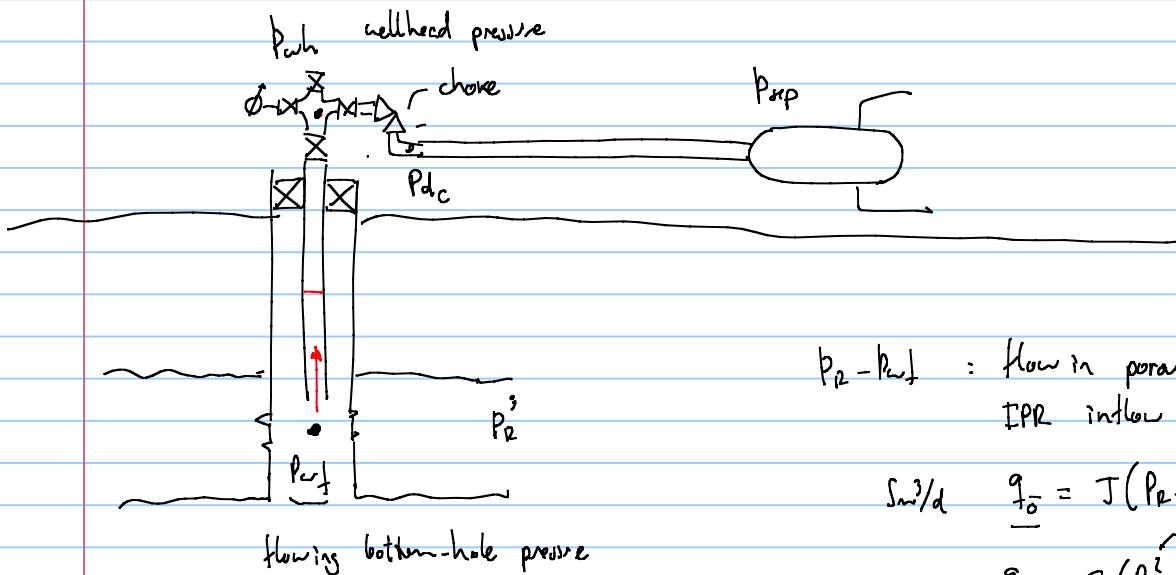


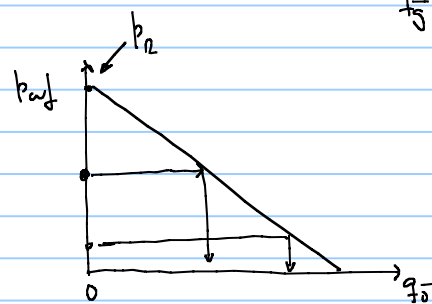
flow equilibrium in dry gas production systems



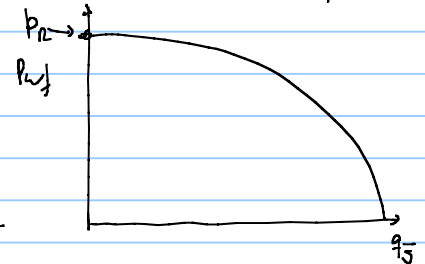
$p_R - p_{wf}$: flow in porous media
IPR inflow performance relationship

$$\text{Sm}^3/\text{d} \quad \underline{q_o} = J(p_R - p_{wf}) \quad \text{PSS}$$

$$\underline{q_o} = C(p_R^2 - p_{wf}^2)^n \quad \text{PSS} \quad \text{low } p$$

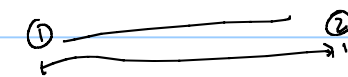


undersaturated oil, water injection



Dry gas
saturated oil
saturated gas

$p_{wf} \rightarrow p_{wlh}$
flow in tubing (pipe)



$$\underbrace{z_1 + \frac{p_1}{\rho \cdot g}}_{h_1} + \frac{V_1^2}{2g} = \underbrace{z_2 + \frac{p_2}{\rho \cdot g}}_{h_2} + \frac{V_2^2}{2g} + f \frac{L}{D} \frac{V^2}{2g}$$

$$Re = \frac{\rho \phi V}{\mu}, \quad E = \frac{c}{\phi}$$

A. THE TUBING RATE EQUATION IN VERTICAL AND DEVIATED GAS-WELLS

Author: Prof. Michael Golan

DERIVATION FROM FIRST PRINCIPLES (PURE SI SYSTEM)

$$p_{wf}^2 = p_t^2 \cdot e^S + \frac{8 \cdot f_M}{\pi^2 \cdot D^5 \cdot g \cdot \cos(\alpha)} \cdot \left(\frac{p}{T}\right)_{sc}^2 \cdot (Z_{av} \cdot T_{av})^2 \cdot (e^S - 1) \cdot q_{sc}^2 \quad \text{Eq. A-19}$$

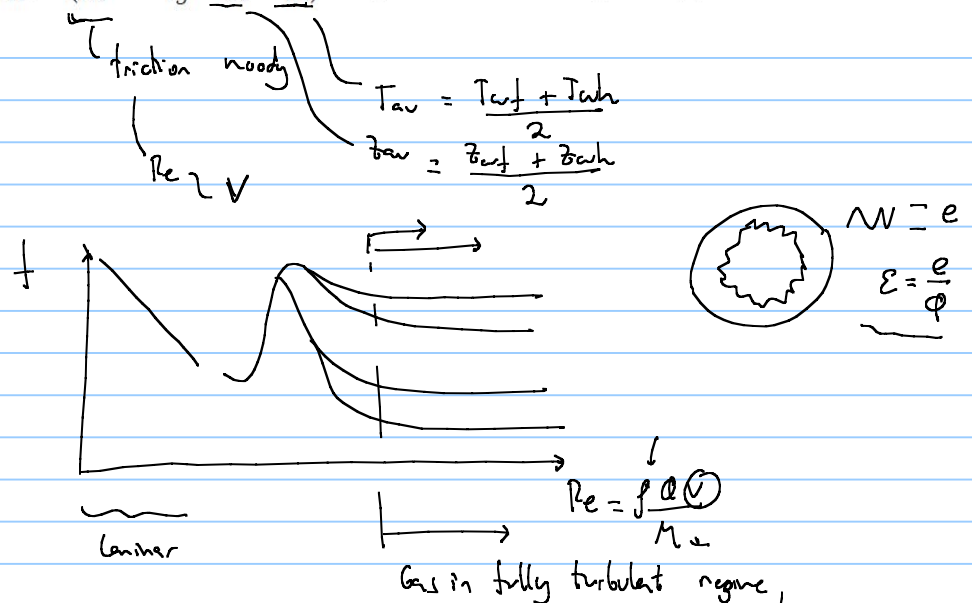
Multiplying and dividing the second term on the right-hand side with:

$$S = 2 \cdot \frac{M_g \cdot g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot \cos(\alpha) = 2 \cdot \frac{28.97 \cdot \gamma_g \cdot g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot \cos(\alpha) \quad \text{Eq. A-20}$$

$$p_{wf}^2 = p_t^2 \cdot e^S + \frac{28.97}{R} \cdot \left(\frac{p}{T}\right)_{sc}^2 \cdot f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av} \cdot \frac{(e^S - 1)}{S \cdot D^5} \cdot q_{sc}^2 \quad \text{Eq. A-21}$$

Solving for the flow rate:

$$q_{sc} = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left[(p_{wf}^2 - p_t^2 \cdot e^S) \cdot \left(\frac{S}{e^S - 1}\right)\right]^{0.5} \quad \text{Eq. A-22}$$



$$f_F = \frac{0.00437}{D^{0.224}}$$

$$f_m = \frac{f_F}{4}$$

fanning

Tubing flow Equation-Dry gas

$$q_{sc} = \left(\frac{\pi}{4}\right) \left(\frac{R}{M_{air}}\right)^{0.5} \left(\frac{T_{sc}}{P_{sc}}\right) \left[\frac{D^5}{\gamma_g f_M Z_{av} T_{av} L}\right]^{0.5} \left(\frac{S e^S}{e^S - 1}\right)^{0.5} \left(\frac{p_1^2}{e^S} - p_2^2\right)^{0.5}$$

$$\frac{S}{2} = \frac{M_g g}{Z_{av} R T_{av}} H = \frac{(28.97) \gamma_g g}{Z_{av} R T_{av}} (H)$$

$$q_{gsc} = C_T \left(\frac{p_1^2}{e^S} - p_2^2\right)^{0.5}$$

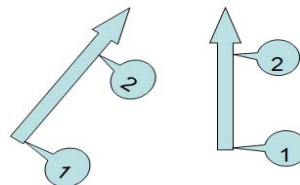
$$p_{inlet} = p_1 = e^{S/2} \left(p_2^2 + \frac{q_g^2}{C_T^2}\right)^{0.5}$$

$$p_{wh} = p_2 = \left(\frac{p_1^2}{e^S} - \frac{q_g^2}{C_T^2}\right)^{0.5}$$

long constant

$$q_g = C_T \left(\frac{p_{wf}^2}{e^S} - p_{wh}^2\right)^{0.5}$$

$\phi \uparrow, C_T \uparrow$



$P_{wh} \rightarrow P_{dc}$ (downstream of choke) $\leadsto$ eg. $f(q_g, P_{wh}, P_{dc})$
page 140 in Compendium

Appendix B: Choke Equations

M. Stanko

B. CHOKE EQUATIONS

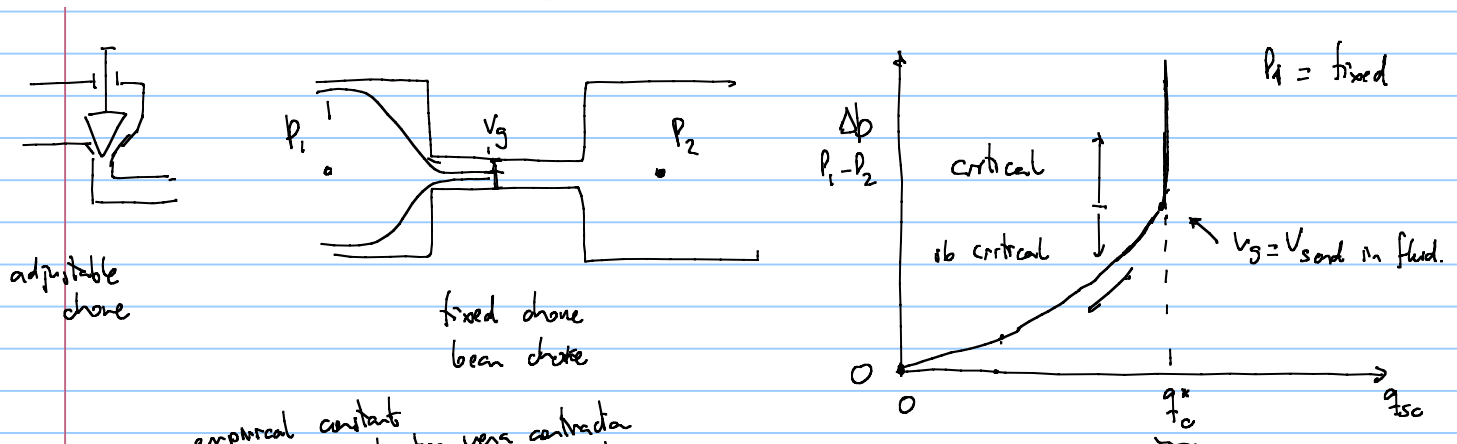
UNDERSATURATED OIL FLOW

Based on a frictionless flow contraction from an upstream point 1 to a downstream point 2.

The single-phase Bernoulli equation for steady state frictionless flow along a streamline, neglecting elevation changes, is:

$$\frac{dp}{\rho} + V \cdot dV = 0$$

Eq. B-1



empirical constant to account for vena contraction effect

$$q_{sc} = c_d A_2 P_1 \left(\frac{T_{sc}}{P_{sc}} \right) \sqrt{2 \frac{R}{M_g Z_1 T_1}} \sqrt{\left(\frac{k}{k-1} \right) \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{k}} - \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} \right]}$$

$$k = \frac{C_p}{C_v}$$

$P_{dc} \rightarrow P_{sep}$ flow in pipeline (pipe, conduit)

$$q_g = C_{FL} \left(\frac{P_{dc}^2}{e^s} - P_{sep}^2 \right)^{0.5} \quad \text{if horizontal line?} \quad s=0$$

$$q_g = C_{FL} \left(P_{dc}^2 - P_{sep}^2 \right)^{0.5} \quad C_{FL} = \text{const.} \frac{se^s}{e^s - 1} \quad s=0$$

L'Hopital

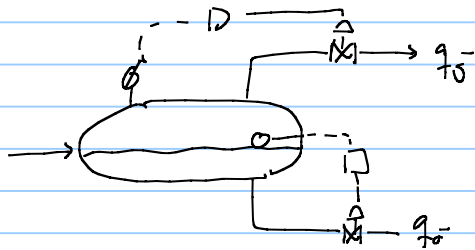
$$\lim_{s \rightarrow 0} \frac{se^s}{e^s - 1} = 1$$

IPR $q_s = C (p_R^2 - p_{wf}^2)^n$

tubing $q_s = C_T \left(\frac{p_{wf}^2}{e^S} - p_{wh}^2 \right)^{0.5}$

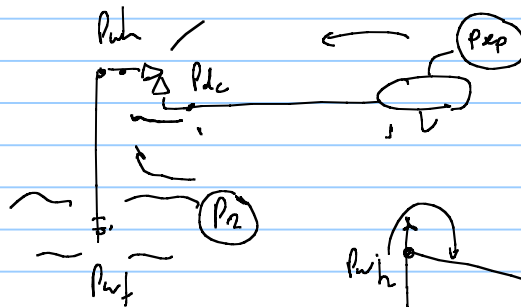
choke $f(q_s, p_{wh}, p_{dc})$

pipeline $q_s = C_{PL} (p_{dc}^2 - p_{sep}^2)^{0.5}$



unknowns	equation
2	1
1	1
1	1
1	1

Graphical solution



$$q_s = C (p_R^2 - p_{wf}^2)^n \quad (1)$$

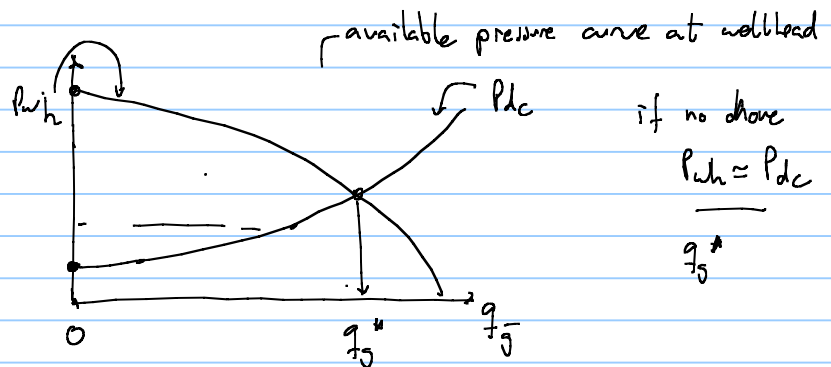
$$q_s = C_T \left(\frac{p_{wf}^2}{e^S} - p_{wh}^2 \right)^{0.5}$$

$$p_{wf}^2 = e^S \left(p_{wh}^2 + \left(\frac{q_s}{C_T} \right)^2 \right) \quad (2)$$

substitute (2) in (1)

$$q_s = C \left(p_R^2 - e^S \left(p_{wh}^2 + \left(\frac{q_s}{C_T} \right)^2 \right) \right)$$

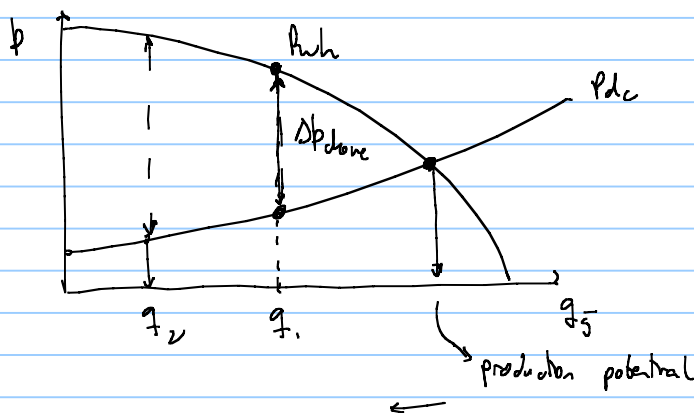
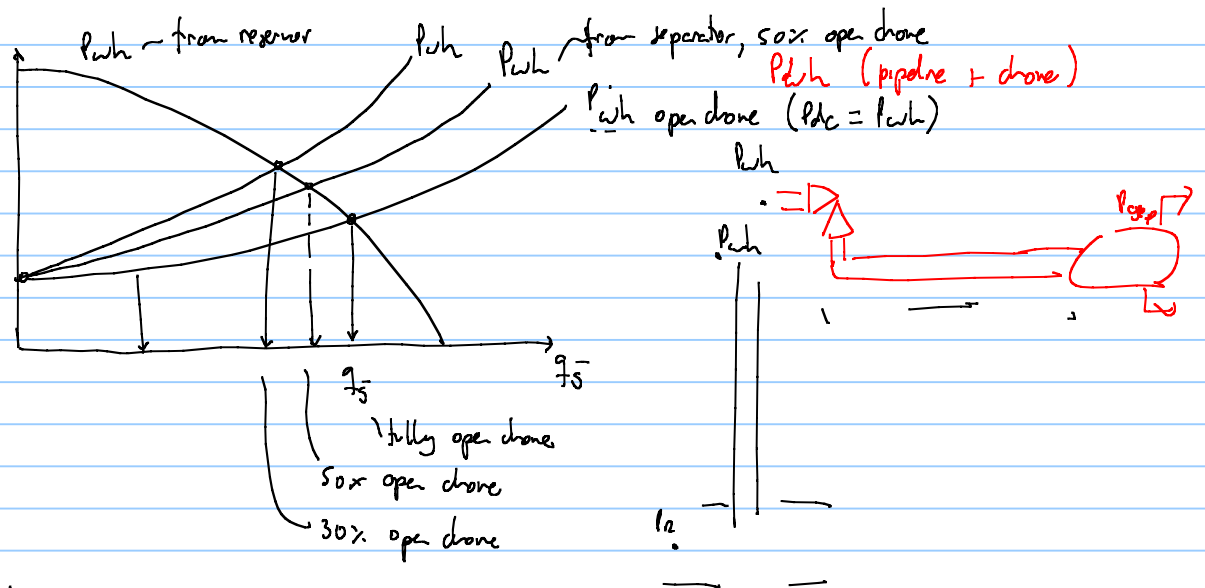
if $q_s = 0$ $p_{wh}^2 = \frac{p_R^2}{e^S}$



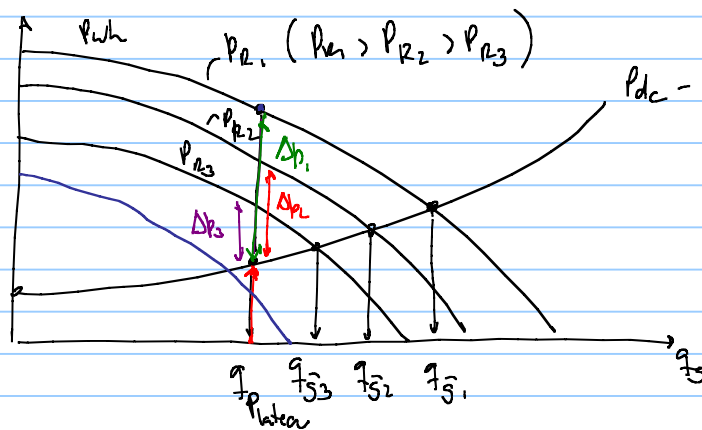
$$q_s = (p_{dc}^2 - p_{sep}^2)^{0.5} \cdot C_{PL}$$

$$p_{dc} = \left(p_{sep}^2 + \left(\frac{q_s}{C_{PL}} \right)^2 \right)^{0.5}$$

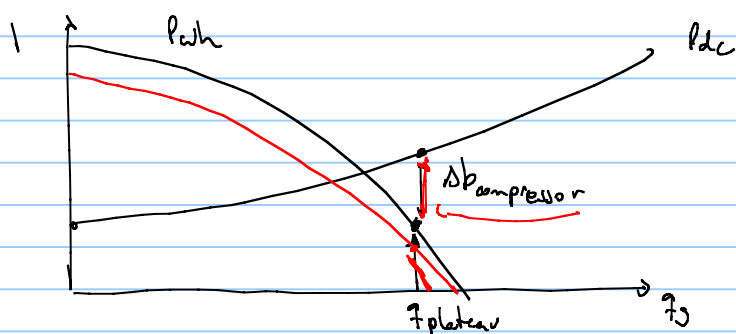




what happens with time?



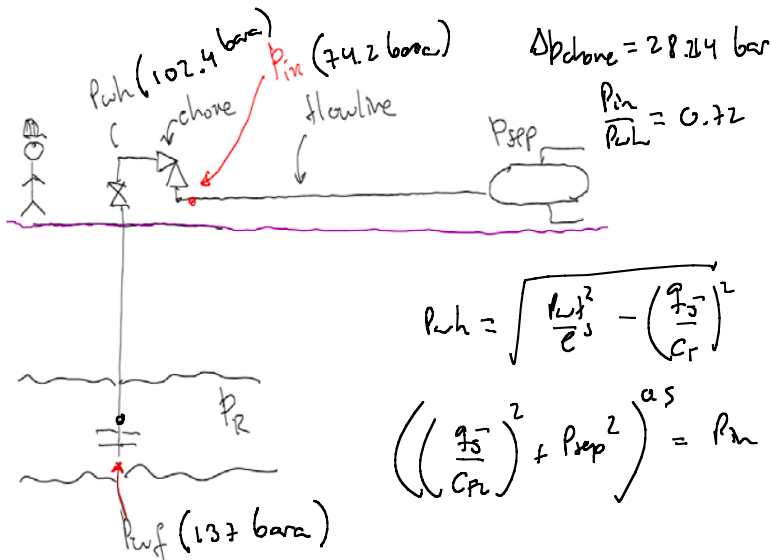
$t_1, p_{R1}, \Delta p_1$
 $t_2, G_{R2}, p_{R2}, \Delta p_2$
 $t_3, G_{R3}, p_{R3}, \Delta p_3$



find a compressor $q_{s, plateau}, \Delta p_{compressor}$.

Class exercise

1. For the dry gas production system shown in the figure below, what is the choke pressure drop required (in bar) for the system to deliver a rate of 2.5 E6 Sm³/d.

**Inflow equation:**

$$q_g = C_R \cdot (p_R^2 - p_{wf}^2)^n$$

With

$$C_R = 104 \text{ Sm}^3/\text{d}/\text{bar}^{2n}$$

$$n = 0.9$$

$$p_R = 304 \text{ bara}$$

Tubing equation:

$$q_g = C_T \cdot \left(\frac{p_{wf}^2}{e^2} - p_{wh}^2 \right)^{0.5}$$

$$C_T = 4.41 \text{ E4 Sm}^3/\text{d}/\text{bar}$$

$$S = 0.31$$

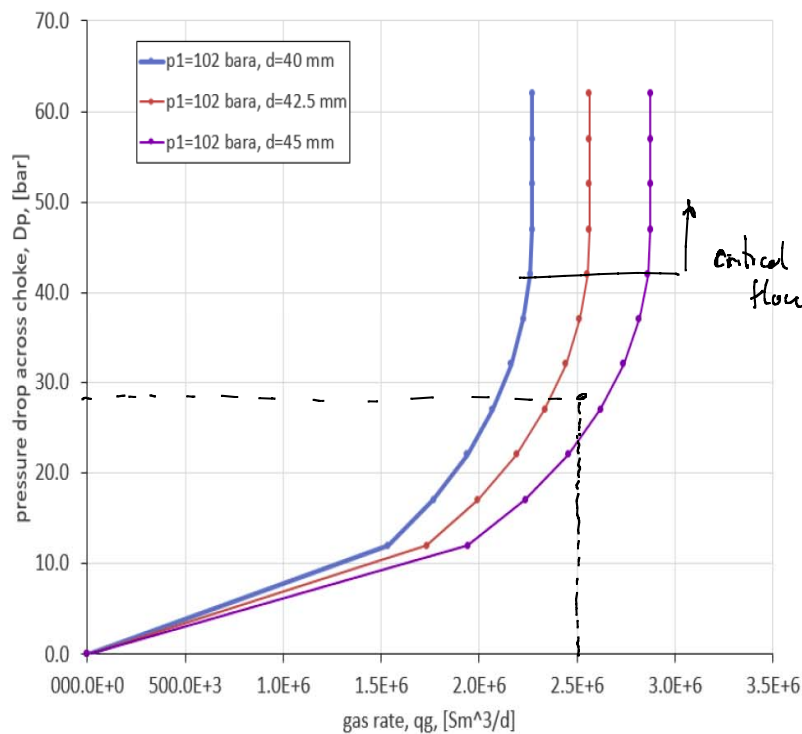
Flowline wellhead-separator:

$$q_g = C_{FL} \cdot (p_m^2 - p_{sep}^2)^{0.5}$$

$$C_{FL} = 4 \text{ E4 Sm}^3/\text{d}/\text{bar}$$

$$p_{sep} = 40 \text{ bara}$$

2. Based on the choke performance maps shown below, what is (approximately) the required choke diameter (in mm) to provide the desired rate of 2.5 E6 Sm³/d. Use the wellhead pressure and choke pressure drop calculated in exercise 2. Will the choke operate in the critical or subcritical regime?



rule of thumb

critical flow occurs when

$$\frac{p_{dc}}{p_{wh}} \leq 0.5 - 0.6$$

$$p_{wh}$$

$$\Delta p_{critical} = p_{wh} - p_{dc}$$

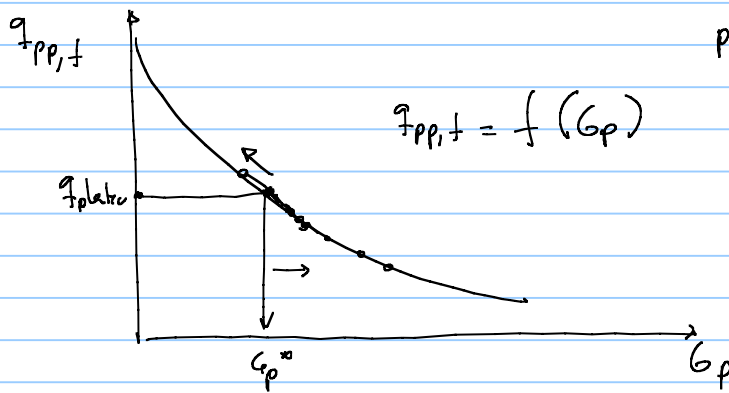
$$p_{wh} = 0.6 \cdot p_{wh}$$

$$0.4 \cdot p_{wh}$$

$$\Delta p = 0.4 \cdot 102 = 40.8 \text{ bara}$$

for open choke $q_g = 2.67 \text{ E6 Sm}^3/\text{d}$

production
potential of
field



plateau period is over when

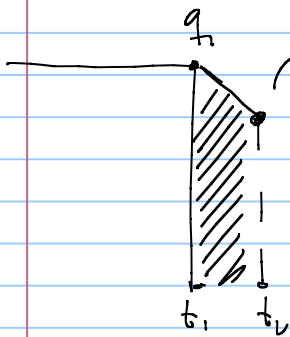
$$q_{\text{plateau}} = q_{pp,t}$$

for $t > t_p$, produce at
potential

$t < t_p$, produce at
plateau

how can this curve be used for predicting post plateau profile?

$$q_{\text{field}} = q_{pp,t}$$



$$(G_{p2} - G_{p1}) = (t_2 - t_1) (q_1 + q_2) \cdot 0.5$$

assume $q_2 (< q_1)$

compute G_{p2}

calculate $q_{pp,t}$ with G_{p2}

check if $q_{pp,t} = q_2$?

yes
solution

no