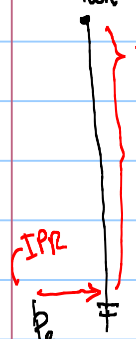
Note Title   30.10.2018

Day 2 All course material is placed here:

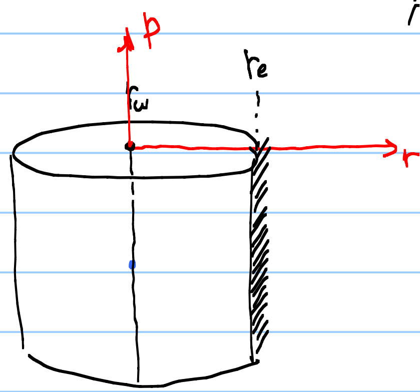
http://www.ipt.ntnu.no/~stanko/files/Courses/POFE_UEM/2018/

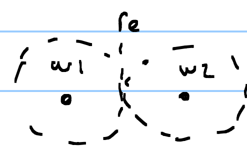
P_{wh} wellhead pressure $\approx P_{sep}$



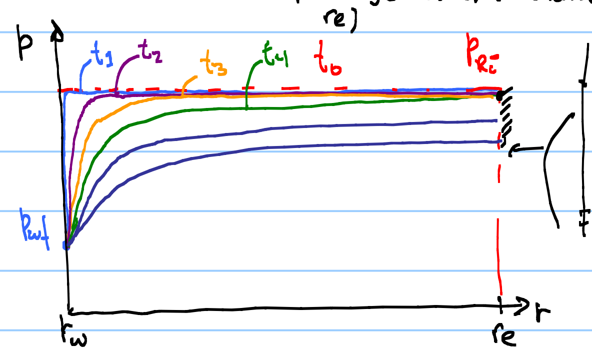
P_{wf} flowing botton-hole pressure

IPR





transient regime (when p changes haven't reached re)



for radial well

$$t = \frac{0.1\ \phi\ \mu\ C\ A}{3.553\ 10^{-9}\ K}$$

K in md
ϕ fraction
μ [Pa s]
C [1/kPa]
A [m²]

$\downarrow K$ $\uparrow t_{to\ PSS}$ or t_{SS} will be ↑

after t_4 there are two options
- PSS pseudo steady state
- steady state SS when P_{re} = constant

conventional reservoirs
$t_{transient}$ is very short (hrs → days)

outtake will happen in PSS / SS

when the well is in PSS
i can remove time and use P_R

$\phi = 0.3$
$\mu = 3E{-5}$ Pa s
$C = 1.67\ E{-5}$ 1/kPa
$A = 647492\ m^2$

t | K md
10000
1000
100
10
1
0.1
0.01

most IPR equation are derived from difussivity equation

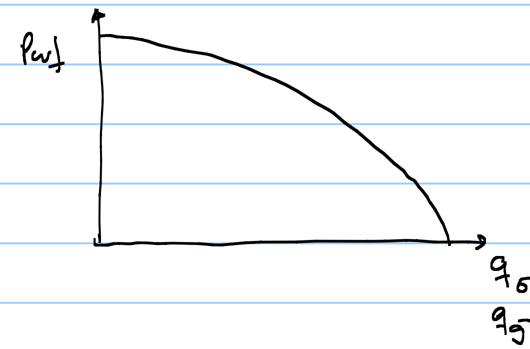
$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\mu\ \phi\ C}{K}\frac{\partial p}{\partial t}$$

for PSS, IPR equations look like:

undersaturated oil

$$q_o = J\,(P_R - P_{wf})$$

gas, saturated oil



USBM Bureau of mines 193~

Backpressure equation

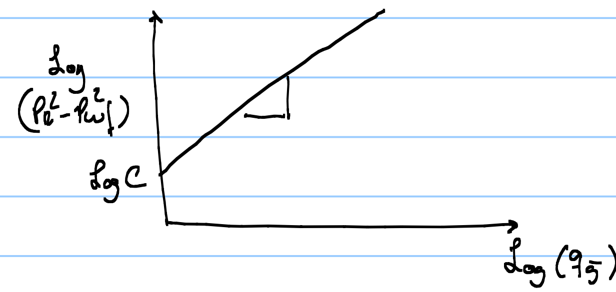
backpressure exponent $0.5 \le n \le 1$ (turbulent ← → Darcy)

$$q_g = \underbrace{C}\,(P_R^2 - P_{wf}^2)^n$$

→ backpressure coefficient

→ fluid properties
reservoir properties
geometry
near wellbore effect

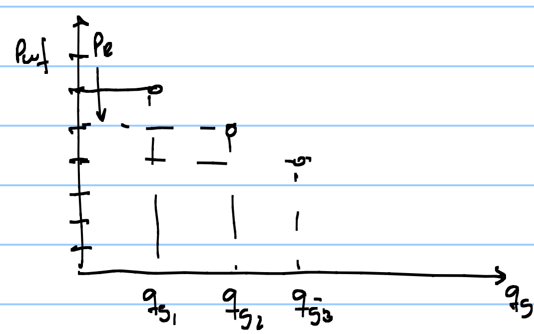
$$\log(q_g) = \log(C) + n \cdot \log(P_R^2 - P_{wf}^2)$$

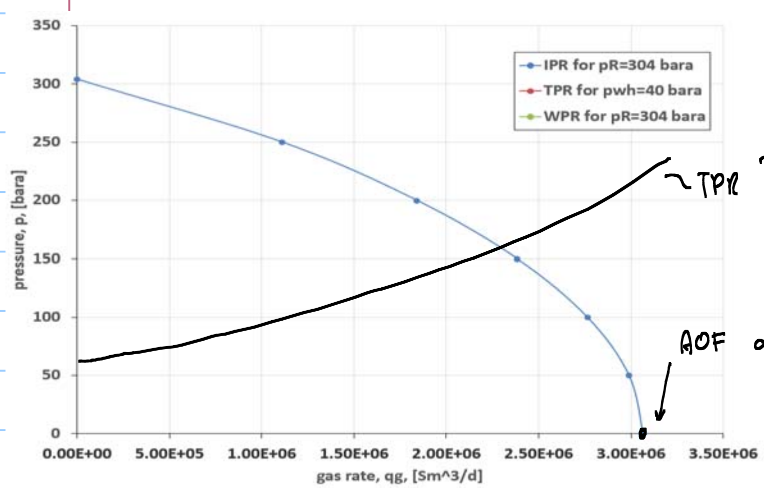


$$q_g = C\,(P_R^2 - P_{wf}^2)^n$$

$P_{wf} \le P_R$

$$P_{wf} = \sqrt{P_R^2 - \left(\frac{q_g}{C}\right)^{1/n}}$$





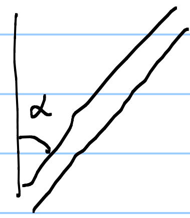
~ TPR ?

AOF absolute open flow

Pressure drop in tubing, tubing equation pipe compressible fluid (gas)

Moody

$$-\frac{dp}{dl} = \rho \cdot g \cdot \cos\alpha + f \frac{\rho}{2\phi} v^2$$



$\rho = f(p)$?

$$\frac{p}{\rho} = zRT \qquad \rho = \frac{p}{zRT}$$

$$-\frac{dp}{dl} = \frac{p}{zRT} g \cos\alpha + f \frac{\frac{p}{zRT}}{2\phi} \qquad v = \frac{q}{A} = \left(\frac{\frac{\dot m}{\rho}}{A}\right)^2$$

$p = y$

$x = l$

$$+ f \frac{16}{2 \cdot \phi} \frac{\dot m^2}{\rho\, \pi^2 \phi^4}$$

$$-\frac{dp}{dl} = \frac{p\, g \cos\alpha}{zRT} + \underbrace{\frac{f \cdot 8}{\phi^5} \frac{\dot m^2}{\pi^2} \frac{zRT}{p}}$$

$$Ca = \frac{g \cos\alpha}{zRT} \qquad Cb = \frac{f \cdot 8\, \dot m^2\, zRT}{\phi^5 \pi^2} \qquad Ca = \frac{g \cos\alpha}{\bar z R \bar T}$$

$$Cb = \frac{f\, 8\, \dot m^2 \cdot \bar z R \bar T}{\phi^5 \pi^2}$$

$$-\frac{dp}{dl} = Ca \cdot p + \frac{Cb}{p} \qquad \bar T = \frac{T_{wf} + T_{wh}}{2}$$

$$\int_{P_{wf}}^{P_{wh}} \frac{dp}{Ca\, p + \frac{Cb}{p}} = \int_0^L - dl \qquad \int_{P_{wf}}^{P_{wh}} \frac{dp}{\left(Ca\, p + \frac{Cb}{p}\right)} = -L \qquad \bar z = \frac{z_{wf} + z_{wh}}{2}$$

$$\int_{P_{wf}}^{P_{wh}} \frac{p\, dp}{Ca\, p^2 + Cb} = -L \qquad U = Ca\, p^2 + Cb$$

$$dU = Ca\, 2p\, dp$$

$$\frac{1}{Ca\, 2} \int_{U_1}^{U_2} \frac{dU}{U} = -L$$

$$-\frac{1}{Ca \cdot 2} \ln U \Big|_{U_1}^{U_2} = L$$

$$\ln\left(\frac{U_1}{U_2}\right) = L\, Ca \cdot 2$$

$$\frac{u_1}{u_2} = e^{\overbrace{L\,Ca\,2}}$$

$$S = L \cdot Ca \cdot 2 = \frac{g \cos\alpha \, 2 \cdot L}{\bar{Z} R \bar{T}}$$

$$u_1 = Ca\, P_{wf}^2 + Cb$$

$$u_2 = Ca\, P_{wh}^2 + Cb$$

$$\frac{Ca\, P_{wf}^2 + Cb}{Ca\, P_{wh}^2 + Cb} = e^{L\,Ca\,2}$$

$$Ca\, P_{wf}^2 + Cb = e^S \cdot Ca\, P_{wh}^2 + Cb\, e^S$$

$$P_{wf}^2 + \frac{Cb}{Ca} = e^S P_{wh}^2 + \frac{Cb}{Ca} e^S$$

$$P_{wf}^2 = e^S P_{wh}^2 + \frac{Cb}{Ca}\left(e^S - 1\right)$$

$$\frac{Cb}{Ca} = \frac{8 f \cdot \dot{m}^2\, \bar{Z} R \bar{T}}{\pi^2 \Phi^5} \cdot \frac{\bar{Z} R \bar{T}}{g \cos\alpha}$$

$$\dot{m} = q_{sc} \cdot \rho_{sc} = q_{sc}\left(\frac{P_{sc}}{T_{sc} \cdot R}\right)$$

elevation coefficient

$$P_{wf}^2 = e^S P_{wh}^2 + q_{sc}^2 \underbrace{\frac{8 f (\bar{Z} \cancel{R} \bar{T})^2}{\pi^2 \Phi^5 g \cos\alpha}\left(\frac{P_{sc}^2}{T_{sc}^2 \cancel{R^2}}\right)}_{\frac{1}{C_T^2}}\left(e^S - 1\right)$$

$C_T^2 \propto \Phi^5$

$\frac{1}{C_T^2}$ tubing coefficient

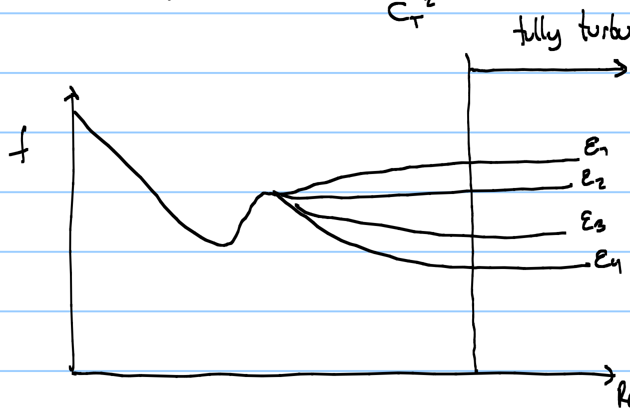
$PV = RT$

↳ $\frac{R_u}{M_w}$

$M_{wg} = \gamma_g \cdot M_{wair}$

$$P_{wf}^2 = P_{wh}^2 e^S + \frac{q_{sc}^2}{C_T^2}$$ dry gas tubing equation

fully turbulent

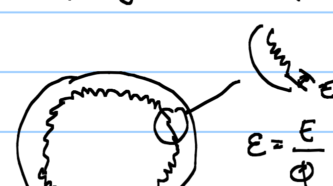


$f \neq Re$

$Re = \frac{\rho \Phi V}{\mu}$ ↑

for gas $1E5$ Pa s

for gas $f = F(\varepsilon)$

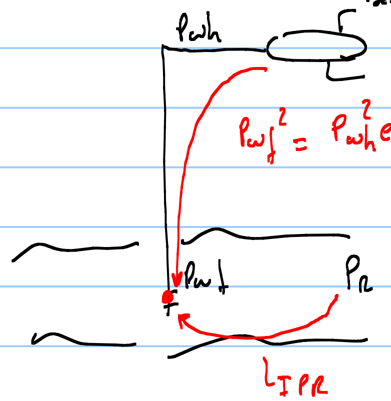


$\varepsilon = \frac{E}{\Phi}$

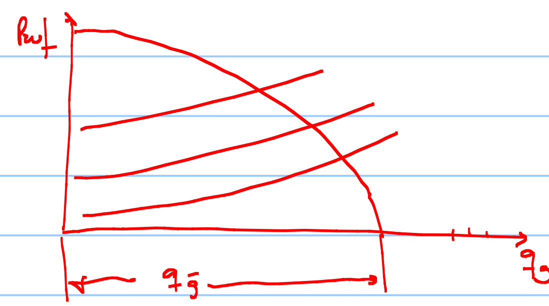
Smith found (1950) $\varepsilon = f(\phi)$

$$\hookrightarrow f_M = \frac{0.0077}{\phi^{0.224}}$$

P_{sep} $\quad P_{wh} \approx P_{sep}$



$$P_{wf}^2 = P_{wh}^2 e^S + \frac{q_g^2}{C_T^2}$$



how to find the equilibrium rate: $\quad \overset{?}{q_g} = C\left(P_R^2 - \overset{?}{P_{wf}}^2\right)^n$ $\quad$ 2 unknowns / 1 eq

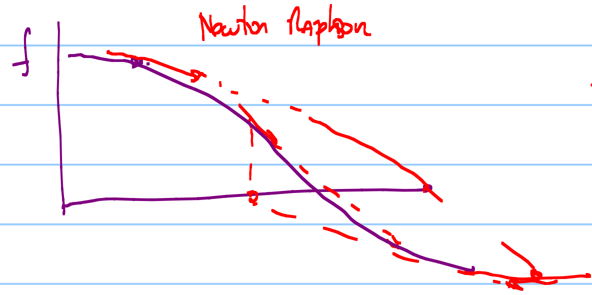
$$\overset{?}{P_{wf}}^2 = e^S P_{wh}^2 + \frac{\overset{?}{q_g}^2}{C_T^2}$$

un / 1 eqa

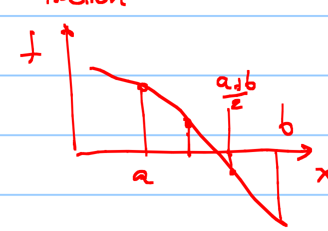
2 − 2

root finding methods $\quad f(x^*) = 0$

Newton Raphson



Bisection



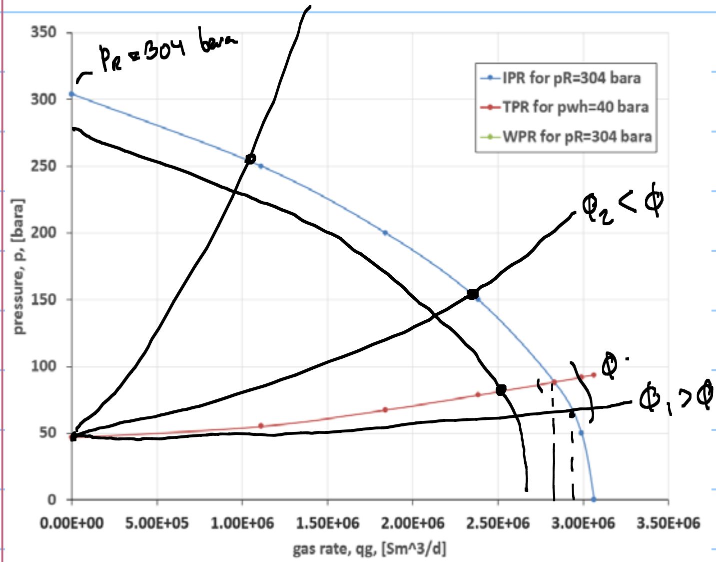
IPR $\rightsquigarrow$ P_{wf} $\quad f_1$ $\qquad f_1 = f_2 \quad P_{wf}^{IPR} = P_{wf}^{TPR}$

TPR $\rightsquigarrow$ $\quad f_2$

$$f_3 = (f_1 - f_2) \qquad x^* \text{ such as } f_3 = 0$$

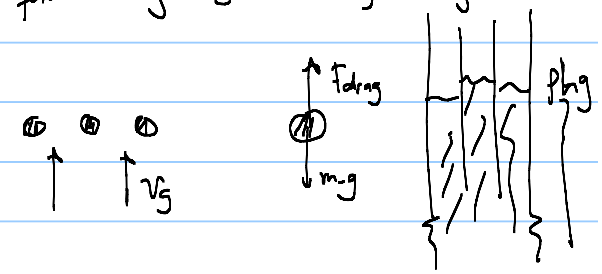
$f_1 = f_2$

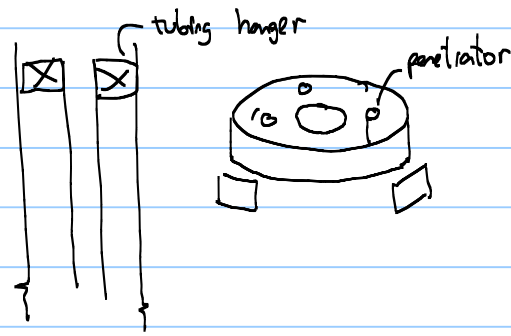
	IPR	TPR	f_3	
pwf_avail	qg	pwf_req	pwf_avail-pwf_req	
[bara]	[Sm^3/d]	[bara]	[bara]	
304	0.00E+00	46.7	257.3	+
250	1.11E+06	55.2	194.8	+
200	1.84E+06	67.5	132.5	+
150	2.38E+06	78.5	71.5	+ $50 < P_{wf}^* < 100$
100	2.76E+06	86.8	13.2	+
50	2.99E+06	91.9	-41.9	−
0	3.06E+06	93.6	-93.6	−



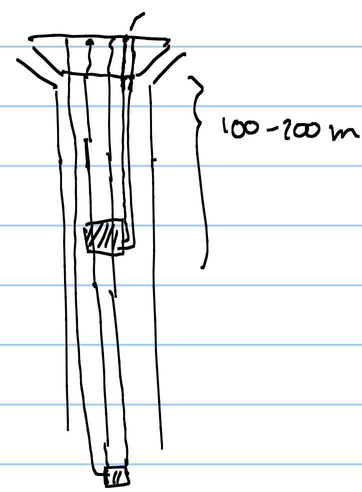
what affects tubing size?

- maximize production (doesn't restrict production)
- fit production casing 9 5/8"
- erosion $V_g \leq V_{erosional}$ API 14E
- liquid loading $V_g \geq V_{unloading\ velocity}$ (Turner)

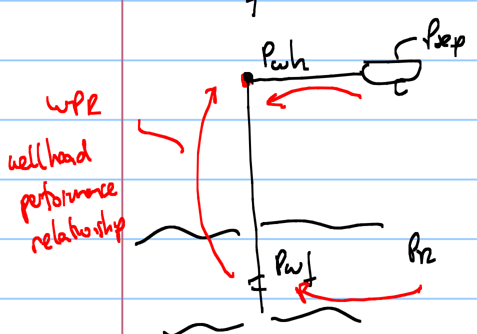




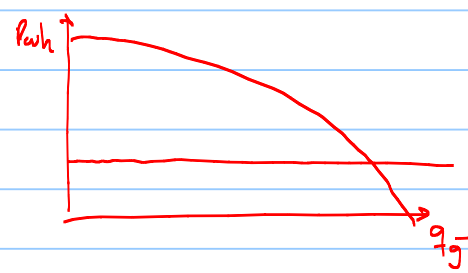
- depends on the size of tubing hanger



make equilibrium at wellhead



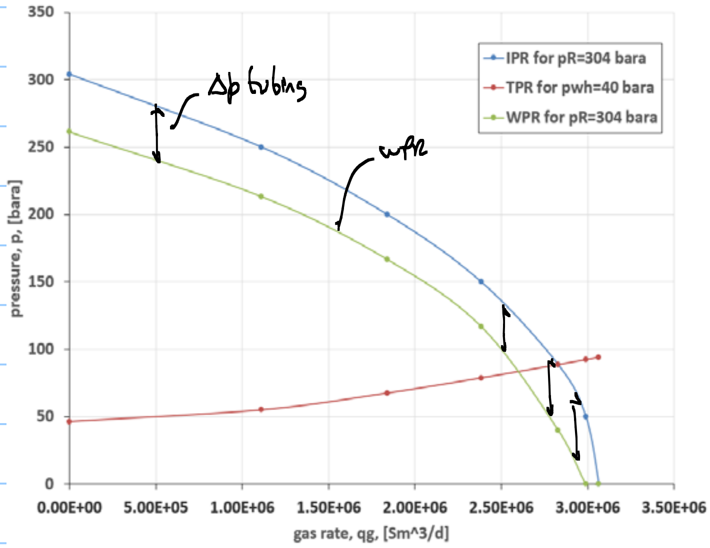
$p_{sep} \approx p_{wh}$

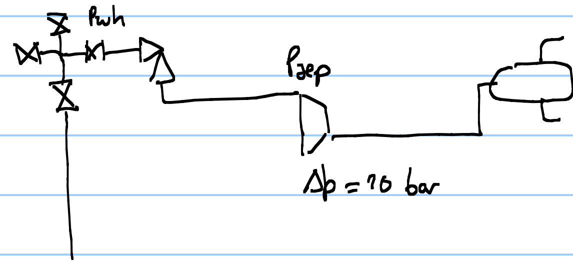


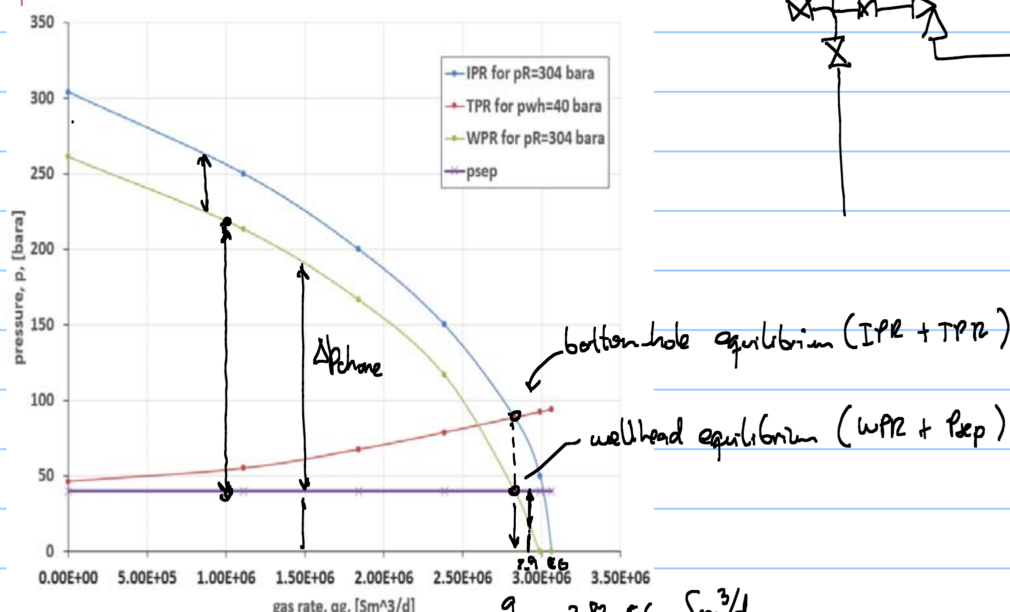
WPR pwh_avail [bara]
261.4
213.4
166.7
116.8
40.0
#VALUE!
#VALUE!

$$p_{wf}^2 = p_{wh}^2 e^S + \frac{q_g^2}{C_T^2}$$

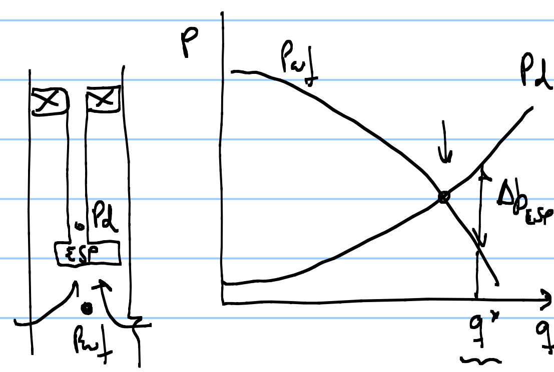
$$p_{wh}^2 = \left(p_{wf}^2 e^{-S} - \frac{q_g^2}{C_T^2} e^{-S}\right)$$





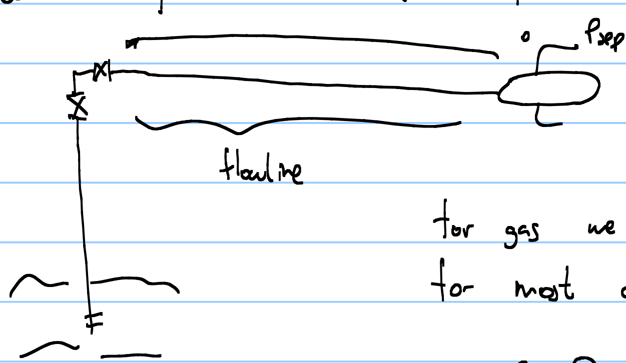


$q_g = 2.83\ E6\ Sm^3/d$



wellhead equilibrium ↝ choke management
compressor planning

bottomhole equilibrium ↝ artificial lift



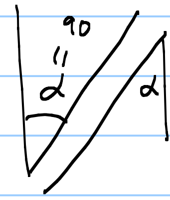
for gas we can use tubing equation for flow in flowline
for most cases flowlines are fairly horizontal

$S = 0$ $L.\cos\alpha$ G ?

$$\underline{s}$$

$$\lim_{x \to 0} \frac{f_1(x)}{f_2(x)} = \text{undefined}$$

L'Hopital



$$\lim_{x \to} \frac{f_1'(x)}{f_2'(x)} =$$

$$\lim_{k} \frac{s}{(e^s - 1)}$$

$$\frac{1}{e^s} = 1$$

for horizontal pipeline

$$q_g = C_{F2} \cdot (P_{in}^2 - P_{out}^2)^{0.5}$$