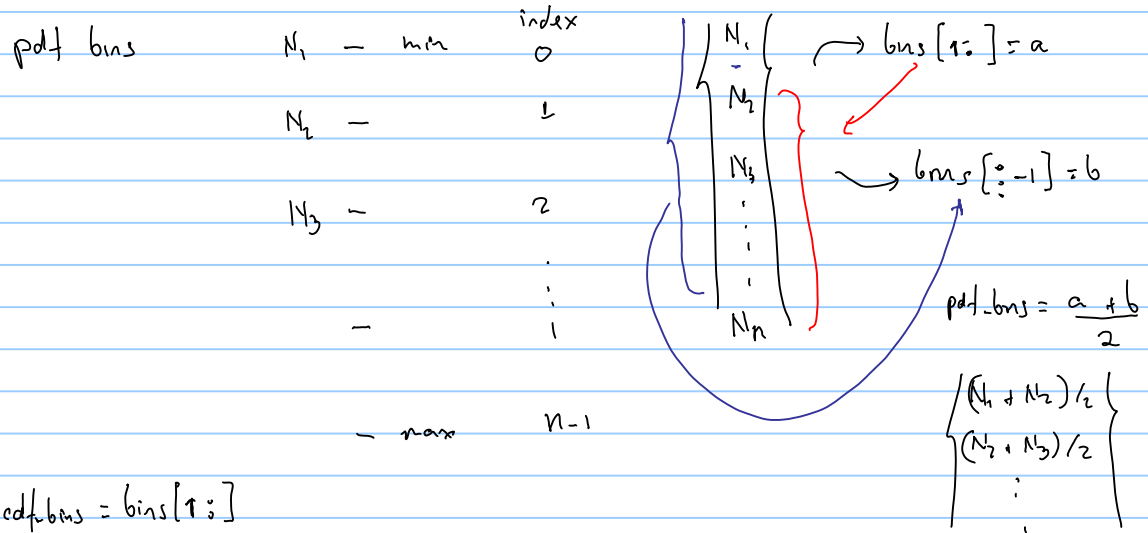


Continuation of class exercise: probabilistic reserve estimation using MC and Jupyter



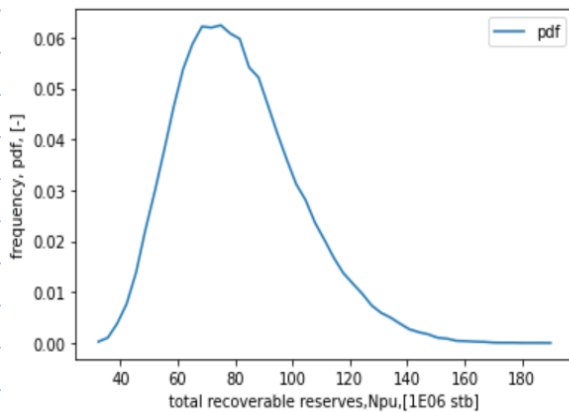
$$\text{cdf_bins} = \text{bins}[1:]$$

How to find the mode?

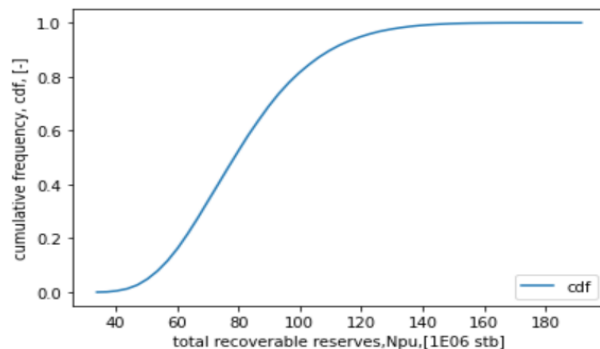
pdf_bins

$$\left\{ \begin{array}{l} N_0 \\ N_1 \\ N_2 \\ \vdots \\ N_n \end{array} \right\}$$

pdf

$$\left\{ \begin{array}{l} p_0 \\ p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_n \end{array} \right\}$$
max $i:3$ mode $\rightarrow \text{pdf_bins}[i]$
`pdf_bins[np.argmax(pdf)],`


This looks very much like a log normal distribution (needs only mean and sigma)

https://en.wikipedia.org/wiki/Log-normal_distribution


The nr. of bins should be adjusted slightly with the number of samples, e.g. more samples ----> more bins. The pdf will look smoother.

Convergence study on the number of iterations required for results not to vary significantly:

Nr iterations	1000	Change % wrt max iter	10000	Change % wrt max iter	100000	Change % wrt max iter	1000000
Mean [1E06 stb]	82.0	1.14	81.0	0.16	81.1	0.01	81.1
Mode [1E06 stb]	75.0	4.36	76.3	6.22	71.0	1.12	71.8
Minimum [1E06 stb]	33.7	21.22	30.7	10.45	31.3	12.51	27.8
Maximum [1E06 stb]	162.1	20.53	172.6	15.38	190.3	6.70	203.9
P90 [1E06 stb]	55.2	0.53	55.0	0.96	55.5	0.13	55.5
P50 [1E06 stb]	79.4	0.86	78.7	0.06	78.6	0.07	78.7
P10 [1E06 stb]	111.7	1.46	109.9	0.12	110.1	0.08	110.1

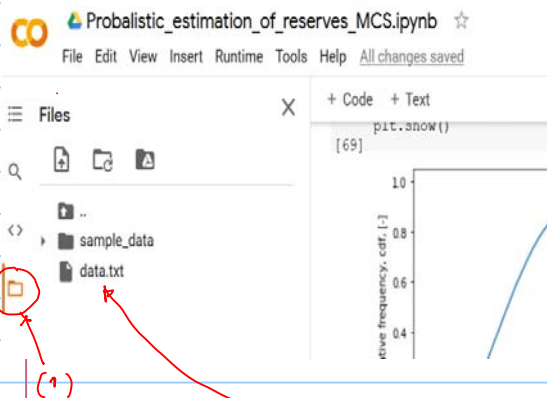
this variable depends on nr. bins and samples!

To output results to a text file:

```
output=np.vstack((pdf_bins,pdf))
np.savetxt('data.txt',output.T)
```

putting arrays together

printing to text file

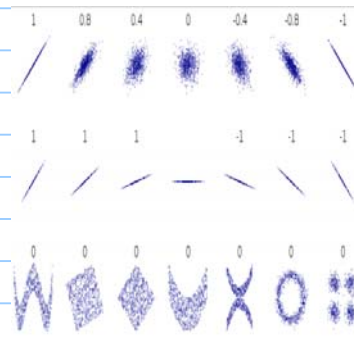


available here (1)

To find correlation between the variables and results and calculate sort of an "importance" for each parameter, the Pearson Correlation coefficient can be used.

https://en.wikipedia.org/wiki/Pearson_correlation_coefficient

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} \quad (\text{Eq.1})$$



for our case:

```
PCC_por=np.corrcoef(por,TRR)[0,1]
print("PCC for Porosity is: ",PCC_por )
PCC_RV=np.corrcoef(RV,TRR)[0,1]
print("PCC for rock volume is: ",PCC_RV )
PCC_NTG=np.corrcoef(NTG,TRR)[0,1]
print("PCC for Net to gross is: ",PCC_NTG )
PCC_Bo=np.corrcoef(Bo,TRR)[0,1]
print("PCC for oil saturation is: ",PCC_Bo )
PCC_Fr=np.corrcoef(Fr,TRR)[0,1]
print("PCC for oil formation volume factor is: ",PCC_Fr )
PCC_Fr=np.corrcoef(Fr,TRR)[0,1]
print("PCC for ultimate recovery factor is: ",PCC_Fr )
```

```
PCC for Porosity is: 0.5450822680244062
PCC for rock volume is: 0.2429501735721322
PCC for Net to gross is: 0.547369047073988
PCC for oil saturation is: 0.13319591901184374
PCC for oil formation volume factor is: -0.18644951517089872
PCC for ultimate recovery factor is: 0.5122102672675487
```

most important : ϕ

NTG

Fr

"intuition"

because they are fractions and their range is relatively broad

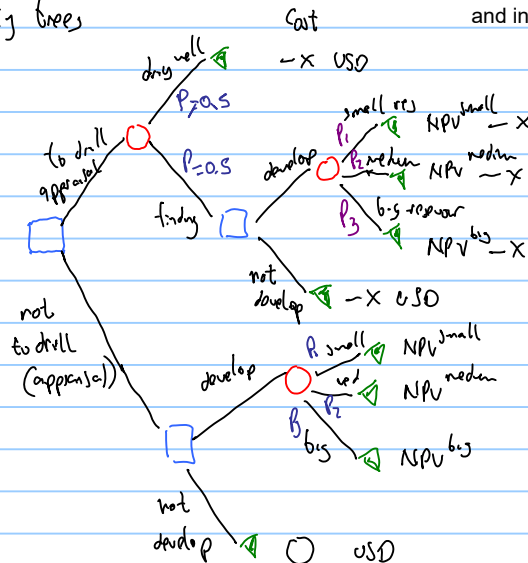
Clarifications about Python

```
#How do we modify an existing array
#appending an array (adding values at the end)
a=np.linspace(1,6,10)
b=np.linspace(1,2,2)
c=np.append(a,b)
#concatenating arrays (order matters!!)
d=np.concatenate((a,b))
```

```
#how to make a function run element-wise for all members of an array
def test_function(a):
    if a>0:
        result=0
    else:
        result=1
    return result
a=np.linspace(-6,6,10)
V_test_function=np.vectorize(test_function)
V_test_function(a)
```

Probability trees

Probability trees are similar to sampling, but they allow us to include decision and integer variables



□ decision node

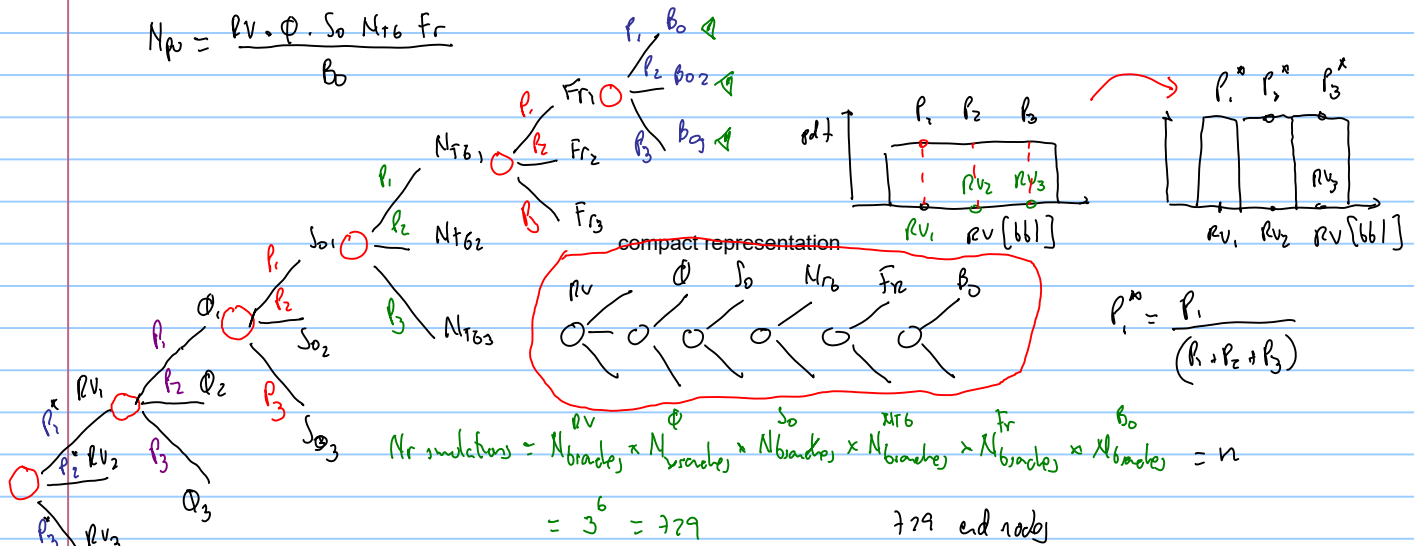
○ chance node ← this can be evaluated with Monte Carlo

△ end node

→ USD value of appraisal cell

How to solve the previous example solved with Python with probability trees?

$$NPV = \frac{RV \cdot \Phi \cdot S_o \cdot N_{r6} \cdot Fr}{b_0}$$



each case (end node) will have an associated probability

Case $PV, Q, S_o, N_{Tg}, Fr, B_o \rightarrow TRR_1$

$$P_{case_1} = P_{Q_1} \cdot P_{S_{o1}} \times P_{B_{o1}} \times P_{Fr_1} \times P_{N_{Tg_1}} = P_{case_1}$$

Case $PV, Q, S_o, N_{Tg}, Fr, B_{o2} \rightarrow TRR_2$

$$P_{case_2} = P_{Q_2} \cdot P_{S_{o2}} \times P_{B_{o2}} \times P_{Fr_2} \times P_{N_{Tg_2}} = P_{case_2}$$

$$\left\{ \begin{array}{cc} TRR_1 & P_{case_1} \\ TRR_2 & P_{case_2} \\ \vdots & \vdots \\ TRR_n & P_n \end{array} \right\}$$

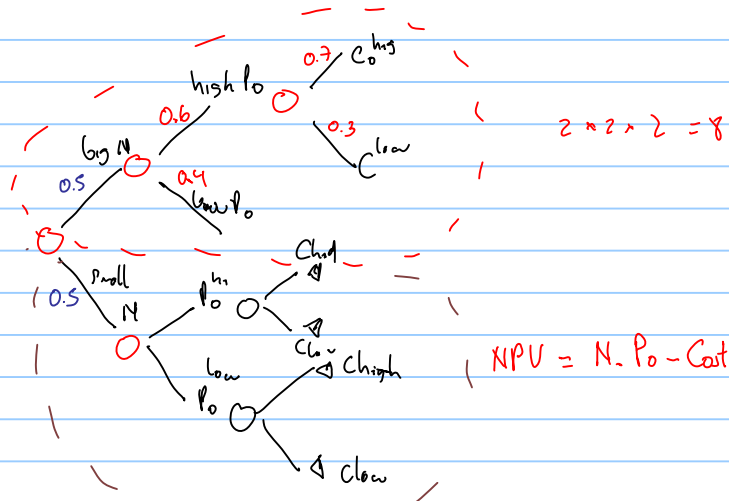
"expected" value of the tree

$$\sum_{i=1}^N (TRR_i \cdot P_{case_i}) = \text{expected TRR}$$

a cdf can be calculated with these results

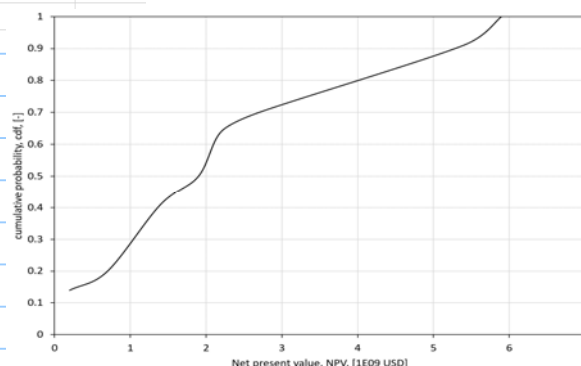
- sort TRR column
- calculate cum probabilities

Spontaneous
Class example



CASE	N [1e06 stb]	Po [usd]	Cost [1e09 usd]	P_N	P_Po	P_Cost	Pcase	NPV [1E09 USD]	Pcase*NPV
1	30	80	1.00E+00	0.5	0.6	0.7	0.21	1.4	0.294
2	30	80	5.00E-01	0.5	0.6	0.3	0.09	1.9	0.171
3	30	40	1.00E+00	0.5	0.4	0.7	0.14	0.2	0.028
4	30	40	5.00E-01	0.5	0.4	0.3	0.06	0.7	0.042
5	80	80	1.00E+00	0.5	0.6	0.7	0.21	5.4	1.134
6	80	80	5.00E-01	0.5	0.6	0.3	0.09	5.9	0.531
7	80	40	1.00E+00	0.5	0.4	0.7	0.14	2.2	0.308
8	80	40	5.00E-01	0.5	0.4	0.3	0.06	2.7	0.162
								EV [1E09 USD]	2.67

NPV [1E09 USD]	P	cdf
0.2	0.14	0.14
0.7	0.06	0.2
1.4	0.21	0.41
1.9	0.09	0.5
2.2	0.14	0.64
2.7	0.06	0.7
5.4	0.21	0.91
5.9	0.09	1



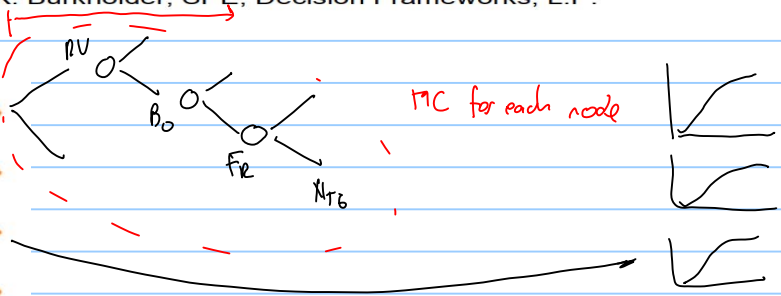
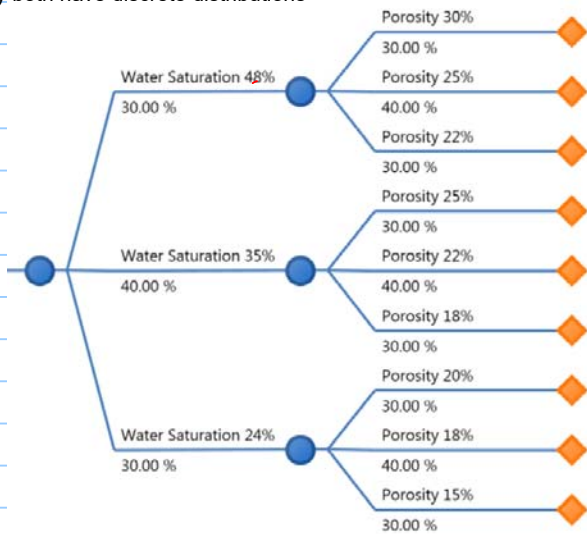
An example combining probability trees and monte carlo

SPE 162883

Integrating the Subsurface and the Commercial: A New Look at Monte Carlo and Decision Tree Analysis

J. H. Schulze, SPE, J. N. Walker, SPE, M. K. Burkholder, SPE, Decision Frameworks, L.P.

In this case, porosity depends on water saturation and they both have discrete distributions



Comments about problem 3

