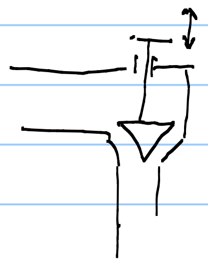
Day 6
→ choke equation for gas ⇝ exercise
→ single phase flow (undersaturated oil, oil + water mixtures) ⇝ exercise
→ ESP basics (electric submersible pump)



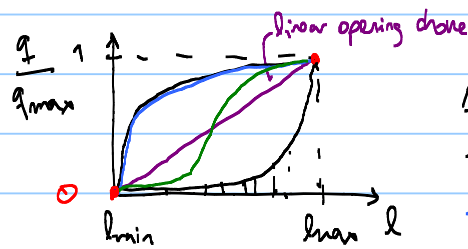
$$q = A_2 C_d \sqrt{\frac{\Delta p}{\rho_L (1-\beta^4)}}$$

A_2 ↳ is changing ! depending on the position of valve stem (l)

$A_2 = f(l)$

$l = l_{max}$ fully open, A_2 is max

$l = l_{min}$ fully closed, A_2 is zero



$q_{max} = 1000$ stb/d

Δp = fixed

– good control for low rates, poor control for hig rates

– good control for high rates - poor control for low rates

chove equation for gas :

① is choke inlet
② is choke outlet

$$dp + \rho u du = 0$$

$$\int_{p_1}^{p_2} \frac{dp}{\rho} = 0.5 (u_1^2 - u_2^2) \qquad u_2 \gg u_1$$

$$\simeq -0.5\, u_2^2$$

$$\rho = \frac{p}{Z R T}$$

$$\int_{p_1}^{p_2} \frac{ZRT}{p} dp = -0.5\, u_2^2$$

the thermodynamic process in choke is modeled as adiabatic (no heat transfer wit environment)

instead of using this I use

$$\underbrace{p \frac{1}{\rho^{k}} = \overbrace{\text{constant}}^{C}}\ \text{from } p_1 \to p_2$$

$$\rho = \frac{p^{1/k}}{C^{1/k}}$$

$$\int_{p_1}^{p_2} \frac{C^{1/k}\, dp}{p^{1/k}} = -0.5\, u_2^2$$

$$C^{1/k} \left(\frac{k}{k-1}\right) \left[p_2^{\frac{k-1}{k}} - p_1^{\frac{k-1}{k}} \right] = -0.5\, u_2^2$$

$$p_1^{\frac{k-1}{k}} \underbrace{\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1 \right]} C^{1/k} \frac{k}{(k-1)} = -0.5\, u_2^2$$

$\frac{p_2}{p_1} = y$ pressure ratio $\qquad C = \frac{p}{\rho^{1/k}}$ $\qquad$ at 1 $\quad C = \frac{p_1}{\rho_1^{1/k}}$

$$C^{1/k} = \frac{p_1^{1/k}}{\rho_1^{1/k^2}}$$

$$\frac{p_1^{1/k}}{\rho_1^{1/k^2}}\, p_1^{\frac{k-1}{k}} \left(y^{\frac{k-1}{k}} - 1\right)\frac{k}{k-1} = -u_2^2$$

$$u_2 = \sqrt{\frac{p_1^{\frac{k-1}{k}}}{\rho_1^{1/k^2}}\,\frac{k}{k-1}\left(1 - y^{\frac{k-1}{k}}\right)}$$

substitute $\rho_1 = \frac{p_1}{R Z_1 T_1}$

$$\frac{p_2}{\rho_2^{1/k}} = \frac{p_1}{\rho_1^{1/k}} \qquad \rho_2 = \left(\frac{p_1}{p_2}\right)^{1/k}\rho_1$$

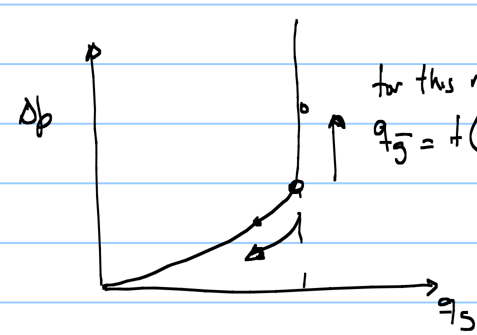
$$u_2 = \frac{\dot m}{\rho_2 A_2}$$

$$\dot m = \rho_2 A_2 \sqrt{\frac{p_1^{\frac{k-1}{k}}}{\rho_1^{1/k^2}}\,\frac{k}{k-1}\left(1 - y^{\frac{k-1}{k}}\right)} \qquad \dot m = q_{\bar g} \cdot \rho_{sc} = q_{\bar g}\,\frac{p_{sc}}{T_{sc} R}$$

$$q_{\bar g} = \rho_2 A_2 \frac{T_{sc} \cdot R}{p_{sc}} \sqrt{\frac{p_1^{\frac{k-1}{k}}}{\rho_1^{1/k^2}}\,\frac{k}{k-1}\left(1 - y^{\frac{k-1}{k}}\right)}$$

$$q_{sc} = c_d A_2 p_1 \left(\frac{T_{sc}}{p_{sc}}\right)\sqrt{2\frac{R}{M_g Z_1 T_1}}\sqrt{\left(\frac{k}{k-1}\right)\left[(y)^{\frac{2}{k}} - (y)^{\frac{k+1}{k}}\right]}$$

↳ valid for subcritical regime



for this regime $q_{\bar g} = f(y^*)$

EVA (day 2)

$p_1 = 205$ bara

$p_{2\,min} = 40$ bara

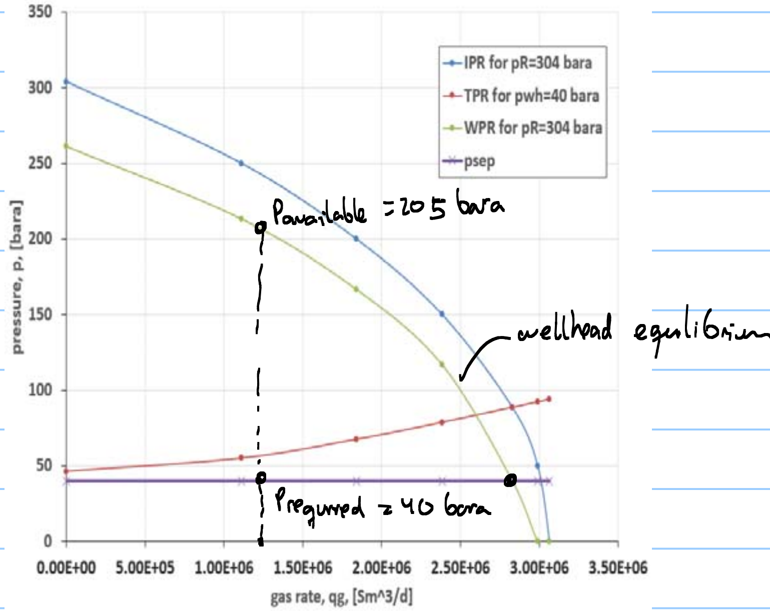
$q_{\bar g} = 1.25\ E6\ Sm^3/d$ (desired)

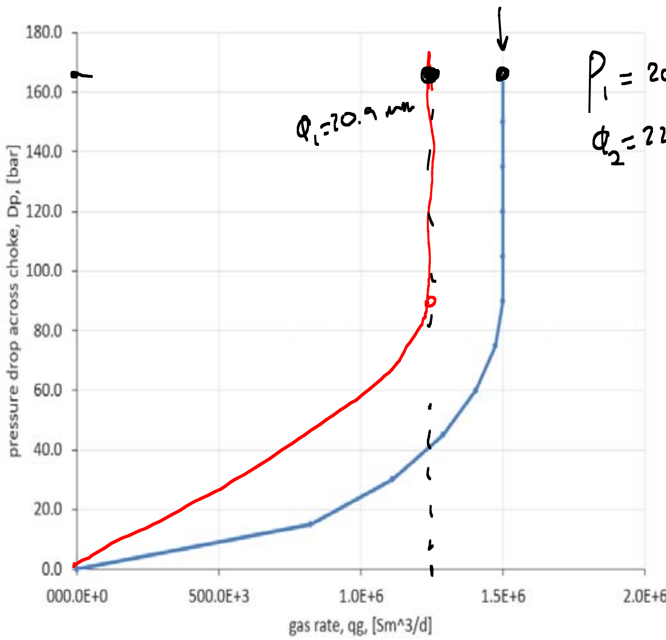
$\Phi_{ch} = 22.3$ mm

$$y = \frac{p_2}{p_1} = \frac{40}{205} = 0.195$$

$y_c \approx 0.5$

$y < y_c$ in critical regime!





P_1 = 205 bara

Φ_2 = 22.3 mm

P_1 = 205

P_2 = 40

Δp = 165 bara

$q_{tg\ desired}$ = 1.25 E6 Sm³/d

1/8" → 2" 1/64"

$\frac{20.9}{25.4} \sim 0.82"$ 3/4" + 1/64"

1 inch — 25.4 mm

chore design !

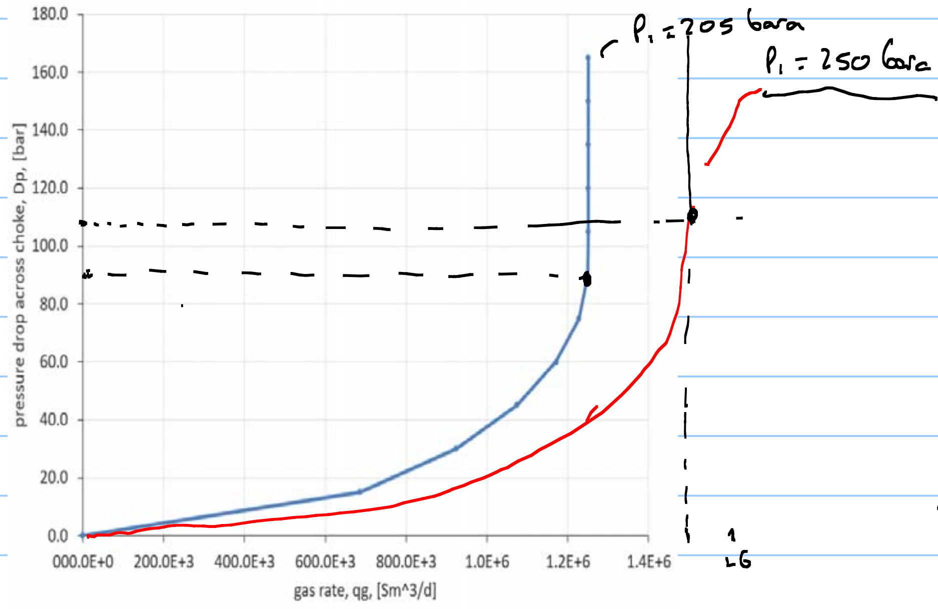
Choke Performance Equation - Dry Gas -Metric							
C_D Discharge Coefficient	0.865						
Cs	1.6259						
p_{sc}, Standard conditions pressure	101.325 kPa						
T_{sc}, Standard conditions temp	288.71 °K						
C_n	4.0075						
k, Adiabatic Constant	1.300						
Gas Gravity, Gamma	0.55 Air=1.0						
T, choke temperature	86.85 C		360 °K				
y_c Critical Ratio	0.546						
d, ChokeDiameter	mm	22.90383					
p1	p2	p1	p2	DP (p1-p2)	y	Z1	qsc
bara	bara	kPa	kPa	bar	[-]	[-]	Sm3/d
205.0	40	20500	4000	165.0	0.195	0.9281	1.5E+6
205.0	55	20500	5500	150.0	0.268	0.9281	1.5E+6
205.0	70	20500	7000	135.0	0.341	0.9281	1.5E+6
205.0	85	20500	8500	120.0	0.415	0.9281	1.5E+6
205.0	100	20500	10000	105.0	0.488	0.9281	1.5E+6
205.0	115	20500	11500	90.0	0.561	0.9281	1.5E+6
205.0	130	20500	13000	75.0	0.634	0.9281	1.5E+6
205.0	145	20500	14500	60.0	0.707	0.9281	1.4E+6
205.0	160	20500	16000	45.0	0.780	0.9281	1.3E+6
205.0	175	20500	17500	30.0	0.854	0.9281	1.1E+6
205.0	190	20500	19000	15.0	0.927	0.9281	823.0E+3

$q_{desired}$ 1.25E6

$$\varepsilon = \left(\frac{q_{calc} - q_{des}}{q_{desired}}\right)^2 = \left(\underbrace{\overbrace{\frac{q_{calc}}{q_{des}}}^{0.5-1.5} - 1}_{0.5}\right)^2$$

$(q_{calc} - q_{des})$

what is the effect of inlet pressure



$y_c = 0.546$ $\quad \frac{P_2}{P_1} = 0.546$

$\Delta p_c = P_1 - P_2 =$

$P_1 - 0.546 P_1 =$

$\Delta p_c = 0.434 P_1 = 108.5$ bara
$\quad\quad\quad\quad\downarrow$
$\quad\quad\quad\quad 250$

$\Delta p_c = 0.434 \cdot 205 = 88.97$ bara

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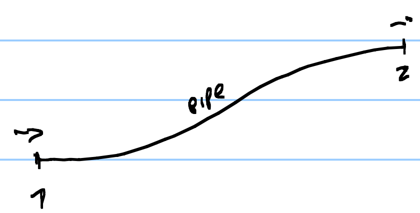
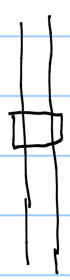
flow of undersaturated oil + water (liquids) → dead oils ↓ p_b low bubble point pressure
→ mixture dead oil + water
→ oil transport flowlines/pipelines



$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} - \underbrace{h_L}_{\text{energy losses}} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$h_f = \begin{cases} h_a \rightarrow \text{flow restrictions change direction} \\ h_f \rightarrow \text{friction in pipe} \end{cases}$$



$P_1 = F(P_2, q)$
$P_2 = F(P_1, q)$
$q = F(P_1, P_2)$

we can model ha as an equivalent length

$$h_f = f \frac{L}{D} \frac{v^2}{2g}$$

usually, for liquid

common is $V_1 \approx V_2$

clear P_2

$$\frac{P_1}{\rho g} + z_1 - \frac{f L v^2}{D\, 2g} = \frac{P_2}{\rho g} + z_2$$

$$P_2' = P_1' + \underbrace{(z_1 - z_2)}_{\text{tubing}}\rho g - f\frac{L}{\Phi}\rho\frac{v^2}{2}$$

$$\frac{q_o}{A} = v \qquad v^2 = \frac{q_o^2\, 16}{\pi^2 \Phi^4}$$

A: cross section area

$$-H = z_1 - z_2$$

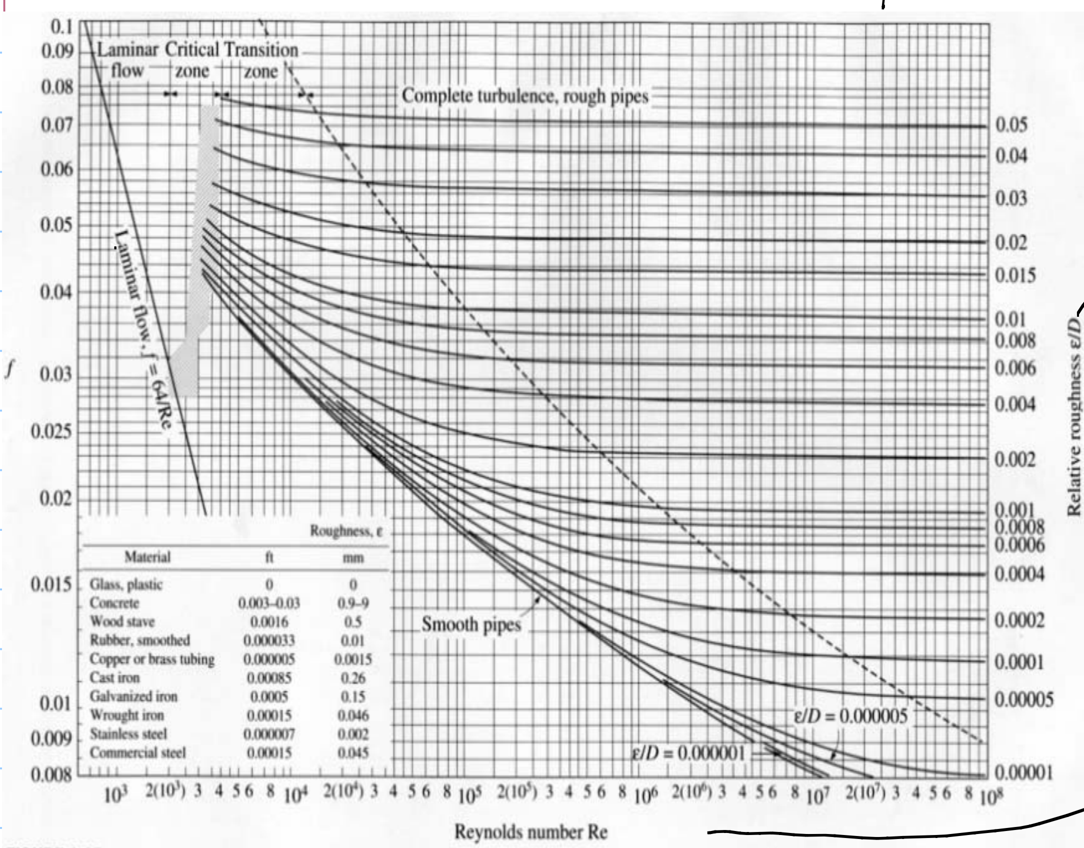
Height of tubing

$$P_2 = P_1 - H\rho g - f\frac{L}{\Phi}\frac{\rho}{2}\frac{q_o^2\, 16}{\pi^2\Phi^4}$$

$$P_2 = P_1 - H\rho g - f\frac{L}{\Phi}\frac{\rho\, 8}{\pi^2\Phi^4}q_o^2$$

normally work with $q_{\bar{o}}$ $\quad q_o = B_o \cdot q_{\bar{o}}$ in this development assuming $B_o \approx 1$

f Darcy weisbach (Moody) friction factor $\begin{cases} \text{laminar} & Re < 2500 \\ \text{turbulent} & Re > 2500 \end{cases}$



Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

ε

$$Re = \frac{v\Phi\rho}{\mu}$$

FIGURE A-27
The Moody chart for the friction factor for fully developed flow in circular tubes.

$$f_{moody} = 4 \cdot f_{FANNING}$$

https://en.wikipedia.org/wiki/Darcy_friction_factor_formulae

Table of Colebrook equation approximations

Equation	Author	Year	Range	Ref
$f = .0055\left[1+\left(2\times10^4\cdot\frac{\varepsilon}{D}+\frac{10^6}{\mathrm{Re}}\right)^{\frac{1}{3}}\right]$	Moody	1947	$Re = 4000 - 5.10^8$ $\varepsilon/D = 0 - 0.01$	
$f = .094\left(\frac{\varepsilon}{D}\right)^{0.225}+0.53\left(\frac{\varepsilon}{D}\right)+88\left(\frac{\varepsilon}{D}\right)^{0.44}\cdot\mathrm{Re}^{-\Psi}$ where $\Psi = 1.62\left(\frac{\varepsilon}{D}\right)^{0.134}$	Wood	1966	$Re = 4000 - 5.10^7$ $\varepsilon/D = 0.00001 - 0.04$	
$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.715}+\frac{15}{\mathrm{Re}}\right)$	Eck	1973		
$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7}+\frac{5.74}{\mathrm{Re}^{0.9}}\right)$	Swamee and Jain	1976	$Re = 5000 - 10^8$ $\varepsilon/D = 0.000001 - 0.05$	
$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.71}+\left(\frac{7}{\mathrm{Re}}\right)^{0.9}\right)$	Churchill	1973	Not specified	
$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.715}+\left(\frac{6.943}{\mathrm{Re}}\right)^{0.9}\right)$	Jain	1976		
$f = 8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12}+\frac{1}{(\Theta_1+\Theta_2)^{1.5}}\right]^{\frac{1}{12}}$ where $\Theta_1 = \left[-2.457\ln\left(\left(\frac{7}{\mathrm{Re}}\right)^{0.9}+0.27\frac{\varepsilon}{D}\right)\right]^{16}$ $\Theta_2 = \left(\frac{37530}{\mathrm{Re}}\right)^{16}$	Churchill	1977		
$\frac{1}{\sqrt{f}} = -2\log\left[\frac{\varepsilon/D}{3.7065}-\frac{5.0452}{\mathrm{Re}}\log\left(\frac{1}{2.8257}\left(\frac{\varepsilon}{D}\right)^{1.1098}+\frac{5.8506}{\mathrm{Re}^{0.8981}}\right)\right]$	Chen	1979	$Re = 4000 - 4.10^8$	
$\frac{1}{\sqrt{f}} = 1.8\log\left[\frac{\mathrm{Re}}{0.135\mathrm{Re}(\varepsilon/D)+6.5}\right]$	Round	1980		
$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7}+\frac{4.518\log\left(\frac{\mathrm{Re}}{7}\right)}{\mathrm{Re}\left(1+\frac{\mathrm{Re}^{0.52}}{29}(\varepsilon/D)^{0.7}\right)}\right)$	Barr	1981		
$\frac{1}{\sqrt{f}} = -2\log\left[\frac{\varepsilon/D}{3.7}-\frac{5.02}{\mathrm{Re}}\log\left(\frac{\varepsilon/D}{3.7}-\frac{5.02}{\mathrm{Re}}\log\left(\frac{\varepsilon/D}{3.7}+\frac{13}{\mathrm{Re}}\right)\right)\right]$ or $\frac{1}{\sqrt{f}} = -2\log\left[\frac{\varepsilon/D}{3.7}-\frac{5.02}{\mathrm{Re}}\log\left(\frac{\varepsilon/D}{3.7}+\frac{13}{\mathrm{Re}}\right)\right]$	Zigrang and Sylvester	1982		
$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}+\frac{6.9}{\mathrm{Re}}\right]$	Haaland[9]	1983		
$\frac{1}{\sqrt{f}} = \Psi_1-\frac{(\Psi_2-\Psi_1)^2}{\Psi_3-2\Psi_2+\Psi_1}$ or $\frac{1}{\sqrt{f}} = 4.781-\frac{(\Psi_1-4.781)^2}{\Psi_2-2\Psi_1+4.781}$ where $\Psi_1 = -2\log\left(\frac{\varepsilon/D}{3.7}+\frac{12}{\mathrm{Re}}\right)$ $\Psi_2 = -2\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51\Psi_1}{\mathrm{Re}}\right)$ $\Psi_3 = -2\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51\Psi_2}{\mathrm{Re}}\right)$	Serghides	1984		
$A = 0.11\left(\frac{68}{Re}+\varepsilon\right)^{0.25}$ if $A \geq 0.018$ then $f = A$ and if $A < 0.018$ then $f = 0.0028+0.85A$	Tsal	1989		
$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7}+\frac{95}{\mathrm{Re}^{0.983}}-\frac{96.82}{\mathrm{Re}}\right)$	Manadilli	1997	$Re = 4000 - 10^8$ $\varepsilon/D = 0 - 0.05$	
$\frac{1}{\sqrt{f}} = -2\log\left\{\frac{\varepsilon/D}{3.7065}-\frac{5.0272}{\mathrm{Re}}\log\left[\frac{\varepsilon/D}{3.827}-\frac{4.657}{\mathrm{Re}}\log\left(\left(\frac{\varepsilon/D}{7.7918}\right)^{0.9924}+\left(\frac{5.3326}{208.815+\mathrm{Re}}\right)^{0.9345}\right)\right]\right\}$	Monzon, Romeo, Royo	2002		
$\frac{1}{\sqrt{f}} = 0.8686\ln\left[\frac{0.4587\mathrm{Re}}{(S-0.31)^{\frac{S}{(S+1)}}}\right]$ where: $S = 0.124\mathrm{Re}\frac{\varepsilon}{D}+\ln(0.4587\mathrm{Re})$	Goudar, Sonnad	2006		
$\frac{1}{\sqrt{f}} = 0.8686\ln\left[\frac{0.4587\mathrm{Re}}{(S-0.31)^{\frac{S}{(S+0.9633)}}}\right]$ where: $S = 0.124\mathrm{Re}\frac{\varepsilon}{D}+\ln(0.4587\mathrm{Re})$	Vatankhah, Kouchakzadeh	2008		
$\frac{1}{\sqrt{f}} = \alpha-\frac{\alpha+2\log\left(\frac{\mathrm{B}}{\mathrm{Re}}\right)}{1+\frac{2.18}{\mathrm{B}}}$ where $\alpha = \frac{0.744\ln(\mathrm{Re})-1.41}{1+1.32\sqrt{\varepsilon/D}}$ $\mathrm{B} = \frac{\varepsilon/D}{3.7}\mathrm{Re}+2.51\alpha$	Buzzelli	2008		
$f = \frac{6.4}{(\ln(\mathrm{Re})-\ln(1+.01\mathrm{Re}\frac{\varepsilon}{D}(1+10\sqrt{\frac{\varepsilon}{D}})))^{2.4}}$	Avci, Kargoz	2009		
$f = \frac{0.2479-0.0000947(7-\log\mathrm{Re})^4}{(\log\left(\frac{\varepsilon/D}{3.615}+\frac{7.366}{\mathrm{Re}^{0.9142}}\right))^2}$	Evangelides, Papaevangelou, Tzimopoulos	2010		
$f = 1.613\left[\ln\left(0.234\varepsilon^{1.1007}-\frac{60.525}{Re^{1.1105}}+\frac{56.291}{Re^{1.0712}}\right)\right]^{-2}$	Fang	2011		
$f = \left[-2\log\left(\frac{2.18\beta}{Re}+\frac{\varepsilon}{3.71}\right)\right]^{-2}, \beta = \ln\frac{Re}{1.816\ln\left(\frac{1.1Re}{\ln(1+1.1Re)}\right)}$	Brkić	2011		
$f = 1.325474505\log_e\left(A-0.8686068432B\log_e\left(A-0.8784893582B\log_e\left(A+(1.665368035B)^{0.8373492157}\right)\right)\right)^{-2}$ where $A = \frac{\varepsilon/D}{3.7065}$ $B = \frac{2.5226}{\mathrm{Re}}$	Alashkar	2012		

some f expressions are implicit ⇝ $f = F(f)$

use numerical solver

explicit expressions

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

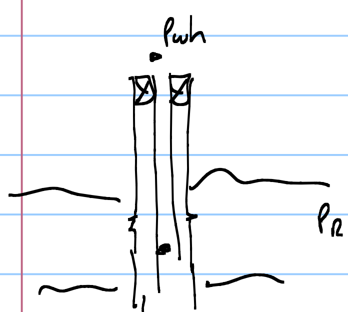
Haaland (198..)

$$f^{-1/2} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

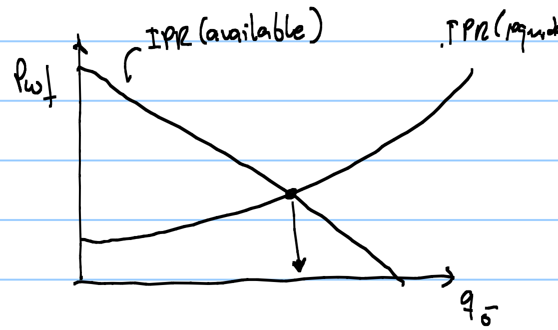
$$P_2 = P_1 - H\rho g - f\frac{L}{D}\rho\frac{8}{\pi^2 D^4}q_o^2$$

Class exercise ⇝ undersaturated oil equilibrium

P_b is very low ⇝ dead oil

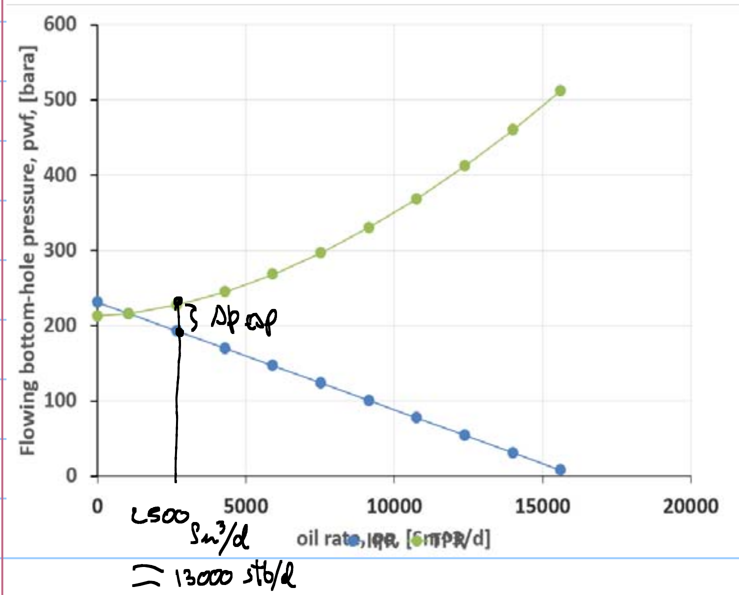


bottom-hole equilibrium

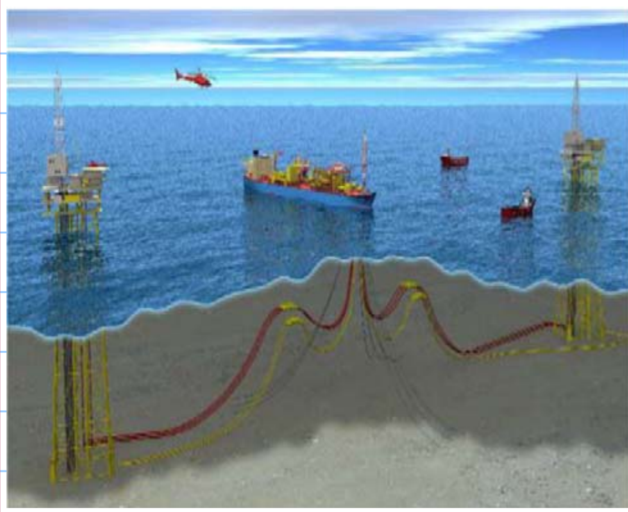


p_R	bara	231
J, for oil flow	Sm^3/d/bar	70
L, tubing length	m	2340
d, tubing diameter	m	0.14
e, tubing roughness	m	0.00010
teta, tubing angle	[deg]	90
muo, oil viscosity	[Pa s]	0.1
rhoo, oil density	[kg/m^3]	897
pwh, wellhead pressure	[bara]	7

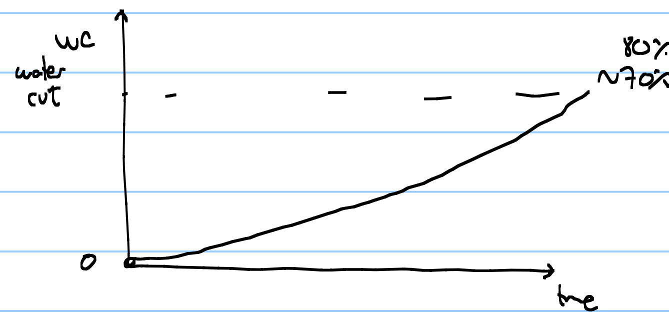
IPR		TPR
pwf	qo	pwf
[bara]	[Sm^3/d]	[bara]
231	0	213
208	1618	218
185	3236	233
162	4853	252
139	6471	277
116	8089	307
92	9707	343
69	11325	383
46	12942	428
23	14560	477
0	16178	531



electric submersible pump



Peregrino in Brazil (offshore)



$$wc = \frac{q_{\bar{w}}}{q_{\bar{o}} + q_{\bar{w}}} \cdot 100$$

local rates

$$\text{volume fraction} = \frac{q_w}{q_o + q_w}$$

one can also use

$$P_2 = P_1 - H \rho g - f \frac{L}{\phi} \rho \frac{8}{\pi^2 \phi^4} q_o^2$$

but using average properties!

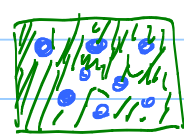
$$\rho_{mixture} = \rho_w \alpha_w + \rho_o (1 - \alpha_w)$$

if $\alpha_w \sim 1$ then

$\rho_{mix} \approx \rho_{water}$

if $\alpha_w \approx 0$

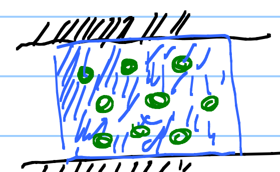
$\rho_{mix} \approx \rho_{oil}$



water is dispersed

oil is continuous

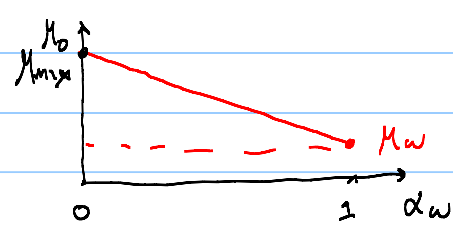
wc = 0% ~> 60%

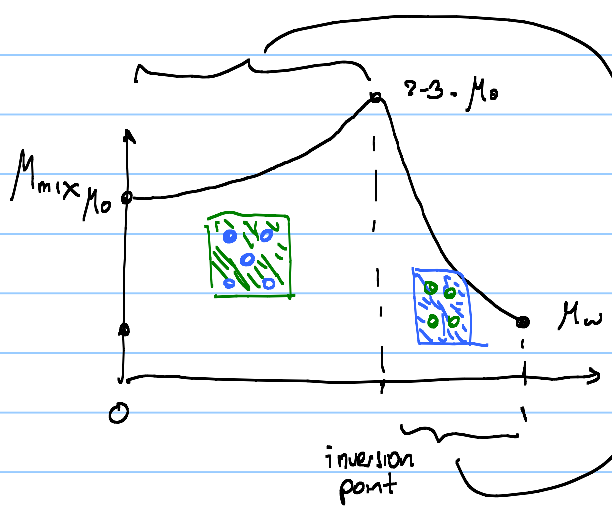


wc ~ 60 – 100%

oil $\mu_o = 100\ cp$ $\mu_w = 1\ cp$

sometimes $\mu_{mix} = \mu_w (\alpha_w) + \mu_o (1 - \alpha_w)$





found from lab test

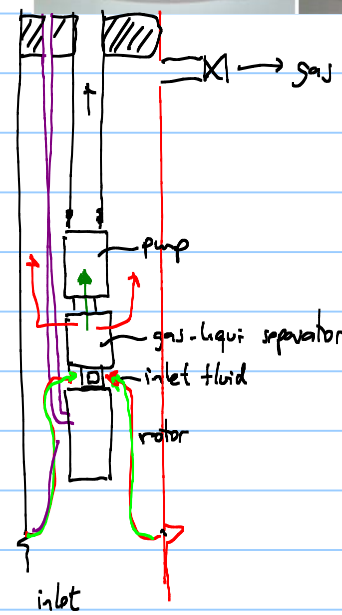
$$\mu_{mix} = \mu_o e^{C_1(\alpha_w)}$$

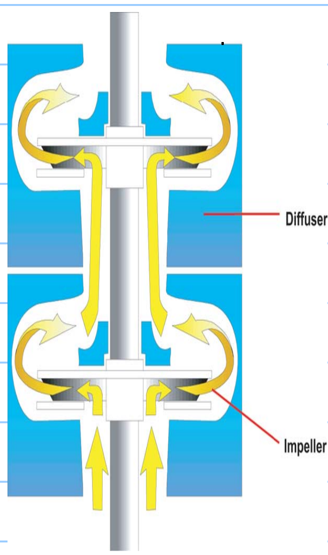
$$\mu_{mix} = \mu_w \cdot e^{C_2(1-\alpha_w)}$$

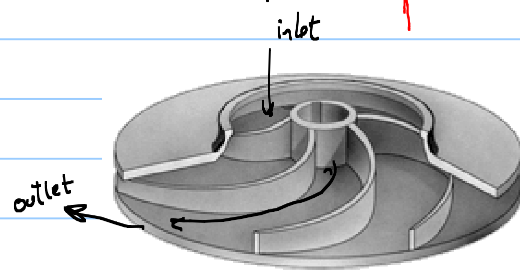
Richardson emulsion model

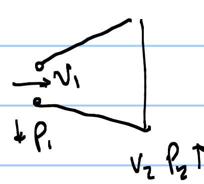
Electric submersible pump

Armais Arutunoff
Oklahoma 1930s Reda

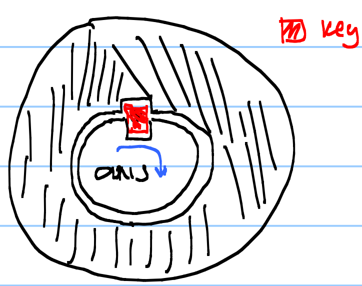


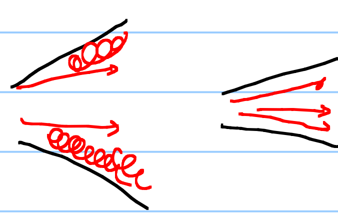


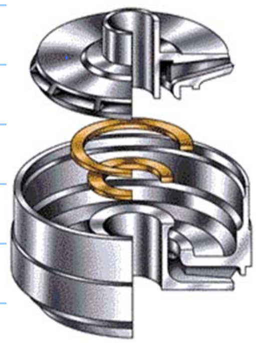


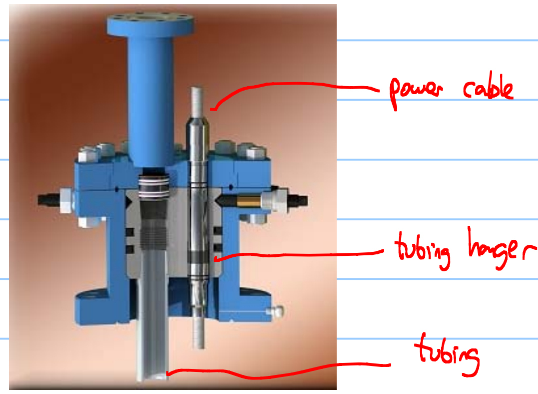


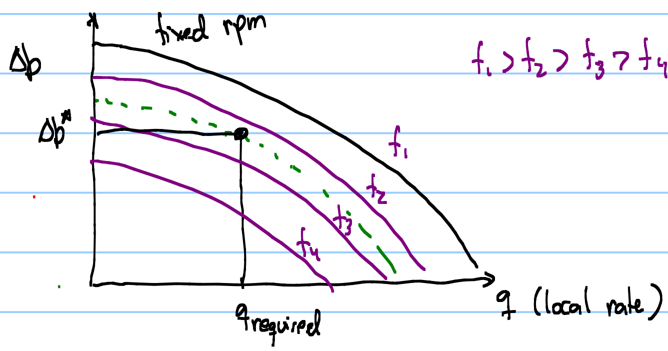
- acceleration of fluid (increasing v)
- pressure recovery








power cable
tubing hanger
tubing


fixed rpm
Δp
Δp*
f1 > f2 > f3 > f4
f1
f2
f3
f4
q required
q (local rate)