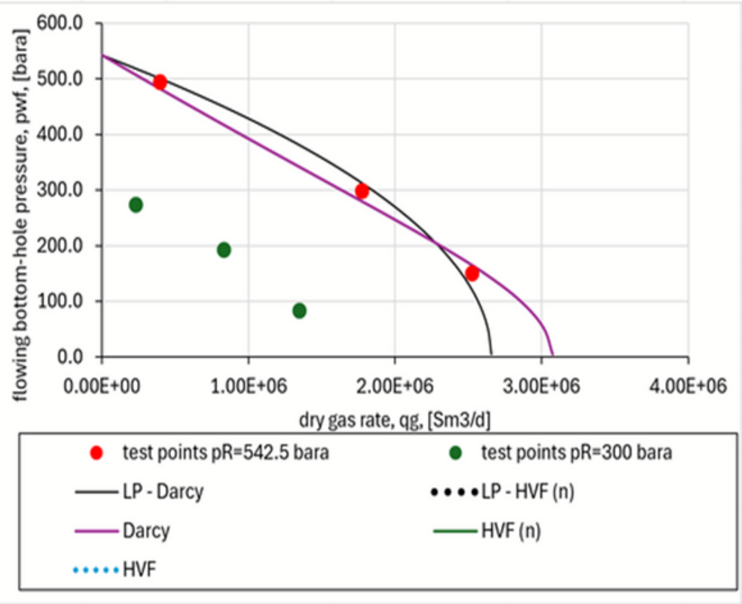
IPR 2

	CR [Sm3/d/bar/cP]	0.78
Integral [bar/cP]	qg_calc [sm3/d]	(qg_test-qg_calc)^2 [(sm3/d)^2]
4.13E+05	3.21E+05	6.34E+09
2.13E+06	1.66E+06	1.47E+10
3.36E+06	2.62E+06	7.48E+09
		2.85E+10

$$\int_{2}^{542.5} \frac{1}{\mu_g B_g} dP = \text{oredinterpol}(541, 3, \text{matrix})$$

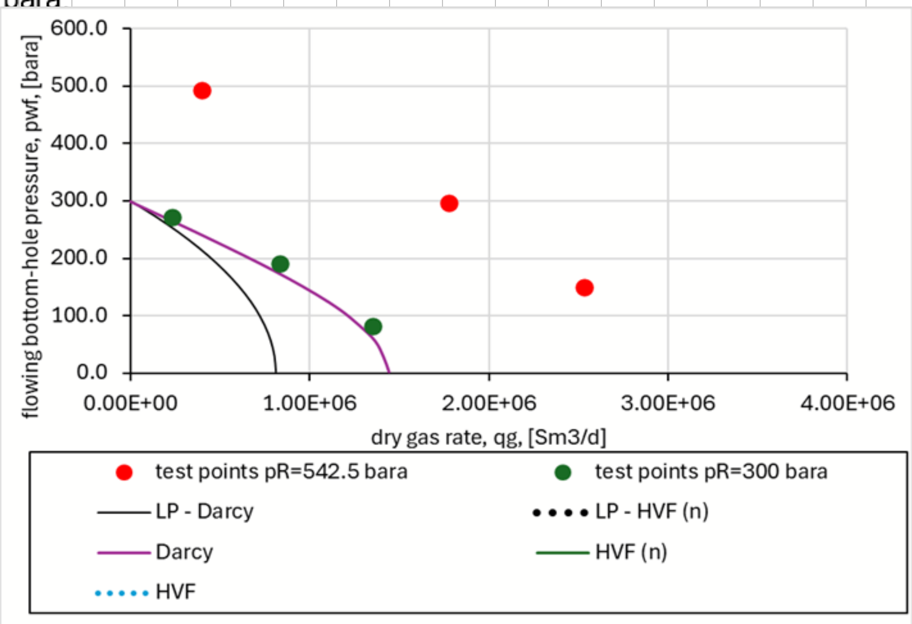
$$- \int_{2}^{493.4} \frac{1}{\mu_g B_g} = \text{oneDinterp}(493.4, 3, \text{matrix})$$

82.8 1.35E+06



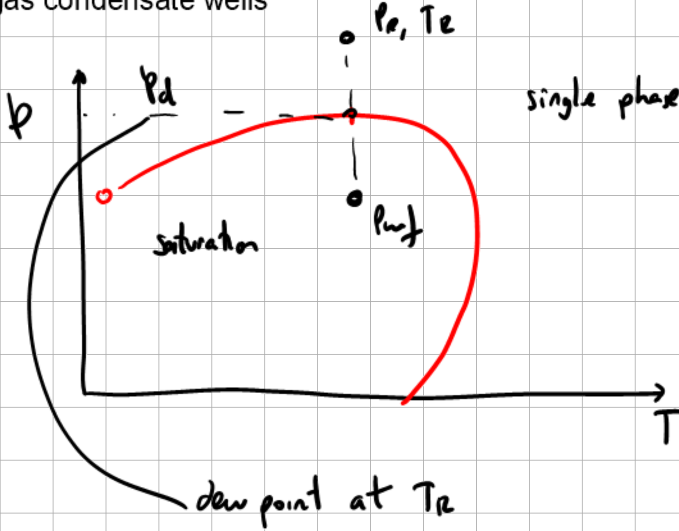
pwf [bara]	IPR 1 LP - Darcy qg [sm3/d]	IPR 3 LP - HVF (n) qg [sm3/d]	Integral [bar/cP]	IPR 2 Darcy qg [sm3/d]
542.5	0.00E+00		0.00E+00	0.00E+00
493.7	4.57E+05		4.11E+05	3.20E+05
444.9	8.71E+05		8.29E+05	6.45E+05
396.0	1.24E+06		1.25E+06	9.75E+05
347.2	1.57E+06		1.68E+06	1.31E+06
298.4	1.85E+06		2.12E+06	1.65E+06
249.6	2.10E+06		2.55E+06	1.98E+06
200.7	2.29E+06		2.96E+06	2.31E+06
151.9	2.45E+06		3.35E+06	2.60E+06
103.1	2.56E+06		3.66E+06	2.85E+06
54.3	2.63E+06		3.87E+06	3.01E+06
5.4	2.66E+06		3.95E+06	3.08E+06

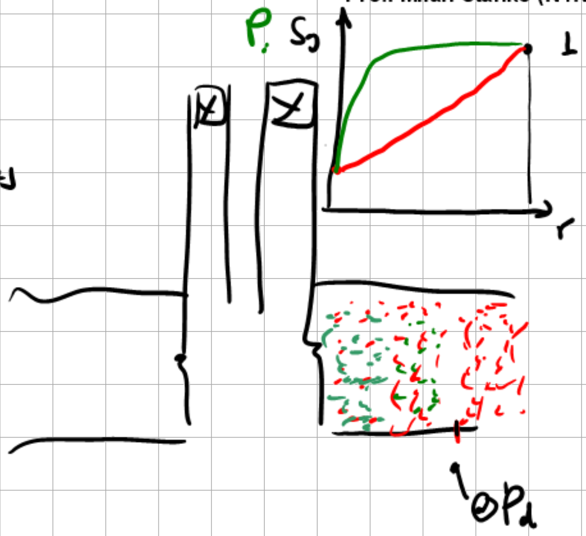
Testing the IPR at pR=300 bara:



Still has good performance. There will be no need to do re-testing :), contrary to if one were going to use the backpressure equation (LP)

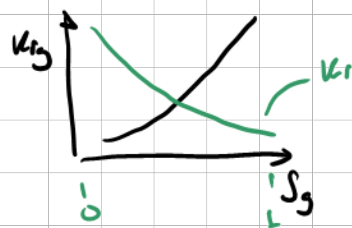
IPR for gas condensate wells





There is no longer single phase flow of gas in the reservoir, but gas and condensate simultaneously

How does this affect the IPR equation?



$K \cdot K_{rg}$

$$\frac{q_s}{2\pi r \cdot h} = \frac{K}{\mu_g}\frac{dp}{dr}$$

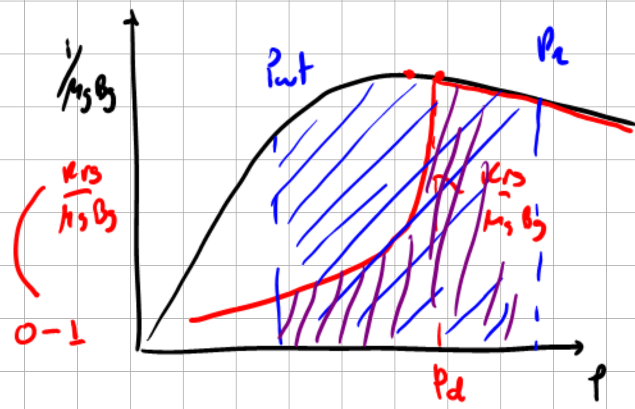
$$q_g = q_{\bar{g}} \cdot B_g$$

$$\frac{q_{\bar{g}}}{K \cdot h\, 2 \cdot \pi}\int \frac{1}{r}\,dr = \int \frac{1}{\mu_g B_g}\,dp$$

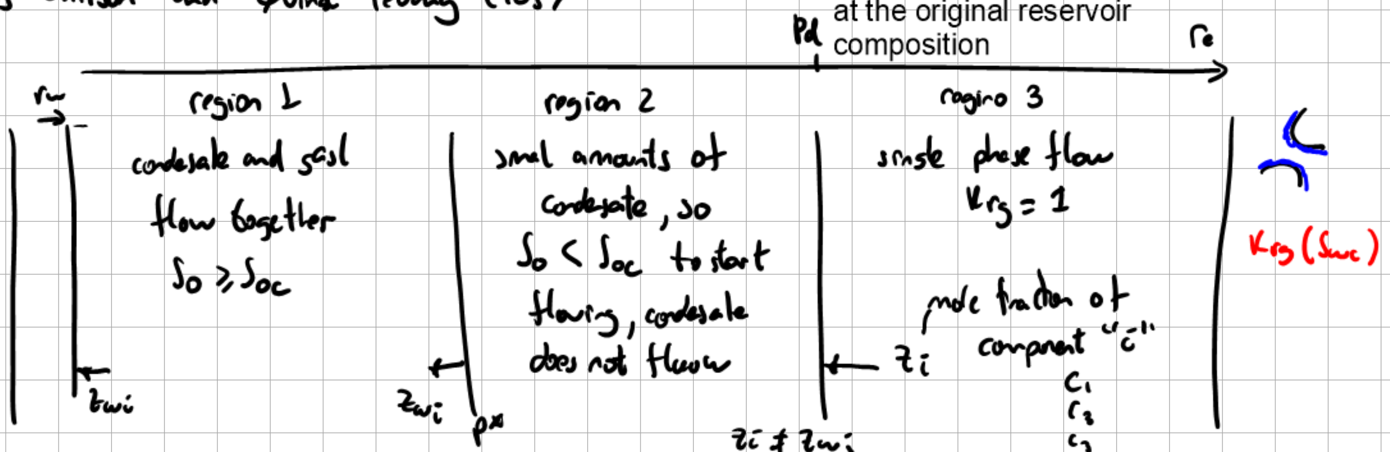
$$\frac{q_{\bar{g}}}{K \cdot h\, 2 \cdot \pi}\int_{r_w}^{r_e}\frac{dr}{r} = \int_{p_{wf}}^{p_R}\frac{k_{rg}}{\mu_g B_g}\,dp$$

krg is a function of radius, but since pressure is also a function of radius, then I can express krg as a function of pressure also (it is more convenient to calculate)

$$q_{\bar{g}} = \frac{1}{18.68}\,\frac{K \cdot h}{\left(\ln\frac{r_e}{r_w} - 0.75 + S\right)}\int_{p_{wf}}^{p_R}\frac{k_{rg}}{\mu_g B_g}\,dp$$



Curtis Whitson and Øivind Fevang (90s)



$$\int_{p_{wf}}^{p_e} \frac{k_{rg}}{\mu_g B_g} = \int_{p_{wf}}^{p^*} \frac{k_{rg}}{\mu_g B_g} dp + \underbrace{\int_{p^*}^{p_d} \frac{k_{rg}}{\mu_g B_g} dp}_{} + \underbrace{\int_{p_d}^{p_e} \frac{1}{\mu_g B_g} dp}_{}$$

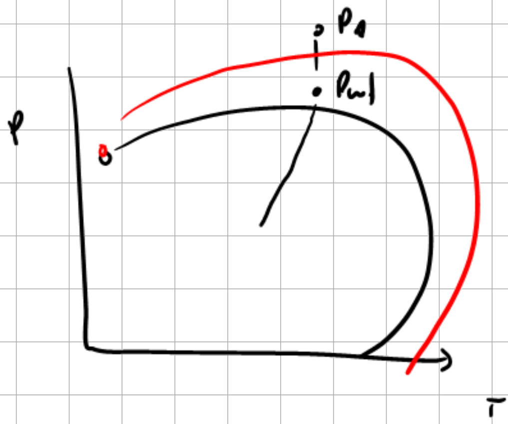
simultaneous flow of condensate and gas

flow of gas only, condensate is trapped

single phase region

$$q_o = \underbrace{CG_e \cdot q_g}$$

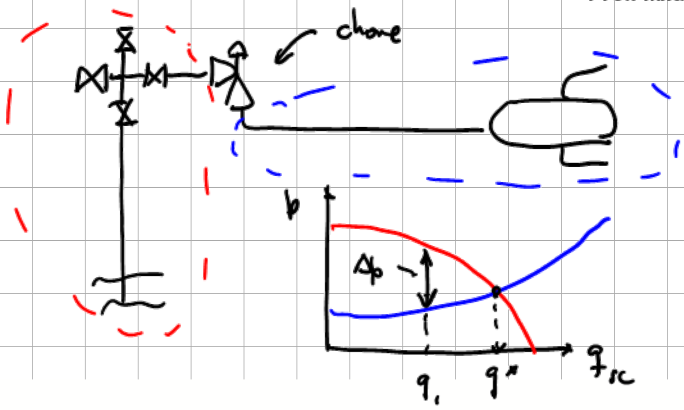
In gas condensate wells, there is a tradeoff, having condensate produced is good because increases revenue, but having condensate reduce the rate of gas is bad because the amount of condesate produced will be reduced



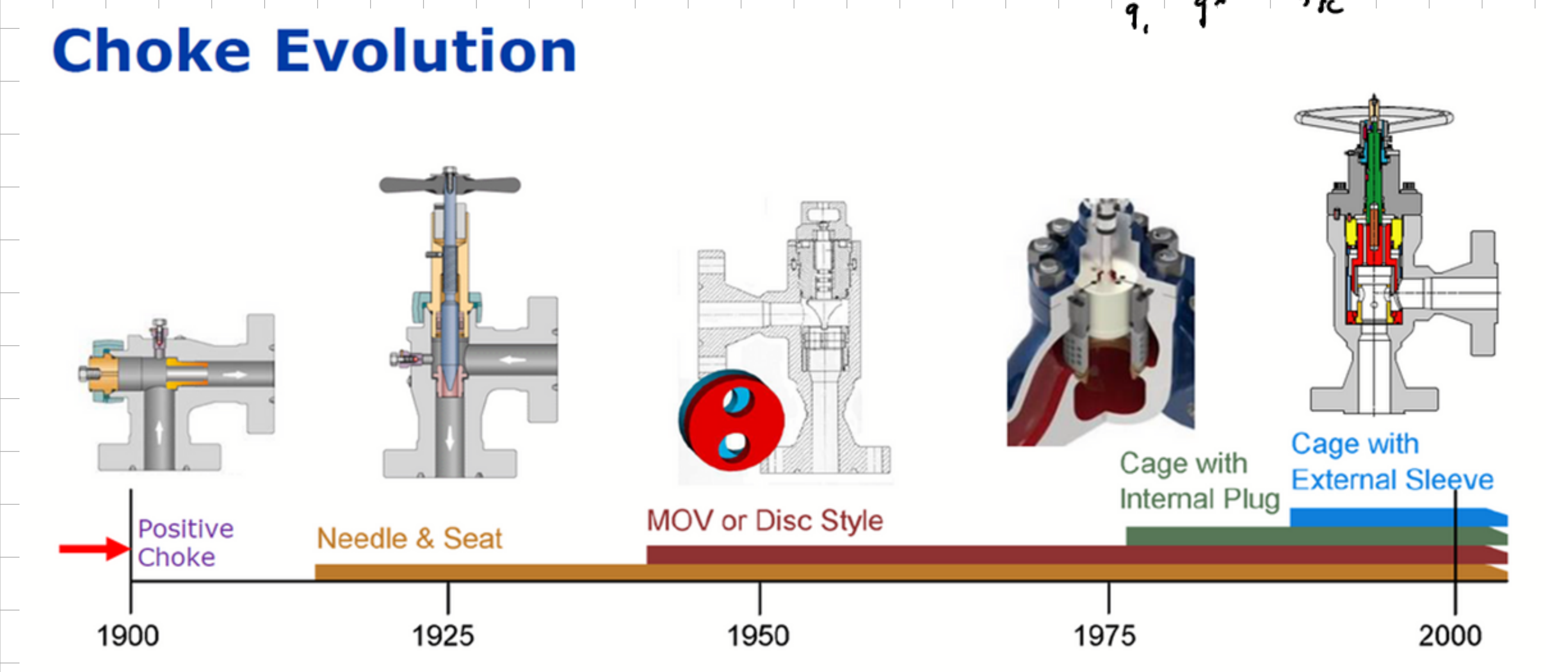
early approach to IPR for gas condensate:

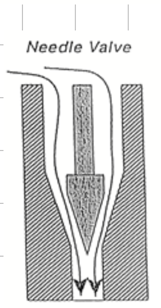
$$q_g = \frac{1}{18.68} \cdot \frac{k \cdot h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + \underbrace{S_{cb}}_{\text{condensate blockage}}\right)} \int_{p_{wf}}^{p_e} \frac{1}{\mu_g B_g} dp$$

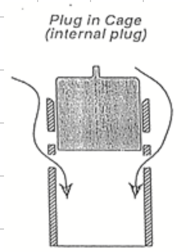
Video 17.1. - Production choke (Wellhead choke)

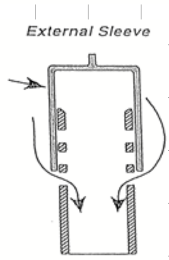


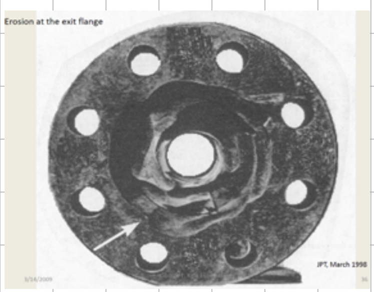
Taken from: https://www.spegcs.org/events/6333/











https://www.gekengineering.com/

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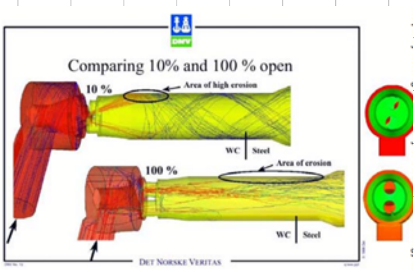
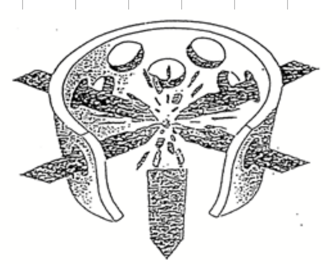
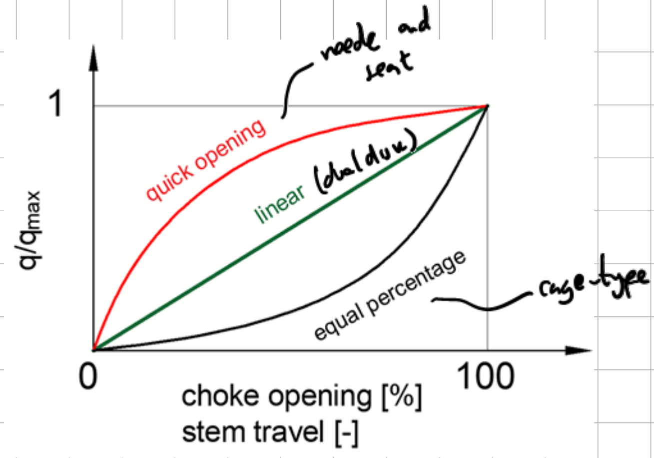
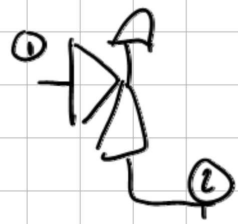




https://www.oedigital.com/news/447270-handling-sand

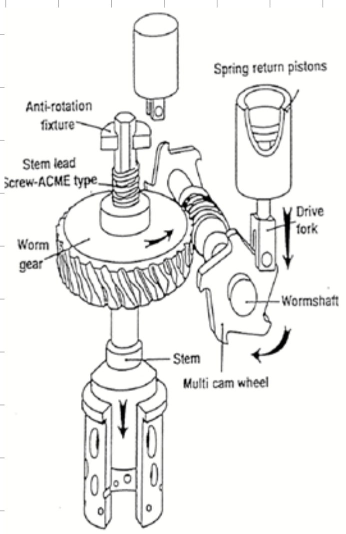




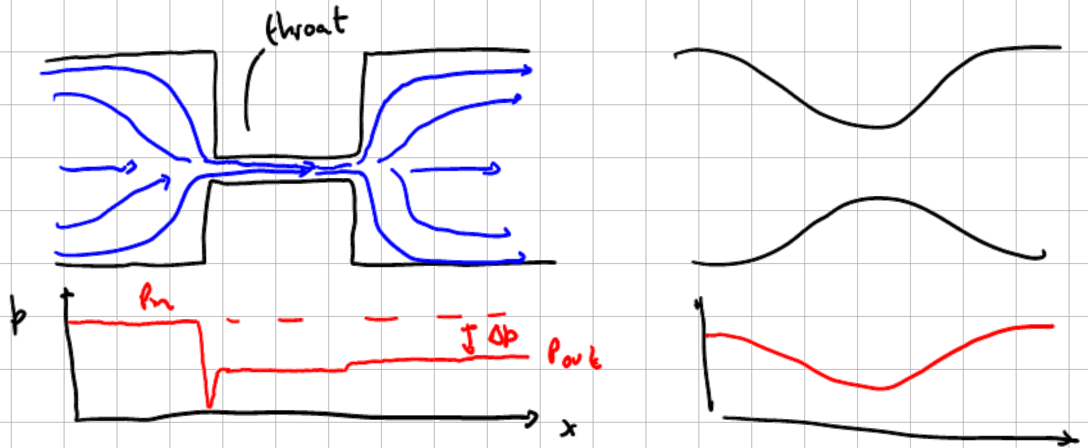
p_1 = constant

Δp = constant

Actuation

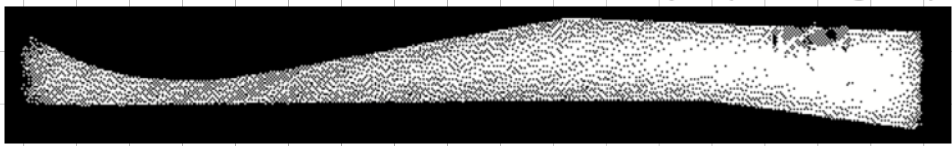


Flow inside a choke

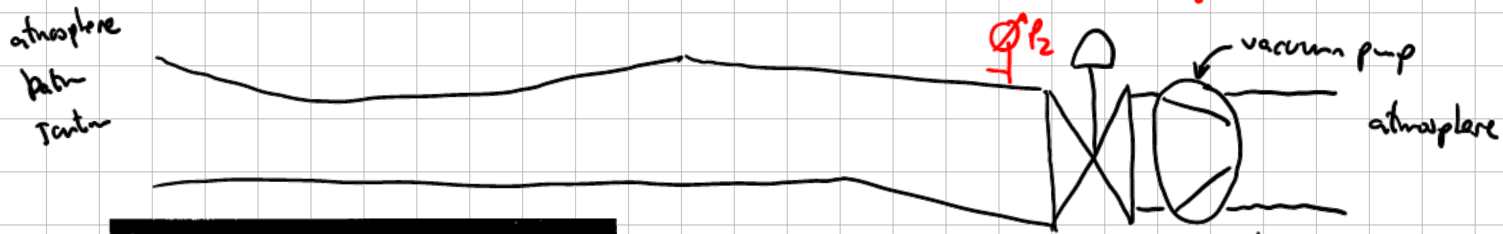


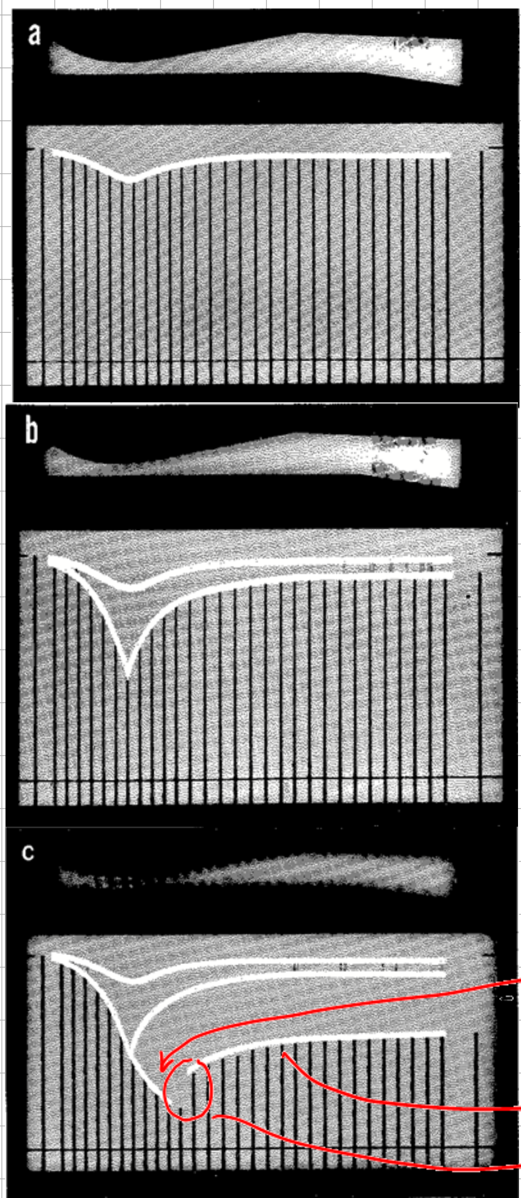
http://web.mit.edu/hml/ncfmf.html

Ascher Shapiro (Mech. Eng. MIT)

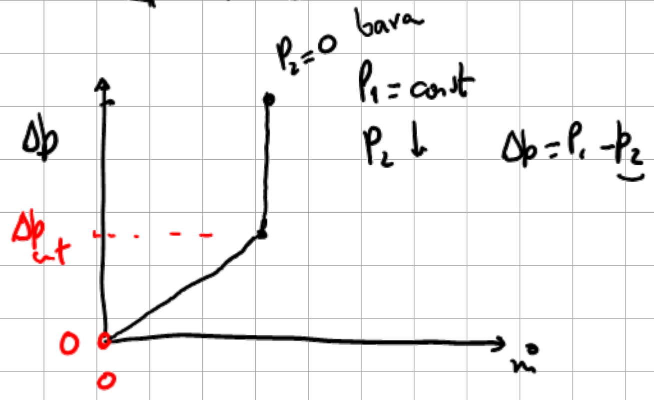


$P_2 \leq P_{atm}$





sub-critical



critical

a : velocity of sound in air ≈ 343 m/s

https://en.wikipedia.org/wiki/Speed_of_sound

c : velocity of sound in the fluid

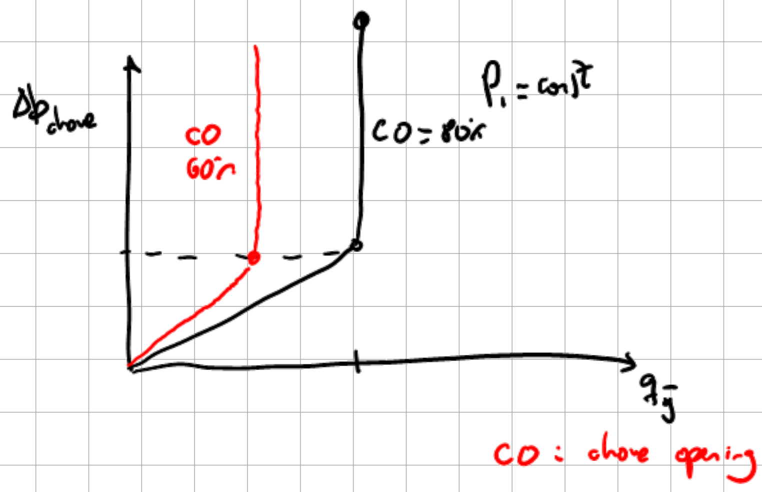
Mach number $\frac{V}{c} = M$

super-critical

$V > c$, supercritical ($M > 1$)

$V < c$, subcritical ($M < 1$)

"shockwave" (discontinuity)

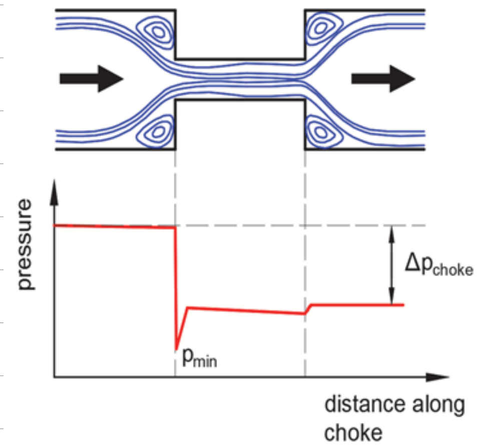


$$\frac{P_2}{P_1} \approx 0.5 \quad \text{gas}$$

$$\Delta p_{crit} = P_1 - P_2 = P_1 \left(1 - 0.5\right)$$

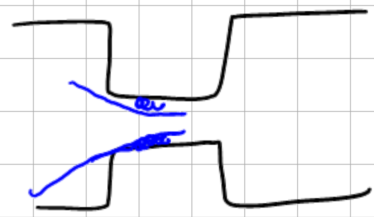
$$\Delta p_{crit} = P_1 \left(0.5\right)$$

Video 17.2. - Choke models



$f(q_{sc}, P_{in}, P_{out}, Ø)$

$\dot{m}$



Undersaturated oil flow

Based on a frictionless flow contraction from an upstream point 1 to a downstream point 2.

The single-phase Bernoulli equation for steady state frictionless flow along a streamline, neglecting elevation changes, is:

$$\frac{dp}{\rho} + V \cdot dV = 0 \qquad \text{Eq. B-1}$$

Where:

p	Pressure
ρ	Density
V	Velocity

Integrating Eq. B-1 from point 1 to 2:

$$\int_{p_1}^{p_2} \frac{dp}{\rho} + \int_{V_1}^{V_2} V \cdot dV = 0 \qquad \text{Eq. B-2}$$

Assuming incompressible flow:

$$\frac{p_2 - p_1}{\rho} + \frac{V_2^2 - V_1^2}{2} = 0 \qquad \text{Eq. B-3}$$

The mass is conserved in the choke, thus:

$$V_1 \cdot A_1 = V_2 \cdot A_2 \qquad \text{Eq. B-4}$$

The area upstream the choke can be expressed with the diameter of the pipe upstream the choke:

$$A_1 = \frac{\pi \cdot Ø_1^2}{4} \qquad \text{Eq. B-5}$$

In a similar way, the cross-section area of 2:

$$A_2 = \frac{\pi \cdot Ø_2^2}{4} \qquad \text{Eq. B-6}$$

Using Eq. B-4, Eq. B-5 and Eq. B-6, it is possible to express V_1 as a function of V_2:

$$V_1 = V_2 \cdot \frac{A_2}{A_1} \cdot \frac{Ø_2^2}{Ø_1^2} \qquad \text{Eq. B-7}$$

To simplify the nomenclature, the ratio between the diameters is named beta (which, in a contraction, is always less than 1):

$$\beta = \frac{Ø_2}{Ø_1} \qquad \text{Eq. B-8}$$

Substituting Eq. B-7 in Eq. B-3:

$$\frac{p_2 - p_1}{\rho} + \frac{V_2^2 - V_2^2 \cdot \beta^4}{2}$$ Eq. B-9

Clearing V_2 in Eq. B-9:

$$V_2 = \sqrt{\frac{2 \cdot (p_2 - p_1)}{\rho \cdot (1 - \beta^4)}}$$ Eq. B-10

$V_2 = \frac{q_{o,2}}{A_2}$

$q_{o,2} = q_{\bar{o}} \cdot B_{o,2}$

For petroleum production calculations, we often require the oil rate at standard conditions, not the velocity, thus, multiplying Eq. B-10 by A_2 and the oil volume factor $B_{o,@2}$:

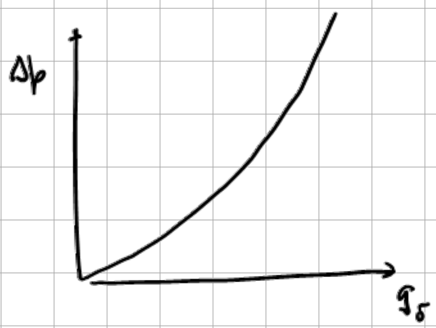
$$q_{\bar{o}} = \frac{A_2}{B_{o,@2}} \cdot \sqrt{\frac{2 \cdot (p_2 - p_1)}{\rho \cdot (1 - \beta^4)}}$$ Eq. B-11

Where $B_{o,@2}$ and ρ are evaluated at p_2 and T_2.

As mentioned earlier, due to the "vena contracta" effect, the effective area at the throat is not exactly A_2, but slightly less. Thus, a correction factor called the flow coefficient is introduced in Eq. B-11:

$$q_{\bar{o}} = \frac{A_2 \cdot C_d}{B_{o,@2}} \cdot \sqrt{\frac{2 \cdot (p_2 - p_1)}{\rho \cdot (1 - \beta^4)}}$$ Eq. B-12

$\Delta p = C q_{\bar{o}}^2$



Dry gas flow

(based on a frictionless flow contraction from an upstream point 1 to a downstream point 2)

Using Eq. B-2 as the starting point, the term related to pressure and density remains valid; however, in gas flow the velocity downstream is usually much higher than the velocity upstream, thus $V_2^2 >> V_1^2$:

$$\int_{p_1}^{p_2} \frac{dp}{\rho} + \frac{V_2^2}{2} = 0$$ Eq. B-13

The density will vary inside the choke. An assumption commonly used is that the contraction process is adiabatic (with an exponent k, the ratio between the specific heats of the gas):

$$p \cdot \rho^{-k} = C$$ Eq. B-14

$k = \frac{C_p}{C_v}$

Where C is a constant. Substituting Eq. B-14 in Eq. B-13:

$$C^{\frac{1}{k}} \cdot \int_{p_1}^{p_2} \frac{dp}{p^{\frac{1}{k}}} + \frac{V_2^2}{2} = 0$$ Eq. B-15

Solving the integral:

$$C^{\frac{1}{k}} \cdot \frac{k}{k-1} \cdot \left(p_2^{\frac{k-1}{k}} - p_1^{\frac{k-1}{k}} \right) + \frac{V_2^2}{2} = 0$$ Eq. B-16

The constant C is expressed in terms of the inlet conditions:

$$C^{\frac{1}{k}} = \frac{p_1^{\frac{1}{k}}}{\rho_1}$$ Eq. B-17

Substituting Eq. B-17 in Eq. B-16 and introducing the pressure ratio $y = p_2/p_1$:

$$\frac{p_1^{\frac{1}{k}}}{\rho_1} \cdot \frac{k}{k-1} \cdot p_1^{\frac{k-1}{k}} \cdot \left(y^{\frac{k-1}{k}} - 1\right) + \frac{V_2^2}{2} = 0$$ Eq. B-18

Clearing V_2 and simplifying p_1:

$$V_2 = \sqrt{2 \cdot \frac{p_1}{\rho_1} \cdot \frac{k}{k-1} \cdot \left(1 - y^{\frac{k-1}{k}}\right)}$$ Eq. B-19

Expressing ρ_1 with the real gas equation:

$$\rho_1 = \frac{p_1 \cdot M_w}{Z_1 \cdot R \cdot T_1}$$ Eq. B-20

Where:

M_w	Molecular weight of the gas
R	Universal gas constant
Z	Generalized compressibility factor

Substituting Eq. B-20 in Eq. B-19:

$$V_2 = \sqrt{2 \cdot \frac{Z_1 \cdot R \cdot T_1}{M_w} \cdot \frac{k}{k-1} \cdot \left(1 - y^{\frac{k-1}{k}}\right)}$$ Eq. B-21

$V_2 = \frac{q_{g,2}}{A_2}$

For petroleum production calculations, we often require the gas rate at standard conditions, not the velocity, thus, multiplying Eq. B-21 by the "effective" cross-section area of 2 gives the local volume rate:

$$q_{g2} = A_2 \cdot C_d \cdot \sqrt{2 \cdot \frac{Z_1 \cdot R \cdot T_1}{M_w} \cdot \frac{k}{k-1} \cdot \left(1 - y^{\frac{k-1}{k}}\right)}$$ Eq. B-22

The local volumetric rate at point 2 is related to the rate at standard conditions by the following equation:

$$q_{g2} \cdot \rho_2 = q_{\bar{g}} \cdot \rho_{sc}$$ Eq. B-23

Substituting Eq. B-23 in Eq. B-22 gives:

$$q_{\bar{g}} = \frac{\rho_2 \cdot A_2 \cdot C_d}{\rho_{sc}} \cdot \sqrt{2 \cdot \frac{Z_1 \cdot R \cdot T_1}{M_w} \cdot \frac{k}{k-1} \cdot \left(1 - y^{\frac{k-1}{k}}\right)}$$ Eq. B-24

ρ_2 is related with ρ_1 by Eq. B-17:

$$\frac{p_2^{\frac{1}{k}}}{\rho_2} = \frac{p_1^{\frac{1}{k}}}{\rho_1}$$ Eq. B-25

Clearing ρ_2 from Eq. B-25 and substituting in Eq. B-24, and using the real gas equation to express the gas density at standard conditions:

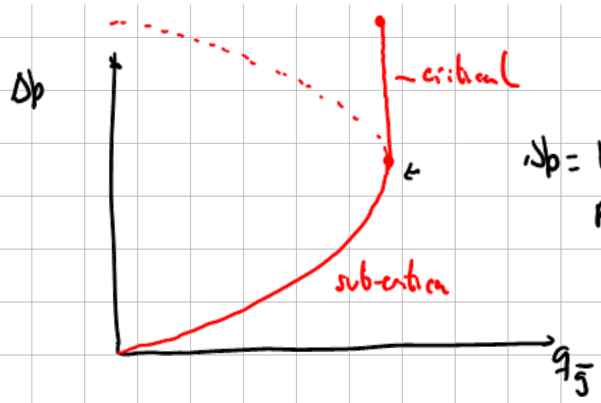
$$q_{\bar{g}} = \frac{\rho_1 \cdot p_2^{\frac{1}{k}} \cdot R \cdot T_{sc} \cdot A_2 \cdot C_d}{p_1^{\frac{1}{k}} \cdot p_{sc} \cdot M_w} \cdot \sqrt{2 \cdot \frac{Z_1 \cdot R \cdot T_1}{M_w} \cdot \frac{k}{k-1} \cdot \left(1 - y^{\frac{k-1}{k}}\right)} \qquad \text{Eq. B-26}$$

Introducing Eq. B-20 for ρ_1:

$$q_{\bar{g}} = \frac{p_1 \cdot M_w \cdot p_2^{\frac{1}{k}} \cdot R \cdot T_{sc} \cdot A_2 \cdot C_d}{Z_1 \cdot R \cdot T_1 \cdot p_1^{\frac{1}{k}} \cdot p_{sc} \cdot M_w} \cdot \sqrt{2 \cdot \frac{Z_1 \cdot R \cdot T_1}{M_w} \cdot \frac{k}{k-1} \cdot \left(1 - y^{\frac{k-1}{k}}\right)} \qquad \text{Eq. B-27}$$

Simplifying and rearranging terms:

$$q_{\bar{g}} = \frac{p_1 \cdot T_{sc} \cdot A_2 \cdot C_d}{p_{sc}} \cdot \sqrt{2 \cdot \frac{R}{Z_1 \cdot T_1 \cdot M_w} \cdot \frac{k}{k-1} \cdot \left(y^{\frac{2}{k}} - y^{\frac{k+1}{k}}\right)} \qquad \text{Eq. B-28}$$



$\Delta p = P_1 - P_2$

$P_1 =$ constant

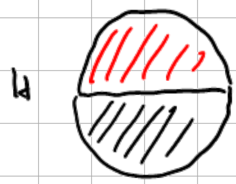
$$\frac{d q_{\bar{g}}}{d P_2} = 0$$

$$y_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$k = 1.3$

$y_c \simeq 0.5$

Choke model for multiphase flow



$H_g = \frac{\boxed{V_l}}{A_x}$

Momentum equation for gas-liquid flow:

$$-dp = H_l \cdot \rho_g \cdot V_g \cdot dV_g + H_g \cdot \rho_l \cdot V_l \cdot dV_l$$

mass fraction at inlet

$$p \cdot (v_g)^{\frac{x_o \cdot C_{v,o} + x_g \cdot C_{p,g} + x_w \cdot C_{v,w}}{x_o \cdot C_{v,o} + x_g \cdot C_{v,g} + x_w \cdot C_{v,w}}} = c$$

$$h_{mix} + x_g \cdot \frac{V_g^{\ 2}}{2} + x_o \cdot \frac{V_o^{\ 2}}{2} + x_w \cdot \frac{V_w^{\ 2}}{2} = const$$

Exercise

$$y_c = \left(\frac{p_2}{p_1}\right)_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \quad k = \frac{c_p}{c_v} \quad k = 1.3 - 0.31 \cdot (\gamma_g - 0.55)$$

$$q_{SC} = \frac{C_n d_{ch}^2}{\sqrt{\gamma_g T_1 Z_1}} P_1 \sqrt{\left(\frac{k}{k-1}\right)\left[(y)^{\frac{2}{k}} - (y)^{\frac{k+1}{k}}\right]}$$

$$C_n = C_s C_d \left(\frac{T_{sc}}{p_{sc}}\right)$$

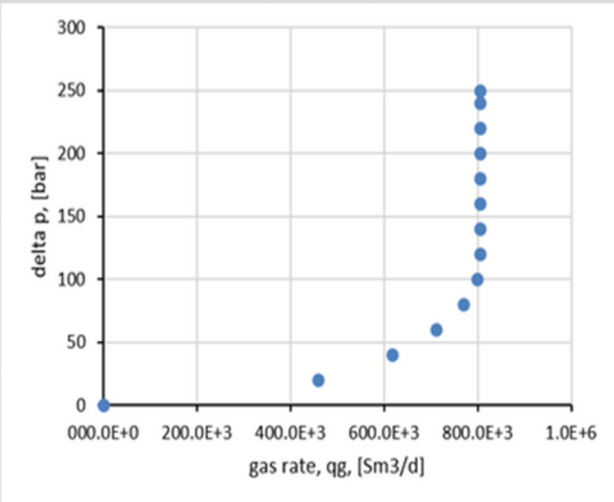
Symbol	Field Units	SI units
q_{sc}	Mscf/D	m^3/d
d_{ch}	in	mm
p	psia	kPa
T	R	K
Cs	27.611	1.6259
C_D	0.865	0.865
p_{sc}	14.696 psia	101.325 kPa
T_{sc}	519.68 R	288.72
c_n	844.57	4.0075

Taken from the book "Gas Production Operations" by Beggs

Prof. Milan Stanko (NTNU). Production wells. Choke Performance Equation for Dry Gas

C_D Discharge Coefficient [-]	0.865
Gas Gravity, [-]	0.55
k, Adiabatic Constant [-]	1.300
T1, choke inlet temperature [C]	70
y_c Critical Ratio [-]	0.546

p1 [bara]	p2 [bara]	d [mm]	y (p2/p1) [-]	qsc [Sm3/d]	dp [bar]
250.0	250.0	15	1.000	000.0E+0	0
250.0	230.0	15	0.920	459.6E+3	20
250.0	210.0	15	0.840	616.5E+3	40
250.0	190.0	15	0.760	712.0E+3	60
250.0	170.0	15	0.680	770.0E+3	80
250.0	150.0	15	0.600	799.1E+3	100
250.0	130.0	15	0.520	804.6E+3	120
250.0	110.0	15	0.440	804.6E+3	140
250.0	90.0	15	0.360	804.6E+3	160
250.0	70.0	15	0.280	804.6E+3	180
250.0	50.0	15	0.200	804.6E+3	200
250.0	30.0	15	0.120	804.6E+3	220
250.0	10.0	15	0.040	804.6E+3	240
250.0	0.0	15	0.000	804.6E+3	250

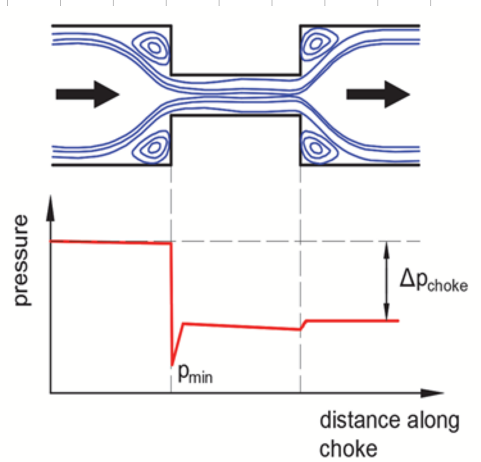


Using the choke model in "metering" mode: p1,p2 and diameter are given, calculate rate

Test yourself!, use the choke model for different purposes:

-Production performance mode: given a rate, p1 and diameter, find p2
-Production performance mode : given a rate, p2 and diameter, find p1
-Design mode: Given a rate, p1,p2, find diameter

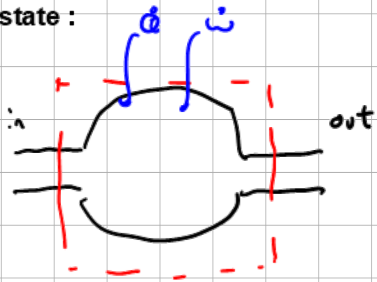
Video 17.3. - Change of temperature across choke



https://www.arab-oil-naturalgas.com/what-does-choke-valve-mean/?quad_cc

OTC26966

Energy conservation equation for open systems, stationary external boundary, steady-state :



shaft work = 0

flow work $p\,dv$

$$\dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{Q}}^{\,0} + \dot{W} = 0$$

variation → 0

$$\dot{m}\Big(\underbrace{u}_{\text{internal energy}} + \underbrace{\cancel{gz}}_{\text{potential energy}} + \underbrace{\frac{v^2}{2}}_{\text{kinetic energy}}\Big)$$

enthalpy

$$h = u + p \cdot v$$

h is a thermodynamic property

$$u = f(p,T)$$

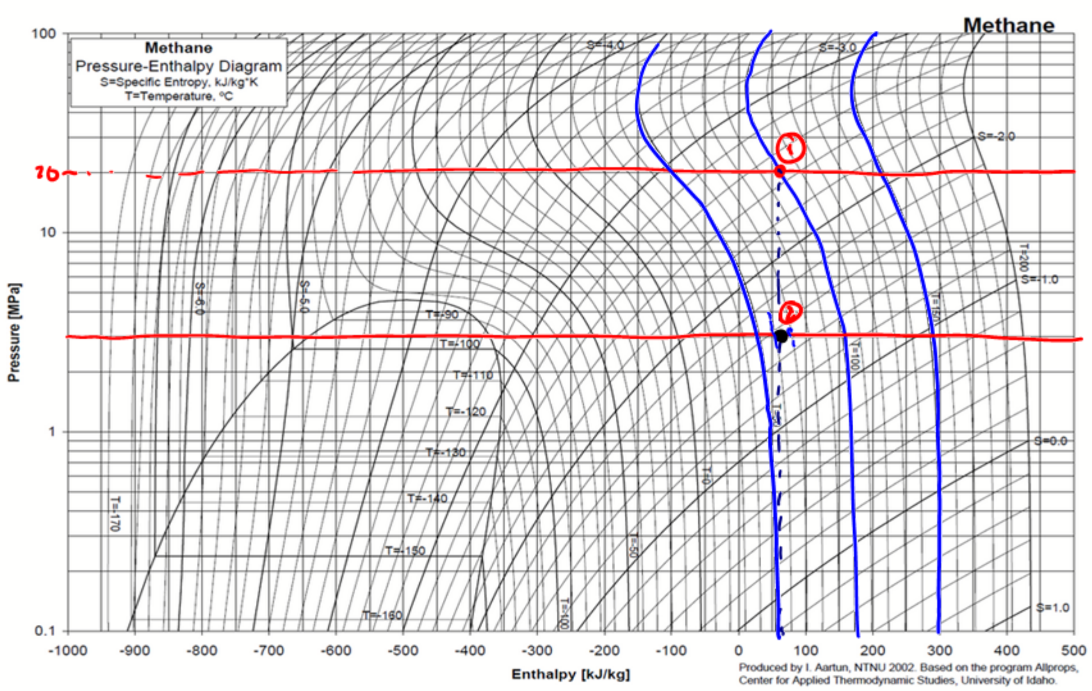
$$h = f(p,T)$$

$$\dot{m}\left(h + \frac{v^2}{2}\right) = \text{constant}$$

$$h + \frac{v^2}{2} = \text{constant}$$

if $\Delta \frac{v^2}{2}$ is neglected (e.g. from in - out)

$$\boxed{h_{in} = h_{out}} \rightarrow \text{isenthalpic}$$



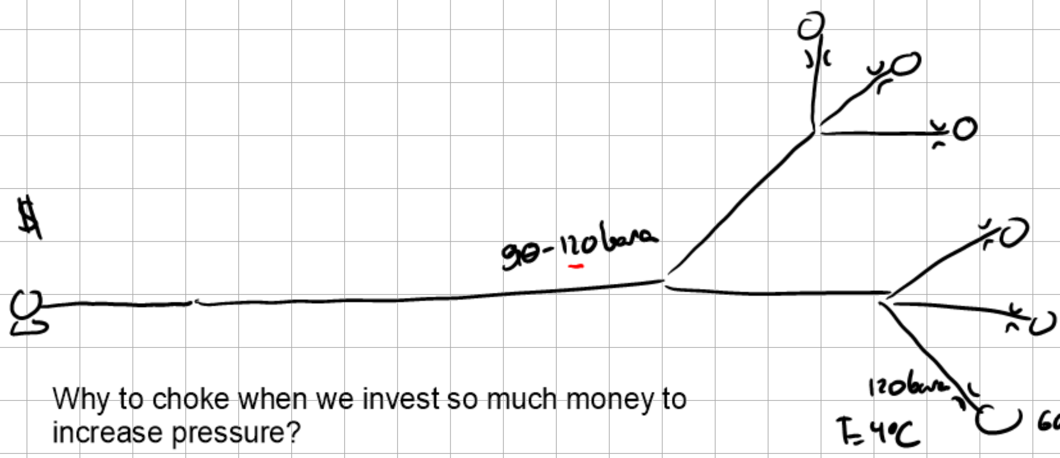
$h_1 = h_2$ CH_4

$h_1 = 50$ kJ/kg

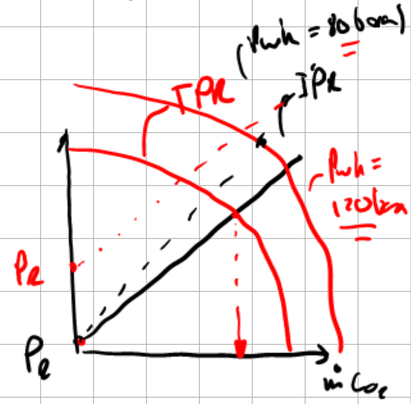
$h_2 = h_1 = 50$ kJ/kg

$T_2 \approx 63$ °C

20240930
OUTLINE
-Recap of YT video assignments of last week (production choke)
-Expansion of CO2 in injection Choke
-Mechanical properties of tubulars

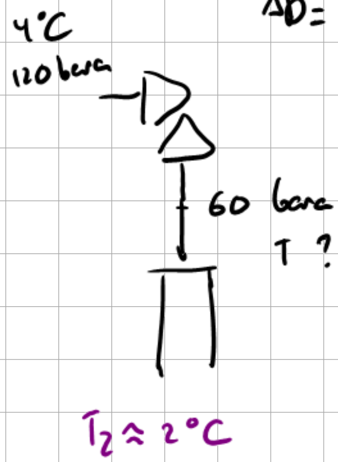


1 Ensure constant rate when reservoir pressure increases



2 heterogeneity in well injectivity
(Not all wells require the same pwh)

Why to choke when we invest so much money to increase pressure?

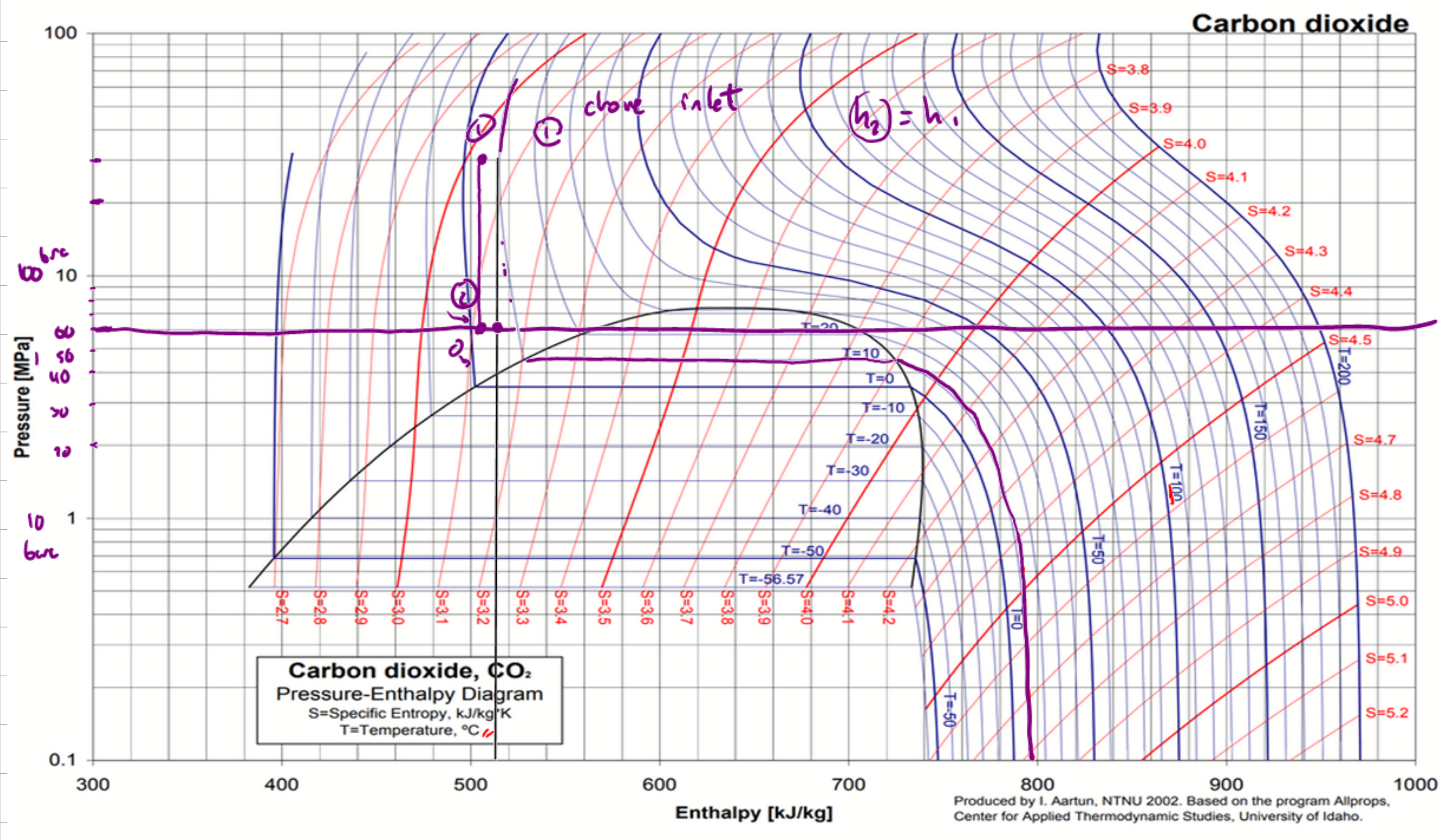


$\Delta p = 60$ bara why do I care?

1 expansion or contraction of tubing and casing

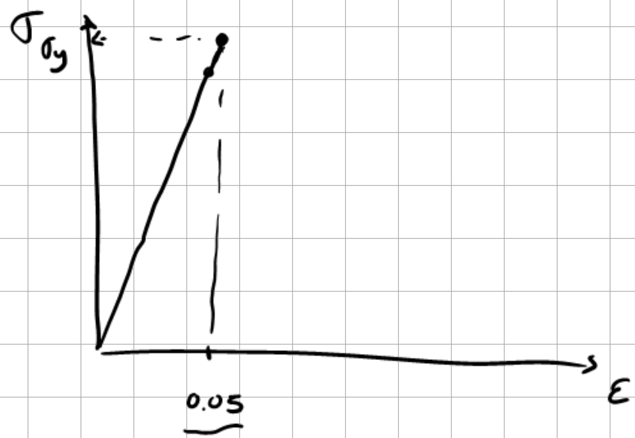
2 change of properties in steel and seals

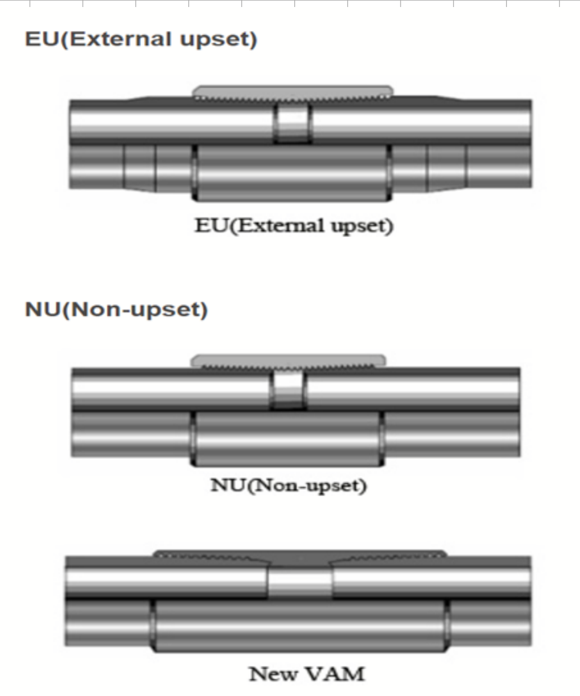
to drop the temperature to below -20 C you would need to have an expansion down to 20 bara or below



-Mechanical properties of tubulars

Tubing Size		Nominal Weight					Threaded Coupling						Joint Yield Strength		Capacity Table	
								Coupling Outside Dia.								
Nom. in.	OD in.	T & C Non-Upset lb/ft	T & C Upset lb/ft	Grade	Wall Thickness in.	Inside Dia. in.	Drift Dia. in.	Non-Upset in.	Upset Reg. in.	Upset Spec. in.	Collapse Resistance psi	Internal Yield Pressure psi	T & C Non-Upset lb	T & C Upset lb	Barrels per Linear ft	Linear ft per Barrel
3/4	1.05	1.14	1.20	H-40	0.113	0.824	0.730	1.313	1.660		7,200	7,530	6,360	13,300	0.0007	1516.13
				J-55							9,370	10,360	8,740	18,290		
				C-75							12,250	14,120	11,920	24,940		
				N-80							12,710	15,070	12,710	26,610		
1	1.315	1.700	1.800	H-40	0.113	1.049	0.955	1.660	1.900		6,820	7,080	10,960	19,760	0.0011	935.49
				J-55							8,860	9,730	15,060	27,160		
				C-75							11,590	13,270	20,540	37,040		
				N-80							12,270	14,160	21,910	39,510		
1 1/4	1.660	2.300	2.400	H-40	0.125	1.410	1.286	2.054	2.200		5,220	5,270			0.0019	517.79
				H-40	0.140	1.380					5,790	5,900	15,530	26,740	0.0018	540.55
				J-55	0.125	1.410					6,790	7,250			0.0019	517.79
				J-55	0.140	1.380					7,530	8,120	21,360	36,770	0.0018	540.55
				C-75	0.140	1.380					9,840	11,070	29,120	50,140	0.0018	540.55
				N-80	0.140	1.380					10,420	11,810	31,060	53,480	0.0018	540.55
1 1/2	1.900	2.750	2.900	H-40	0.125	1.650	1.516	2.200	2.500		4,450				0.0026	378.11
				H-40	0.145	1.610					5,290		19,090	31,980	0.0025	397.14
				J-55	0.125	1.650					5,790				0.0026	378.11
				J-55	0.145	1.610					6,870		26,250	43,970	0.0025	397.14
				C-75	0.145	1.610					8,990	10,020	35,800	59,960	0.0025	397.14
				N-80	0.145	1.610					9,520	10,680	38,180	63,960	0.0025	397.14
2 1/16	2.063			H-40	0.156	1.751					5,240	5,290			0.0030	335.75
				J-55							6,820	7,280				
				C-75							8,910	9,920				
				N-80							9,440	10,590				





What's The Difference Between Upset Tubing And Non-Upset Tubing?

World Iron & Steel Co.,Ltd | Apr 18, 2018

What's The Difference Between Upset Tubing And Non-Upset Tubing?

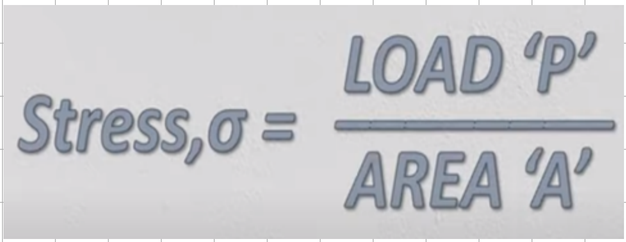
APT 5CT specified the requirements of Casing and tubing for the purpose of oil or gas production, must be cased with material with sufficient strength and functionality.

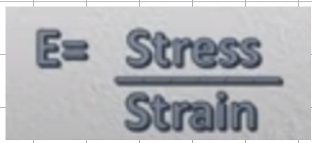
Oil tubing are the main parts of the well drilled construction. They are selected for the specific conditions anticipated in a given well. The anticipated production flow rates and economics of the well determine tubing size, which then determines the necessary size of each previous hole and tubular. Once the tubular size and setting depths are determined, the wall thickness and grade of material are then chosen by the well designer to ensure the strength is adequate for the expected loads. API 5CT developed specifications for three different connectors for use as casing tubing joints: External-upset tubing and coupling (Upset tubing),Non-upset tubing and couplings,Integral-joint tubing.

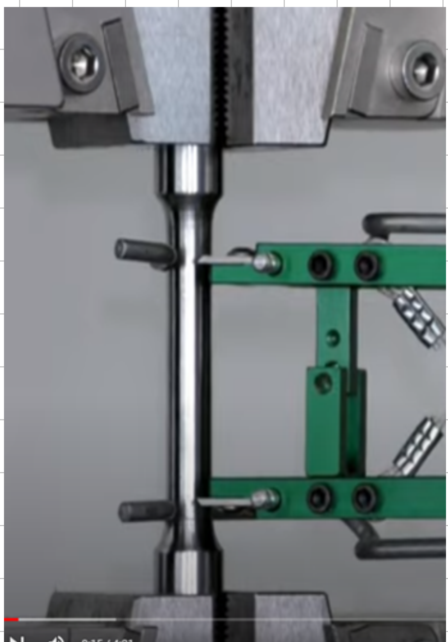
Upsetting is a forging process that makes for a thicker wall on the tube ends. **Upset Tubing** (EUE) is a long tubing with joint installed in a wellbore to facilitate the extraction of oil and gas. The tubing joints are designed to suit a certain well type, based on the underground conditions and fluids.It acts as a conduit, through which the produced fluids or gas get transmitted from the bottom to the surface. Tubing joint is relatively a single length pipe that can vary in length anywhere from 25 to 30 ft. This ideal length allows easy production as the tubing joint is fitted with threads on both sides and can run down through a well of any depth.

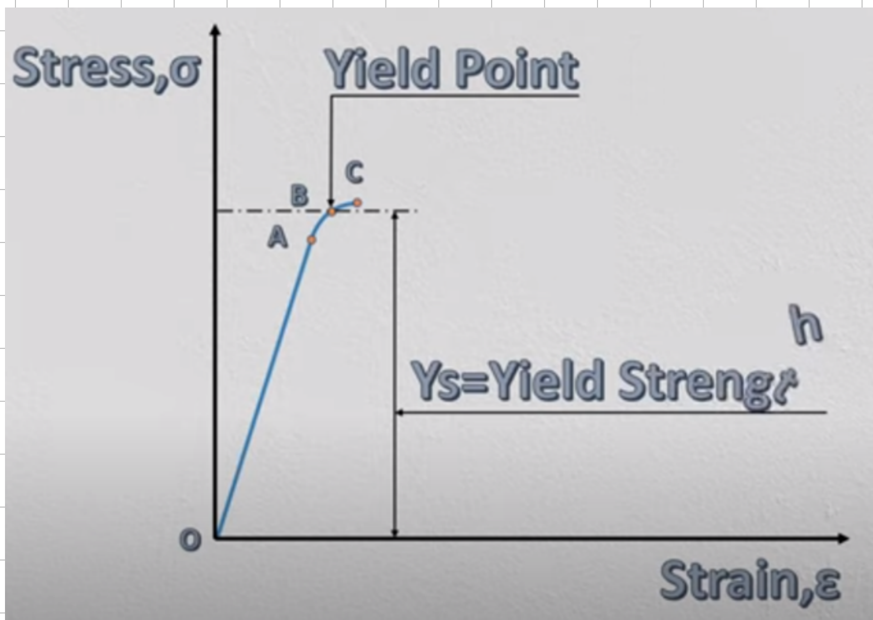
Important when doing wireline/tractor operations in the tubing, to avoid getting stuck
https://www.youtube.com/watch?v=i77v2snWZ1c

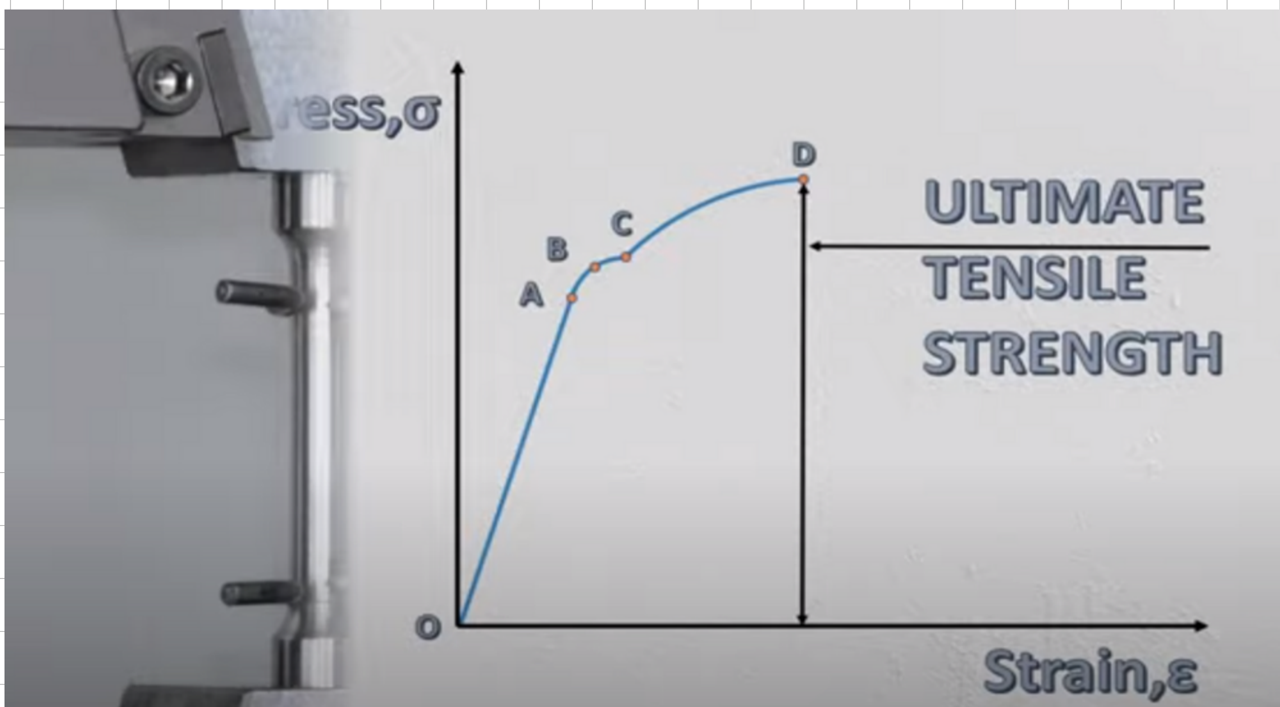
What is yield stress: https://www.youtube.com/watch?v=RY9X_O8is-k





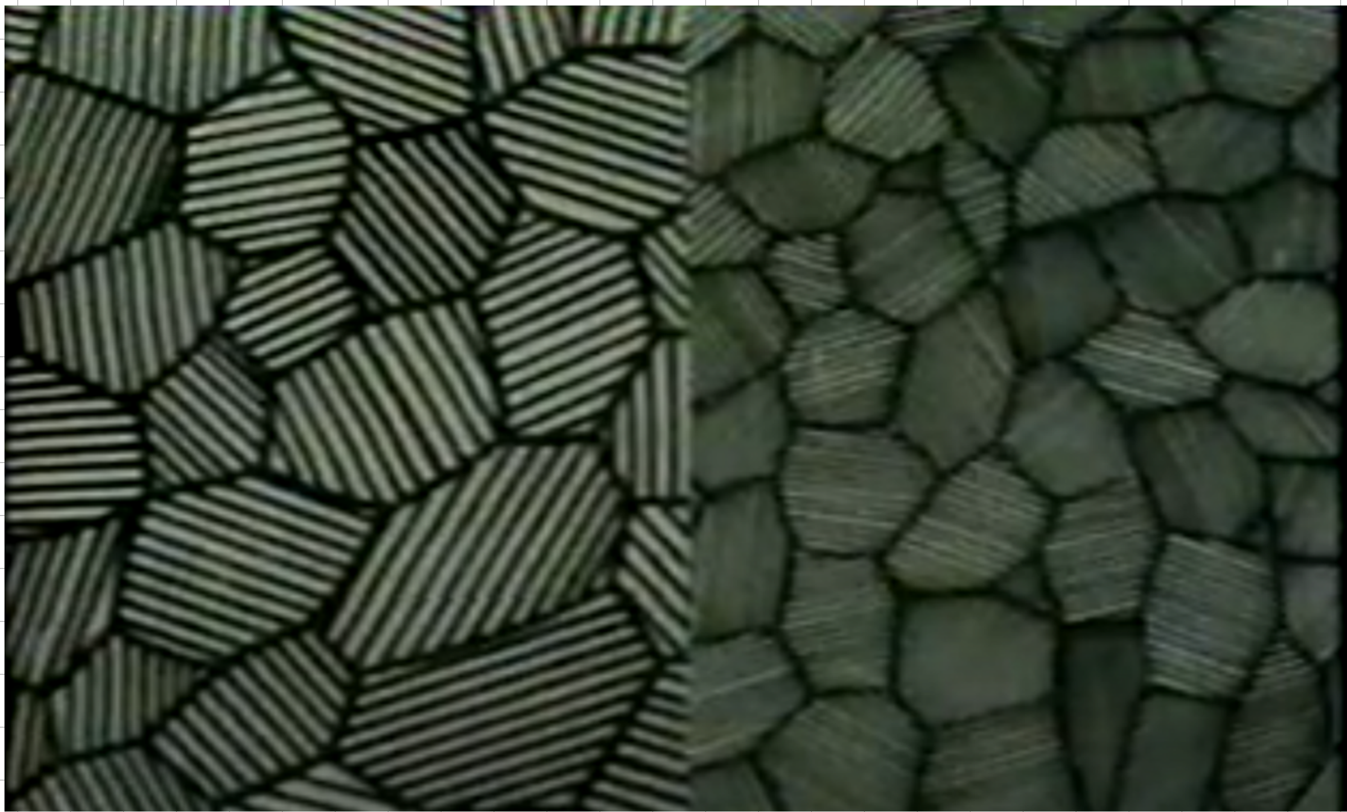




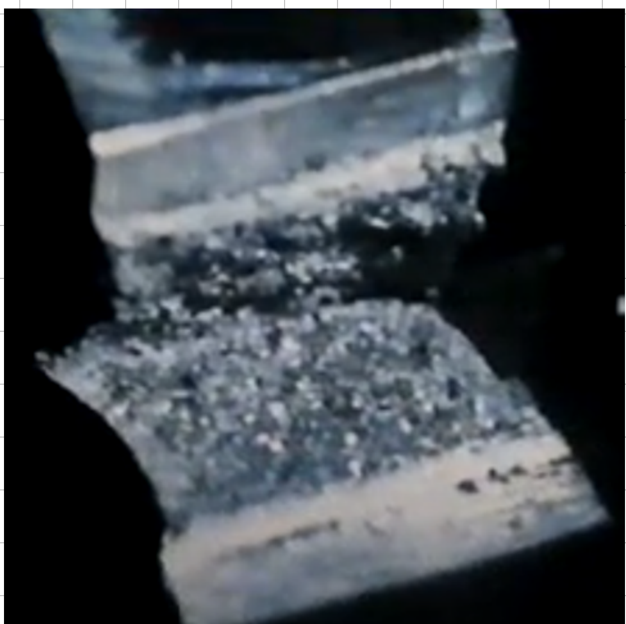


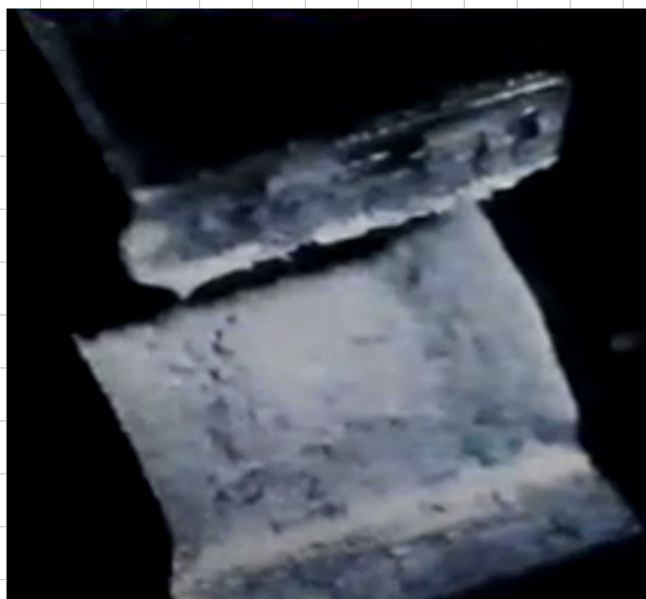
Prof. Milan Stanko (NTNU)

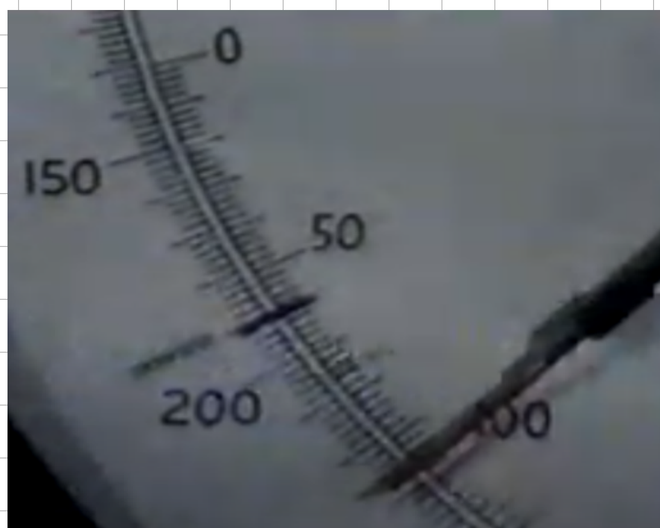
Effect of heat treatment on material: https://www.youtube.com/watch?v=0SIr2sBHxA4



Normalization







Nominal diameter often is neither the ID nor the OD, but when diameters start to go up it coincides with OD

The yield point is not easy to measure exactly. In the industry standard (API-5 series) most often used for well tubular goods, the yield point is defined as the stress corresponding to 0.5% elongation. Such an elongation causes permanent deformation.

The API (API Specification 5CT, 2005) defines the API yield strength (somewhat arbitrarily) as the minimum stress required to elongate the pipe by 0.5% for all grades up to T95, 0.6% for grade P110 and 0.65% for grade Q125. Elongation is measured using an extensometer according to ASTM A370-5 standard (2005). The API yield stress is above the yield point. The API yield stress defines the minimum strength of the grade. For example, L80 pipe has a minimum API yield stress of 80 ksi, that is 80,000 psi. As well as the grade providing the yield strength, tubulars are frequently designated with a singular or double letter prefix, for example L or HC. API grades use the single letter, while proprietary grades use

API-pipe is referred to by steel grade and dimensions. The steel grade is designated by letters and numbers (e.g. N-80). The letter refers to requirements of the manufacturing process. The number refers to the minimum yield strength in thousands of psi, Tab. 3.1. The minimum yield strength is commonly used as load limit. Thus, N-80 and L-80 tubulars have the same load capacity. The manufacturing process differs, resulting in different corrosion resistance. Some manufacturers produce N-80 tubulars approaching L-80 quality.

Example:

P-105 and P-110 are highest yield strength tubulars covered by the API specifications. The manufacturing process may according to the cification API- 5AX be either quenched and tempered or normalized and tempered. Experience seems to indicate that normalized and tempered pipes is less suitable for these grades.

Steel properties can be modified with carbon content, heat treatment and:

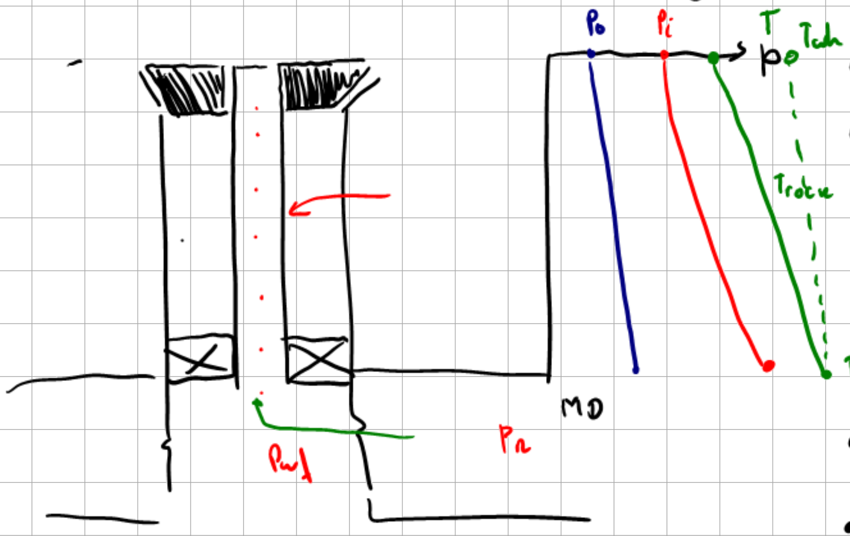
Alloy steel is a type of steel alloyed with several elements such as molybdenum, manganese, nickel, chromium, vanadium, silicon, and boron. These alloying elements are added to increase strength, hardness, wear resistance, and toughness.

Premium tubular = NON API

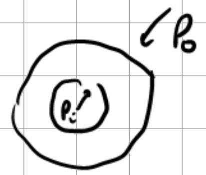
20241004

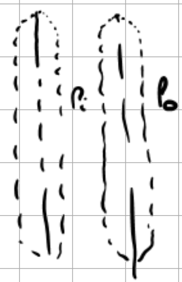
OUTLINE:

-Class exercise - Stress calculations in tubing

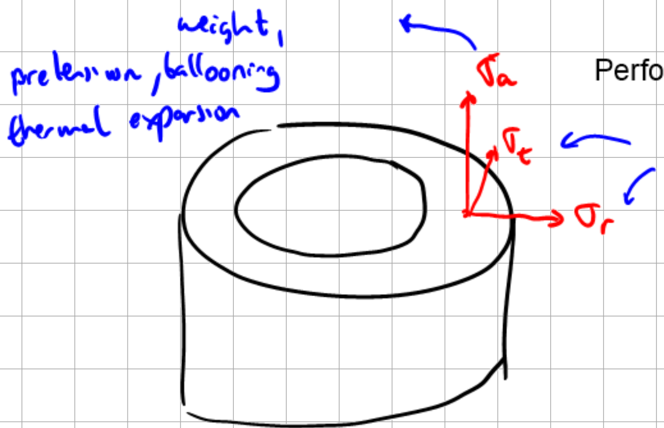


- tubing weight
- pre-tension
- internal/external pressure





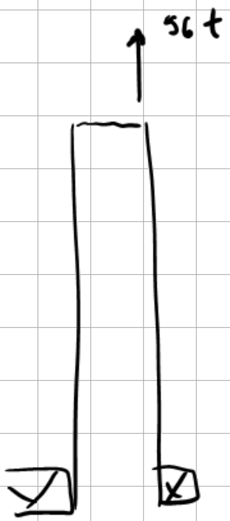
- thermal expansion (expansion)
- ballooning (contraction)



Perform a stress analysis during production

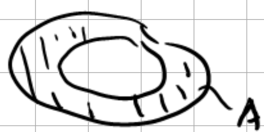
calculate for all depths, for inner and outer radius

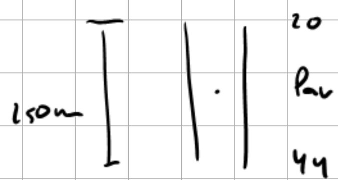
$$\uparrow F_x = (L-x)\cdot \text{tubing weight}\cdot g$$



$$56000 \cdot 9.81 = N$$

$$\sigma_{a\,\text{pre-tension}} = \frac{56000 \cdot 9.81}{A}$$





$$\sigma = E \cdot \frac{\Delta L}{L}$$

$$\Delta L_{ballooning} + \Delta L_{Thermal} \longrightarrow \sigma_{axial}\ ?$$

Finally, solving the general equations with A & B gives Lamé's equations:

- Hoop Stress,

$$\sigma_h = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2) r^2} \quad (5)$$

- Radial Stress,

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{(p_i - p_o) r_o^2 r_i^2}{(r_o^2 - r_i^2) r^2} \quad (6)$$

expansion/contraction due to ballooning

Length Change due to Ballooning Formula (If tubing is free to move)

$$\Delta L_{ballooning} = -\frac{2\mu L}{E(A_o - A_i)} \times (A_i \Delta P_i - A_o \Delta P_o)$$

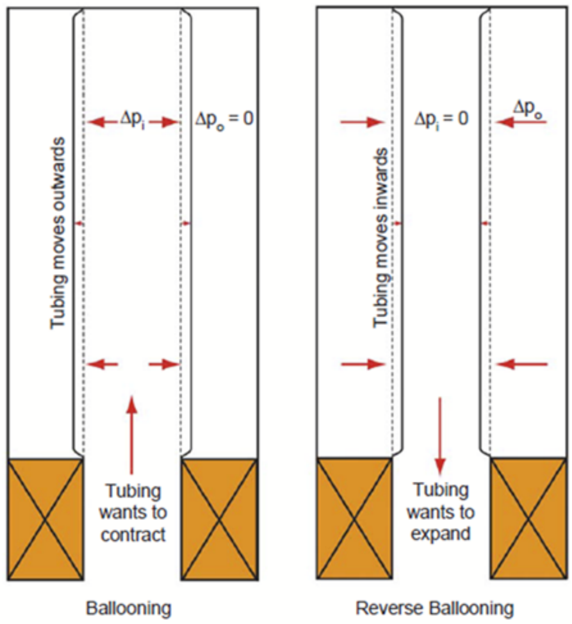


Figure 9.12 Ballooning effects.

Thermal expansion

$$\Delta L_T = L \alpha \Delta T \quad (3\text{-}14)$$

where:
- L : initial length
- ΔT : temperature change
- α : thermal expansion coefficient (for steel, $\alpha = 11.5 \cdot 10^{-6}\ K^{-1}$)

How to convert deformation to stress?

$$\sigma = E \cdot \varepsilon = E \cdot \frac{\Delta L}{L}$$

$$\sigma_a = \frac{\Delta L_{te}}{L} \cdot E$$

thermal expansion/ compression

The most widely used yielding criterion is the Huber–Hencky–Mises (abbreviated as Von Mises equivalent or VME) yield condition, which is based on the maximum distortion energy theory. Ignoring torque, the yielding criterion is calculated from the three stresses:

$$\sigma_{\mathrm{VME}} = \frac{1}{\sqrt{2}}\left[(\sigma_a - \sigma_t)^2 + (\sigma_t - \sigma_r)^2 + (\sigma_r - \sigma_a)^2\right]^{0.5} \tag{9.43}$$

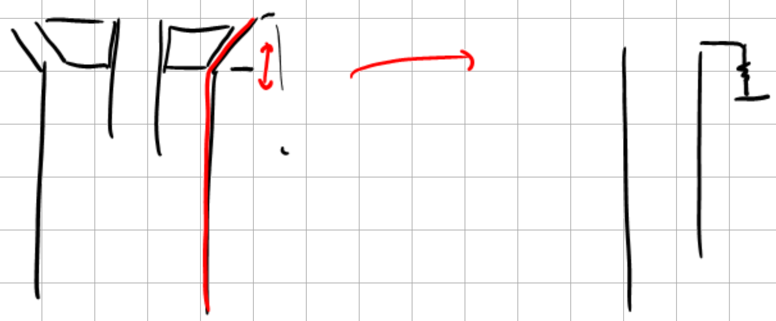
Yielding occurs when the VME stress (σ_{VME}) exceeds the yield stress (Y_p). Note that the VME stress is a combination of all three stresses, but not simply a vector addition of these stresses.

$$\frac{\sigma_{vmE}}{\sigma_{yield}} = F \quad < 1$$

$$\frac{\sigma_{yield}}{F_{sa}} \leftarrow \quad > 1$$

What have we neglected?

wellhead movement

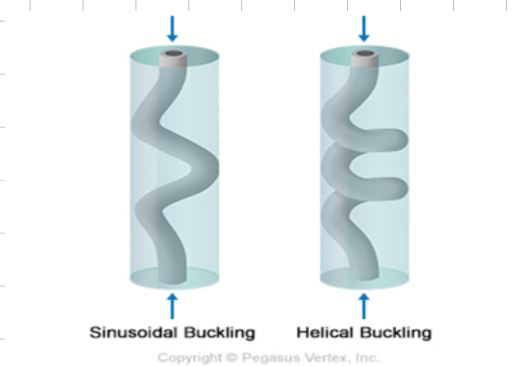


-Buckling (bending, torsion)

-Deviation effect (bending, torsion)

-Buoyancy

-fluid friction



Heat treatment in metals analog: https://www.youtube.com/watch?v=xuL2yT-B2TM

Why are metals so stretchy: https://www.youtube.com/watch?v=sn1Y6zIS91g

Tubing stress analysis (TPG4245), Prof Milan Stanko, NTNU

Steel properties

Tubing yield stress	[bara]	5515.8	8.00E+04	[psi]
Young modulus	[bara]	2.10E+06		
Poisson ratio	[-]	0.3		
thermal expansion coefficient	[1/K]	1.15E-05		

Tubing dimensions and properties

Tubing ID	[m]	0.101	Tubing inner r	[m]	0.050
tubing OD	[m]	0.114	Tubing outer r	[m]	0.057
Tubing weight	[kg/m]	18.75			
Tubing cross section	[m^2]	2.32E-03			

Uniform axial stresses

Pre-load of tubing	[t]	56.3
Stress due to tubing pre-load	[bara]	2376.37
DeltaL thermal expansion	[m]	1.04
DeltaL ballooning	[m]	-0.72
DeltaL total	[m]	0.32
Constant axial stress due to length change	[bara]	-223
Total Constant axial stress	[bara]	2153

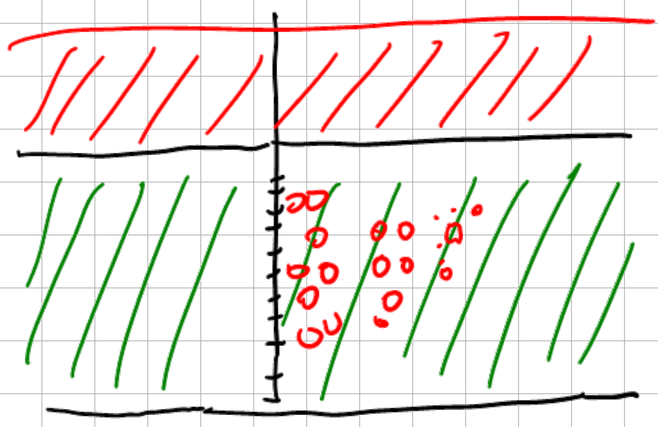
TVD	Tformation	Tfluid (prod)	pannulus	ptubing	tangential stress at ri	tangential stress at ro	radial stress at ri	radial stress at ro	Axial force (weight)	Axial stress (weight)	Axial stress (all)	Equivalent stress @ ri	Equivalent stress @ ri	Safety factor at ri
[m]	[C]	[C]	[bara]	[bara]	[bara]	[bara]	[Pa]	[Pa]	[N]	[bara]	[bara]	[bara]	[bara]	
0	10	70	20.0	323.4	2357	2054	-323	-20	5.52E+05	2376	4529	4210	3945	0.76
250	18	73	44.5	338.1	2256	1962	-338	-45	5.06E+05	2178	4331	4052	3794	0.73
500	25	75	69.1	352.9	2154	1871	-353	-69	4.60E+05	1980	4133	3894	3643	0.71
750	33	78	93.6	367.6	2053	1779	-368	-94	4.14E+05	1782	3935	3736	3492	0.68
1000	40	80	118.1	382.3	1952	1687	-382	-118	3.68E+05	1584	3737	3578	3341	0.65
1250	48	83	142.6	397.0	1850	1596	-397	-143	3.22E+05	1386	3539	3420	3190	0.62
1500	55	85	167.2	411.7	1749	1504	-412	-167	2.76E+05	1188	3341	3262	3039	0.59
1750	63	88	191.7	426.4	1648	1413	-426	-192	2.30E+05	990	3143	3105	2889	0.56
2000	70	90	216.2	441.1	1546	1321	-441	-216	1.84E+05	792	2945	2947	2738	0.53
2250	78	93	240.7	455.9	1445	1230	-456	-241	1.38E+05	594	2747	2790	2588	0.51
2500	85	95	265.3	470.6	1343	1138	-471	-265	9.20E+04	396	2549	2633	2437	0.48
2750	93	98	289.8	485.3	1242	1046	-485	-290	4.60E+04	198	2351	2476	2287	0.45
3000	100	100	314.3	500.0	1141	955	-500	-314	0.00E+00	0	2153	2319	2137	0.42

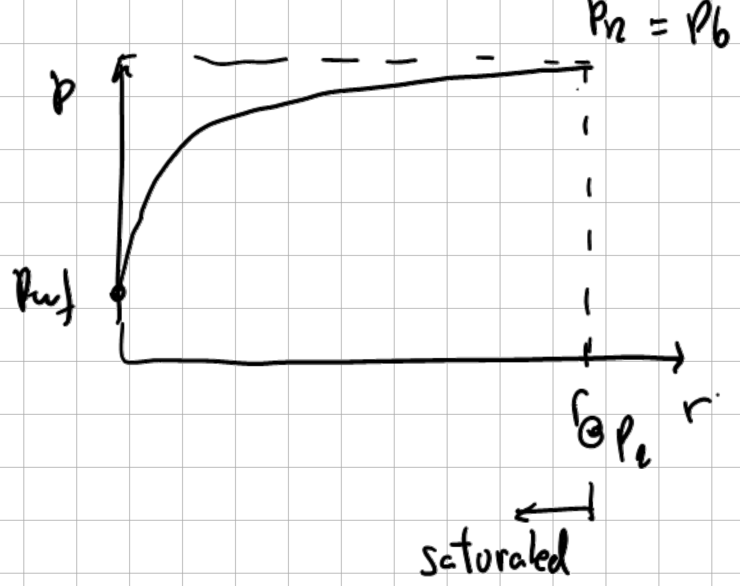
Tubing section	Length	Tav_initial	Tav_final	DeltaL_thermal expansion	pi_initial	pi_final	po_initial	po_final	DeltaL_ballooning
	[m]	[C]	[C]	[m]	[bara]	[bara]	[bara]	[bara]	[m]
1	250	13.75	71.25	0.17	32.3	330.8	32.3	32.3	-0.07
2	250	21.25	73.75	0.15	56.8	345.5	56.8	56.8	-0.07
3	250	28.75	76.25	0.14	81.3	360.2	81.3	81.3	-0.07
4	250	36.25	78.75	0.12	105.8	374.9	105.8	105.8	-0.07
5	250	43.75	81.25	0.11	130.4	389.6	130.4	130.4	-0.06
6	250	51.25	83.75	0.09	154.9	404.4	154.9	154.9	-0.06
7	250	58.75	86.25	0.08	179.4	419.1	179.4	179.4	-0.06
8	250	66.25	88.75	0.06	203.9	433.8	203.9	203.9	-0.06
9	250	73.75	91.25	0.05	228.5	448.5	228.5	228.5	-0.05
10	250	81.25	93.75	0.04	253.0	463.2	253.0	253.0	-0.05
11	250	88.75	96.25	0.02	277.5	477.9	277.5	277.5	-0.05
12	250	96.25	98.75	0.01	302.0	492.6	302.0	302.0	-0.05
			TOTAL-->	1.04				TOTAL-->	-0.72

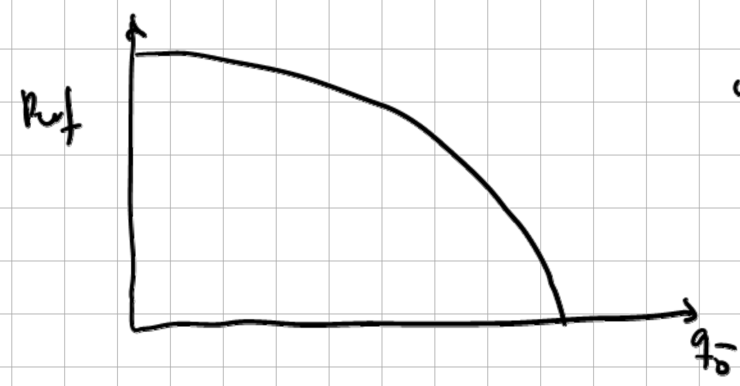
Saturated oil

$p_R = p_b$ @ T_R

solution-gas drive







what equation ?

$$\sim q_o = C\left(p_R^2 - p_{wf}^2\right)$$

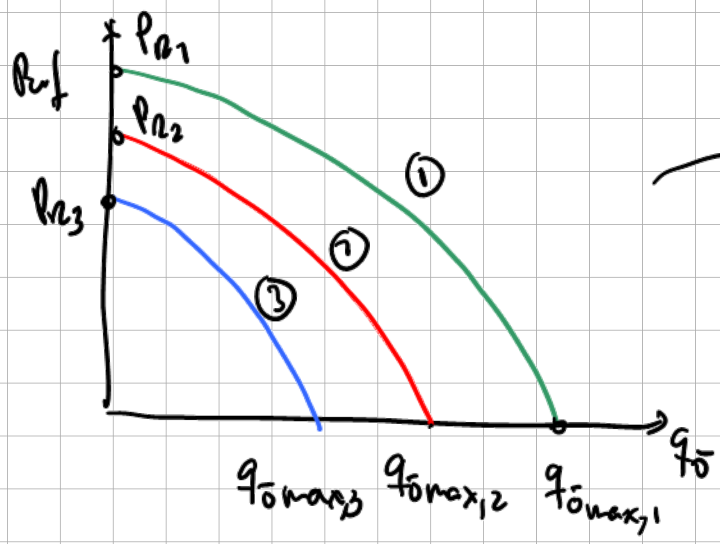
$$q_o = J\left(p_R - p_{wf}\right)$$

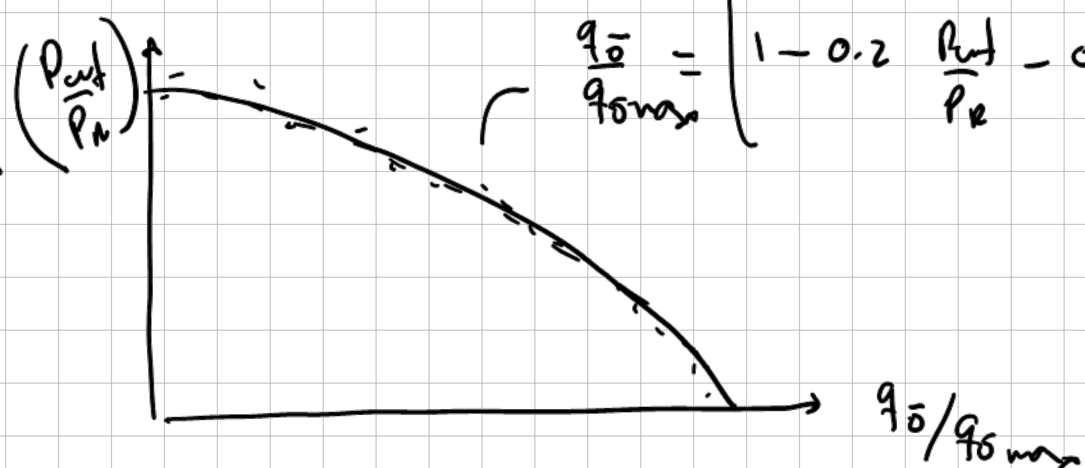
?

Reservoir simulation → 1968 Vogel

Inflow Performance Relationships for Solution-Gas Drive Wells

J. V. VOGEL
MEMBER AIME
SHELL OIL CO.
BAKERSFIELD, CALIF.





$$\frac{q_o}{q_{o\,max}} = \left[1 - 0.2\,\frac{p_{wf}}{p_R} - 0.8\left(\frac{p_{wf}}{p_R}\right)^2\right]$$

q_o	p_{wf}
□	□
□	□
□	□

→

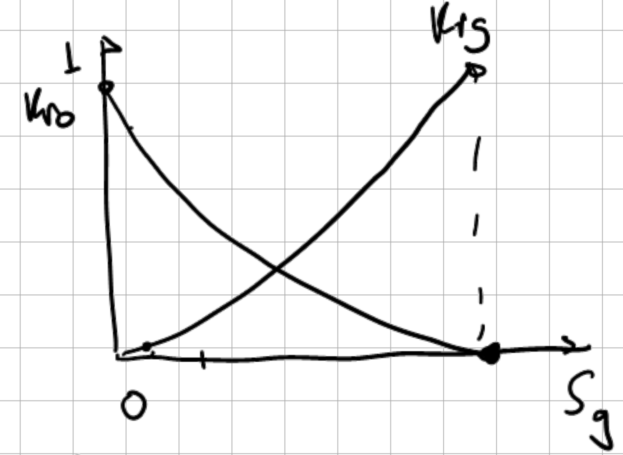
$\frac{q_o}{q_{o\,max}}$	$\frac{p_{wf}}{p_R}$

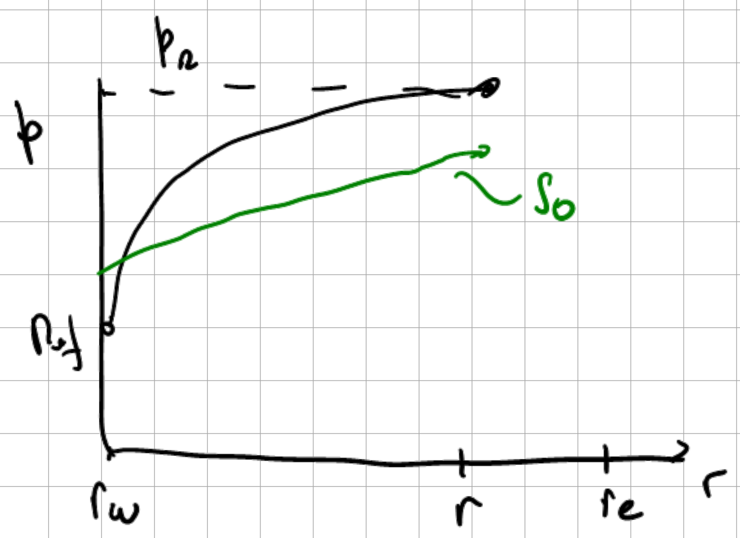
Analytical derivation

$$q_o = \frac{h\ k}{18.68\left(\ln\frac{r_e}{r_w} - 0.75 + S\right)} \qquad \int_{P_{wf}}^{P_R} \frac{k_{ro}}{\mu_o B_o}\,dp$$

effective permeability $(k \cdot k_{ro})$

$k_{ro} = f(S_o)$

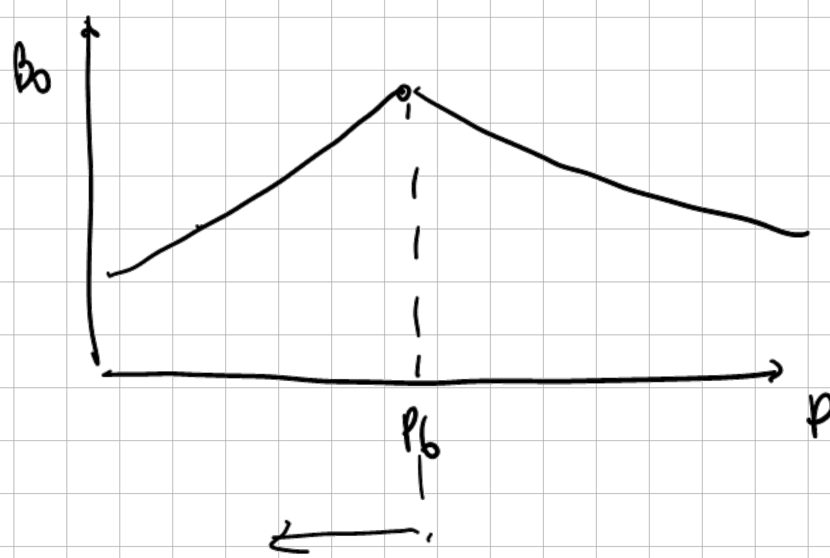


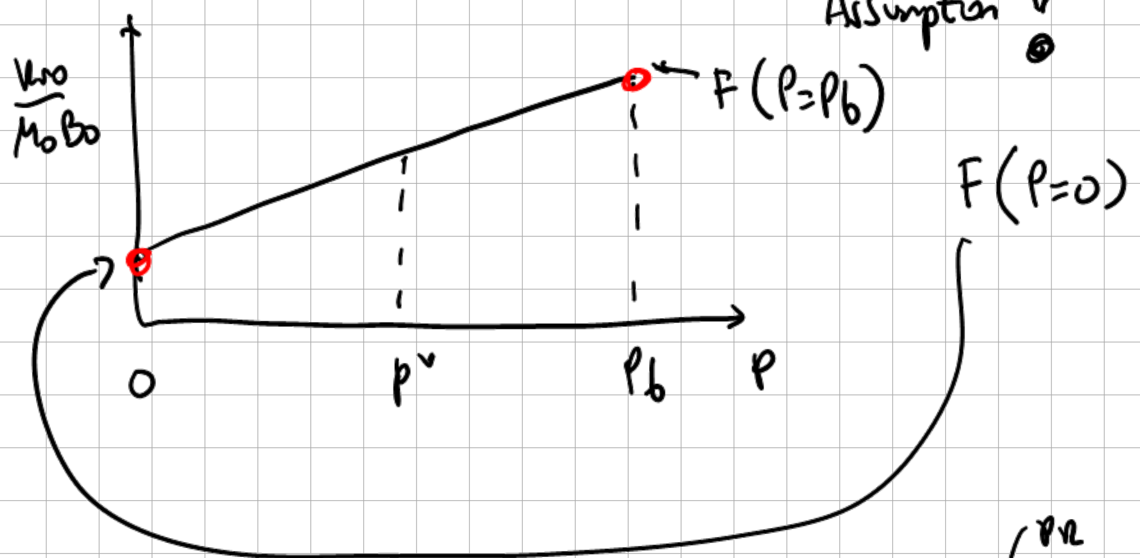


$S_o = f(p) \longrightarrow k_{ro} = f(p)$

$$\int_{P_{wf}}^{P_R} \frac{k_{ro}}{\mu_o B_o}\,dp$$

$k_{ro} = f(S_o) = f(p)$





Assumption

$P \geq P_b$ single phase oil

$k_{ro} = 1$

$$\int_{P_{wf}}^{P_R} F(p)\,dp$$

$$F(p) = F(p=0) + \frac{F(P_R) - F(P=0)}{(P_R - 0)}(p^* - 0)$$

$$\int_{P_{wf}}^{P_R} (a + b\,x)\,dx$$

$$F(p) = F(p=0) + [F(p_R) - F(p=0)] \cdot \frac{p}{p_R} \qquad \text{Eq. 2-16}$$

Therefore, the solution of the pressure function integral will have a linear term in addition to the quadratic term:

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0) \cdot (p_R - p_{wf}) + [F(p_R) - F(p=0)] \cdot \frac{1}{p_R \cdot 2} (p_R{}^2 - p_{wf}{}^2) \qquad \text{Eq. 2-17}$$

Expanding terms:

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0) \cdot p_R - F(p=0) \cdot p_{wf} + [F(p_R) - F(p=0)] \cdot \frac{1}{p_R \cdot 2} (p_R{}^2 - p_{wf}{}^2) \qquad \text{Eq. 2-18}$$

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0) \cdot p_R - F(p=0) \cdot p_{wf} + F(p_R) \cdot \frac{p_R}{2} - F(p_R) \cdot \frac{p_{wf}{}^2}{p_R \cdot 2} - F(p=0) \cdot \frac{p_R}{2} + F(p=0) \cdot \frac{p_{wf}{}^2}{p_R \cdot 2} \qquad \text{Eq. 2-19}$$

Grouping terms by pressure:

$$\int_{p_{wf}}^{p_R} F(p)dp = [F(p=0) + F(p_R)] \cdot \frac{p_R}{2} - F(p=0) \cdot p_{wf} - \frac{[F(p_R) - F(p=0)]}{2} \cdot \frac{p_{wf}{}^2}{p_R} \qquad \text{Eq. 2-20}$$

Dividing by $[F(p=0) + F(p_R)] \cdot \frac{p_R}{2}$

$$\frac{2}{[F(p=0) + F(p_R)] \cdot p_R} \cdot \int_{p_{wf}}^{p_R} F(p)dp = 1 - \frac{F(p=0) \cdot 2}{[F(p=0) + F(p_R)]} \cdot \frac{p_{wf}}{p_R} - \frac{[F(p_R) - F(p=0)]}{[F(p=0) + F(p_R)]} \cdot \left(\frac{p_{wf}}{p_R}\right)^2 \qquad \text{Eq. 2-21}$$

Defining a variable "V"

$$V = \frac{F(p=0) \cdot 2}{[F(p=0) + F(p_R)]} \qquad \text{Eq. 2-22}$$

Therefore:

$$1 - V = \frac{F(p_R) - F(p=0)}{[F(p=0) + F(p_R)]} \qquad \text{Eq. 2-23}$$

Substituting back in the integral of the pressure function:

$$\frac{2}{[F(p=0) + F(p_R)] \cdot p_R} \cdot \int_{p_{wf}}^{p_R} F(p)dp = 1 - V \cdot \frac{p_{wf}}{p_R} - (1 - V) \cdot \left(\frac{p_{wf}}{p_R}\right)^2 \qquad \text{Eq. 2-24}$$

Substituting Eq. 2-24 back in the IPR equation:

$$q_{\bar{o}} = \frac{k \cdot h \cdot [F(p=0) + F(p_R)] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} \left[1 - V \cdot \frac{p_{wf}}{p_R} - (1 - V) \cdot \left(\frac{p_{wf}}{p_R}\right)^2\right] \qquad \text{Eq. 2-25}$$

Making $q_{\bar{o},max}$:

$$q_{\bar{o},max} = \frac{k \cdot h \cdot [F(p=0) + F(p_R)] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} \qquad \text{Eq. 2-26}$$

The following expression is obtained:

$$q_o = q_{o,max}\left[1 - V \cdot \frac{p_{wf}}{p_R} - (1 - V) \cdot \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

Vogel says $V = 0.2$ →

Vogel found this same equation using data points generated with reservoir simulator, with V = 0.2.

Using Eq. 2-22, and assuming V = 0.2, $F(p=0)$ is then:

$$F(p=0) = \frac{F(p_R)}{9} \qquad \text{Eq. 2-28}$$

Eq. 2-26 can then be further simplified:

$$q_{o,max} = \frac{k \cdot h \cdot \left[\frac{10}{9} \cdot F(p_R)\right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o}\right)_{@pR}\right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 1.8} = \frac{J}{1.8} \cdot pR \qquad \text{Eq. 2-29}$$

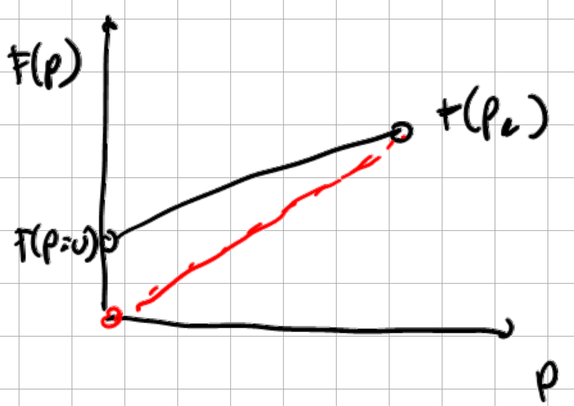
The Isochronal Testing of Oil Wells

By

M. J. Fetkovich, Member AIME, Phillips Petroleum Co.

1973

Fetkovich proposed



$v = 0$

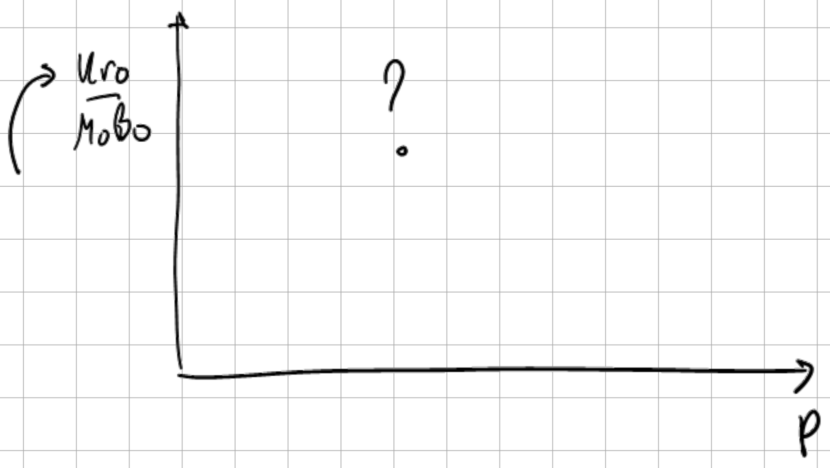
$$\frac{q_o}{q_{o\,max}} = \left[1 - \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

$$q_{o,max} = \frac{k \cdot h \cdot [F(p_R)] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o}\right)_{@p_R}\right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{J}{2} \cdot pR$$

$$q_o = \frac{J}{2} p_R \left(1 - \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

$$q_o = \frac{J}{2} \left[p_R - \frac{(p_{wf})^2}{p_R}\right]$$

$$q_o = \frac{J}{2 p_R} \left[p_R^2 - p_{wf}^2\right] \rightsquigarrow q_g = C\left(p_R^2 - p_{wf}^2\right)^n$$



(1941) Evinger and Muskat

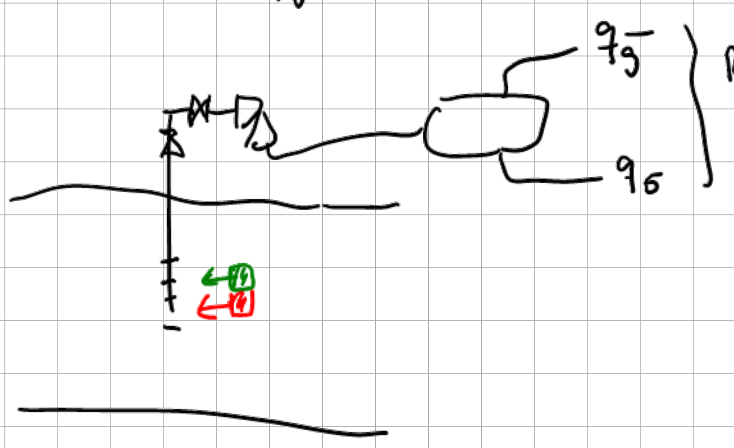
Calculation of Theoretical Productivity Factor

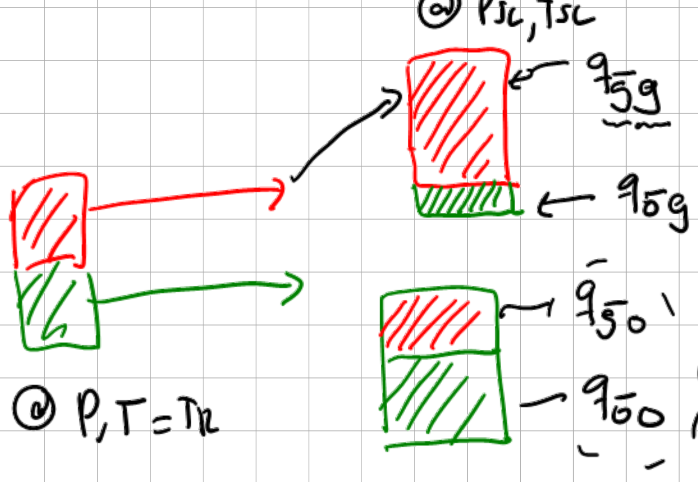
By H. H. Evinger* and M. Muskat*

(New York Meeting, February 1941)

$k_{ro} = f(p)$

$$\frac{q_{\bar{g}}}{q_{\bar{o}}} \qquad GOR = R_p = \frac{q_{\bar{g}}}{q_{\bar{o}}} = \frac{q_{\bar{g}g} + q_{\bar{g}o}}{q_{\bar{o}o}} = \frac{q_{\bar{g}g}}{q_{\bar{o}o}} + \frac{q_{\bar{g}o}}{q_{\bar{o}o}} = \frac{q_{\bar{g}g}}{q_{\bar{o}o}} + R_s(p)$$





$q_{\bar{g}} = q_{\bar{g}g} + q_{\bar{g}o}$

$q_{\bar{o}} = q_{\bar{o}g} + q_{\bar{o}o}$

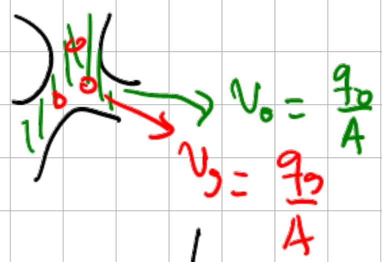
very small

$q_{\bar{o}g} << q_{\bar{o}o}$

$q_{\bar{o}} = q_{\bar{o}o}$

$$R_s = \frac{q_{\bar{g}o}}{q_{\bar{o}o}} \qquad R_s(p) \text{ solution gas oil ratio}$$

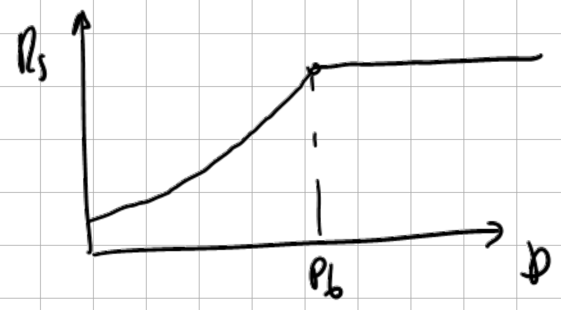
$$\frac{q_{\bar{g}g}}{q_{\bar{o}o}} = \frac{q_g}{B_g}\frac{B_o}{q_o} = \frac{k_{rg}}{k_{ro}}\frac{\mu_o B_o}{\mu_g B_g}$$



$$\frac{q_g}{q_o} = \frac{k_{rg}}{\mu_g}\frac{\mu_o}{k_{ro}}$$

$$v_g = \frac{k_{rg} k}{\mu_g}\frac{dp}{dx}$$

$$v_o = \frac{k_{ro} k}{\mu_o}\frac{dp}{dx}$$



$$\frac{V_o}{V_{\bar{o}}} = B_o = \frac{q_o}{q_{\bar{o}o}}$$

$$\frac{V_g}{V_{\bar{g}}} = B_g = \frac{q_g}{q_{\bar{g}g}}$$

$$R_p = \frac{k_{rg}}{k_{ro}}\frac{\mu_o B_o}{\mu_g B_g} + R_s(p)$$

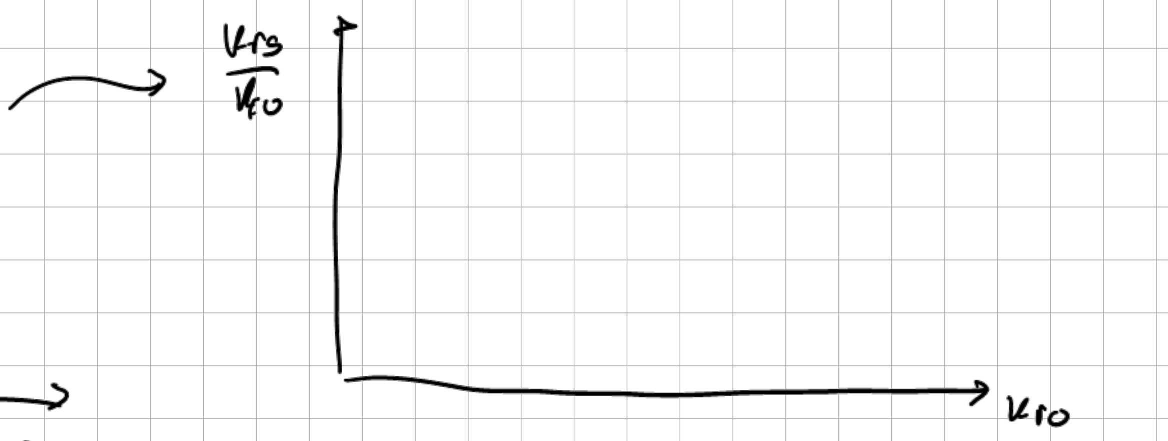
measured

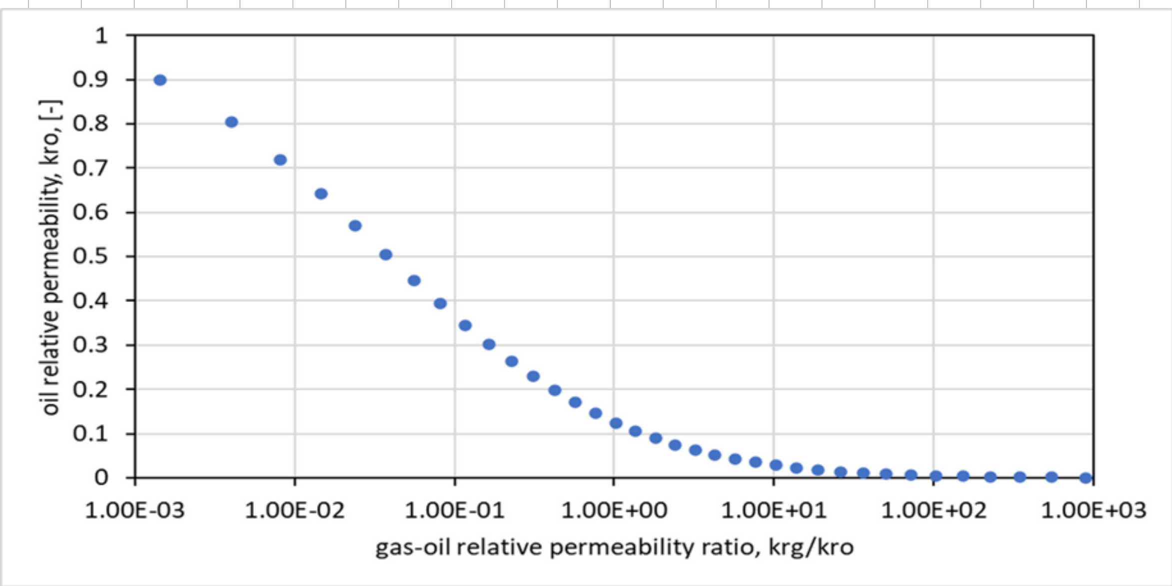
f(p)

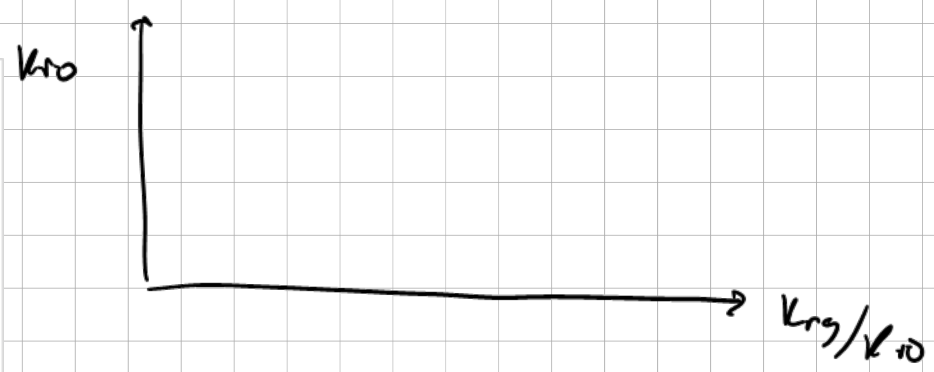
$$\frac{k_{rg}}{k_{ro}} = \frac{\mu_g B_g}{\mu_o B_o}(R_p - R_s) \qquad (1)$$

f(p) !

But we need k_{ro}, NOT $\frac{k_{rg}}{k_{ro}}$!





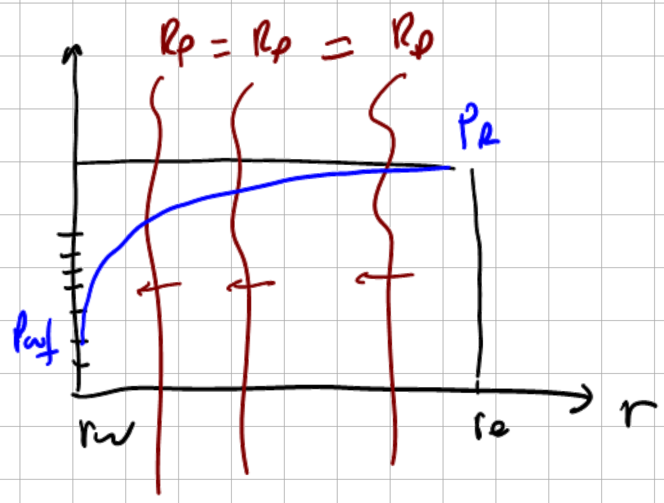


(2)

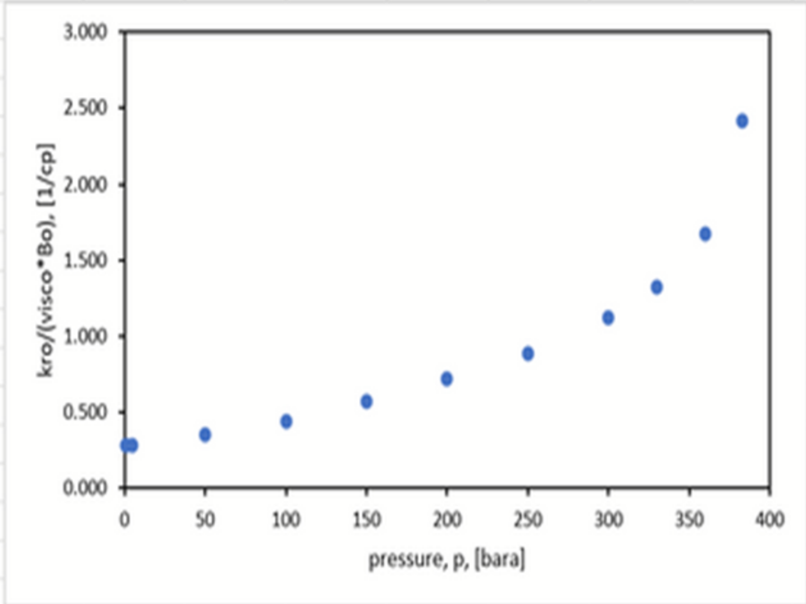
1) from (1) at p ~> $\frac{k_{rg}}{k_{ro}}$

2) from (2) with $\frac{k_{rg}}{k_{ro}}$ ~> kro

3) kro f(p)



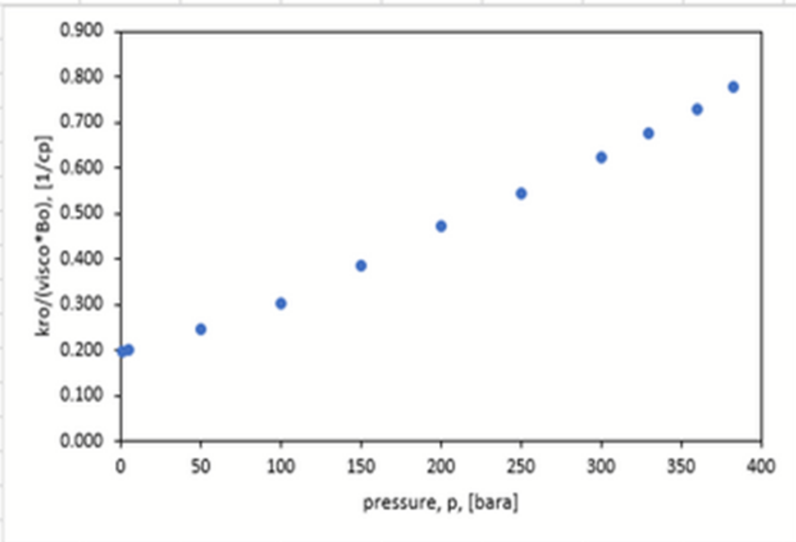
Rp	[Sm3/Sm3]	300							
COLUMN	4	6	3	5	2				
p	Rs	viscg	Bg	Visco	Bo	krg/kro	kro	kro/(visco*Bo)	So
[bara]	[Sm3/Sm3]	[cp]	[m3/Sm3]	[cp]	[m3/Sm3]	[-]	[-]	[1/cp]	[-]
1	0.5	1.12E-02	2.66E-02	1.4	1.2	5.54E-02	4.49E-01	0.277	0.60
5	2.6	1.14E-02	2.59E-02	1.4	1.2	5.50E-02	4.50E-01	0.282	0.60
50	25.5	1.32E-02	1.81E-02	1.1	1.2	4.96E-02	4.67E-01	0.352	0.60
100	48.8	1.58E-02	1.22E-02	0.9	1.2	4.46E-02	4.82E-01	0.443	0.60
150	76.4	1.82E-02	8.20E-03	0.7	1.3	3.81E-02	5.03E-01	0.573	0.61
200	107.2	2.14E-02	6.25E-03	0.5	1.4	3.65E-02	5.09E-01	0.722	0.61
250	143.9	2.51E-02	5.29E-03	0.4	1.5	3.59E-02	5.12E-01	0.888	0.61
300	187.9	2.90E-02	4.69E-03	0.3	1.6	3.23E-02	5.29E-01	1.121	0.62
330	220.4	3.15E-02	4.45E-03	0.3	1.7	2.65E-02	5.57E-01	1.322	0.62
360	258.3	3.41E-02	4.26E-03	0.2	1.8	1.61E-02	6.29E-01	1.672	0.64
383	292.6	3.60E-02	4.14E-03	0.2	1.9	3.19E-03	8.34E-01	2.414	0.67



NOT LINEAR!!!

For higher GOR

Rp	[Sm3/Sm3]	800							
COLUMN	4	6	3	5	2				
p	Rs	viscg	Bg	Visco	Bo	krg/kro	kro	kro/(visco*Bo)	So
[bara]	[Sm3/Sm3]	[cp]	[m3/Sm3]	[cp]	[m3/Sm3]	[-]	[-]	[1/cp]	[-]
1	0.5	1.12E-02	2.66E-02	1.4	1.2	1.48E-01	3.17E-01	0.196	0.56
5	2.6	1.14E-02	2.59E-02	1.4	1.2	1.47E-01	3.17E-01	0.199	0.56
50	25.5	1.32E-02	1.81E-02	1.1	1.2	1.40E-01	3.24E-01	0.244	0.56
100	48.8	1.58E-02	1.22E-02	0.9	1.2	1.33E-01	3.30E-01	0.304	0.56
150	76.4	1.82E-02	8.20E-03	0.7	1.3	1.23E-01	3.40E-01	0.387	0.56
200	107.2	2.14E-02	6.25E-03	0.5	1.4	1.31E-01	3.32E-01	0.471	0.56
250	143.9	2.51E-02	5.29E-03	0.4	1.5	1.51E-01	3.14E-01	0.545	0.56
300	187.9	2.90E-02	4.69E-03	0.3	1.6	1.77E-01	2.95E-01	0.624	0.55
330	220.4	3.15E-02	4.45E-03	0.3	1.7	1.93E-01	2.85E-01	0.675	0.54
360	258.3	3.41E-02	4.26E-03	0.2	1.8	2.09E-01	2.75E-01	0.730	0.54
383	292.6	3.60E-02	4.14E-03	0.2	1.9	2.19E-01	2.69E-01	0.778	0.54



LINEAR??

$$q_{\bar{o}} = \frac{k \cdot h}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)} \underbrace{\int_{p_{wf}}^{p_R} \frac{k_{ro}}{\mu_o \cdot B_o} dp}_{\left(m(p_R) - m(p_{wf})\right)} =$$

$$\int_0^{p_{wf}} \frac{k_{ro}}{\mu_o B_o} dp = m(p_{wf})$$

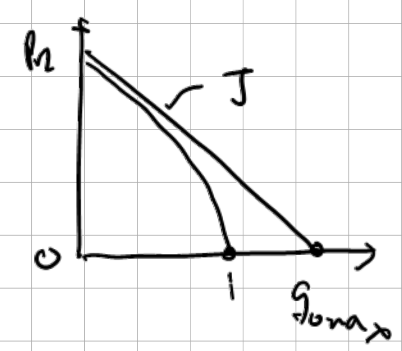
Not the same gas m function !

$$\int \frac{p \, dp}{Z \mu_g}$$

Vogel $(V = 0.2)$

$$q_{\bar{o}} = q_{\bar{o},max} \left[1 - V \cdot \frac{p_{wf}}{p_R} - (1 - V) \cdot \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

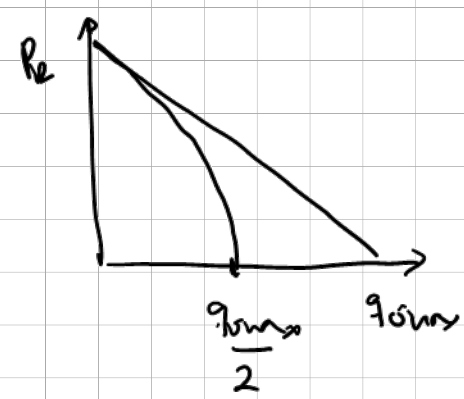
$$q_{\bar{o},max} = \frac{\overbrace{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o}\right)_{@pR}\right]}^{J} \cdot (p_R - 0)}{18.68 \cdot \left(ln\left(\frac{r_e}{r}\right) - 0.75 + s\right) \cdot 1.8} = \frac{J}{1.8} \cdot pR$$



Fetkovich

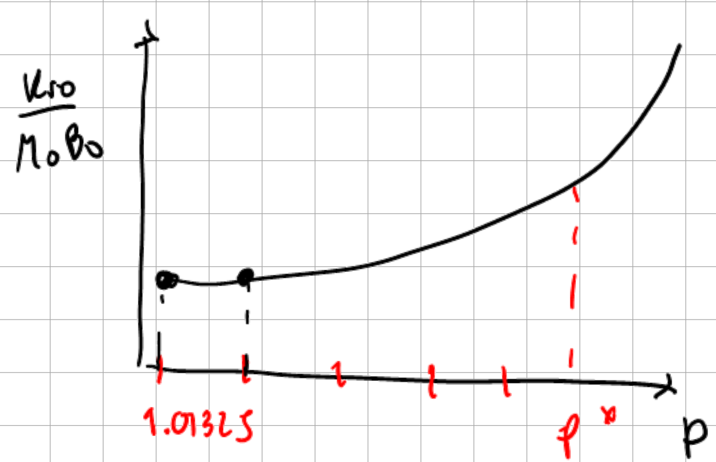
$$q_{\bar{o}} = q_{\bar{o},max} \left[1 - \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

$$\frac{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o}\right)_{@pR}\right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{J}{2} \cdot pR$$

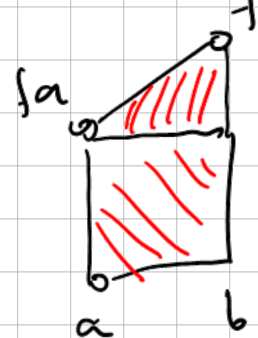


oil m function in VBA:

```
Function m_function_oil(p, col, Matrix As Range)
    'p in bara
    'n number of intervals to use in the integration
    p1 = 1.01325
    p2 = p
    If p2 - p1 > 40 Then
        n = Round((p2 - p1) / 10, 0)
    Else
        n = 10
    End If
    DP = (p2 - p1) / n
    pj = p1
    Sum = 0
    fa = tabinterpol(pj, col, Matrix)
    For J = 1 To n
        pj = pj + DP
        fb = tabinterpol(pj, col, Matrix)
        Sum = Sum + (DP * (fa + fb)) * 0.5
        fa = fb
    Next
    m_function_oil = Sum
End Function
```

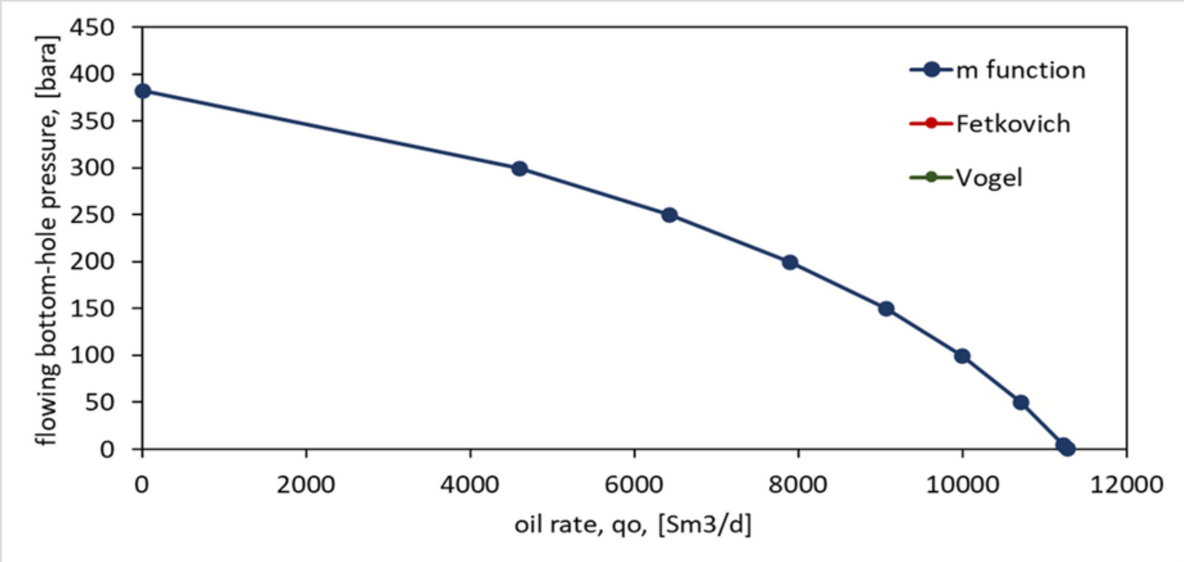


fb (from interpolation on table)

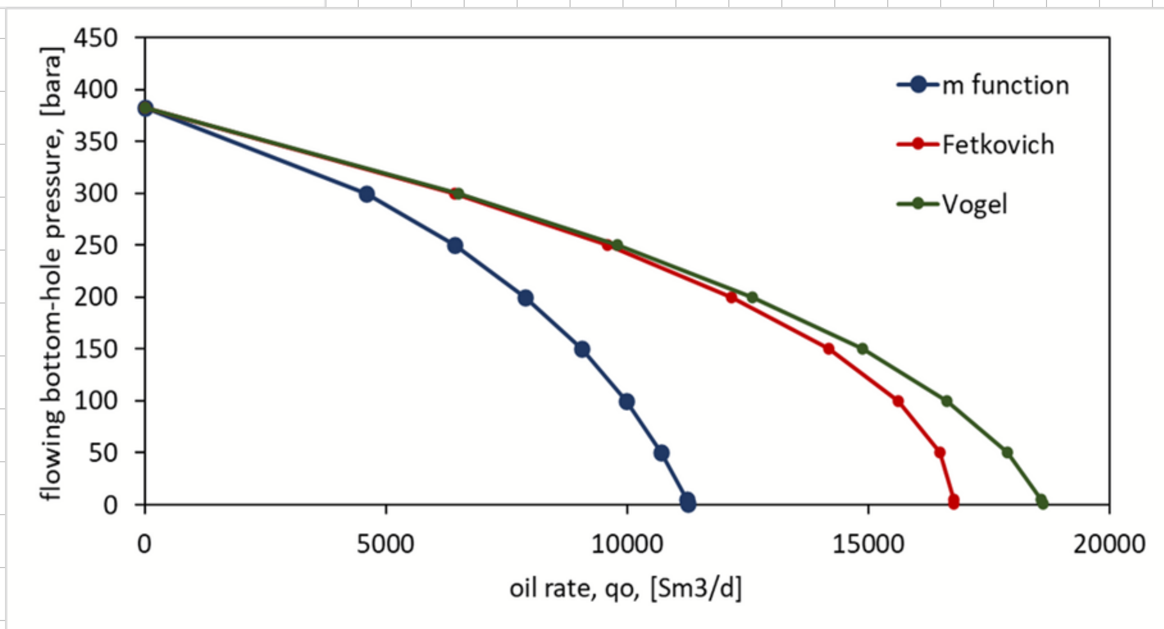


$$Area = (fa + fb) \cdot \frac{(b - a)}{2}$$

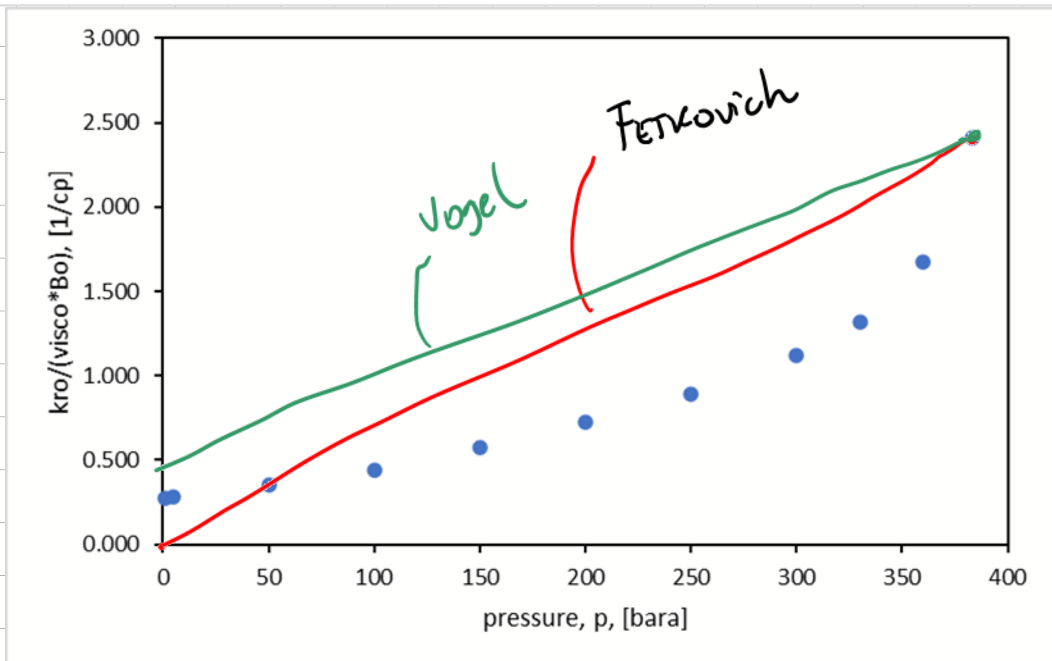
```
Function ipr_sat_oil_qo_m_function(k, h, re, rw, s, mpR, mPwf)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'mpR, m function at reservoir pressure [bara/cp]
    'mpwf, m functino at flowing bottom-hole pressure [bara/cp]
    ipr_sat_oil_qo_m_function = k * h * (mpR - mPwf) / ((Log(re / rw) - 0.75 + s) * 18.68)
End Function
```



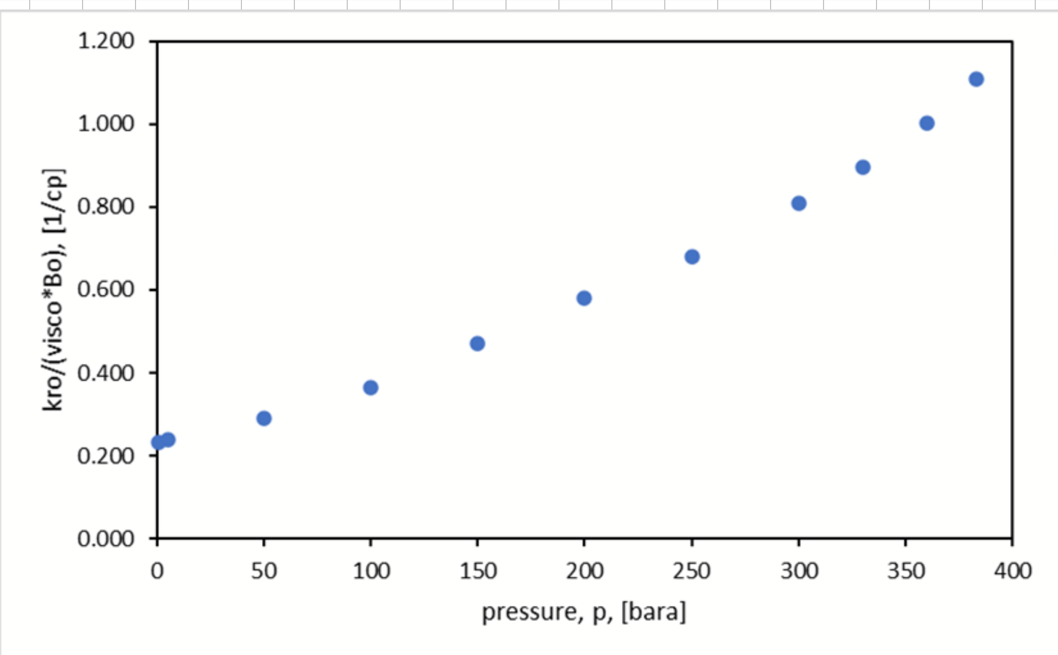

```
Function ipr_sat_oil_qo_Fetkovich(k, h, re, rw, s, pR, Pwf, Bo_pR, visco_pR, kro_pR)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'pR, reservoir pressure [bara]
    'pwf, flowing bottom-hole pressure [bara]
    'kro_pR oil relative permeability at reservoir pressure, [-]
    'visco_pR oil viscosity at reservoir pressure [cp]
    'Bo_pR oil formation volume factor at reservoir pressure, [m3/Sm3]
    J = k * h * (kro_pR / (visco_pR * Bo_pR)) / ((Log(re / rw) - 0.75 + s) * 18.68)
    qomax = J * (pR - 0) / 2
    ipr_sat_oil_qo_Fetkovich = qomax * (1 - (Pwf / pR) ^ 2)
End Function
```




```
Function ipr_sat_oil_qo_Vogel(k, h, re, rw, s, pR, Pwf, Bo_pR, visco_pR, kro_pR)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'pR, reservoir pressure [bara]
    'pwf, flowing bottom-hole pressure [bara]
    'kro_pR oil relative permeability at reservoir pressure, [-]
    'visco_pR oil viscosity at reservoir pressure [cp]
    'Bo_pR oil formation volume factor at reservoir pressure, [m3/Sm3]
    J = k * h * (kro_pR / (visco_pR * Bo_pR)) / ((Log(re / rw) - 0.75 + s) * 18.68)
    qomax = J * (pR - 0) / 1.8
    ipr_sat_oil_qo_Vogel = qomax * (1 - 0.2 * (Pwf / pR) - 0.8 * (Pwf / pR) ^ 2)
End Function
```



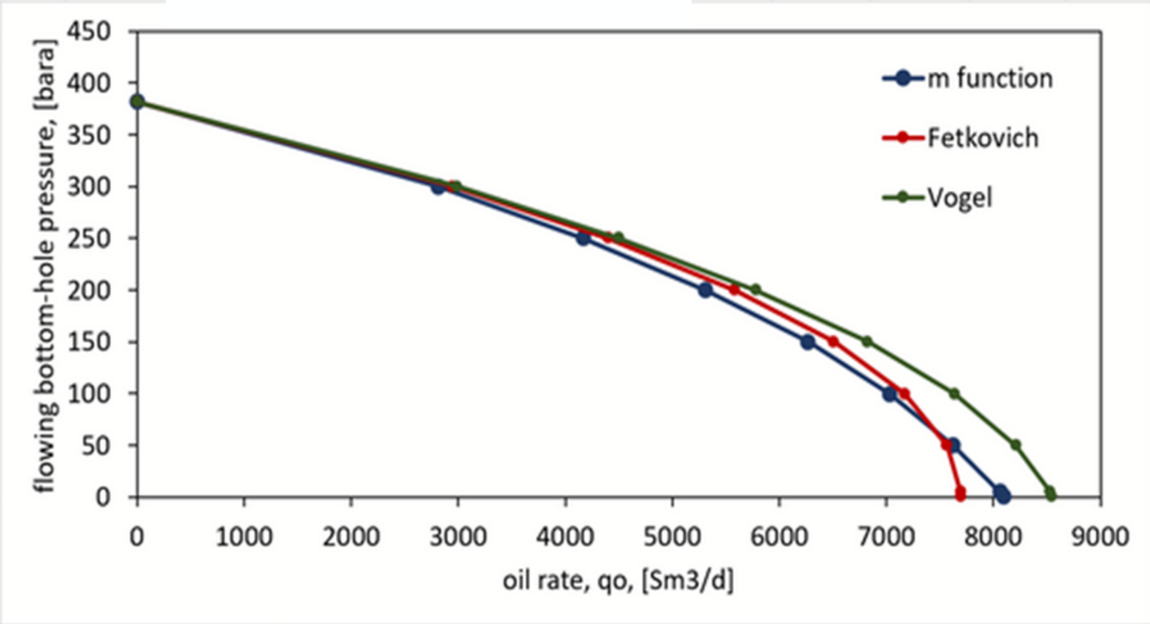
Changing the GOR (Rp) manually to 500 Sm3/Sm3

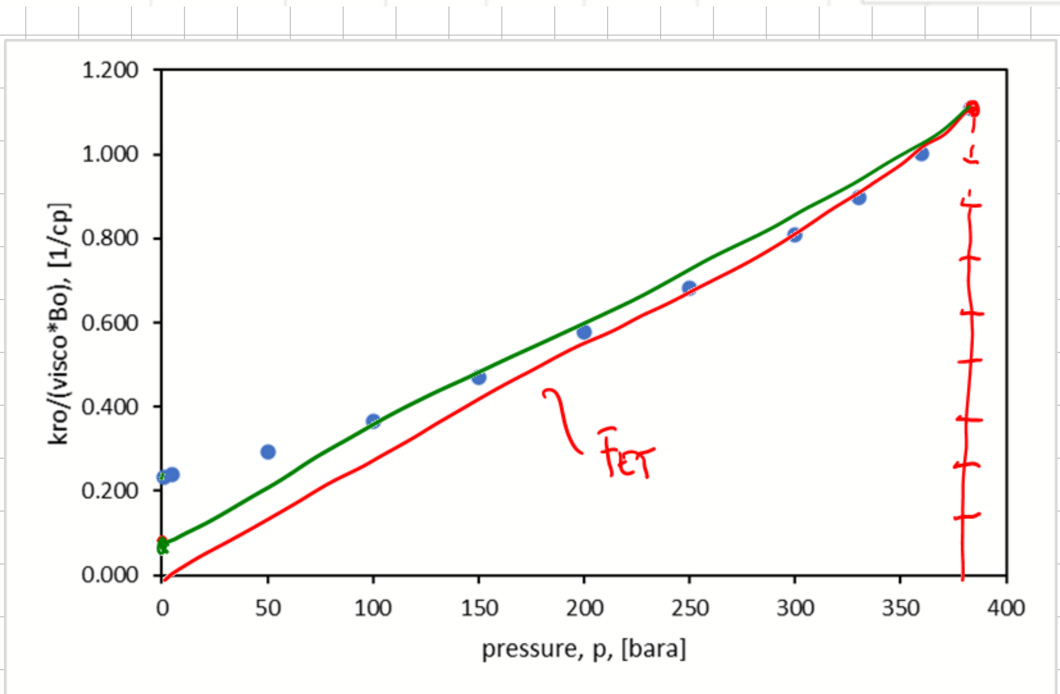


MORE LINEAR....

THE PREDICTION IS BETTER

pwf [bara]	m(p) [bara/cp]	qo [Sm3/d]	qo - Fet [Sm3/d]	qo -Vogel [Sm3/d]	qo - Fet-v2 [Sm3/d]	qo -Vogel-v2 [Sm3/d]
382	222.6	0	0	0		
300	145.3	2810	2950	2990		
250	108.0	4166	4401	4503		
200	76.5	5312	5588	5782		
150	50.2	6268	6511	6826		
100	29.3	7028	7170	7637		
50	12.9	7625	7566	8212		
5	0.9	8059	7697	8530		
1	0.0	8094	7698	8549		





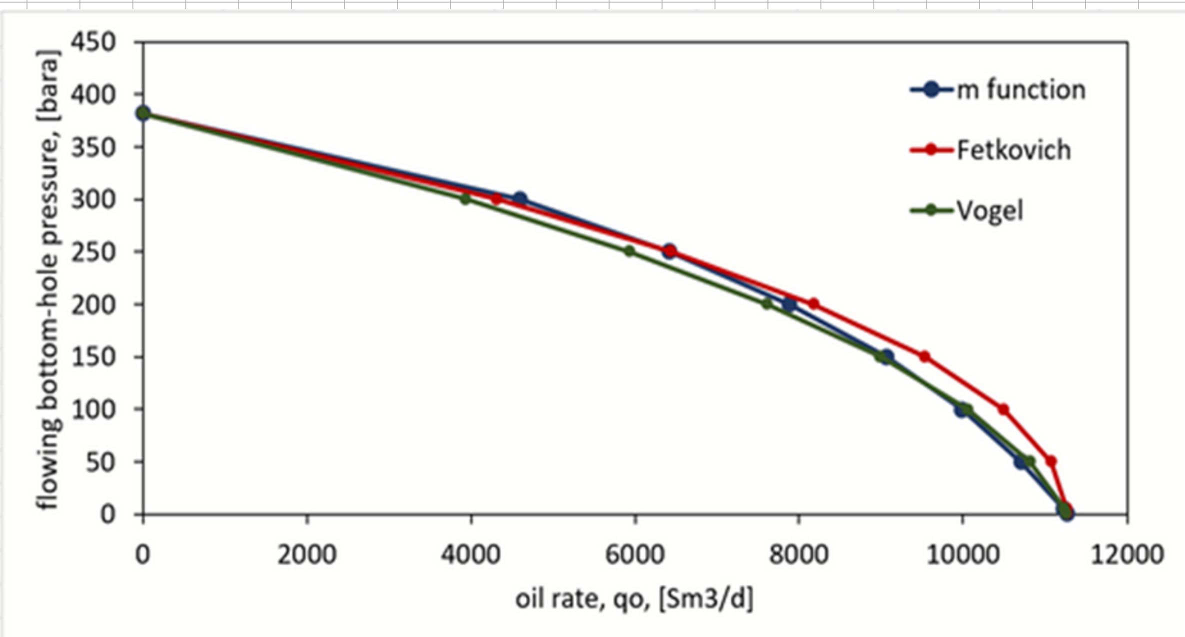
WE CHANGE THE RP BACK TO 300...

But, what if we use the q_{omax} from the m function equation? ($11268\ Sm^3/d$)

$$q_o = q_{omax}\left(1-\left(\frac{P_{wf}}{P_R}\right)^2\right)$$

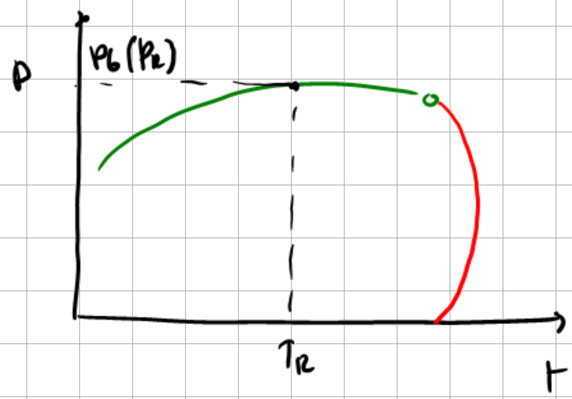
$$q_o = q_{omax}\left(1-0.2\left(\frac{P_{wf}}{P_R}\right)-0.8\left(\frac{P_{wf}}{P_R}\right)^2\right)$$

pwf [bara]	m(p) [bara/cp]	qo [Sm3/d]	qo - Fet [Sm3/d]	qo -Vogel [Sm3/d]	qo - Fet-v2 [Sm3/d]	qo -Vogel-v2 [Sm3/d]
382	309.9	0	0	0	0	0
300	183.5	4596	6425	6511	4318	3938
250	133.3	6421	9584	9807	6442	5932
200	93.1	7884	12169	12592	8179	7617
150	60.7	9061	14180	14867	9530	8993
100	35.3	9985	15616	16631	10495	10060
50	15.4	10708	16477	17884	11075	10818
5	1.1	11227	16762	18576	11266	11236
1	0.0	11268	16765	18617	11267	11261



the prediction is improved considerably!

20241011
OUTLINE
-Re-cap of last week's video lectures
-Composite IPR
-variation of V in time
-Solving problem 1, exercise set 1

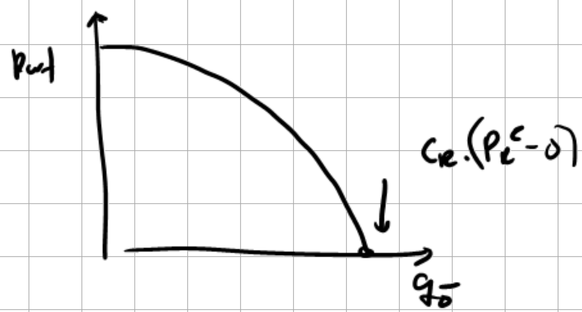


Fetkovich use the same equation used for gas for saturated oil

$$q_g = C_R\left(p_R^2 - p_{wf}^2\right)$$

$$q_o = \underbrace{C_R\, p_R^2}_{q_{o\,max}}\underbrace{\left(1 - \frac{p_{wf}^2}{p_R^2}\right)}$$

$$\left[1 - \underbrace{V}_{V=0}\cdot\frac{p_{wf}}{p_R} - (1-V)\frac{p_{wf}^2}{p_R^2}\right]$$



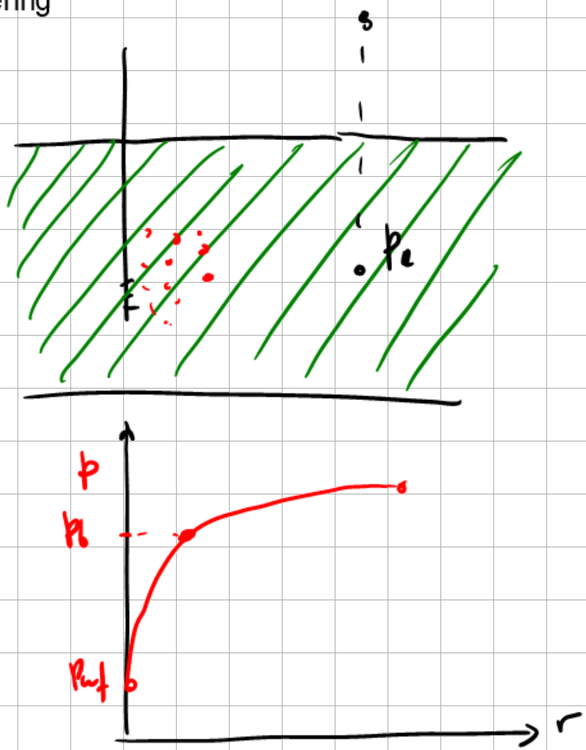
Morris Muskat Father of petroleum engineering

Composite IPR $\quad p_R > p_b$

$p_{wf} < p_b$

$$q_o = C\int_{p_{wf}}^{p_R}\frac{k_{ro}}{\mu_o B_o}\,dp$$

$$q_o = C\left[\underbrace{\int_{p_{wf}}^{p_b}\frac{k_{ro}}{\mu_o B_o}\,dp}_{\left[1 - v\frac{p_{wf}}{p_b} - (1-v)\left(\frac{p_{wf}}{p_b}\right)^2\right]} + \underbrace{\int_{p_b}^{p_R}\frac{1}{\mu_o B_o}\,dp}_{(p_R - p_b)}\right]$$



Composite IPR

if $P_{wf} > P_b \longrightarrow q_{\bar{o}} = J\,(P_R - P_{wf})$

if $P_{wf} \leq P_b$

$$q_{\bar{o}} = q_{\bar{o}max}\left[1 - V\frac{P_{wf}}{P_b} - (1-V)\left(\frac{P_{wf}}{P_b}\right)^2\right] + J\,(P_R - P_b)$$

$V = 0.2$ (Vogel)

$q_{\bar{o}max}$? $\quad q_{\bar{o}max} = \frac{J}{1.8} \cdot P_b$

$V=0$ (Fetkovich) $\quad q_{\bar{o}max} = \frac{J}{2} \cdot P_b$

what happens if $\frac{k_{ro}}{\mu_o B_o}$ is not linear with pressure?

$q_{\bar{o}max}$ must be obtained from test data!

PROBLEM 1

A test has been performed on an oil well and the following pairs of oil rate and flowing bottomhole pressure are reported:

Test point	qo [Sm3/d]	pwf [bara]
1	1080	270
2	2050	180
3	2485	120

The reservoir pressure is 360 bara and the bubble point pressure at reservoir temperature is 250 bara.

Propose an IPR equation to use for this well and calculate all the parameters in the equation suggested using the test data. Justify your answer.

Additional information:

Generic saturated oil IPR equation

$$q_{\bar{o}} = q_{\bar{o},max}\left[1 - V \cdot \frac{p_{wf}}{p_R} - (1-V)\cdot\left(\frac{p_{wf}}{p_R}\right)^2\right]$$

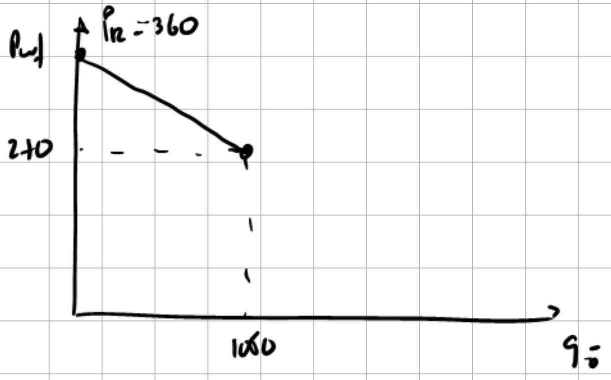
Undersaturated oil

$$q_{\bar{o}} = J \cdot (p_R - p_{wf})$$

-Flow in undersaturated and saturated conditions. (pwf>pb, pwf<pb),
-Reservoir pressure is above pb
-Oil well

IPR proposed:
-Linear for pwf>pb
-Fetkovich or Vogel, or generic V function for pwf<pb, not enough data for Evinger and Muskat method.
-The combination of these two is called Composite IPR.

What needs to be determined?:
for pwf>pb
-J (linear)
For pwf<pb
-maybe qomax, if kro/moBo is not linear!, because if it is linear then qomax = J*pb/2 or J*pb/1.8.
-Maybe V, if Vogel/Fetkovich do not match properly.



$$\overset{1080}{q_{o_{test_1}}} = J \cdot \left(P_R - \overset{270}{P_{wf_{test_1}}}\right)$$

$$J = \frac{1080}{(P_R - 270)} = \frac{1080}{90} = 12$$

pR	[bara]	360
pb	[bara]	250
qo	pwf	
[Sm3/]	[bara]	
1080	270	
2050	180	
2485	120	
Undersaturated		
J	[Sm3/d/bar]	12
Saturated		
V	[-]	0.2
qomax	[Sm3/d]	1666.7

I use Vogel first, V=0.2

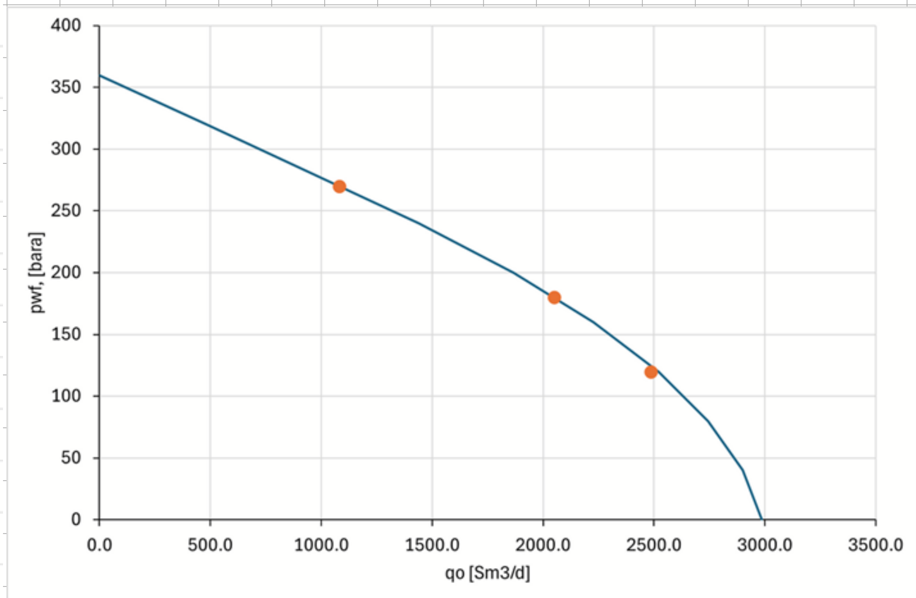
---> J*pb/1.8

I then plot this IPR to see how does it match the test data. To make the plot, I need to calculate qo for several values of pwf. For that I program in VBA a function called composite IPR:

(General)

```
Function composite_IPR_qo(J, qomax, V, pwf, pb, pR)
    If pwf >= pb Then
        composite_IPR_qo = J * (pR - pwf)
    Else
        sum_part_1 = J * (pR - pb)
        pressure_ratio = (pwf / pb)
        sum_part_2 = qomax * (1 - V * pressure_ratio - (1 - V) * (pressure_ratio ^ 2))
        composite_IPR_qo = sum_part_1 + sum_part_2
    End If
End Function
```

For IPR plotting	
pwf [bara]	qo [Sm3/d]
360	0.0
320	480.0
280	960.0
240	1437.9
200	1866.7
160	2227.2
120	2519.5
80	2743.5
40	2899.2
0	2986.7

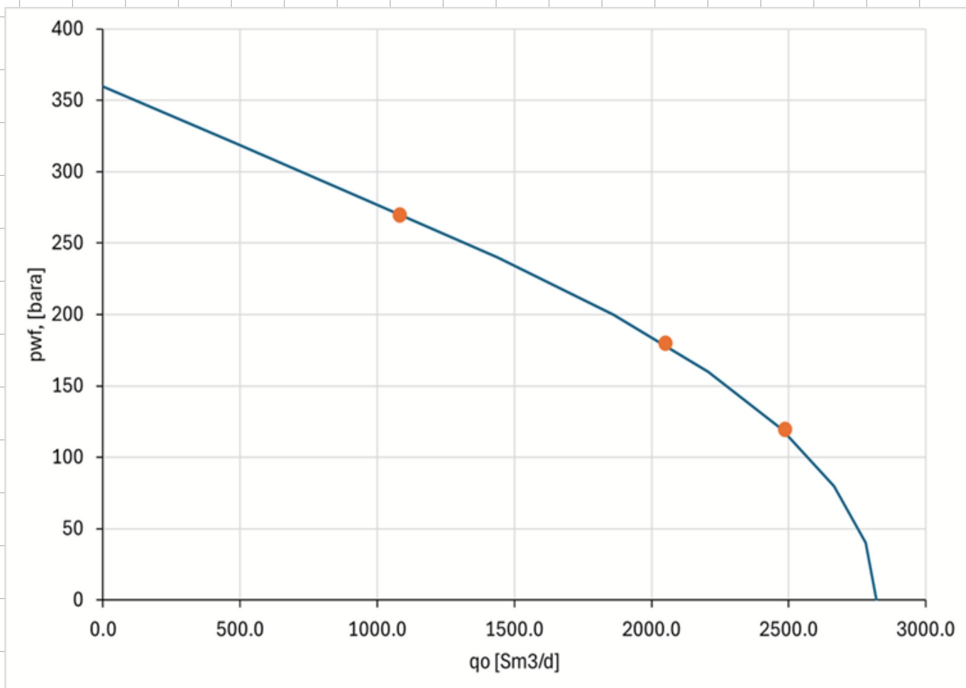


Good match!

What about Fetkovich?

V	[-]	0
qomax	[Sm3/d]	1500.0

For IPR plotting	
pwf [bara]	qo [Sm3/d]
360	0.0
320	480.0
280	960.0
240	1437.6
200	1860.0
160	2205.6
120	2474.4
80	2666.4
40	2781.6
0	2820.0



Very good match also!

To find out which approach is better, one can calculate the mean average percentage error (MAPE) and compare both

$$M = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right|$$

M = mean absolute percentage error
n = number of times the summation iteration happens
A_t = actual value
F_t = forecast value

		V	0.2		0	
qo	pwf	qomax [Sm3/	1666.7		1500	
[Sm3/]	[bara]		qo [Sm3/]	$\left\lvert \frac{A_t - F_t}{A_t} \right\rvert$	qo [Sm3/]	$\left\lvert \frac{A_t - F_t}{A_t} \right\rvert$
1080	270					
2050	180		2055.467	0.003	2042.4	0.004
2485	120		2519.467	0.014	2474.4	0.004
			MAPE	0.008	**MAPE**	0.004
Undersaturated						
J	[Sm3/d/bar]	12				

It seems Fetkovich is slightly better.

One could change qomax, or qomax and V to obtain a better match, for example:

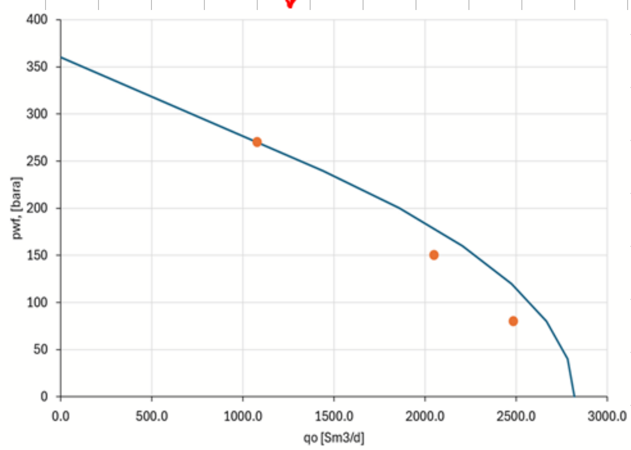
Changing qomax only:

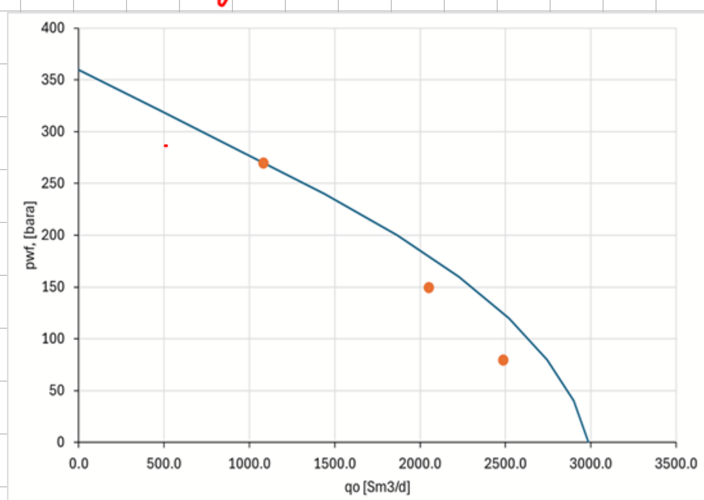
V	0	
qomax [Sm3/d]	1513.774	
	qo [Sm3/]	$\left\lvert \frac{A_t - F_t}{A_t} \right\rvert$
	2049	4.72E-04
	2485	4.56E-08
	MAPE	2.36E-04

For example, if the test data would have been this one:

qo	pwf
[Sm3/]	[bara]
1080	270
2050	150
2485	80

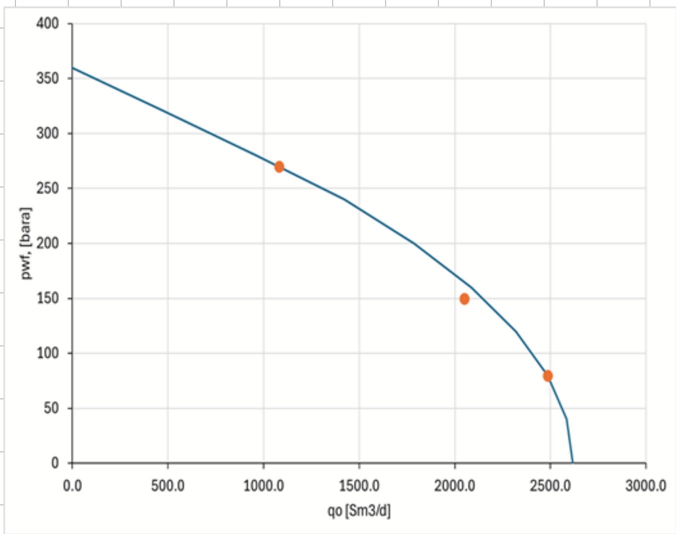
Not Fetkovich (with qomax=J*pb/2) nor Vogel (with qomax=J*pb/1.8) work very well



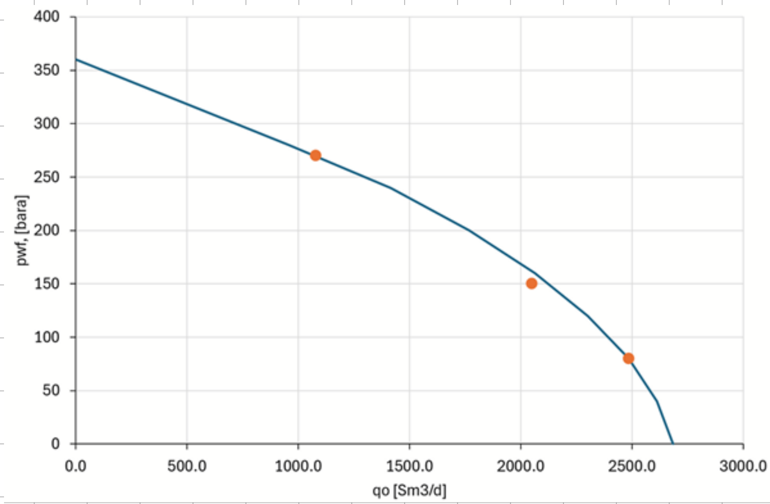


But if we use the solver to obtain qomax that gives a good match:

Using V = 0, qomax = 1297 Sm3/d



Using V = 0.2, qomax = 1364 Sm3/d

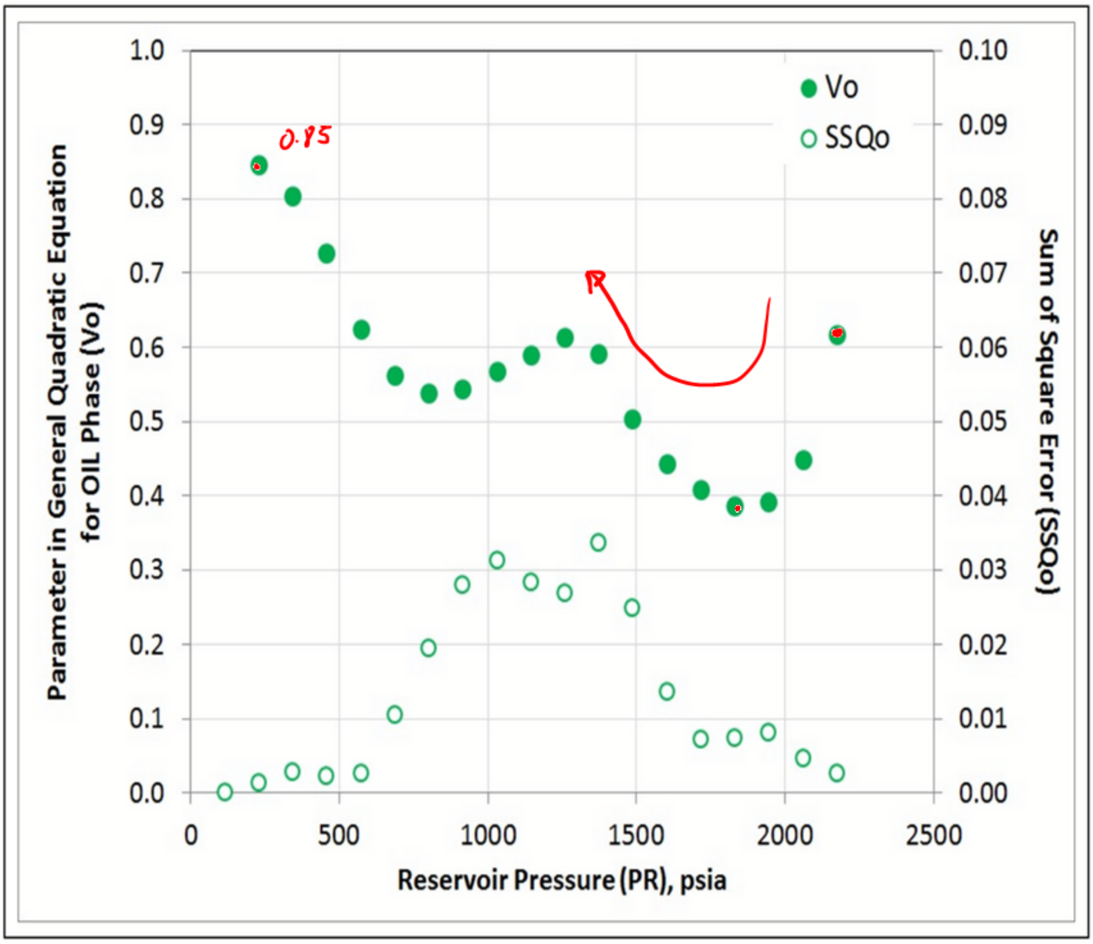


Is V allowed to change apart from 0 and 0.2?

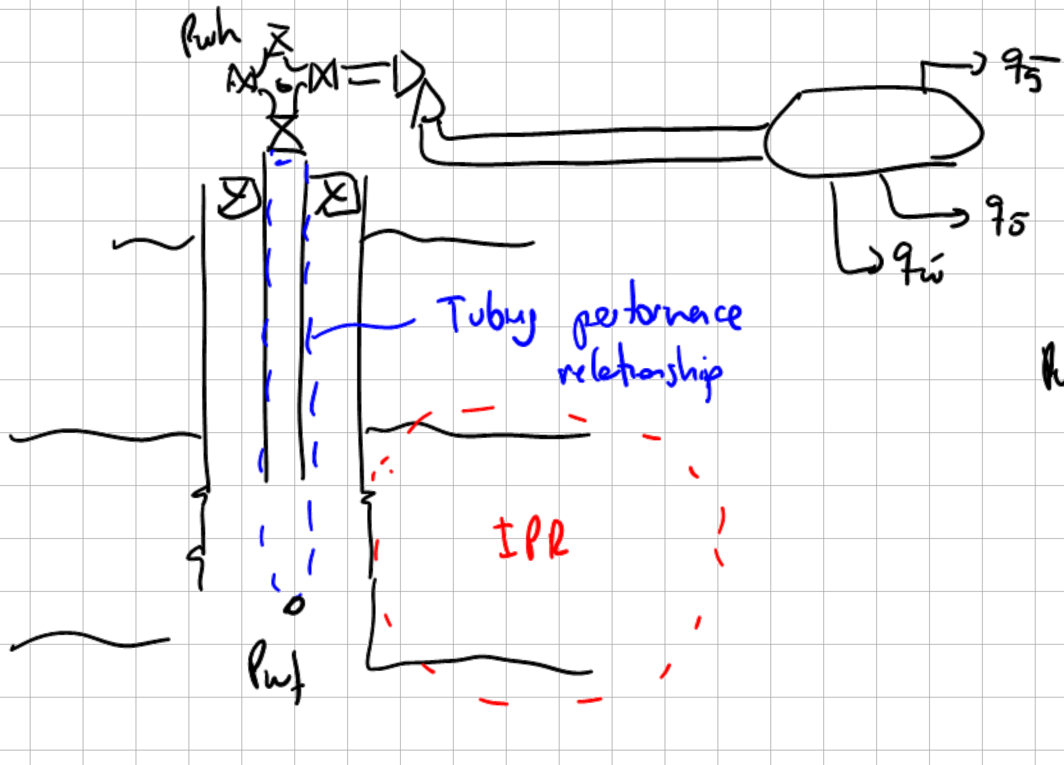
MSc, finished in 2015

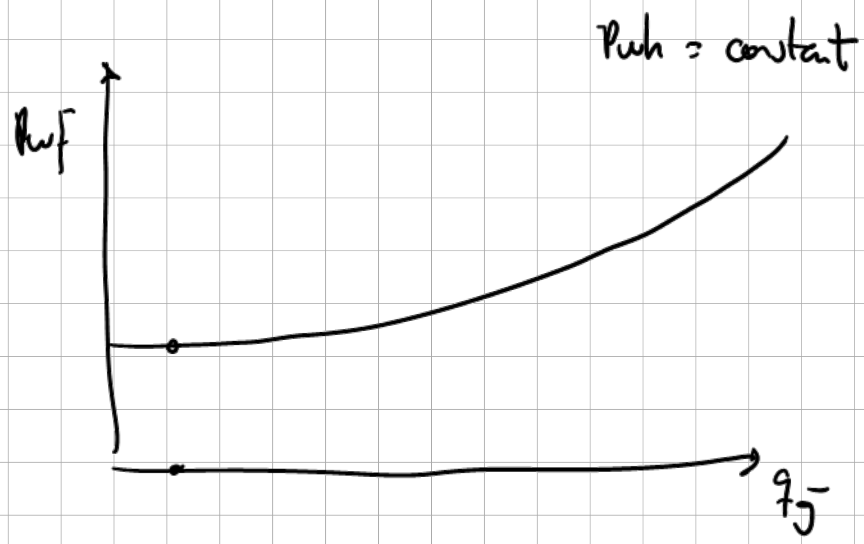
WYNDA ASTUTIK

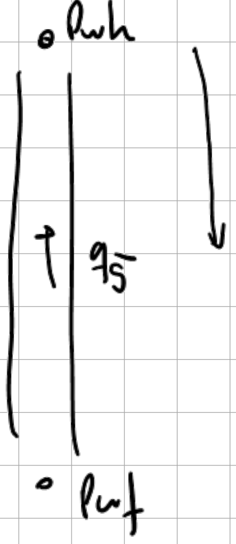
IPR Modeling for Coning Wells



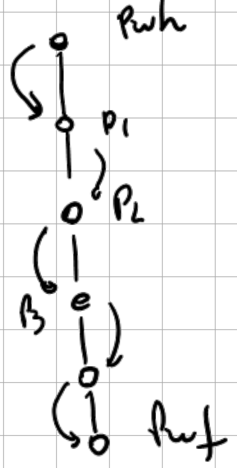
If you have a lot of test points, V can be adjusted to better fit the IPR to the test values. But if there are very few points, it is better to keep V fixed (either Fetkovich or Vogel) and change qomax (in case a linear assumption qomax=J*pR/1.8 or qomax = J*pR/2 does not match well). qo max has a much bigger effect on the IPR than V.

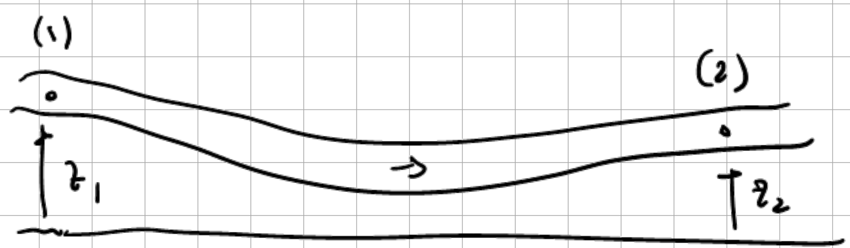






Counter current to calculate Pwf (pressure integration)





head → $h_1 = h_2$ (no energy losses) $\quad h = z + \frac{p}{\rho g} + \frac{v^2}{2g}$ incompressible

$h_1 = h_2 + \Delta h_{losses}$ $\quad \Delta h_{losses} = \underbrace{\cancel{\Delta h_{accessories}}}_{\text{valves, restrictions}} + \Delta h_{friction}$ $\quad \Delta h_{friction} = f \frac{L}{D} \frac{v^2}{2g}$

f: moody friction factor

$$h_1 = z_1 + \frac{p_1}{\rho g} + \cancel{\frac{v_1^2}{2g}} = z_2 + \frac{p_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + f\frac{L}{D}\frac{v^2}{2g}$$

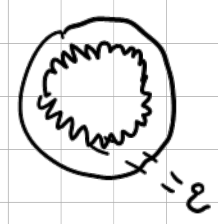
its incompressible, so $v_1 = v_2 = v$, Q = constant

$$\Delta p = (p_1 - p_2) = \rho g \cdot \left((z_2 - z_1) + f \frac{L}{D} \frac{\rho g}{2g} v^2\right)$$

$$(p_1 - p_2) = \underbrace{\Delta z \cdot \rho g}_{\text{hydrostatic}} + \underbrace{f \frac{L}{D} \rho \frac{v^2}{2}}_{\text{friction}}$$

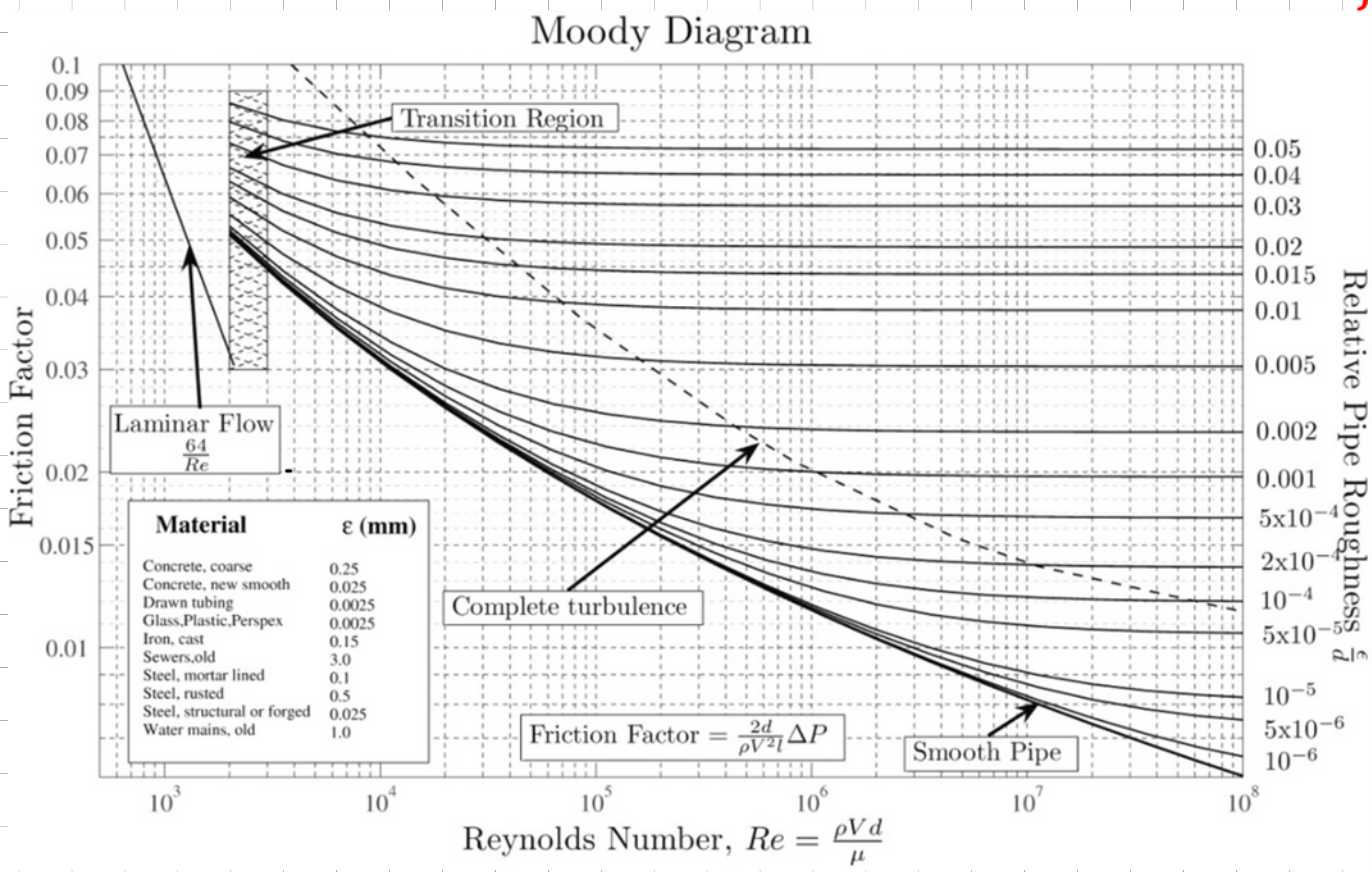
for gas $\rho = f(p, T)$

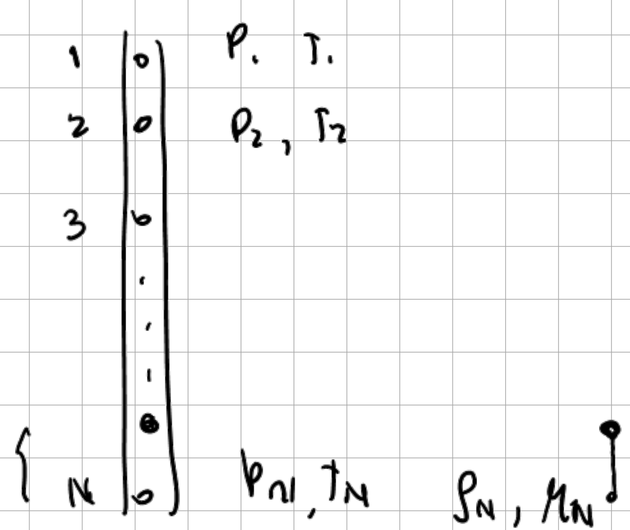
ε/D dimensionless roughness



$$Re = \frac{\rho v D}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

ρ, μ: $f(p,T)$





$N \to N-1$, gas properties don't change much

valid if ΔL is small

ΔT, Δp is small

$$v_g = \frac{q_g}{A} = \frac{q_{\bar{g}} B_g(p,T)}{A}$$

$$B_g = \frac{q_g}{q_{\bar{g}}}$$

$$q_{\bar{g}} \cdot \rho_{\bar{g}} = q_g \cdot \rho_g$$

$$B_g = \frac{q_g}{q_{\bar{g}}} = \frac{\rho_{\bar{g}}}{\rho_g} = \frac{zT}{p}\left(\frac{p_{sc}}{T_{sc}}\right)$$

$$\rho_{\bar{g}} = \frac{p_{sc}}{R\, z_{sc} T_{sc}}$$

$$\rho_g = \frac{p}{zRT}$$

m^3/d $\quad \frac{1\,d}{24\,h} \quad \frac{1\,h}{3600\,s}$

qg [Sm3/d]	2.85E+06
Gas gravity	0.7
Tubing ID [m]	0.157
Tubing cross section area [m2]	0.019
Tubing roughness [m]	1.50E-06

TVD	p	T	Z	deng	Bg	viscg	qg	vg	p-calc
[m]	[bara]	[C]	[-]	[kg/m3]	[m3/Sm3]	[cp]	[m3/d]	[m/s]	[bara]
0	40	87	0.948	28.6	3.00E-02	1.36E-02	8.54E+04	51.03	
284	46	89	0.942	32.9	2.60E-02	1.38E-02	7.41E+04	44.28	
567	51	90	0.938	36.8	2.32E-02	1.39E-02	6.63E+04	39.61	
851	56	92	0.934	40.4	2.12E-02	1.41E-02	6.04E+04	36.14	
1135	61	94	0.930	43.6	1.96E-02	1.43E-02	5.59E+04	33.43	
1418	66	96	0.928	46.7	1.83E-02	1.45E-02	5.23E+04	31.25	
1702	70	98	0.926	49.5	1.73E-02	1.46E-02	4.92E+04	29.44	
1986	74	99	0.924	52.2	1.64E-02	1.48E-02	4.67E+04	27.92	
2269	78	101	0.922	54.8	1.56E-02	1.49E-02	4.45E+04	26.61	
2553	81	103	0.921	57.3	1.49E-02	1.51E-02	4.26E+04	25.47	
2837	85	105	0.920	59.6	1.44E-02	1.52E-02	4.09E+04	24.46	

2837 - 2553

- p_{N-1}
- p_{wf}

$$p_{N-1} = p_{wf} - \underbrace{\rho \cdot g\,\Delta z - f\,\frac{L}{D}\,\rho\,\frac{v^2}{2}}_{\text{using conditions at } p_{wf}\,!}$$

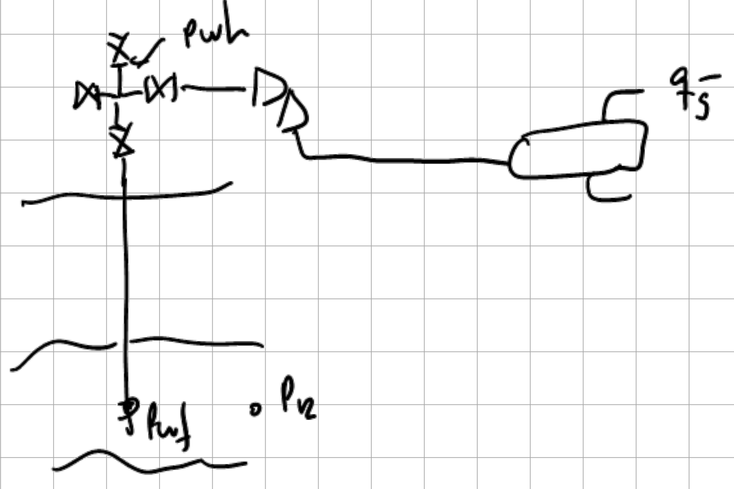
```
Function Pout(qt1, ID, den, visc, Length, teta, pin, roughness)
    'Function that give pressure available at the oulet of a pipe with a flow qt and inlet pressure pressure pin
    'Calculation made for liquid single phase flow
    'Takes in data in SI
    'qt flow [m^3/d]
    'ID inner diameter of pipe [m]
    'den density of fluid, [kg/m^3]
    'visc viscosity of fluid, [Pa s]
    'Length, pipe length, [m]
    'teta inclination angle of pipe with respect to horizontal [°]
    'pin, discharge pressure required, [bara]
    'roughness of pipe [m]

    'Gravitational acceleration g, [m/s(2]
    g = 9.81
    'Pi number
    Pi = 4 * Atn(1)
    qt = qt1 / (3600# * 24#)  '[m^3/s]

    'Calculating area and velocity
    Area = Pi * (ID ^ 2) / 4
    v = qt / Area

    Presscalc1 = pin - (Length * Sin(teta * Pi / 180) * den * g / 100000#) - (ffactor(den, visc, ID, roughness, v) * Length * (v ^ 2) * den / (ID * 200000#))
    Pout = Presscalc1

End Function
```

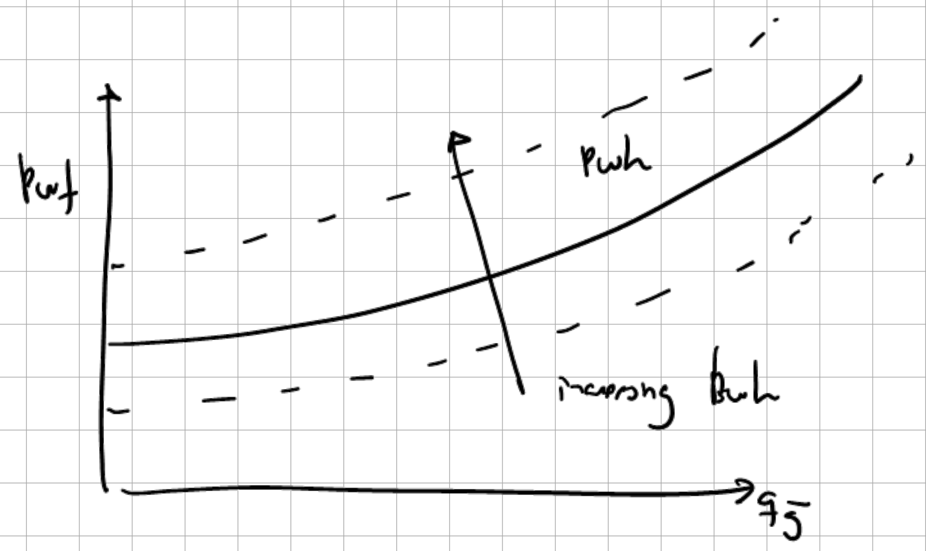
change for m3/d to m3/s

dividing by 1E5 to take from Pa to bara

TVD	p	T	Z	deng	Bg	viscg	qg	vg	p-calc
[m]	[bara]	[C]	[-]	[kg/m3]	[m3/Sm3]	[cp]	[m3/d]	[m/s]	[bara]
0	40	87	0.948	28.6	3.00E-02	1.36E-02	8.54E+04	51.03	35.55
284	46	89	0.942	32.9	2.60E-02	1.38E-02	7.41E+04	44.28	41.50
567	51	90	0.938	36.8	2.32E-02	1.39E-02	6.63E+04	39.61	47.03
851	56	92	0.934	40.4	2.12E-02	1.41E-02	6.04E+04	36.14	52.27
1135	61	94	0.930	43.6	1.96E-02	1.43E-02	5.59E+04	33.43	57.29
1418	66	96	0.928	46.7	1.83E-02	1.45E-02	5.23E+04	31.25	62.15
1702	70	98	0.926	49.5	1.73E-02	1.46E-02	4.92E+04	29.44	66.88
1986	74	99	0.924	52.2	1.64E-02	1.48E-02	4.67E+04	27.92	71.52
2269	78	101	0.922	54.8	1.56E-02	1.49E-02	4.45E+04	26.61	76.08
2553	81	103	0.921	57.3	1.49E-02	1.51E-02	4.26E+04	25.47	80.59
2837	85	105	0.920	59.6	1.44E-02	1.52E-02	4.09E+04	24.46	85.04

surprisingly close!





$$q_{\bar{s}} = \underset{\text{tubing coefficient}}{C_T} \cdot \left(\frac{P_{wf}^2}{e^{S}} - P_{wh}^2 \right)^{0.5}$$

tubing coefficient, tubing elevation coefficient

for same Pwf, Pwh:

- $C_T \uparrow \quad q_s \uparrow$
- $S \uparrow \quad q_s \downarrow$

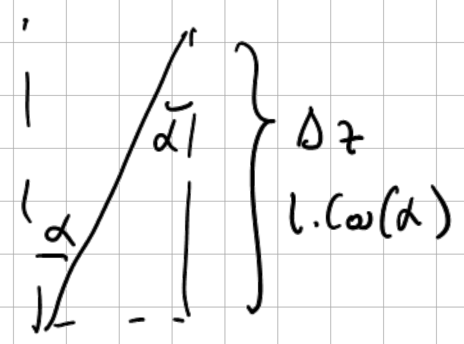
$$u_g = \frac{q_g}{A} = \frac{\dot{m}_g}{\rho_g \cdot A} \qquad \rho_g = \frac{p}{Z R T}$$

$R \rightarrow \frac{R_0}{MW}$

$$u_g^2 = \left(\frac{\dot{m}_g}{\rho_g A} \right)^2$$

$$-\frac{dp}{dl} = \frac{p \cdot M_g}{\underbrace{Z}_{f(p)} \cdot R \cdot T} \cdot g \cdot \cos(\alpha) + \frac{8 \cdot \overbrace{f_M}^{f(v,\rho,\mu)} \cdot \dot{m}^2}{\pi^2 \cdot D^5} \cdot \frac{Z \cdot R \cdot T}{p \cdot M_g}$$

"Ru"



$$S = 2 \cdot L \cdot C_a = 2 \cdot \frac{M_g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot g \cdot \cos(\alpha)$$

$$q_{sc} = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left[\left(p_{wf}^2 - p_t^2 \cdot e^S\right) \cdot \left(\frac{S}{e^S - 1}\right)\right]^{0.5}$$

By defining the tubing constant C_T:

$$C_T = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left(\frac{S \cdot e^S}{e^S - 1}\right)^{0.5} \qquad \text{EQ. A-24}$$

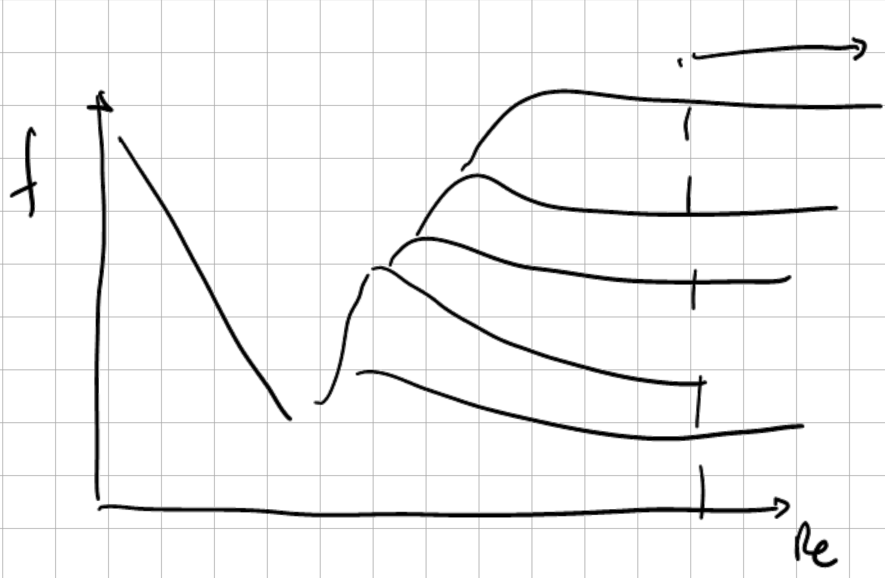
This yields:

$$q_{sc} = C_T \cdot \left[\left(\frac{p_{wf}^2}{e^S} - p_t^2 \cdot\right)\right]^{0.5} \qquad \text{EQ. A-25}$$

→ to calculate C_T, S, a speed Pwf, Pwh, Twf, Twh are required !

Zav, Tav

Luckily the effect of speed is modest



fully turbulent regime $\rightarrow$ $f = f(e/D)$

$Re = \frac{V D \rho}{\mu}$

ρ is low

V is high

μ is high

$\underbrace{\qquad\qquad}$

Re is very high

$e = f(D)$ Smith (1950)

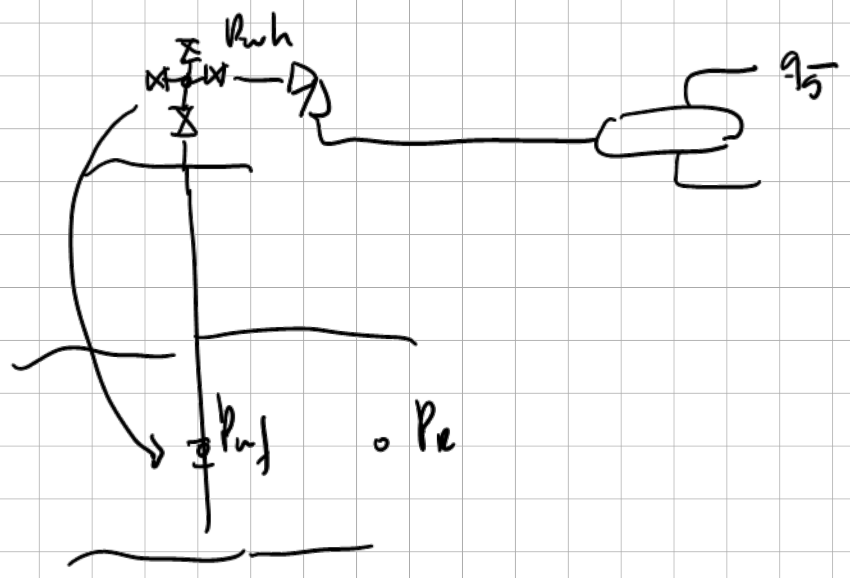
$\hookrightarrow f = F(D)$

DETERMINING FRICTION FACTORS FOR MEASURING PRODUCTIVITY OF GAS WELLS

R. V. SMITH, U. S. BUREAU OF MINES, BARTLESVILLE, OKLAHOMA

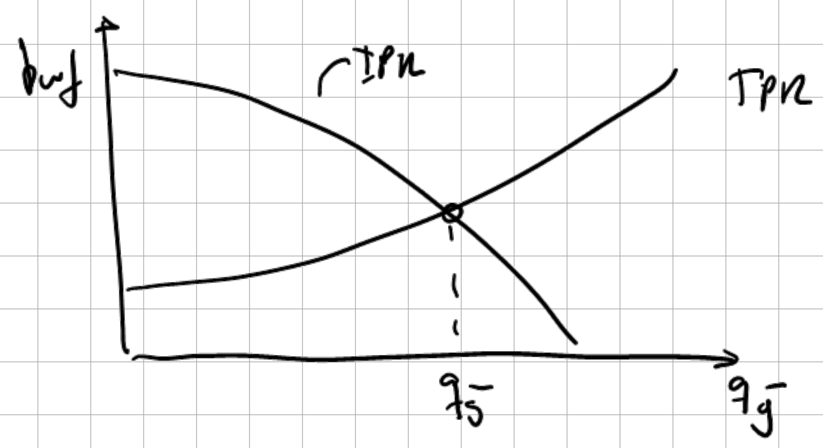
$$f_M = \frac{0.01748}{D^{0.224} \cdot \left(|1\ m| \cdot \left|\frac{39.37\ in}{1\ m}\right|\right)^{0.224}} = \frac{0.0077}{D^{0.224}}$$

Video 23 - Dry gas flow equilibrium, exercise



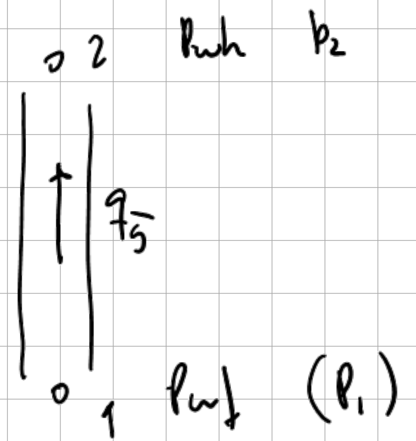
1) neglecting flowline $\rightarrow$ $p_{wh} = p_{sep}$
fully open choke

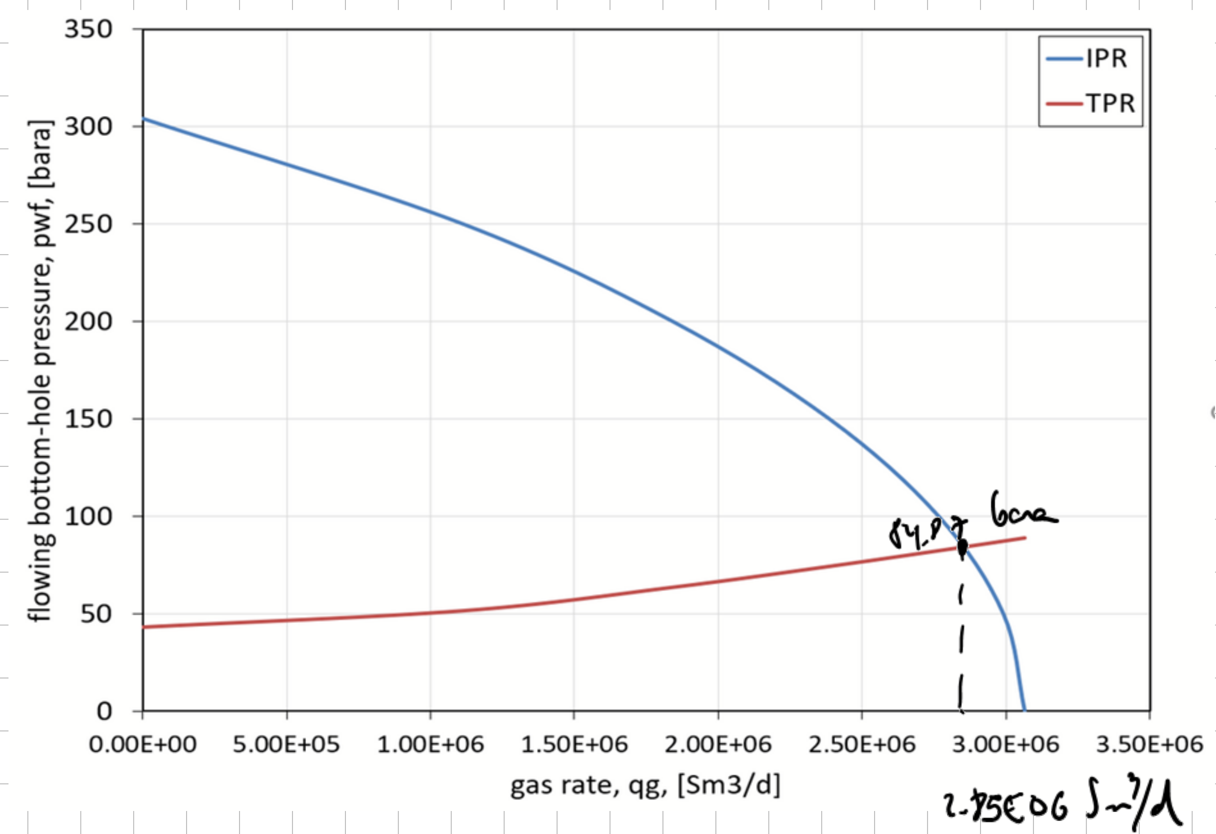
- select an equilibrium point $\rightarrow$ bottom-hole

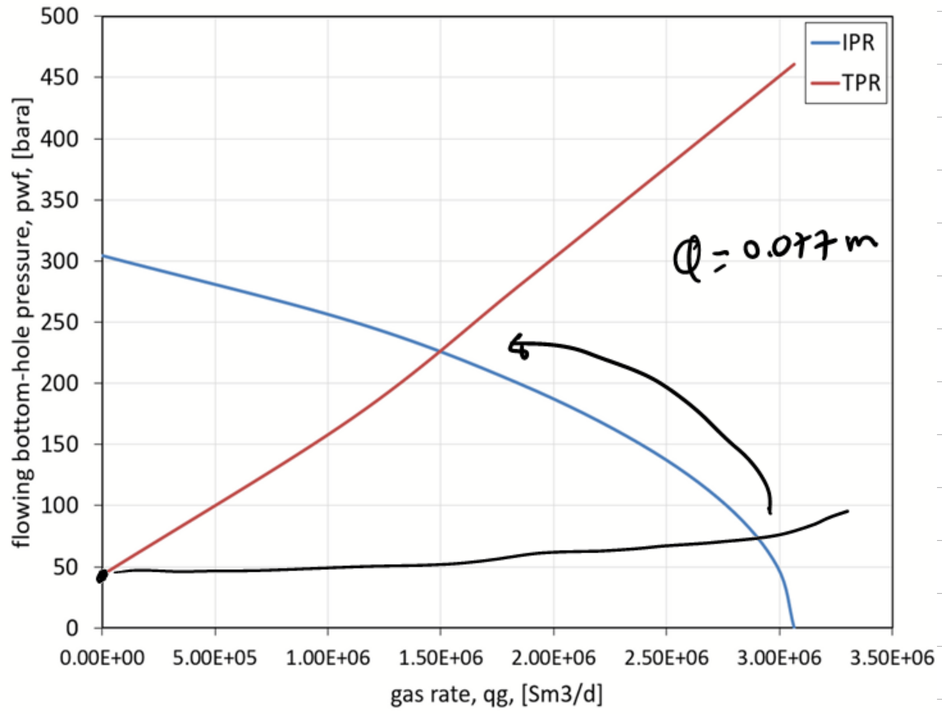


$$q_g = C_R \left(p_R^2 - p_{wf}^2\right)^{n}$$

$$q_g = C_T \left(\frac{p_{wf}^2}{e^S} - p_{wh}^2\right)^{0.5} \rightarrow p_{wf} = \sqrt{\left(p_{wh}^2 + \left(\frac{q_g}{C_T}\right)^2\right) e^S}$$







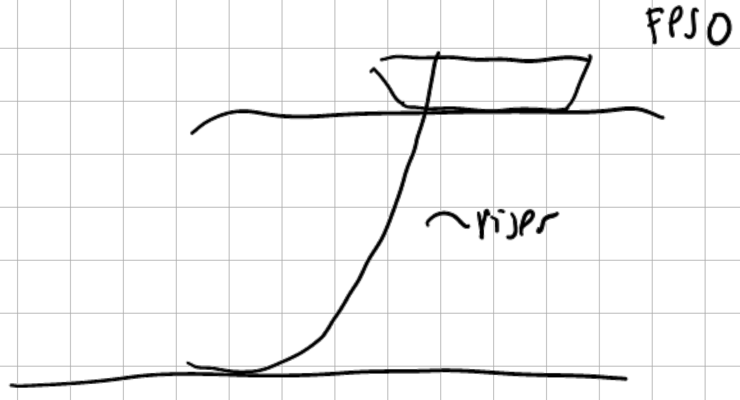
effect of tubing size

$D \downarrow \rightarrow G \downarrow \rightarrow q_g^* \downarrow$

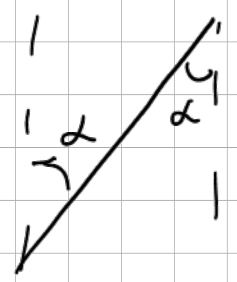
$C_{PL}, C_{FL}, C_{riser}$

$p_1 \rightarrow p_2$

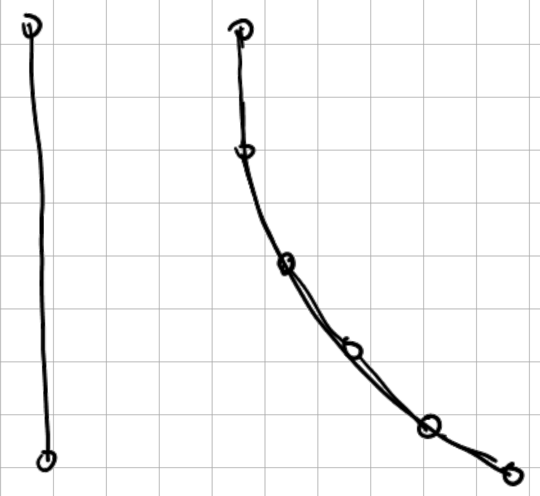
$$q_{sc} = C_T \cdot \left[\left(\frac{p_{wf}^2}{e^S} - p_t^2 \cdot\right)\right]^{0.5}$$

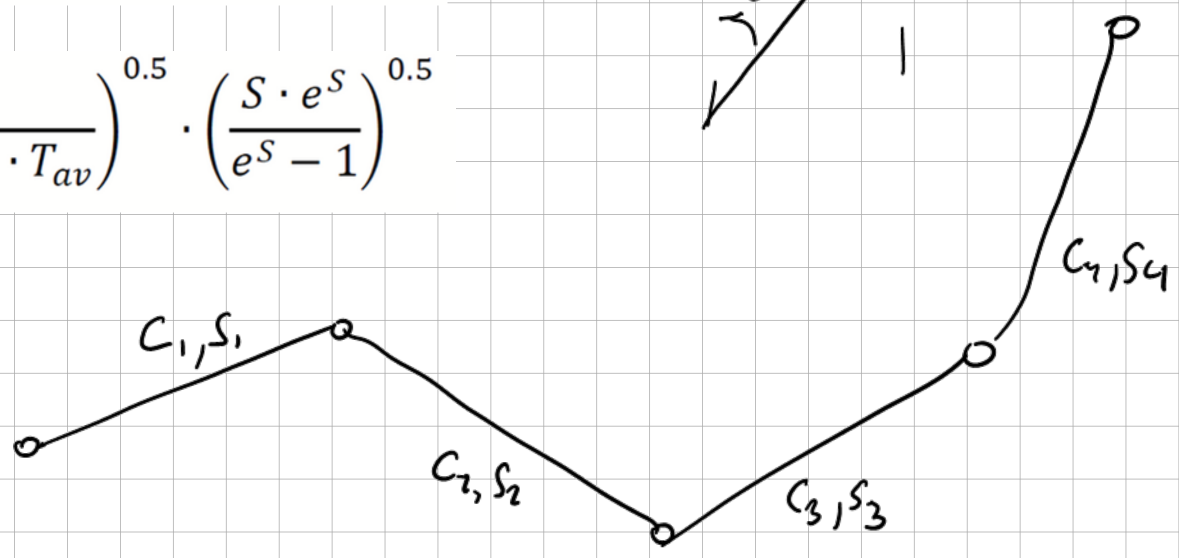


$$S = 2 \cdot L \cdot C_a = 2 \cdot \frac{M_g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot g \cdot \cos(\alpha)$$



$$C_T = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left(\frac{S \cdot e^S}{e^S - 1}\right)^{0.5}$$





Special case of horizontal pipe $S=0$

however when calculating $C \rightarrow \frac{S \cdot e^S}{(e^S - 1)}$ $\quad S=0 \quad \frac{0 \cdot 1}{(1-1)} = \frac{0}{0}$?

use L'Hopital rule

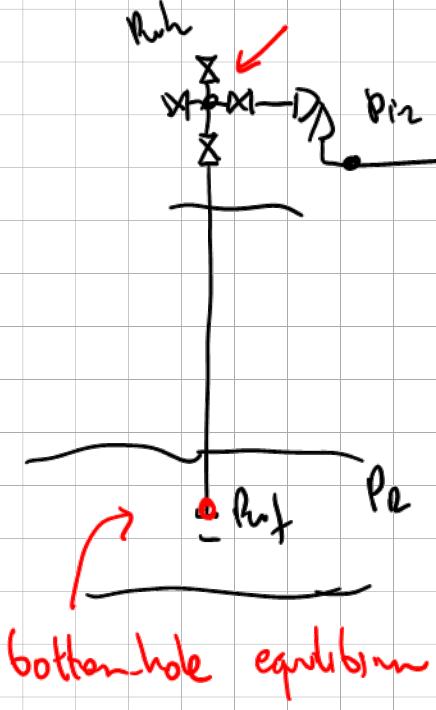
$$\lim_{x \to 0} \frac{f(x)}{g(x)} \rightarrow \text{undefined}$$

$$\lim_{x \to 0} \frac{f'(x)}{g'(x)} \quad \frac{(e^S + Se^S)}{(e^S)} \overset{S \to 0}{=} \frac{1+0}{1} = 1$$

in horizontal pipe $\frac{Se^S}{(e^S-1)} = 1$

if $D \uparrow \rightarrow C_{FL} \uparrow \rightarrow$ lower Δp (or larger rate)

if $L \uparrow \rightarrow C_{FL} \downarrow \rightarrow$ higher Δp (or lower rate)

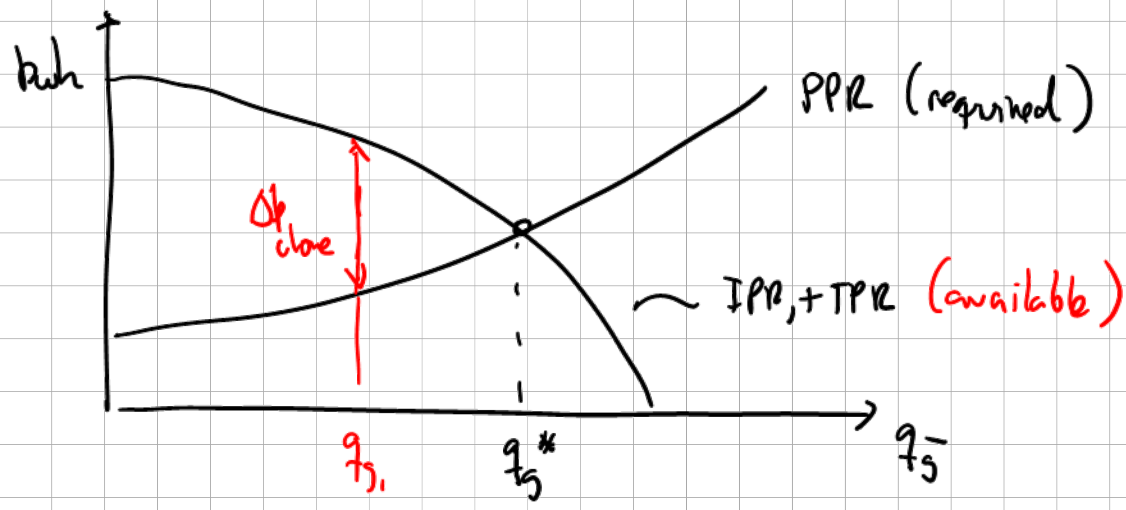


$\Delta p_{choke} = (P_{wh} - P_{in})$

when $\Delta p_{choke} = 0$

$P_{wh} = P_{in}$

to do equilibrium with flowline



p_R, Res pressure	304	bara
C_R	104	Sm3/d/bar^2n
n, exponent	0.9	
C_t, tubing	4.25E+04	$Sm^3/d/bar$
s, elevation	0.155	
C_{fl}, flowline	1.25E+05	4.00E+04 $Sm^3/d/bar$
p_{sep}	40	bara

$$q_g = c(p_R^2 - p_{wf}^2)^n$$

Tubing performance relationship (TPR)

$$p_{wh} = p_2 = \left(\frac{p_1^2}{e^s} - \frac{q_g^2}{C_T^2}\right)^{0.5}$$

Flowline performance relationship (FPR)

$$\boxed{q_{\bar{g}} = C_{FL} \cdot \left(p_{in}{}^2 - p_{out}{}^2\right)}$$

assumed

IPR-qg

pwf [bara]	qg-IPR [Sm3/d]	pwf-TPR [Sm3/d]	WPR [Sm3/d]	PPR - 1 [bara]	PPR - 2 [bara]
304	0.00E+00	43			
250	1.11E+06	52			
200	1.84E+06	64			
150	2.38E+06	74			
100	2.76E+06	82			
50	2.99E+06	87			
0	3.06E+06	89			
84.378937	2.85E+06	84			

wellhead performance relationship

pwf → pwh

TPR

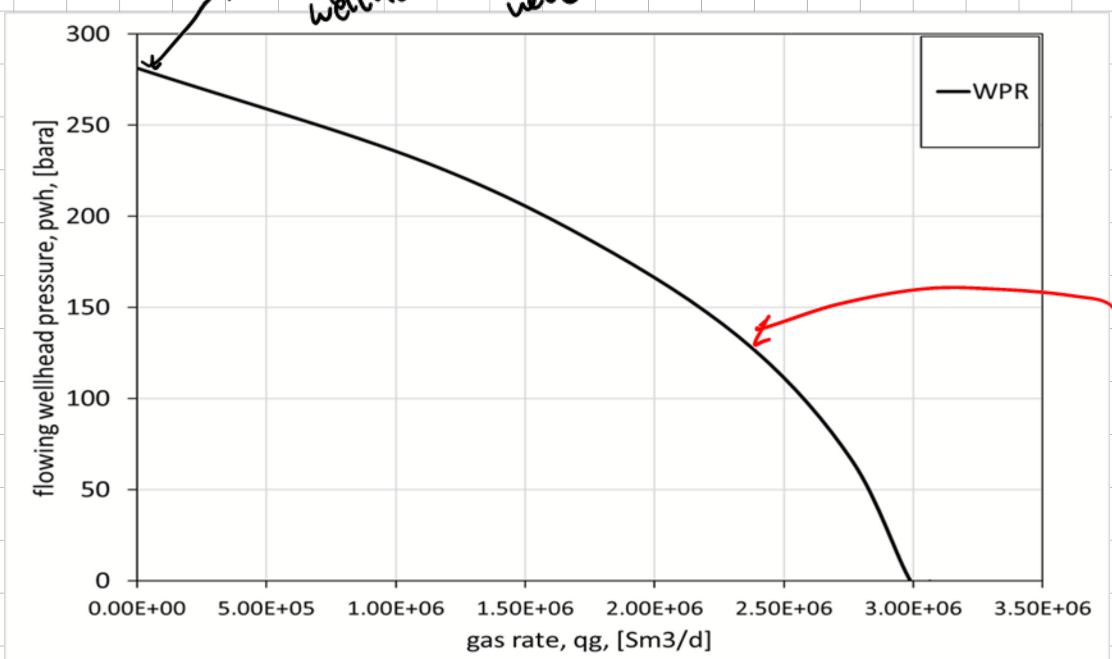
required, with TPR, assuming Pwh = 40 bara

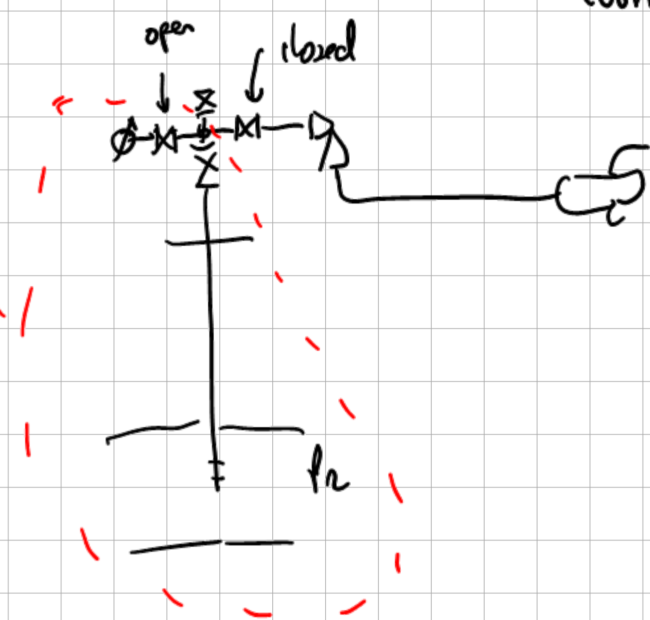
tubing_qg (P1, P2)

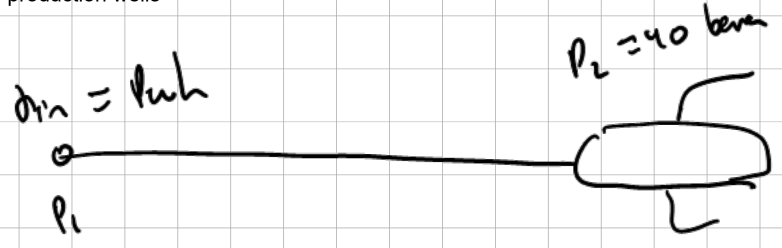
calculate → tubing_P1 (P2, qg)

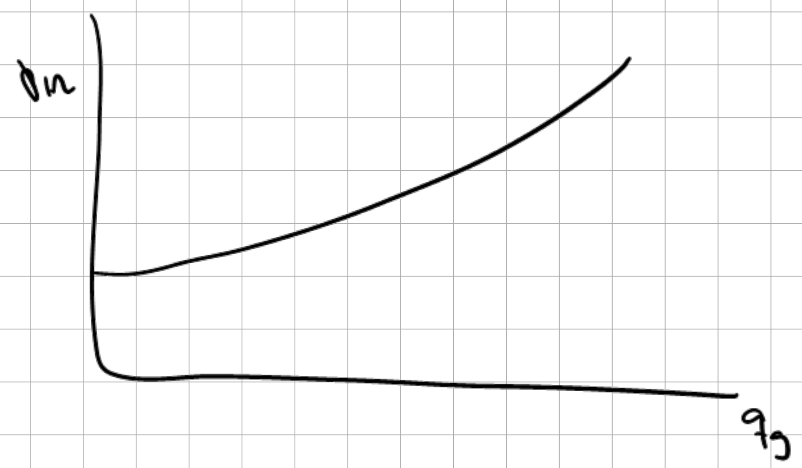
tubing_P2 (P1, qg)

pressure measured at wellhead with closed well









$$q_g = C_{FL} \cdot \left(\frac{p_1^2}{e^s} - p_2^2 \right)^{0.5}$$

horizontal $s \to 0$

$$q_g = C_{FL} \left(p_1^2 - p_2^2 \right)^{0.5}$$

required pressure

$$p_1 = \sqrt{p_2^2 + \left(\frac{q_g}{C_{FL}} \right)^2}$$

```
Function Linep1(Cfl, p2, qg)
Linep1 = (p2 ^ 2 + (qg / Cfl) ^ 2) ^ 0.5
End Function
```

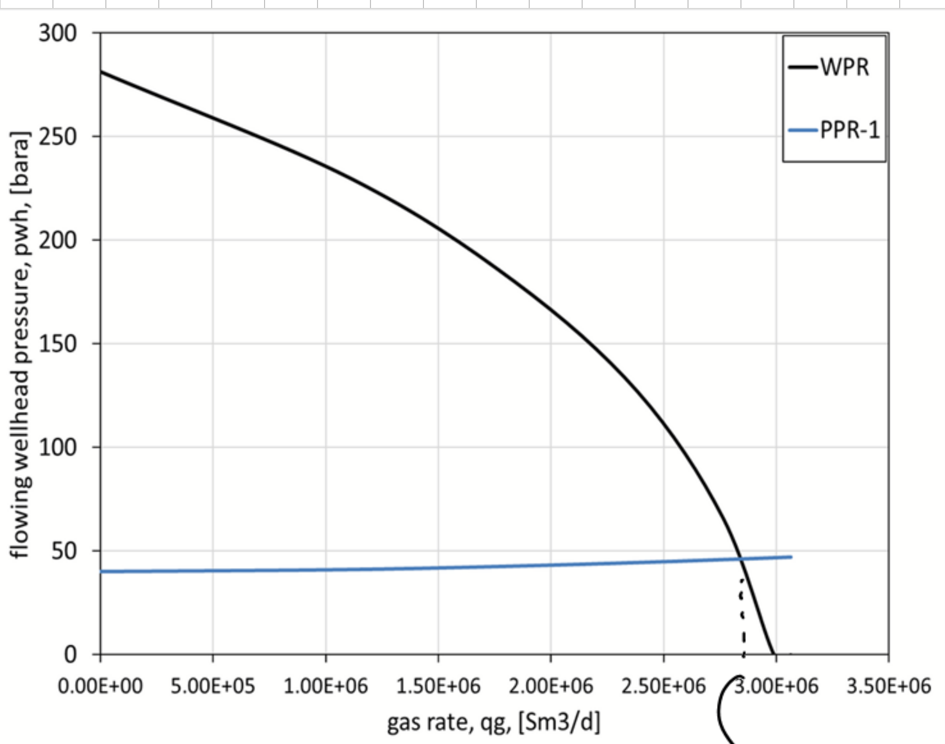
pwf [bara]	qg-IPR [Sm3/d]	pwf-TPR [Sm3/d]	WPR [Sm3/d]
304	0.00E+00	43	281
250	1.11E+06	52	230
200	1.84E+06	64	180
150	2.38E+06	74	127
100	2.76E+06	82	66
50	2.99E+06	87	#VALUE!
0	3.06E+06	89	#VALUE!

$$q_g = C_T \left(\frac{p_{wf}^2}{e^s} - p_{wh}^2 \right)^{0.5}$$

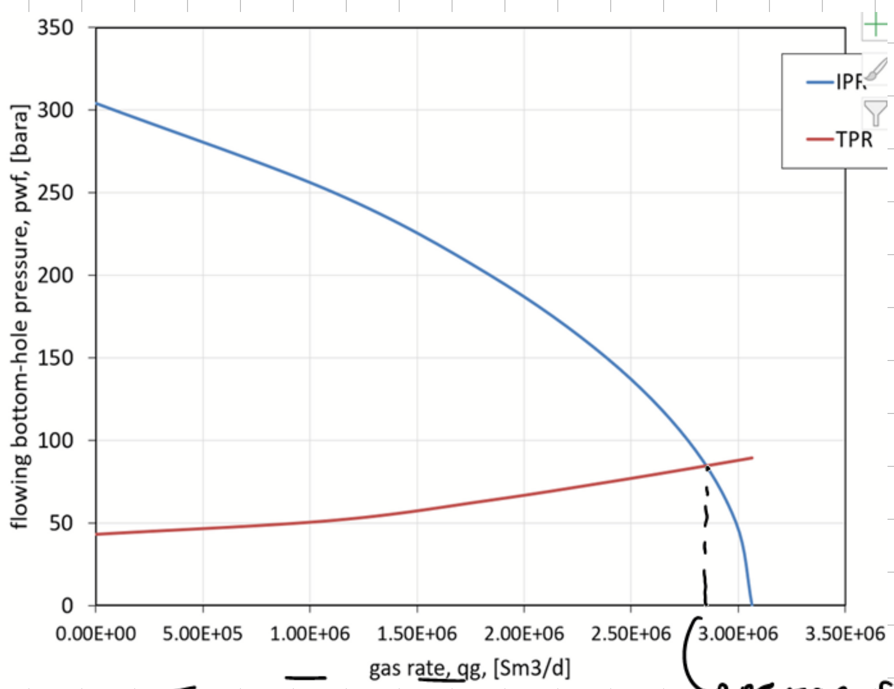
$$p_{wh} = \sqrt{\underbrace{\frac{(p_{wf})^2}{e^s} - \left(\frac{q_g}{C_T} \right)^2}}$$

(+)

(−)



a bit lower

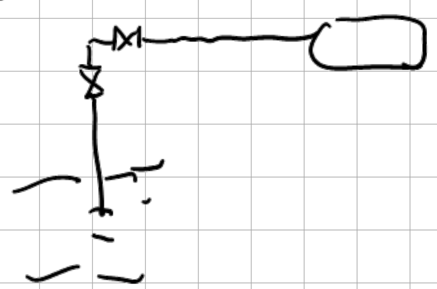


→ 2.85E06 Sm^3/d

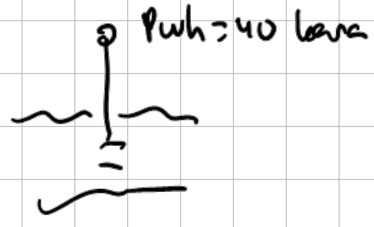
bottom-hole eq

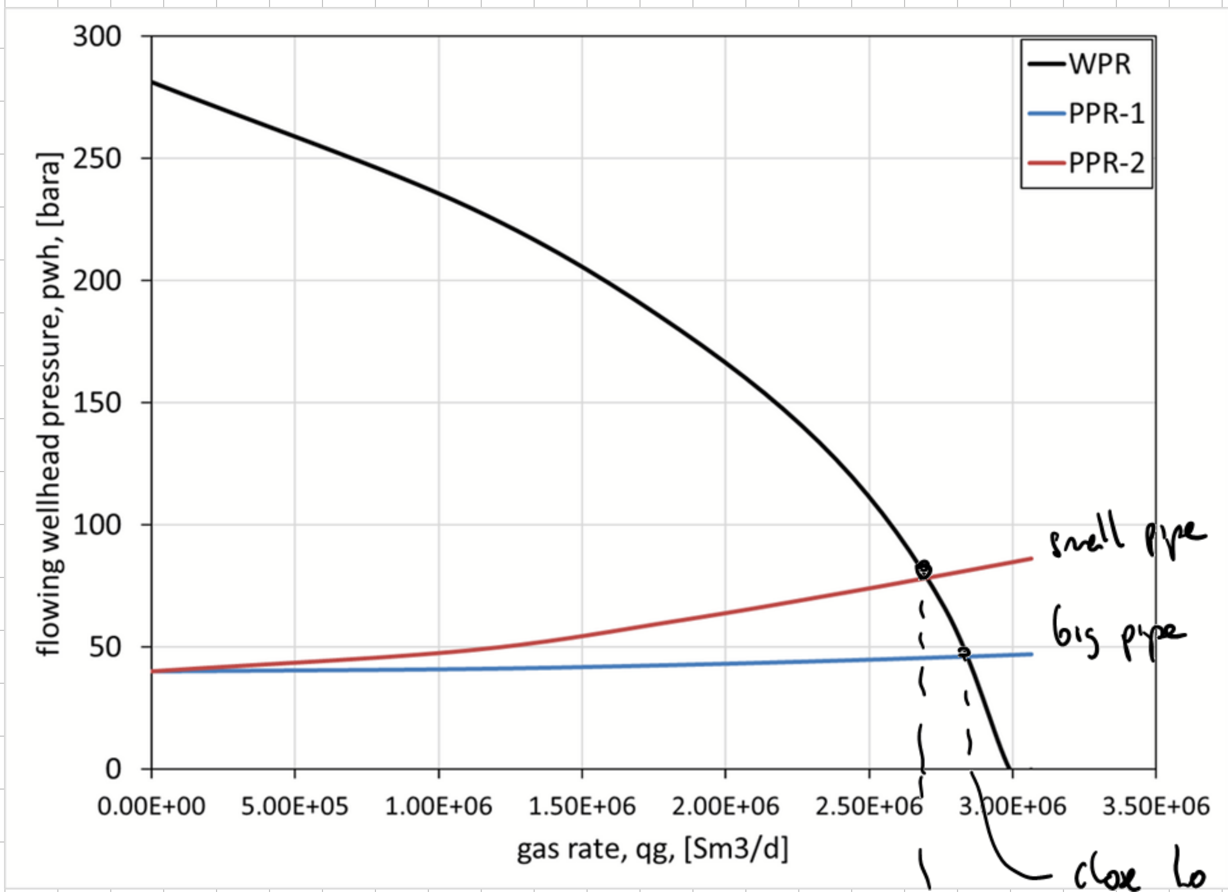
Psep = 40 bara

wellhead eq



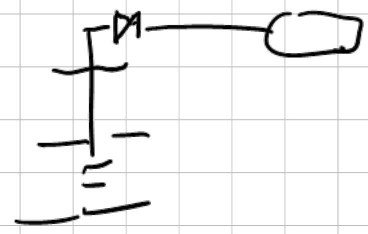
$P_{wh} = 40$ bara





wellhead eq

pipeline included



small pipeline

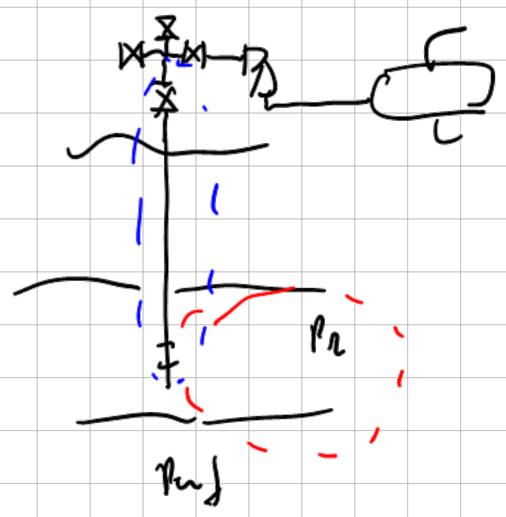
long pipeline

close to 2.85 E06 Sm³/d

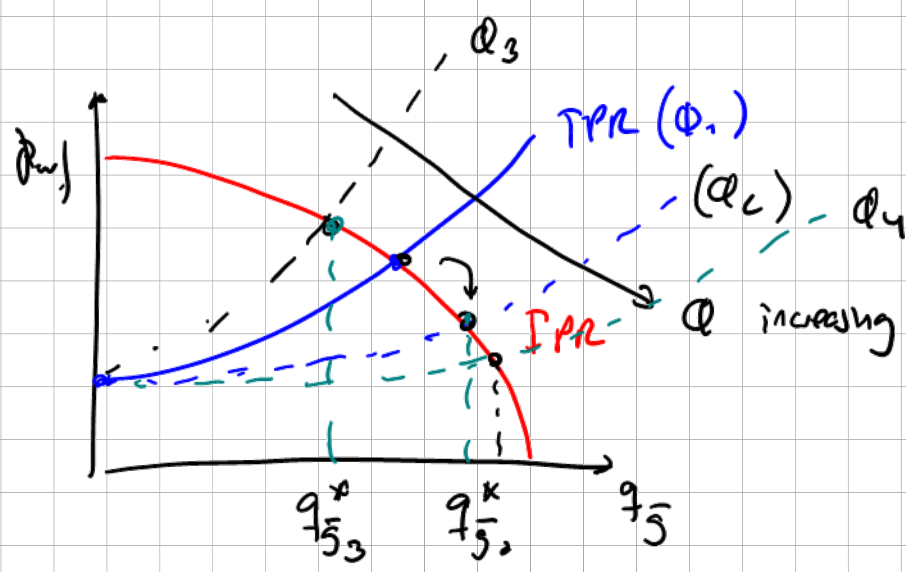
much lower than 2.85 E06 Sm³/d

Tubing size considerations: (ϕ)

- Maximize q_o, q_g → increase equilibrium rate
  ↳ revenue



$p_{wh} = p_{sep}$



- minimize cost $\downarrow \phi$, $\downarrow$ cost

tubing comes in discrete sizes $\phi \to \begin{cases} \phi_1 \\ \phi_2 \\ \phi_3 \end{cases}$

tubing according to API specs

used in pipe flow calculations

Tubing Size		Nominal Weight			Wall	Inside	Drift	Threaded Coupling			Col-	Internal	Joint Yield Strength		Capacity Table	
		T & C						Coupling Outside Dia.								
Nom. in.	OD in.	Non-Upset lb/ft	T & C Upset lb/ft	Grade	Thick-ness in.	Dia. in.	Dia. in.	Non-Upset in.	Upset Reg. in.	Upset Spec. in.	lapse Resis-tance psi	Yield Pres-sure psi	T & C Non-Upset lb	T & C Upset lb	Barrels per Linear ft	Linear ft per Barrel
3/4	1.05	1.14	1.20	H-40 J-55 C-75 N-80	0.113	0.824	0.730	1.313	1.660		7,200 9,370 12,250 12,710	7,530 10,360 14,120 15,070	6,360 8,740 11,920 12,710	13,300 18,290 24,940 26,610	0.0007	1516.13
1	1.315	1.700	1.800	H-40 J-55 C-75 N-80	0.113	1.049	0.955	1.660	1.900		6,820 8,860 11,590 12,270	7,080 9,730 13,270 14,160	10,960 15,060 20,540 21,910	19,760 27,160 37,040 39,510	0.0011	935.49
1 1/4	1.660	2.300	2.400	H-40 H-40 J-55 J-55 C-75 N-80	0.125 0.140 0.125 0.140 0.140 0.140	1.410 1.380 1.410 1.380 1.380 1.380	1.286	2.054	2.200		5,220 5,790 6,790 7,530 9,840 10,420	5,270 5,900 7,250 8,120 11,070 11,810	15,530 21,360 29,120 31,060	26,740 36,770 50,140 53,480	0.0019 0.0018 0.0019 0.0018 0.0018 0.0018	517.79 540.55 517.79 540.55 540.55 540.55

Non-API tubing (tubulars)

VAM – Vallourec (French)

	VAM® 21	VAM® 21 HT	VAM TOP ®	VAM TOP ® HT	VAM TOP ® HC	VAM® HTTC	VAM® HP	VAM® HW ST	VAM® LOX	VAM® BOLT-II	VAM® HTF-NR	VAM® FJL	VAM® MUST	VAM® SG	VAM® EDGE SF	VAM® SLIJ-II	VAM® LIFT	DINO VAM®	BIG OMEGA ®	VAM TOP ® FE	VAM® TTR
2 3/8			✓									✓									
2 7/8			✓									✓									
3 1/2	✓		✓			✓						✓									
4			✓									✓									
4 1/2	✓	✓	✓	✓	✓	✓					✓	✓		✓		✓					
5	✓	✓	✓	✓	✓	✓		✓			✓	✓		✓	✓	✓					
5 1/2	✓	✓	✓	✓	✓	✓		✓			✓	✓	✓	✓	✓	✓					
5 3/4			✓																		
6	✓	✓	✓	✓	✓																
6 5/8	✓	✓	✓	✓	✓	✓		✓			✓	✓				✓				✓	
6 7/8																✓					
7	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓				✓		✓		✓	
7 5/8	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓			✓				✓	
7 3/4	✓		✓		✓						✓					✓				✓	
8																				✓	
8 1/8												✓									
8 5/8	✓		✓				✓					✓				✓				✓	✓
8 3/4	✓																				
9							✓														
9 3/8											✓	✓									
9 5/8	✓	✓	✓			✓		✓			✓	✓				✓		✓		✓	✓
9 3/4																✓		✓			
9 7/8	✓		✓								✓	✓				✓		✓		✓	
10	✓		✓													✓					
10 1/8	✓		✓				✓			✓						✓					
10 1/2							✓														
10 3/4	✓		✓				✓	✓				✓	✓			✓		✓		✓	✓

tub

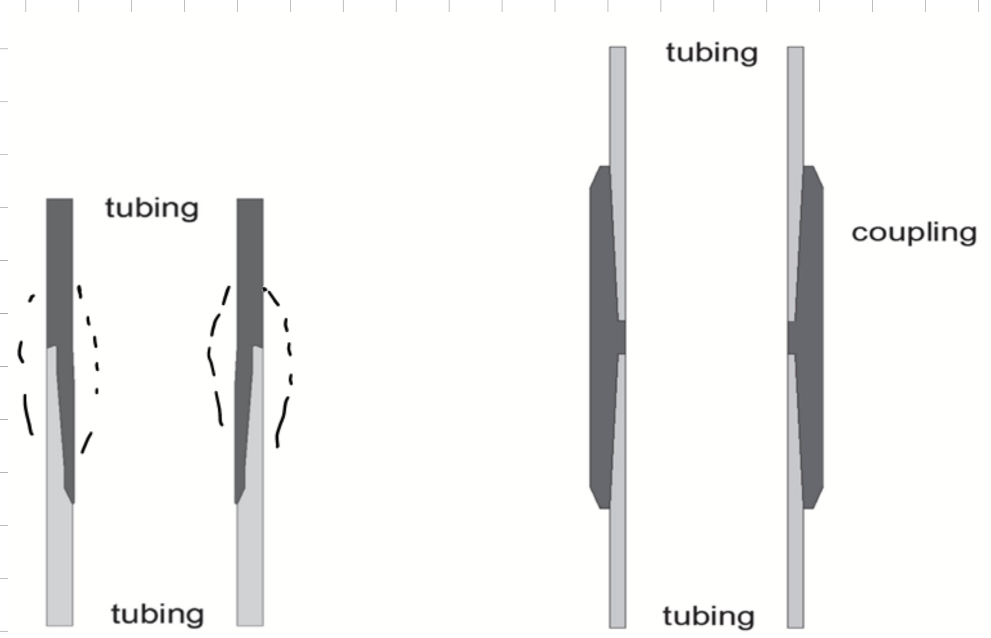
casing

riser flowline

⋮

26

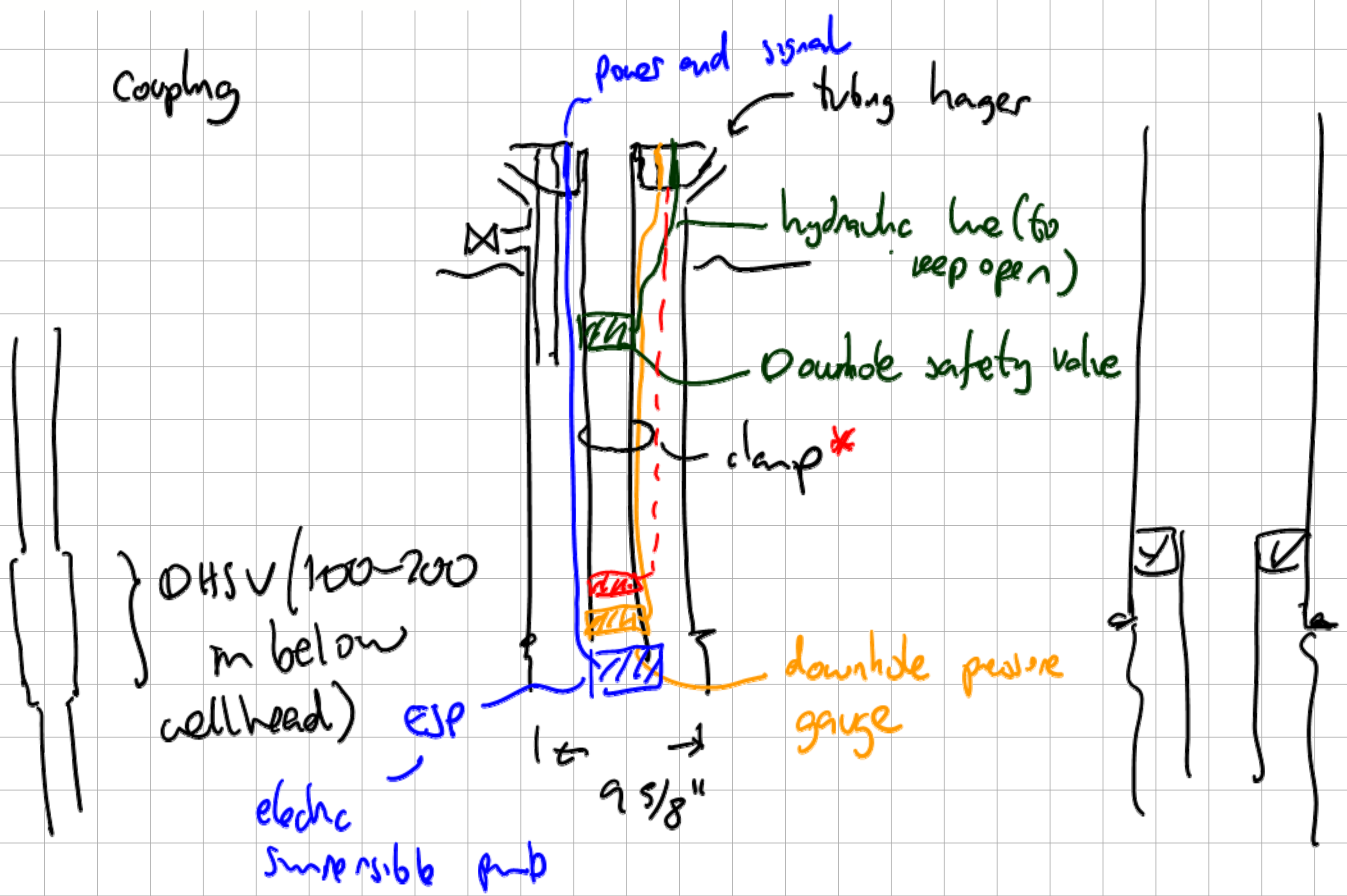
7''	-	32#	-	P-110	-	BTC/LTC
nominal size		weight per length pound per foot ppf		steel "type" grade resistance		type of thread



integrated joint

Coupling

- tubing hanger considerations



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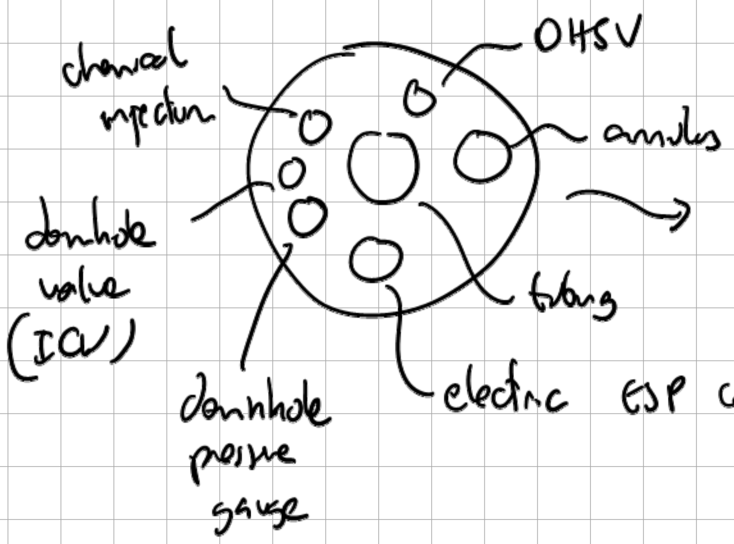
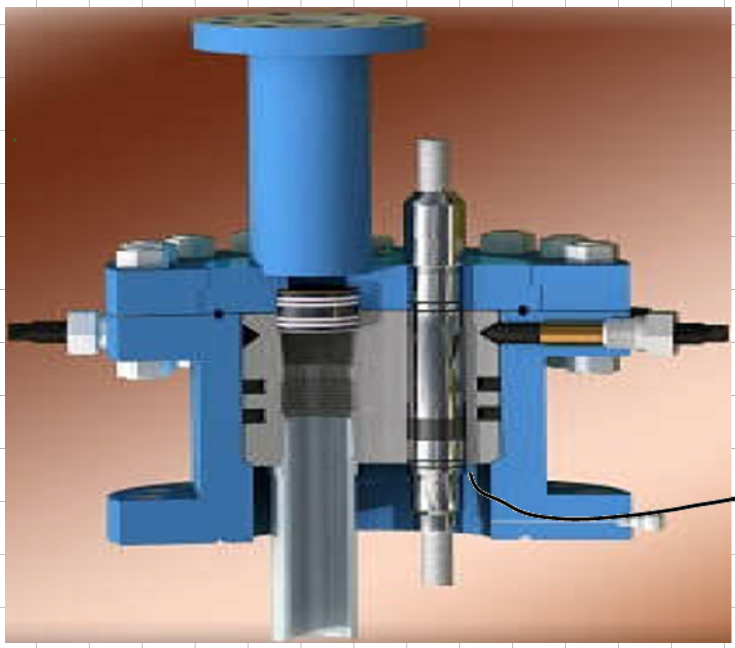


structurally "solid"



penetrator

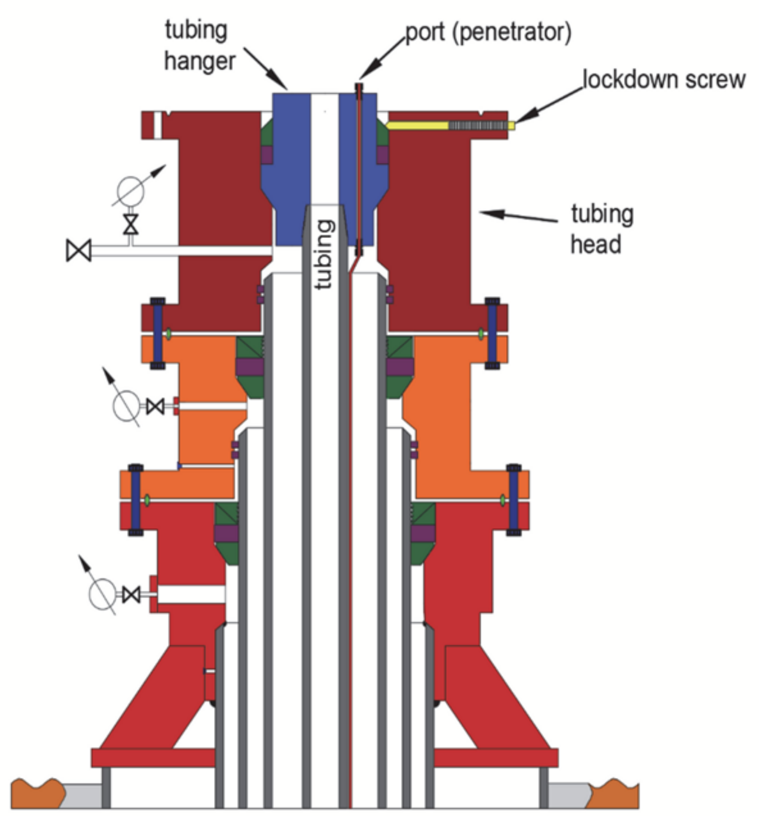
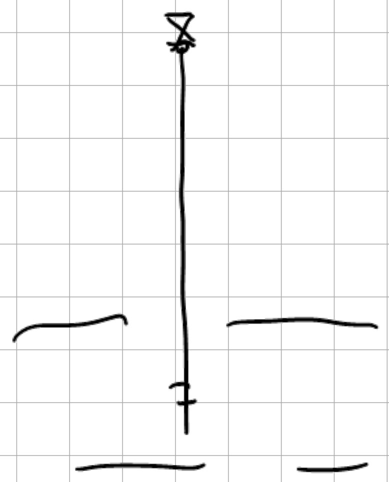


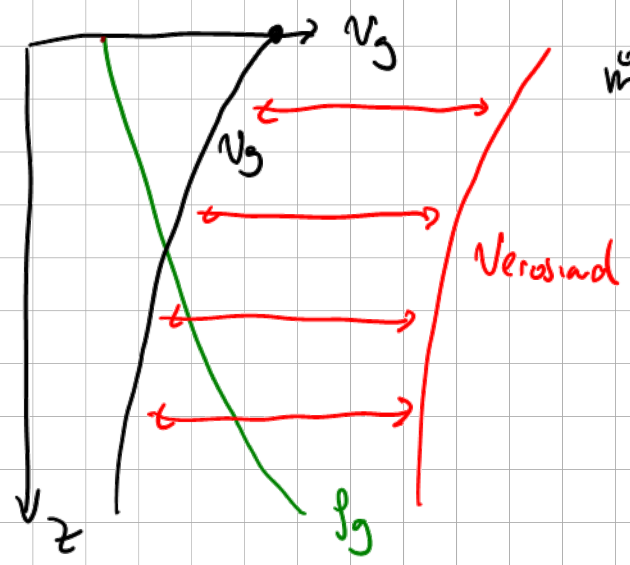
FIGURE 6-10. FINAL CONFIGURATION OF THE WELLHEAD

- erosional velocity

$v_f < \underbrace{V_{erosional}}_{\text{velocity}}$

gas well $\quad v_f = v_g = \dfrac{q_g}{A}$





$\dot{m}_g = \bar{\rho}_g \bar{q}_g = q_g \cdot \rho_g = v_g \cdot A \cdot \rho_g$

$v_g = \dfrac{\bar{\rho}_g \bar{q}_g}{A\,\rho_g}$ — constant with depth

API 14E $\sim$ $V_{erosional} = \dfrac{C}{\sqrt{\rho_m}} \rightsquigarrow$ (100 – 125) "severe" $\left[ft/s \right]$

conservative $\qquad \sqrt{\rho_m} \sim \left[lb/ft^3 \right]$

$\rho_m = f(\rho_L, \rho_g)$

e.g. $\quad \rho_m = \dfrac{q_o \rho_o + q_g \rho_g + q_w \rho_g}{q_o + q_g + q_w}$

what happens if there is more than 1 fluid in the tubing?

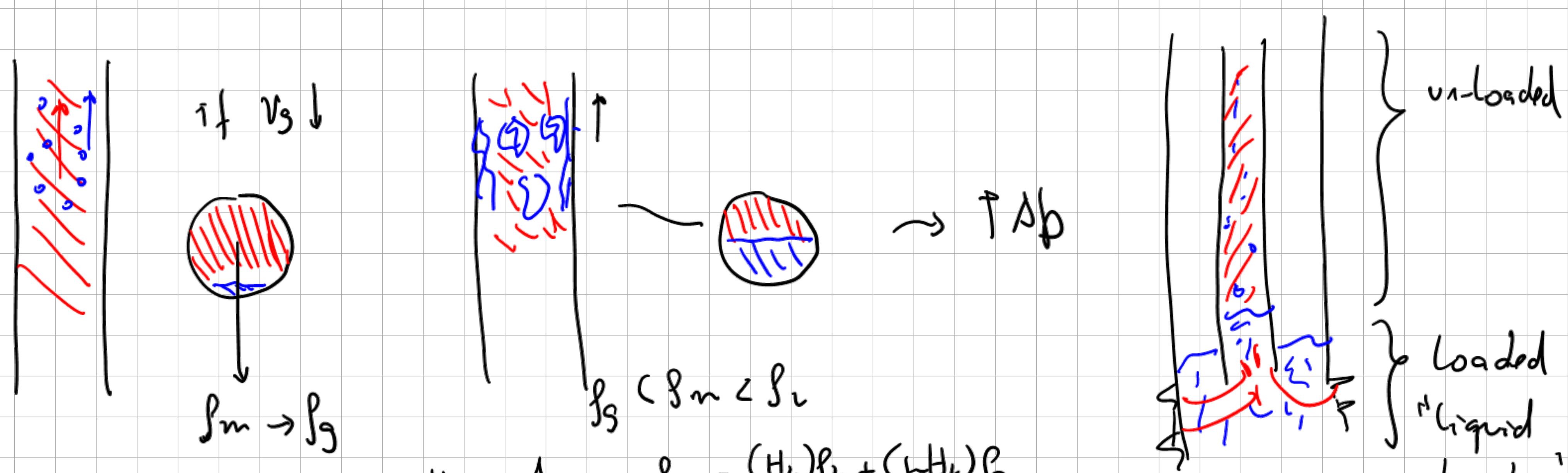
o + g + w

$V_m = \dfrac{(q_o + q_g + q_w)}{A} = \dfrac{q_o}{A} + \dfrac{q_g}{A} + \dfrac{q_w}{A}$

$u_{so} \quad u_{sg} \quad u_{sw}$

↳ superficial velocity

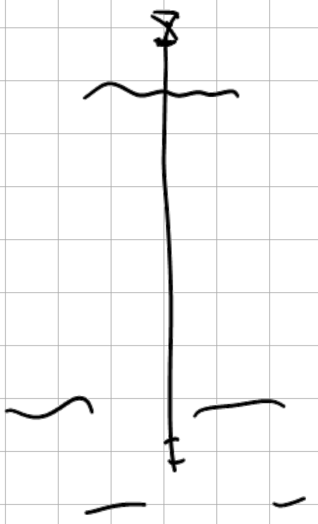
- only in gas wells → avoid liquid loading

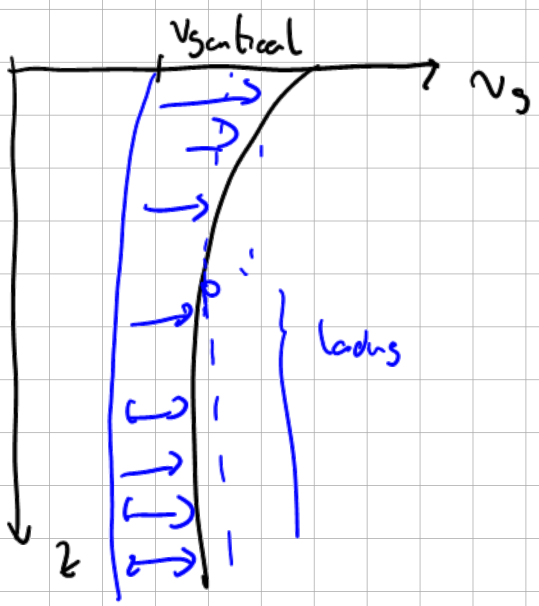


$H_L = \dfrac{A_L}{A} \qquad \rho_m = (H_L)\rho_L + (1-H_L)\rho_g$

in general $H_L \neq \frac{q_L}{q_L + q_g}$

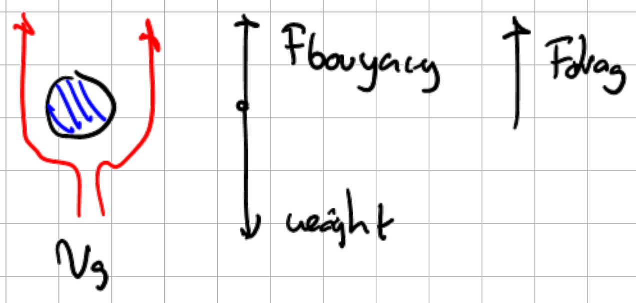
$v_g > v_{critical}$ for all tubing depths





How to calculate $v_{critical}$?

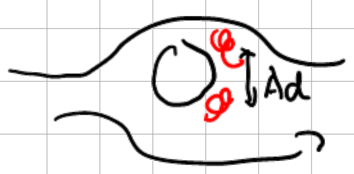
Turner criteria



$$F_{drag} + F_{bouyancy} = F_{weight}$$

$$\frac{1}{2}\rho_g C_D v_g^2 A_d + V_d \rho_g g = V_d \cdot \rho_L \cdot g$$

$C_D \rightarrow 0.44$



$$V_d = \frac{\pi \phi^3}{6} \qquad A_d = \frac{\pi \phi^2}{4}$$

$$\frac{1}{2}\rho_g \, 0.44 \, v_g^2 \frac{\cancel{\pi}\phi^{\cancel{2}}}{\cancel{4}_{2}} = (\rho_L - \rho_g) g \frac{\cancel{\pi}\phi^{\cancel{3}}}{\cancel{6}_{3}}$$

$$v_{crit} = \sqrt{\frac{4}{3}\frac{(\rho_L - \rho_g)}{\rho_g}\frac{\phi \cdot g}{0.44}}$$

Analysis and Prediction of Minimum Flow Rate for the Continuous Removal of Liquids from Gas Wells

R. G. Turner, SPE-AIME, Baker Oil Tools, Inc.
M. G. Hubbard, SPE-AIME, U. of Houston
A. E. Dukler, U. of Houston

1969

$$N_{we} = \frac{\overbrace{\rho_g u_g^2 d}^{\text{Drag}}}{\underbrace{\sigma_{gL}}_{\text{interfacial tension}}}$$

Weber

Fundamentals of the Hydrodynamic Mechanism of Splitting in Dispersion Processes

J. O. Hinze, Royal Dutch Shell-Laboratory, Delft, Holland

1955

$N_{we} = 30 \rightarrow \phi_{max}$

$$30 = \frac{\rho_g u_g^2 \phi}{\sigma_{Lg}}$$

$$\phi = \frac{\sigma_{Lg} 30}{\rho_g u_g^2}$$

for every depth in tubing

$v_g \geq v_{crit}$

$$v_{crit} = 5.46\left[\frac{\sigma_{Lg}^{0.25}(\rho_L - \rho_g)^{0.25}}{\rho_g^{0.5}}\right] \quad [m/s]$$

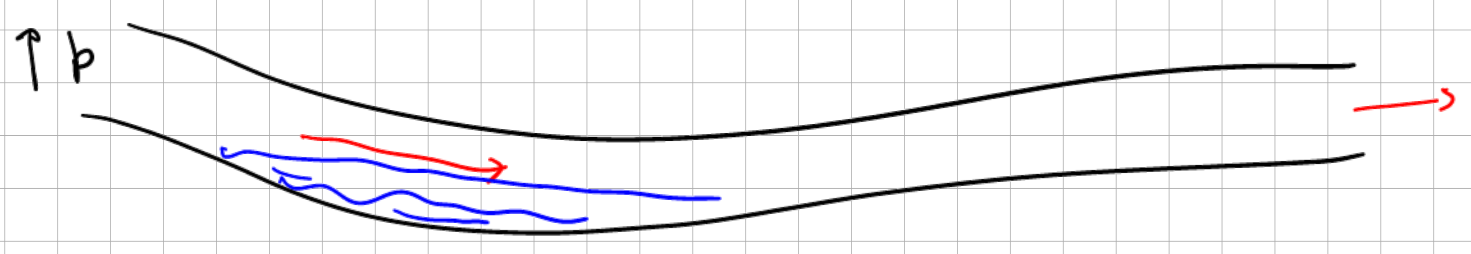
σ in $\left[\frac{N}{m}\right]$

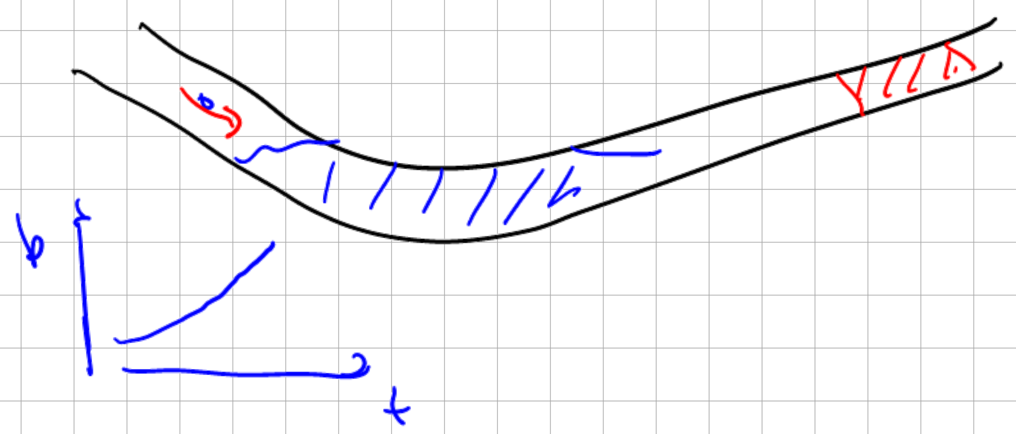
ρ in $\left[kg/m^3\right]$

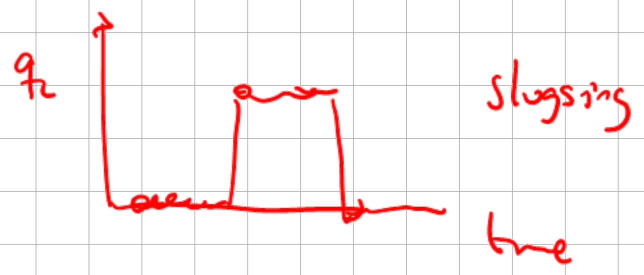
- pipeline sizing considerations

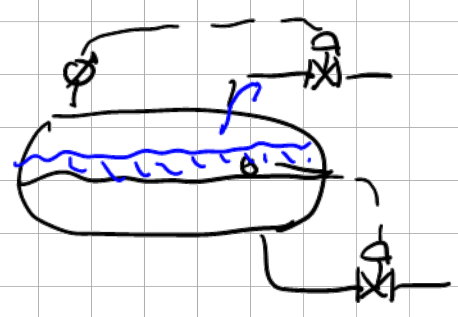
$\uparrow \hat{q}_{f\bar{s}}$, $\uparrow \hat{q}_{f\bar{o}}$ $\downarrow$ cost

- v_g high enough to avoid accumulation of liquid (gas condensate pipeline)









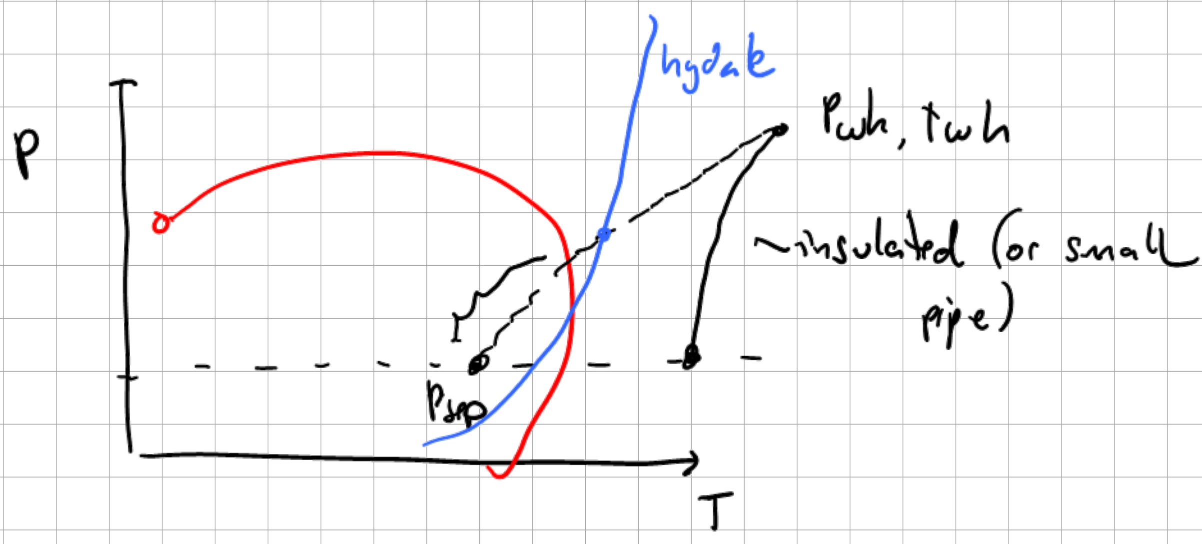
- heat transfer

$$\dot{q} = A \cdot U \left(T_f - T_{amb}\right)$$

$$\pi \phi \cdot L \; U \left(T_f - T_{amb}\right)$$

$$\phi \uparrow \quad \dot{q} \uparrow \quad T_f \rightarrow \downarrow$$

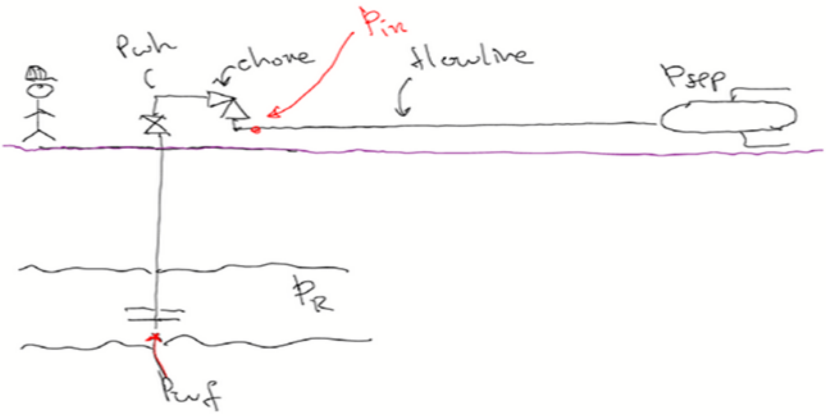
can also cause hydrate formation



20241014
OUTLINE
-Recap of last week video lectures
-Class work on Problem 2, exercise set 3

PROBLEM 2.

Consider the dry gas production system shown in the figure below:



The pressure drop in the pipeline can be neglected, therefore, the pressure at the inlet of the flowline can be assumed equal to the separator pressure, 40 bara.

An excel file is provided with all the information and VBA functions you need to make your calculations.

Task 1. If the well system is producing a dry gas rate of 1 E5 Sm³/d, and wellhead pressure is 80 bara, estimate the backpressure coefficient of the formation. Regarding n, assume values in the range 0.8-1 are possible.

1.5E5 Sm³/d ? 120 bara ?

$$q_{\bar{g}} = C_R \left(P_R^2 - P_{wf}^2\right)^n$$

$P_{wh} = 80$ bara

$P_{dc} = P_{in} = 40$ bara

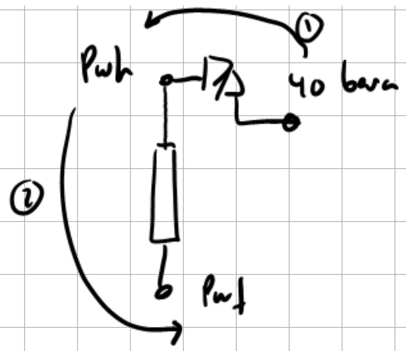
$P_R = 120$ bara

calculate from P_{wh}, $q_{\bar{g}}$ and tubing equation

$$q_{\bar{g}} = C_T \left(\frac{P_{wf}^2}{e^S} - P_{wh}^2\right)^{0.5}$$

Task 2. Consider that the sensor of the wellhead pressure is damaged and unavailable. The only information available is pressure downstream the choke (40 bara) and choke opening (9.4 mm). Would it still be possible to estimate the backpressure coefficient of the formation? (assume that n = 1)? Is the choke operating in the critical or subcritical regime?

$$q_{\bar{g}} = \overset{?}{C_R} \left(P_R^2 - P_{wf}^2\right)^n$$



$P_{dc} \rightsquigarrow P_{wh}$?

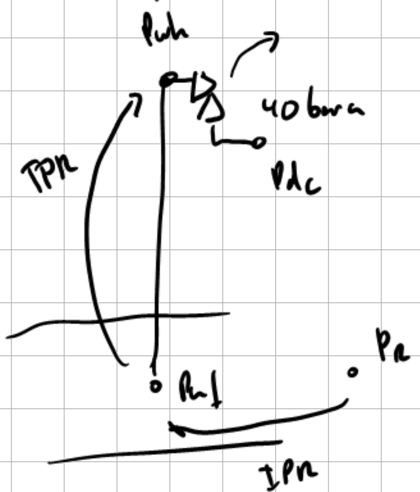
$$q_{\bar{g}} = \frac{\rho_2 \cdot A_2 \cdot C_d}{\rho_{sc}} \cdot \sqrt{2 \cdot \frac{Z_1 \cdot R \cdot T_1}{M_w} \cdot \frac{k}{k-1} \cdot \left(1 - y^{\frac{k-1}{k}}\right)}$$

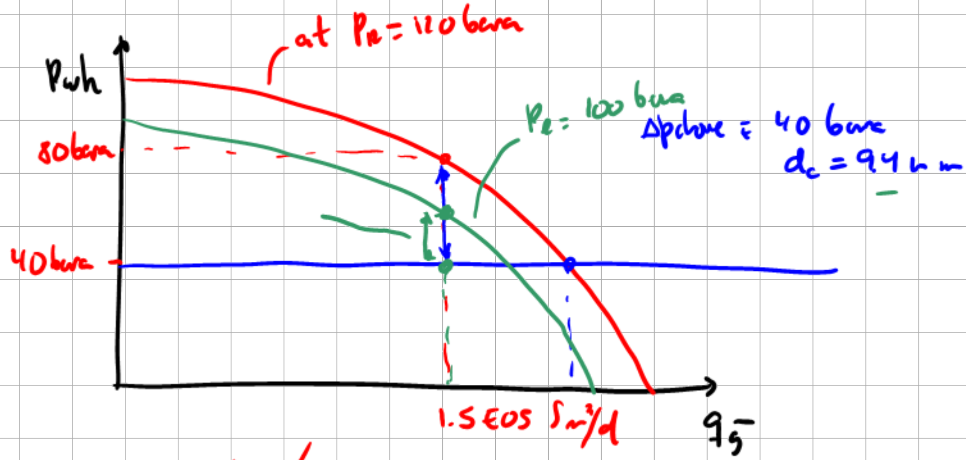
$$y = \frac{P_{dc}}{P_{wh}} \text{ - ?}$$

repeat same procedure as task 1, pwh-->pwf-->CR

Task 3. Assume reservoir pressure has changed to 100 bara. Estimate choke adjustment (new choke opening) to ensure the dry gas rate remains constant. Use the values of dry gas IPR estimated in task 2. Is the choke operating in the critical or subcritical regime?

well head equilibrium





$$q_{\bar{g}} = C_R \cdot (P_R^2 - P_{wf}^2)$$

$P_{wf} = P_R = 100$ bara

$$q_{\bar{g}} = C_T \left(\frac{P_{wf}^2}{e^s} - P_{wh}^2\right)^{0.5}$$

$$\frac{P_{wf}^2}{e^s} = P_{wh}^2$$

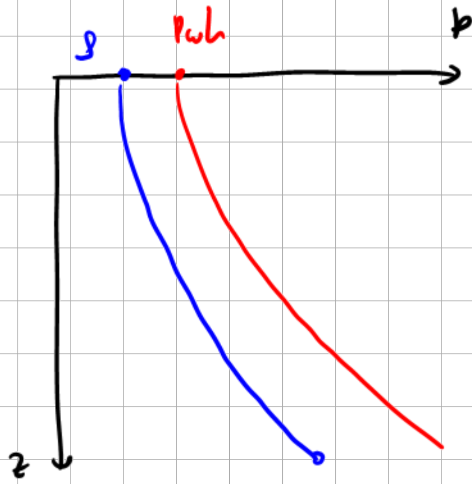
$$q_{\bar{g}} = \frac{\rho_2 \cdot A_2 \cdot C_d}{\rho_{sc}} \cdot \sqrt{2 \cdot \frac{Z_1 \cdot R \cdot T_1}{M_w} \cdot \frac{k}{k-1} \cdot \left(1 - y^{\frac{k-1}{k}}\right)}$$

$$y = \frac{P_{dc}}{P_{wh}}$$

Task 4. At this new reservoir pressure, you are concerned about liquid loading. Check if there could be risk of liquid loading at well bottom, using the Turner equation:

$$v_{gc} = 5.46 \cdot \left(\frac{(\rho_L - \rho_G) \cdot \sigma}{\rho_G^2}\right)^{1/4}$$

Assume that liquid density is 1000 kg/m3, and interfacial tension liquid-gas is 0.028 N/m. Assume the tubing inner diameter is 0.07 m.



$$\frac{p}{\rho} = R Z T$$

$$p = R Z \cdot T \rho$$

$$v_{gc} = 5.46 \cdot \overbrace{\left(\frac{(\rho_L - \rho_g) \cdot \sigma}{\rho_g^{2}} \right)^{0.25}}^{\text{smaller}}$$

with $\uparrow p$ $v_{gc} \downarrow$

v_{gc} v_g

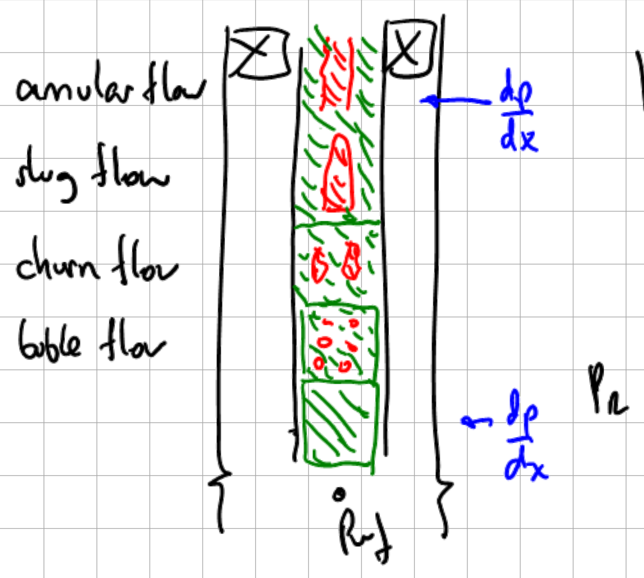
$$v_g = \frac{q_g}{A_T} = \frac{q_g \cdot B_g}{A_T}$$

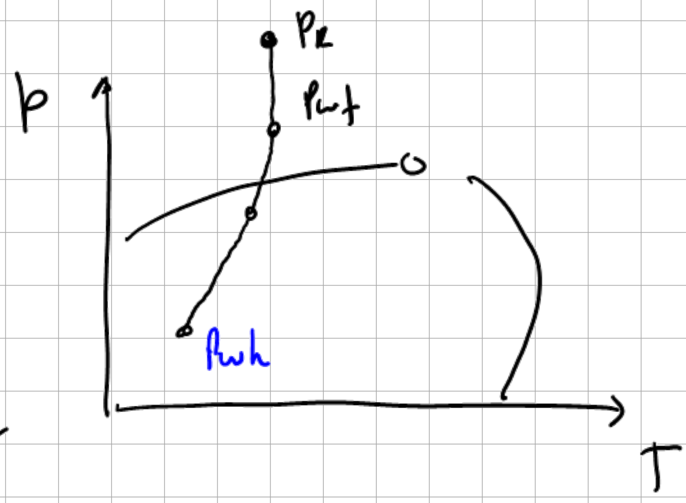
How do I solve this task?

v_g ρ_g @ p_{wf}, T_{wf}

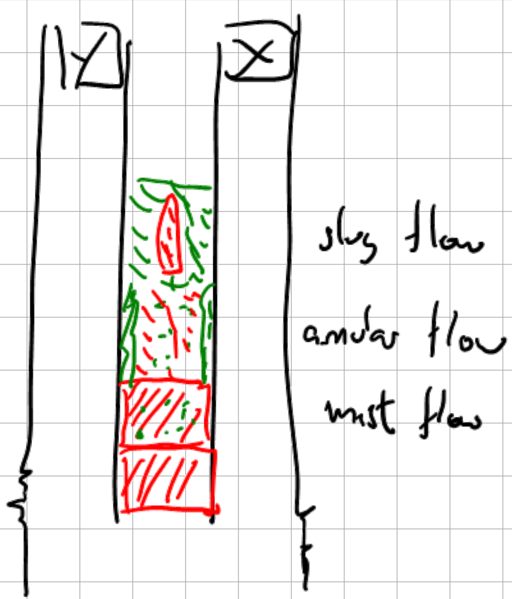
v_{gc}

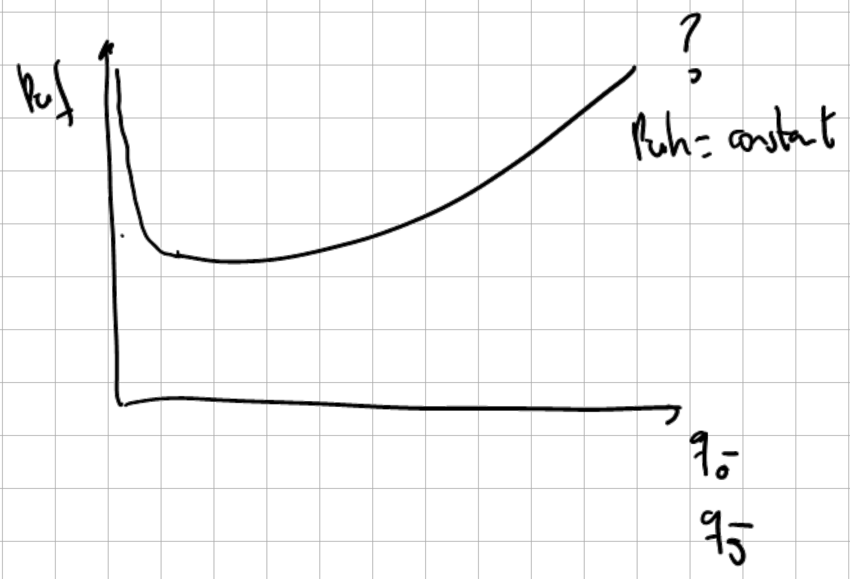
B_g @ p_{wf}, T_{wf} check if $v_g \geq v_{gc}$

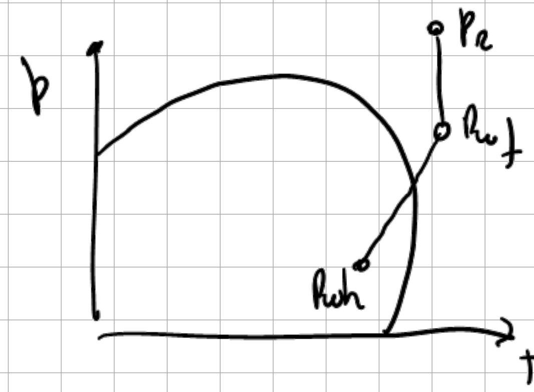




flow patterns

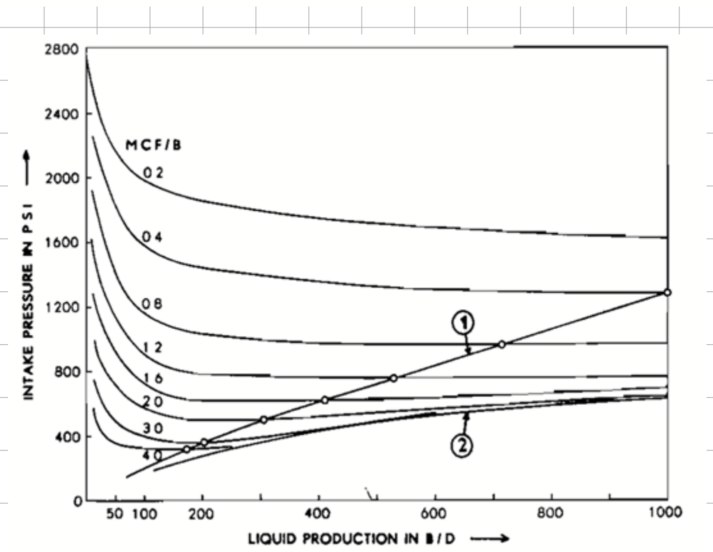






Flowing and Gas-lift Well Performance†

W. E. GILBERT*



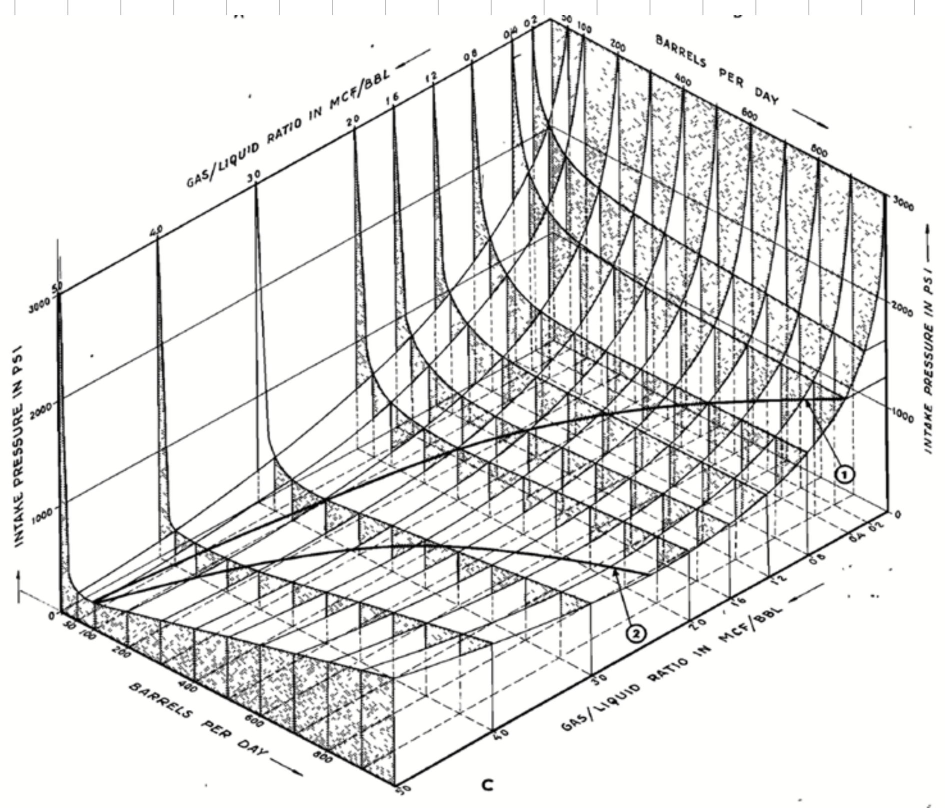
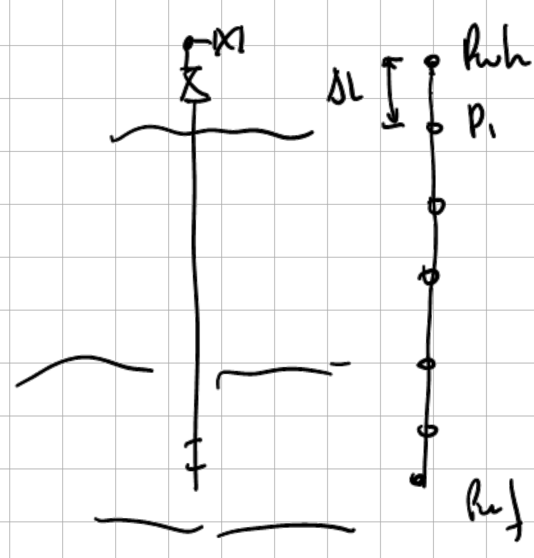


Fig. 6–The Two-phase Vertical-lift Function for 2.875-in. Tubing Set at 8,000 Ft
(Tubing Pressure = Zero Psi Gage)

$$\Delta p_{tubing} = \Delta p_{hydrostatic} + \Delta p_{friction}$$



$$\left.\frac{dp}{dx}\right|_{P_{wh}} \rightarrow P_1$$

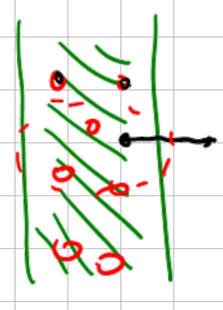
$$P_1 = P_{wh} + \left.\frac{dp}{dx}\right|_{P_{wh}} \cdot \Delta L$$

?

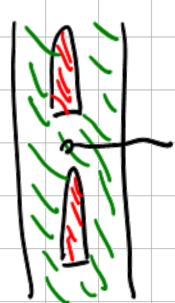
$$P_2 = P_1 + \left.\frac{dp}{dx}\right|_{P_1} \cdot \Delta L$$

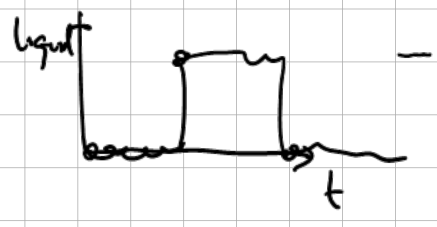
integration

marching algorithm



$$\frac{d}{dt} \neq 0$$





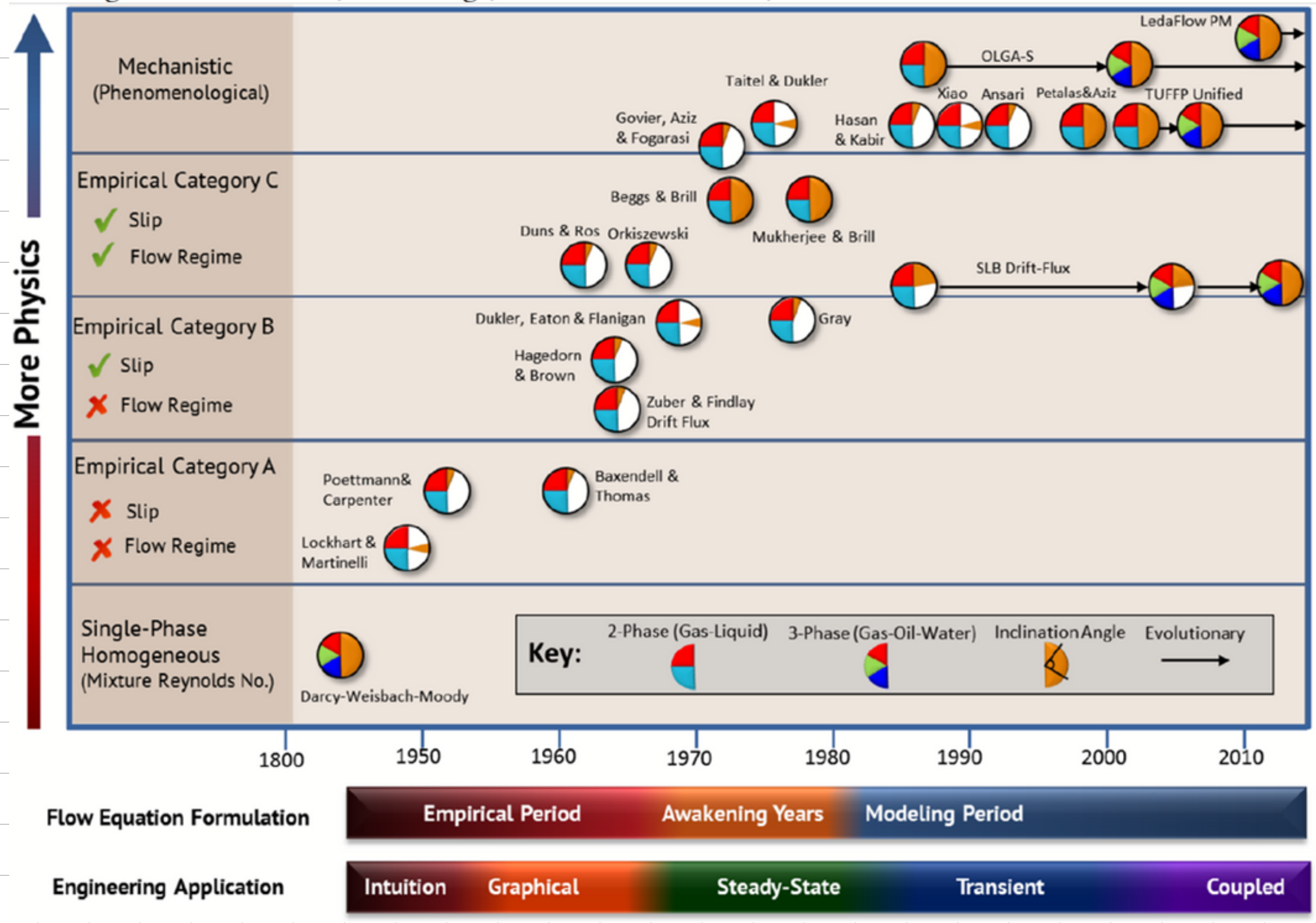
$x \qquad \bar{x} \quad \int_t x \; dt \qquad \bar{x} = \int_A x \; dA$

$\frac{dp}{dx}$ ~> PVT properties $(\rho_L, \rho_g, \mu_L, \mu_g, \sigma_{Lg}, x_g)$ ← mass fraction
→ velocity (q_g, q_L)
→ A, ϕ, θ, e
roughness

Steady-State Multiphase Flow—Past, Present, and Future, with a Perspective on Flow Assurance

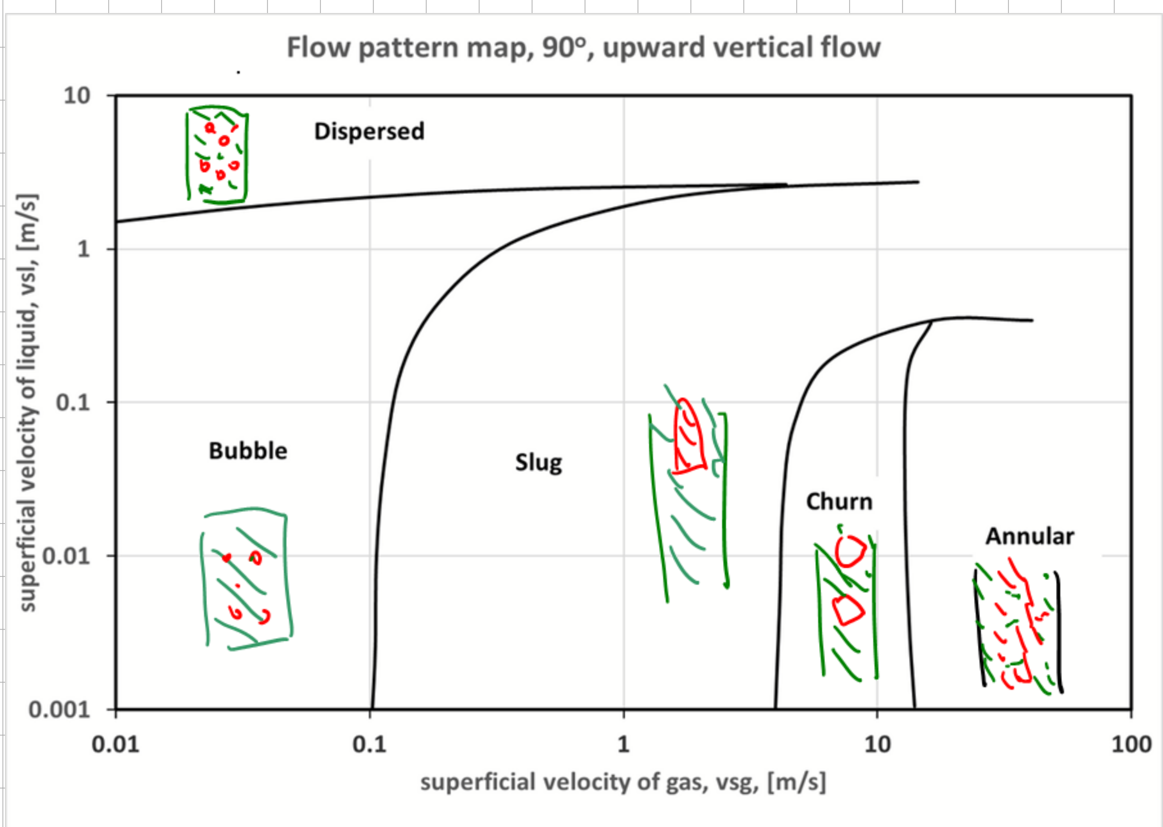
Mack Shippen*
Schlumberger SIS, Houston, Texas 77056, United States

William J. Bailey
Schlumberger–Doll Research, Cambridge, Massachusetts 02139, United States



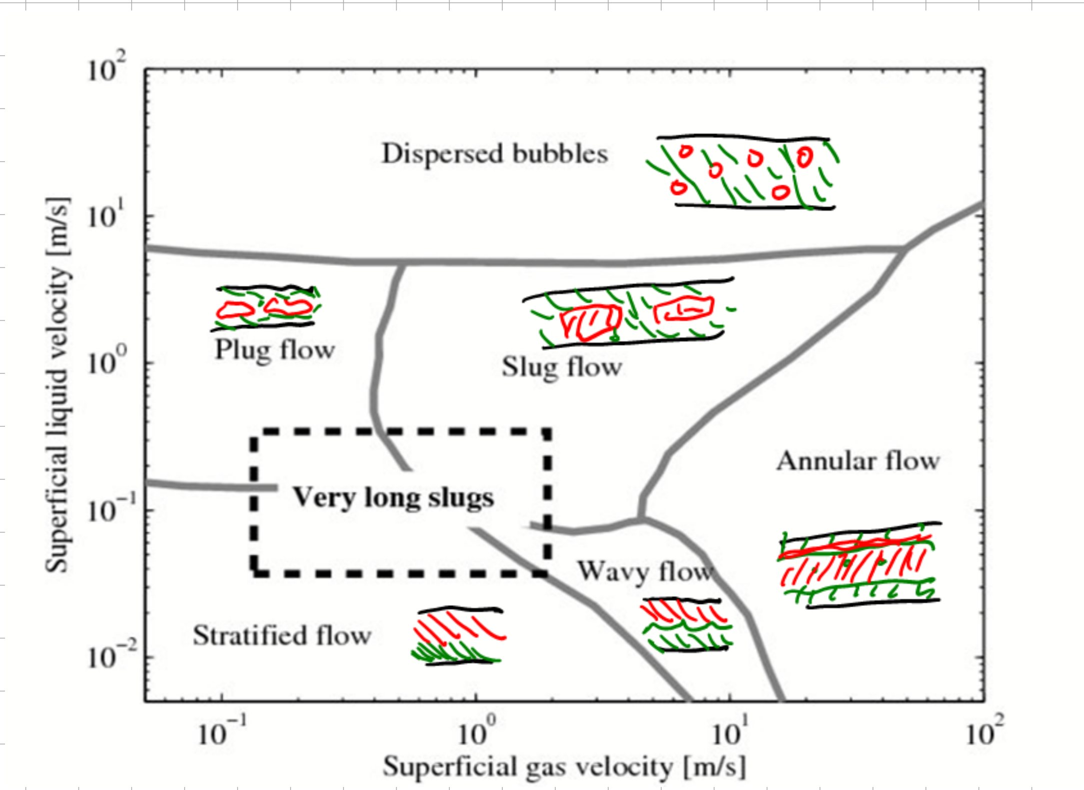
flow pattern map

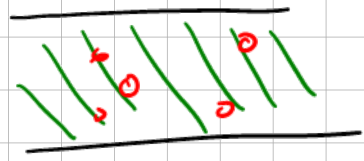
← for a given fluid, pipe, inclination, etc



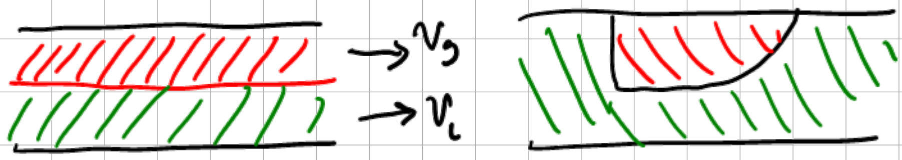
$u_{sL} = \frac{q_L}{A}$

$u_{sg} = \frac{q_g}{A}$

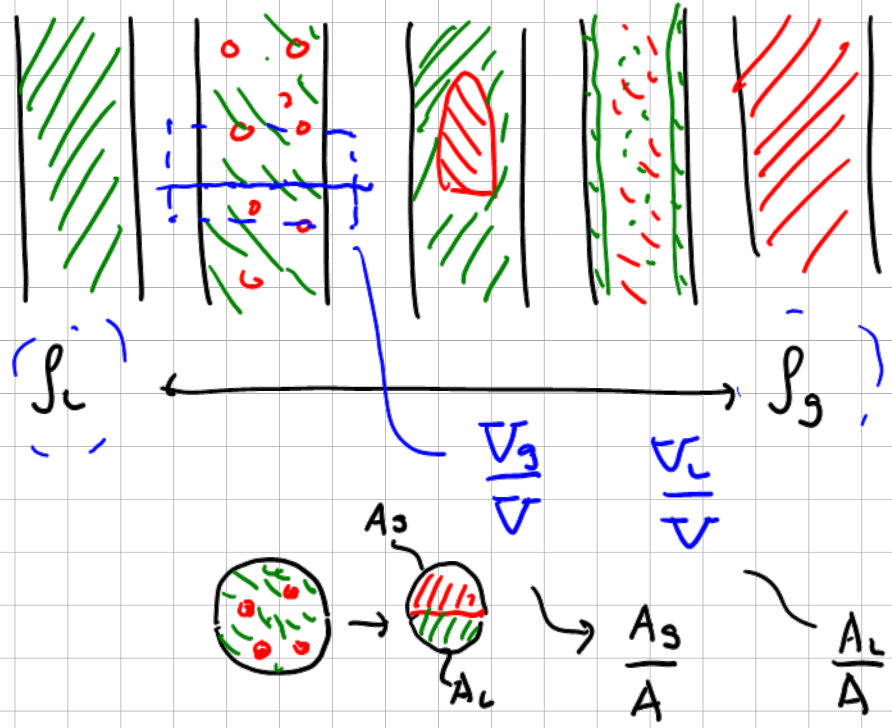

Dispersed bubbles
Plug flow
Slug flow
Annular flow
Very long slugs
Wavy flow
Stratified flow
Superficial liquid velocity [m/s]
Superficial gas velocity [m/s]
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}



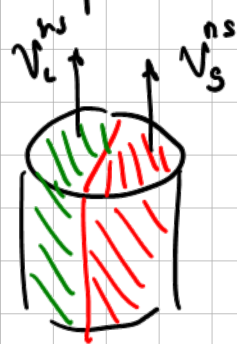
- flow patterns



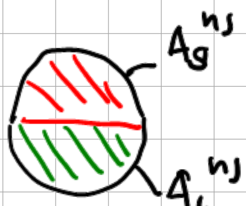
- phase velocity $\rightsquigarrow$ friction Δp
- phase spatial distribution $\rightsquigarrow$ hydrostatic Δp



Liquid and gas travel at the same velocity (no-slip)



$$V_L^{ns} = V_g^{ns} = V_m = \frac{q_g + q_L}{A} = \frac{q_g}{A} + \frac{q_L}{A} = u_{sg} + u_{sL}$$



$$\lambda_g = \frac{A_g^{ns}}{A} = 1 - \lambda_L = \frac{q_g}{q_g + q_L} \qquad q_g \uparrow\uparrow \gg q_L \longrightarrow \lambda_g \rightarrow 1$$

$$\lambda_L = \frac{A_L^{ns}}{A} = 1 - \lambda_g = \frac{q_L}{q_g + q_L} \qquad q_L \gg q_g \longrightarrow \lambda_L \rightarrow 1$$

gas and liquid move at different velocities $V_g \neq V_L$ (slip condition)

$$\overbrace{\varepsilon}^{\text{void fraction}} = \underbrace{H_g}_{\text{gas holdup}} = \frac{A_g}{A} = 1 - H_L$$

$$\underbrace{H_L}_{\text{liquid holdup}} = \frac{A_L}{A} = 1 - H_g$$

V_g ?
V_L ?

mass conservation between non-slip condition and slip condition

$$q_g = V_g^{ns} \cdot A_g^{ns} = V_g \cdot A_g \qquad \text{divide by } A$$

$$V_g^{ns} \lambda_g = V_g \cdot \varepsilon$$

$$\boxed{V_g = V_m \frac{\lambda_g}{\varepsilon}}$$

$\lambda_g < \varepsilon$

$V_g < V_m$

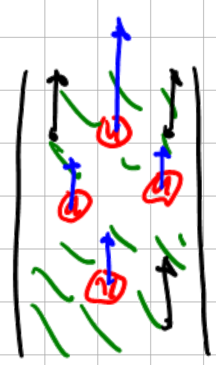
$\lambda_g = \varepsilon$
$V_g = V_m$

$\lambda_g > \varepsilon$
$V_g > V_m$

$$\boxed{v_L = v_m \frac{\lambda_L}{H_L}}$$

$$v_g = \frac{v_m \lambda_g}{\varepsilon} = \frac{\cancel{q_g+q_L}}{A}\cdot\frac{q_g}{\cancel{q_g+q_L}}\cdot\frac{1}{\varepsilon} = \frac{u_{sg}}{\varepsilon}$$

$$\boxed{v_g = \frac{u_{sg}}{\varepsilon} \qquad v_L = \frac{u_{sL}}{H_L}}$$



$\varepsilon, H_L, v_L, v_g$ is a result of solving our $\left\{\begin{array}{l}\text{mass conservation}\\ \text{momentum conservation}\end{array}\right\}$

one example

$$v_g = C_0 v_m + \underbrace{u_0}_{\text{velocity of a bubble in stagnant liquid}}$$

TWO-PHASE FLOW IN VERTICAL TUBES

By D. J. NICKLIN, B.Sc. App.,* J. O. WILKES, M.A.*† and J. F. DAVIDSON, M.A., Ph.D., A.M.I.Mech.E.,*

SUMMARY

A study has been made of the properties of long bubbles in vertical tubes. It has been shown that these bubbles rise relative to the liquid ahead of them at a velocity exactly equal to the rising velocity of wakeless bubbles of the type studied by Dumitrescu and by Davies and Taylor. For 1 in. tubes, this velocity is closely predicted by Dumitrescu's theory and equals 0·35 $(gD)^{\frac{1}{2}}$ where g is the acceleration of gravity and D the tube diameter. The motion of the bubbles in moving liquid streams has been studied, and the results applied to the problem of two-phase slug flow. An expression for the voidage in steady two-phase slug flow has been derived, and this predicted voidage agrees well with results reported here and elsewhere.

* University of Cambridge, Department of Chemical Engineering, Pembroke Street, Cambridge.

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TRANS. INSTN CHEM. ENGRS, Vol. 40, 1962

$$u_s = 1{\cdot}2\,\bar{u}_L + 0{\cdot}35\,(gD)^{\frac{1}{2}}$$

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Advanced Technology Laboratories.
Mem. ASME

J. A. FINDLAY
Knolls Atomic Power Laboratory.
Mem. ASME

General Electric Co.,
Schenectady, N. Y.

Average Volumetric Concentration in Two-Phase Flow Systems

A general expression which can be used either for predicting the average volumetric concentration or for analyzing and interpreting experimental data is derived. The analysis takes into account both the effect of nonuniform flow and concentration profiles as well as the effect of the local relative velocity between the phases. The first effect is taken into account by a distribution parameter, whereas the latter is accounted for by the weighted average drift velocity. Both effects are analyzed and evaluated. The results predicted by the analysis are compared with experimental data obtained for various two-phase flow regimes, with various liquid-gas mixtures in adiabatic, vertical flow over a wide pressure range. Good agreement with experimental data is shown.

[2] Numbers in brackets designate References at end of paper.
Contributed by the Heat Transfer Division and presented at the Winter Annual Meeting, New York, N. Y., November 29–December 3, 1964, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received at ASME Headquarters, September 15, 1964.

"v_0"

THE FLOW OF LIQUID-GAS MIXTURES IN VERTICAL TUBES.

By

Hans Behringer

ZEITSCHRIFT FUR DIE GESAMTE KALTE-INDUSTRIE, 43, 55-58, 1936.

Z. angew. Math. Mech. Bd. 23 Nr. 3 Juni 1943 — Dumitrescu, Strömung an einer Luftblase im senkrechten Rohr — 139

Strömung an einer Luftblase im senkrechten Rohr.

Von *D. T. Dumitrescu* in Bukarest.

The mechanics of large bubbles rising through extended liquids and through liquids in tubes

By R. M. DAVIES AND SIR GEOFFREY TAYLOR, F.R.S.

(*Received* 13 *September* 1949)

$$\varepsilon = \frac{U_{SG}}{U_{SG}\left(1+\left(\frac{U_{SL}}{U_{SG}}\right)^{\left(\frac{\rho_G}{\rho_L}\right)^{0.1}}\right)+2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L-\rho_G)}{\rho_L^2}\right]^{0.25}(1.22+1.22\sin\theta)^{\frac{P_{atm}}{P_{system}}}}$$

Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

Melkamu A. Woldesemayat, Afshin J. Ghajar *

School of Mechanical and Aerospace Engineering, Oklahoma State University, Stillwater, OK 74078, USA

Received 1 June 2006; received in revised form 13 September 2006

$$H_g = \frac{\lambda_g + 1 - \left[(\lambda_g+1)^2 - 4\cdot\lambda_g^{\,2}\right]^{0.5}}{2\cdot\lambda_g}$$

Pipe Fractional Flow Theory: Principles and Applications

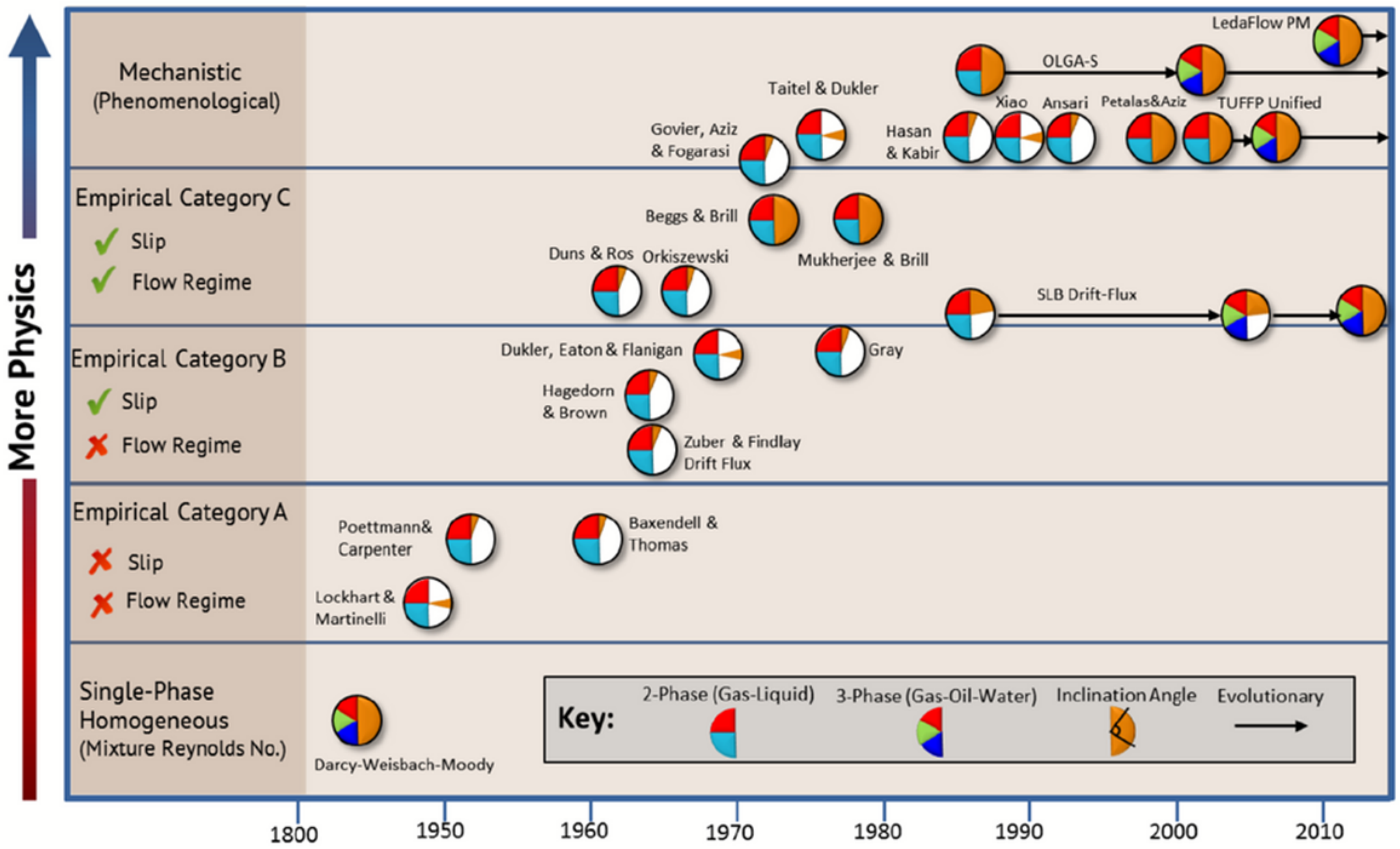
by

Anand Subhash Nagoo, B.Sc., M.S., M.S.

2013

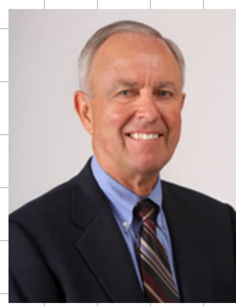
- $U_s = v_g - v_L$ slip velocity
- $S = \frac{v_g}{v_L}$ slip ratio

Video 29 - Some examples of pressure drop models for multiphase flow



A Study of Two-Phase Flow in Inclined Pipes

H. Dale Beggs,* SPE-AIME, U. of Tulsa
James P. Brill, SPE-AIME, U. of Tulsa

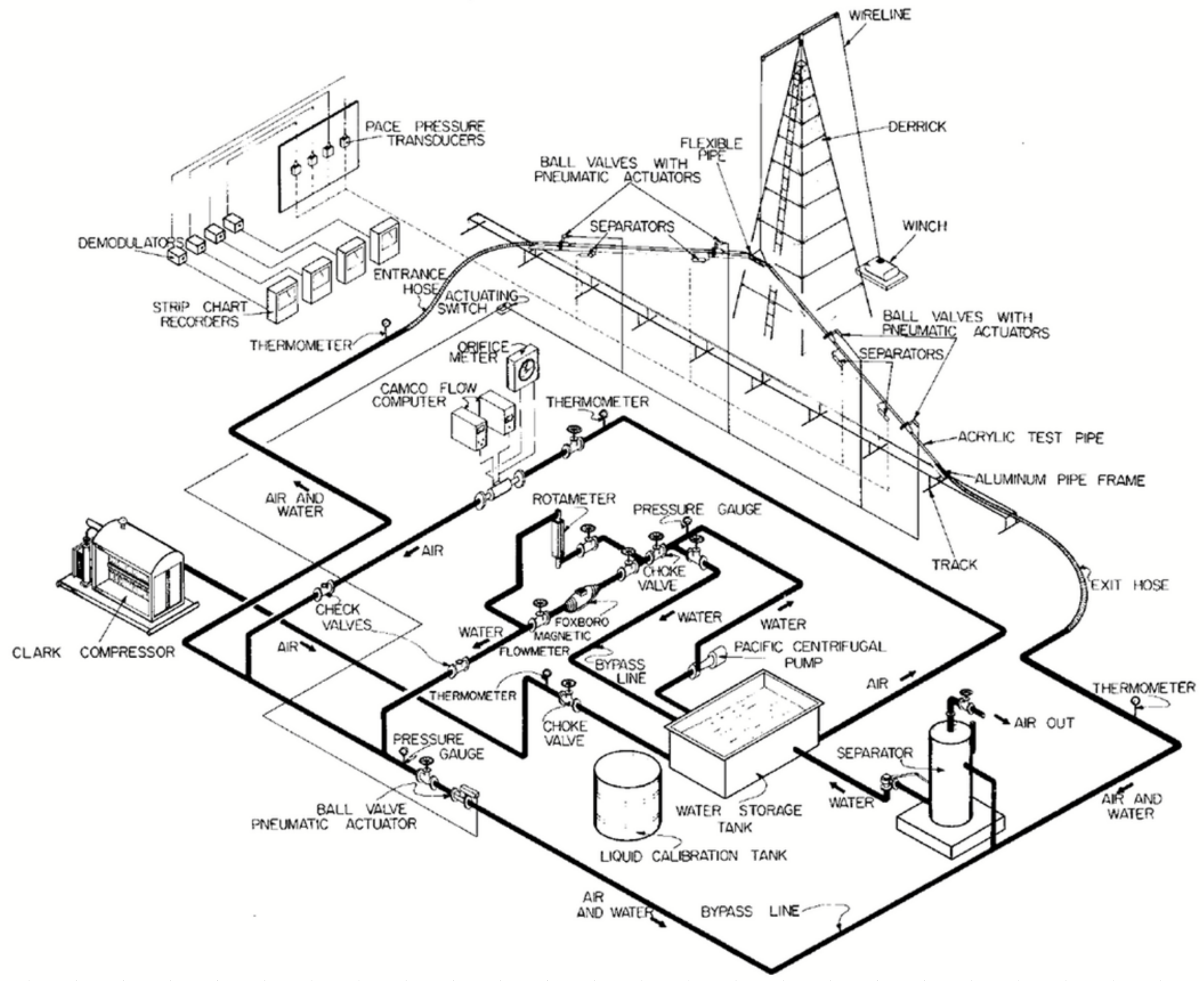


1973

$$-\frac{dp}{ds} = \left.\frac{dp}{ds}\right|_{grav.} + \left.\frac{dp}{ds}\right|_{fric.} + \left.\frac{dp}{ds}\right|_{accel.}$$

$$-\frac{dp}{dZ} = \frac{\frac{g}{g_c}\sin\theta\,[\rho_L H_L + \rho_g(1-H_L)] + \frac{f_{tp} G_m v_m}{2 g_c d}}{1 - \{[\rho_L H_L + \rho_g(1-H_L)]\, v_m v_{sg}\}/g_c p}$$

https://wiki.whitson.com/pipeflow/correlations/beggs_brill/



A UNIFIED MODEL FOR PREDICTING FLOW-PATTERN TRANSITIONS FOR THE WHOLE RANGE OF PIPE INCLINATIONS

D. Barnea

Faculty of Engineering, Department of Fluid Mechanics and Heat Transfer, Tel-Aviv University, Ramat-Aviv 69978, Israel

(Received 2 February 1986; in revised form 9 June 1986)

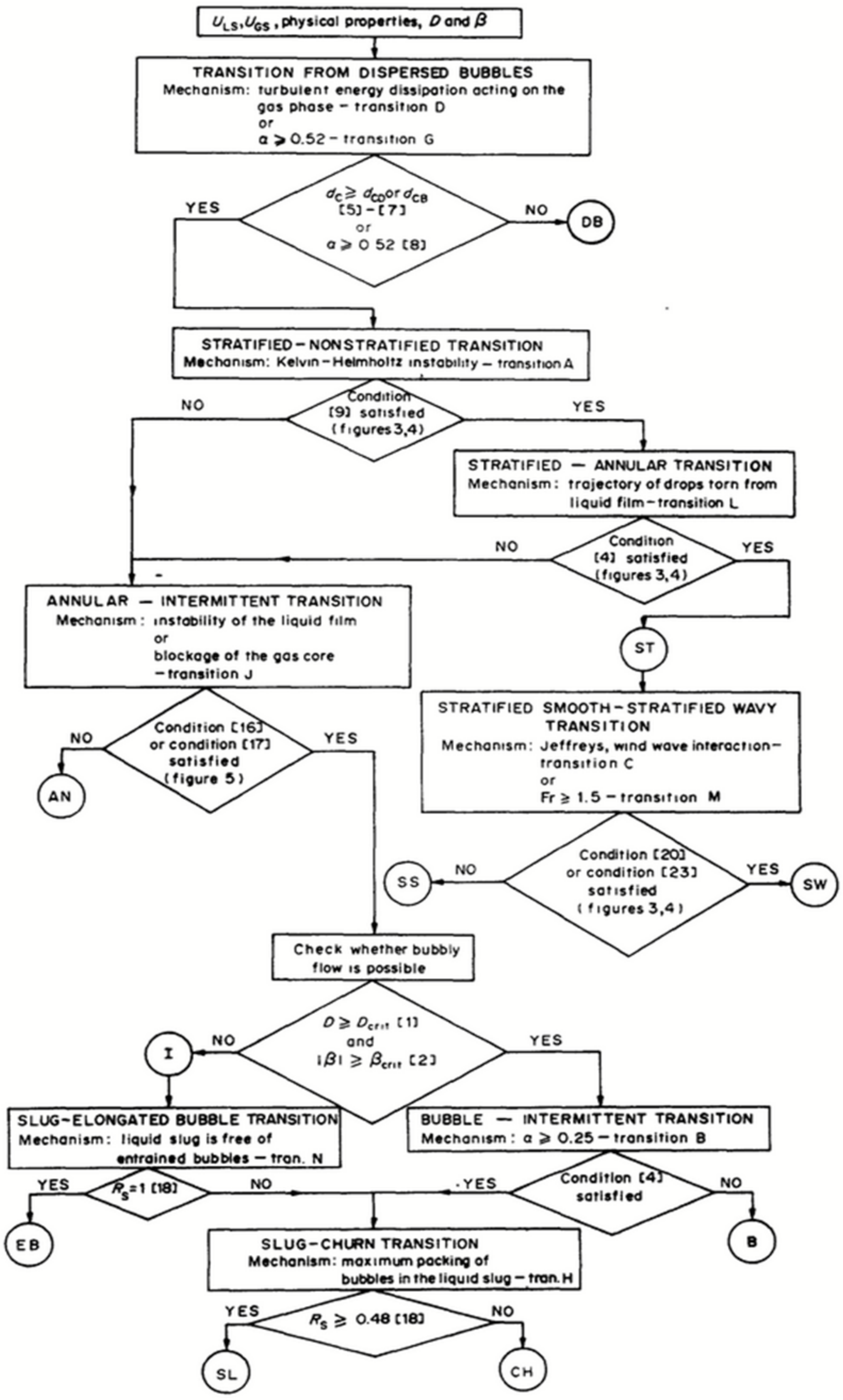
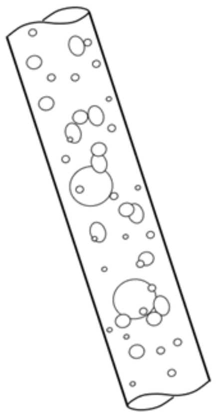


Figure 6. Logical pass for flow-pattern determination.

Bubble Flow-Pattern

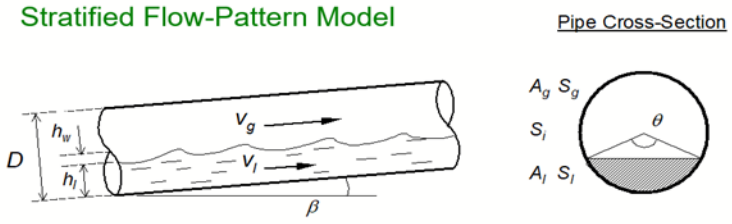


- Turbulent forces prevent bubble agglomeration and slip effect.
- Transition from bubble flow is given in the work of Barnea et al. (1987).
- The bubble flow-pattern is modeled as homogenous single fluid flow with averaged properties of liquid and gas.

Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = f_m \frac{2\rho_m v_m^2}{D} + \rho_m g \sin\beta$$

Stratified Flow-Pattern Model



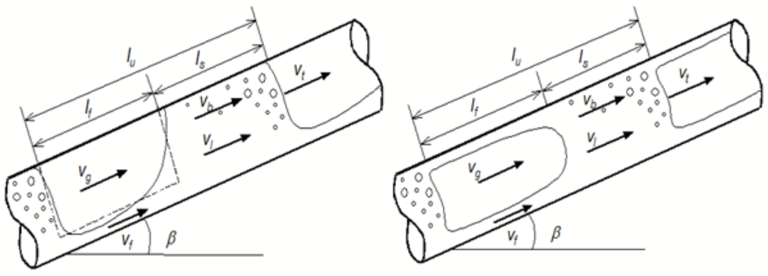
Combined momentum equation:

$$\frac{\tau_g S_g}{A_g} - \frac{\tau_l S_l}{A_l} + \tau_i S_i\left(\frac{1}{A_l}+\frac{1}{A_g}\right) - (\rho_l-\rho_g)\, g\sin\beta = 0$$

Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = \frac{\tau_l S_l + \tau_g S_g}{A} + \left(\frac{A_l}{A}\rho_l + \frac{A_g}{A}\rho_g\right) g\sin\beta$$

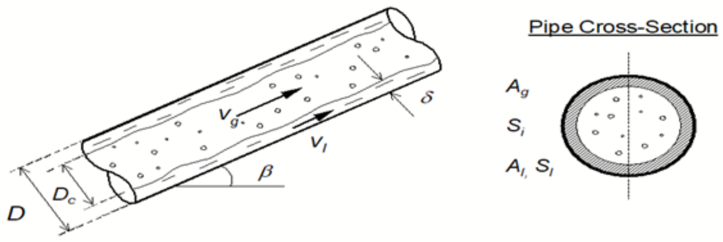
Intermittent Flow-Pattern Models



Pressure gradient equation:

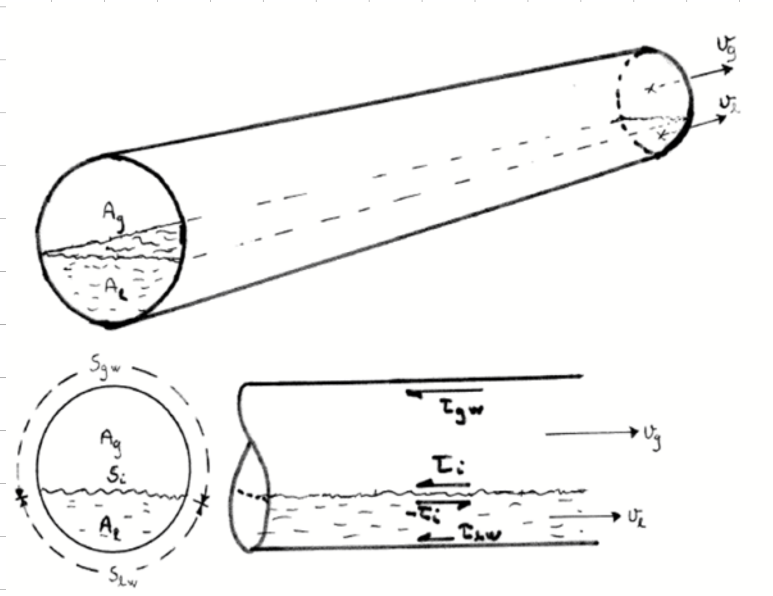
$$-\left(\frac{dP}{dx}\right) = \rho_u\, g\sin\beta + \frac{1}{l_u}\left[\left(\frac{\tau_s \pi D}{A} l_s\right) + \left(\frac{\tau_f S_f + \tau_g S_g}{A} l_f\right)\right]$$

Annular Flow-Pattern Model



Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = \frac{\tau_l S_l}{A} + \left(\frac{A_l}{A}\rho_l + \frac{A_g}{A}\rho_{gc}\right) g\sin\beta$$



Harald Asheim's drift flux model

$$A_g\, dp + A_g\, \rho_g\, g_x\, dx + A_g\, \rho_g\, v_g\, dv_g + \tau_{gw}\, S_{gw}\, dx + \tau_i\, S_i dx = 0$$

+

$$A_l\, dp + A_l\, \rho_l\, g_x\, dx + A_l\, \rho_l\, v_l\, dv_l + \tau_{lw}\, S_{lw}\, dx - \tau_i\, S_i dx = 0$$

$$dp + \left(\rho_g y_g + \rho_l y_l\right) g_x dx + \rho_g v_g y_g dv_g + \rho_l v_l y_l dv_l + \frac{\tau_g S_{gw} + \tau_{lw} S_{lw}}{A} dx = 0$$

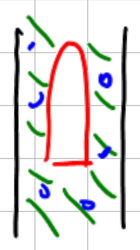
$$\tau_g = \frac{1}{8} f_g \rho_g v_g |v_g|$$

$$\tau_l = \frac{1}{8} f_l \rho_l v_l |v_l|$$

$$S_{gw} = \pi d y_g$$

$$S_{gw} = \pi d y_l$$

$$\frac{dp}{dx} + \rho_{TP} g_x + \rho_g v_{sg} \frac{dv_g}{dx} + \rho_l v_{sl} \frac{dv_l}{dx} + \frac{1}{2 \cdot d}\left(f_g \rho_g v_g |v_g| y_g + f_l \rho_l v_l |v_l| y_l\right) = 0$$



$$p^*, T^* \rightarrow \begin{array}{lll} o & g & w \\ \hline \rho & & \\ M & & \\ \sigma_{og} & \sigma_{wg} & \sigma_{ow} \\ h & & \\ \dot m_o & \dot m_g & \dot m_w \end{array} \rightarrow q_o \; q_g \; q_w \rightarrow u_{so} \; u_{sg} \; u_{sw} \rightarrow \frac{dp}{dx}, \frac{dT}{dx}$$

2 approaches:

- Compositional z_i — mole frac: C_1, C_2, C_3, ⋮, i, pseudo components + EOS (Peng Robinson) + EOS parameters
  - mixing rules
  - Binary interaction parameters

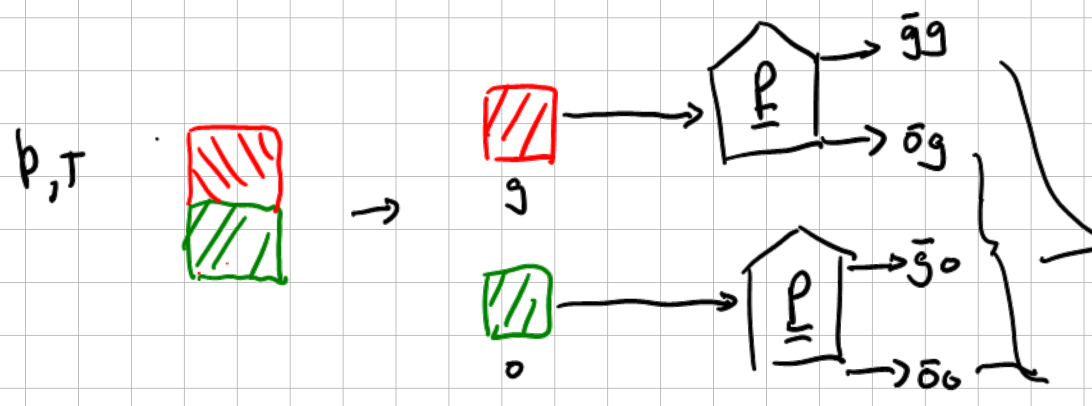
  → mass fraction o, g, w $\quad X_o = \dfrac{\dot m_o}{\dot m_o + \dot m_g} \quad X_g = (1 - X_o)$

  $\dot m_o = \dot m \, X_o$

  $\dot m_g = \dot m \, X_g$

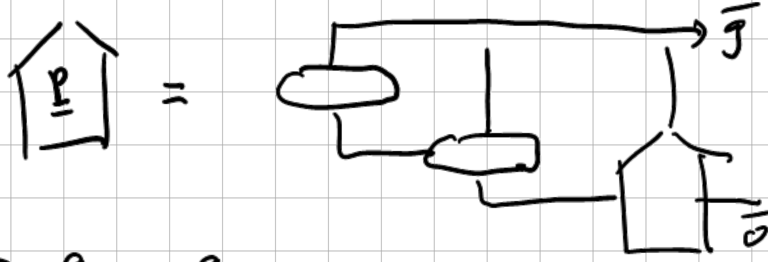
  run in two ways
  - live: flash calculation (equilibrium calculation) → property estimation
  - precompute tables and interpolate on table

- Black oil tables generated by:
  - EOS
  - correlations
  - lab experiments



$q_{\bar g} = q_{\bar g g} + q_{\bar g o}$

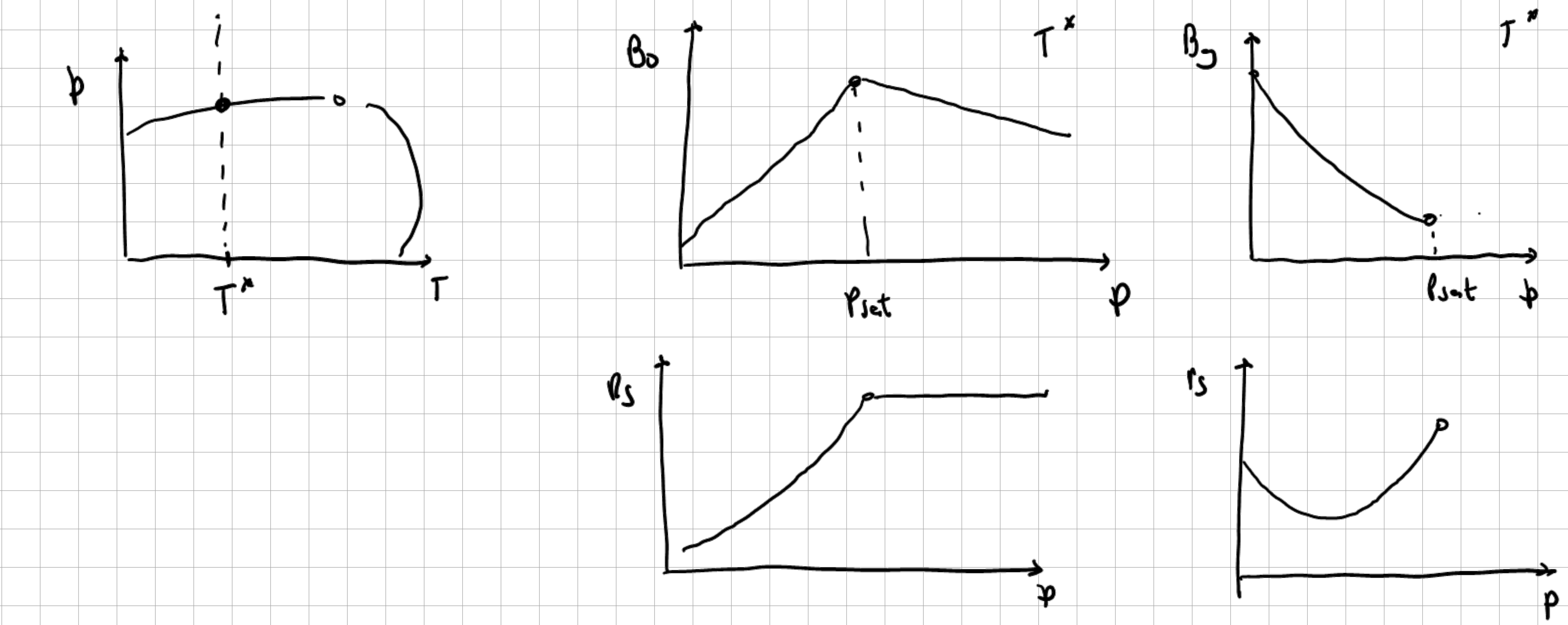
$q_{\bar o} = q_{\bar o o} + q_{\bar o g}$

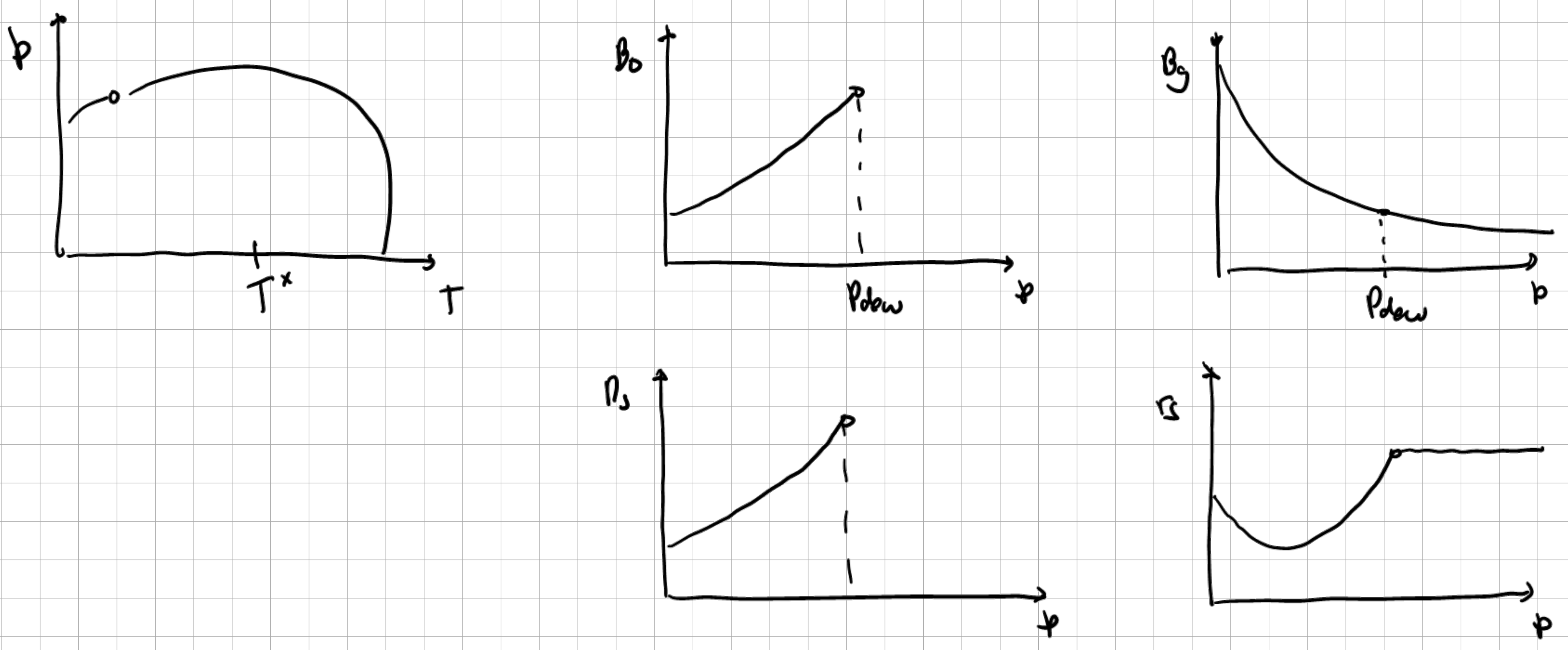


$B_o = \dfrac{V_o}{V_{\bar o o}} \qquad B_g = \dfrac{V_g}{V_{\bar g g}} \qquad R_s = \dfrac{V_{\bar g o}}{V_{\bar o o}} \qquad r_s\,(r_v) = \dfrac{V_{\bar o g}}{V_{\bar g g}}$

solution gas-oil ratio

solution oil-gas ratio





$\begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix} = \begin{bmatrix} \frac{1}{B_g} & \frac{R_s}{B_o} & 0 \\ \frac{r_s}{B_g} & \frac{1}{B_o} & 0 \\ 0 & 0 & \frac{1}{B_w} \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix}$	$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \frac{B_g}{1-R_s \cdot r_s} & \frac{-R_s \cdot B_g}{1-R_s \cdot r_s} & 0 \\ \frac{-B_o \cdot r_s}{1-R_s \cdot r_s} & \frac{B_o}{1-R_s \cdot r_s} & 0 \\ 0 & 0 & B_w \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix}$
Standard conditions calculated from local conditions	Local conditions calculated from standard conditions

$q_{\bar{g}} = q_{\bar{g}g} + q_{\bar{g}o}$

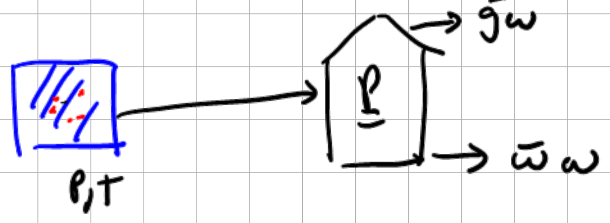
$q_{\bar{o}} = q_{\bar{o}o} + q_{\bar{o}g}$

for conventional / heavy oil $r_s = 0$

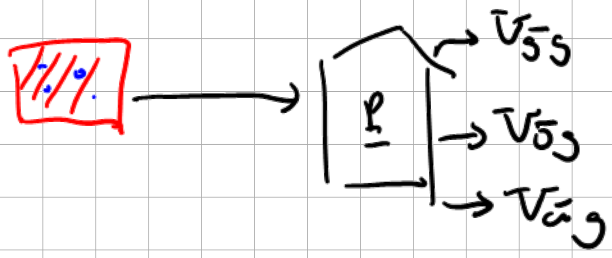
not adequate for volatile oils

gas condensate

water BO properties



$$B_w = \frac{V_w}{V_{\bar{w}w}}$$

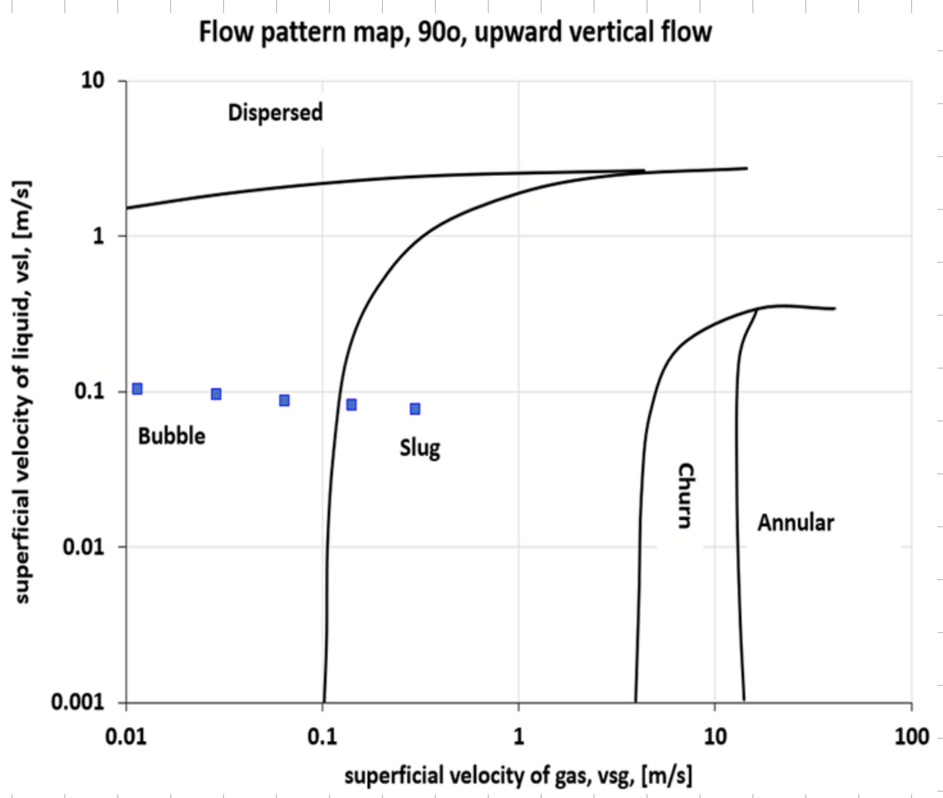


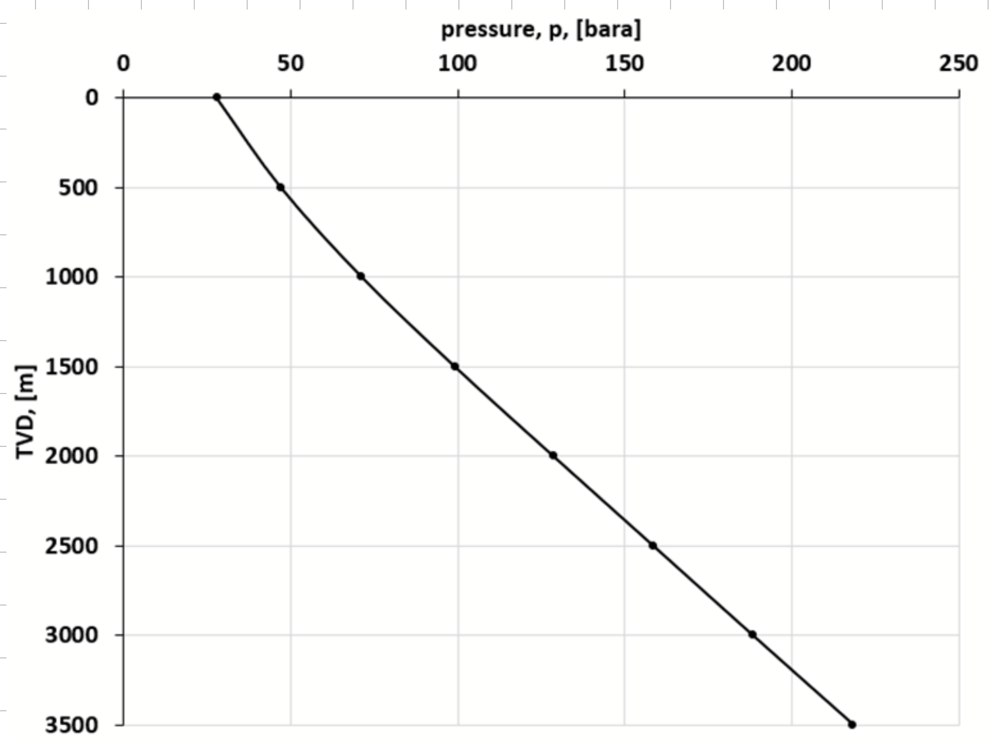
Some cases it could be important to include surface gas from local water and surface water from local gas.

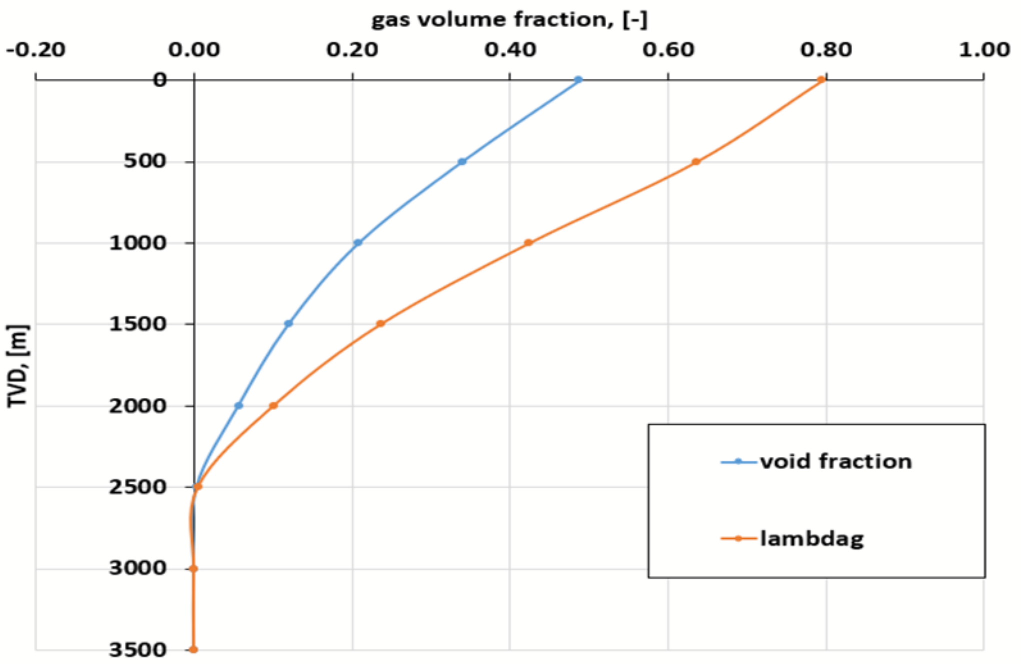
Exercise: Pressure drop calculation in saturated oil well, Prof. Milan Stanko (NTNU)

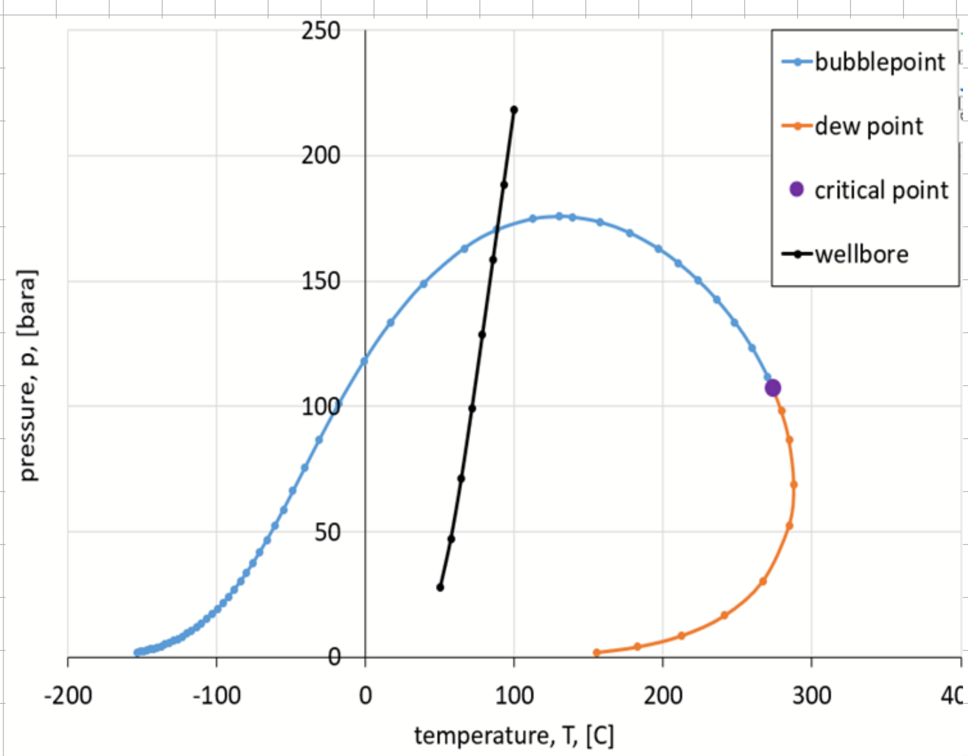
GOR	[Sm3/Sm3]	155.1
Pipe ID	[m]	0.15
Pipe cross section area	[m2]	0.0177
Pipe roughness	[m]	1.50E-05
Pipe inclination from hor	[deg]	90
qo	[Sm3/d]	100
qg	[Sm3/d]	1.55E+04

		BO table column	3	4	5	6	8	10	7	9	11						Woldesemayat and Ghajar	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	117.4	4.566E+02	0.077	0.299	0.80	0.49	0.0384
500	57.1	47.2	41.1	1.31E-05	1.2	1.90E-02	70.8	1.25E-02	708.8	1.2	8.37E-03	124.2	2.173E+02	0.081	0.142	0.64	0.34	0.0483
1000	64.3	71.4	65.3	1.43E-05	1.3	1.09E-02	119.4	1.49E-02	684.3	0.8	5.12E-03	133.2	9.832E+01	0.087	0.064	0.42	0.21	0.0556
1500	71.4	99.2	93.9	1.69E-05	1.4	7.29E-03	178.7	1.91E-02	657.3	0.6	2.64E-03	144.4	4.468E+01	0.095	0.029	0.24	0.12	0.0589
2000	78.6	128.6	124.4	2.13E-05	1.6	5.71E-03	228.2	2.38E-02	630.8	0.5	1.33E-03	156.8	1.761E+01	0.103	0.012	0.10	0.06	0.0597
2500	85.7	158.4	153.2	2.41E-05	1.7	4.49E-03	229.5	2.44E-02	607.9	0.4	6.71E-04	169.0	8.799E-01	0.111	0.001	0.01	0.00	0.0595
3000	92.9	188.2	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.3	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602
3500	100.0	218.3	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.4	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602



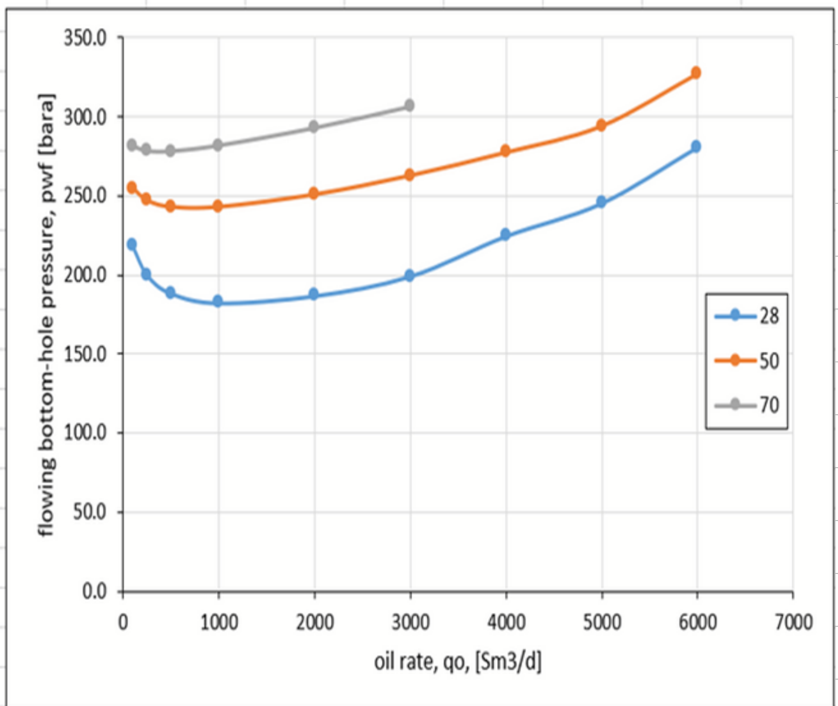






With Wolgha model

pwh[bara]	28	50	70
qo	pwf	pwf	pwf
[Sm3/d]	[bara]	[bara]	[bara]
100	218.3	254.9	281.4
250	200.0	247.1	278.3
500	188.5	243.0	278.2
1000	182.4	243.0	281.8
2000	186.9	250.8	292.9
3000	199.2	262.8	306.3
4000	224.5	277.3	
5000	245.2	294.0	
6000	280.5	327.0	



lculation in saturated oil well, Prof. Milan Stanko (NTNU)

[Sm3/Sm3]	155.1
[m]	0.15
[m2]	0.0177
[m]	1.50E-05
[deg]	90
[Sm3/d]	100
[Sm3/d]	1.55E+04

		BO table column	3	4	5	6	8	10	7	9	11						Woldesemayat and Ghajar	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	117.4	4.566E+02	0.077	0.299	0.80	0.49	0.0384
500	57.1	47.2	41.1	1.31E-05	1.2	1.90E-02	70.8	1.25E-02	708.8	1.2	8.37E-03	124.2	2.173E+02	0.081	0.142	0.64	0.34	0.0483
1000	64.3	71.4	65.3	1.43E-05	1.3	1.09E-02	119.4	1.49E-02	684.3	0.8	5.12E-03	133.2	9.832E+01	0.087	0.064	0.42	0.21	0.0556
1500	71.4	99.2	93.9	1.69E-05	1.4	7.29E-03	178.7	1.91E-02	657.3	0.6	2.64E-03	144.4	4.468E+01	0.095	0.029	0.24	0.12	0.0589
2000	78.6	128.6	124.4	2.13E-05	1.6	5.71E-03	228.2	2.38E-02	630.8	0.5	1.33E-03	156.8	1.761E+01	0.103	0.012	0.10	0.06	0.0597
2500	85.7	158.4	153.2	2.41E-05	1.7	4.49E-03	229.5	2.44E-02	607.9	0.4	6.71E-04	169.0	8.799E-01	0.111	0.001	0.01	0.00	0.0595
3000	92.9	188.2	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.3	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602
3500	100.0	218.3	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.4	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602

```
Function e_wolgha(usl, usg, denl, deng, sigma_lg, teta_deg, p, D)
    'p in bar
    'D in m
    'usl in m/s
    'usg in m/s
    'denl kg/m^3
    'deng kg/m^3
    'teta deg in deg
    'sigma_lg in N/m
    If usg = 0 Then
        e_wolgha = 0
    Else
        Pi = Atn(1) * 4
        teta = teta_deg * Pi / 180
        'void fraction correlation by Woldesemayat and Ghajar (2006)
        a = usg * (1 + ((usl / usg) ^ ((deng / denl) ^ 0.1)))
        B = 2.9 * ((9.81 * sigma_lg * D * (1 + Cos(teta)) * (denl - deng) / (denl ^ 2)) ^ 0.25)
        C = (1.22 + 1.22 * Sin(teta)) ^ (1.01325 / p)
        e_wolgha = usg / (a + (B * C))
    End If
End Function
```

```
Function dpdx_mpf(roughness, viscl, viscg, denl, deng, usl, usg, D, angle, voidfraction)
    'dpdx_mpf pressure gradient, in bar/m, for multiphase flow
    'denl, liquid density, [kg/m3]
    'deng, gas density, [kg/m3]
    'usl superficial liquid velocity, [m/s]
    'usg superficial gas velocity, [m/s]
    'angle, inclination angle of pipe with respect to horizontal [deg]
    'D hydraulic diameter of pipe [m]
    'roughness pipe roughness, [m]
    'viscl, liquid viscosity [cP]
    'viscg, gas viscosity, [cP]
    'voidfraction [-]
    Pi = Atn(1) * 4
    denm = voidfraction * deng + (1 - voidfraction) * denl
    If voidfraction = 0 Or usg = 0 Then
        ug = 0
        ul = usl
        fg = 0
        fl = ffactor(denl, viscl, D, roughness, ul)
    ElseIf voidfraction = 1 Or usl = 0 Then
        ug = usg
        ul = 0
        fl = 0
        fg = ffactor(deng, viscg, D, roughness, ug)
    Else
        ug = usg / voidfraction
        ul = usl / (1 - voidfraction)
        fg = ffactor(deng, viscg / 1000#, D, roughness, ug)
        fl = ffactor(denl, viscl / 1000, D, roughness, ul)
    End If
    dpdx_f = (fg * deng * (ug * Abs(usg)) * 0.5 / D) + (fl * denl * (ul * Abs(usl)) * 0.5 / D)
    dpdx_h = denm * 9.81 * Sin(angle * Pi / 180)
    dpdx_mpf = dpdx_f + dpdx_h
    dpdx_mpf = dpdx_mpf / 100000#
End Function
```

lculation in saturated oil well, Prof. Milan Stanko (NTNU)

[Sm3/Sm3]	155.1
[m]	0.15
[m2]	0.0177
[m]	1.50E-05
[deg]	90
[Sm3/d]	1000
[Sm3/d]	1.55E+05

		BO table column	3	4	5	6	8	10	7	9	11							Nagoo
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	1174.3	4.566E+03	0.769	2.991	0.80	0.61	0.0313
500	57.1	43.7	37.3	1.29E-05	1.2	2.08E-02	63.4	1.22E-02	711.9	1.2	8.90E-03	1229.4	2.451E+03	0.805	1.605	0.67	0.50	0.0387
1000	64.3	63.0	56.3	1.36E-05	1.3	1.34E-02	100.1	1.40E-02	691.6	0.9	6.20E-03	1301.4	1.320E+03	0.852	0.865	0.50	0.38	0.0461
1500	71.4	86.1	79.2	1.53E-05	1.4	8.91E-03	146.3	1.68E-02	668.7	0.6	3.75E-03	1390.6	6.776E+02	0.911	0.444	0.33	0.26	0.0525
2000	78.6	112.3	105.7	1.84E-05	1.5	6.67E-03	195.8	2.08E-02	644.3	0.5	2.05E-03	1497.1	3.306E+02	0.981	0.217	0.18	0.16	0.0567
2500	85.7	140.7	134.2	2.32E-05	1.6	5.56E-03	234.6	2.48E-02	620.2	0.4	1.12E-03	1615.5	1.167E+02	1.058	0.076	0.07	0.06	0.0588
3000	92.9	170.1	154.6	1.91E-06	1.7	3.87E-04	17.9	1.91E-03	604.6	0.4	6.30E-05	1701.8	2.006E-01	1.115	0.000	0.00	0.00	0.0597
3500	100.0	200.0	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	605.0	0.4	0.00E+00	1702.0	0.000E+00	1.115	0.000	0.00	0.00	0.0649

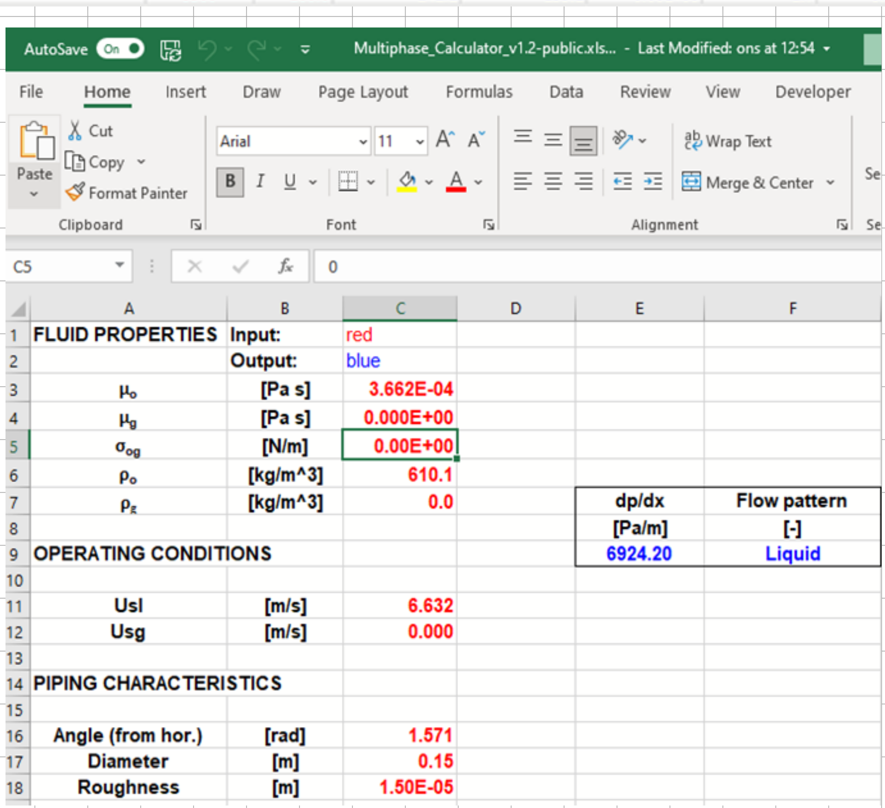
```
Function e_Nagoo(lambdag)
    ' e_Nagoo, the void fraction of gas, in fraction, using the ANSLIP equation by Nagoo, 2013
    'lambdag is non slip volume fraction of gas, in fraction
    If lambdag = 0 Then
        e_Nagoo = 0
    Else
        e_Nagoo = (lambdag + 1 - ((lambdag + 1) ^ 2 - 4 * (lambdag ^ 2)) ^ 0.5) / (2 * lambdag)
    End If
End Function
```


```
Function dpdx_mpf(roughness, viscl, viscg, denl, deng, usl, usg, D, angle, voidfraction)
    'dpdx_mpf pressure gradient, in bar/m, for multiphase flow
    'denl, liquid density, [kg/m3]
    'deng, gas density, [kg/m3]
    'usl superficial liquid velocity, [m/s]
    'usg superficial gas velocity, [m/s]
    'angle, inclination angle of pipe with respect to horizontal [deg]
    'D hydraulic diameter of pipe [m]
    'roughness pipe roughness, [m]
    'viscl, liquid viscosity [cP]
    'viscg, gas viscosity, [cP]
    'voidfraction [-]
    Pi = Atn(1) * 4
    denm = voidfraction * deng + (1 - voidfraction) * denl
    If voidfraction = 0 Or usg = 0 Then
        ug = 0
        ul = usl
        fg = 0
        fl = ffactor(denl, viscl, D, roughness, ul)
    ElseIf voidfraction = 1 Or usl = 0 Then
        ug = usg
        ul = 0
        fl = 0
        fg = ffactor(deng, viscg, D, roughness, ug)
    Else
        ug = usg / voidfraction
        ul = usl / (1 - voidfraction)
        fg = ffactor(deng, viscg / 1000#, D, roughness, ug)
        fl = ffactor(denl, viscl / 1000, D, roughness, ul)
    End If
    dpdx_f = (fg * deng * (ug * Abs(usg)) * 0.5 / D) + (fl * denl * (ul * Abs(usl)) * 0.5 / D)
    dpdx_h = denm * 9.81 * Sin(angle * Pi / 180)
    dpdx_mpf = dpdx_f + dpdx_h
    dpdx_mpf = dpdx_mpf / 100000#
End Function
```

Exercise: Pressure drop calculation in saturated oil well, Prof. Milan Stanko (NTNU)

GOR	[Sm3/Sm3]	155.1
Pipe ID	[m]	0.15
Pipe cross section area	[m2]	0.0177
Pipe roughness	[m]	1.50E-05
Pipe inclination from hor	[deg]	90
qo	[Sm3/d]	6000
qg	[Sm3/d]	9.31E+05

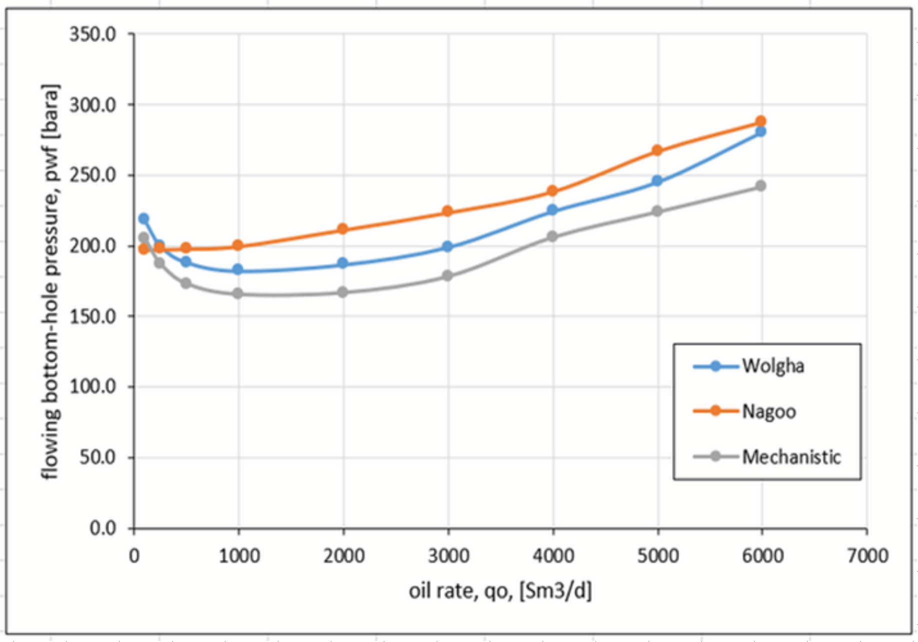
		BO table column	3	4	5	6	8	10	7	9	11						Mechanistic model	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	flowpattern	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	7046.0	2.740E+04	4.615	17.944	0.80	Slug	0.0492
500	57.1	52.6	46.8	1.34E-05	1.3	1.64E-02	82.1	1.29E-02	704.1	1.1	7.56E-03	7564.5	1.065E+04	4.954	6.976	0.58	Slug	0.0417
1000	64.3	73.5	67.6	1.45E-05	1.3	1.05E-02	125.0	1.53E-02	682.4	0.8	4.88E-03	8044.2	5.540E+03	5.269	3.628	0.41	Bubble	0.0605
1500	71.4	103.7	99.2	1.76E-05	1.5	6.90E-03	189.5	2.00E-02	653.3	0.6	2.35E-03	8781.2	2.322E+03	5.751	1.521	0.21	Bubble	0.0675
2000	78.6	137.5	134.8	2.33E-05	1.6	5.37E-03	243.1	2.55E-02	623.6	0.4	1.07E-03	9650.4	6.558E+02	6.321	0.430	0.06	Bubble	0.0708
2500	85.7	172.9	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	609.0	0.4	0.00E+00	10144.6	0.000E+00	6.644	0.000	0.00	Liquid	0.0693
3000	92.9	207.5	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	610.1	0.4	0.00E+00	10127.0	0.000E+00	6.633	0.000	0.00	Liquid	0.0693
3500	100.0	242.1	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	610.1	0.4	0.00E+00	10125.7	0.000E+00	6.632	0.000	0.00	Liquid	0.0692



	A	B	C	D	E	F
1	FLUID PROPERTIES	Input:	red			
2		Output:	blue			
3	μ_o	[Pa s]	3.662E-04			
4	μ_g	[Pa s]	0.000E+00			
5	σ_{og}	[N/m]	0.00E+00			
6	ρ_o	[kg/m^3]	610.1			
7	ρ_g	[kg/m^3]	0.0		dp/dx	Flow pattern
8					[Pa/m]	[-]
9	OPERATING CONDITIONS				6924.20	Liquid
10						
11	Usl	[m/s]	6.632			
12	Usg	[m/s]	0.000			
13						
14	PIPING CHARACTERISTICS					
15						
16	Angle (from hor.)	[rad]	1.571			
17	Diameter	[m]	0.15			
18	Roughness	[m]	1.50E-05			

pwh=28 bara

Models	Wolgha	Nagoo	Mechanistic
qo	pwf	pwf	pwf
[Sm3/d]	[bara]	[bara]	[bara]
100	218.3	197.5	205.5
250	200.0	197.6	187.5
500	188.5	198.2	173.2
1000	182.4	200.0	165.6
2000	186.9	211.5	166.8
3000	199.2	223.7	178.4
4000	224.5	238.5	206.2
5000	245.2	266.9	224.2
6000	280.5	287.5	242.1



- g
- l: oil, water →

$\dfrac{q_w}{q_o+q_w}$

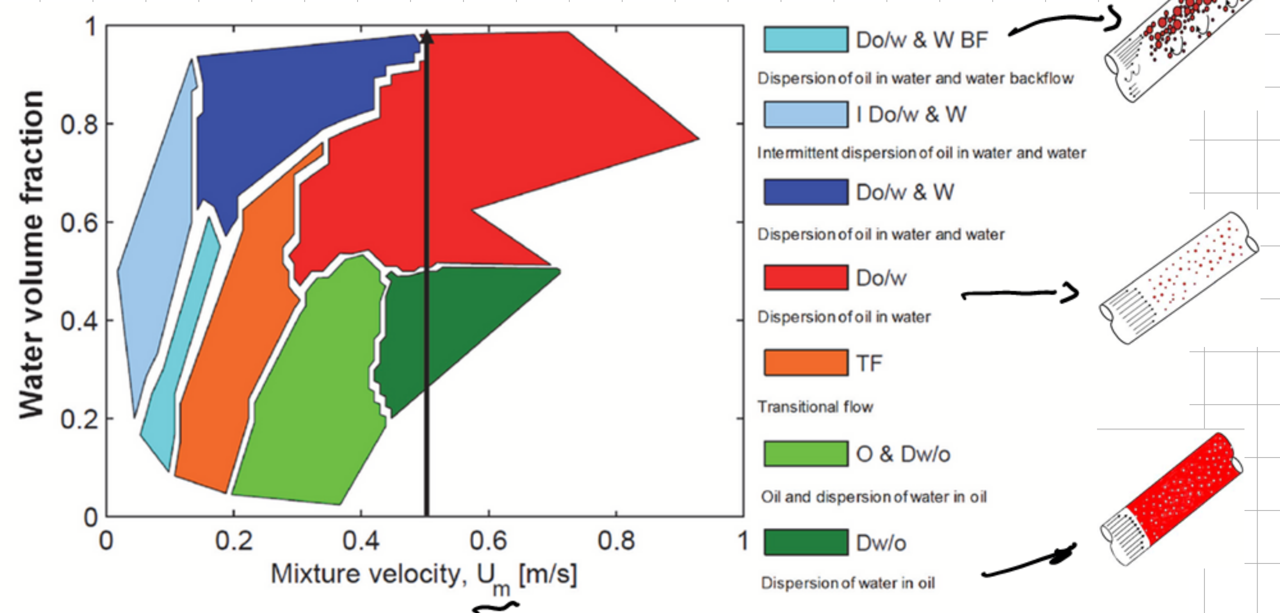


FIGURE 7-24. OIL-WATER FLOW PATTERN MAP OF WATER VOLUME FRACTION VERSUS MIXTURE VELOCITY FOR AN UPWARD PIPE INCLINATION OF 45°. FIGURE ADAPTED FROM RIVERA[7-5] [7-1].

$u_{so} + u_{sw}$

g	u_{sg}	λ_g	H_g	V_g
o	u_{so}	λ_o	H_o	V_o
w	u_{sw}	λ_w	H_w	V_w

$V_o \approx V_w$ — little slip

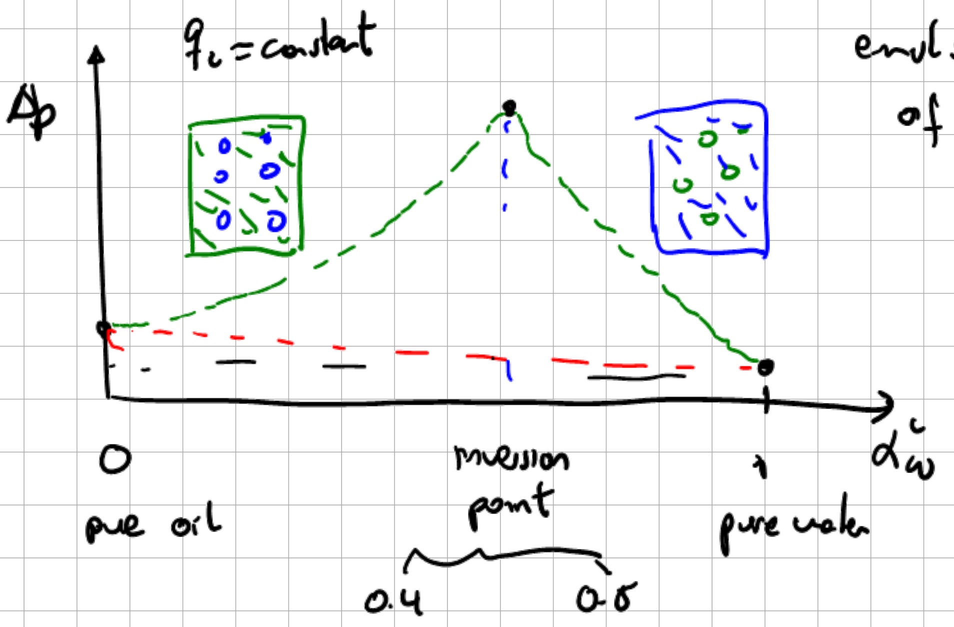
assume both travel at the same speed V_L

$$\rho_L = \rho_o \alpha_o^L + \rho_w \alpha_w^L$$

$$\mu_L = \mu_o \alpha_o^L + \mu_w \cdot \alpha_w^L$$

$$\alpha_o^L = \frac{q_o}{q_o+q_w} \qquad \alpha_w^L = \frac{q_w}{q_o+q_w} = 1-\alpha_o^L$$

$\underset{\sim}{q_L}$ / q_o+q_w →



emulsion behavior of oil + water mixture

$$\mu_L = \mu_o \cdot e^{c_1(\alpha_w^L)} \qquad \mu_L = \mu_w \cdot e^{c_2(1-\alpha_w^L)}$$

$$wc = \frac{q_{\bar{w}}}{q_{\bar{o}}+q_{\bar{w}}} \qquad \alpha_w^L = \frac{q_w}{q_o+q_w}$$

sometimes in the literature, if $q_{\bar{o}} = q_o$

$wc = \alpha_w^L$

Pressure integration method

p_3
p_2
p_1 $\Big\}\Delta L$

$$p_2 = p_1 + \left.\frac{dp}{dx}\right|_{@p_1} \cdot \Delta L$$ } not so accurate for large ΔL

$$p_2 = p_1 + \left.\frac{dp}{dx}\right|_{@p_{av}} \Delta L$$

$$p_{av} = \frac{p_1 + p_2}{2}$$

implicit calculation

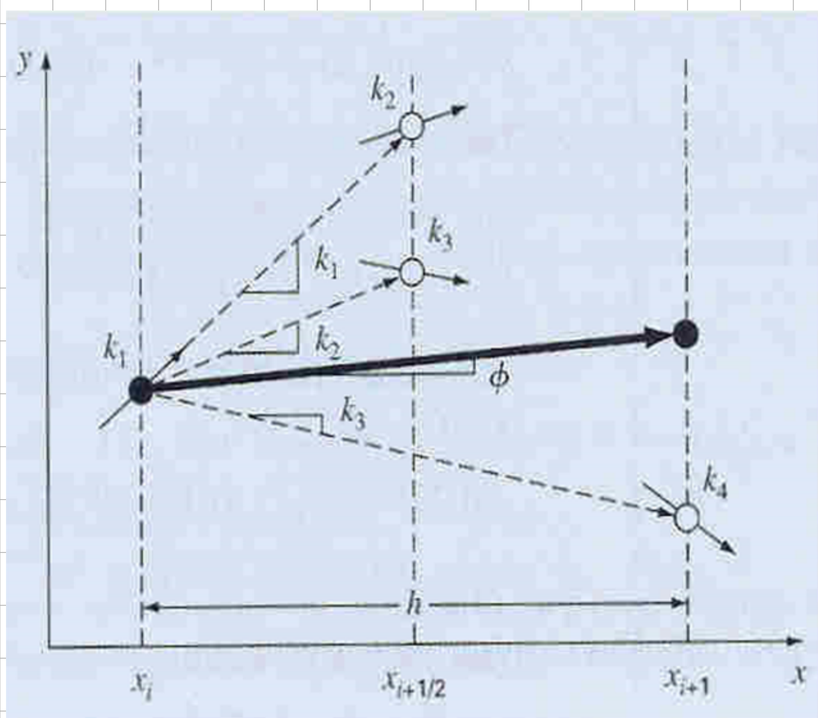
- assume a value of p_2^*
- compute $\left.\frac{dp}{dx}\right|_{p_{av}}$
- compute $p_2^{calc} = p_1 + \left.\frac{dp}{dx}\right|_{@p_{av}} \Delta L$
- check $p_2^{calc} = p_2^*$

not — yes

proceed to next step

explicit approach (higher order)

Runge-Kutta 4th



p_1 p^* p_2

$$y_{n+1} = y_n + \frac{1}{6} \cdot h \cdot (k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4)$$

$$p_2 = p_1 + \frac{1}{6} \Delta L (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = \left.\frac{dp}{dx}\right|_{@p_1}$$

$$p^* = p_1 + \left.\frac{dp}{dx}\right|_{@p_1} \cdot \frac{\Delta L}{2}$$

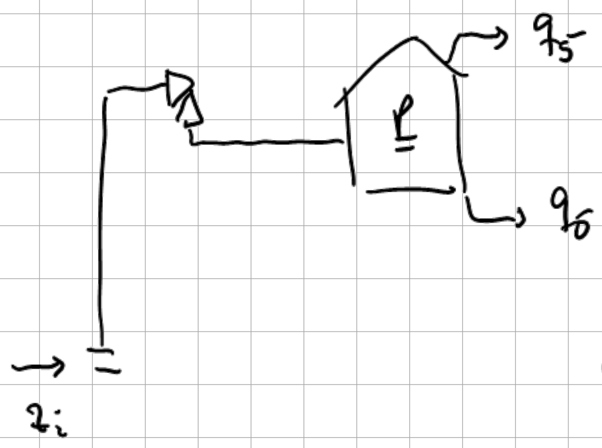
$$k_2 = \left.\frac{dp}{dx}\right|_{@p^*}$$

$$p^{**} = p_1 + k_2 \frac{\Delta L}{2}$$

$$k_3 = \left.\frac{dp}{dx}\right|_{@p^{**}}$$

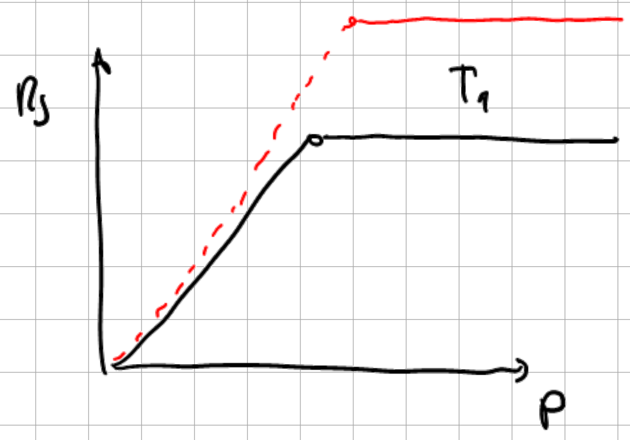
$$p^{***} = p_1 + k_3 \Delta L$$

$$k_4 = \left.\frac{dp}{dx}\right|_{@p^{***}}$$



$$R_p = \frac{q_g}{q_o}$$

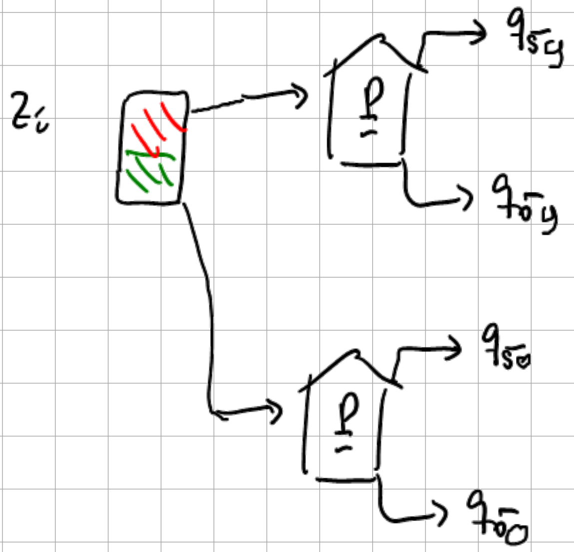
If z_i changes then R_p should also change

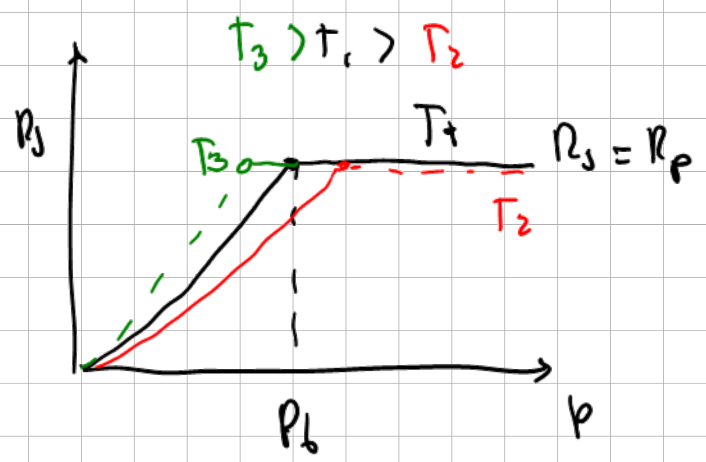


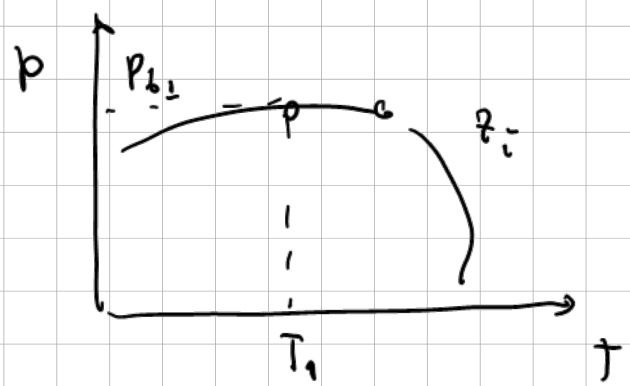
If z_i changes

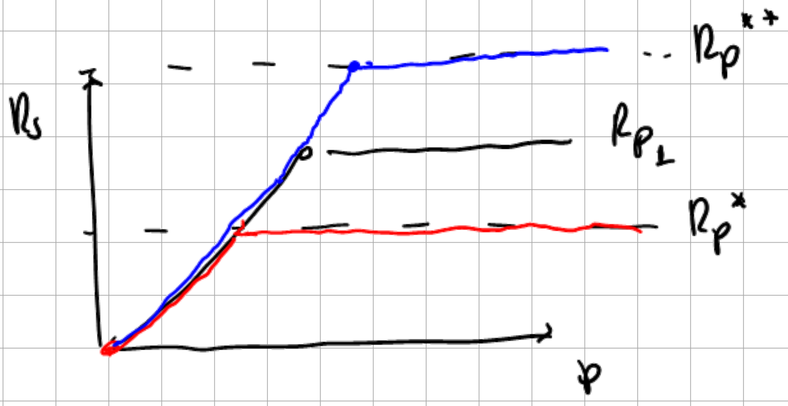
z_i^*

If z_i changes (or R_p) it is usually necessary to generate a new black oil table









20241021

OUTLINE

-Recap of last week video lectures

-Quick overview of exam questions with Multiphase flow

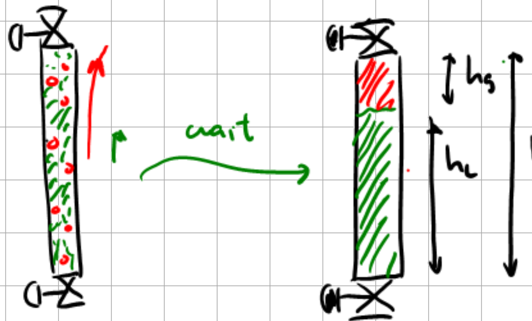
-Meeting with Reference group

How do we measure void fraction ε $\left(\frac{A_g}{A}\right.$ liquid holdup H_L $\frac{A_L}{A}$?

$\sim \left.\frac{dP}{dx}\right|_{hydrostatic}$

① quick closing valves

not suited for field test

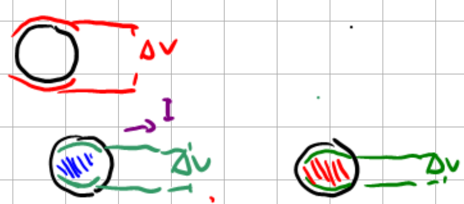


$$\varepsilon = \frac{h_g}{h} \neq \frac{q_g}{q_L + q_g}$$

$$H_L = \frac{h_L}{h} \neq \frac{q_L}{q_L + q_g}$$

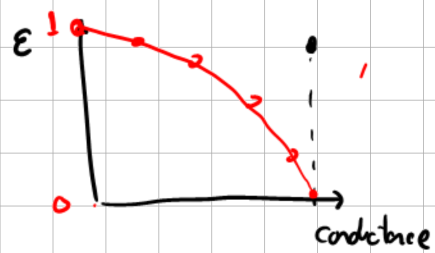
② Impedance probes ~ capacitance

conductance



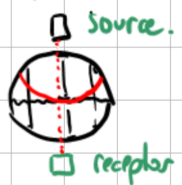
for dielectric fluids

for conductive fluids



this calibration curve is often flow pattern-dependent.

③ Gamma densitometer (single beam)

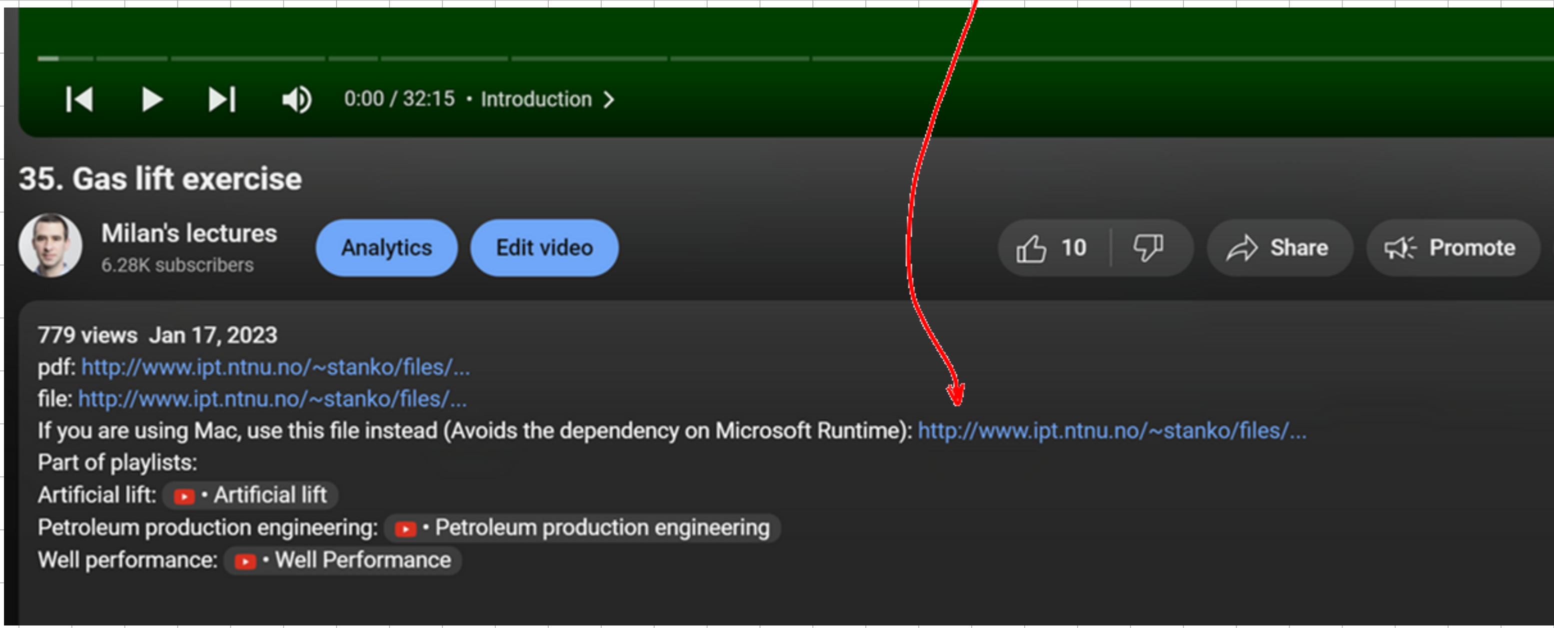


Considered to be more robust

20241025
OUTLINE
-Q&A
-Exercises related to Multiphase flow in tubing

In YT video 35 (gas lift exercise), some students report that the mpf_p function does not work. This function uses dictionary, and requires to enable the library Microsoft runtime. MAC users do not have this library, unfortunately. Milan will upload a alternative version of the Excel file that is using self-programmed dictionary function and does not require Microsoft runtime.

Milan already did last year!! check all the information in the YT video

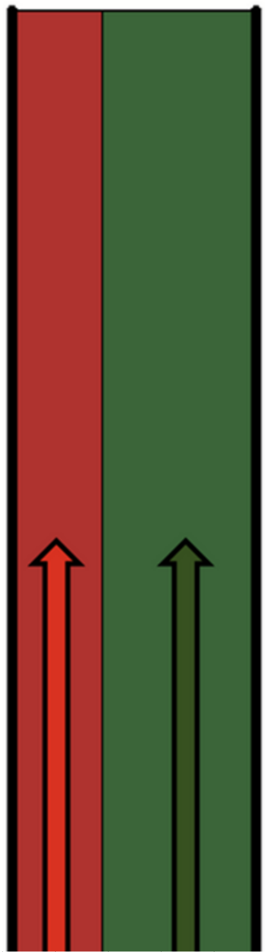


From last year's resit:

PROBLEM 6 (20 POINTS).

Consider a vertical tubing, 2000 m long and with an internal diameter of 0.1 m that has gas and oil circulating through it. Assume both are incompressible and that there is no mass transfer between them.
The mass flow of the oil is 8 kg/s while the mass flow of gas is 4 kg/s. The density of the oil is 800 kg/m³, while the density of the gas is 100 kg/m³.
Calculate:

- Total amount of liquid (in kg) that is in the tubing
- How long does it take a particle of oil to travel from the bottom of the well to the top
- How long does it take a particle of gas to travel from the bottom of the well to the top.



Additional information:

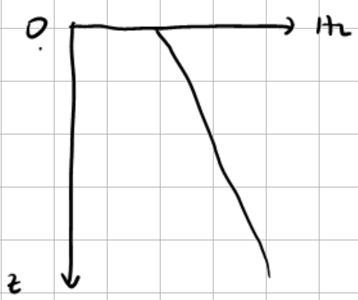
- The liquid holdup (H_L) can be estimated with the Chisholm correlation:

$$H_L = \frac{1}{\dfrac{\rho_L \cdot x}{\left(\dfrac{\rho_L}{\rho_g}\right)^{0.25} \cdot \rho_g \cdot (1-x)} + 1}$$

Where x is the mass fraction (mg/(ml+mg))

Asking how much liquid is in the tubing is equivalent to asking: "if there are two quick closing valves, one at bottom-hole and one at wellhead, if I activate them, how much liquid will be trapped in the tubing?" This is essentially the holdup*tubing volume. Since both phases are considered incompressible, and have the same velocity through the tubing, then the holdup will be constant along the tubing. If that were not the case, one needs to integrate segment-wise.

potential exam question?



$$V_L = \int_0^{TVD} H_L \cdot A \cdot dz \qquad dV_L = H_L \cdot A \cdot dz$$

$\sim H_L \cdot A$

To calculate the time it takes for a particle to travel from bottom to top, you need to use the length and the velocity of the phase. The velocity of the phase can be calculated from the holdup, the tubing cross section area and the volumetric rate of the phase.

Use the provided equation to calculate HL

$$H_L = \frac{1}{\dfrac{\rho_L \cdot x}{\left(\dfrac{\rho_L}{\rho_g}\right)^{0.25} \cdot \rho_g \cdot (1-x)} + 1}$$

ρ_L

ρ_g

$x = \frac{m_g}{m_L + m_L} = \frac{4}{12}$

$H_L = 0.3$

$V_L = H_L \cdot V_T = H_L \cdot A \cdot L$

$m_L = \rho_L \cdot V_L = 800 \cdot V_L = 3.7$ t oil

$t_o = \frac{L}{V_o}$ 1000m

$t_g = \frac{L}{V_g}$

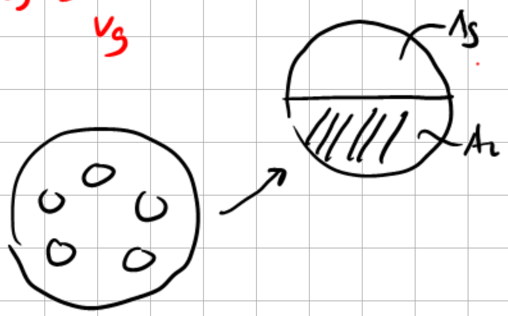
$\dot{m}_L = 8$ kg/s

$\dot{m}_g = 4$ kg/s

$q_L = \frac{\dot{m}_L}{\rho_L} = \frac{[kg/s]}{[kg/m^3]} = m^3/s$

$q_L = \frac{8}{800} = 1E-2 \ m^3/s$

$q_g = \frac{4}{100} = 4E-2 \ m^3/s$



$v_L = \frac{q_L}{A_L} = \frac{q_L}{H_L \cdot A} = \frac{1E-2}{0.3 \cdot \pi \ 0.1^2 \cdot 0.25} = 4.2 \ m/s$

$v_g = 7.2 \ m/s$

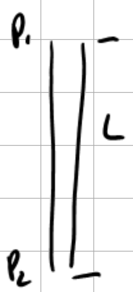
$t_o = \frac{1000}{4.2} =$

$t_g = \frac{1000}{7.2}$

PROBLEM 12 (10 POINTS).

Consider an oil and gas mixture flowing upwards in a vertical well. The local rates of oil and gas are q_o = 0.07 m^3/s and q_g = 0.3 m^3/s. The inner diameter of the tubing is 0.15 m. The density of the oil and the gas are 700 kg/m^3 and 200 kg/m^3 respectively.

Task 1. Calculate the non-slip gas volume fraction.
Task 2. Calculate the hydrostatic pressure gradient (dp/dx in bara/m) using the value of the density of the mixture. To calculate the density of the mixture use the non-slip gas volume fraction calculated in Task 1.
Task 3. Assume that the real velocity of the gas is twice the real velocity of the liquid. Calculate the gas holdup (gas volume fraction of the mixture considering slip).
Task 4. For the condition presented in Task 3, will the hydrostatic pressure gradient of the mixture be higher than the value calculated in task 2 or lower? Explain your answer



ΔP: $P_2 - P_1 = \rho \cdot g \cdot L$

$$\lambda_g = \frac{q_g}{q_T} = \frac{0.3}{0.37} = \underline{0.81}$$

$(q_g + q_L)$

Potential exam question: if there was slip between liquid and gas, will the void fraction be higher than the non-slip gas fraction or lower?

$\varepsilon < \lambda_g$

$$\left.\frac{dP}{dx}\right|_{\underline{Hyd}} = g\,\rho$$

2. $\rho_{mix} = \rho_L \cdot H_L + (1 - H_L)\rho_g$

$\rho_L(1-\lambda_g) + \lambda_g \rho_g = 294\ ?$

Task 2

Density of the mixture rho_m = rho_l * lamda_l + rho_g* lamgda_g = 700*0.19+200*0.81 = 294.6 kg/m3

Hydrostatic pressure gradient = rho_m*g = 294.6 * 9.81/1e5 = 0.0289 bar/m

task 3 $\boxed{v_g = 2 v_L} \rightarrow H_L$

$H_L = \frac{A_g}{A}$

$$\boxed{v_L = \frac{q_L}{(A - A_g)} = \frac{q_L}{(1-\varepsilon)A}} \qquad \boxed{v_g = \frac{q_g}{A_g} = \frac{q_g}{A \cdot \varepsilon}}$$

$$\frac{q_g}{\cancel{A}\cdot\varepsilon} = \frac{2 q_L}{(1-\varepsilon)\cancel{A}}$$

$\varepsilon = 0.68$

task 4 $v_g = 2 v_L$

$\varepsilon \downarrow < 0.81$

$\rho_m > 294\ ?$

$\rho_m < 294\ ?$

$\rho_m = \rho_g \cdot \varepsilon + \rho_L(1-\varepsilon)$ (0.81, 0.68)

$\rho_m = 359\ kg/m^3$

PROBLEM 3.

A pressure survey has been performed in a producing oil well (production rate of 1394 Sm3/d, GOR = 155.1 and water cut of 30%), and pressure and temperature at several depths have been recorded. The values are provided in the Excel file attached. Assume that the water density is constant and equal to 1000 kg/m^3, the water viscosity is constant and equal to 0.6 cP, and the liquid-gas interfacial tension is constant and equal to 0.01 N/m. Assume there is no slip between the oil and water.

Task 1. Calculate the following parameters along the well:

- Non-slip gas volume fraction
- Gas void fraction
- Liquid and gas real velocities
- The gas-liquid slip ratio
- The hydrostatic pressure gradient
- The gas-liquid flow pattern (for this use the file Problem_3_Multiphase_Calculator_v1.3-public.xlsm)

For this task, assume the viscosity of the oil and water mixture can be calculated as
$\mu_m = WC \cdot \mu_w + (1 - WC) \cdot \mu_o$
With WC in fraction

p-T along the well--> immediately associate with the exercise solved on YT video

-No need to integrate p/calculate dp/dx, since p is given.

-Water is given!!, but we know how to deal with water (equivalent liquid with equivalent density and viscosity)

-To calculate lambda_g, qg, qo and qw are needed.
Workflow:
-Calculate BO properties at p,T (twoDiminterp)
-Caculate local rates at p,T
-calculate void fraction at p,T
-Calculate real gas velocities
-Calculate.......

$q_{\bar{g}} = q_{\bar{o}} \cdot GOR$

$q_{\bar{w}} = q_{\bar{o}} \left(\frac{wc}{1-wc}\right)$ in fraction   please check !!

Since B_w is not provided $B_w = \frac{q_w}{q_{\bar{w}}}$ assumption $B_w = 1$ $q_w = q_{\bar{w}}$

$\lambda_w = \frac{q_w}{q_o + q_w}$

$\rho_L = \lambda_w \cdot \rho_w + (1-\lambda_w)\rho_o$

$\mu_L = \lambda_w \cdot \mu_w + (1-\lambda_w)\mu_o$

$\frac{dp}{dx}\bigg|_{hydrostatic} = \rho_m \cdot g \quad \left[\rho_L \cdot (1-\varepsilon) + \varepsilon \rho_g\right]$

$u_{sL} = \frac{q_L}{A}$

$u_{sg} = \frac{q_g}{A}$

Task 2. How are you results affected if the oil-water mixture exhibits an emulsion behavior as indicated below:
The oil+water mixture exhibits an emulsion behavior where its viscosity is a function of the water volume fraction. The cutoff watercut is 60%.

Regime	Richardson emulsion viscosity
Oil continuous (WC < 60%)	$\mu_m = \mu_o \cdot e^{3.215 \cdot WC}$
Water continuous (WC > 60%)	$\mu_m = \mu_w \cdot e^{3.089 \cdot (1-WC)}$

With WC in fraction

$\mu_L = \mu_o(1-\lambda_w) + \mu_w \lambda_w$

change μ_L value

FLUID PROPERTIES	Input:	red
	Output:	blue
μ_o	[Pa s]	5.690E-04
μ_g	[Pa s]	2.150E-05
σ_{og}	[N/m]	0.003
ρ_o	[kg/m^3]	657.6
ρ_g	[kg/m^3]	164.1
OPERATING CONDITIONS		
Usl	[m/s]	1.000
Usg	[m/s]	1.000
PIPING CHARACTERISTICS		
Angle (from hor.)	[rad]	1.571
Diameter	[m]	0.1
Roughness	[m]	1.50E-05

dp/dx	Flow pattern	Liquid holdup
[Pa/m]	[-]	[-]
3973.89	Slug	Film:1.24E-1 Slug:2.78E-1

different flow pattern

Task 3. Is it important to consider the effect of r_s in your calculations? Can it be neglected?

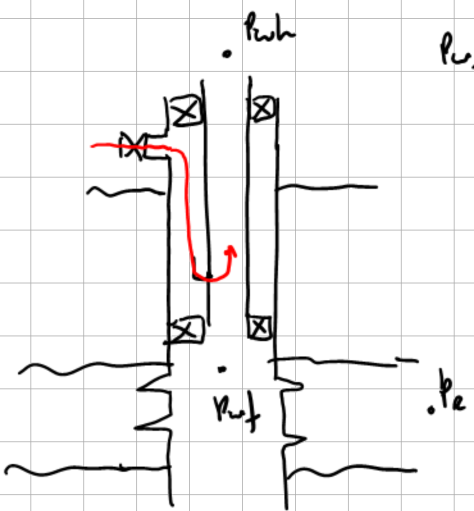
Task 4. Consider that gas lift is applied to the bottom of the well (rate of 300 000 Sm3/d), and the temperature and pressure is 93 C and 160 bar bara. The oil rate is 1450 Sm3/d. Can the table provided be used to estimate black oil properties for these conditions? If they can, estimate local rates of oil and gas.

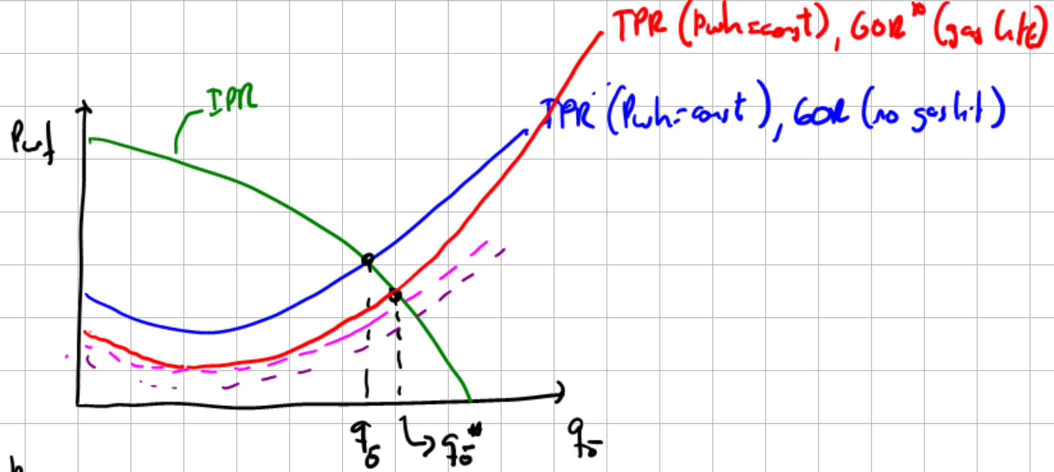
$$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \dfrac{B_g}{1-R_s\cdot \cancel{r_s}\,0} & \dfrac{-R_s\cdot B_g}{1-R_s\cdot \cancel{r_s}\,0} & 0 \\ \dfrac{-B_o\cdot \cancel{r_s}\,0}{1-R_s\cdot \cancel{r_s}\,0} & \dfrac{B_o}{1-R_s\cdot \cancel{r_s}\,0} & 0 \\ 0 & 0 & B_w \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix}$$

if new q_g and q_o are very different from the previous then r_s will have a high impact

Lecture 34, Gas lift intro

Artificial lift technique to:
- Increase production rate from natural flow
- enable production (natural flow is zero)
- Improve well flow (e.g. avoid instabilities)





$$\Delta p = P_{wf} - P_{wh} = \underbrace{\Delta p_{Hydrostatic}}_{\rho_m \cdot g \cdot \Delta h \quad (1)} + \underbrace{\Delta p_{friction}}_{\rho_m \cdot f \frac{L}{D} \frac{v^2}{2} \quad (2)} + \Delta p_{acceleration}$$

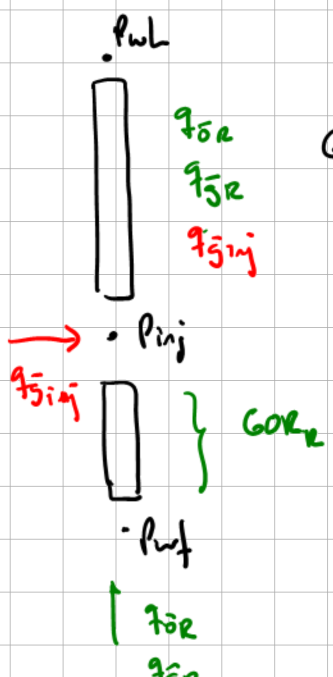
$$\rho_g \le \rho_m \le \rho_L$$

$$\rho_m \to \rho_g$$

$$H_g = \frac{A_g}{A}$$

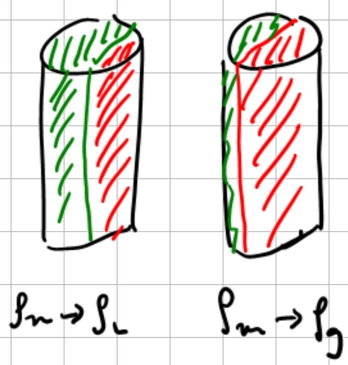
the goal of gas lift : reduce (1)) increasing (2)

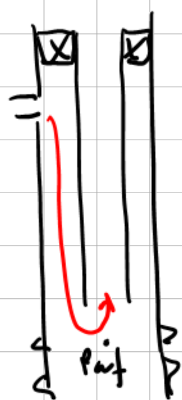
$$\rho_m = H_g \cdot \rho_g + H_L \cdot \rho_L$$



$$GOR^* = \frac{q_{g\,inj} + q_{gR}}{q_{oR}}$$

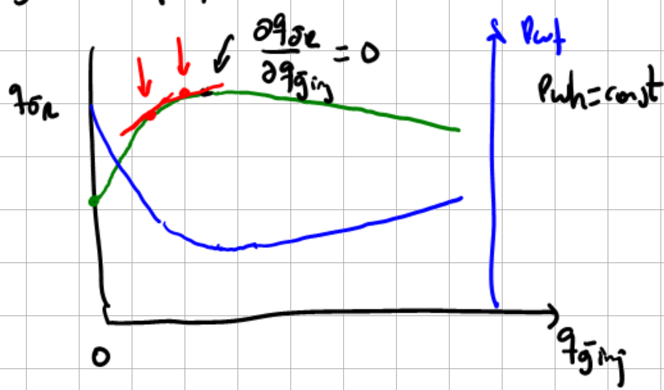
$$GOR^* > GOR_R$$





all tubing has GOR*

gas lift performance curve



$$\text{Profit} = \text{revenue} - \text{cost}$$

$$= q_{o,R} \cdot P_o - \text{cost}(q_{g,inj})$$

$$= q_{o,R} \cdot P_o - P_g \cdot q_{g,inj}$$

max Profit

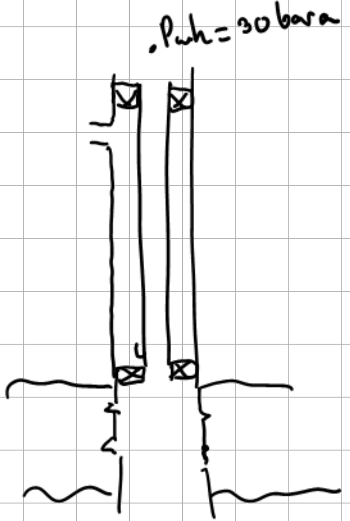
$$\frac{\partial \text{Profit}}{\partial q_{g,inj}} = \frac{\partial q_{o,R}}{\partial q_{g,inj}} \cdot P_o - P_g = 0 \quad \Rightarrow \quad \frac{\partial q_{o,R}}{\partial q_{g,inj}} = \frac{P_g}{P_o} > 0 \quad \text{close to zero}$$

Lecture 35: Gas lift exercise

Exercise goals:

1. -Learn how to use a VBA function to estimate flowing bottom-hole pressure by performing tubing pressure drop calculations from wellhead
2. -Calculate IPR and TPR and visualize results. Calculate TPR for several Rp and see the effect on the intersection and estimate how much gas lift gas is needed
3. -Calculate the gas lift performance curve of the well

Well details



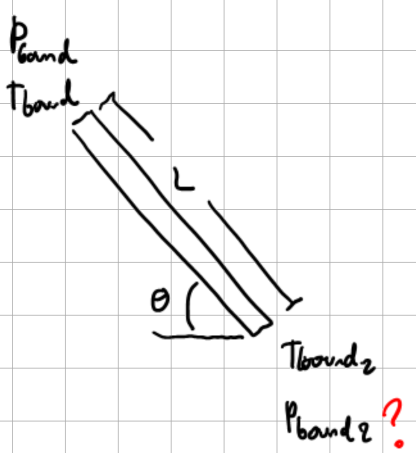
GOR = 53.7 Sm^3/Sm^3

Tubing diameter, D	[m]	0.1
Tubing angle from hor, angle	[deg]	90
Tubing length, L	[m]	2500
Tubing roughness	[m]	2.00E-05
wellhead pressure, pwh	[bara]	30
Wellhead temperature, Twh	[C]	70
Flowing bottom-hole temperature, Twf	[C]	100

P_R = 200 bara

use Fetkovich equation for IPR ($q_{o\,max}$ = 3000 Sm^3/d)

Part 1. For estimation of flowing bottom-hole pressure, we will use a VBA function




```
Function mpf_p(pBound, BoundType, qo_sc, qg_sc, qw_sc, L, angle, roughness, D, TBound, TBound2, PVTMatrix, Profile)
    'mpf_p, function that calculates pressure at inlet or outlet of pipe, in bara, depending on the input
    'pBound, pressure at the boundary, [bara]
    'BoundType, type of boundary, -1 for inlet, 1 for outlet
    'qo_sc, oil rate at standard conditions, [Sm3/d]
    'qg_sc, gas rate at standard conditions, [Sm3/d]
    'qw_sc, water rate at standard conditions, [Sm3/d]
    'L, pipe length, [m]
    'angle, pipe inclination angle, in deg, measured from the horizontal
    'roughness, pipe roughness, in [m],
    'D, pipe hydraulic radius, [m]
    'TBound, fluid temperature at the boundary, [C]
    'TBound2, fluid temperature at the other boundary, [C]
    'PVTMatrix, matrix with BO properties for the flowing composition, arranged in the following manner
    ''GOR(1)    p(2)      T(3)      PROP1   PROP2   PROP3   PROP4
    ' value   value   value   value   value   value
    ' value   value   value   value   value   value
    ' value   value   value   value   value   value
    ' value   value   value   value   value   value
    'profile, provides the profile along the conduit, 1 yes, 0, no
    'Preparing to perform interpolation in the property table
    ColRs = 4
    ColRv = 5
    ColBo = 6
    ColBg = 7
    ColViscg = 8
    ColDeng = 9
    ColVisco = 10
    ColDeno = 11
```

This function does the following:

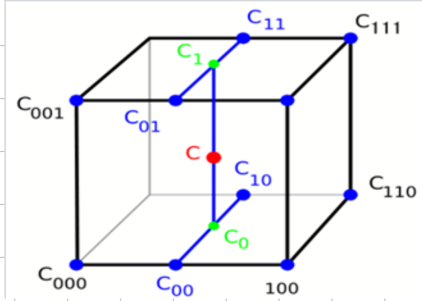
-Estimation of PVT properties by interpolating on a BO table.

As Gas lift increases the GOR, then properties for several GORs, pressures and temperatures must be provided

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
GOR [Sm3/Sm3]	Pressure [bara]	Temperature [C]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	viscg [cp]	deng [kg/m3]	visco [cp]	deno [kg/m3]	IFTog [N/m]	rsw [Sm3/Sm3]	Bw [m3/Sm3]	viscw [cp]	denw [kg/m3]
54	10	20	10.4	0.00E+00	1.030	1.00E-01	1.12E-02	7.8	0.65	711.8	2.04E-02	0.00E+00	1.000	1.0	997.0
54	10	60	7.1	2.63E-05	1.072	1.15E-01	1.25E-02	7.6	0.43	680.6	1.74E-02	0.00E+00	1.000	1.0	997.0
54	10	100	5.2	2.45E-04	1.123	1.33E-01	1.39E-02	8.1	0.30	647.8	1.44E-02	0.00E+00	1.000	1.0	997.0
54	20	20	21.0	0.00E+00	1.054	4.88E-02	1.15E-02	15.4	0.60	706.1	1.92E-02	0.00E+00	1.000	1.0	997.0
54	20	60	16.0	8.90E-06	1.096	5.63E-02	1.28E-02	14.4	0.40	674.5	1.64E-02	0.00E+00	1.000	1.0	997.0
54	20	100	12.8	1.18E-04	1.148	6.45E-02	1.41E-02	14.6	0.28	640.8	1.36E-02	0.00E+00	1.000	1.0	997.0
54	30	20	30.1	0.00E+00	1.073	3.18E-02	1.17E-02	23.2	0.56	700.3	1.81E-02	0.00E+00	1.000	1.0	997.0
54	30	60	24.0	5.53E-06	1.116	3.69E-02	1.30E-02	21.3	0.38	668.8	1.56E-02	0.00E+00	1.000	1.0	997.0
54	30	100	20.3	8.28E-05	1.171	4.23E-02	1.44E-02	21.0	0.27	634.4	1.29E-02	0.00E+00	1.000	1.0	997.0
54	40	20	39.2	0.00E+00	1.092	2.33E-02	1.20E-02	31.4	0.53	694.2	1.72E-02	0.00E+00	1.000	1.0	997.0
54	40	60	31.6	4.79E-06	1.134	2.73E-02	1.33E-02	28.4	0.36	663.0	1.49E-02	0.00E+00	1.000	1.0	997.0
54	40	100	27.2	6.89E-05	1.191	3.13E-02	1.46E-02	27.5	0.26	628.1	1.24E-02	0.00E+00	1.000	1.0	997.0
54	50	20	48.5	0.00E+00	1.112	1.82E-02	1.24E-02	40.0	0.50	687.9	1.64E-02	0.00E+00	1.000	1.0	997.0
54	50	60	39.4	5.08E-06	1.154	2.15E-02	1.35E-02	35.6	0.35	657.1	1.43E-02	0.00E+00	1.000	1.0	997.0
54	50	100	34.3	6.32E-05	1.213	2.48E-02	1.48E-02	34.1	0.25	621.7	1.18E-02	0.00E+00	1.000	1.0	997.0
54	60	20	53.7	0.00E+00	1.123	0.00E+00	0.00E+00	0.0	0.49	684.8	0.00E+00	0.00E+00	1.000	1.0	997.0
54	60	60	47.3	5.99E-06	1.174	1.77E-02	1.38E-02	43.1	0.34	651.0	1.36E-02	0.00E+00	1.000	1.0	997.0
54	60	100	41.5	6.15E-05	1.235	2.05E-02	1.51E-02	40.8	0.24	615.1	1.13E-02	0.00E+00	1.000	1.0	997.0
54	70	20	53.7	0.00E+00	1.121	0.00E+00	0.00E+00	0.0	0.50	686.0	0.00E+00	0.00E+00	1.000	1.0	997.0
54	70	60	53.7	0.00E+00	1.190	0.00E+00	0.00E+00	0.0	0.33	646.4	0.00E+00	0.00E+00	1.000	1.0	997.0
54	70	100	49.0	6.22E-05	1.258	1.74E-02	1.54E-02	47.7	0.23	608.3	1.09E-02	0.00E+00	1.000	1.0	997.0
54	80	20	53.7	0.00E+00	1.119	0.00E+00	0.00E+00	0.0	0.50	687.2	0.00E+00	0.00E+00	1.000	1.0	997.0
54	80	60	53.7	0.00E+00	1.187	0.00E+00	0.00E+00	0.0	0.33	648.1	0.00E+00	0.00E+00	1.000	1.0	997.0

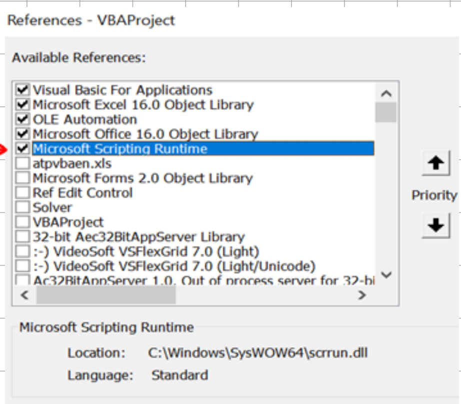
Interpolation is performed using a tri-linear interpolation (GOR, p and T must be provided)

https://en.wikipedia.org/wiki/Trilinear_interpolation




```
Rs = TrilinearInterpol(GOR, p, T, GOR_bounds(0), GC
Rv = TrilinearInterpol(GOR, p, T, GOR_bounds(0), GC
Bo = TrilinearInterpol(GOR, p, T, GOR_bounds(0), GC
Bg = TrilinearInterpol(GOR, p, T, GOR_bounds(0), GC
viscg = TrilinearInterpol(GOR, p, T, GOR_bounds(0),
deng = TrilinearInterpol(GOR, p, T, GOR_bounds(0),
visco = TrilinearInterpol(GOR, p, T, GOR_bounds(0),
deno = TrilinearInterpol(GOR, p, T, GOR_bounds(0),
IFTog = TrilinearInterpol(GOR, p, T, GOR_bounds(0),
'rsw = TrilinearInterpol(GOR, p, T, GOR_bounds(0),
Bw = TrilinearInterpol(GOR, p, T, GOR_bounds(0), GC
denw = TrilinearInterpol(GOR, p, T, GOR_bounds(0),
Viscw = TrilinearInterpol(GOR, p, T, GOR_bounds(0),
```

To make the interpolation efficient, a "dictionary" is used in the VBA code. Therefore, you must have this library active (inside the VBA menu Tools--> references)




```
Set dictx = CreateObject("Scripting.Dictionary")
Set dictx1 = CreateObject("Scripting.Dictionary")
Set dictx2 = CreateObject("Scripting.Dictionary")
Set dictx3 = CreateObject("Scripting.Dictionary")
```

-pressure gradients (dp/dx) are calculated using the drift flux model (neglecting the acceleration term)

$$\frac{dp}{dx} + \rho_{TP} g_x + \rho_g v_{sg} \frac{dv_g}{dx} + \rho_l v_{sl} \frac{dv_l}{dx} = 0$$

-The gas holdup (void fraction) was calculated using the correlation by Woldesemayat and Ghajar

$$\varepsilon = \frac{U_{SG}}{U_{SG}\left(1 + \left(\frac{U_{SL}}{U_{SG}}\right)^{\left(\frac{\rho_G}{\rho_L}\right)^{0.1}}\right) + 2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}(1.22 + 1.22\sin\theta)^{\frac{P_{atm}}{P_{system}}}}$$

Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

Melkamu A. Woldesemayat, Afshin J. Ghajar *

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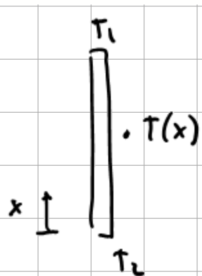
Received 1 June 2006; received in revised form 13 September 2006

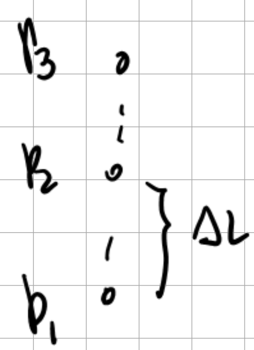
-Temperature of an intermediate point was calculated by doing a linear interpolation between inlet and outlet and using the position of the point



The pressure integration was performed using Euler's method. 10 intervals were used

```
Nsec = 10
deltaL = L / Nsec
```

Pressure integration method



$$p_2 = p_1 + \left.\frac{dp}{dx}\right|_{@\, p_1} \cdot \Delta L$$

PART 1: Learning how to use the VBA tubing function to calculate pwf

qo [Sm3/d]	qw [Sm3/d]	qg [Sm3/d]	pwf [bara]
500	500	50000	271

-Calculate IPR and TPR and visualize results. Calculate TPR for several Rp and see the effect on the intersection and estimate how much gas lift gas is needed

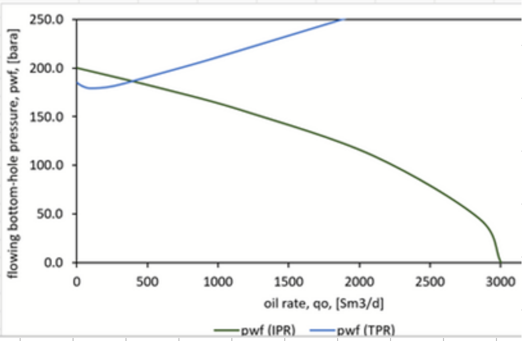
Natural flow

PART 2: IPR vs TPR

WC= 0 Rp_R= 54 Rp^*= 54

qo [Sm3/d]	WC [-]	qw [Sm3/d]	Rp_R [Sm3/Sm3]	qg_R [Sm3/d]	Rp^* [Sm3/Sm3]	qg_{total} [Sm3/d]	qg_{inj} [Sm3/d]	pwf (TPR) [bara]
0.1	0	0	54	5.37E+00	54	5.37E+00	0.00E+00	185
1	0	0	54	5.37E+01	54	5.37E+01	0.00E+00	185
10	0	0	54	5.37E+02	54	5.37E+02	0.00E+00	184
50	0	0	54	2.69E+03	54	2.69E+03	0.00E+00	181
100	0	0	54	5.37E+03	54	5.37E+03	0.00E+00	179
250	0	0	54	1.34E+04	54	1.34E+04	0.00E+00	181
500	0	0	54	2.69E+04	54	2.69E+04	0.00E+00	191
750	0	0	54	4.03E+04	54	4.03E+04	0.00E+00	201
1000	0	0	54	5.37E+04	54	5.37E+04	0.00E+00	211
2000	0	0	54	1.07E+05	54	1.07E+05	0.00E+00	256

PART 3: gas lift performance curve



Natural flow approx. 400 Sm3/d oil

Gas lift, GOR in tubing = 100

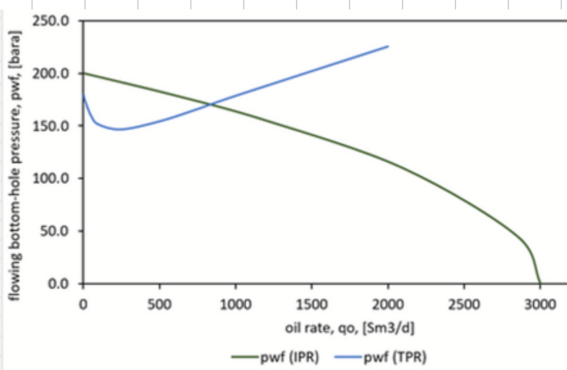
PART 2: IPR vs TPR

WC= 0 Rp_R= 54 Rp^*= 100

qo [Sm3/d]	WC [-]	qw [Sm3/d]	Rp_R [Sm3/Sm3]	qg_R [Sm3/d]	Rp^* [Sm3/Sm3]	qg_{total} [Sm3/d]	qg_{inj} [Sm3/d]	pwf (TPR) [bara]
0.1	0	0	54	5.37E+00	100	1.00E+01	4.63E+00	180
1	0	0	54	5.37E+01	100	1.00E+02	4.63E+01	180
10	0	0	54	5.37E+02	100	1.00E+03	4.63E+02	174
50	0	0	54	2.69E+03	100	5.00E+03	2.31E+03	160
100	0	0	54	5.37E+03	100	1.00E+04	4.63E+03	151
250	0	0	54	1.34E+04	100	2.50E+04	1.16E+04	147
500	0	0	54	2.69E+04	100	5.00E+04	2.31E+04	154
750	0	0	54	4.03E+04	100	7.50E+04	3.47E+04	166
1000	0	0	54	5.37E+04	100	1.00E+05	4.63E+04	179
2000	0	0	54	1.07E+05	100	2.00E+05	9.25E+04	226

PART 3: gas lift performance curve

SOLVER



Intersection approx. 800 Sm3/d oil
qginj between 34 700-46 300 Sm3/d gas

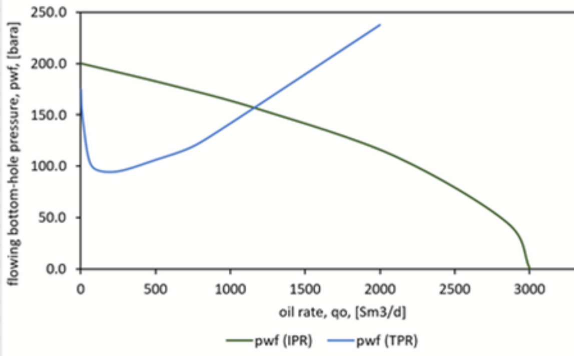
gas lift, GOR in tubing =300

PART 2: IPR vs TPR

WC=	0	Rp_R=	54	Rp^*=	300			
qo	WC	qw	Rp_R	qg_R	Rp^*	qg_{total}	qg_{inj}	pwf (TPR)
[Sm3/d]	[-]	[Sm3/d]	[Sm3/Sm3]	[Sm3/d]	[Sm3/Sm3]	[Sm3/d]	[Sm3/d]	[bara]
0.1	0	0	54	5.37E+00	300	3.00E+01	2.46E+01	175
1	0	0	54	5.37E+01	300	3.00E+02	2.46E+02	172
10	0	0	54	5.37E+02	300	3.00E+03	2.46E+03	149
50	0	0	54	2.69E+03	300	1.50E+04	1.23E+04	109
100	0	0	54	5.37E+03	300	3.00E+04	2.46E+04	97
250	0	0	54	1.34E+04	300	7.50E+04	6.16E+04	95
500	0	0	54	2.69E+04	300	1.50E+05	1.23E+05	106
750	0	0	54	4.03E+04	300	2.25E+05	1.85E+05	119
1000	0	0	54	5.37E+04	300	3.00E+05	2.46E+05	141
2000	0	0	54	1.07E+05	300	6.00E+05	4.93E+05	237

PART 3: gas lift performance curve

SOLVER



Intersection approx. 1200 Sm3/d oil
qinj betweeen 246 000-493 000 Sm3/d gas

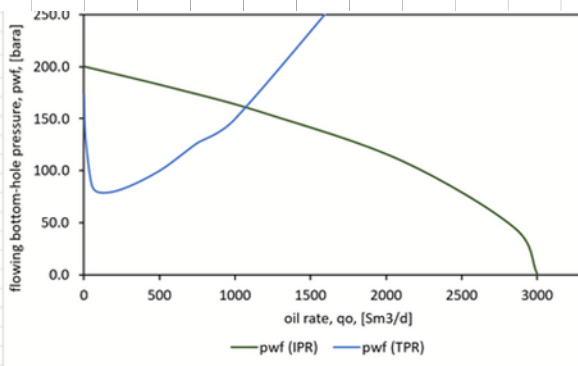
GOR = 500 (gas lift)

PART 2: IPR vs TPR

WC=	0	Rp_R=	54	Rp^*=	500			
qo	WC	qw	Rp_R	qg_R	Rp^*	qg_{total}	qg_{inj}	pwf (TPR)
[Sm3/d]	[-]	[Sm3/d]	[Sm3/Sm3]	[Sm3/d]	[Sm3/Sm3]	[Sm3/d]	[Sm3/d]	[bara]
0.1	0	0	54	5.37E+00	500	5.00E+01	4.46E+01	173
1	0	0	54	5.37E+01	500	5.00E+02	4.46E+02	168
10	0	0	54	5.37E+02	500	5.00E+03	4.46E+03	132
50	0	0	54	2.69E+03	500	2.50E+04	2.23E+04	87
100	0	0	54	5.37E+03	500	5.00E+04	4.46E+04	79
250	0	0	54	1.34E+04	500	1.25E+05	1.12E+05	82
500	0	0	54	2.69E+04	500	2.50E+05	2.23E+05	100
750	0	0	54	4.03E+04	500	3.75E+05	3.35E+05	127
1000	0	0	54	5.37E+04	500	5.00E+05	4.46E+05	150
2000	0	0	54	1.07E+05	500	1.00E+06	8.93E+05	320

PART 3: gas lift performance curve

SOLVER

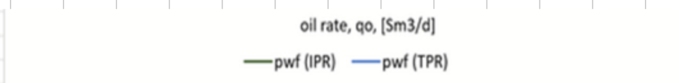


Intersection approx. 1100 Sm3/d of oil

-Calculate the gas lift performance curve of the well

Flow equlibrium-> pwf calculated from TPR should be equal to pwf calculated from IPR by adjusting the rate

PART 3: gas lift performance curve

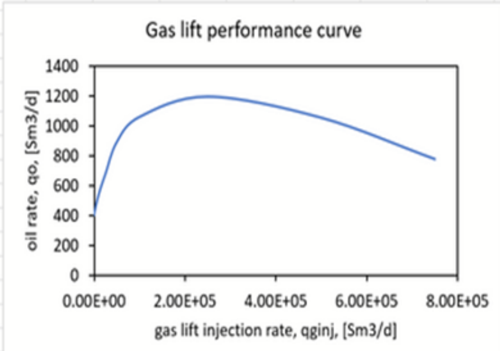


SOLVER

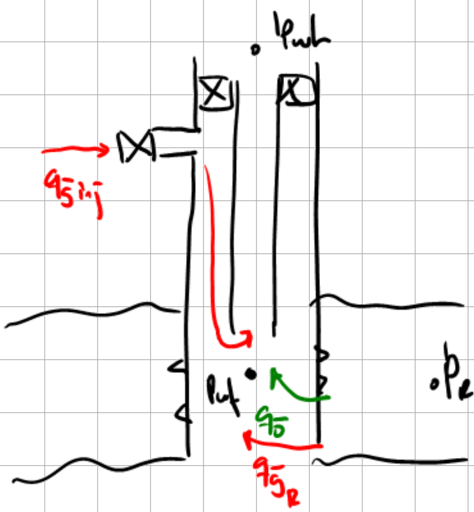
qo	WC	qw	Rp_R	qg_R	qg_{inj}	qg_{total}	Rp^*	pwf (TPR)	pwf (IPR)	error
[Sm3/d]	[-]	[Sm3/d]	[Sm3/Sm3]	[Sm3/d]	[Sm3/d]	[Sm3/d]	[Sm3/Sm3]	[bara]	[bara]	[bara]
777.0667545	0	0	54	4.18E+04	7.50E+05	7.92E+05	1019	172	172	1

POINTS

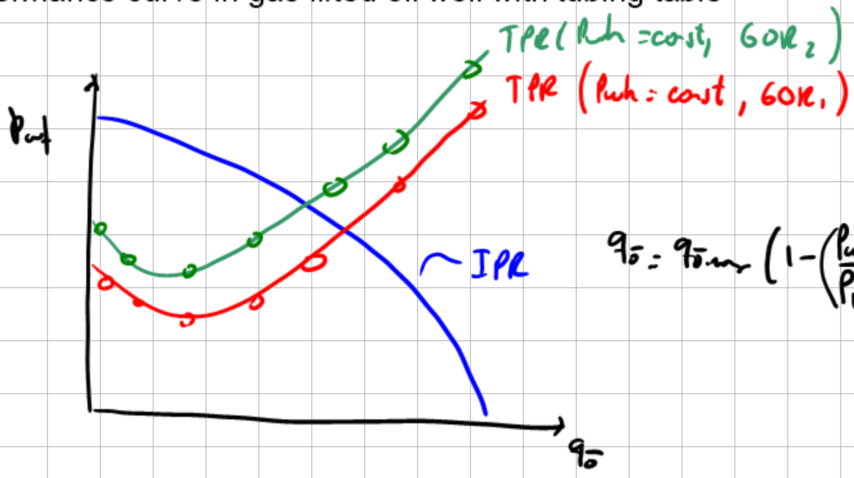
qg_{inj}	qo_R	Rp^*
[Sm3/d]	[Sm3/d]	[Sm3/Sm3]
0.00E+00	400	54
1.00E+03	423	56
1.00E+04	539	72
2.50E+04	679	91
5.00E+04	893	110
1.00E+05	1061	148
2.50E+05	1197	263
5.00E+05	1055	528
7.50E+05	777	1019



35.1. Flow equilibrium and gas lift performance curve in gas-lifted oil well with tubing table



$$GOR_R = R_{pR} = \frac{q_{gR}}{q_o}$$



$$q_o = q_{omax}\left(1-\left(\frac{p_{wf}}{p_R}\right)^2\right)$$

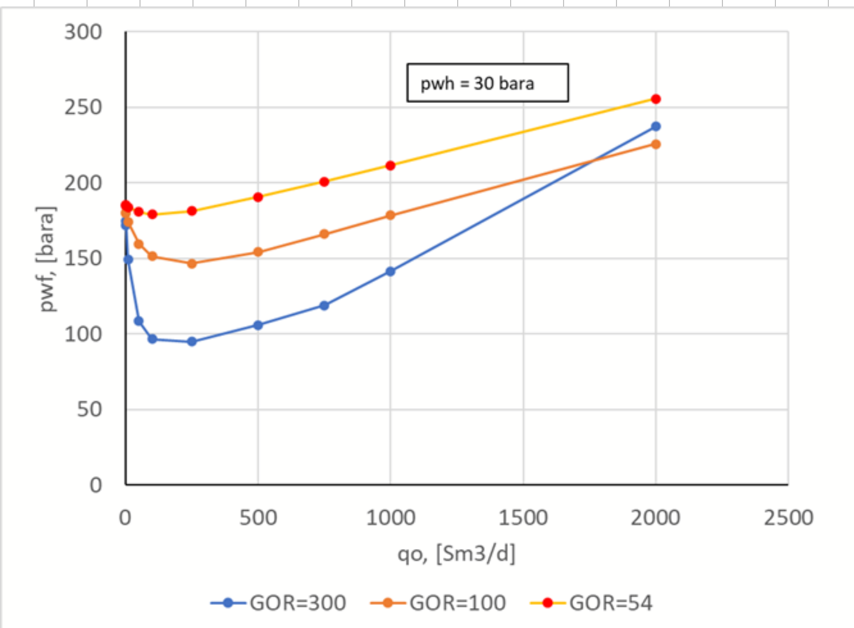
p_{wh} = const, GOR_1

q_o	p_{wf}
.	.
.	.
.	.

p_{wh} = const, GOR_2

q_o	p_{wf}
.	.
.	.
.	.

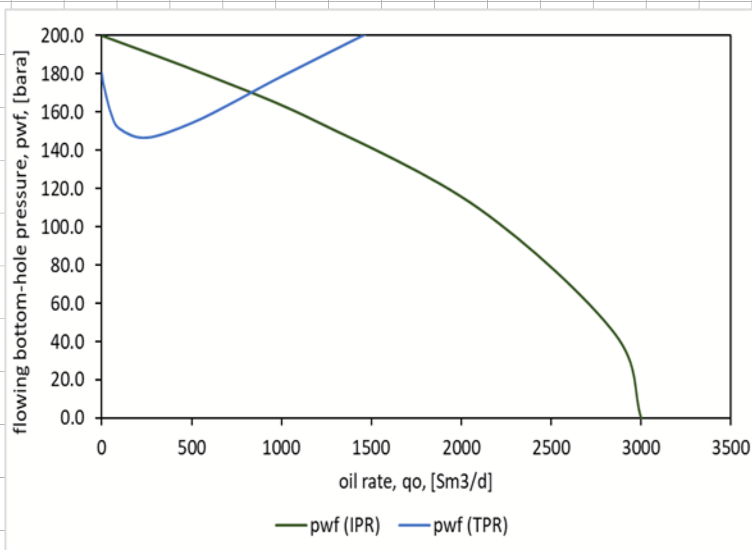
qo [Sm3/d]	GOR [Sm3/Sm3]	pwf [bara]
0.1	54	185.3
1	54	185.1
10	54	183.9
50	54	180.7
100	54	179.2
250	54	181.2
500	54	190.8
750	54	200.9
1000	54	211.4
2000	54	255.6
0.1	100	180.4
1	100	179.7
10	100	174.3
50	100	159.7
100	100	151.3
250	100	146.7
500	100	154.3
750	100	166.1
1000	100	178.6
2000	100	225.6
0.1	200	175.4
1	200	173.8
10	200	159.7
50	200	127.9
100	200	115
250	200	110.1
500	200	119.4



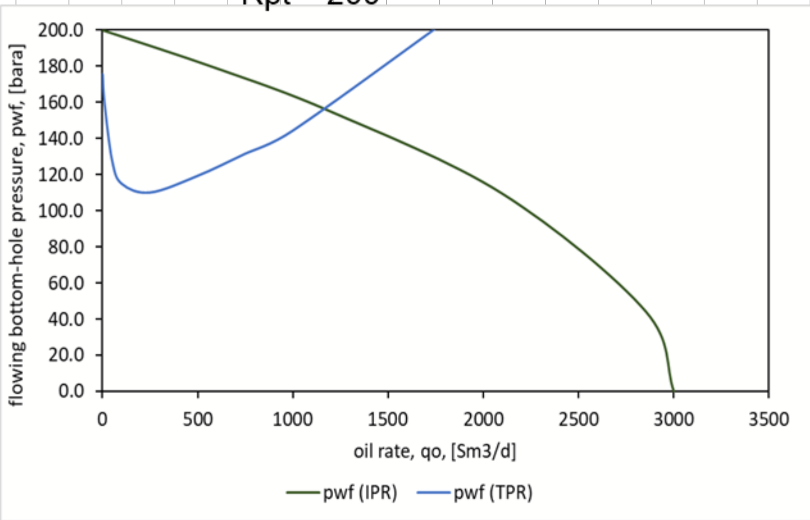
Gas lifted well exercise, Prof Milan Stanko (NTNU)

IPR			TPR		
pR	[bara]	200	pwh	[bara]	30
qomax	[Sm3/d]	3000			
WC	[-]	0			
GOR (Rp)	[-]	100			
			R_{pT}	100	

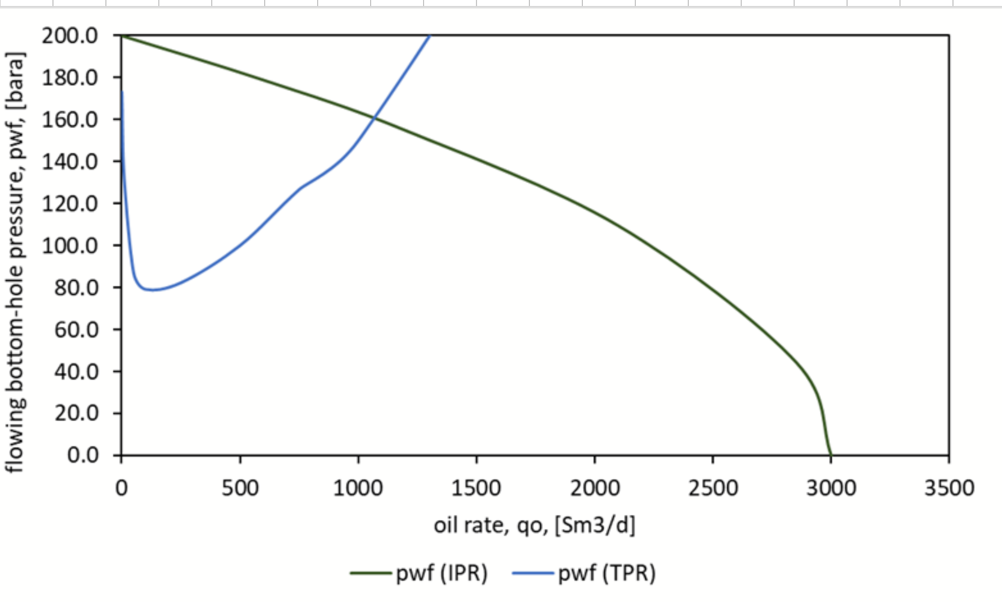
qo/qomax [-]	qo [Sm3/d]	pwf (IPR) [bara]	qo [Sm3/d]	pwf (TPR) [bara]	qgT [Sm3/d]	qgR [Sm3/d]	qginj [Sm3/d]
0.00	0	200.0	0.1	180	1.00E+01	1.00E+01	0.00E+00
0.20	600	178.9	1	180	1.00E+02	1.00E+02	0.00E+00
0.40	1200	154.9	10	174	1.00E+03	1.00E+03	0.00E+00
0.70	2100	109.5	50	160	5.00E+03	5.00E+03	0.00E+00
0.95	2850	44.7	100	151	1.00E+04	1.00E+04	0.00E+00
1.00	3000	0.0	250	147	2.50E+04	2.50E+04	0.00E+00
			500	154	5.00E+04	5.00E+04	0.00E+00
			750	166	7.50E+04	7.50E+04	0.00E+00
			1000	179	1.00E+05	1.00E+05	0.00E+00
			2000	226	2.00E+05	2.00E+05	0.00E+00



Rpt = 200



Rpt = 500



Determine the equilibrium point:

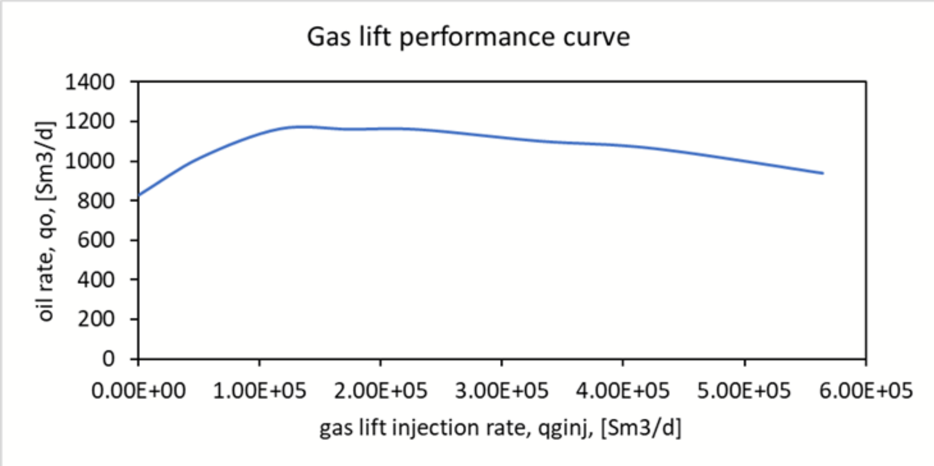
Flow equilibrium solver

qo	R_{pT}	pwf (IPR)	pwf (TPR)	error	qg_R	qg_{total}	qg_{inj}	qo	R_{pT}
[Sm3/d]	[Sm3/Sm3]	[bara]	[bara]	[bara]	[Sm3/d]	[Sm3/d]	[Sm3/d]	[Sm3/d]	[Sm3/Sm3]
939.97	700	166	166	0	9.40E+04	6.58E+05	5.64E+05	939.97	700

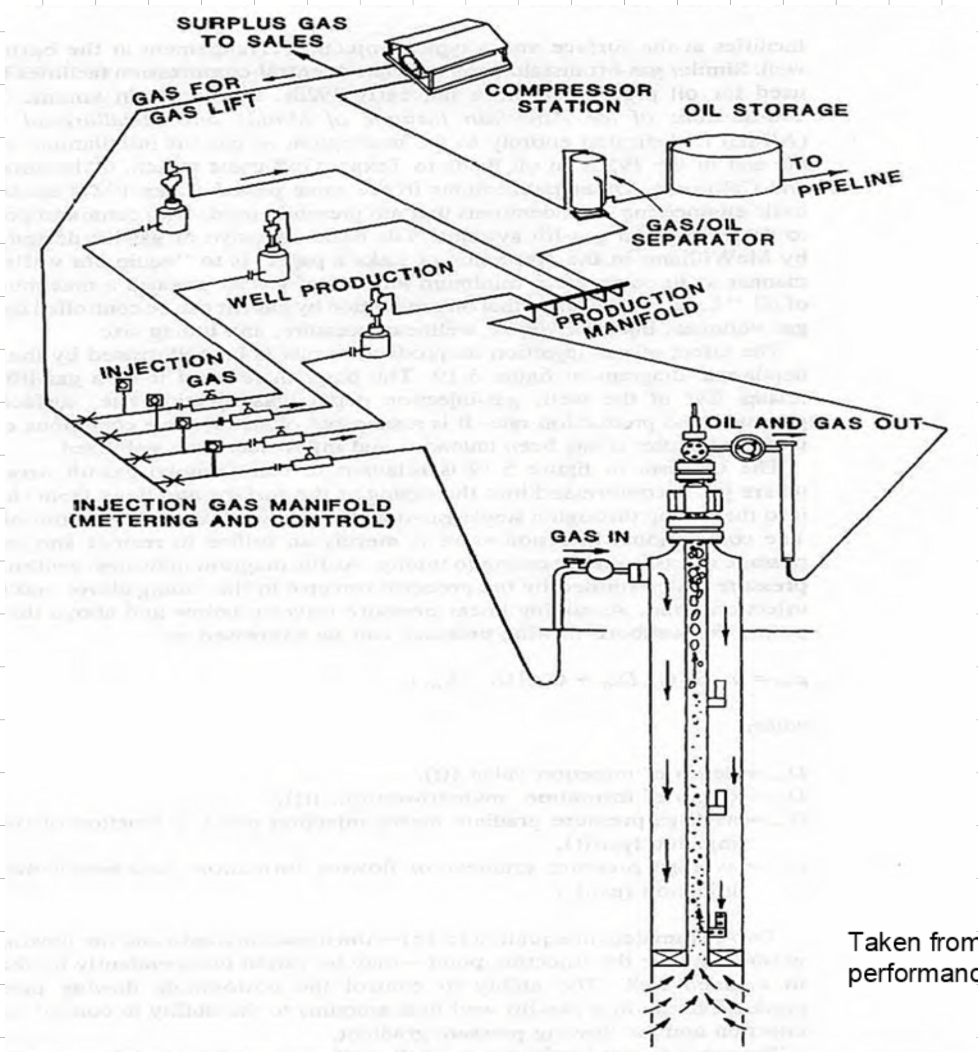
change, until

equals zero

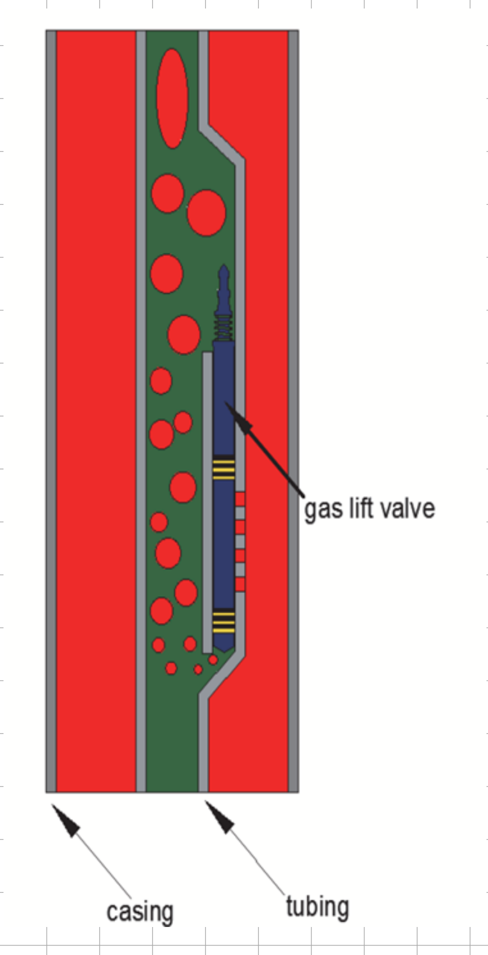
Repeat for Rpt from 100 to 700:

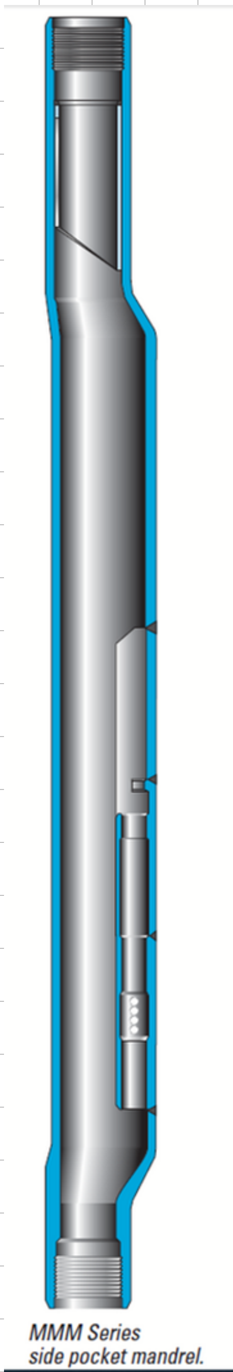

Gas lift performance curve
oil rate, qo, [Sm3/d]
1400
1200
1000
800
600
400
200
0
0.00E+00
1.00E+05
2.00E+05
3.00E+05
4.00E+05
5.00E+05
6.00E+05
gas lift injection rate, qginj, [Sm3/d]

Lecture 36: Hardware for gas lift well



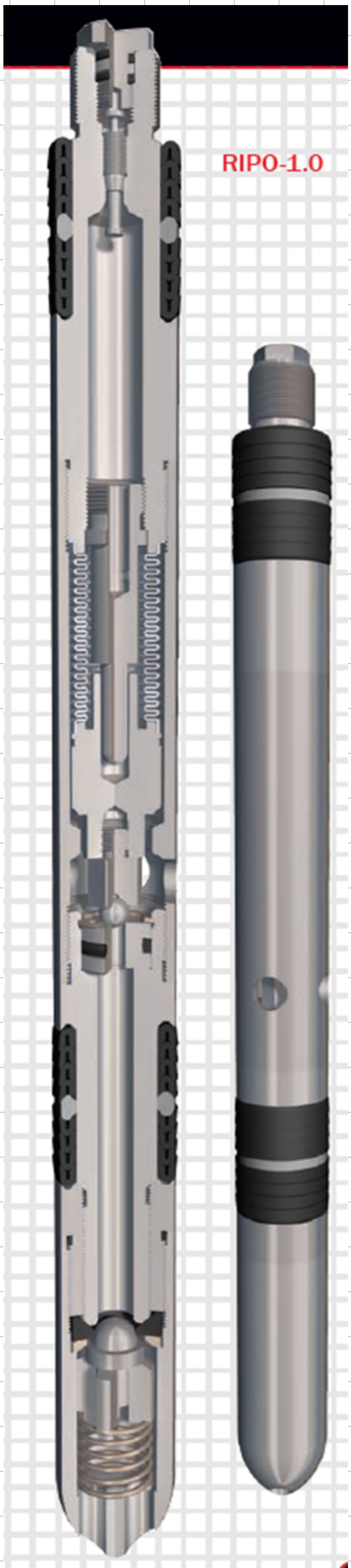
Taken from Book Well performance (Golan& Whitson)



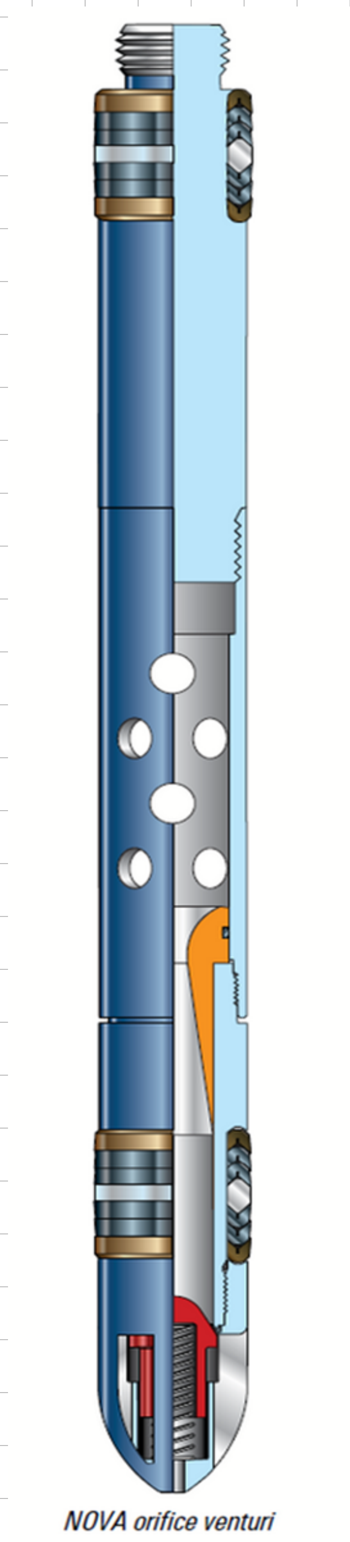


From Schlumberger catalog

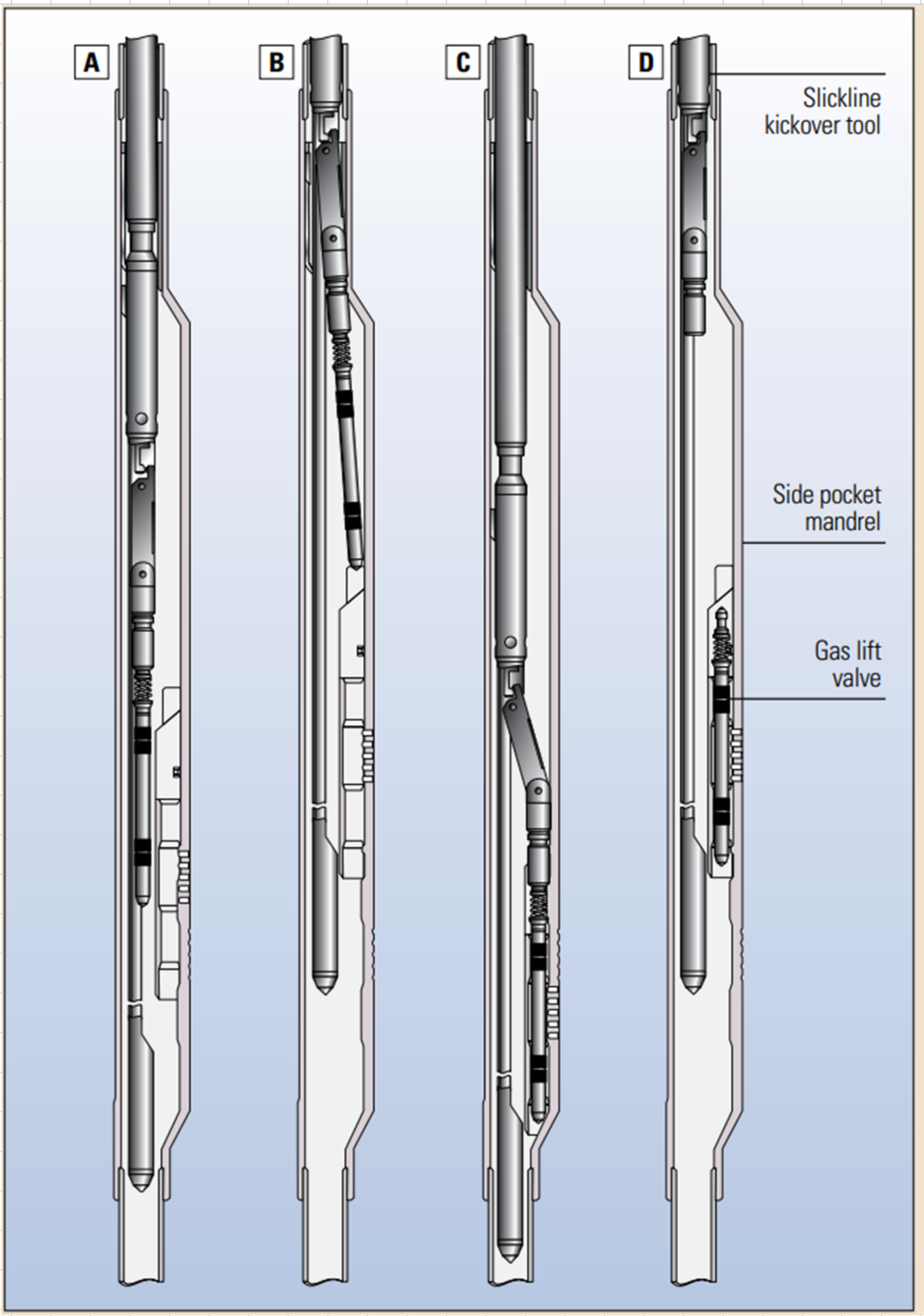
From Lufkin catalog



From Lufkin catalog

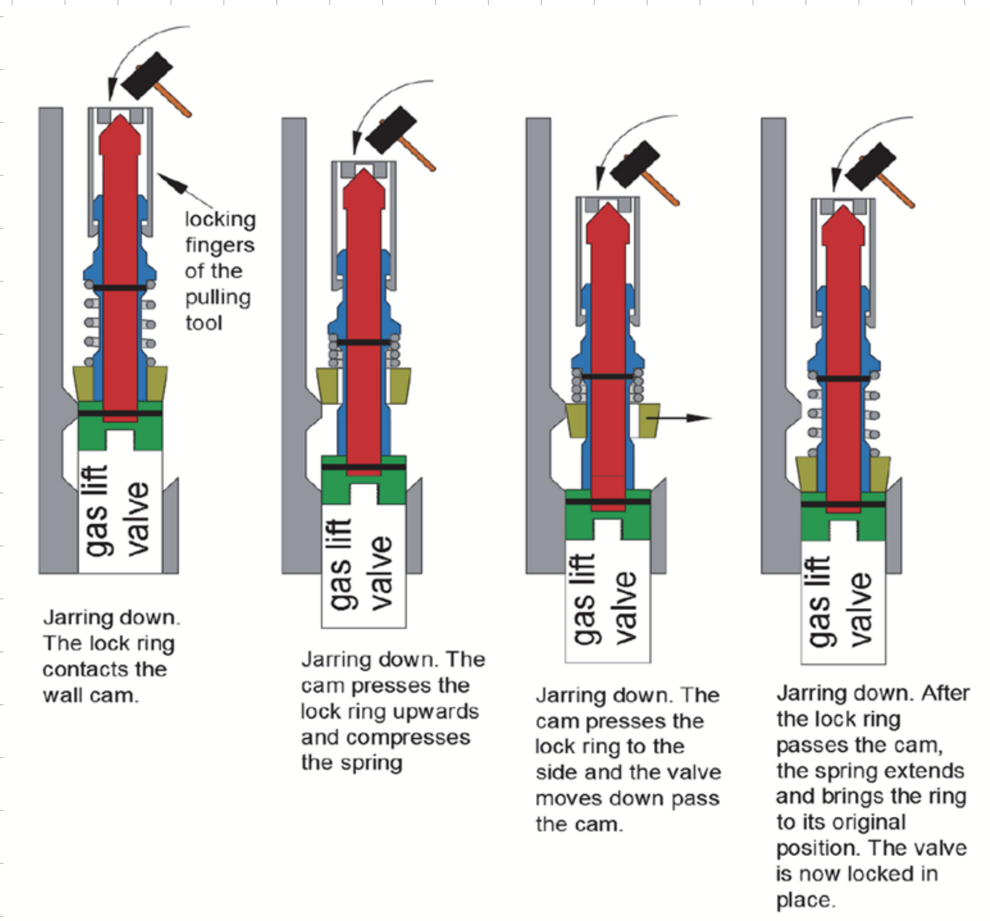


From Schlumberger catalog



https://www.slb.com/-/media/files/oilfield-review/defining-gas-lift.ashx

For an animation check: https://www.youtube.com/watch?v=RA3V42bdrDk at 01:00



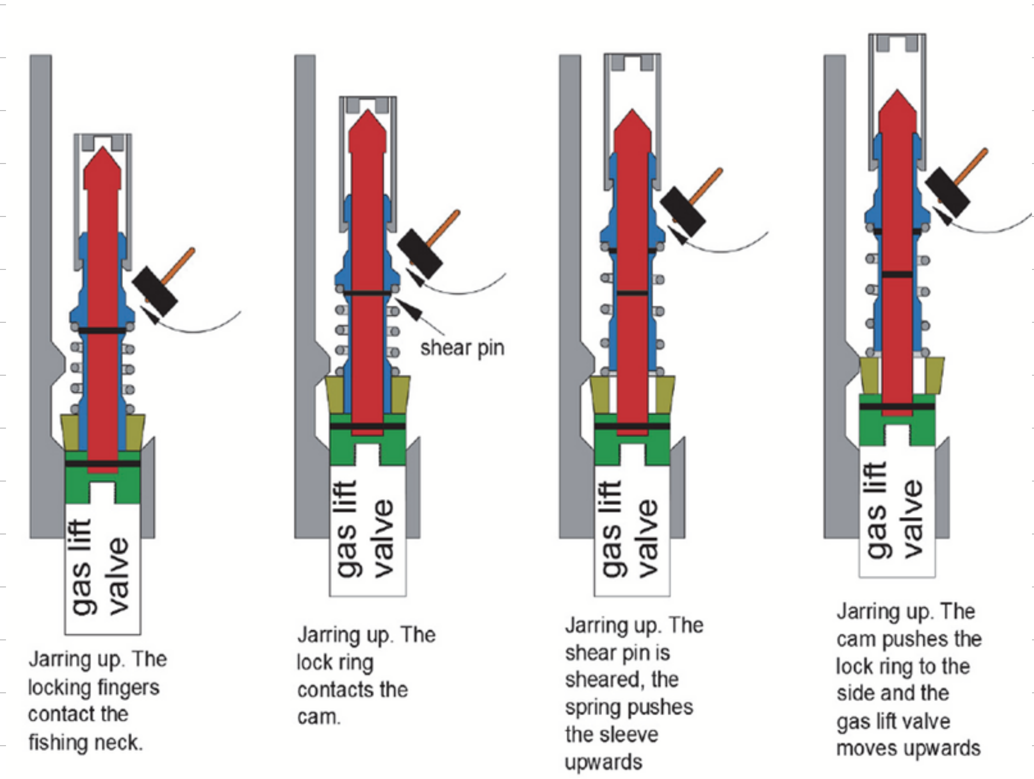
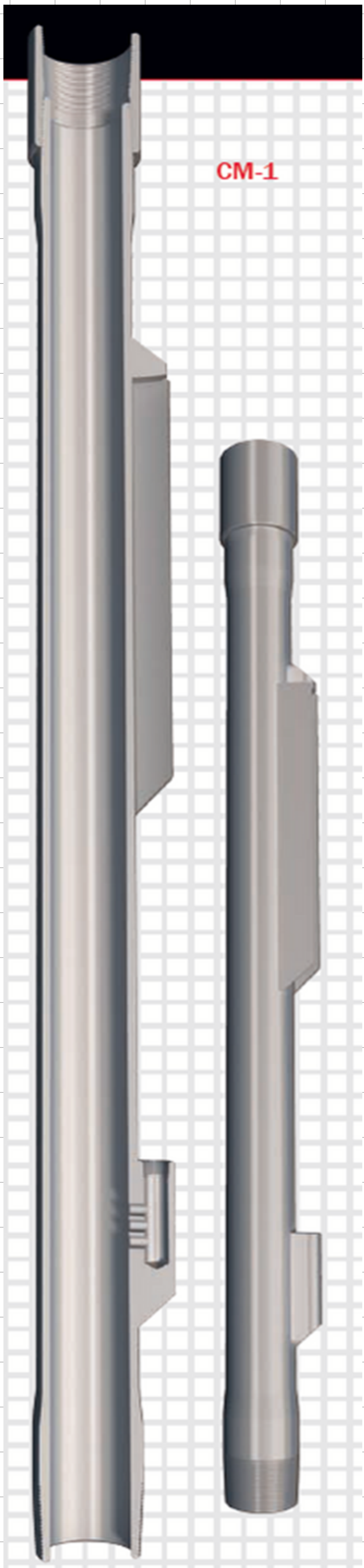
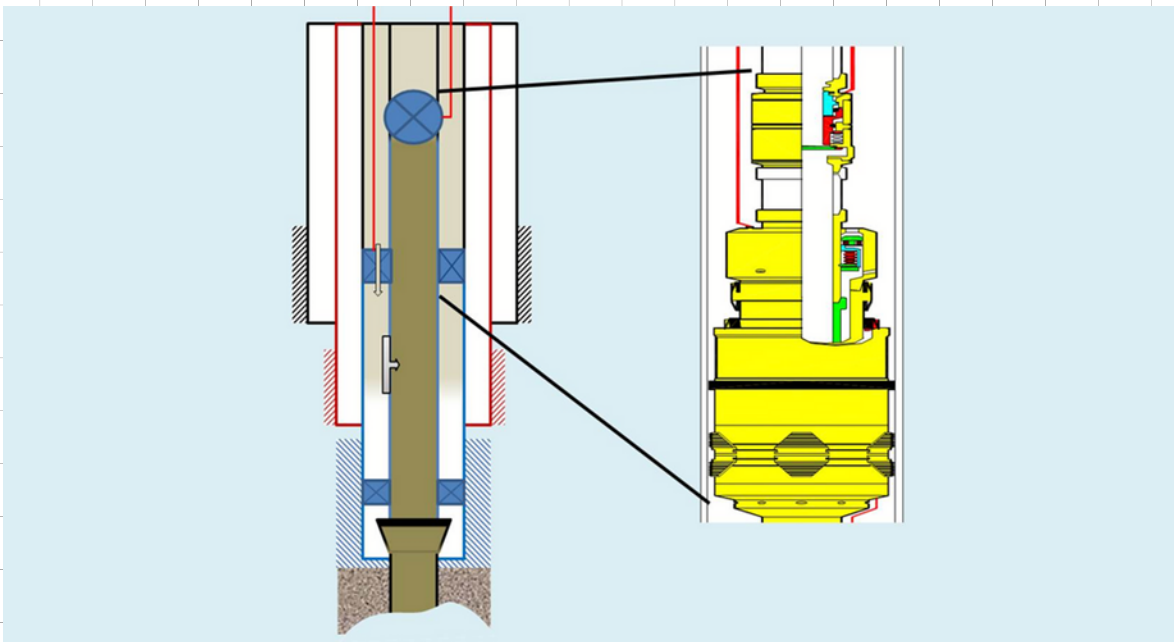


FIGURE 3-8. SEQUENCE TO RETRIEVE A GAS-LIFT VALVE FROM THE MANDREL POCKET



Tubing-retrievable gas lift mandrels (From Lufkin catalogue)

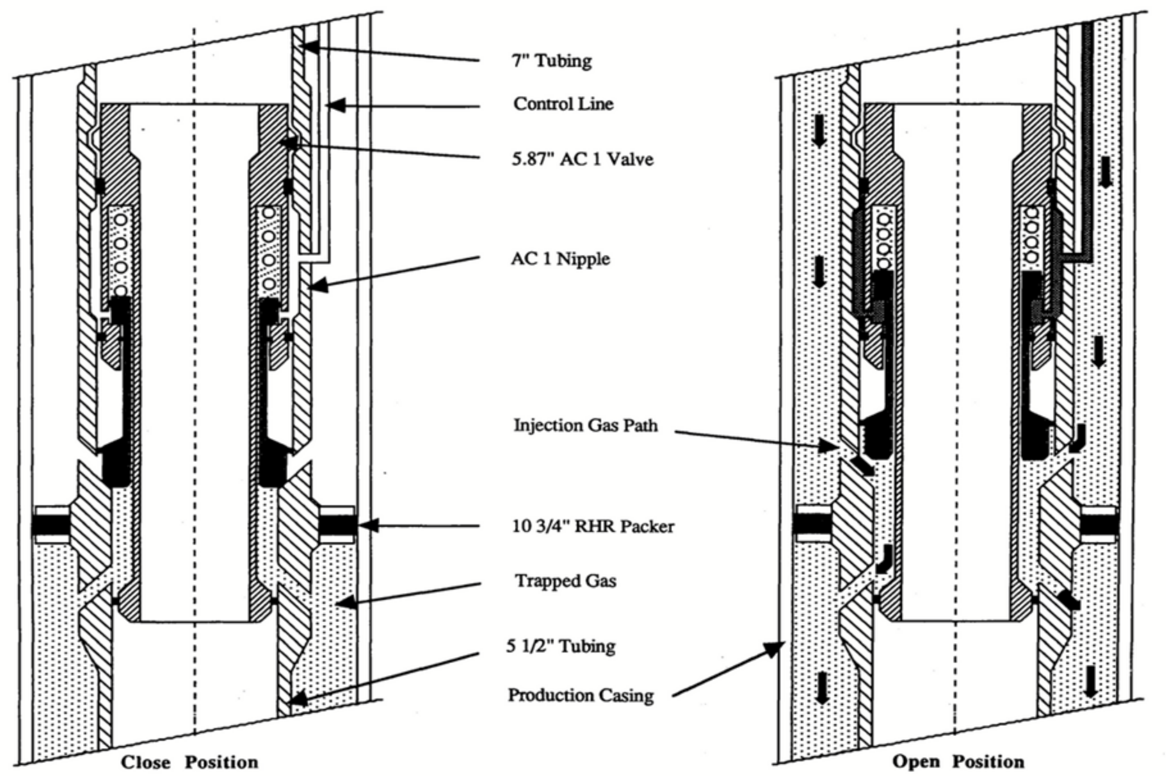
Annular safety valve



https://ndla.no/subject:1:01c27030-e8f8-4a7c-a5b3-489fdb8fea30/topic:2:182061/topic:2:151959/resource:1:182399/332

ASV skal alltid plasseres under DHSV i kompletteringen. Det er fordi kontrollinjen til DHSV ikke skal føres gjennom ASV.

Fig 1 - PRINCIPLE OF AC 1 ANNULUS VALVE



SPE number/page 19278/1

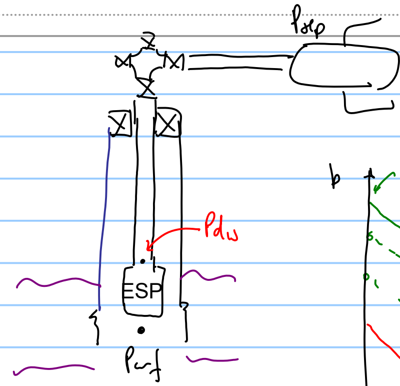
Title FIELD EXPERIENCE IN DOWN HOLE ANNULUS SAFETY VALVE

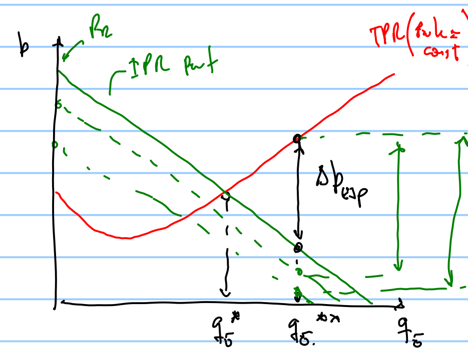
Authors J L GEYELIN

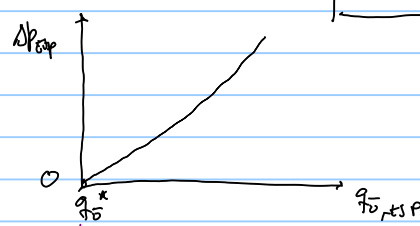
TOTAL-CFP

Note Title

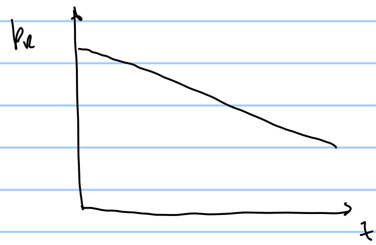
ESP electric submersible pump

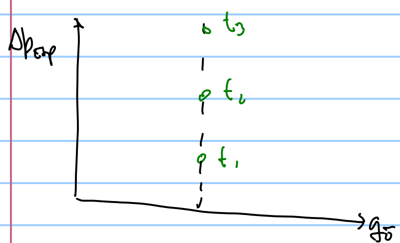






Depletion

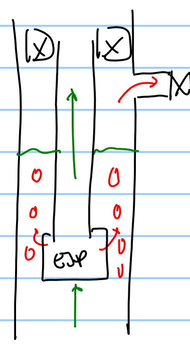




the pump has operational constraints

- limited available motor capacity (maximum power)
- operational map (envelope)
- $P_{suc} \geq P_b(T_R)$
  ↳ buble point pressure (no gas is allowed in the pump)

onshore



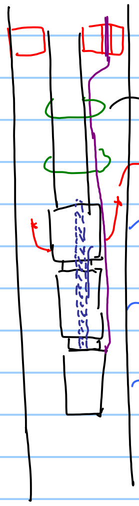
offshore

typical gas tolerance in esp is GVF = 10%
gas volume fraction

$$GVF = \frac{q_g}{\underbrace{q_L}_{q_o + q_w} + q_g} \times 100\%$$

Armais Arutunoff

https://jpt.spe.org/legends-artificial-lift-part-3-history



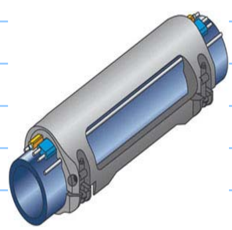
clamps

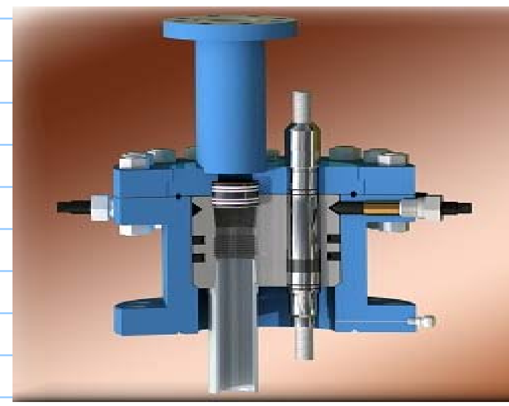
gas

pump might have a gas separator

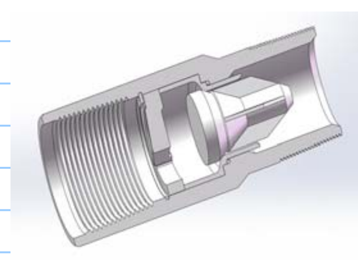
protector / seal

motor

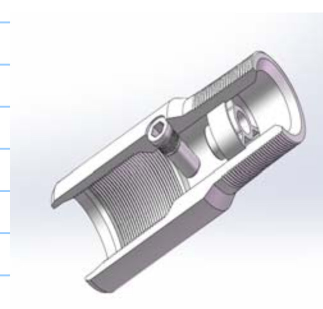




Other important components:
-other gas-handling equipment (after the gas separator)
-Check valve (above the pump)
-Bleeder valve (above the check valve)



http://www.orisun.asia/Products.asp?dl=111&xl=165

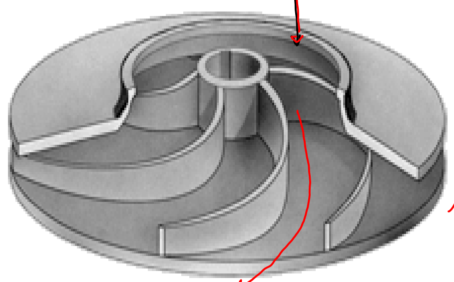


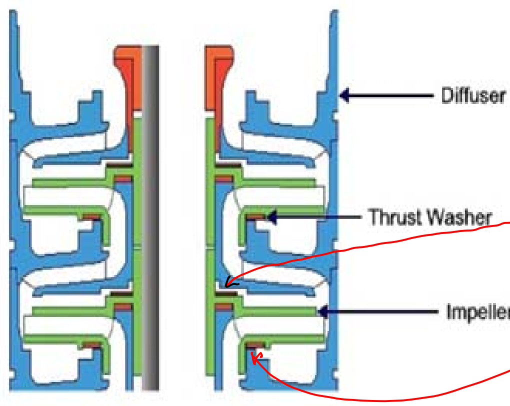
http://www.orisun.asia/Products.asp?dl=111&xl=164

Stage: impeller+diffuser

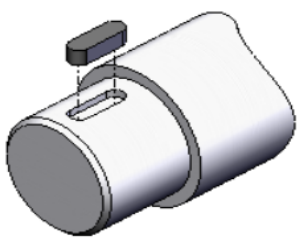
p_1 p_2

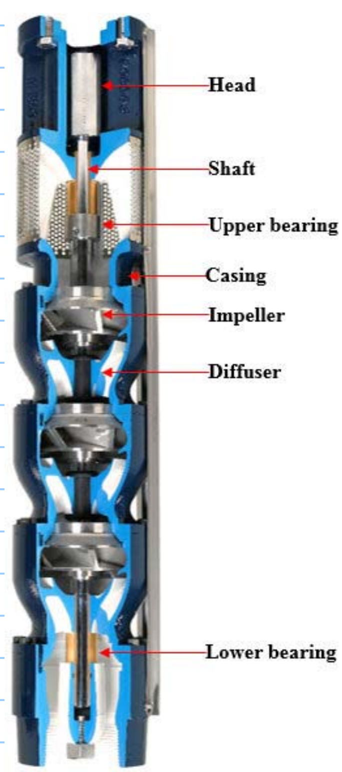
50 - 100 "stages"

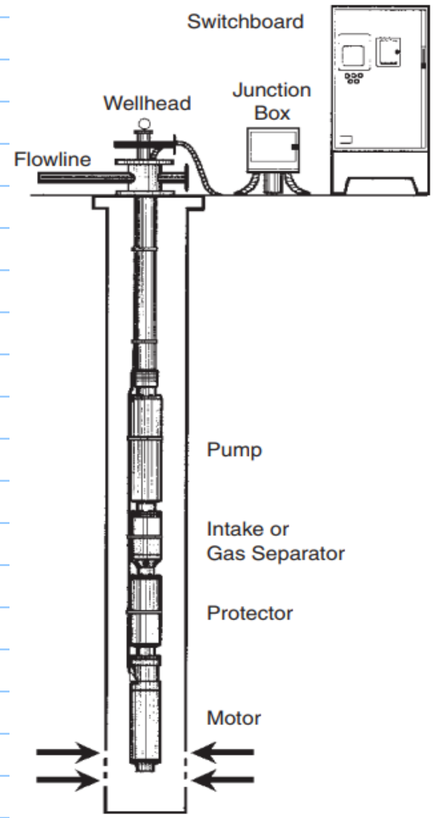


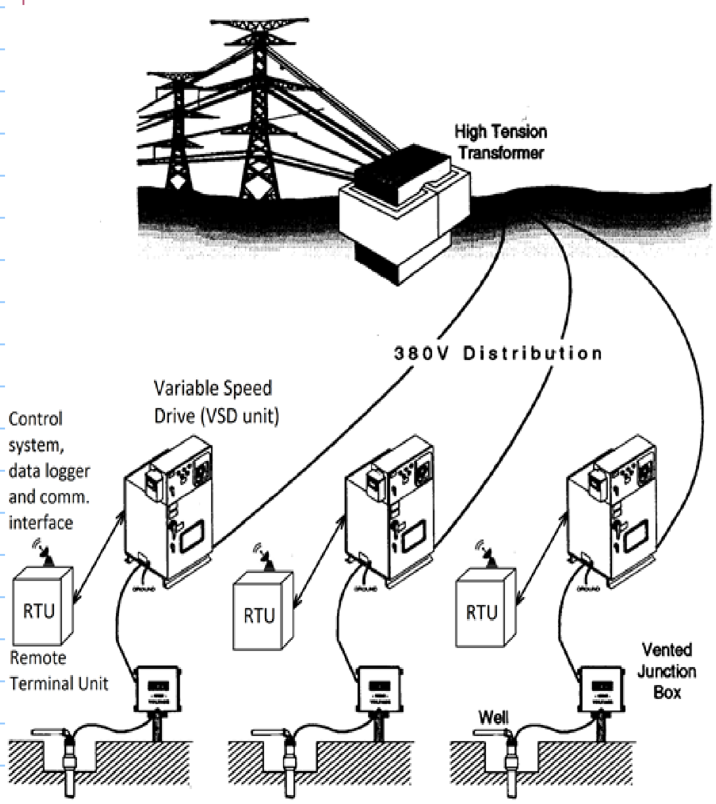


-Upthrust q is high
-Downthrust q is low





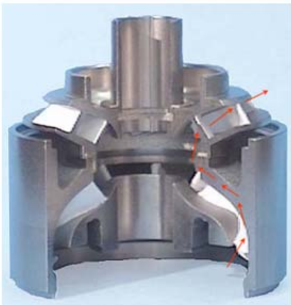




pump frequency

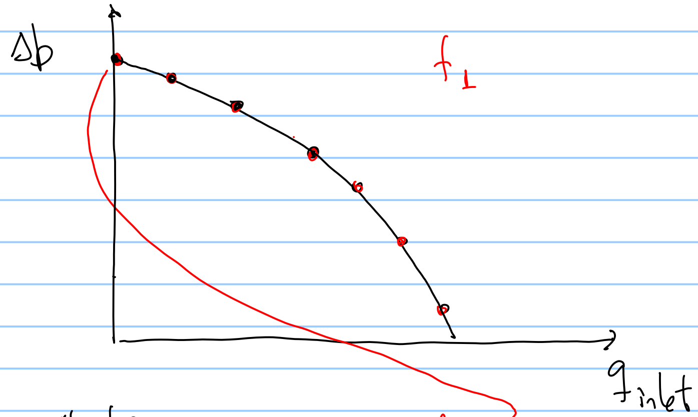
$f = 30 - 70\ Hz$

gas impeller and diffuser more
(tolerant to gas



Note Title

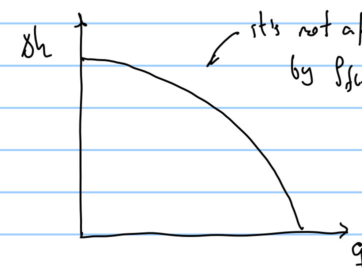
operational map of ESP



-Bench test — 1 stage, f_{ref} {50 Hz, 60 Hz}, water
-Definition of head

head

$$\delta h = h = \frac{\Delta p}{\rho_{fluid} \cdot g}$$



$$\Delta p = \delta h \cdot \rho_{fluid} \cdot g$$

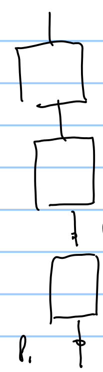
affected by viscosity

$\mu \approx \mu_w = 1\,cP$

Δp	q
Δp_1	q_1
Δp_2	q_2
Δp_3	q_3
⋮	⋮

$$\delta h = a \cdot q^2 + b\, q + c$$

Single stage vs multi-stage (50 - 100)



$\dot{m}$ = constant

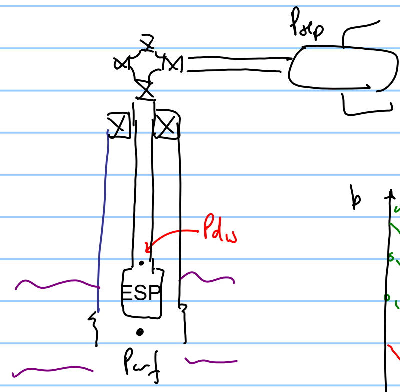
$$\Delta p_{esp} = \sum_{i=1}^{N_{stage}} \Delta p_i = N_{stages} \cdot \Delta p_{stage} \cdot F_{pacane}$$

$F_{pacane} < 1$

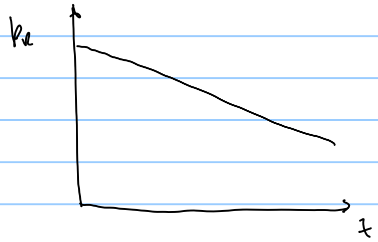
$q_{\bar{o}}$

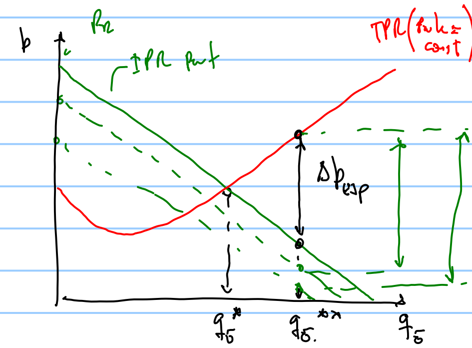
$$q_{o_i} = q_{\bar{o}} \cdot Bo_i (P_i, T_r)$$

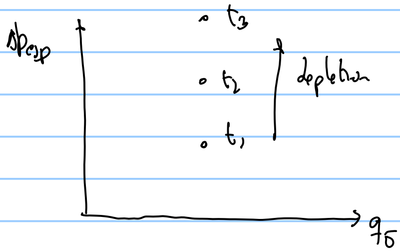
Variation of frequency - why?

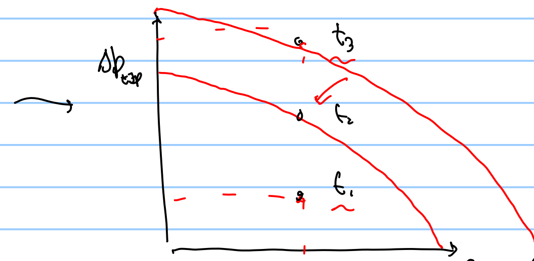


Depletion

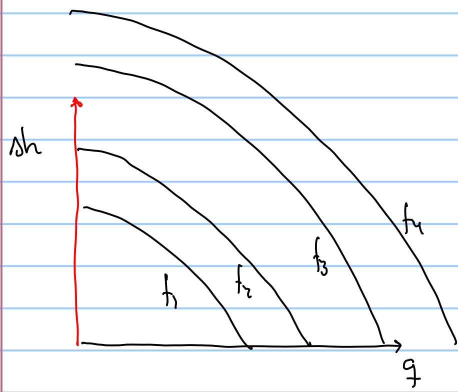








$q_o = q_\sigma \cdot B_o(p_{wc}, t_{wc})$



$f_4 > f_3 > f_2 > f_1$

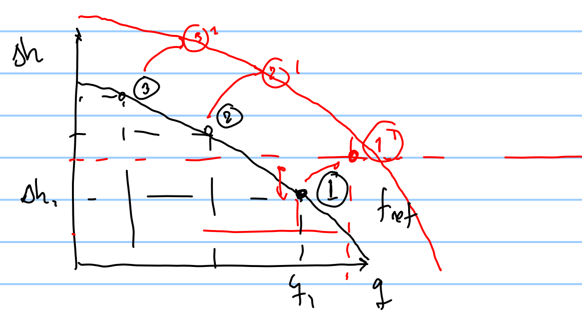
Analytical estimation:

Affinity laws
Similarity laws

$$Q_B = Q_A \cdot \frac{n_B}{n_A}$$

$$H_B = H_A \cdot \left(\frac{n_B}{n_A}\right)^2$$

$$P_B = P_A \cdot \left(\frac{n_B}{n_A}\right)^3$$



$$\frac{\Delta h_1}{\Delta h_{1*}} = \left(\frac{f_{ref}}{f}\right)^2 \Rightarrow \Delta h_{1*} = \Delta h_1 \cdot \left(\frac{f}{f_{ref}}\right)^2$$

$$\frac{q_1}{q_{1*}} = \left(\frac{f_{ref}}{f}\right) \Rightarrow q_1^* = q_1 \left(\frac{f}{f_{ref}}\right)$$

f_{ref}

Δh	q
D	D
D	D
D	D

$a q^2 + b q + c$

→ f

Δh	q
D	D
D	D
D	D

$$\frac{q_{ref}}{q} = \frac{f_{ref}}{f}$$

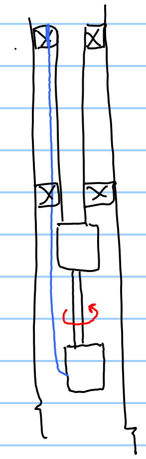
$$\Delta h_{ref} = a\, q_{ref}^2 + b\, q_{ref} + c$$

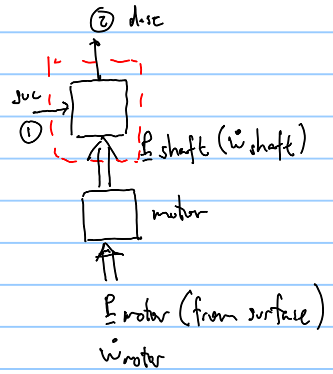
$$\Delta h_{ref} = a \cdot \left(\frac{f_{ref}}{f}\right)^2 q^2 + b \cdot \frac{f_{ref}}{f} q + c$$

$$\Delta h = \Delta h_{ref} \cdot \left(\frac{f}{f_{ref}}\right)^2 = a q^2 + b \left(\frac{f}{f_{ref}}\right) q + \left(\frac{f}{f_{ref}}\right)^2 C$$

Lecture 39 - Flow performance of ESP - part 2

Hydraulic efficiency





$$\eta_H = \frac{P_{pump}}{P_{shaft}}$$

$$\eta_m = \frac{P_{shaft}}{P_{motor}} \approx 0.95-0.98$$

$$\cancel{\dot{Q}} + \dot{W} + \dot{E}_1 - \dot{E}_2 = 0$$

$$\dot{W}_{shaft} = \dot{E}_2 - \dot{E}_1 = \dot{m}\,(e_2 - e_1)$$

$$e = \left(\cancel{g \cdot z} + \cancel{\frac{v^2}{2}} + h\right)$$

$h \to u + p\,v$

ideal pumping process $\Delta s = 0$ isentropic

$\Delta u = 0 \quad u = f(T)$

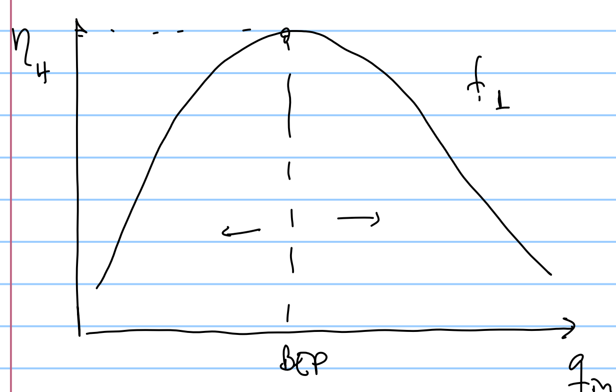
$$\dot{W}_{ideal} = \dot{m}\,(P_2 v_2 - P_1 v_1)$$

$$v = \text{constant} = \frac{1}{\rho}$$

$$\dot{W}_{ideal} = q\,(P_2 - P_1)$$

$$\eta_H = \frac{q\,(P_2 - P_1)}{P_{shaft}} = 0.4 - 0.8$$

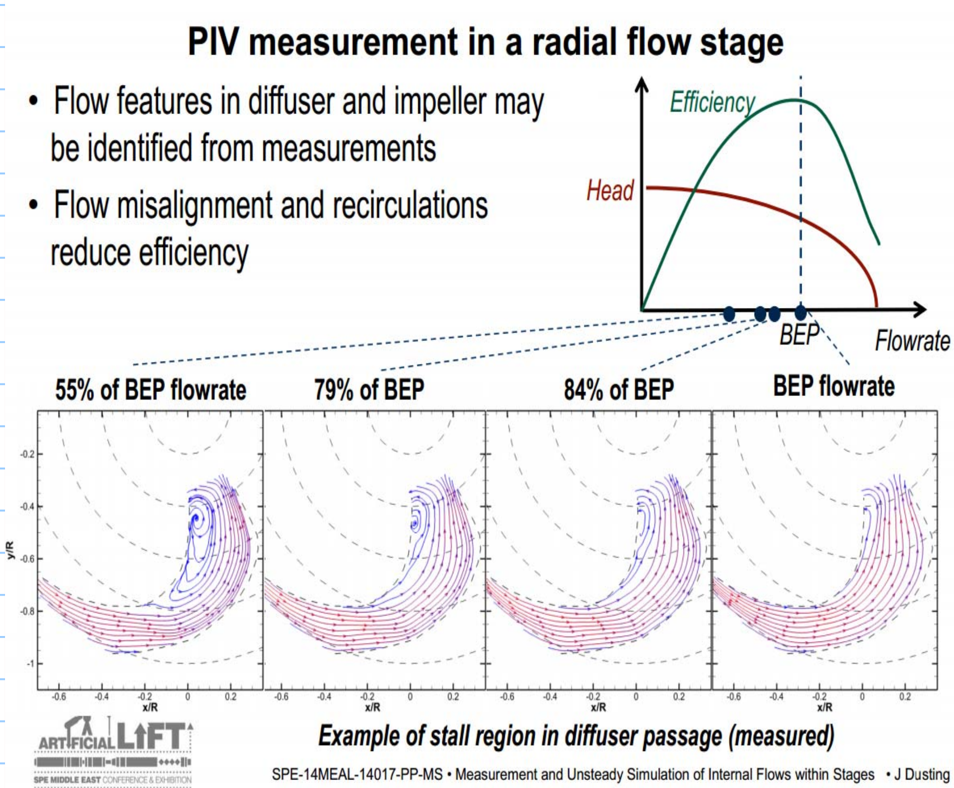
- friction: vanes, bearings, impeller
- volumetric: backflow in pump
- turbulence, eddies



$$\eta_H = b_1 q^2 + b_2 q + b_3$$

$$\dot{W}_{shaft} = \frac{\Delta p \cdot q}{\eta_H}$$

$$\dot{W}_{motor} = \frac{\dot{W}_{shaft}}{\eta_m}$$


PIV measurement in a radial flow stage
• Flow features in diffuser and impeller may be identified from measurements
• Flow misalignment and recirculations reduce efficiency
Efficiency
Head
BEP
Flowrate
55% of BEP flowrate
79% of BEP
84% of BEP
BEP flowrate
y/R
x/R
Example of stall region in diffuser passage (measured)
ARTIFICIAL LIFT
SPE MIDDLE EAST CONFERENCE & EXHIBITION
SPE-14MEAL-14017-PP-MS • Measurement and Unsteady Simulation of Internal Flows within Stages • J Dusting

analytical

$$Q_B = Q_A \cdot \frac{n_B}{n_A}$$

$$H_B = H_A \cdot \left(\frac{n_B}{n_A}\right)^2$$

$$P_B = P_A \cdot \left(\frac{n_B}{n_A}\right)^3$$

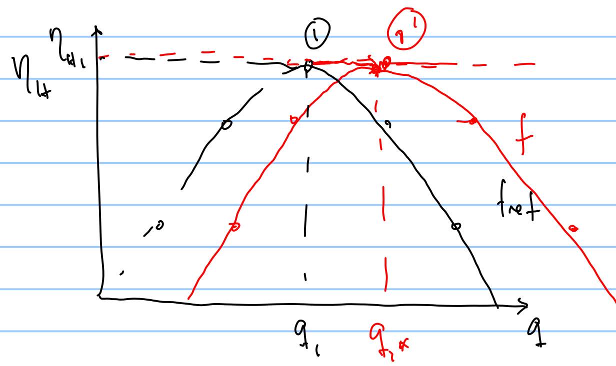
Similarity law

$$\eta_{H\,ref} = \frac{q_{ref} \cdot \Delta h_{ref}}{P_{shaft\,ref}} \longrightarrow \eta_{H@f} = \frac{q\,\Delta h}{P_{shaft}}$$

$$\eta_{H@f} = \frac{q_{ref}\cancel{\frac{f}{f_{ref}}}\,\Delta h_{ref}\cancel{\left(\frac{f}{f_{ref}}\right)^2}}{P_{shaft\,ref}\cancel{\left(\frac{f}{f_{ref}}\right)^3}}$$

ω_{shaft}

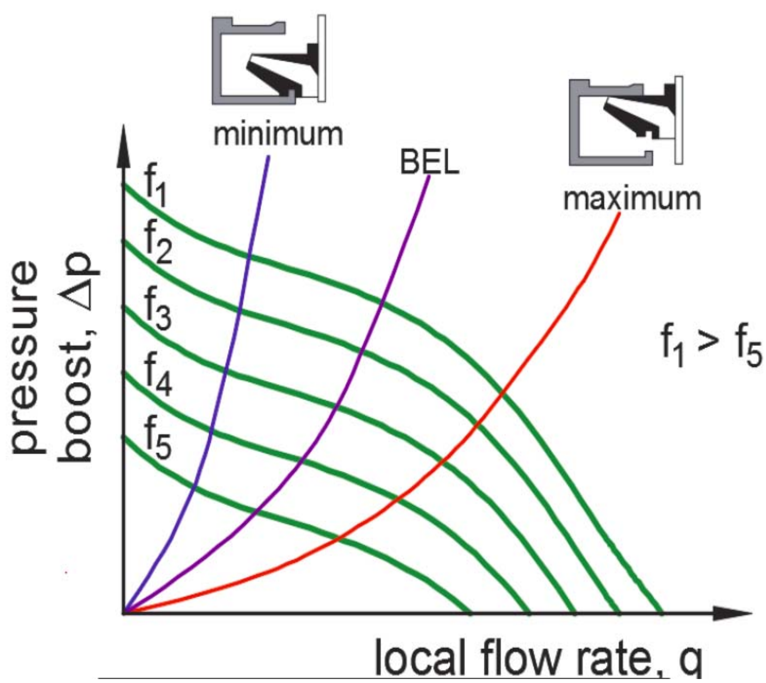
$$\eta_{H@f} = \eta_{H@f_{ref}}$$

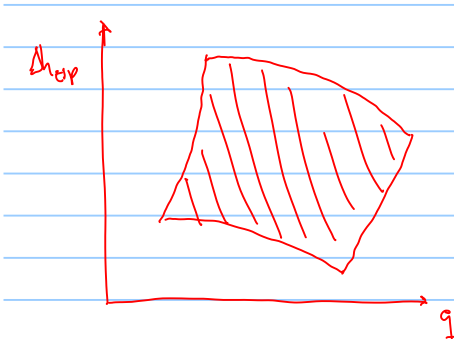


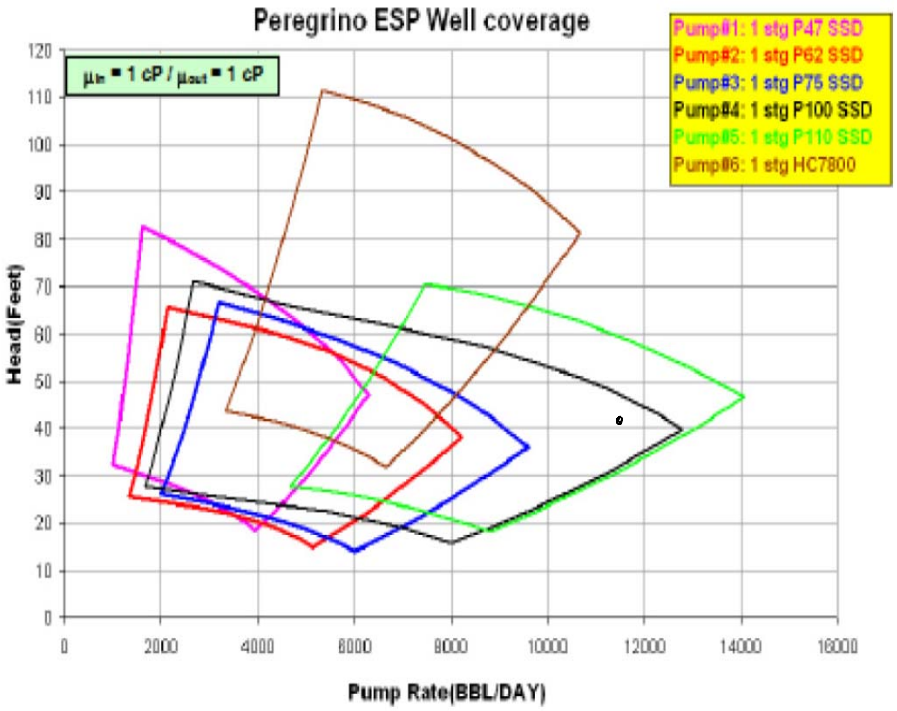
$$\eta_{H,q^*@f} = \eta_{H,1@f_{ref}}$$

$$q_{1@f_{ref}}$$

$$q_{2^*@f} = q_{1@f_{ref}} \cdot \frac{f}{f_{ref}}$$





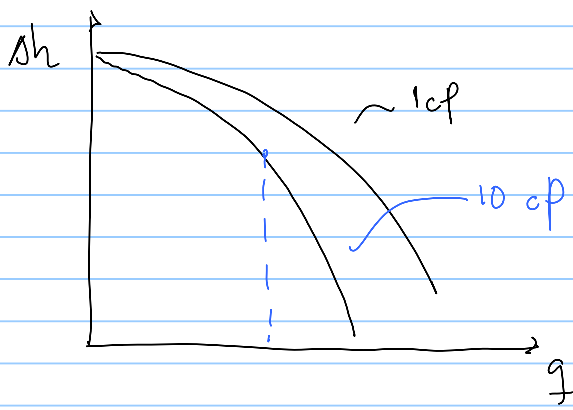


SPE-173948-MS

ESP Application on Heavy Oil in Peregrino Field

Vinicius Castro, and Daniela Leite, Statoil; Daniel Lemos, Jean Marins, Rui Pessoa, and João Magalhães, Baker Hughes

the viscosity of fluid also affects the performance of pump



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ANSI/HI 9.6.7-2010

American National Standard (Guideline) for

Effects of Liquid Viscosity on Rotodynamic (Centrifugal and Vertical) Pump Performance

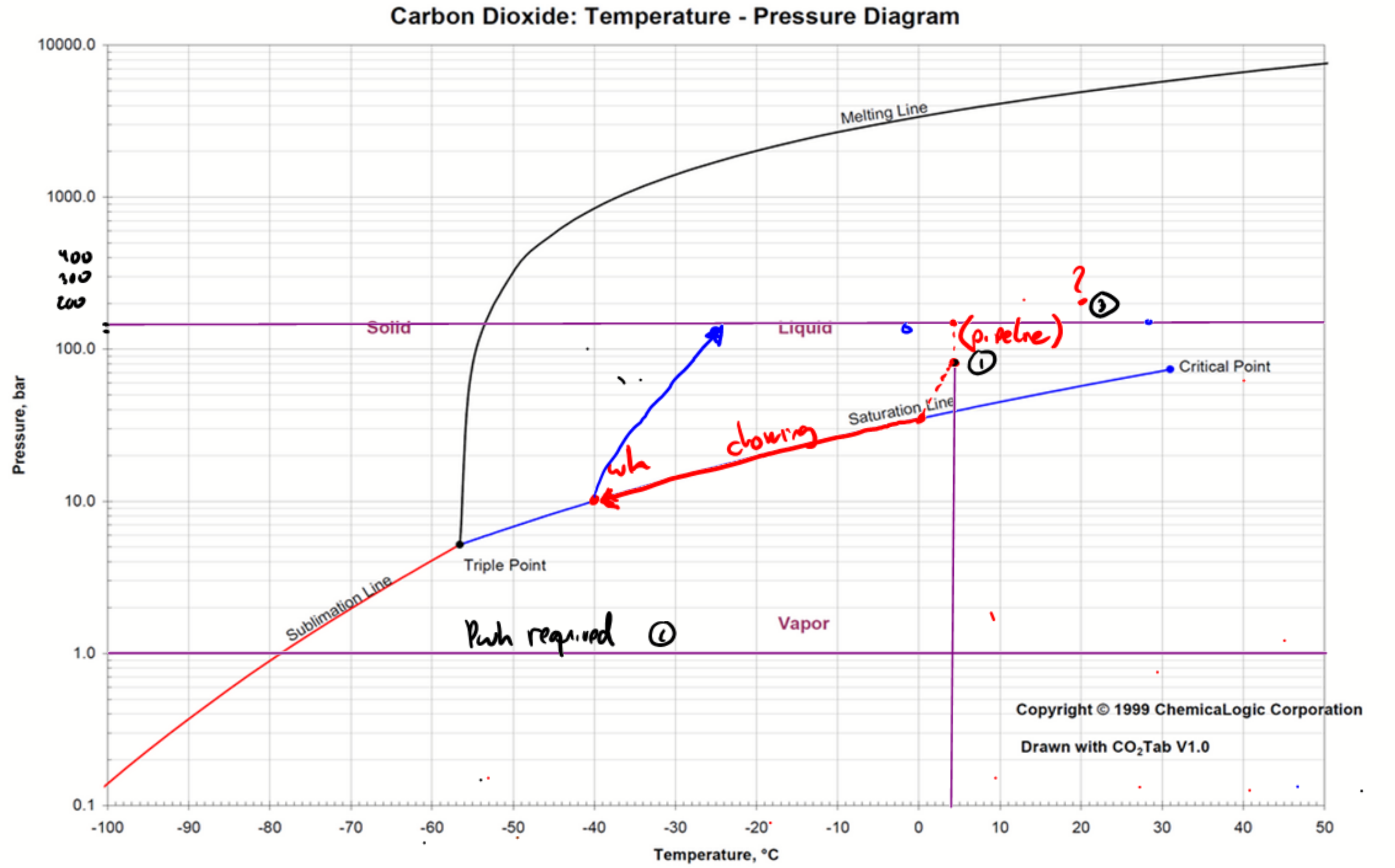
20241028

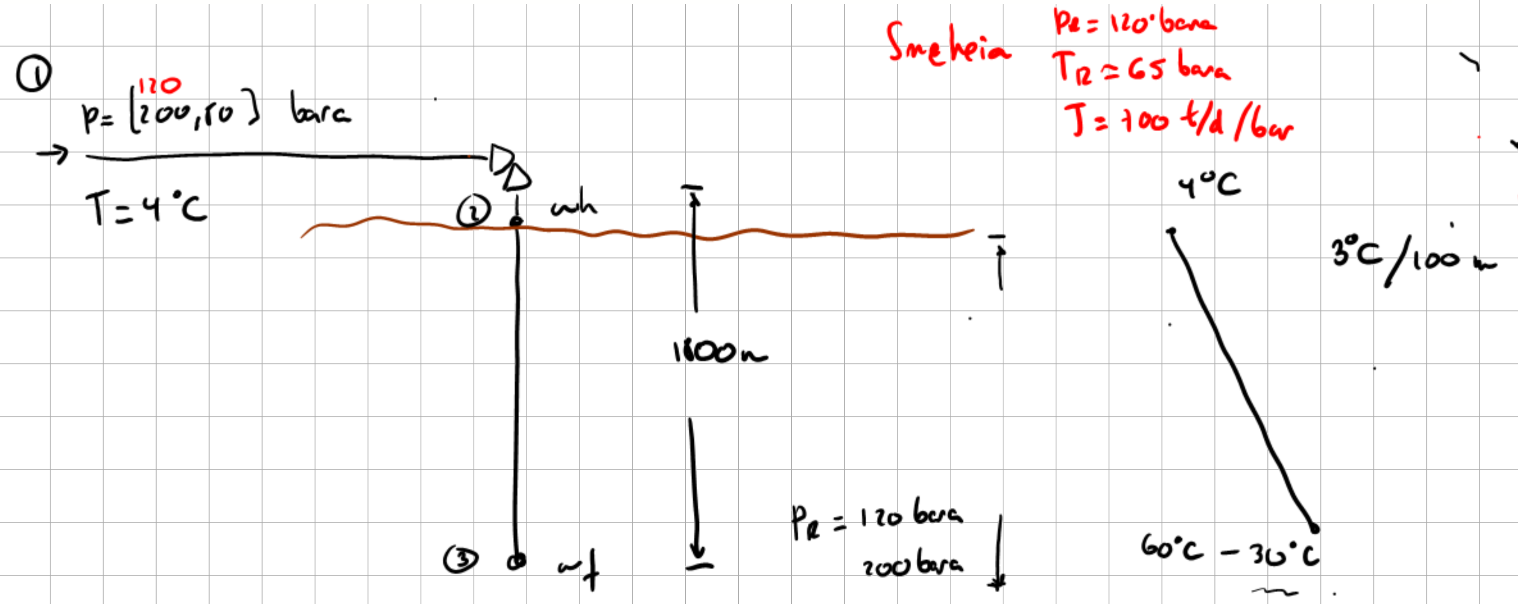
OUTLINE

-Occurrence of multiphase flow of CO2 in injection wells

-Recap of video lectures on artificial lift

-Exam questions





1.5.E6 t/y ≈ 4110 t/d

$$\underbrace{\dot{m}_{CO_2}}_{4110\ t/d} = \underbrace{J}_{700\ t/d/bar}\left(P_{wf} - \underbrace{P_R}_{120\ bara}\right)$$

Pwh required to have Pwf = 126 bara?

↓ p? ↓ 4110 t/d

126 bara

$$\frac{\dot{m}_{CO_2}}{J} + P_R = P_{wf}$$

$$4110/700 + 120 = P_{wf} = 126\ bara$$

$\mu = 0.05$ cP $\quad \Delta p_f << \Delta p_H \quad \Delta p_f \approx 0$

$$p_{wf} = p_{wh} - \Delta p_f - \Delta p_H$$

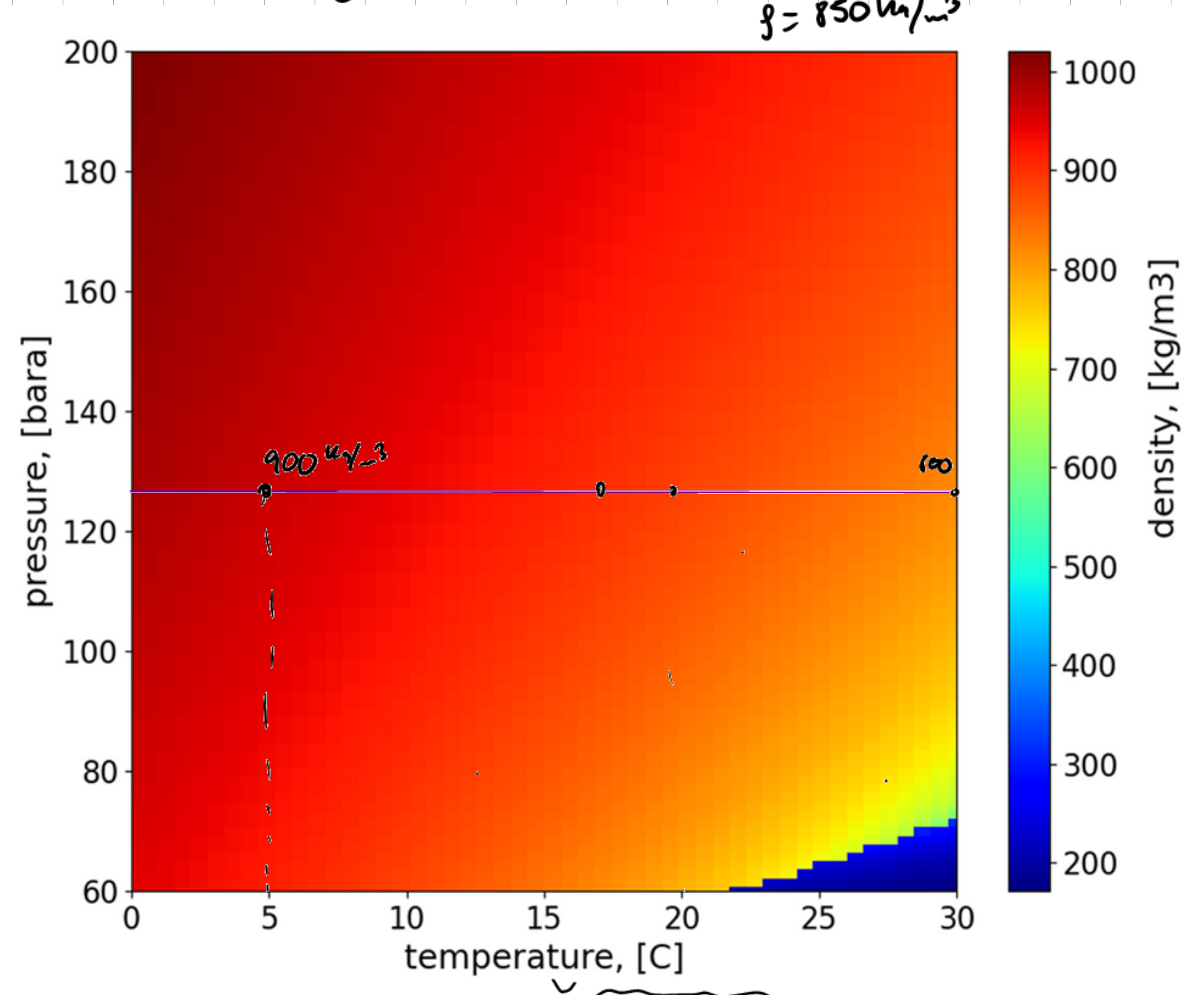
$v = 0.5 - 1$ m/s

$$126 = p_{wh} - \Delta p_H = p_{wh} - \frac{\rho \cdot \Delta h \cdot g}{1E5} = p_{wh} - \rho \cdot 1800\text{m} \cdot g$$

$$\Delta h = (z_{wf} - z_{wh}) = -1800 \text{ m}$$

at bottom-hole

$\rho = 850$ kg/m^3



$$-\frac{850 \cdot (-1800) \cdot 9.81}{1E5} + p_{wh} = 126 \text{ bara}$$

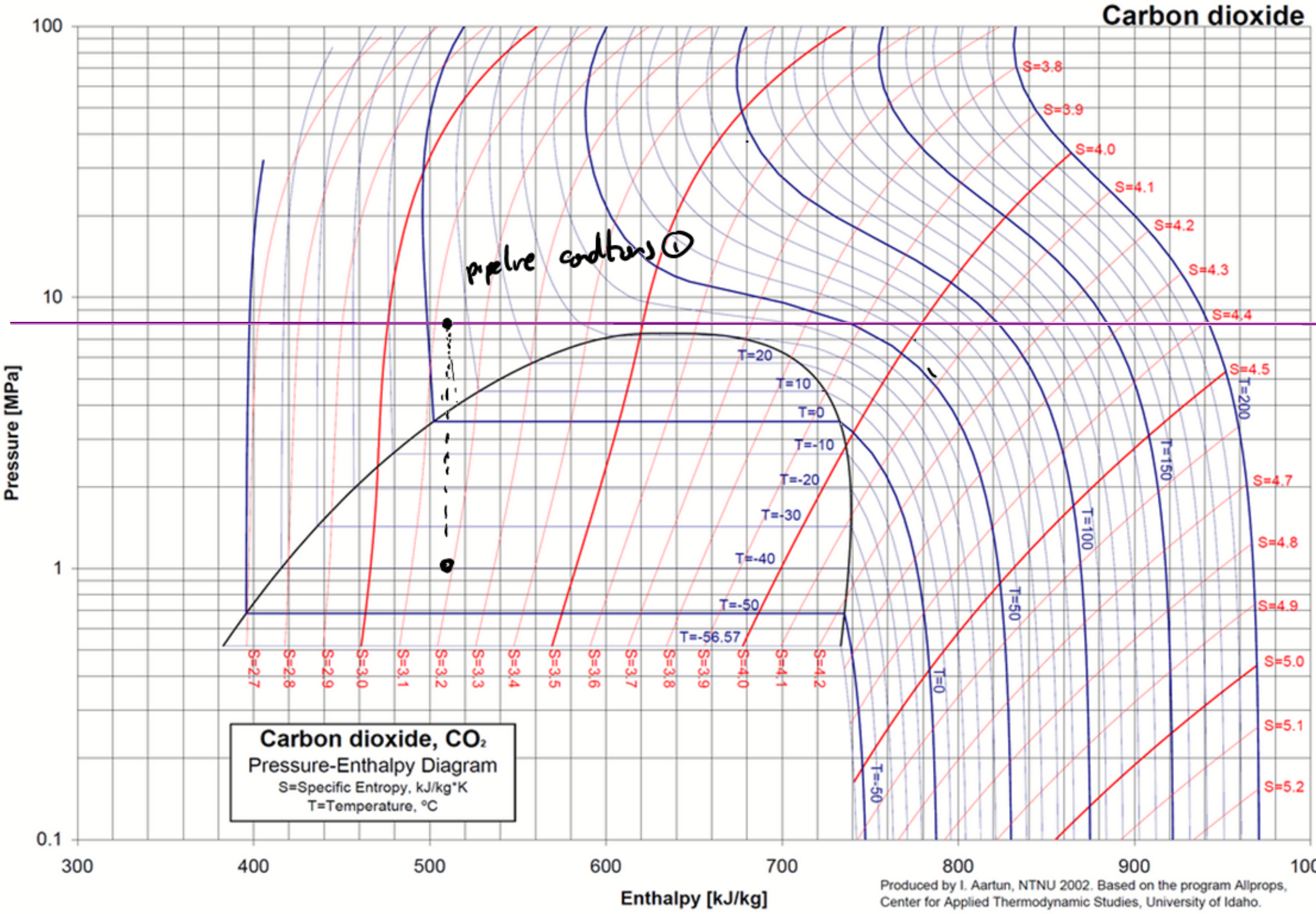
$p_{wh} = 126$ bara $- 150$ bara

$p_{wh} = -34$ bara

until $\Delta h = 1500$ m

$p_{wh} \geq 0$ bara

I use the choke to bring down the pressure from pipeline pressure to the pressure that I need at the wellhead, which is much lower than 80 bara.



Conditions for which I might have CO2 multiphase flow in the wellbore:

-Low pR(saline aquifer hydrostatic column (120 bara)
-Low pwf, high injectivity index (J)
-High pressure in the pipeline (higher than 80 bara)
-Wellhead choking

Why is it a concern to have two phase flow in the tubing?

$\dot{m}$ 4110 t/d $\quad \rho = 850 \ kg/m^3 \quad V = ? \quad ID = 0.168 \ m$

$$V = \frac{4110 \cdot 1000}{(3600 \cdot 24) \cdot 850 \ \pi (0.168)^2 \cdot 0.25} = 2.5 \ m/s$$

$$\rho_{vapor} = 100 \ kg/m^3$$

$$V = 21.25 \ m/s$$

$\Delta p_f \uparrow$ i need higher P_{wh} to inject 4110 t/d effect of impurities:

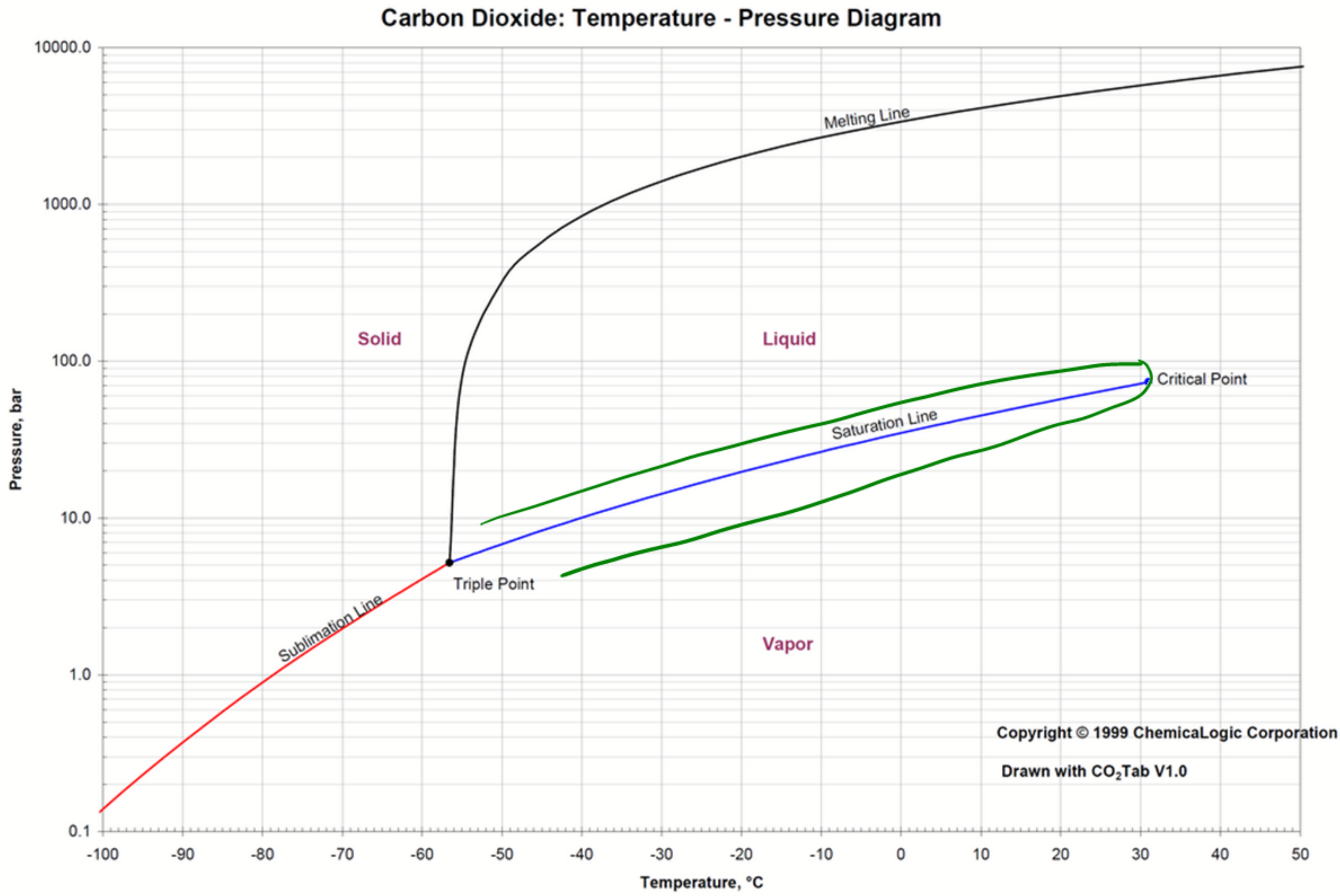
- two specs for injection CO_2

mole fraction $x_{CO_2} \geq 99.8\%$ food grade for tanker transport.

$x_{CO_2} \geq 96\%$ pipeline transport

if it is CO_2 captured from combustion N_2, NO_x, SO_x

makes the two phase region slightly bigger, and higher chance to fall into it



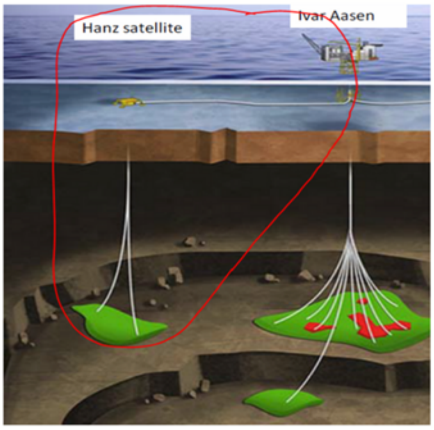
20241101
OUTLINE:
-Exam questions on artificial lift.

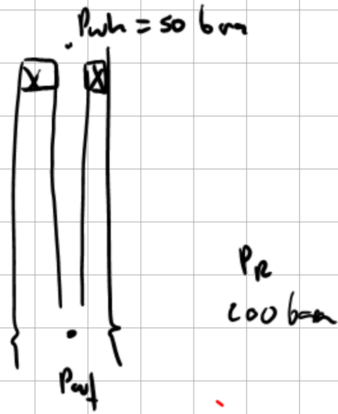
2021 - resitting examination

PROBLEM 3 (6 POINTS).

Hanz is a small undersaturated oil reservoir satellite to the Ivar Aasen platform. The reservoir will be developed using one single oil producer and a flowline connected to the platform. The flowline from the Xmas tree to the platform exhibits a very low pressure drop, thus, as a first approximation, the wellhead pressure can be safely assumed to have a constant value of 50 bara.

The well will be produced with open choke. The well doesn't produce any water.





The IPR of the well is provided:

$q_{\bar{o}} = J \cdot (p_R - p_{wf})$

available pressure

$P_{wf} = P_R - \frac{q_{\bar{o}}}{J}$

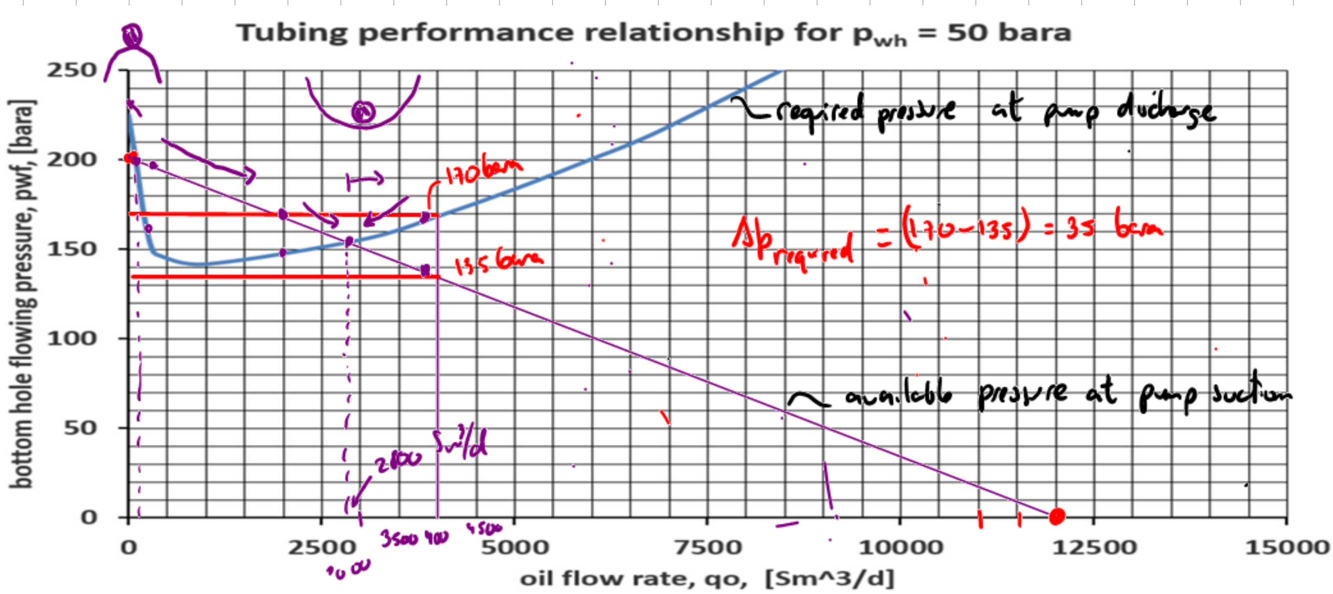
0

$q_{\bar{o}max} @ P_{wf} = 0 = 60 \cdot 200 = 12000$

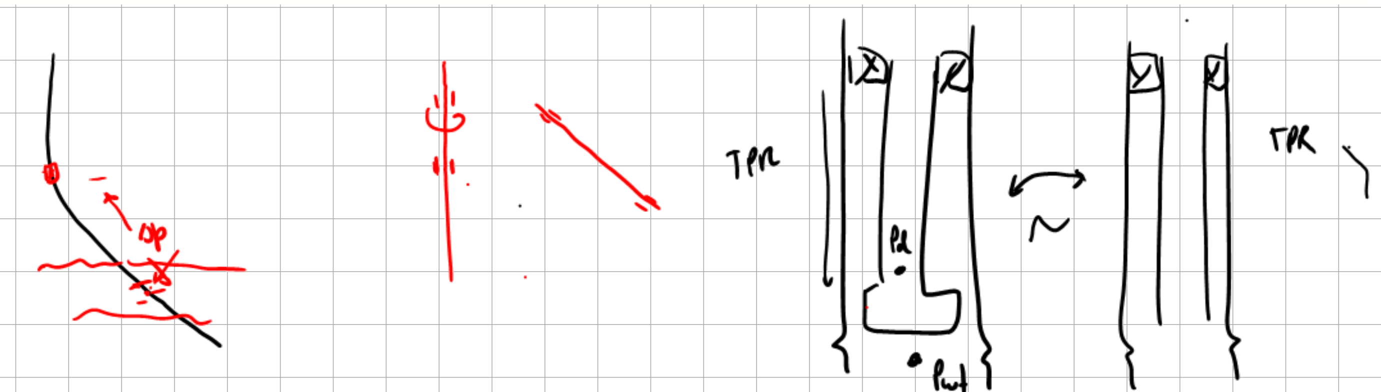
J = 60 Sm³/d/bar

With p_R= 200 bara.

The tubing performance relationship (required flowing bottom-hole pressure at constant wellhead pressure of 50 bara) is provided in the figure below.



Task 1. Is is possible for the well to produce 4000 Sm3/d by natural flow? If not, estimate the pressure boost required by the ESP pump located downhole.



Power required by the ESP?

$$\underline{P} = \frac{\Delta p \cdot q}{\eta_H \, \eta_m} \quad [W]$$

Δp in Pa; q in m³/s at pump suction; $\eta_H \eta_m$ in fraction

$\Delta p = 35$ bara

$\eta_m \geq 0.95$

$\eta_H \sim (0.6 - 0.8)$

$$q = q_L = q_o + \cancel{q_w}^{\,0} = q_o$$

$q_{\bar{o}}$

$$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \dfrac{B_g}{1 - R_s \cdot r_s} & \dfrac{-R_s \cdot B_g}{1 - R_s \cdot r_s} & 0 \\ \dfrac{-B_o . r_s}{1 - R_s \cdot r_s} & \dfrac{B_o}{1 - R_s \cdot r_s} & 0 \\ 0 & 0 & B_w \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix}$$

$r_s \approx 0$

m³/d

$q_o = B_o \, q_{\bar{o}}$

B_o: (1.05 ~ 1.7)

PROBLEM. (100 POINTS)

You are part of the well planning team in AkerBP that is tasked with designing a vertical production well for the Noaka development. The reservoir consists of an undersaturated oil layer.

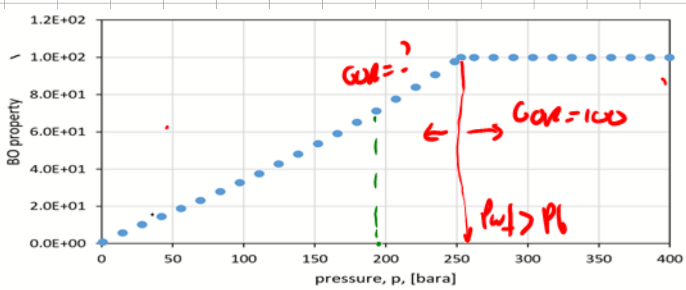
The well is vertical, it has a tubing and a bottom packer, both placed close to the formation and to the perforations, therefore the pressure drop from the perforations to the bottom of the tubing can be neglected. The lower completion consists of a perforated cemented casing.

4. **(20 POINTS)** The reservoir engineers have determined that the natural flow rate is not high enough and that artificial lift is required. Consider a downhole electric submersible pump located at the end of the tubing, in front of the perforations. Estimate pump delta pressure, oil volumetric rate at pump suction and required pumping power to produce as much oil as possible. Make sure that the suction pressure does not go below the bubble point pressure of the oil at reservoir temperature.

This question is similar to the one solved above. The only difference is that TPR is not given graphically, but in tables. But which TPR curve to use? (for which GOR), what is the reservoir GOR?

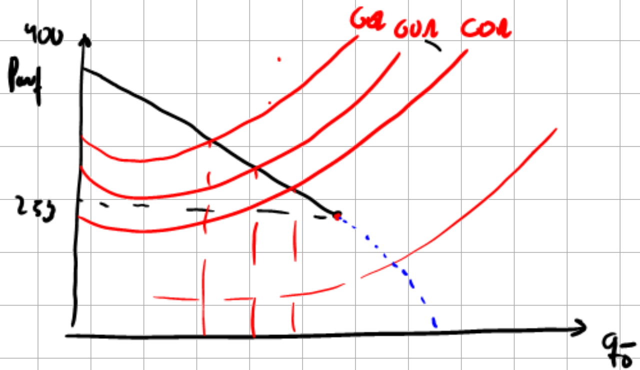
-Undersaturated oil reservoir --> single phase flow of oil entering into the wellbore
-BO properties at TR versus p are available.

$$GOR_{Reservoir} = R_s \,@\, p_R = 100$$



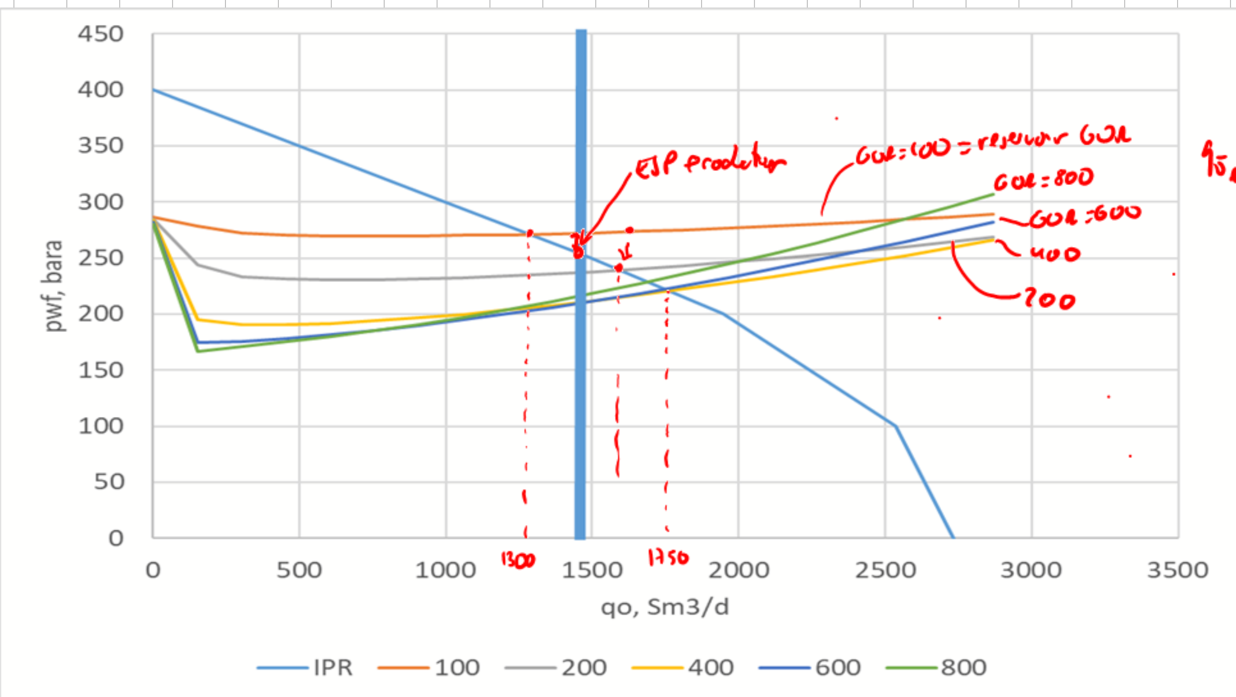
qo will be maximum, when pwf is minimum, but pwf>=pb (253 bara), then pwf=253 bara

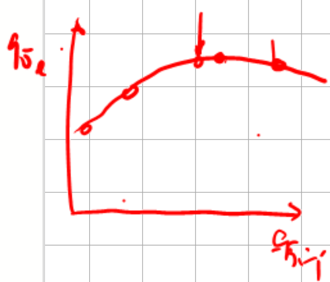
5. **(20 POINTS)** An engineer in the office has suggested using gas lift, because then the suction pressure can go below the bubble point pressure and one can potentially produce more from the reservoir. The gas lift valve will be placed as close as possible to the bottom of the tubing, and therefore, the gas-lift analysis can be performed considering that the tubing GOR is changing. In the Excel file, several TPR curves are given for different values of Rp (GOR). Determine if using gas lift is a better idea to increase production and estimate the optimal amount of gas lift rate required.



IPR for $P_{wf} \geq P_b$ $J(P_R - P_{wf})$

$P_{wf} < P_b$





GOR: 400

$q_o \approx 1750 \ Sm^3/d$

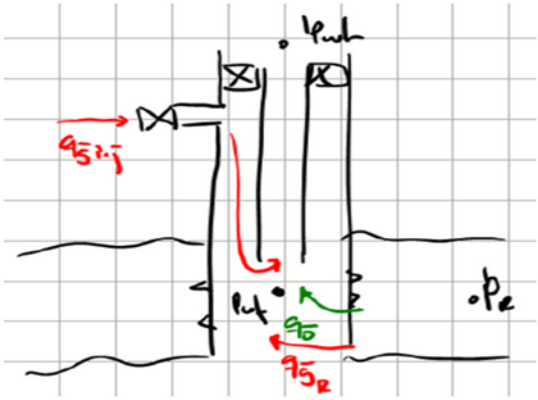
$q_{g_R} = 1750 \cdot 100 = 175000 \ Sm^3/d$

$q_{g_{TOTAL}} = \underline{1750 \cdot 400} =$

$GOR_T - GOR_R$

$q_{g_{inj}} = q_{g_{TOTAL}} - q_{g_R} = \underline{1750\ (400 - 100)} = 52500 \ Sm^3/d$

Consider a gas-lifted oil well. The injection point is very close to the bottom of the tubing, so it is reasonable to assume that the lift gas is injected at the end of the tubing (see the figure below). The end of the tubing is very close to the perforations.

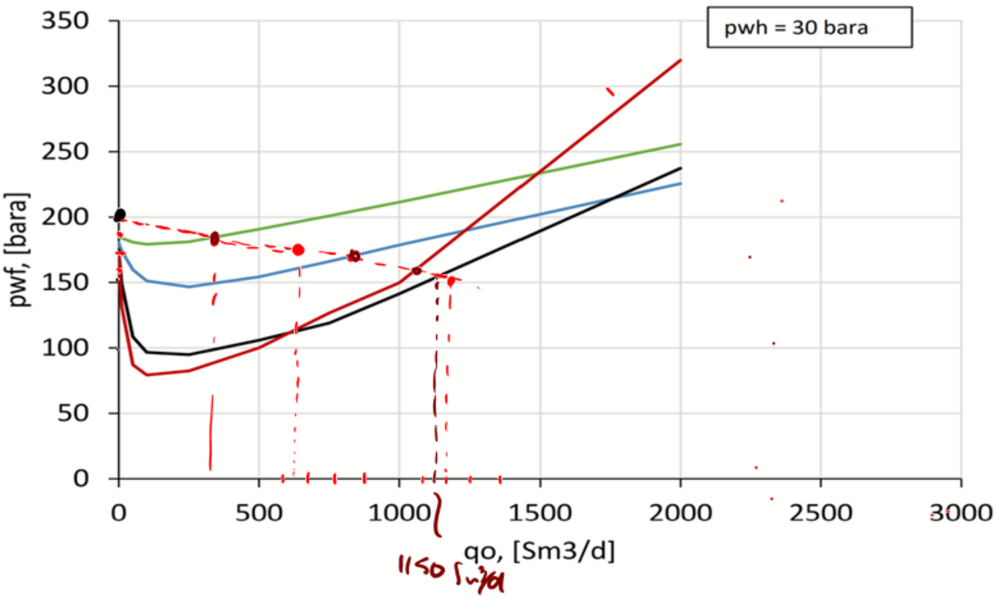


The reservoir GOR is equal to 54. The reservoir IPR can be modeled with Vogel equation:

$$q_{\bar{o}} = q_{\bar{o},max}\left[1 - 0.2 \cdot \frac{p_{wf}}{p_R} - 0.8 \cdot \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

using a pR = 200 bara, and a qomax = 3000 Sm3/d.

The figure below shows the curves of Tubing performance relationship at a constant wellhead pressure of 30 bara, for different values of GOR in the tubing.



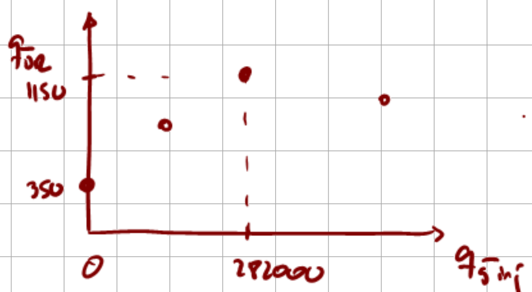
pwf	pwf/pR	qo
175	0.875	637.5
150	0.75	1200
125	0.625	1687.5
100	0.5	2100

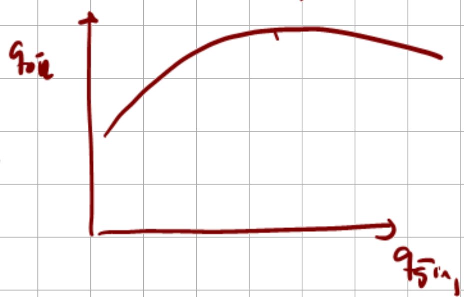
1150 (300-54) = 282000

Task 1. Determine the optimal gas lift injection rate (i.e. the one that gives the highest reservoir oil rate). Use only the tubing GORs that are provided in the figure. Explain the procedure you followed.

Additional information:

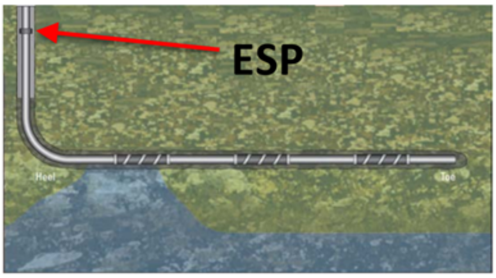
- Solve this problem graphically.





PROBLEM 2 (20 POINTS). Estimation and verification of ESP requirements

The Rio Ariari complex is a field currently under development in the region of “Los llanos” in Colombia. The reservoir has a thin layer (19 m) containing undersaturated oil and a very strong bottom aquifer. There is coning from the water layer into the well
The field will be produced with ESP-lifted horizontal wells like the one shown in the figure below.



The Wood company has proposed a unique ESP model (TE7000) with 50 stages which they claim has a wide operational envelope to handle all possible well operational conditions.
Your task is to estimate and verify the ESP requirements for a well in the Ariari Field.

TASKS:

Task 1 (5 POINTS). What is the effective viscosity (in cP) of the oil-water mixture (using the Richardson emulsion equation) when the well is producing a total liquid rate of 250 Sm³/d with 54% water cut?
Explain how you have performed this task.
Task 2 (10 POINTS). -For the total liquid rates of 250 Sm³/d, estimate the required pump pressure boost (DP in bar, input a positive number) and pump power (in kW) to deliver the rate if the wellhead pressure is constant and equal to 40 bara.
Explain how you have performed this task.
Task 3 (5 POINTS). -According to the ESP envelope given below, will the ESP model suggested be able to deliver the desired rate of 250 Sm³/d?
Explain how you have performed this task.

WC is input in fraction.
This equation is programmed in a VBA function called “Avprop” provided in the Excel sheet.

- The oil+water mixture exhibits an emulsion behavior where its viscosity is a function of the water volume fraction. The cutoff watercut is 60%.

Regime	Richardson emulsion viscosity
Oil continuous (WC < 60%)	$\mu_m = \mu_o \cdot e^{3.215 \cdot WC}$
Water continuous (WC > 60%)	$\mu_m = \mu_w \cdot e^{3.089 \cdot (1-WC)}$

The viscosity of the oil is 10 cp and viscosity of the water is 1 cp (1 cp = 1 E-3 Pa s). **WC is input in fraction.**

local rate.

$$wc = \frac{q_w}{q_o + q_w}$$

$q_{\bar w} \to q_w$

$q_{\bar o} \to q_o$

- Assume that the oil compressibility and GOR can be neglected such as the rate at standard conditions is equal to the rate at local conditions p and T.

$q_{\bar w} = q_w$

$q_{\bar o} = q_o$

$q_{\bar L} = q_{\bar o} + q_w = 250$

$q_{\bar w} = 0.54 \cdot 250 = 135$

$q_{\bar o} = 0.46 \cdot 250 = 115$

$\mu_m = \underbrace{\mu_o}_{10} \cdot e^{3.215 \cdot 0.54} =$

Pwf available (from IPR)

Pwf required (from TPR)

$\Delta p_{pum} = P_{wf\,req} - P_{wf\,available}$

$P = \dfrac{\Delta P \cdot q}{\eta_A \cdot \eta_m}$

$\rho_m = \rho_o (1 - wc) + \rho_w (wc)$

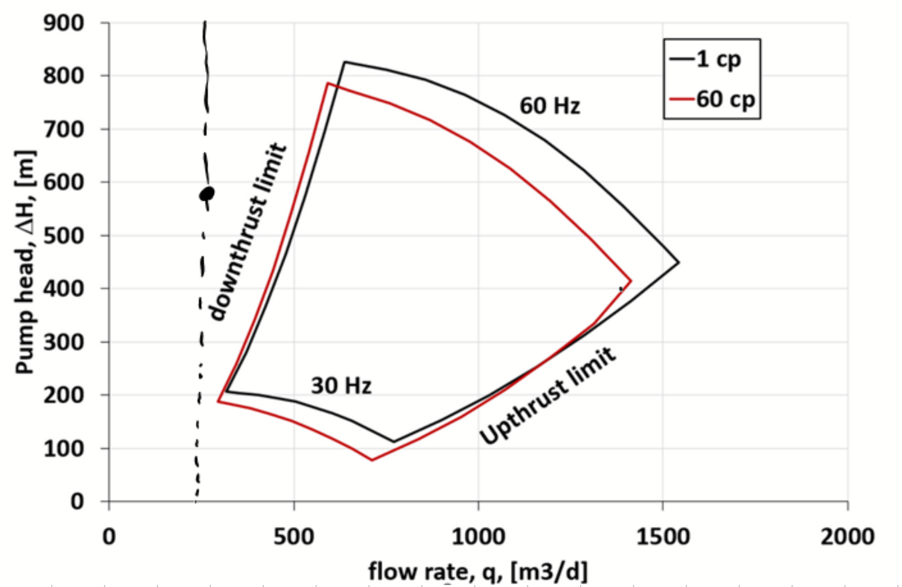
```
Function pin(qt, ID, den, visc, Length, teta, Pout, roughness)
'Function that give pressure required at the inlet of a pipe in order to have the flow qt and discharge pressure Pout
'Calculation made for liquid single phase flow
'Takes in data in SI
'qt flow [Sm^3/d]
'ID inner diameter of pipe [m]
'den density of fluid, [kg/m^3]
'visc viscosity of fluid, [Pa s]
'Length, pipe length, [m]
'teta inclination angle of pipe with respect to horizontal [°]
'Pout, discharge pressure required, [bara]
'roughness of pipe [m]

'Gravitational acceleration g, [m/s{2]
g = 9.81
'Pi number
Pi = 4 * Atn(1)
qt = qt / (3600# * 24#)  '[m^3/s]

'Calculating area and velocity
Area = Pi * (ID ^ 2) / 4
v = qt / Area

Presscalc1 = Pout + (Length * Sin(teta * Pi / 180) * den * g / 100000#) + (ffactor(den, visc, ID, roughness, v) * Length * (v ^ 2) * den / (ID * 200000#))
pin = Presscalc1

End Function
```



$\Delta H = \frac{\Delta P}{\rho \cdot 10} = \frac{56.4E5}{966 \cdot 9.81} = 595\ m$

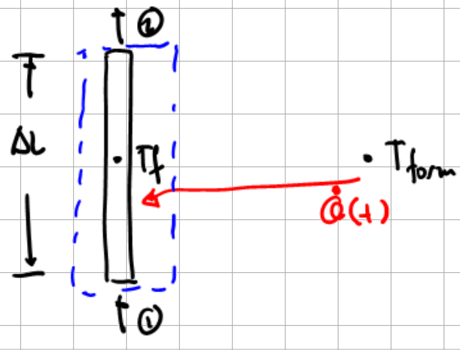
$\mu_m = 57\ cP$

Lecture 40: ESP exercise

Goal of the exercise: verify that the ESP model selected for the application will work properly for 5 times of production profile
-Verify that the points of dh vs q fall inside the operational envelope of the pump
-Estimate ESP frequency and hydraulic efficiency for all years
-Compute required pumping power

Study case: Peregrino field, offshore Brazil

Lecture 41: Temperature calculations in wellbore



8. HEAT TRANSFER FOR FLOW IN CONDUITS

The equation for conservation of energy for a section of a conduit is

$$\dot{Q} + \dot{W} = \dot{m} \cdot (e_{out} - e_{in}) \qquad \text{EQ. 8-1}$$

The specific energy that the stream has is usually split in internal energy (u), potential energy (z·g) and kinetic energy ($V^2/2$).

A conduit doesn't exchange work with the surroundings, but the fluid must perform work to enter and leave the system. This specific work is: $(p_{in} \cdot v_{in} - p_{out} \cdot v_{out})$ (Here v is specific volume).

By combining the inlet and outlet specific internal energy "u" with the specific work to enter and leave the system to obtain specific enthalpy, the energy conservation equation is written as:

$$\dot{Q} = \dot{m} \cdot \left(h_{out} + z_{out} \cdot g + \frac{(V_{out})^2}{2} - h_{in} - z_{in} \cdot g - \frac{(V_{in})^2}{2} \right) \qquad \text{EQ. 8-2}$$

Or, alternatively

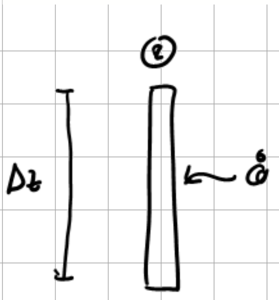
$$\dot{Q} = \dot{m} \cdot \left(\Delta h + \Delta z \cdot g + \frac{(V_{out})^2}{2} - \frac{(V_{in})^2}{2} \right) \qquad \text{EQ. 8-3}$$

Here Δ represents outlet minus inlet.

In differential form (for an infinitesimally small length of pipe) the equation can be expressed as follows:

$$\frac{d\dot{Q}}{dL} = \dot{m} \cdot \left(\frac{dh}{dL} + g \cdot \frac{dz}{dL} + \cancel{v \cdot \frac{dv}{dL}}^{\approx 0} \right) \qquad \text{EQ. 8-4}$$

Heat leaving the system is negative (the temperature of the outlet fluid is lower than the temperature at the inlet and the term Δh is usually negative). Heat entering the system is positive.



$$\dot{Q} = \dot{m}(h_2 - h_1 + \Delta z \cdot g)$$

p distribution in known $p(z)$

$$\dot{Q} = f(T_f, T_{form})$$
$\hookrightarrow T_1$

$$h_2 = \frac{\dot{Q}}{\dot{m}} + h_1 - \Delta z \cdot g$$

with P_2, assume $T_2 \rightarrow h_2^{calc}$

① $P_{wf}, T_{wf} (T_R), h_1 \{ x_{g1}, x_{o1}, x_{w1}, h_{g1}, h_{o1}, h_{w1})$

specific enthalpy (h):

-Another BO property, just like viscosity, density, etc
-calculated at p-T with e.g. EOS

Mixture specific enthalpy:

$$h_{mix} = x_g \cdot h_g + x_o h_o + x_w h_w$$

$$x_g = \frac{\dot{m}_g}{\dot{m}_T} \quad \text{mass fraction}$$

p [bara]	T [C]	ho [kJ/kg]	hg [kJ/kg]
300.0	148.0	-2108.26	-3529.99
300.0	137.1	-2136.36	-3602.31
300.0	126.2	-2164.67	-3672.1
300.0	115.3	-2193.21	-3739.34
300.0	104.4	-2221.97	-3803.95
300.0	93.6	-2250.97	-3865.83
300.0	82.7	-2280.24	-3924.82
300.0	71.8	-2305.53	0
300.0	60.9	-2329	0
300.0	50.0	-2351.97	0
285.7	148.0	-2094.13	-3547.64
285.7	137.1	-2122.44	-3619.91
285.7	126.2	-2150.93	-3689.79
285.7	115.3	-2179.62	-3757.24

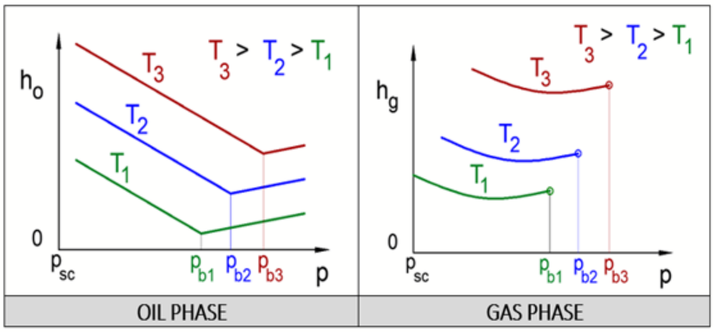


FIGURE 8-6. BEHAVIOR OF SPECIFIC ENTHALPY OF GAS AND OIL VS. PRESSURE FOR THREE TEMPERATURES

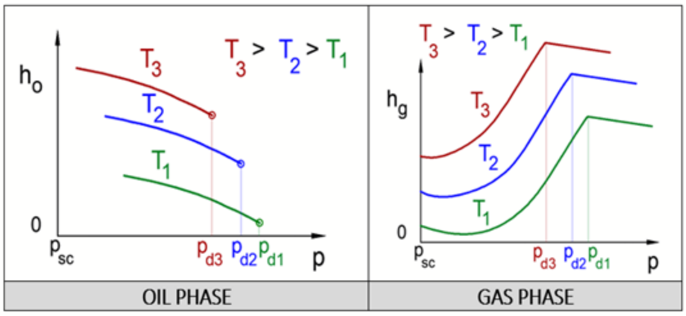
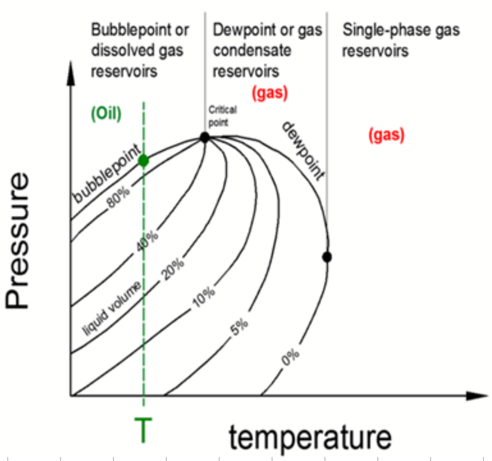
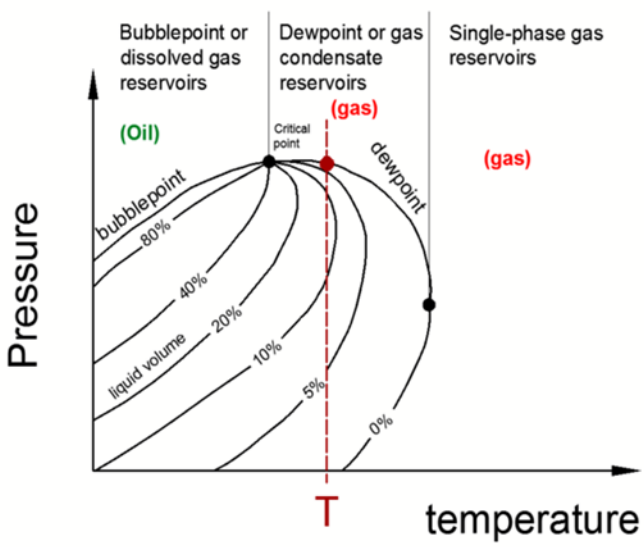


FIGURE 8-8. BEHAVIOR OF SPECIFIC ENTHALPY OF GAS AND OIL VS. PRESSURE FOR THREE TEMPERATURES





$\Delta h = C_p \cdot \Delta T$ for liquids

for gasses $C_p = f(p,T)$ and requires frequent update

$\boxed{\text{Heat term } (\dot{Q})}$

sign

$$\dot{Q} = -2 \cdot \pi \cdot L \cdot r \cdot U \cdot (T_f - T_\infty) \qquad \text{Eq. 8-20}$$

Where:

r	reference radius [m]
U	Overall heat transfer coefficient, expressed in terms of the reference radius r [W/m^2.K]
T_∞	Mean ambient temperature [K or °C]
T_f	Mean fluid temperature in the section [K or °C]

In this section we will work with the heat by unit of conduit length $\frac{\dot{Q}}{L} = \frac{d\dot{Q}}{dL}$.

Heat transfer mechanisms:

-Forced convection
-Free convection
-Conduction

In pipes:

$$\mathrm{Nu}_L = \frac{\text{Convective heat transfer}}{\text{Conductive heat transfer}} = \frac{h}{k/L} = \frac{hL}{k}$$

$$\frac{d\dot{Q}}{dL} = -2 \cdot \pi \cdot r_i \cdot h_i \cdot (T_f - T_i)$$

forced: $\mathrm{Nu} = f(\mathrm{Re}, \mathrm{Pr})$

free: Nu= f(Gr,Pr)

IMPLICIT! T is required to find h!!

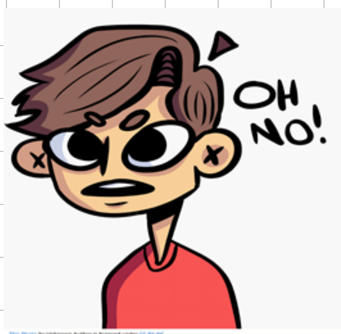
steady state $$\frac{d\dot{Q}}{dL} = -2 \cdot \pi \cdot k_p \cdot \frac{(T_i - T_o)}{\ln\left(\frac{r_o}{r_i}\right)}$$

Transient?

ATTENTION!!!

-h can be pump head (deltap/(den*g))
-h can be specific enthalpy (u+p*v)
-h can be forced/free convection coefficient

BE CAREFUL!!!



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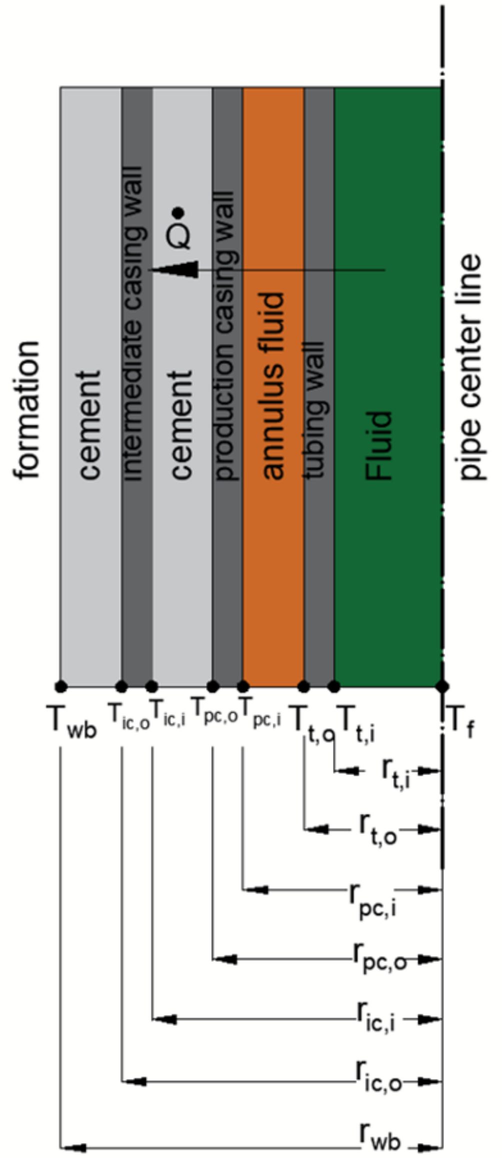
Example:



Overall heat transfer coefficient:

$$(T_f - T_{t,i}) + (T_{t,i} - T_{t,o}) + (T_{t,o} - T_{pc,i}) + (T_{pc,i} - T_{pc,o}) + (T_{pc,o} - T_{ic,i}) + (T_{ic,i} - T_{ic,o}) + (T_{ic,o} - T_{wb}) = \frac{-\frac{d\dot{Q}}{dL}}{2\cdot\pi\cdot r_{t,i}\cdot h_i} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2\cdot\pi\cdot k_t}{\ln\left(\frac{r_{t,o}}{r_{t,i}}\right)}} + \frac{-\frac{d\dot{Q}}{dL}}{2\cdot\pi\cdot r_{t,o}\cdot h_{ann}} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2\cdot\pi\cdot k_{pc}}{\ln\left(\frac{r_{pc,o}}{r_{pc,i}}\right)}} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2\cdot\pi\cdot k_c}{\ln\left(\frac{r_{ic,i}}{r_{pc,o}}\right)}} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2\cdot\pi\cdot k_{ic}}{\ln\left(\frac{r_{ic,o}}{r_{ic,i}}\right)}} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2\cdot\pi\cdot k_c}{\ln\left(\frac{r_{wb}}{r_{ic,o}}\right)}}$$

Clearing the temperature difference between fluid and wellbore wall:

$$(T_f - T_{wb}) = -\frac{d\dot{Q}}{dL} \cdot \left[\frac{1}{2\cdot\pi\cdot r_{t,i}\cdot h_i} + \frac{1}{\frac{2\cdot\pi\cdot k_t}{\ln\left(\frac{r_{t,o}}{r_{t,i}}\right)}} + \frac{1}{2\cdot\pi\cdot r_{t,o}\cdot h_{ann}} + \frac{1}{\frac{2\cdot\pi\cdot k_{pc}}{\ln\left(\frac{r_{pc,o}}{r_{pc,i}}\right)}} + \frac{1}{\frac{2\cdot\pi\cdot k_c}{\ln\left(\frac{r_{ic,i}}{r_{pc,o}}\right)}} + \frac{1}{\frac{2\cdot\pi\cdot k_{ic}}{\ln\left(\frac{r_{ic,o}}{r_{ic,i}}\right)}} + \frac{1}{\frac{2\cdot\pi\cdot k_c}{\ln\left(\frac{r_{wb}}{r_{ic,o}}\right)}}\right]$$

If the inner tubing radius will be used as reference radius, we then we divide by the inner perimeter of the inner tubing:

$$(T_f - T_{wb}) = -\frac{d\dot{Q}}{dL}\cdot\frac{1}{2\cdot\pi\cdot r_{t,i}} \cdot \left[\frac{1}{h_i} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{t,o}}{r_{t,i}}\right)}{k_t} + \frac{r_{t,i}}{r_{t,o}\cdot h_{ann}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{pc,o}}{r_{pc,i}}\right)}{k_{pc}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{ic,i}}{r_{pc,o}}\right)}{k_c} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{ic,o}}{r_{ic,i}}\right)}{k_{ic}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{wb}}{r_{ic,o}}\right)}{k_c}\right]$$

Then:

$$U = \left(\frac{1}{h_i} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{t,o}}{r_{t,i}}\right)}{k_t} + \frac{r_{t,i}}{r_{t,o}\cdot h_{ann}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{pc,o}}{r_{pc,i}}\right)}{k_{pc}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{ic,i}}{r_{pc,o}}\right)}{k_c} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{ic,o}}{r_{ic,i}}\right)}{k_{ic}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{wb}}{r_{ic,o}}\right)}{k_c}\right)^{-1}$$

- Inner forced convection: The inner forced-convection coefficient (h_i) is usually in the range 100-50 000 W/m^2 K.[23] It is lower for low velocities and for gas flow. This gives a term in the range O(1E-5) to O(1E-2).
- Conduction in metal: Inner radii of well tubulars and pipelines are usually in the range 0.01-0.25 m. The ratio between inner and outer radius is usually between 1.05-1.3 (thickest pipe walls are usually for the small pipe diameters), thus the natural log of it is between 0.04-0.24. Lastly, the conductivity of the steel is around 45 W/m^2 K. This gives a term O(1E-4).

Free convection in the annulus (Term 3): The free convection coefficient in the annulus usually has values around 100 W/m^2 K. The ratio between outer and inner tubing diameter can range from 1.05 to 1.3. Therefore, this term is usually O(1E-2).

Conduction in cement (terms 5 and 7): The thermal conductivity of cement (k_c) is usually in the range between 0.3 to 2 W/m K. The ratio between the outer and inner diameter of the annular space is usually around 1.2. The inner tubing diameter is usually 0.02-0.2. Therefore, this term is usually O(1E-2).

Heat transfer in formation or soil

$$\frac{\partial^2 T_e}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T_e}{\partial r} = \frac{c_e \cdot \rho_e}{k_e} \cdot \frac{\partial T_e}{\partial t}$$

$$T_e(r, t=0) = T_{ei}$$

$$\frac{\partial T_e}{\partial r}(r \to \infty, t) = 0$$

$$\frac{d\dot{Q}}{dz} = -2 \cdot \pi \cdot k_e \cdot r_{wb} \cdot \left.\frac{\partial T_e}{\partial r}\right|_{r=r_{wb}}$$

input

k_e	Thermal conductivity, soil [W/m.K]
C_e	Specific heat capacity, soil [J/K.kg]
t	Time [s]

Transient!!!!

An approximate, analytical solution:

Wellbore Heat Transmission

H. J. RAMEY, JR.
MEMBER AIME

MOBIL OIL CO.
SANTA FE SPRINGS, CALIF.

SPE 22866

Heat Transfer During Two-Phase Flow in Wellbores: Part I—Formation Temperature

A.R. Hasan, U. of North Dakota, and C.S. Kabir, Chevron Oil Field Research Co.
SPE Members

Using that solution, and doing some math:

$$U_{eff}(t) = \left(\frac{U \cdot k_e}{k_e + T_D \cdot r_{t,i} \cdot U}\right)$$

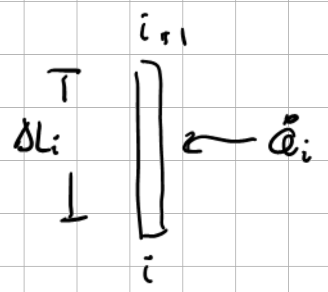
$$\begin{cases} T_D = 1.1281 \cdot \sqrt{t_D} \cdot (1 - 0.3 \cdot \sqrt{t_D}), & for \quad t_D \leq 1.5 \\ T_D = (0.4063 + 0.5 \cdot ln(t_D))\left(1 + \frac{0.6}{t_D}\right) & for \quad t_D > 1.5 \end{cases}$$

Dimensionless time, $t_D = \frac{\alpha_e \cdot t}{{r_{wb}}^2}$

And α_e is the thermal diffusivity of the soil, [m2/s], equal to $\frac{k_e}{\rho_e \cdot C_e}$

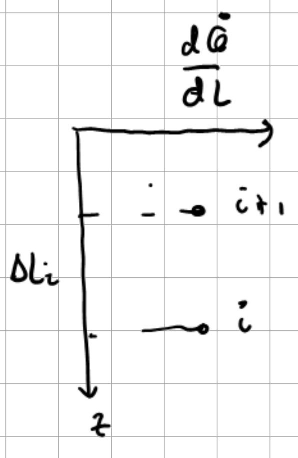
Lecture 42 : Temperature calculations in wellbore - exercise

		BO table column
TVD [m]	T [C]	p[bara]
0	50.0	28
500	57.1	39.2
1000	64.3	58.6
1500	71.4	82.5
2000	78.6	109.4
2500	85.7	138.4
3000	92.9	168.9
3500	100.0	200.2



$$h_{i+1} = \frac{\dot{Q}_i}{\dot{m}} + h_i - \Delta z \cdot g$$

$$\left.\frac{d\dot{Q}}{dL}\right|_i = -2\pi r \cdot U_i \left(T_{f_i} - T_{form,i}\right)$$



$$\underbrace{\dot{Q}_i = \left(\left.\frac{d\dot{Q}}{dL}\right|_i + \left.\frac{d\dot{Q}}{dL}\right|_{i+1}\right)\frac{1}{2} \cdot \Delta L_i}_{\text{trapezoidal rule!}}$$

residual $\quad \varepsilon_i = 0 = \frac{\dot{Q}_i}{\dot{m}} + h_i - h_{i+1} - \Delta z\, g$

		BO table column
TVD [m]	T [C]	p[bara]
0	50.0	28
500	57.1	39.2
1000	64.3	58.6
1500	71.4	82.5
2000	78.6	109.4
2500	85.7	138.4
3000	92.9	168.9
3500	100.0	200.2

TVD [m]	T [C]	p[bara]
0	65.6	28
500	74.5	44.8
1000	81.2	65.3
1500	86.3	89.1
2000	90.9	115.8
2500	94.9	144.5
3000	97.9	174.7
3500	100.0	205.9

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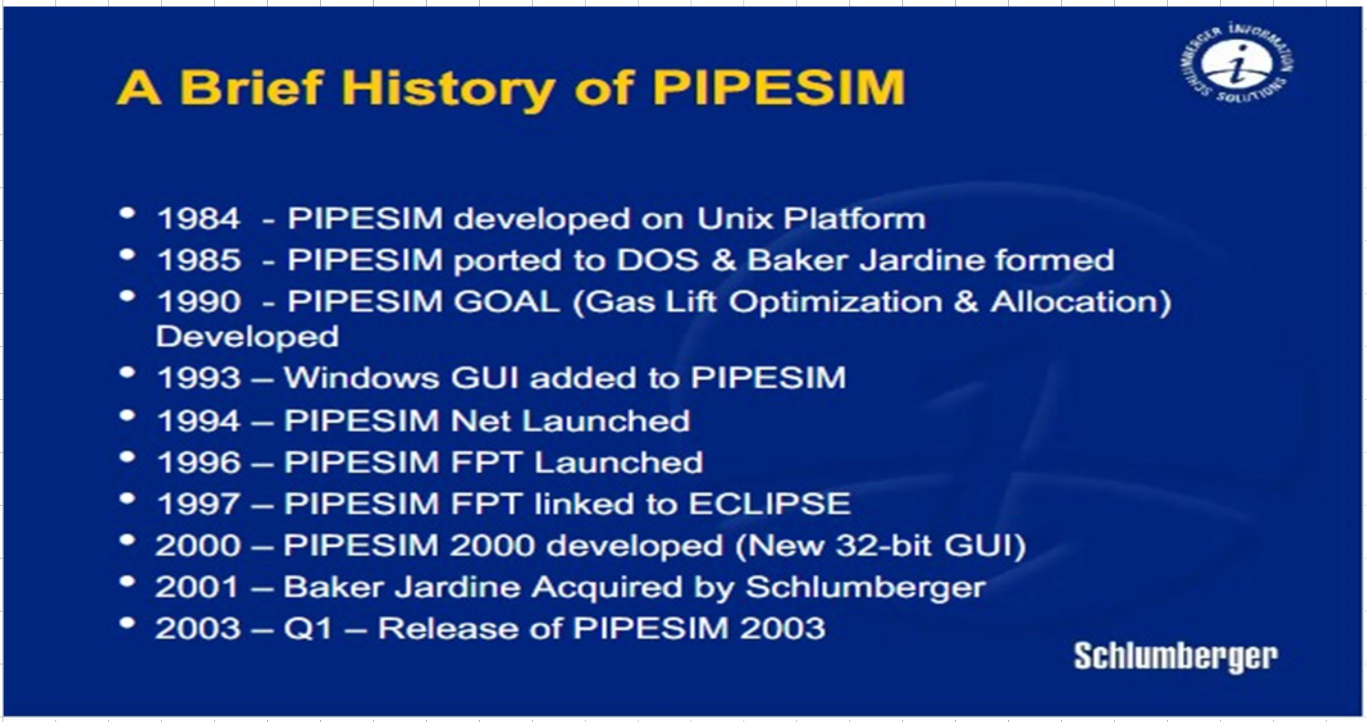
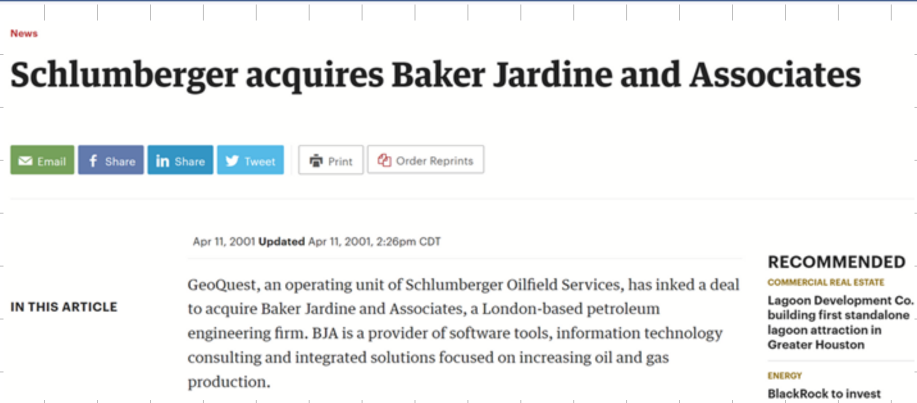
OUTLINE:
-Introduction to PROSPER and Pipesim





News

Schlumberger acquires Baker Jardine and Associates

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Apr 11, 2001 **Updated** Apr 11, 2001, 2:26pm CDT

IN THIS ARTICLE

GeoQuest, an operating unit of Schlumberger Oilfield Services, has inked a deal to acquire Baker Jardine and Associates, a London-based petroleum engineering firm. BJA is a provider of software tools, information technology consulting and integrated solutions focused on increasing oil and gas production.

RECOMMENDED

COMMERCIAL REAL ESTATE

Lagoon Development Co. building first standalone lagoon attraction in Greater Houston

ENERGY

BlackRock to invest

PETEX (1990)

FOUNDER: GUEDROUDJ, HAMID (working earlier at EDINBURGH PETROLEUM SERVICES LIMITED (EPS)

https://www.petex.com/the-company/

1. Subsea oil well modeling

Goal: set up a computational model of the well and perform some production engineering analysis.

Fluid information:

Use a black oil correlation of Glasø (p_b, R_s, B_o) and Beal (viscosity) to characterize your PVT behavior.

Solution GOR = 142 Sm^3/Sm3 Producing GOR = 142 Sm^3/Sm^3 Oil gravity = 37 API (840 Kg/m^3) Gas gravity = 0.76 At initial conditions no water.	Formation Water salinity = 23000 ppm No H2S, CO2, N2. Heat capacity of oil = 2.219 KJ/Kg/K Heat capacity of gas = 2.1353 KJ/Kg/K Heat capacity of water = 4.1868 KJ/Kg/K

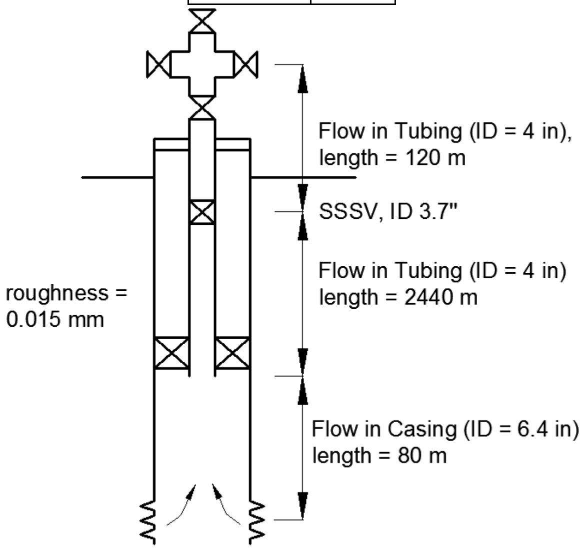
Well layout:

Deviation survey

MD [m]	TVD [m]
0	0
123	122
1059	1036
2164	2103
2640	2560

Geothermal gradient

MD [m]	T [C]
0	4
2640	100



Overall heat transfer coefficient = 45 W/m^2 K

Reservoir info:

Producing from a single layer
Reservoir pressure = 360 bara
Reservoir temperature = 100 C
Water cut = 0%
Productivity index = 12 Sm^3/d/bara

Tasks

PVT

- Determine the bubble point pressure of the oil and gas mixture at reservoir temperature
- Plot Bo, Rs and visco versus pressure at reservoir temperature. Export the curves to Excel.
- Perform a calibration of the BO correlations. Assume that the viscosity of the oil at reservoir pressure and temperature is known and equal to 1.3 cP.

Pressure transverse calculations

- Perform a calculation assuming wellhead pressure is 35 bara and oil rate is 1000 Sm3/d. examine the results and plot versus measured depth the following variables:
    - Total pressure gradient, hydrostatic, frictional and acceleration pressure gradient components
    - Liquid holdup and non-slip volume fraction. Compute slip between liquid and gas.
    - Gas and liquid velocities
    - Temperature
- Repeat the calculations above for a wellhead pressure of 70 bara and oil rate of 1000 Sm3/d. How does this change affect your results?
- Change the overall heat transfer coefficient to 10 W/m2 K and repeat your calculations. How does this change affect your results?
- Assume the well is producing with a water cut of 20%. How does this affect your results?

TPR

- Calculate TPR curves for wellhead pressures equal to 35 bara, 70 bara, and 100 bara.
- Calculate TPR curves for GOR equal to 200, 300 and 600 Sm3/Sm3.

Flow equilibrium

- Estimate the producing rate using flow equilibrium and wellhead pressure is 35 bara.
- The team is considering using a bigger tubing. Evaluate the effect this could have in the equilibrium rate.
- Assume the well is producing with a water cut of 20%. How does this affect your results?

Transient flow

20241111

Presentation by Prof emeritus Harald Asheim

Transient, multiphase flow in pipe

.

- In multiphase piplelines, liquid slugs, large enough to «drown» the processing plant, had been experienced. This led to development of the OLGA-program, supported and verified by measurements from the large scale experimental facility at Tiller. It enabled prediction of transient multiphase performance, providing basis for safe operation. So, the task of predicting transient multiphase flow may be considered solved… ?
- Is further R&D is worthwhile? This old professor-emeritus thinks so. And it may be supported by field measurements.
- This presentation outlines the basics of transient multiphase flow and how it may be predicted…..

- **Content**

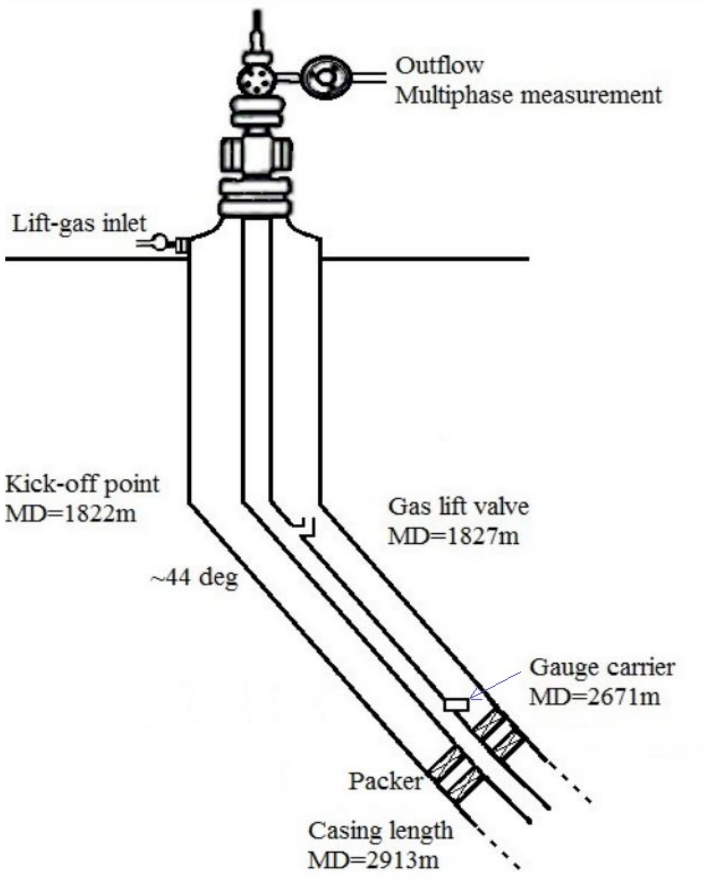
1 Field measurements

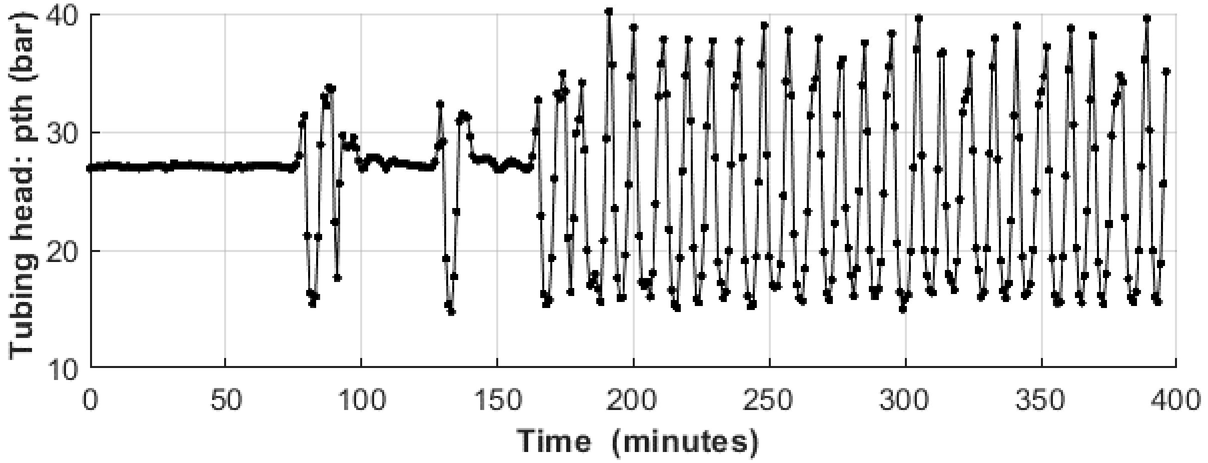
2 Physics

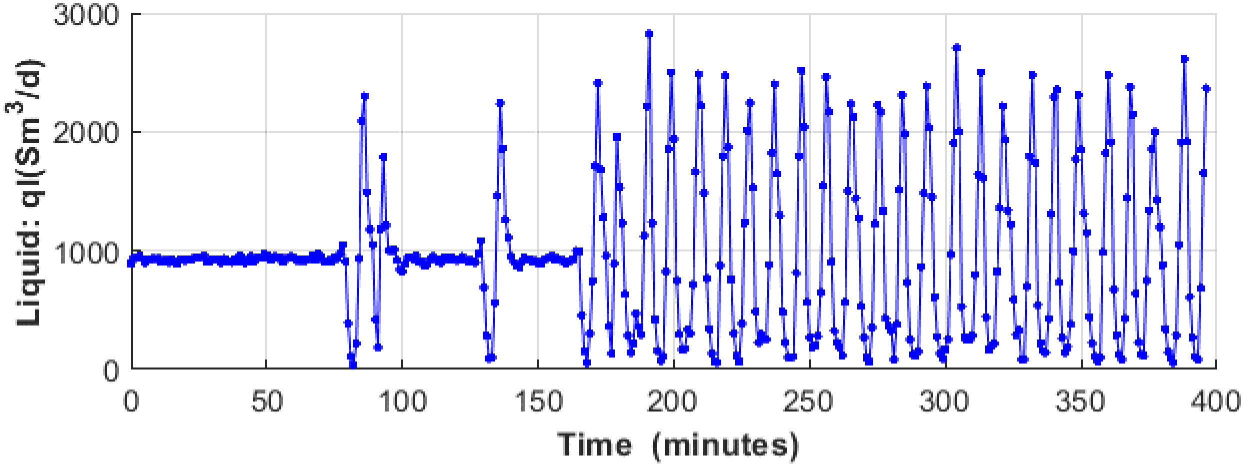
3 Mathematical modelling

4 Numerical solution

Field measurements, Heidrun well







Ref.: Verification of Transient, Multi-Phase Flow Simulation for Gas Lift Applications, SPE 56659.

Physics

Wave=Some recognizable change that moves through space

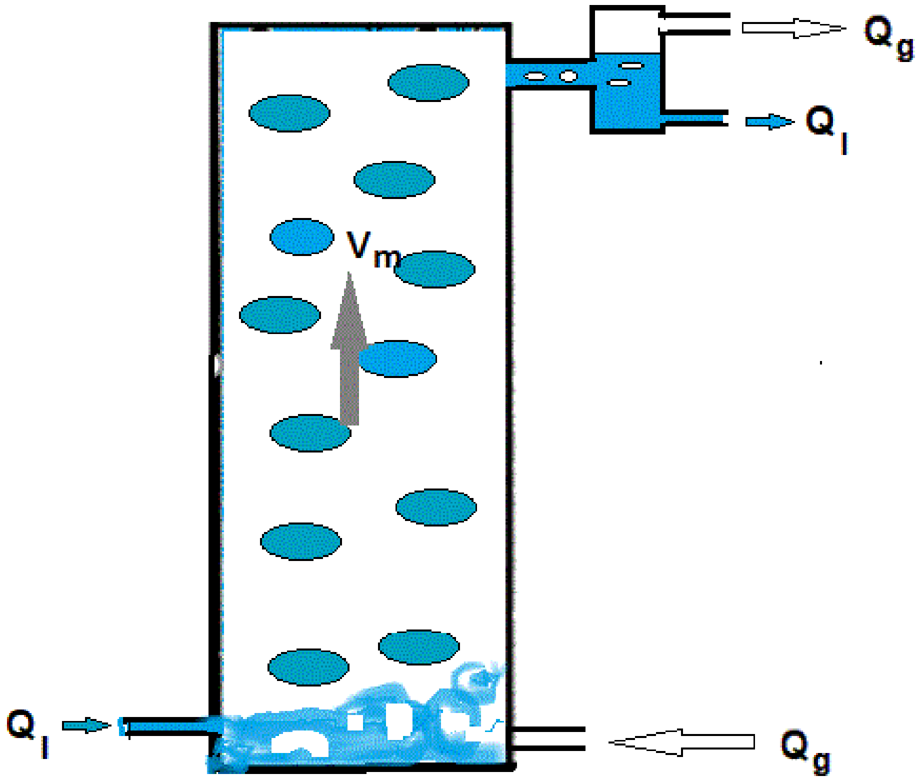
In 1 phase phase flow: compression waves (change of pressure and velocity)

In 2 phase flow

- Pressure & velocity change : Compression waves. Fast
- Fraction change : Kinematic waves*. Slow

* aka: «continuity waves» , «density waves»

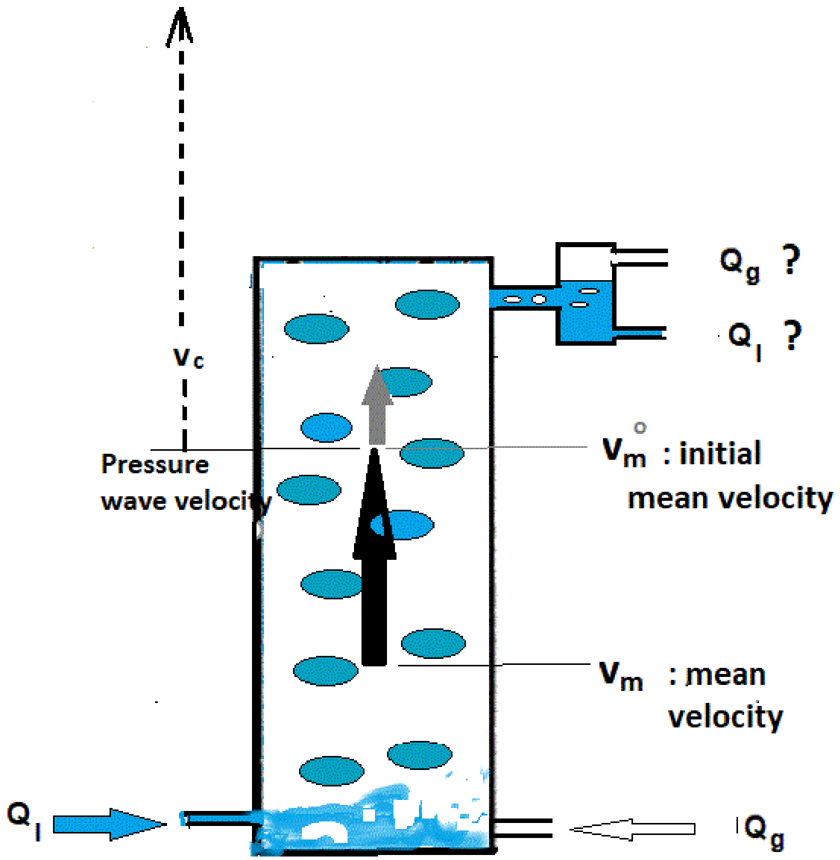
Initial: Steady state flow



Gas inflow twice as high as liquid

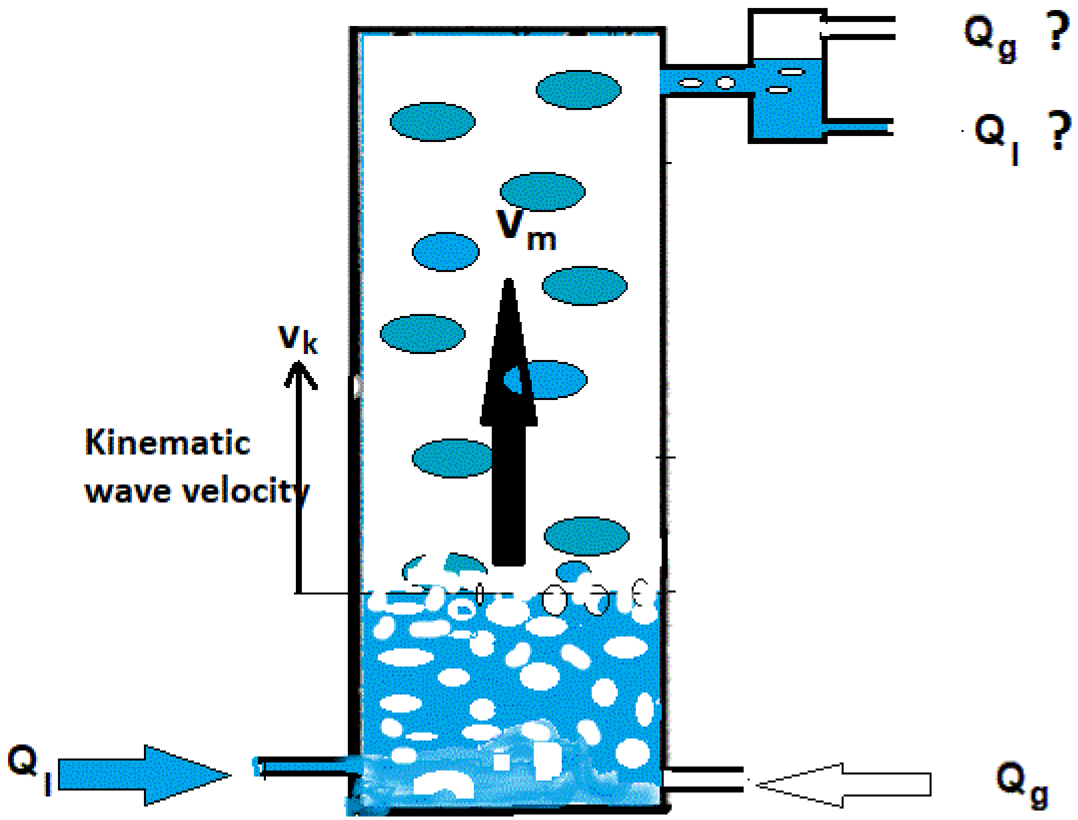
Liquid fraction in pipe 50%.
This due to slippage

5 seconds after increased water inflow



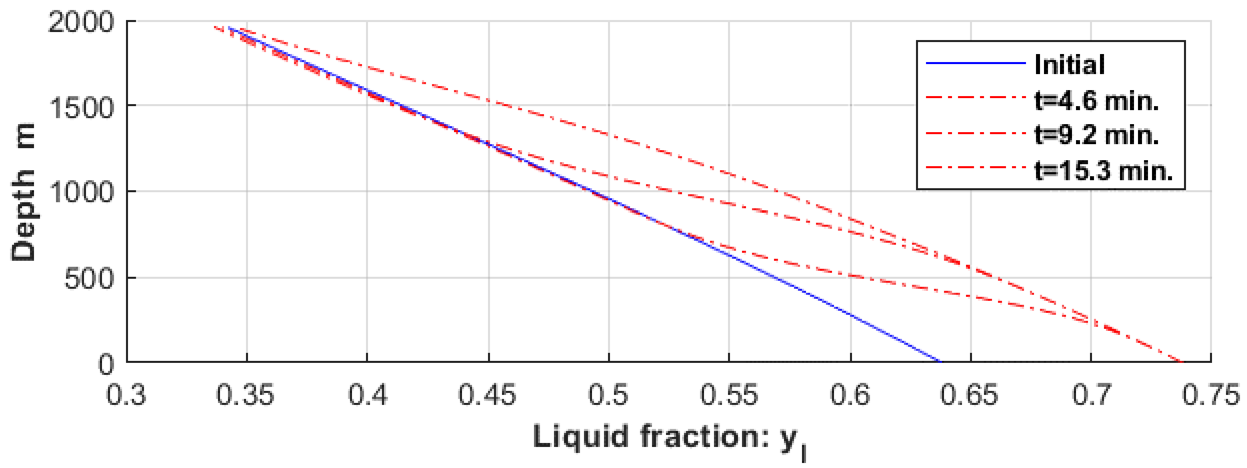
10 minutes after increased water inflow

The pressure wave has passed
Fraction change proceeds along the pipe



Performance

The first minutes, the water inflow increase has little effect on the outflow.
After 15 minutes, the increased fraction reaches the outlet and liquid outflow will increase
After some more time, liquid outflow will stabilize and new steady state reached



Mathematical modelling

(Transient contributions in red frames)

1 Pressure balance of flowing mixture

$$\boxed{\rho_g \frac{\partial v_g}{\partial t} + \rho_l \frac{\partial v_l}{\partial t}} + \frac{\partial p}{\partial x} + \rho_m v_m \frac{\partial v_m}{\partial x} + \rho_{TP} g_x + \frac{1}{2} f_m \frac{\rho_m}{d} v_m^2 = 0$$

2 Mass balances for gas and liquid phases

$$\boxed{\frac{\partial}{\partial t}\left(\rho_g y_g\right)} + \frac{\partial}{\partial x}\left(\rho_g v_{sg}\right) + w_s = \dot{m}_g$$

$$\boxed{\frac{\partial}{\partial t}\left(\rho_l y_l\right)} + \frac{\partial}{\partial x}\left(\rho_l v_{sl}\right) - w_s = \dot{m}_l$$

3 Drift flux relation: $\mathrm{v_g = C_o v_l + v_o}$

4 Density relations: $\rho_g\left(p, T, \ldots\ldots\right) \quad \rho_l\left(p, T, \ldots\ldots\right)$

«Processed Equations »

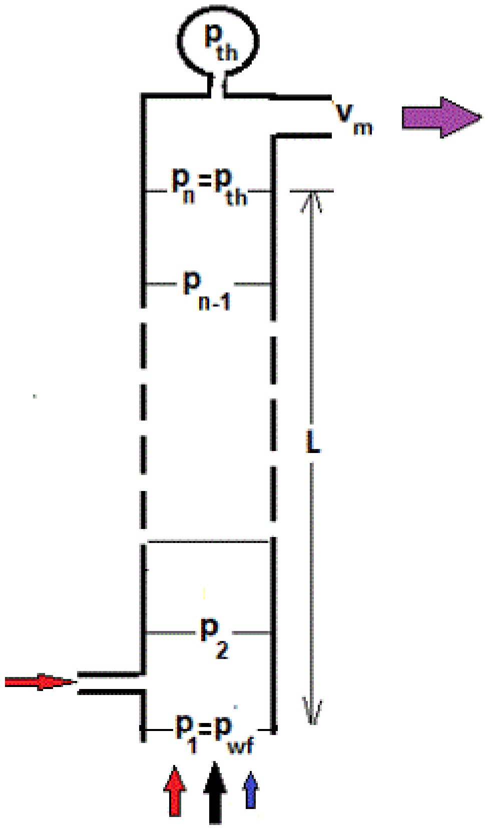
Mathematical manipulations of the modelling equations above results in:

Reduction from the 3 pdf equations to 2: One pressure equation and a total mass balance

Numerical solution

Discretization of the 2 pdf equations leads to the matrix: Ax=b

Prediction of pressure and velocity change in time and space by inversion:



$$
\begin{Bmatrix}
a_{2,1} & a_{3,1} & a_{4,1} & & & \\
b_{2,1} & b_{3,1} & b_{4,1} & 0 & & \\
0 & a_{1,2} & a_{2,2} & a_{3,2} & a_{4,2} & \\
 & b_{1,2} & b_{2,2} & b_{3,2} & b_{4,2} & 0 \\
 & & 0 & a_{3,1} & a_{3,2} & a_{3,3} \\
 & & & - & - & -
\end{Bmatrix}^{t}
\begin{bmatrix} p_1 \\ v_2 \\ p_2 \\ v_3 \\ p_3 \\ v_4 \end{bmatrix}^{t+1}
=
\begin{bmatrix} \alpha_1 - a_{1,1}v_1 \\ \beta_1 - b_{1,1}v_1 \\ \alpha_2 \\ \beta_2 \\ \alpha_3 \\ - \end{bmatrix}^{t}
$$

Bandwidth (5)

Diagonal

Given pressure and velocity, fraction is updated explicitly (no matrix inversion)

1. Snøhvit subsea gas well modeling

Fluid information:

Use the compositional PVT model for your PVT behavior.

Component	Moles
Water	0
Methane	78
Ethane	8
Propane	3.5
Isobutane	1.5
Butane	1.2
Isopentane	0.8
Pentane	0.5
Hexane	0.5
C7+	6

Properties for pseudo component C7+: Molecular weight: 115, specific gravity: 0.683

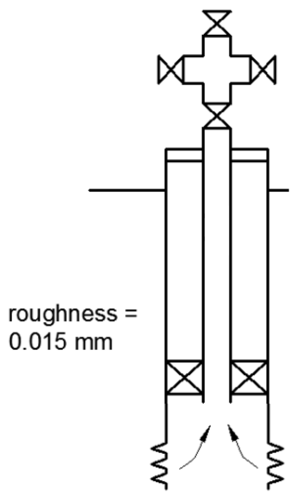
Well layout:

Deviation survey

MD [m]	TVD [m]
0	0
2100	2100

Geothermal gradient

TVD [m]	T [C]
0	4
2100	92



Flow in tubing only, tubing diameter 0.15 m

Overall wellbore heat transfer coefficient = 30 W/m^2 K

Reservoir info:

Producing from a single layer
Reservoir pressure = 276 bara
Reservoir temperature = 92 C
Backpressure coefficient = 1000 Sm^3/d/bara
Backpressure exponent = 1

Tasks:

- Determine the saturation pressure of the fluid at reservoir temperature.
- Plot the phase envelope of reservoir fluids.
- What is the condensate gas ratio, and the water gas ratio of the well?
- Estimate the producing rate using flow equilibrium assuming that the well is producing against a constant wellhead pressure of 100 bara.
- Calculate equilibrium rates for wellhead pressures equal to 100 bara, 120 bara, and 150 bara.
- Add a wellhead choke with size 16/64". Report the new equilibrium rate and temperature downstream the choke.
- **Pressure gradient calculations:** Perform a calculation assuming wellhead pressure is 100 bara and gas rate is 3 E06 Sm3/d. Examine the results and plot versus measured depth the following variables:
    - Total pressure gradient, hydrostatic, frictional and acceleration pressure gradient components
    - Liquid holdup and non-slip volume fraction. Compute slip between liquid and gas.
    - Gas and liquid velocities
    - Temperature