- Some comments from last class

time	qwell	Gp	Z	RF	PR	pwf	pwh	ptemp	qtemp	qfield	Pplem	Psep	Deltpchoke
[years]	[Sm^3/d]	[Sm^3]			[bara]	[bara]	[bara]	[bara]	[Sm^3/d]	[Sm^3/d]	[bara]	[bara]	[bara]
0	2.22E+06	000.0E+0	0.967	000.0E+0	276	271.95	245.568	83.96564	6.66E+06	2.00E+07	81	35	162
1	2.22E+06	7.3E+9	0.963	27.0E-3	268.548	264.3839	238.387	83.96564	6.66E+06	2.00E+07	81	35	154
2	2.22E+06	14.6E+9	0.957	54.0E-3	259.8348	255.5287	229.9684	83.96564	6.66E+06	2.00E+07	81	35	146
3	2.22E+06	21.9E+9	0.953	81.0E-3	251.0633	246.6041	221.4669	83.96564	6.66E+06	2.00E+07	81	35	138
4	2.22E+06	29.2E+9	0.948	108.0E-3	242.4487	237.8281	213.0883	83.96564	6.66E+06	2.00E+07	81	35	129
5	2.22E+06	36.5E+9	0.944	135.0E-3	234.0167	229.2262	204.8559	83.96564	6.66E+06	2.00E+07	81	35	121
6	2.22E+06	43.7E+9	0.941	162.0E-3	225.7596	220.79	196.7603	83.96564	6.66E+06	2.00E+07	81	35	113

no decimal points!

$$q = C_T \left(\frac{P_1^2}{e^S} - P_2^2 \right)^{0.5}$$

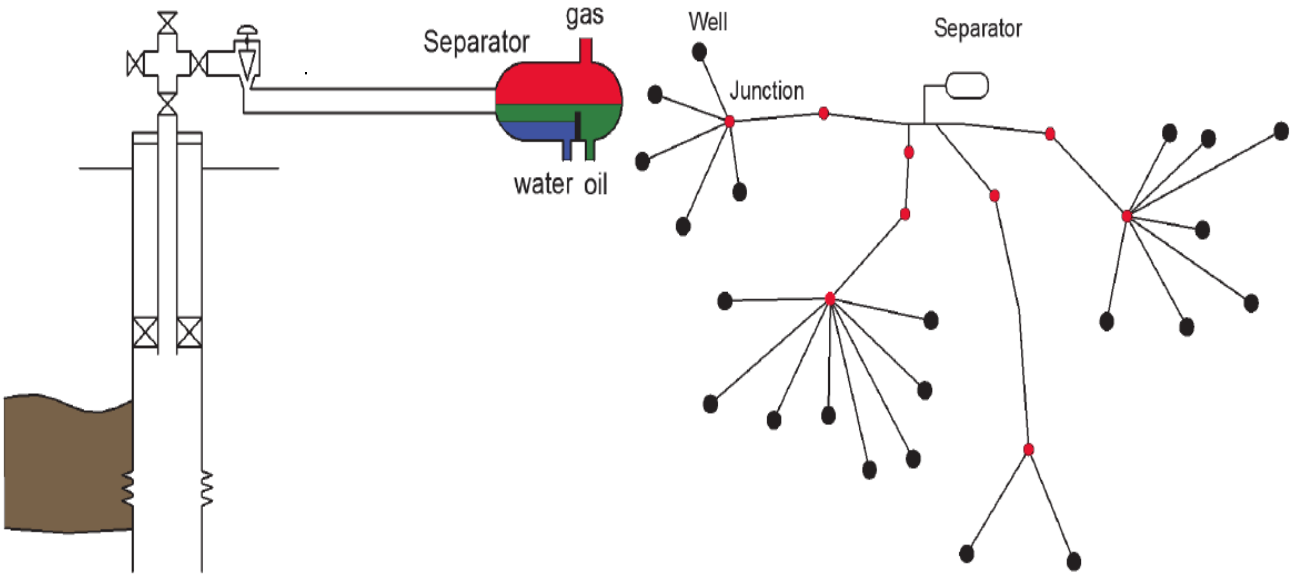
$$P_2 = \left(\frac{P_1^2}{e^S} - \left(\frac{q}{C_T} \right)^2 \right)^{0.5}$$ will give error if

$$\frac{P_1^2}{e^S} < \frac{q^2}{C_T}$$

- be careful with initial assumptions of variables for calculating equilibrium!

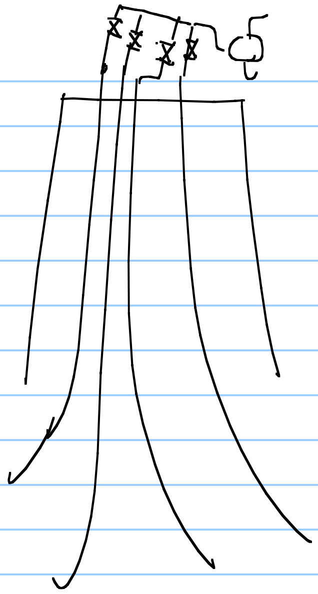
might cause problems when computing co-current calculations of Δp.!

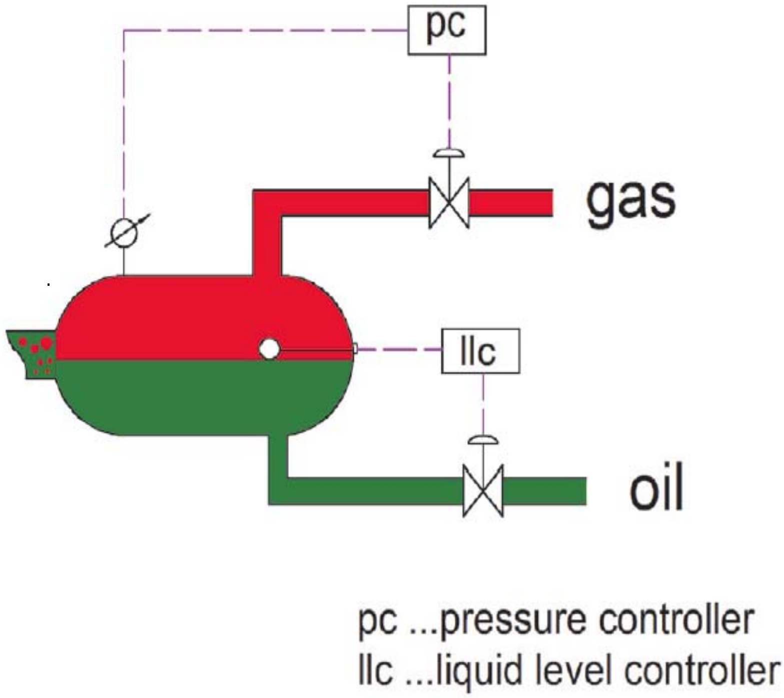
Architecture of a production system.



(a) standalone wells

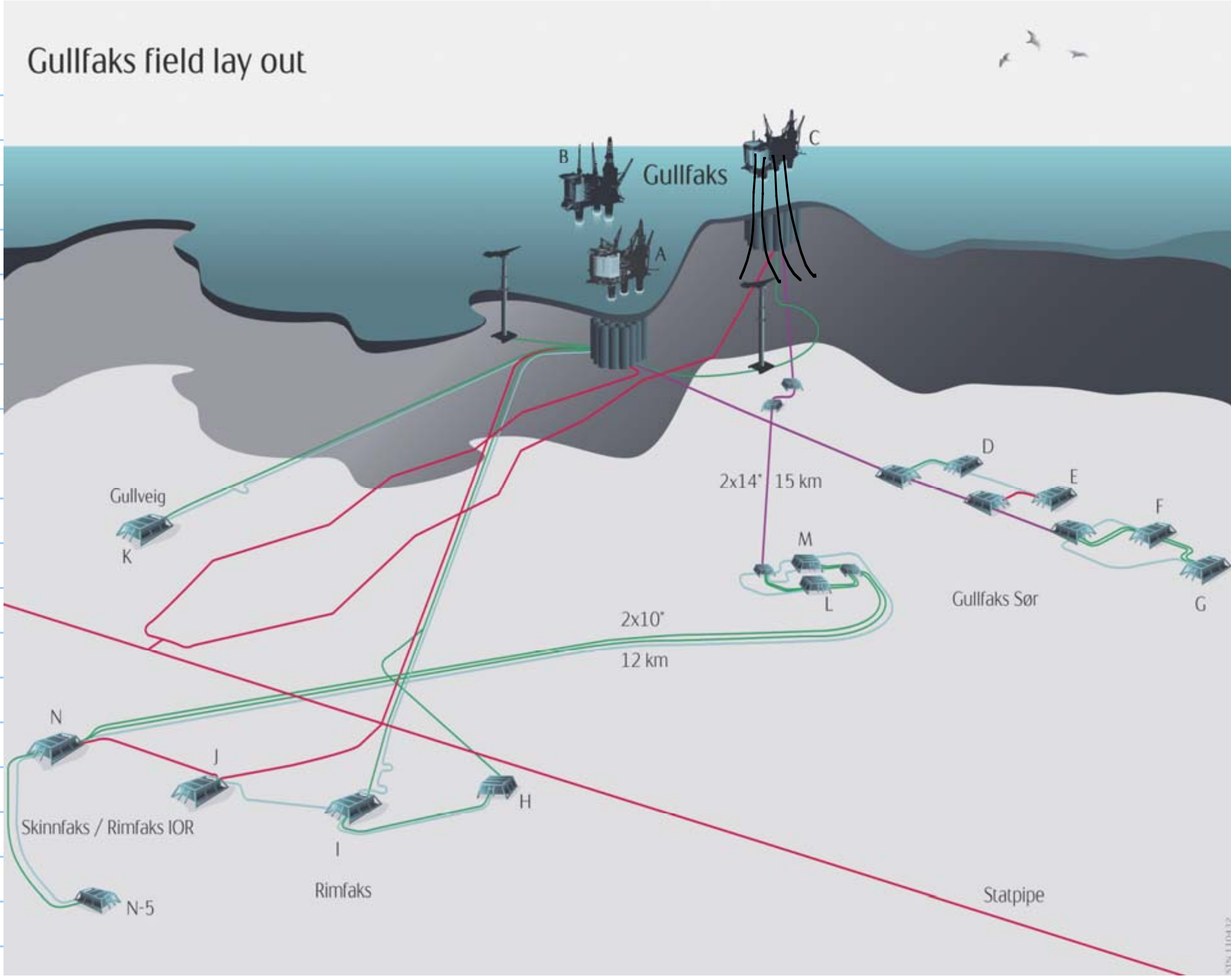
(b) commingling network

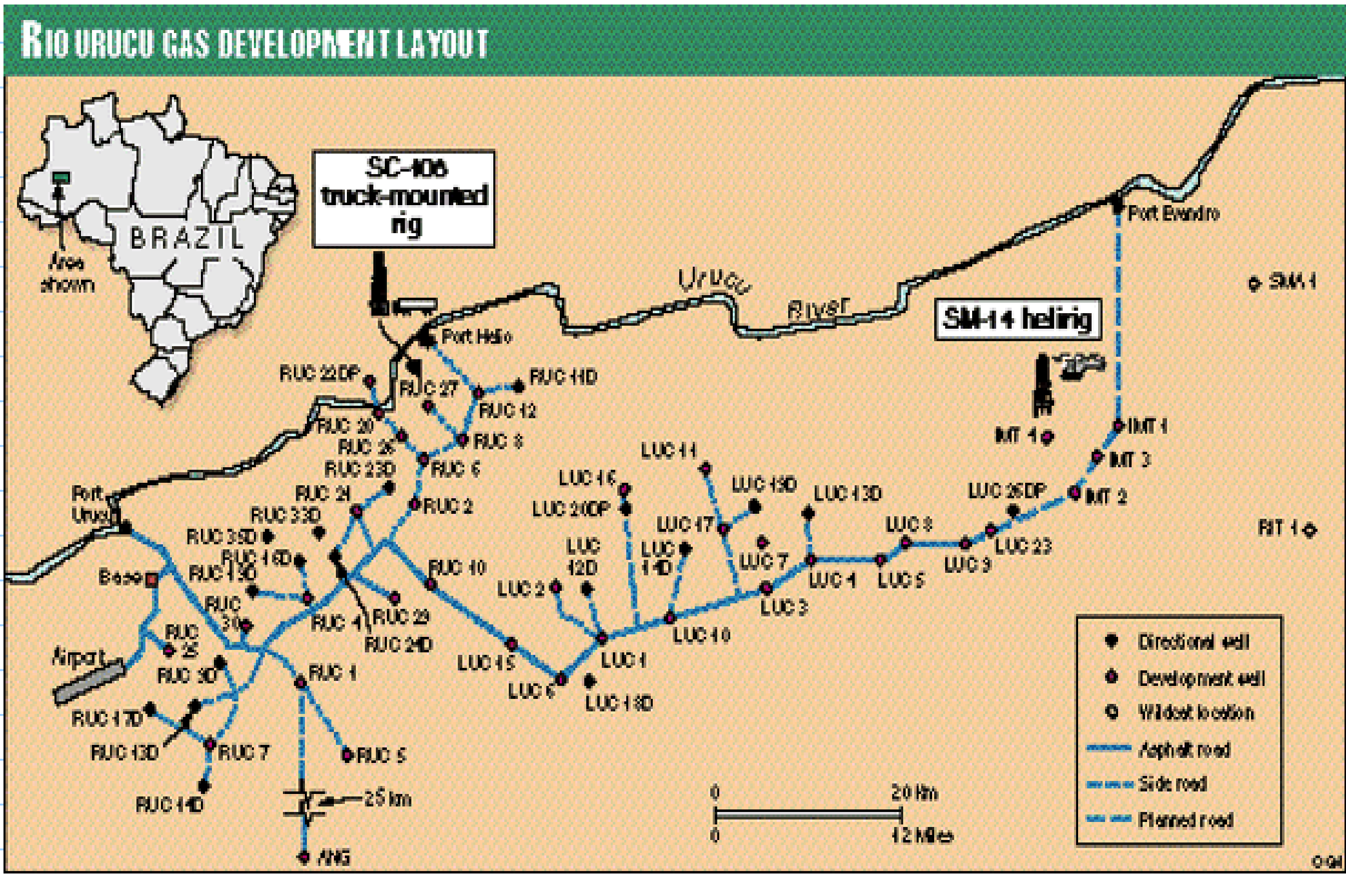


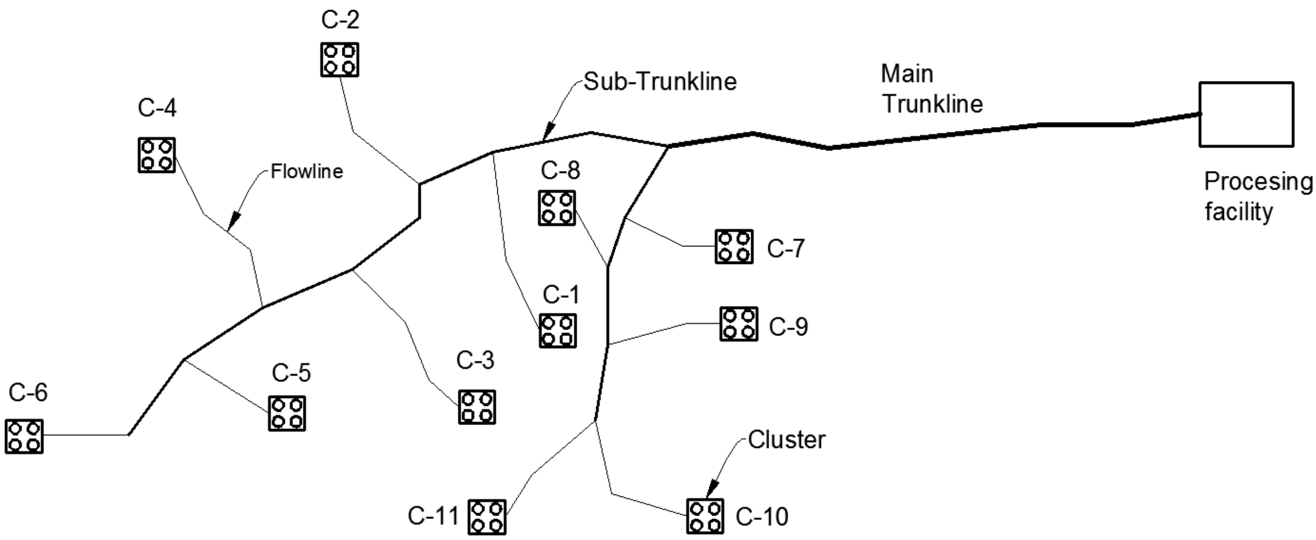


hydraulic dependency ends up at the separator.

Salym

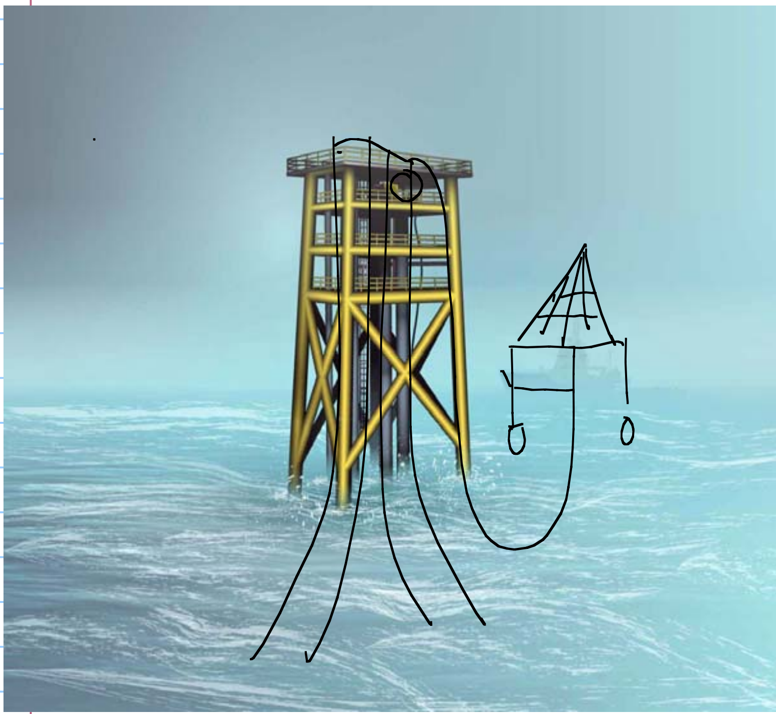

Gullfaks field lay out
Gullfaks
B
C
A
Gullveig
K
2x14" 15 km
D
E
F
G
M
L
Gullfaks Sør
2x10"
12 km
N
J
H
I
Skinnfaks / Rimfaks IOR
N-5
Rimfaks
Statpipe


RIO URUCU GAS DEVELOPMENT LAYOUT
BRAZIL
Area shown
SC-106 truck-mounted rig
SM-14 helirig
Urucu River
Port Evandro
Port Urucu
Base
Airport
SM-1
RIT 1
IMT 1
IMT 3
IMT 2
LUC 25DP
LUC 23
LUC 3
LUC 8
LUC 5
LUC 4
LUC 13D
LUC 15D
LUC 7
LUC 11
LUC 16
LUC 20DP
LUC 17
LUC 14D
LUC 12D
LUC 2
LUC 10
LUC 1
LUC 15
LUC 6
LUC 18D
RUC 22DP
RUC 27
RUC 11D
RUC 12
RUC 8
RUC 20
RUC 6
RUC 23D
RUC 21
RUC 2
RUC 33D
RUC 35D
RUC 16D
RUC 13D
RUC 10
RUC 30
RUC 4
RUC 29
RUC 24D
RUC 25
RUC 1
RUC 17D
RUC 7
RUC 14D
RUC 5
25 km
ANG
0
20 Km
12 Miles
Directional well
Development well
Wildcat location
Asphalt road
Side road
Planned road
OGJ

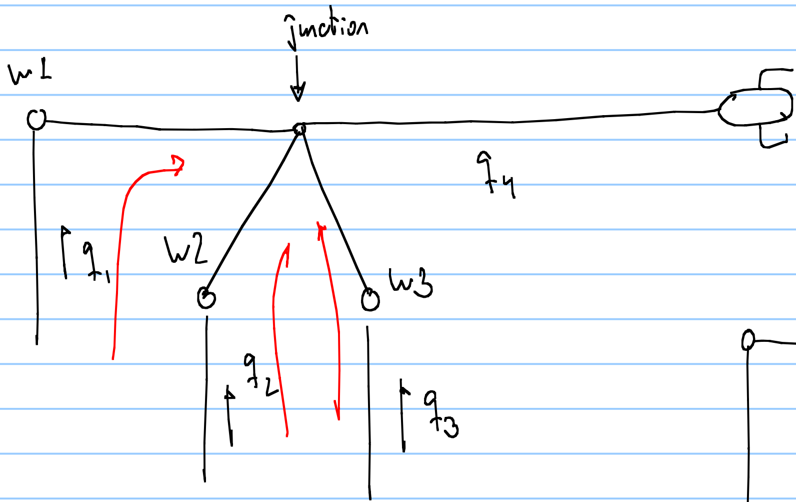

C-2
C-4
Sub-Trunkline
Main
Trunkline
Flowline
C-8
Procesing
facility
C-7
C-1
C-9
C-3
C-5
C-6
Cluster
C-11
C-10

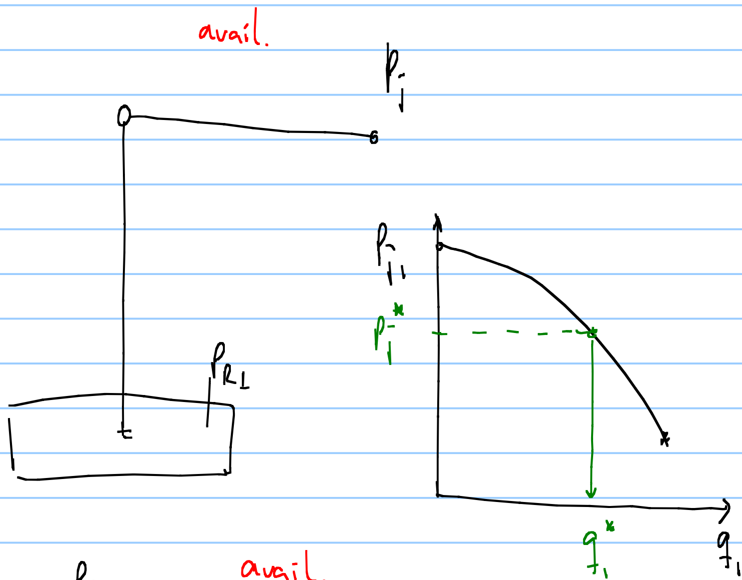

Caribbean Sea
Barranquilla
Cartagena
(Puerto Bahia)
PROMIGAS
Pipeline
Coveñas
Colon
TRANSISTMICO
Pipeline
Taboguilla
PANAMA
CR-1
SSJN-3
Guama
La Creciente
SSJN-7
SSJN-9*
Catguas
Ayacucho
Carbonera La Silla
Santa Cruz
Caño Limon-Coveñas
Pipeline
Carbonera
Cerrito
VENEZUELA
ODC
Pipeline
BICENTENARIO
Pipeline
Caño Limon
Arauca
Banadia
Lla-41
Lla-7
Lla-55
CPE-1
Cor-24
OCENSA
Pipeline
Las Quinchas
VMM-35
VMM-11
Cor-15*
Yamu
Araguaney
Arrendajo
Cubiro
La Dorada
Dindal
Guaduas-Dorada
Pipeline
Rio Seco
Rio Magdalena
Guaduas
Bogota
Monterrey
Moriche
La Punta
CPO-1
Abanico
Buganviles
Guasimo
Cor-33
ODL
Pipeline
Sabanero*
Chipalo
OAM
Pipeline
CPO-12
CPO-14
Rubiales/Pirin
CPE-6
VSM-12
Tenay
VSM-13
CPO-17*
Pacific Ocean
TRANSANDINO
Pipeline
Tumaco
Portofino
Topoyaco
Cag-6
Cag-5
Mecaya
Mansoya
Put-9
Tacacho
Terecay
COLOMBIA
ECUADOR
LEGEND
Exploration
E&P
Production
2012 JV & Acquisitions
Oil Pipelines
Gas Pipelines
Projects
Port (Oil & Liquids Facilities)
Capital City
Station
*The Company holds an indirect ownership interest of 49.999% in Maurel et Prom Colombia B.V.

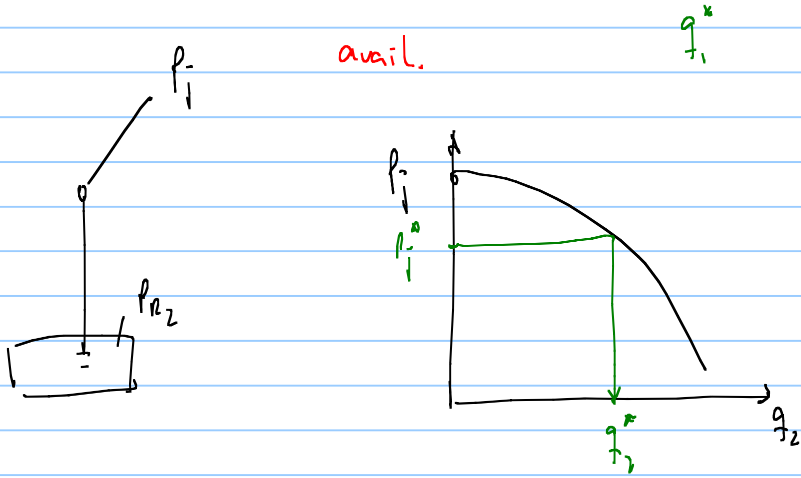

Bahrain
Qatar
Sharjah
Abu Dhabi
Hatta
Al Ain
United Arab Emirates
Riyadh
Saudi Arabia
Al Majmaah
Ar Rass
Al Duwadimi
Kharj
Al Hofuf
Mesaieed
Ghayathi
Al Qu'a
Al Aflaj
SHAYBAH OIL FIELD
Wadi Addawasir
Bishah
Sabt Alalayah
Khamis Mushait
Abha
Oroug Bani M'aradh Wildlife Sanctuary
Oman

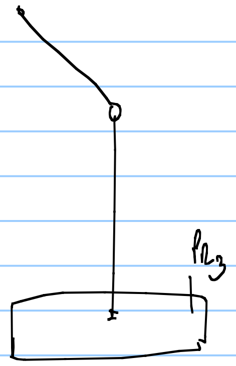


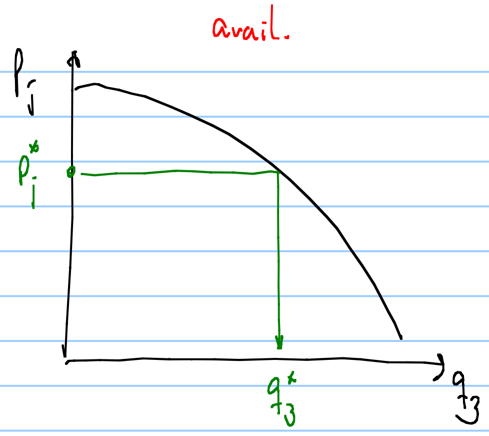
Hydraulic performance of production networks.

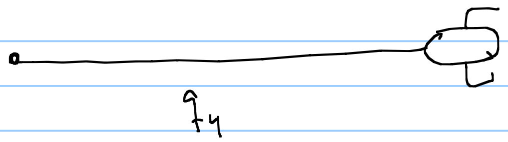


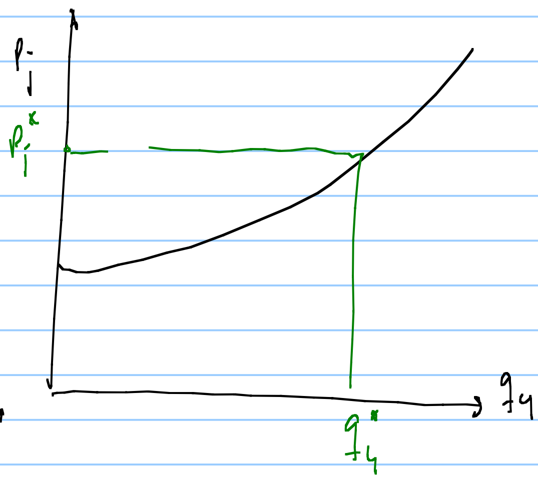












mass conservation in the junction

$$\boxed{q_4 = q_1 + q_2 + q_3} \to P_{j1} = P_{j2} = P_{j3} = P_{j4}$$

assumed Method to solve

1. Assume P_j^*

2: Read $q_1^*, q_2^*, q_3^*, q_4^*$

avail avail avail required

3: Check for mass conservation $q_1^* + q_2^* + q_3^* - q_4^* = 0$

---

iterating with rate

1. assume $q_1^* \to P_{j1}^*$

2: assume $q_2^* \to P_{j2}^*$

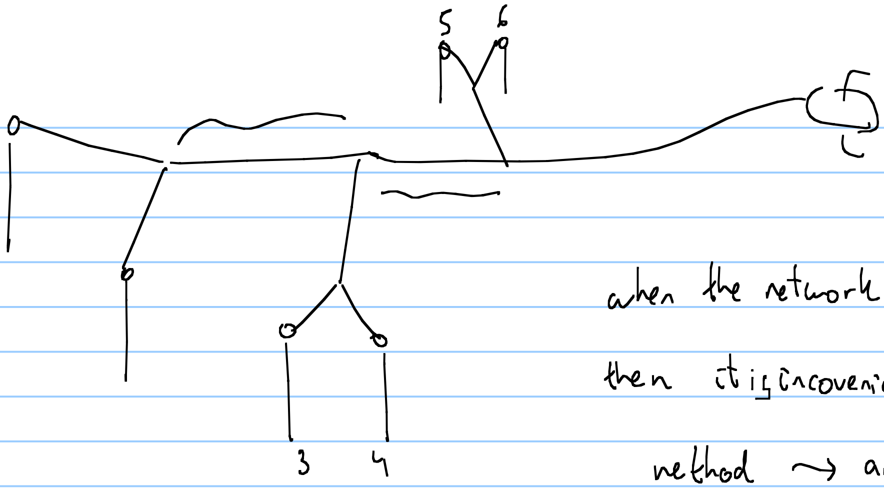
3: assume $q_3^* \to P_{j3}^*$

4: $q_4^* = q_1^* + q_2^* + q_3^*$

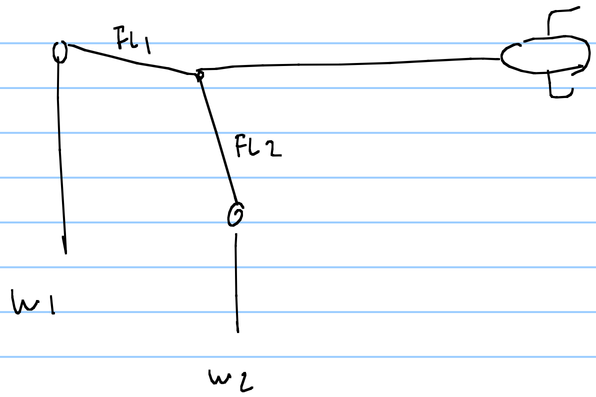
5: with q_4^* read P_{j4}^*

6: check if

$P_{j1}^* = P_{j2}^* = P_{j3}^* = P_{j4}^*$



when the network is too complex then it is inconvenient to use the graphical method $\rightarrow$ analytical method



dry gas

eq	unk.		
2	4	IPR	$q_1 = C_{R_1}\left(P_{R_1}^2 - P_{wf_1}^2\right)^{n_1}$ $q_2 = C_{R_2}\left(P_{R_2}^2 - P_{wf_2}^2\right)^{n_2}$
2	2	tubing	$q_1 = C_{T_1}\left(\frac{P_{wf_1}^2}{e^{S_1}} - P_{wh_1}^2\right)^{0.5}$ $q_2 = C_{T_2}\left(\frac{P_{wf_2}^2}{e^{S_2}} - P_{wh_2}^2\right)^{0.5}$
2	1	flowlines	$q_1 = C_{FL_1}\left(P_{wh_1}^2 - P_j^2\right)^{0.5}$ $q_2 = C_{FL_2}\left(P_{wh_2}^2 - P_j^2\right)^{0.5}$
1	0	pipeline	$q_1 + q_2 = C_{PL}\left(P_j^2 - P_{sep}^2\right)^{0.5}$
7	7		

$\rightarrow$ can solve this system of equations using a numerical method

$\rightarrow$ Newton-Raphson for a system of equations

Network solving

Nonlinear system of equations

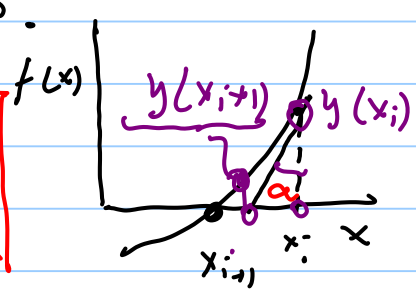
given a set of non-Liner equations (NLE).

$$f_1(x_1, x_2, \ldots x_n) = 0$$
$$f_2(x_1, x_2 \ldots x_n) = 0$$
$$\vdots$$
$$f_n(x_1, x_2 \ldots x_n) = 0$$

find a set of values $x_1 x_2 \ldots x_n$ that simultaneously results in all equation equaling zero

Recall Newton-Raphson root finding procedure of $f(x)=0$.

$$\boxed{tg\,\alpha = f'(x_i) = \frac{f(x_i)-0}{x_i - x_{i+1}}}$$



Rearranging

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}} \quad \text{or} \quad \boxed{f' = \frac{f}{-dx}} \quad \text{or} \quad \boxed{f'(x) \cdot dx = -f(x)}$$

This is Newton Raphson formula

solution procedure (for one NLE)

1. guss x
2. find $f(x)$ and $f'(x)$
3. calculate $dx = (x_{i+1} - x_i) = -\dfrac{f(x)}{f'(x)}$
4. update $x^{new} = x^{old} + dx$
5. calculate $f(x)^{new}$
6. If $f(x)^{new} \leq \varepsilon \rightarrow$ yes $\rightarrow$ Exit; no $\rightarrow$ go to 2

The graphical procedure is based on first-order Taylor series expansion

$$f(x_{i+1}) = f(x_i) + (x_{i+1} - x_i) f'(x_i)$$

One approach for solving a set of NLE is to use multi-dimensional version of Newton Raphson method for one equation above:
This is based on multi-dimension Taylor series expansion:

for example,

$f_1(x_1, x_2) =$

$f_2(x_1, x_2)$

$$f_1^{i+1} = f_1^i + \underbrace{(x_1^{i+1} - x_1^i)}_{dx_1} \frac{\partial f_1^i}{\partial x_1} + \underbrace{(x_2^{i+1} - x_2^i)}_{dx_2} \frac{\partial f_1^i}{\partial x_2}$$

$$f_2^{i+1} = f_2^i + \underbrace{(x_1^{i+1} - x_1^i)}_{dx_1} \frac{\partial f_2^i}{\partial x_1} + \underbrace{(x_2^{i+1} - x_2^i)}_{dx_2} \frac{\partial f_2^i}{\partial x_2}$$

Recal for single equation

$$f'(x) \cdot dx = -f(x)$$

we can prove (soon) that this equation is valid for set of equations and can be written as an Array Formula As matrix

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{df_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} = - \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix}$$

$$\boxed{\frac{\partial \bar{f}}{\partial x} \cdot d\bar{x} = -\bar{f}}$$

The partial derivative metrix is called the <u>Jacobian</u> of the system. Thus

$$J \begin{bmatrix} dx_1 \\ dy_1 \end{bmatrix} = -\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} = -J^{-1}\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\begin{bmatrix} dx_1 \\ dx_2 \end{bmatrix} = -\frac{\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}}{J} =$$

In algebric form:

$$dx_1 = \underbrace{x_1^{i+1} - x_1^i}_{dx_1} = -\frac{f_1 \frac{\partial f_2}{\partial x_2} - f_2 \frac{\partial f_1}{\partial x_2}}{\underbrace{\frac{\partial f_1}{\partial x_1} \cdot \frac{\partial f_2}{\partial x_2} - \frac{\partial f_1}{\partial x_2} \cdot \frac{\partial f_2}{\partial x_1}}_{\text{Determinant of the Jacobian}}}$$

$$dx_2 = x_2^{i+1} - x_2^i = -\frac{f_2 \cdot \frac{\partial f_1}{\partial x_1} - f_1 \frac{\partial f_2}{\partial x_1}}{\underbrace{\frac{\partial f_1}{\partial x_1} \frac{\partial f_2}{\partial x_2} - \frac{\partial f_1}{\partial x_2} \cdot \frac{\partial f_2}{\partial x_1}}_{\text{determinant of } J}}$$

in more general form (Jacobian form)

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{dx_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_2} \end{bmatrix}$$

The array equation is then

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_2} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

$$\{\bar{x}_i\}^T = \lfloor x_1^i, x_2^i, \cdots x_n^i \rfloor$$

$$\{\bar{x}_{i+1}\}^T = \lfloor x_1^{i+1}, x_2^{i+1}, \cdots x_n^{i+1} \rfloor$$

$$\{\bar{f}_i\}^T = \lfloor f_1^i, f_2^i, \cdots f_n^i \rfloor$$

$$[J]\{\bar{x}_{i+1}] = -\{\bar{f}_i\} + [J]\{\bar{x}_i\}$$

This equation can be solved using technique such as Gaus Eliminatio

$$J_{ij}\,\bar{\Delta x}_j = -\bar{f}_i$$

$$\Delta x_j = -J_{ij}^{-1} \cdot f_i$$

$$x^{n+1} = x^n + \Delta x$$

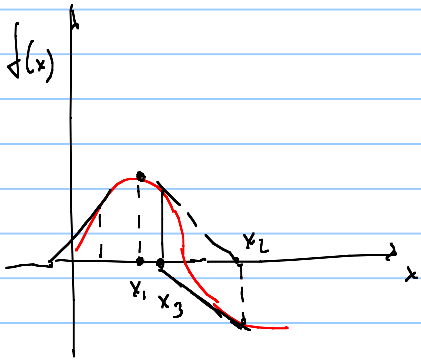
1. guss x_j
2. find $f(x)$ and $f'(x)$
3. calculate $dx = -\frac{f(x)}{f'(x)}$
4. update $x^{new} = x^{old} + dx$
5. got to 2 if necessary

Anothe apprach to solution of
a set of NLE

1. Formulate the non-linear system
as a single function

$$F(x) = \sum_{i=1}^{n} \left[w_i f_i (x_1, x_2 \cdots x_n) \right]^2$$

The value of $\bar{x}$ that minimize the
function represent the solution
of the non-linear system.
This formulation is a class of
Problem call <u>non-linear regrassion</u>



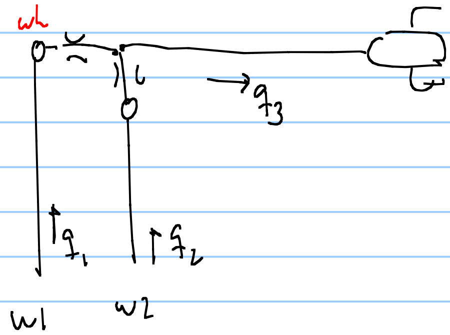
$$|x_{i+1} - x_i| \leq TOL$$

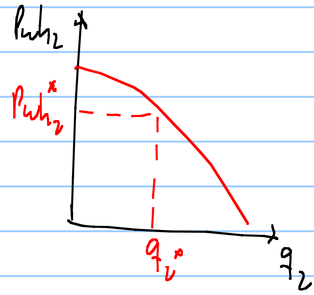
We have to make our equations "bulletproof" $\rightsquigarrow$ avoid crashing of the solving algorithm

↳ useful for using commercial software !

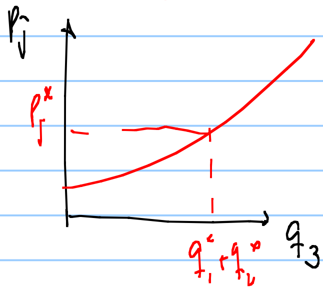
---

wells close to the junction → flowline can be neglected





- e.g. with production rates from the reservoir engineer obtained to ↑ RF $\Bigg\}$ q_1^*, q_2^*



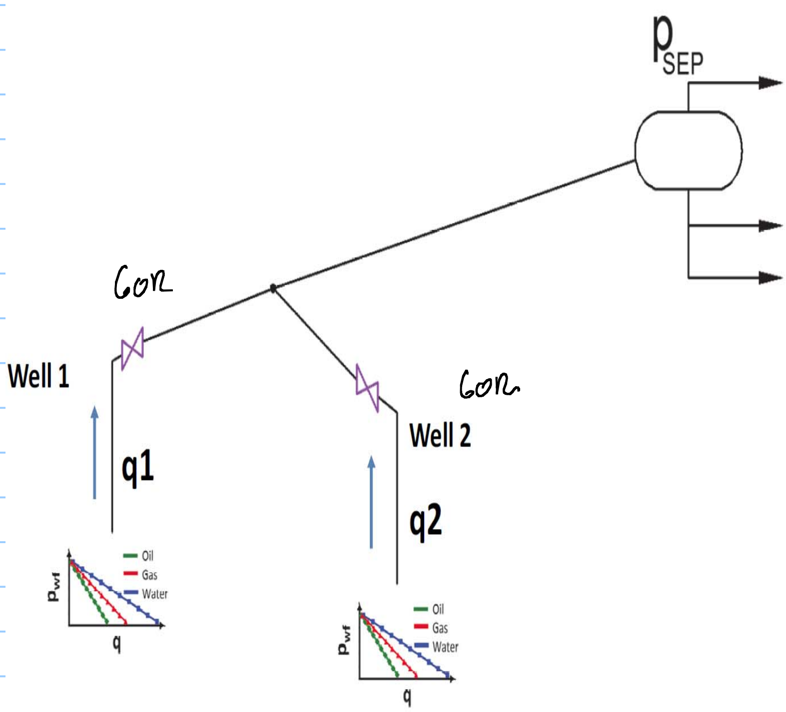
- use $q_1^* \overset{\text{read}}{\rightsquigarrow} P_{wh_1}^*$
- use $q_2^* \overset{\text{read}}{\sim} P_{wh_2}^*$
- use $q_1^* + q_2^* \overset{\text{read}}{\sim} P_j^*$
- Calculate choke Δp

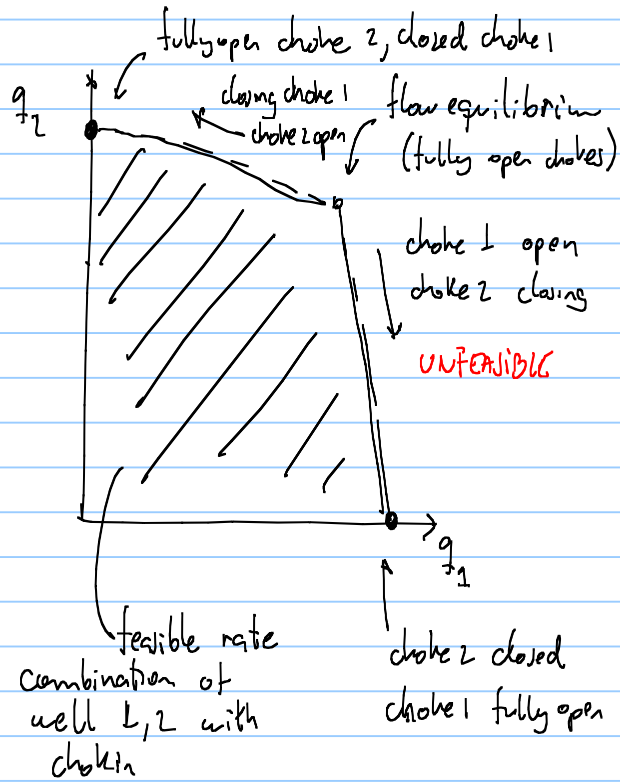
- $\Delta p_{choke_1} = P_{wh_1}^* - P_j^*$
- $\Delta p_{choke_2} = P_{wh_2}^* - P_j^*$

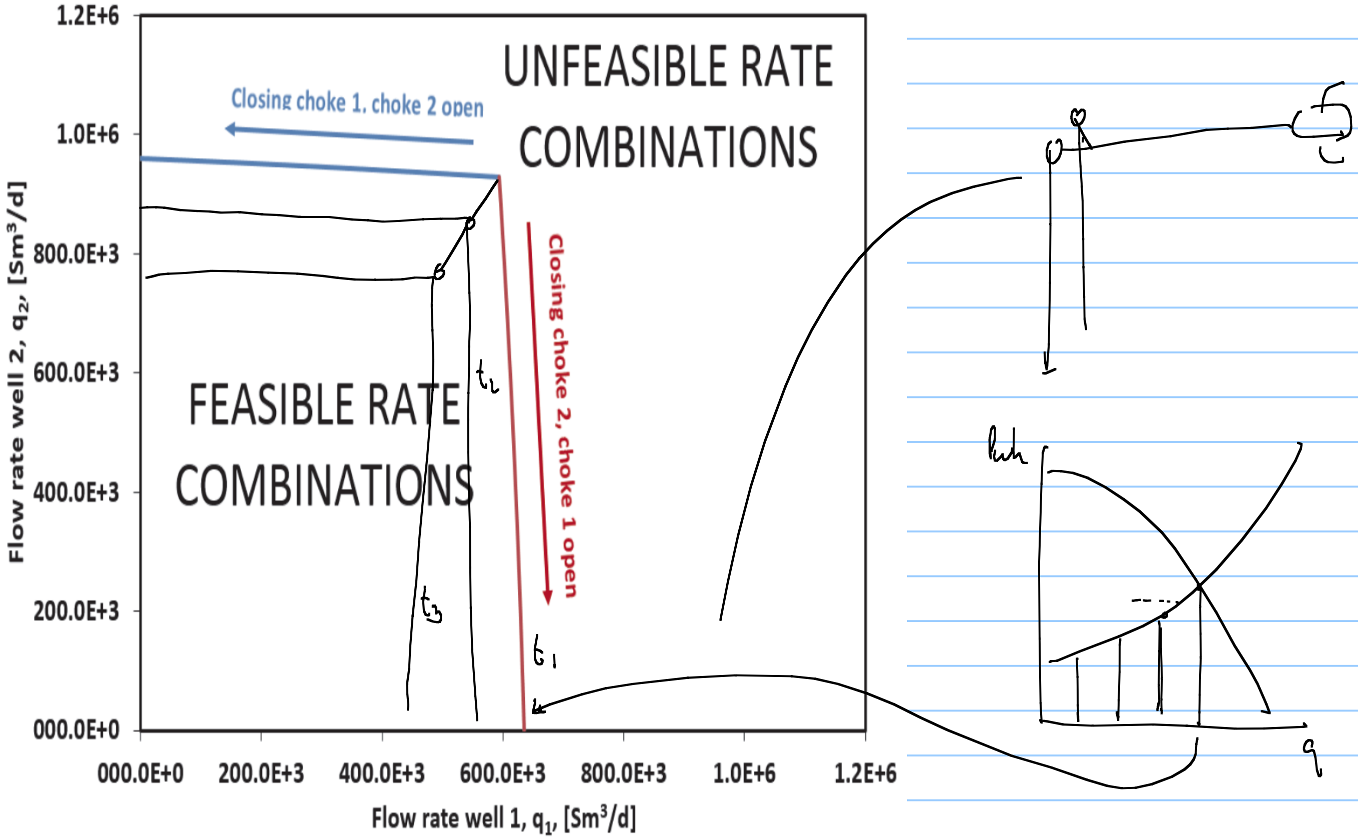
If the rates are feasible the $\Delta p_{choke_1} \geq 0$

$\Delta p_{choke_2} \geq 0$


LAXMI BLESSING
NW5630


p SEP
GOR
Well 1
q1
GOR
Well 2
q2
Oil
Gas
Water
pwf
q


fully open choke 2, closed choke 1
q2
closing choke 1
choke 2 open
flow equilibrium
(fully open chokes)
choke 1 open
choke 2 closing
UNFEASIBLE
q1
feasible rate
combination of
well 1,2 with
chokin
choke 2 closed
choke 1 fully open



UNFEASIBLE RATE COMBINATIONS
Closing choke 1, choke 2 open
Closing choke 2, choke 1 open
FEASIBLE RATE COMBINATIONS
1.2E+6
1.0E+6
800.0E+3
600.0E+3
400.0E+3
200.0E+3
000.0E+0
Flow rate well 2, q_2, [Sm³/d]
000.0E+0
200.0E+3
400.0E+3
600.0E+3
800.0E+3
1.0E+6
1.2E+6
Flow rate well 1, q_1, [Sm³/d]
t_1
t_2
t_3
p_{wh}
q