Note Title solve excercise / plots 07.11.2018

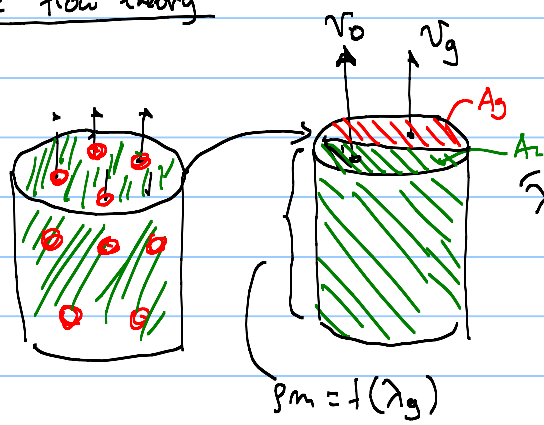
Class 9

evaluation : 20% home exercises → to be delivered by 18 Nov 2018 midnight
→ only ONE email per group
email subject:
"POFE exercise group last name 1, last name 2, last name 3"

40% Quiz 19–23 Nov 2018 ~ 3 hrs theory/practice

40% exam 26–30 Nov 2018 ~ 3 hrs theory/practice

multiphase flow theory



the fluid is homogeneous

$v_L = v_g$ → no-slip condition

$\lambda_g = \frac{A_g}{A}$ cross-section Area

→ no-slip gas volume fraction

$\lambda_L = \frac{A_L}{A}$

$$v_m = v_L = v_g = \frac{q_L + q_g}{A} = \frac{q_L}{A} + \frac{q_g}{A}$$

$u_{sL} + u_{sg}$

$$v_m = \frac{q_L}{A} + \frac{q_g}{A} = v_L = v_g$$

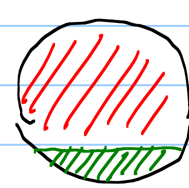
$$A_L v_L = \frac{A_L q_L}{A} + \frac{q_g A_L}{A}$$

$$q_L = \lambda_L q_L + \lambda_L q_g$$

$$q_L = \lambda_L (q_L + q_g)$$

$$\frac{q_L}{q_L + q_g} = \lambda_L \longrightarrow \rho_m^H = \lambda_L \rho_L + (1-\lambda_L)\rho_g$$

$$\lambda_g = \frac{A_g}{A} = \frac{A - A_L}{A} = 1 - \lambda_L$$



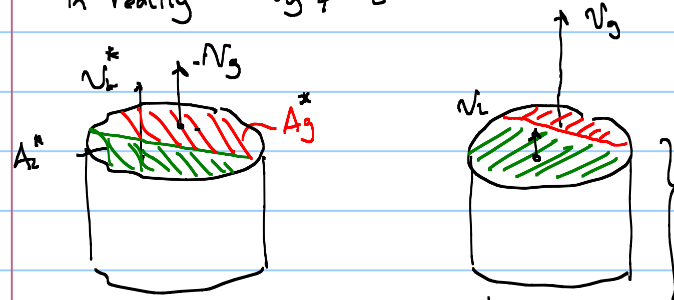
$\sim \lambda_g = 80\%$

$$\frac{dp}{dl} = -f \cdot \rho_m^H \frac{v_m^2}{2\phi} - \rho_m^H \sin\theta \, g$$

→ teta is angle from horizontal

homogeneous model

in reality $v_g \neq v_L$



homogeneous case

inhomogeneous (slip) case

$q_L = v_L^* A_L^* = v_L \cdot A_L$

$q_g = v_g^* A_g^* = v_g A_g$

$\rho_m \rightsquigarrow \rho_L$

q_L = constant

q_g = constant

"real" volume fraction

$\varepsilon = \frac{A_g}{A} \neq \lambda_L$ — void fraction

$\underbrace{H_L}_{\text{liquid Holdup}} = \frac{A_L}{A}$

- slip ratio $S = \frac{v_g}{v_L}$
- relative velocity $v_R = v_g - v_L$

the first step to calculate $\frac{dp}{dx}$ is to calculate $\varepsilon (H_L)$
- lab measurements → correlation
- mechanistic model (force balance)
- mix

Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

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Lockhart and Martinelli (1949)	$\varepsilon = \left[1 + 0.28\left(\frac{1-x}{x}\right)^{0.64}\left(\frac{\rho_G}{\rho_L}\right)^{0.36}\left(\frac{\mu_L}{\mu_G}\right)^{0.07}\right]^{-1}$
Fauske (1961)	$\varepsilon = \left[1 + \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{0.5}\right]^{-1}$
Fujie (1964)	$\varepsilon = \left[1 + \left(\sqrt{(68947.57/P)\varepsilon} + 1\right)\left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)\right]^{-1}$
Thom (1964)	$\varepsilon = \left[1 + \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{0.89}\left(\frac{\mu_L}{\mu_G}\right)^{0.18}\right]^{-1}$
Zivi (1964)	$\varepsilon = \left[1 + \left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)^{0.67}\right]^{-1}$
Turner and Wallis (1965)	$\varepsilon = \left[1 + \left(\frac{1-x}{x}\right)^{0.72}\left(\frac{\rho_G}{\rho_L}\right)^{0.4}\left(\frac{\mu_L}{\mu_G}\right)^{0.08}\right]^{-1}$
Baroczy (1966)	$\varepsilon = \left[1 + \left(\frac{1-x}{x}\right)^{0.74}\left(\frac{\rho_G}{\rho_L}\right)^{0.65}\left(\frac{\mu_L}{\mu_G}\right)^{0.13}\right]^{-1}$
Smith (1969)	$\varepsilon = \left[1 + A_{SM}\left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)\right]^{-1}$
	where $A_{SM} = 0.4 + 0.6\sqrt{\left[\frac{\rho_L}{\rho_G} + 0.4\left(\frac{1-x}{x}\right)\right] / \left[1 + 0.4\left(\frac{1-x}{x}\right)\right]}$
Premoli et al. (1970)	$\varepsilon = \left[1 + A_{PRM}\left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)\right]^{-1}$
	where $A_{PRM} = 1 + F_1\left\{\frac{y}{1+yF_2} - yF_2\right\}$, $F_1 = 1.578 Re_L^{-0.19}\left(\frac{\rho_L}{\rho_G}\right)^{0.22}$
	$F_2 = 0.0273 We_L Re_L^{-0.51}\left(\frac{\rho_L}{\rho_G}\right)^{-0.08}$, $y = \left[\left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)\right]^{-1}$, $We_L = \frac{G^2 D}{\sigma \rho_L}$, $Re_L = \frac{GD}{\mu_L}$
Chisholm (1973)	$\varepsilon = \left[1 + \sqrt{1 - x\left(1 - \frac{\rho_L}{\rho_G}\right)}\left(\frac{1-x}{x}\right)\left(\frac{\rho_G}{\rho_L}\right)\right]^{-1}$
Madsen (1975)	$\varepsilon = \left[1 + \left(\frac{1-x}{x}\right)^{b}\left(\frac{\rho_G}{\rho_L}\right)^{-0.5}\right]^{-1}$
	where $b = 1 + \log\left(\frac{\rho_L}{\rho_G}\right)\left(\log\left(\frac{1-x}{x}\right)\right)^{-1}$
Spedding and Chen (1984)	$\varepsilon = \left[1 + 2.22\left(\frac{1-x}{x}\right)^{0.65}\left(\frac{\rho_G}{\rho_L}\right)^{0.65}\right]^{-1}$

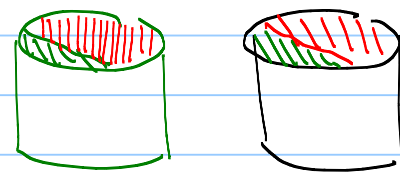
$$\varepsilon = \frac{U_{SG}}{U_{SG}\left(1+\left(\frac{U_{SL}}{U_{SG}}\right)\left(\frac{\rho_G}{\rho_L}\right)^{0.1}\right)+2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L-\rho_G)}{\rho_L^2}\right]^{0.25}(1.22+1.22\sin\theta)^{\frac{p_{atm}}{p_{system}}}}$$

$H_L = f(u_{sL}, u_{sg}, \theta, \rho_g, \rho_L, \Phi, \sigma, \mu_L, \mu_g, p)$

θ ↳ pipe inclination

lambdag[-]	e[-]
0.80	0.70
0.70	0.62
0.58	0.52
0.46	0.43
0.34	0.33
0.24	0.24
0.15	0.16
0.08	0.09

$A_g^* = 0.80$ 0.70



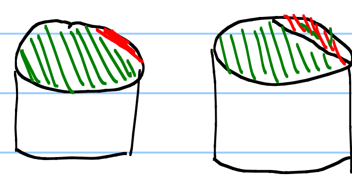
A_L when $u_g = v_L$ / A

A_L when slip $v_g \neq v_L$ / A

gas flows slower

$\lambda_g < \varepsilon$

0.09 > 0.08

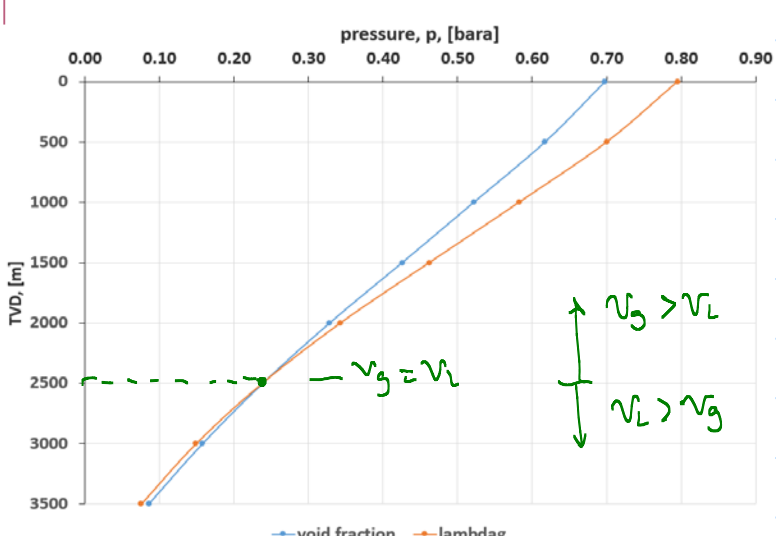


$A_g^* < A_g$

$q_g = A_g^* v_g^* = A_g v_g$

v_L^*

v_m



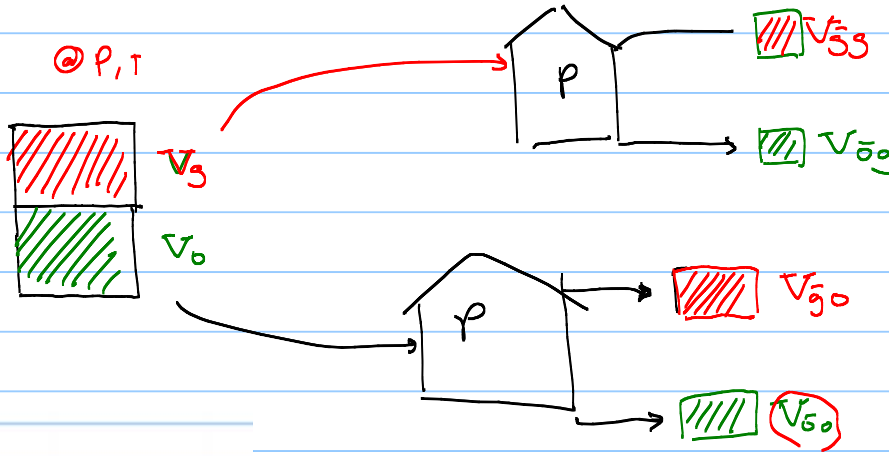
$\rho_m^{H} = \lambda_g \rho_g + (1-\lambda_g)\rho_L$ (0.7)

$\rho_m = \varepsilon \rho_g + (1-\varepsilon)\rho_L$ (0.8)

~ %

$\rho_g(P,T)$

$\rho_o(P,T)$?



@ S.C

$$\rho_o = \frac{M_o}{V_o} = \frac{V_{\bar{g}} \cdot \rho_{\bar{g}} + V_{\bar{o}} \cdot \rho_{\bar{o}}}{V_o} \qquad V_{\bar{g}} = GOR \cdot V_{\bar{o}}$$

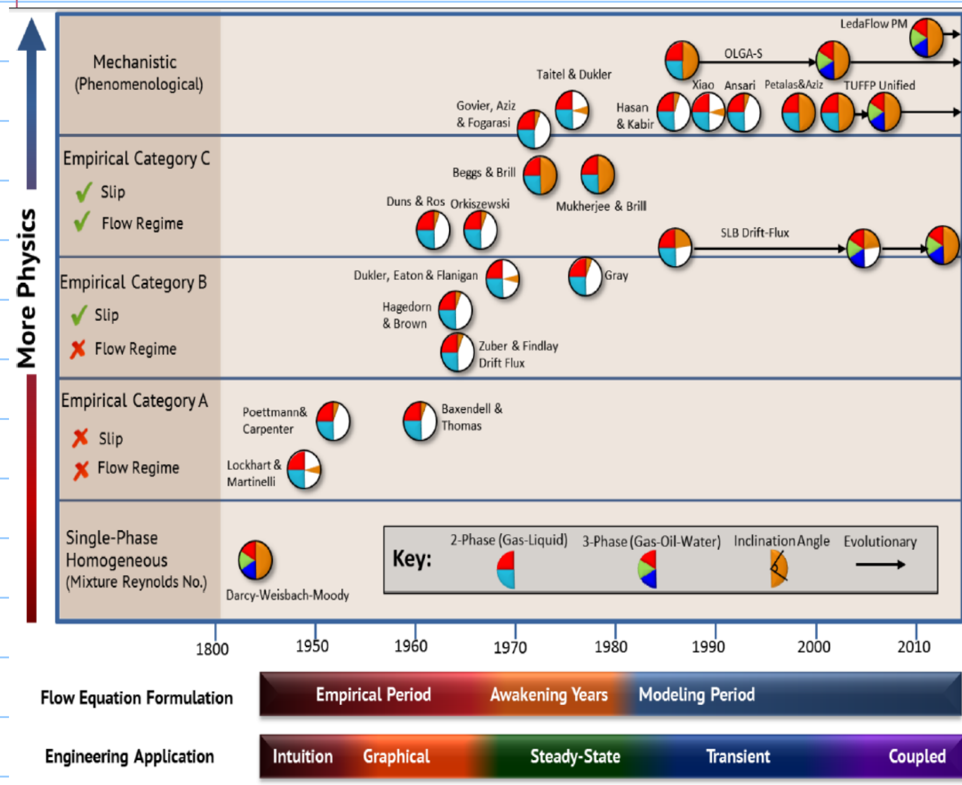
$$\rho_o = \frac{SG_g \cdot 1.224 \; GOR \cdot V_{\bar{o}} + V_{\bar{o}} \; SG_o \cdot 1000}{V_o}$$

$$\Rightarrow \rho_o(P,T) = \frac{SG_g \cdot 1.224 \; GOR + SG_o \cdot 1000}{B_o}$$

$$\begin{Bmatrix} \rho_g \\ \rho_o \end{Bmatrix}_{(P,T)} = \begin{bmatrix} 1/B_g & r_s/B_g \\ R_s/B_o & 1/B_o \end{bmatrix}_{(P,T)} \begin{Bmatrix} \rho_{\bar{g}} \\ \rho_{\bar{o}} \end{Bmatrix} \rightarrow \begin{Bmatrix} \rho_{\bar{g}} \\ \rho_{\bar{o}} \end{Bmatrix} = \begin{bmatrix} \frac{B_g}{1 - r_s R_s} & \frac{-r_s B_o}{1 - r_s R_s} \\ \frac{-R_s B_g}{1 - r_s R_s} & \frac{B_o}{1 - r_s R_s} \end{bmatrix}_{P,T} \begin{Bmatrix} \rho_g \\ \rho_o \end{Bmatrix}$$

calculations of pressure drop in conduits (tubing)

two approaches
- → empirical ⇝ test data (field / lab) ⇝ correlation, adimensional numbers
- → mechanistic ⇝ force balance (momentum equation)
  - mass balance (mass conservation)
  - ⇝ correlation because n equations < n unknowns
    - ↳ closure model

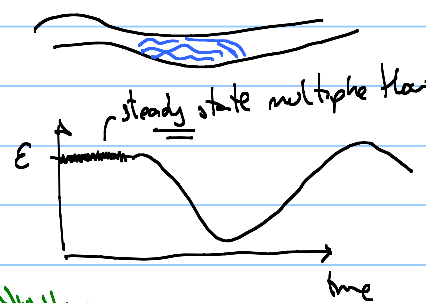


Shippen

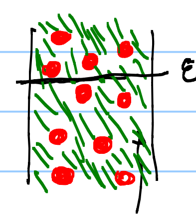
$\frac{dp}{dx}$ multiphase flow is also studied in other applications

- flow assurance (P, T, H_L along the line), liquid accumulation
- study transient phenomena (severe slugging)

$\frac{\partial}{\partial t} = 0$



$\frac{\partial}{\partial t} \neq 0$ then OLGA, Leda



- DRIFT FLUX model

similar homogeneous model single momentum equation

$$\cdot \frac{dP}{dx} = -\frac{\tilde{f}_{TP}\, \tilde{\rho}_{TP}\, V_{TP}^2}{2\phi} - \rho_{TP}\, g \sin\theta \qquad V_m = u_{SL} + u_{SG}$$

f_{moody} friction factor

$\cdot\ f = F(Re, \text{roughness})$

is calculated with liquid Holdup (considering slip!)

$$\rho_{TP} = \varepsilon \rho_g + (1-\varepsilon)\rho_L$$

while homogeneous $\rho_m = \lambda_g \rho_g + (1-\lambda_g)\rho_L$

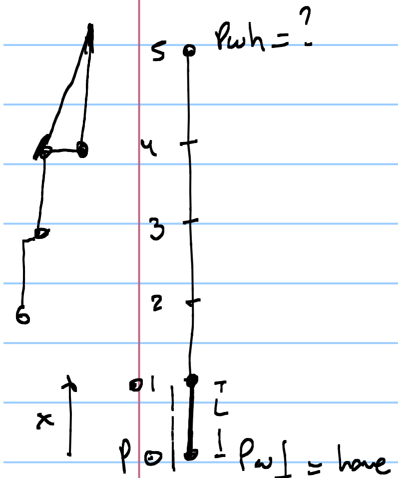
$$Re_m = \frac{v\, d\, \rho}{\mu}$$

$$v = v_m = u_{sl} + u_{sg}$$

$$\rho_{TP} = \varepsilon(\rho_g) + (1-\varepsilon)\rho_o$$

$$\mu_m = \varepsilon(\mu_g) + (1-\varepsilon)\mu_o$$

how to compute p along tubing (pressure integration procedure)



- depart from a place of know pressure
- discretize in segments
- for point of know pressure calculate q_o, q_g → BO tables
- calculate $\rho_g, \rho_o, \mu_g, \mu_o, \sigma_{og}$
- calculate u_{so}, u_{sg}, ⟶ calculate ε

$$\left.\frac{dp}{dx}\right|_{P_{wf}} = C_1$$

- numerical integration

$$\frac{dp}{dx} = \frac{P_o - P_1}{L} \qquad \text{Euler integration}$$

$$P_1 = P_{wf} - \left.\frac{dp}{dx}\right|_{P_{wf}} \cdot L$$

for next section

$$\left.\frac{dp}{dx}\right|_{P_1} = \frac{P_1 - P_2}{L}$$

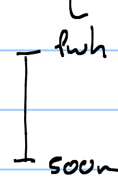
to improve prediction : • reduce spacing (increase number of points)
• use a more sofisticated integration { - Runge-Kutta, - implicit

```
Function dpdx(denl, deng, voidfraction, usl, usg, teta, ID, roughness, viscl, viscg)
Pi = Atn(1) * 4
denm = voidfraction * deng + (1 - voidfraction) * denl
viscm = (voidfraction * viscg + (1 - voidfraction) * viscl) * 0.001
um = usl + usg
f = ffactor(denm, viscm, ID, roughness, um)
dpdx = (f * denm * (um ^ 2) * 0.5 / ID) + denm * 9.81 * Sin(teta * Pi / 180)
dpdx = dpdx / 100000#
End Function
```

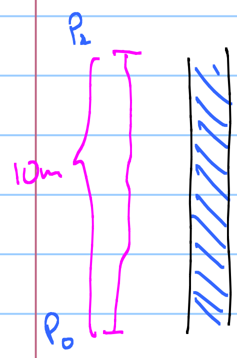
ρ_{TP}

μ_{TP}

V_m



$$\frac{P_{@500} - P_{wh}}{L} = \left.\frac{dp}{dx}\right|_{P_{wh}}$$



$\rho = \rho_w = 1000$

10m – 1.01325 bara

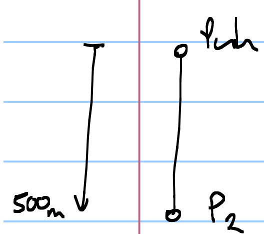
$$\frac{dP}{dx} = \frac{P_0 - P_1}{L} = \frac{1.01325}{10} = 0.1 \text{ bar/m}$$

$\frac{dP}{dx}$ 0.001 bar/m

static column of air

$\rho_{air} = 1.224 \text{ kg/m}^3$

0.02



$$\frac{P_2 - P_{wh}}{L} = \left.\frac{dP}{dx}\right|_{P_{wh}}$$

$$P_2 = P_{wh} + \left.\frac{dP}{dx}\right|_{P_{wh}} \cdot L$$

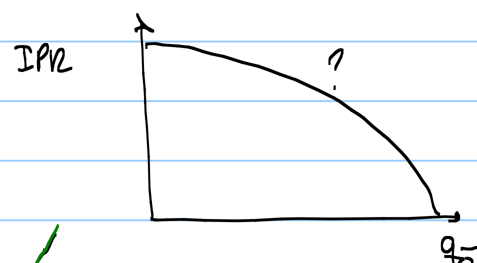
dp/dx [bara/m]
0.0260
0.0312
0.0373
0.0428
0.0476
0.0512
0.0537
0.0552

$$\frac{dp}{dx} = \rho_m \cdot g \sin\theta$$

$\rho_m \rightarrow \rho_L$

flow equilibrium for saturated oil well

IPR



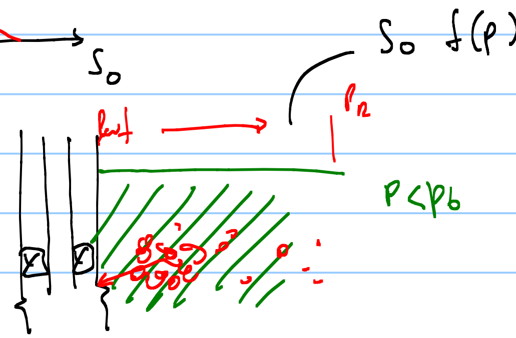
$$\frac{q_o}{A} = \frac{K}{\mu}\frac{dP}{dr}$$

k_{ro}

$q_o = B_o q_{\bar{o}}$

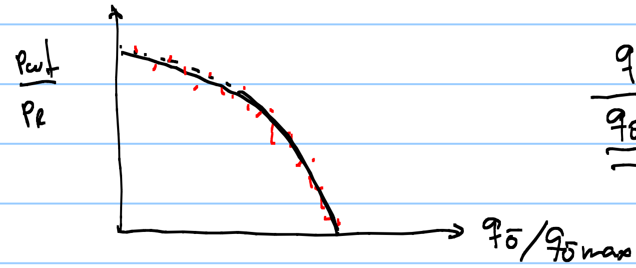
S_o $f(P)$

$$q_{\bar{o}} = \frac{K}{\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right]} \int_{P_{wf}}^{P_R} \frac{k_{ro} B_o}{\mu_o} dP$$



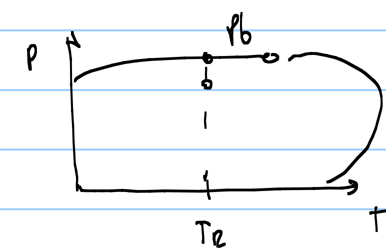
Vogel (1968 .) ~ generated IPR from reservoir simulator

for all cases



$$\frac{q_{\bar{o}}}{q_{\bar{o}max}} = 1 - 0.2\left(\frac{p_{wf}}{p_R}\right) - 0.8\left(\frac{p_{wf}}{p_R}\right)^2$$

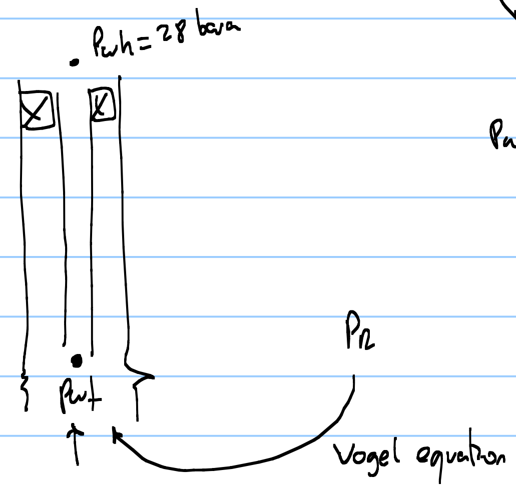
$p_{wf} < p_b(T_R)$

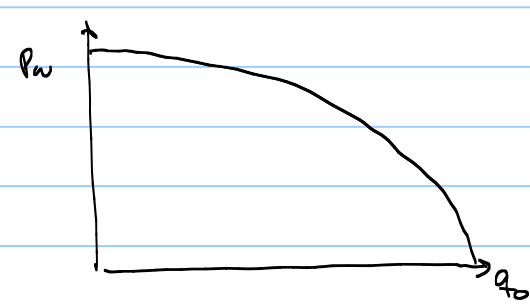


to use Vogel equation, from test ~> $p_{wf_{test}}$, $q_{\bar{o}\,test}$, p_R

$$q_{\bar{o}max} = \frac{q_{\bar{o}\,test}}{1 - 0.2\left(\frac{p_{wf\,test}}{p_R}\right) - 0.8\left(\frac{p_{wf\,test}}{p_R}\right)^2}$$

$$q_{\bar{o}} = C_R\,(p_R^2 - p_{wf}^2)^n$$? ?



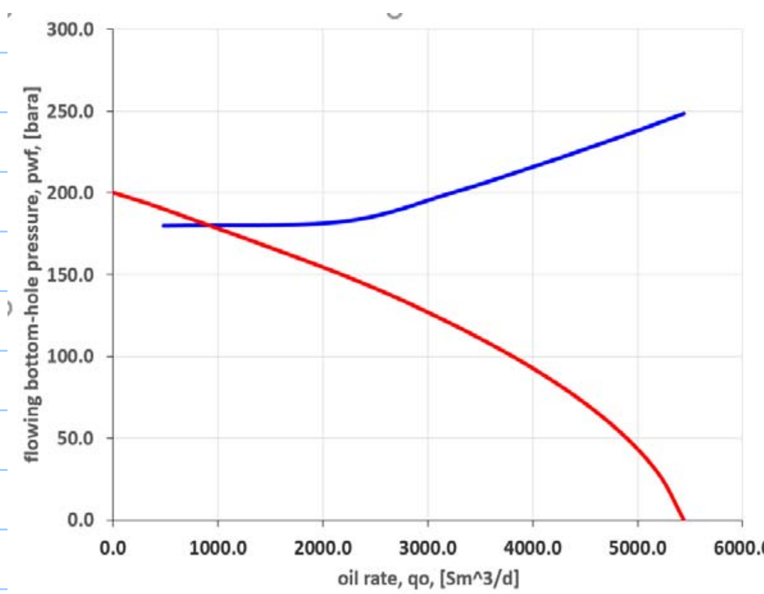


IPR INFORMATION		
pR [bara]	200	
qmax [Sm^3/d]	5441	
	IPR	TPR*
pwf	qo	qo
[bara]	[Sm^3/d]	[Sm^3/d]
200	0.0	
190	478.8	
150	2176.4	
120	3221.1	
90	4069.9	
60	4722.8	
30	5179.8	
0	5441.0	

*calculate in multiphase tubing exercise

IPR INFORMATION		
pR [bara]	200	
qmax [Sm^3/d]	5441	
	IPR	TPR*
pwf	qo	qo
[bara]	[Sm^3/d]	[Sm^3/d]
200	0.0	
190	478.8	179.7
150	2176.4	182.2
120	3221.1	199.8
90	4069.9	217.4
60	4722.8	231.8
30	5179.8	242.4
0	5441.0	248.7

*calculate in multiphase tubing exercise



tubing tables ≃ pressure traverse curves

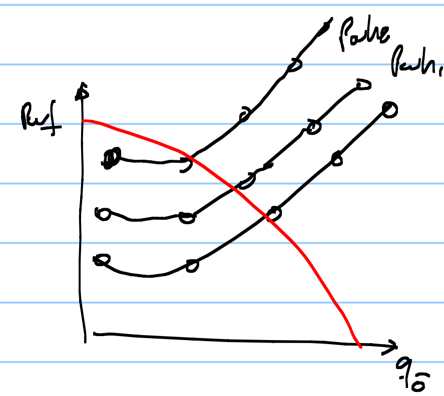
in tubing tables i precompute for multiple cases Pwf

P_{wh1}, GOR_1, WC_1

q_o	P_{wf}
q_{o1}	
q_{o2}	
q_{o3}	
q_{o4}	
q_{o5}	

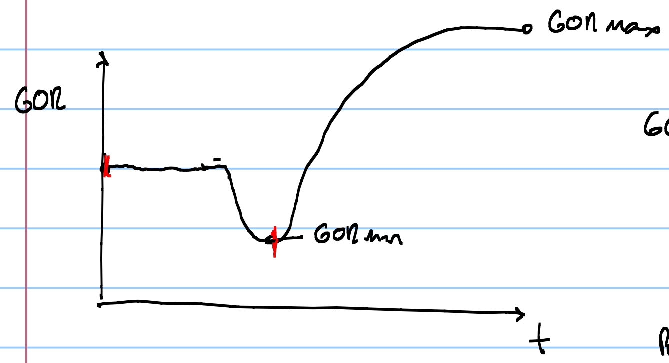
P_{wh2}, GOR_1, WC_1

q_o	P_{wf}
q_{o1}	
q_{o2}	
q_{o3}	
q_{o4}	
q_{o5}	



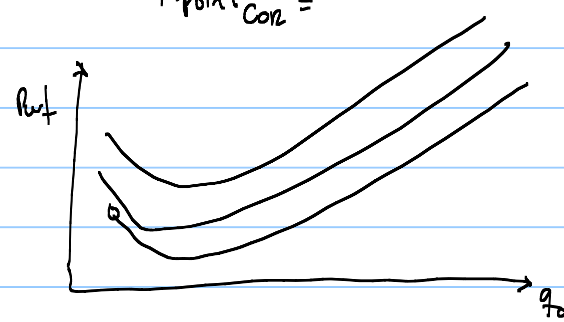
$P_{wh\,min} < P_{wh} < P_{wh\,max}$

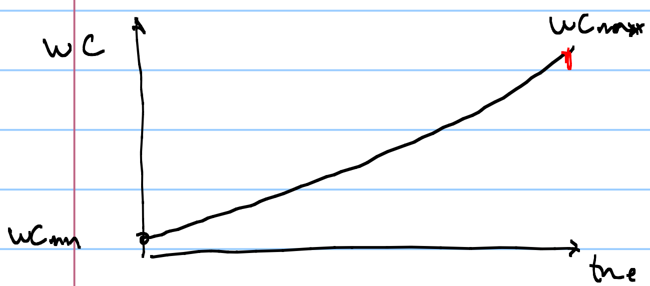
$N_{point\ P_{wh}} = 5$



$GOR_{min} < GOR < GOR_{max}$

$N_{point_{GOR}} =$





$WC_{min} < WC < WC_{max}$

total of points to simulate : $\underbrace{N_{Pwh}}_{10} \cdot \underbrace{N_{GOR}}_{10} \cdot \underbrace{N_{WC}}_{10} \cdot \underbrace{N_{q_{TO}}}_{10} = 10^4$ simulations

simulator → tubing table generation

Pipesim (schlumberger)

Prosper (petroleum experts)

Wellflow (landmark)

⋮

Download DWSim open source process simulator install