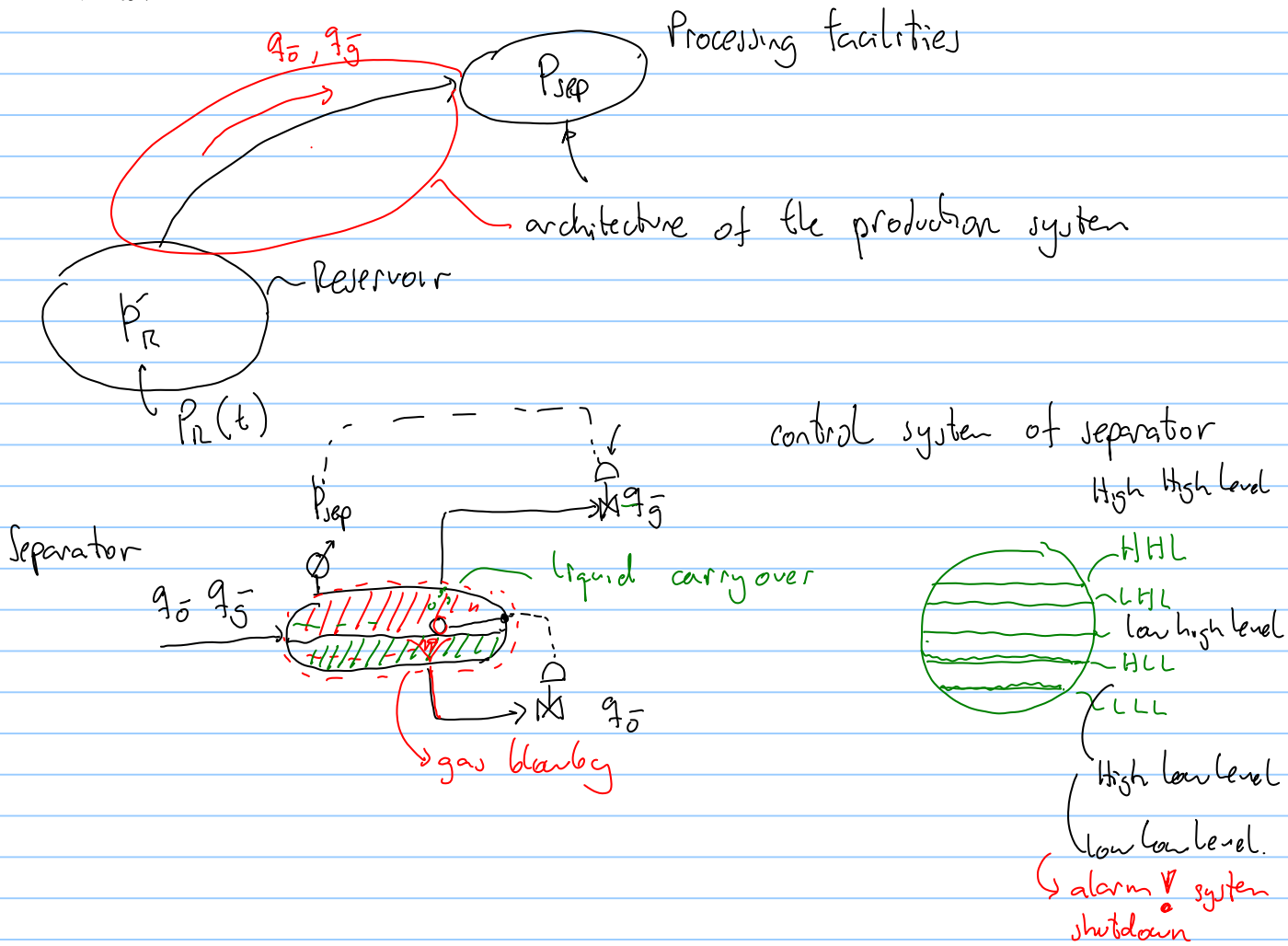


Prof. Milan Stanko

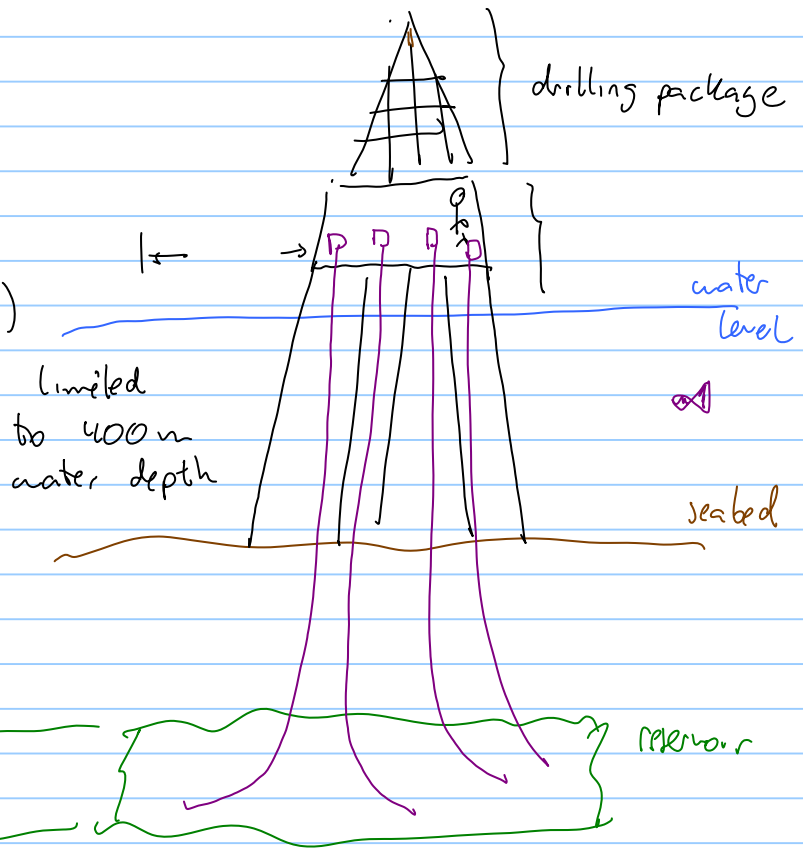


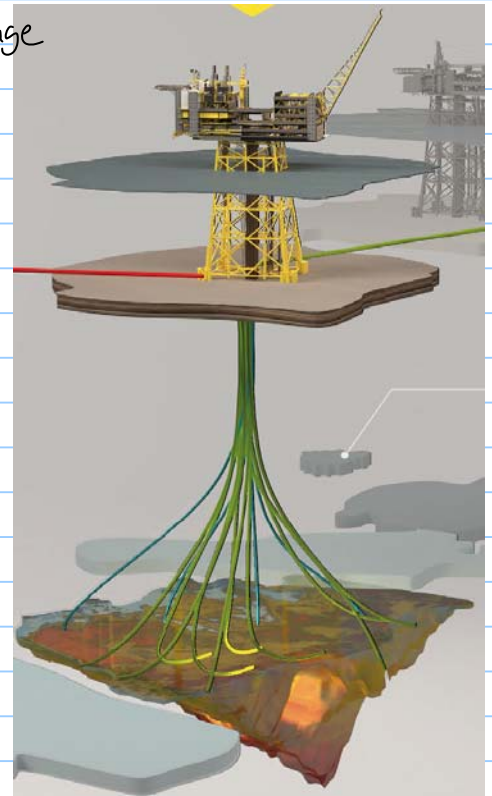
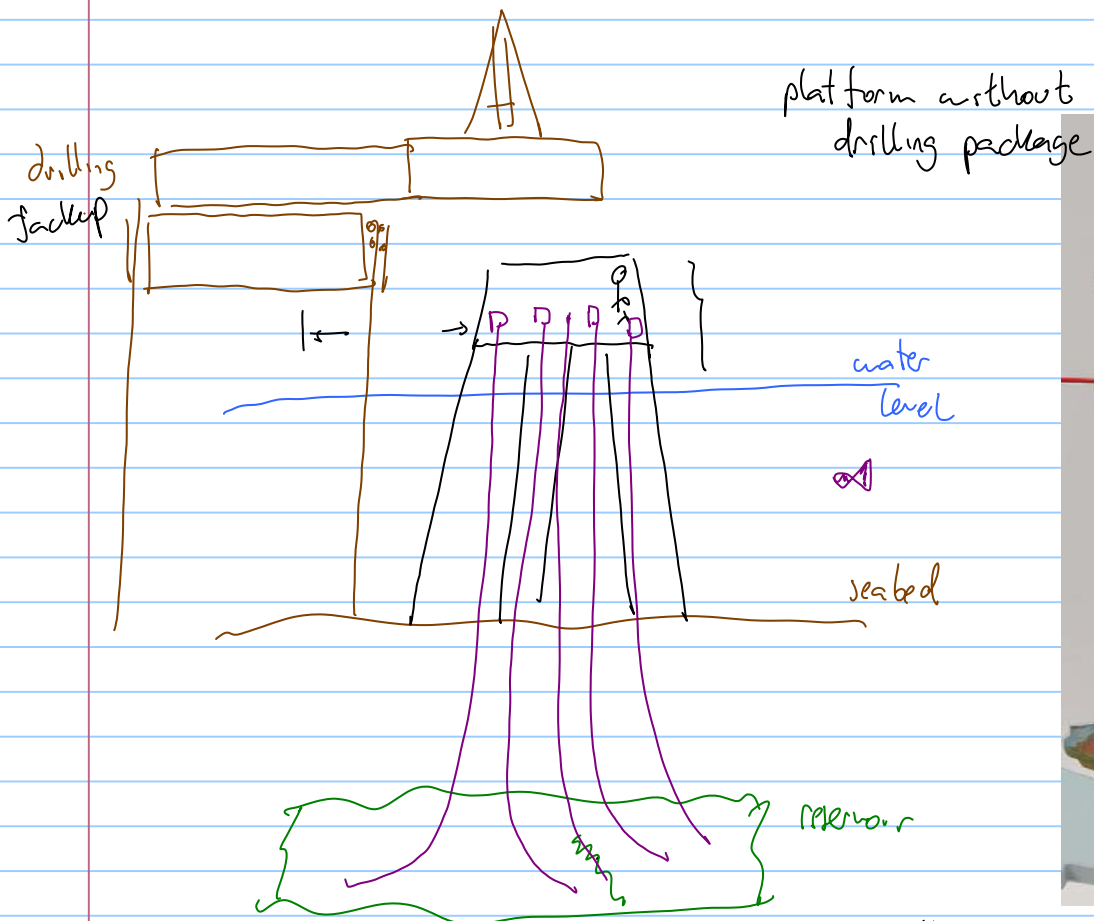
layout of production systems:

- offshore { platform wells (dry christmas trees)

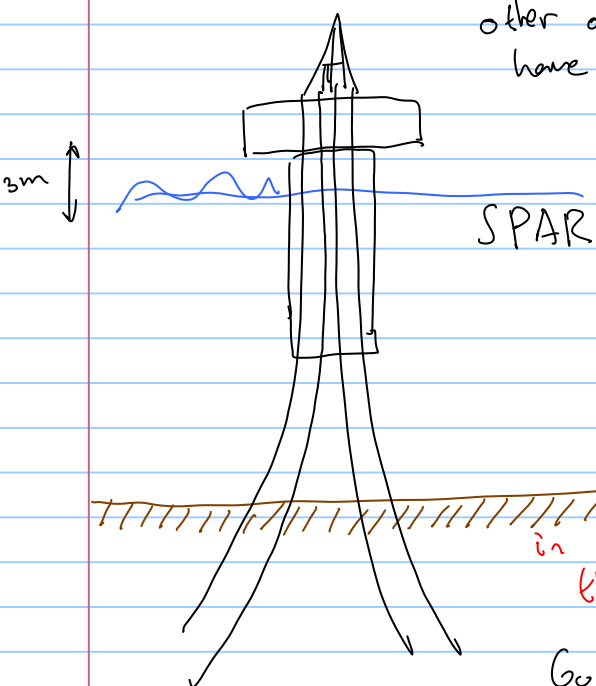
- Easy access to well for intervention

- Limited well slots for in-fill drilling



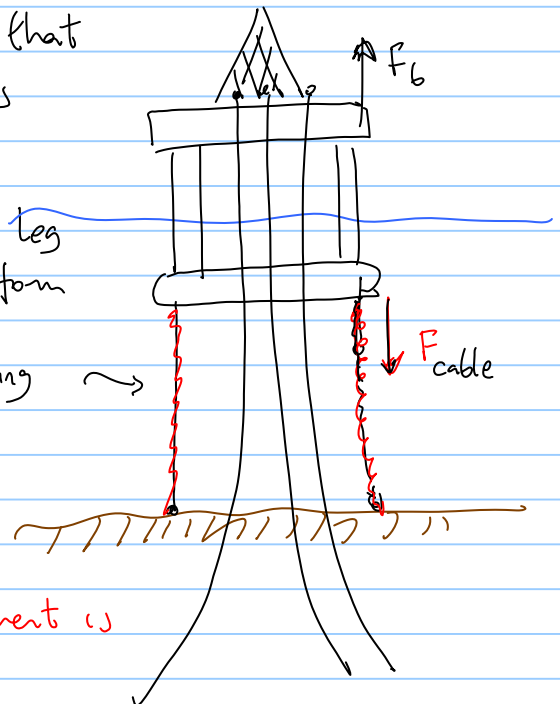


other offshore structures that have dry christmas trees



TLP
tension leg
platform

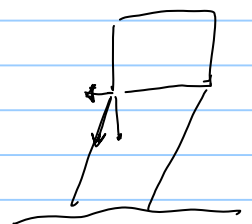
tensioning
cable



in these structures
the vertical movement is
limited

Gulf of Mexico

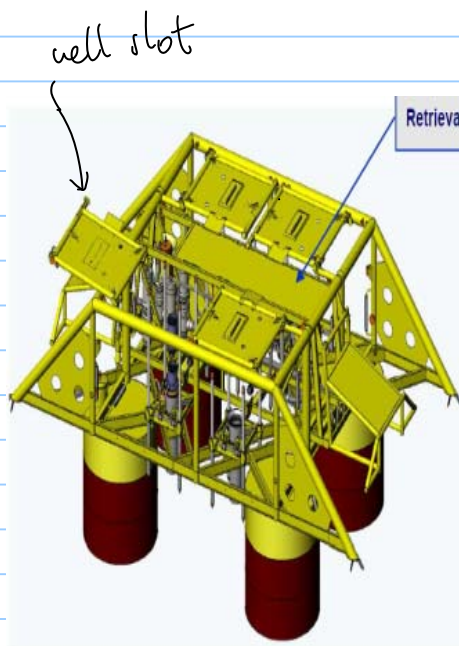
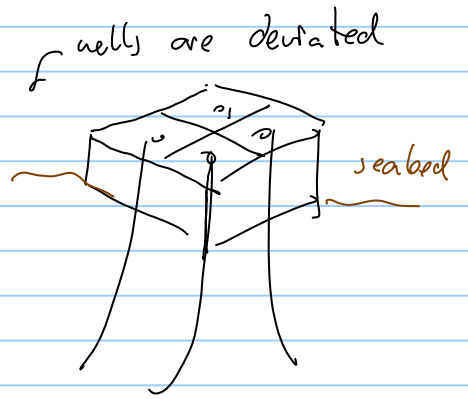
≈ 1700 m water depth max



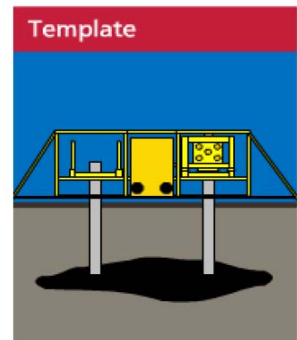
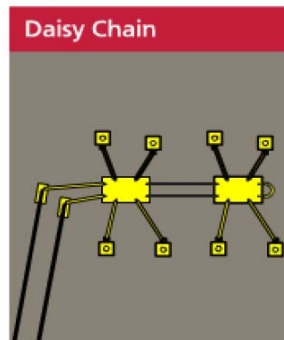
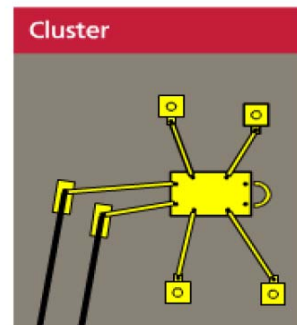
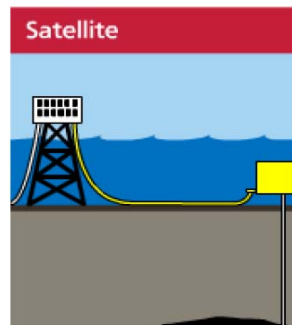
- Subsea wells
net christmas trees

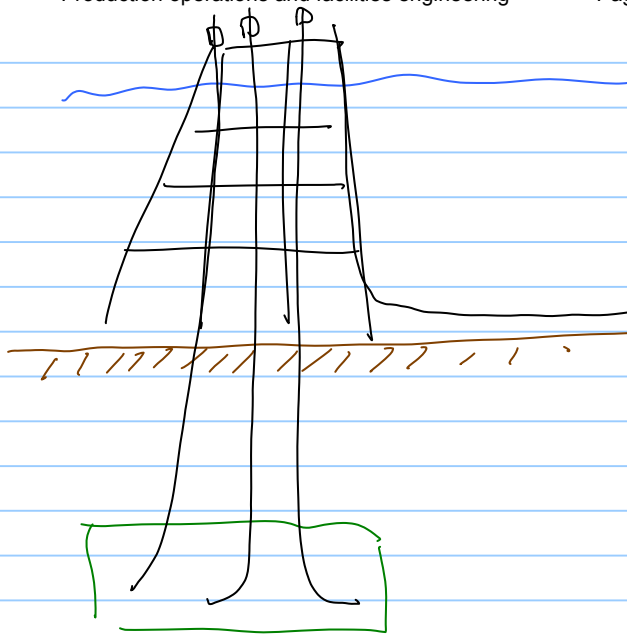
template drilled wells
wells are deviated
seabed

satellite wells
wells are usually drilled
above target.

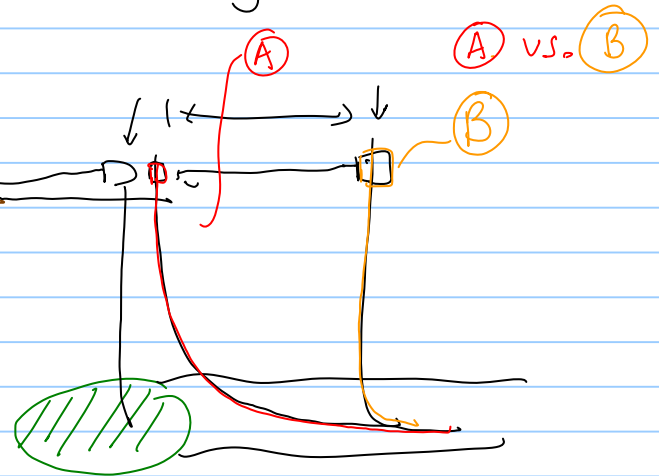


subsea template

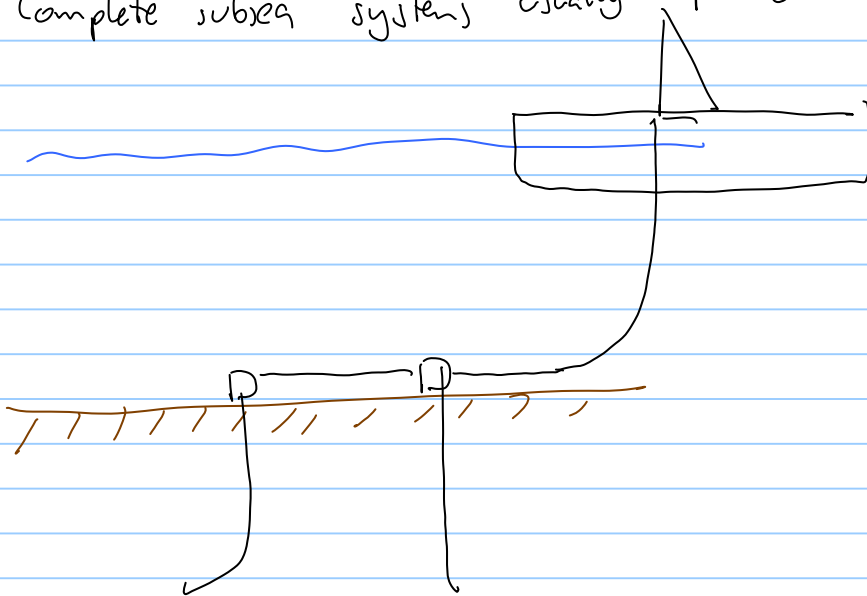




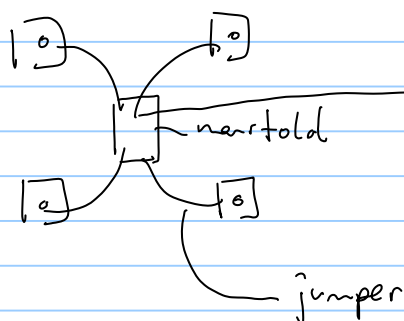
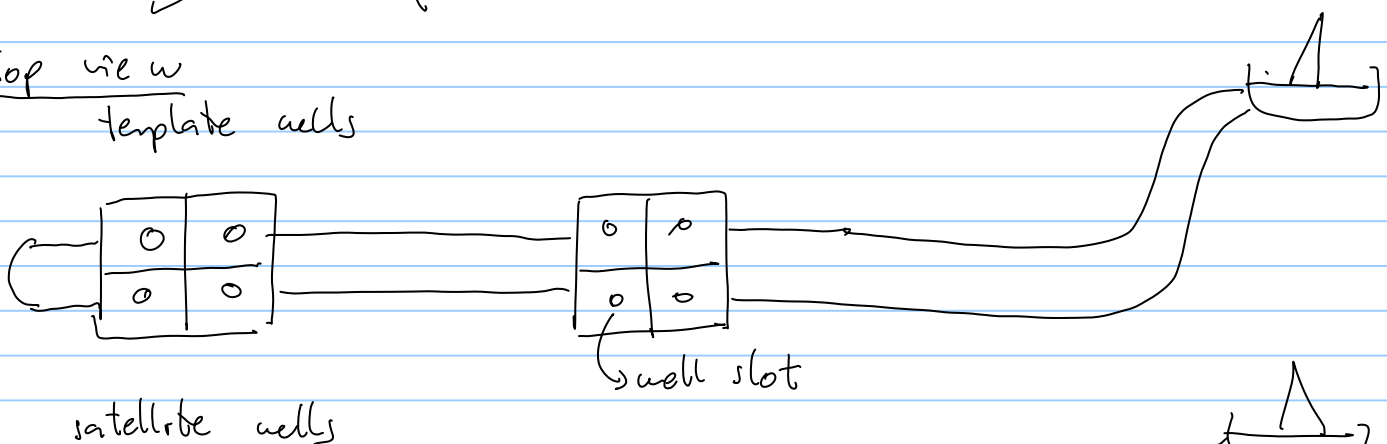
subsea well producing to existing facilities.

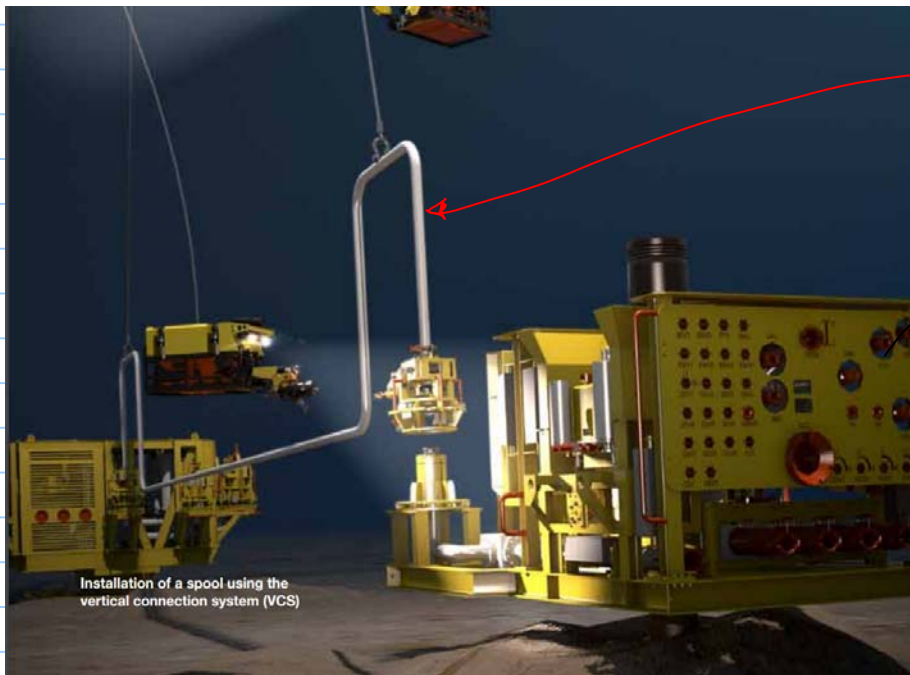


Complete subsea systems usually produce to { FPSO, Semi-sub } similar to TLP without the tensioning cables.
(FPSO) floating production storage offloading



top view
template wells

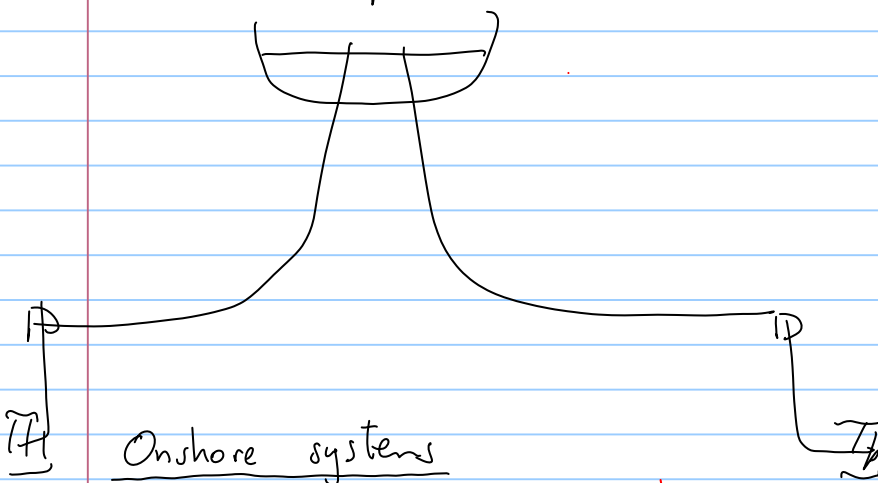




jumper

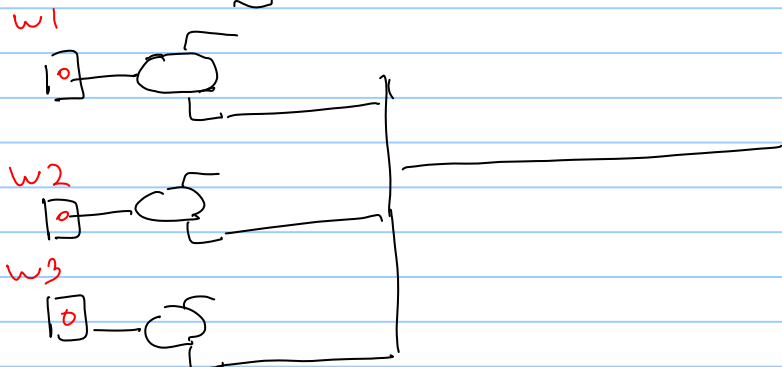
x-may tree

* for example

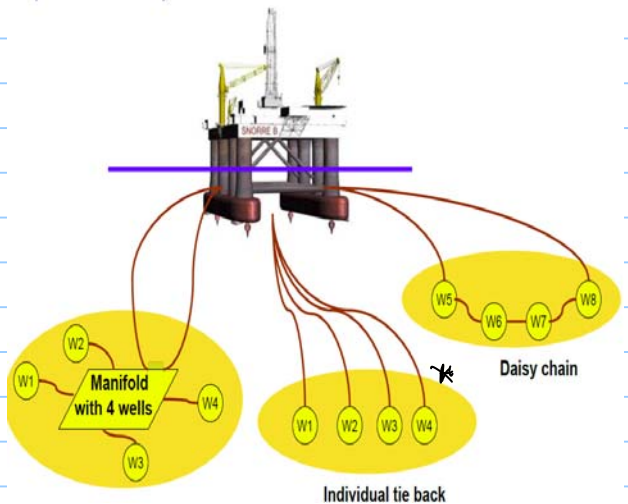
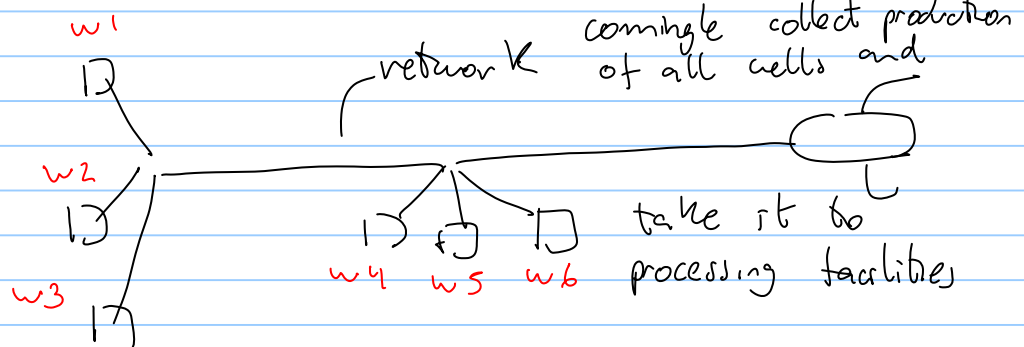


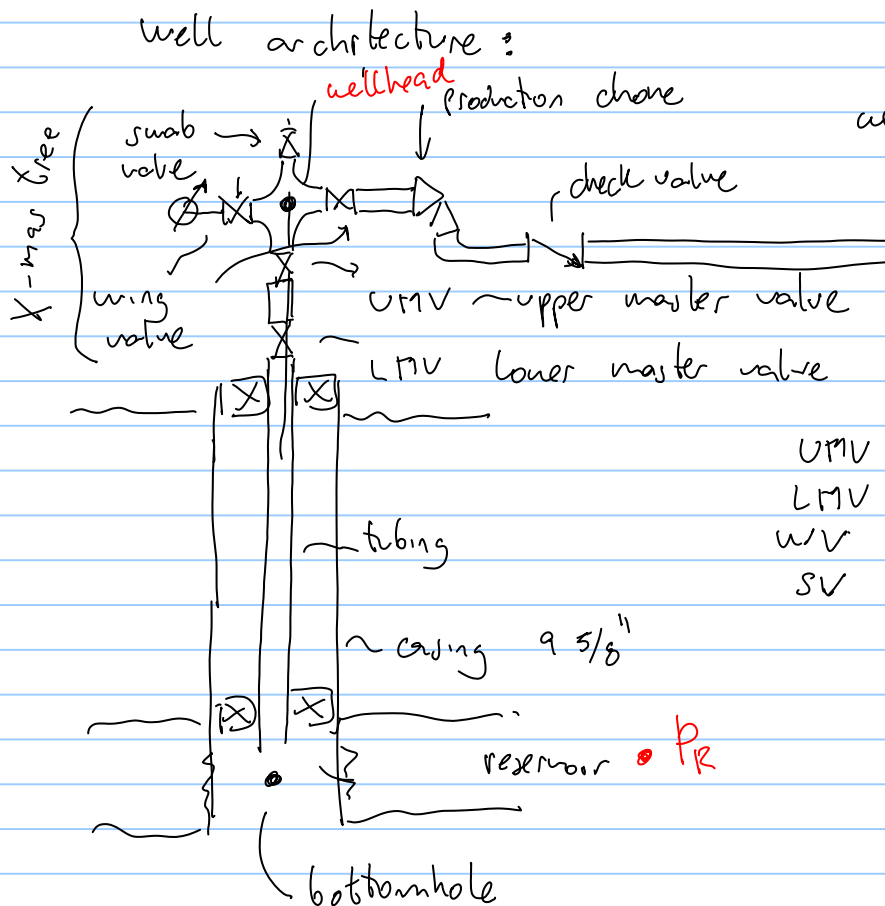
Onshore systems

• Standalone wells

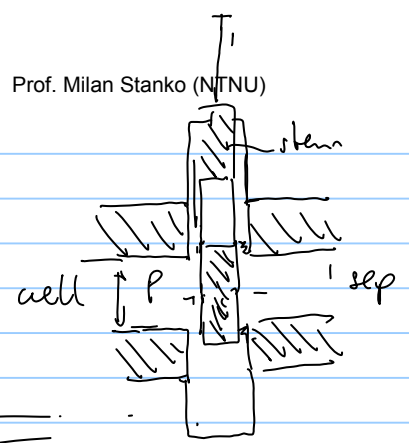


• network



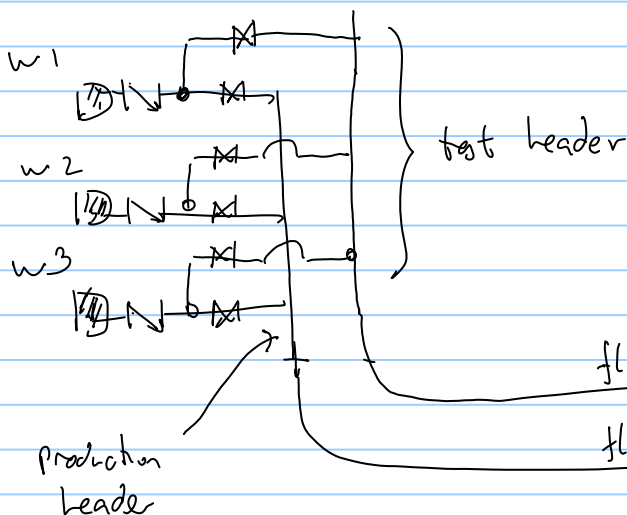


UMV - on-off valves
 LMV - on-off valves
 WV - on-off valve
 SV - on-off valve



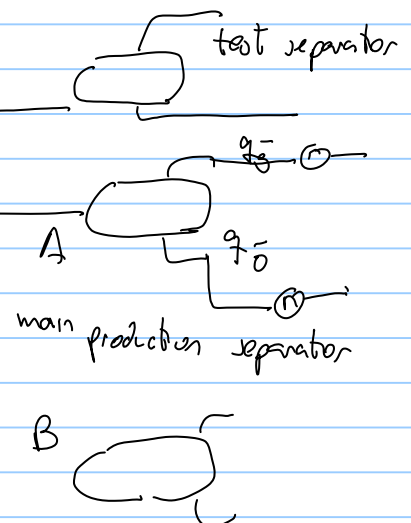
Production manifold:

- commingle the production of several wells
- test the well — measure q_o , q_g , P_{wh}

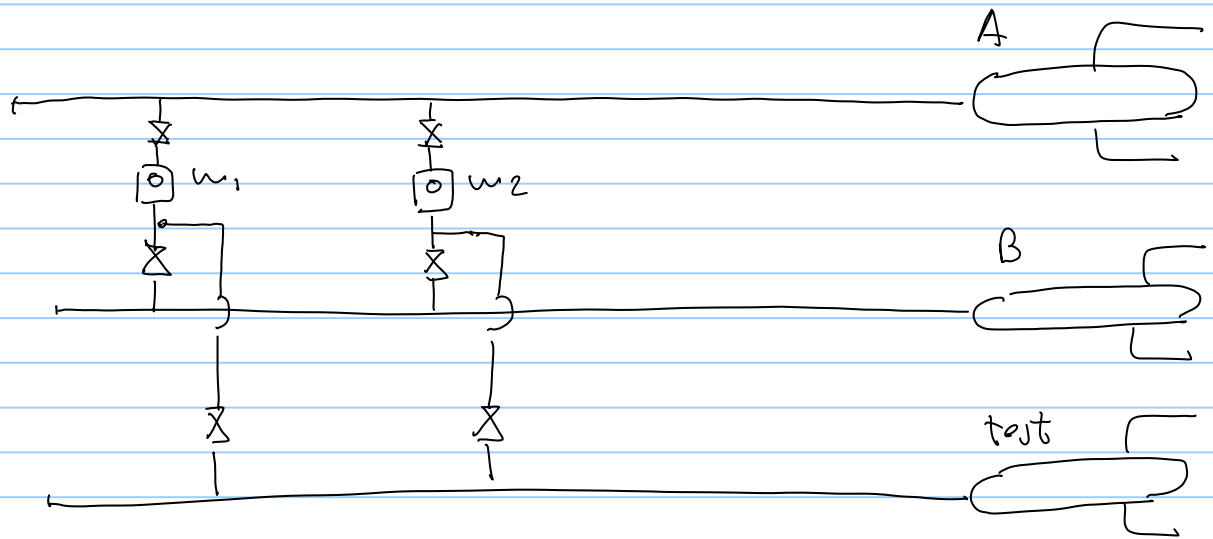


- Send the well to different production trains

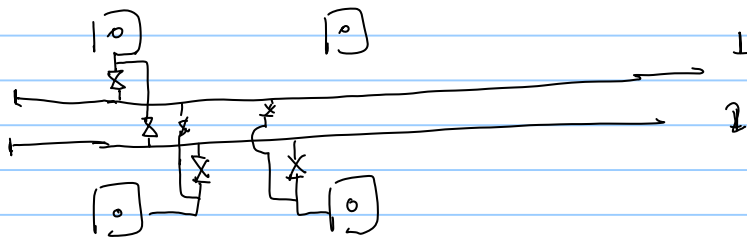
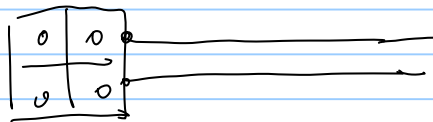
- allocation — split production to different
- determine well productivity
- Reservoir modeling to perform history match

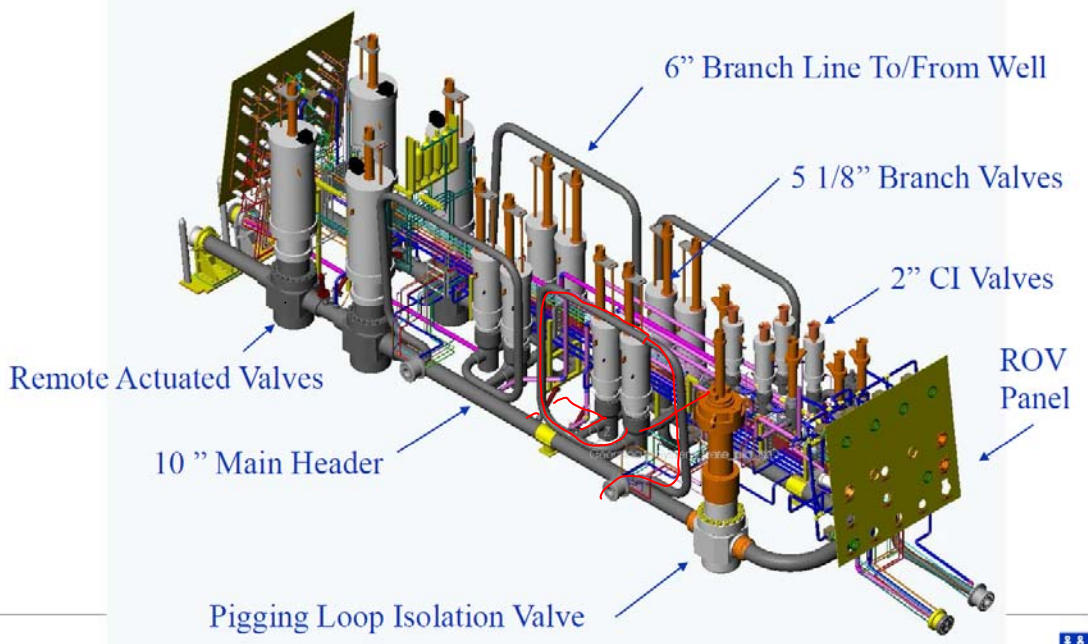


(class exercise) 3 wells to 3 separators

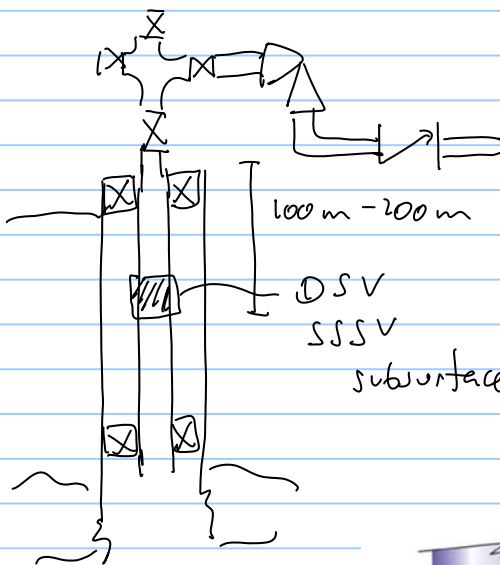


Subsea wells arranged in template. Subsea manifold



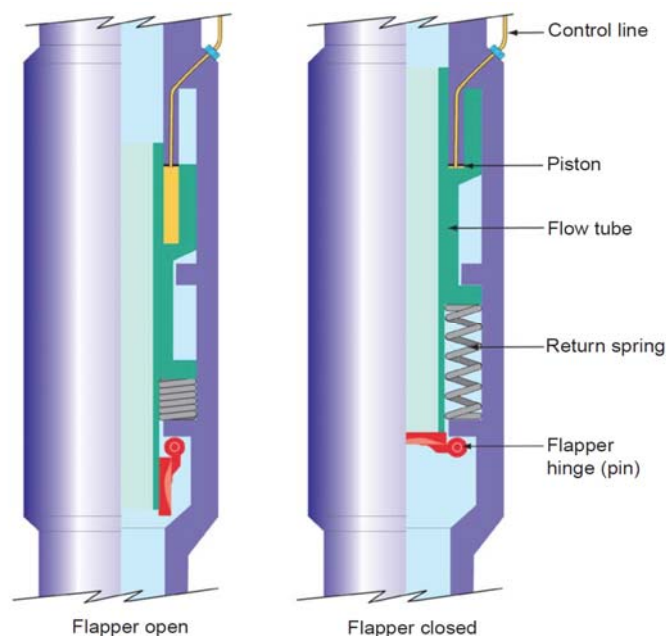


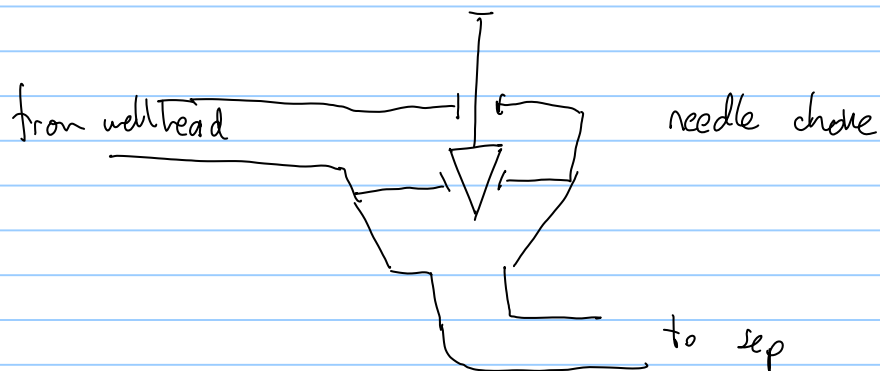
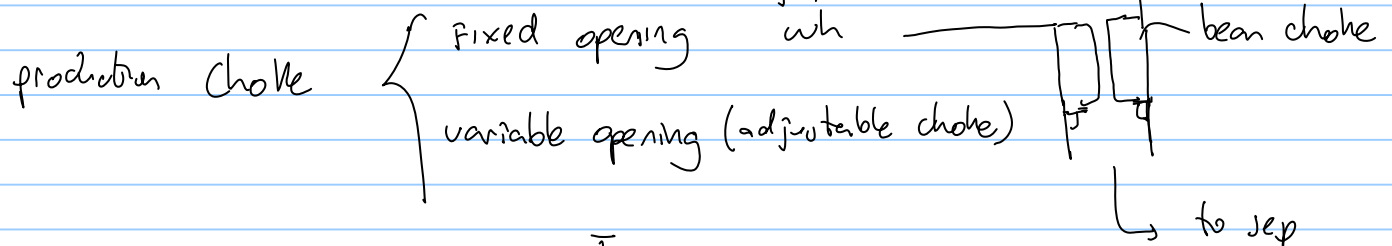
values of a well DSV downhole safety valve



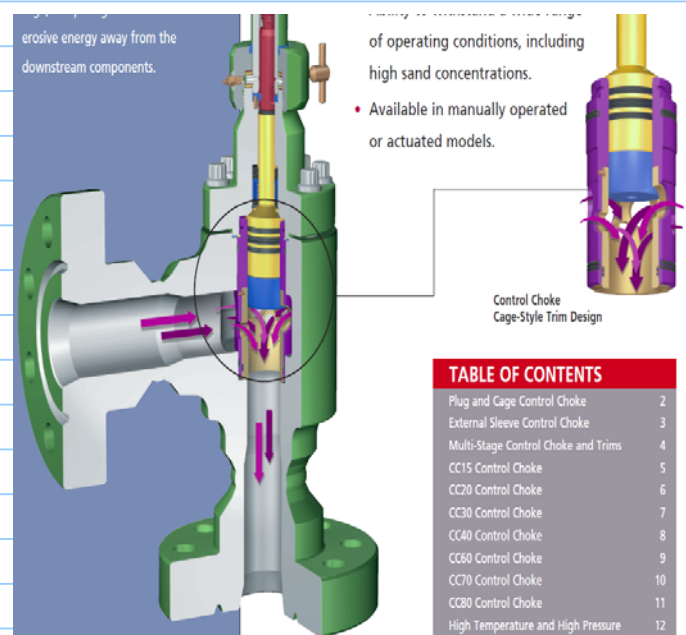
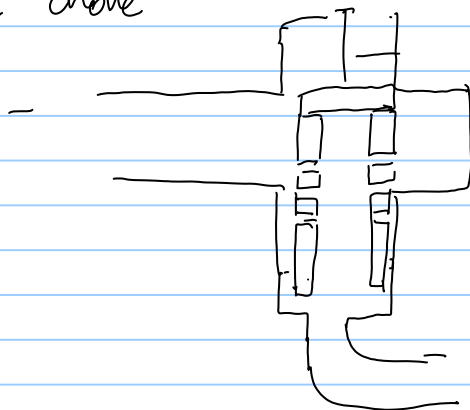
Barrier philosophy

- always 2 barriers between reservoir and environment
- barriers must be physically in different places
- action mechanism must be independent

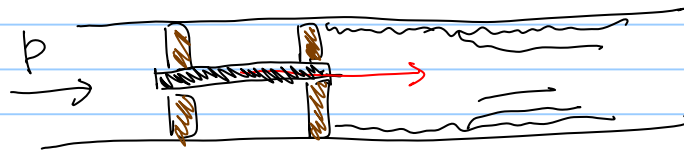




Cage choke



Need for pigging.



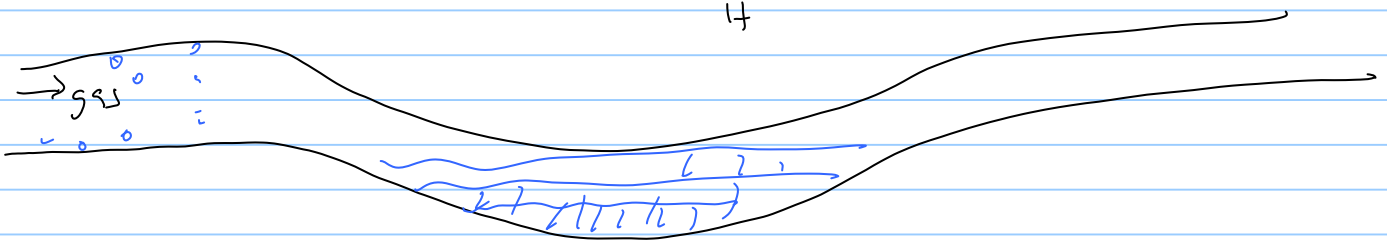
- remove accumulations on the pipe wall

wax heavy hydrocarbon fractions

$C_1, C_2, \dots, C_{11}, C_{20}$

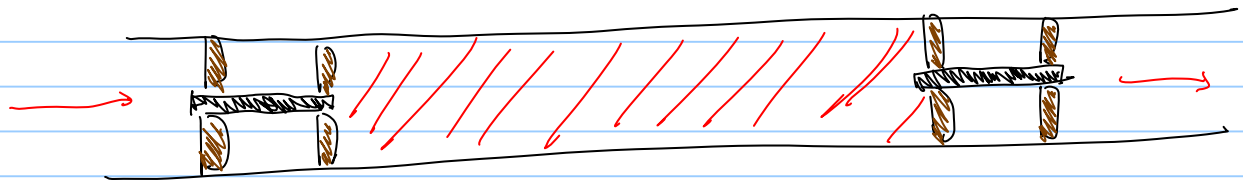
- remove liquid accumulation

H_1, H_2, \dots, H_n



- inspection of pipeline integrity (corrosion) thickness, etc

- treatment of inner pipe wall

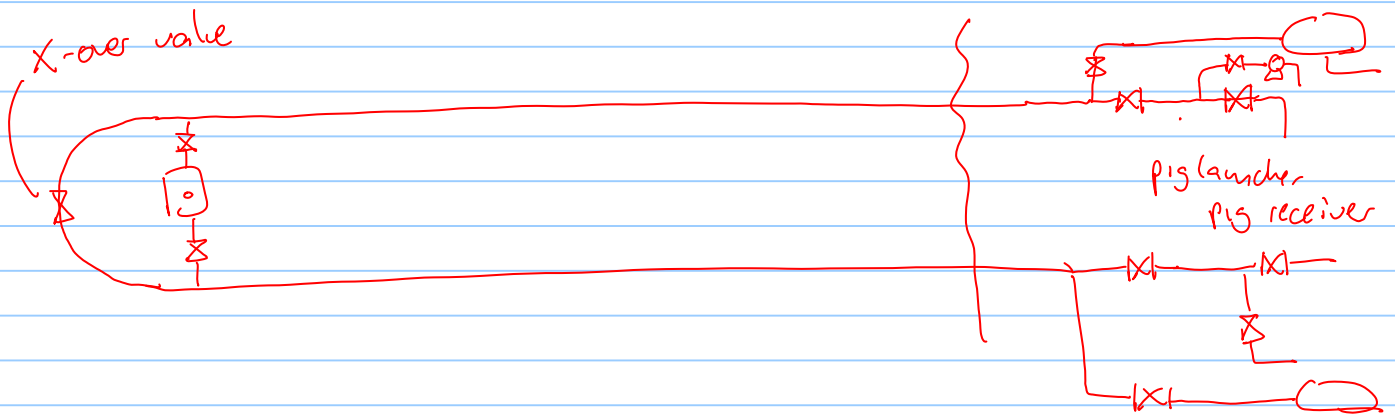
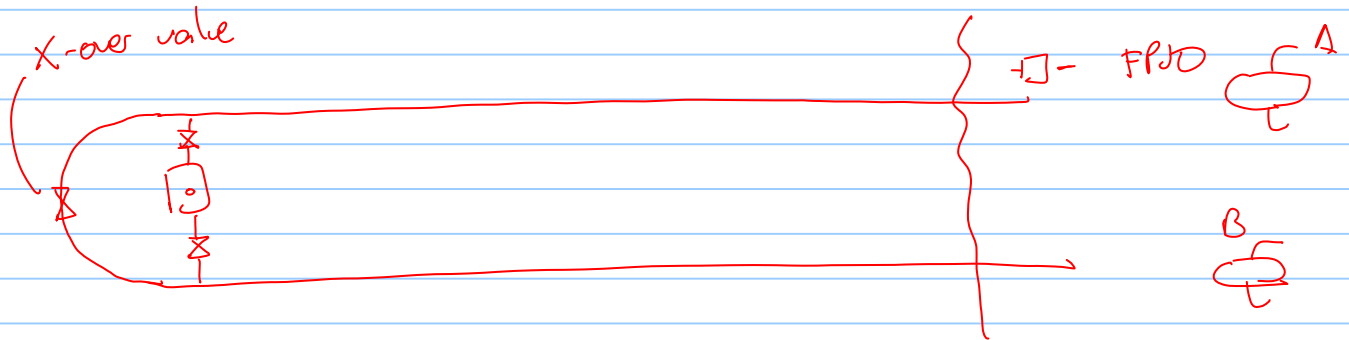
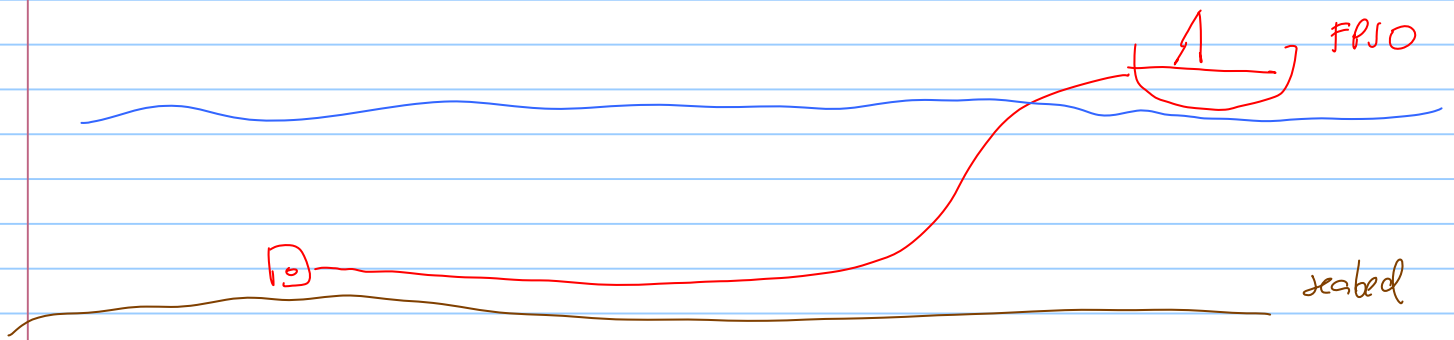


Various pig types

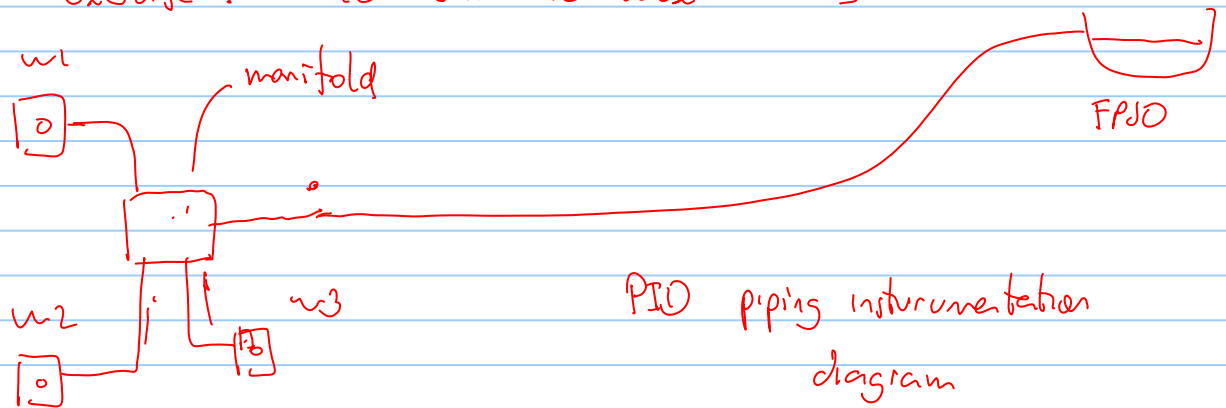


Wax plug-North Sea line pigging





• Home exercise: three satellite subsea wells

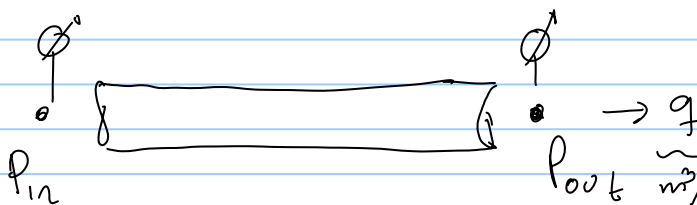


1 multiphase meter inside the manifold
please provide a solution (flow diagram)
to be able to test individually each well

flow equilibrium : MODAL Analysis

Inflow - outflow balance

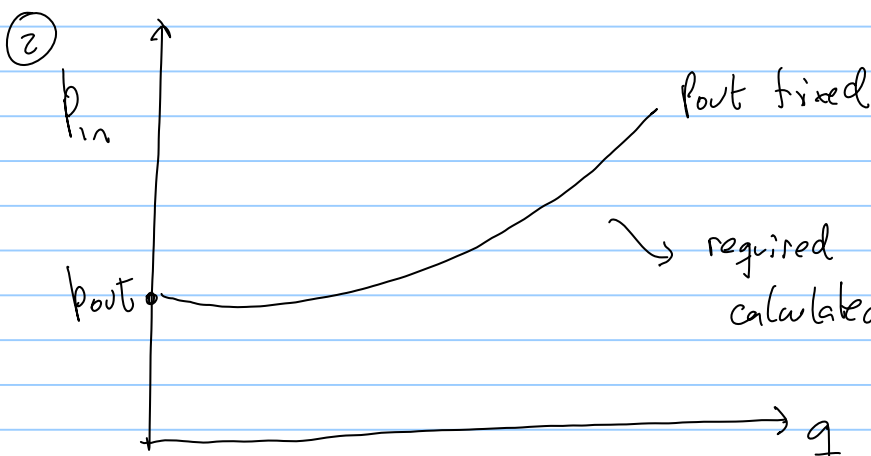
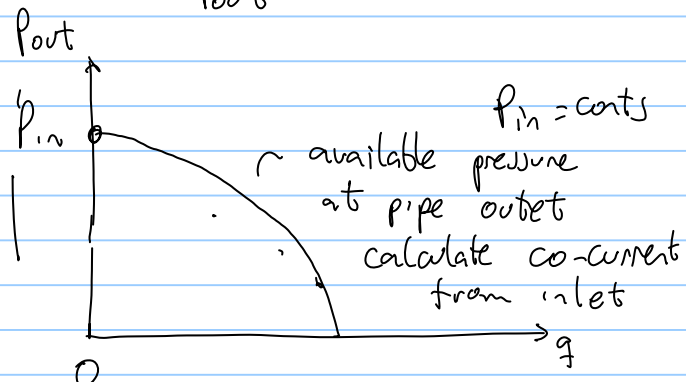
Horizontal pipe



$$P_{out} = P_{in} - |\Delta P_{friction}(q)|$$

fix P_{out} , change q obtain P_{in}

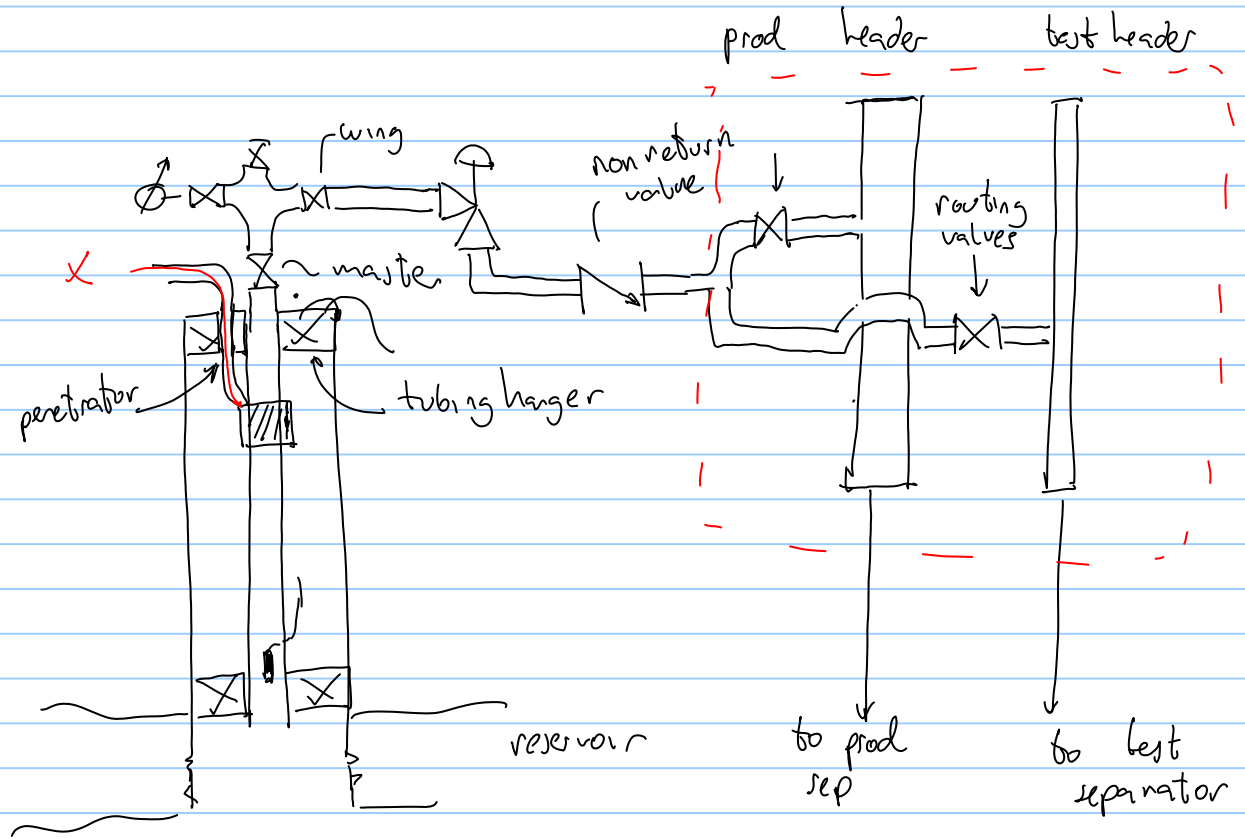
① fix P_{in} , change q , obtain P_{out}



$$P_{in} = P_{out} + |\Delta p(q)|$$

required pressure at pipe inlet
calculated counter-current from pipe outlet

Day 2 :



Christmas Tree Systems



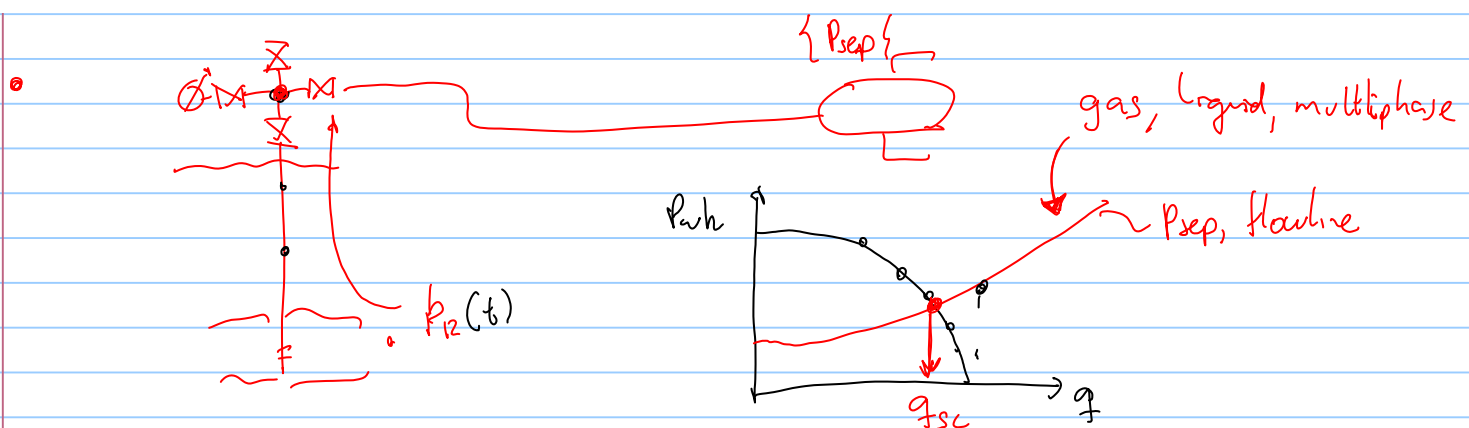
Onshore tree

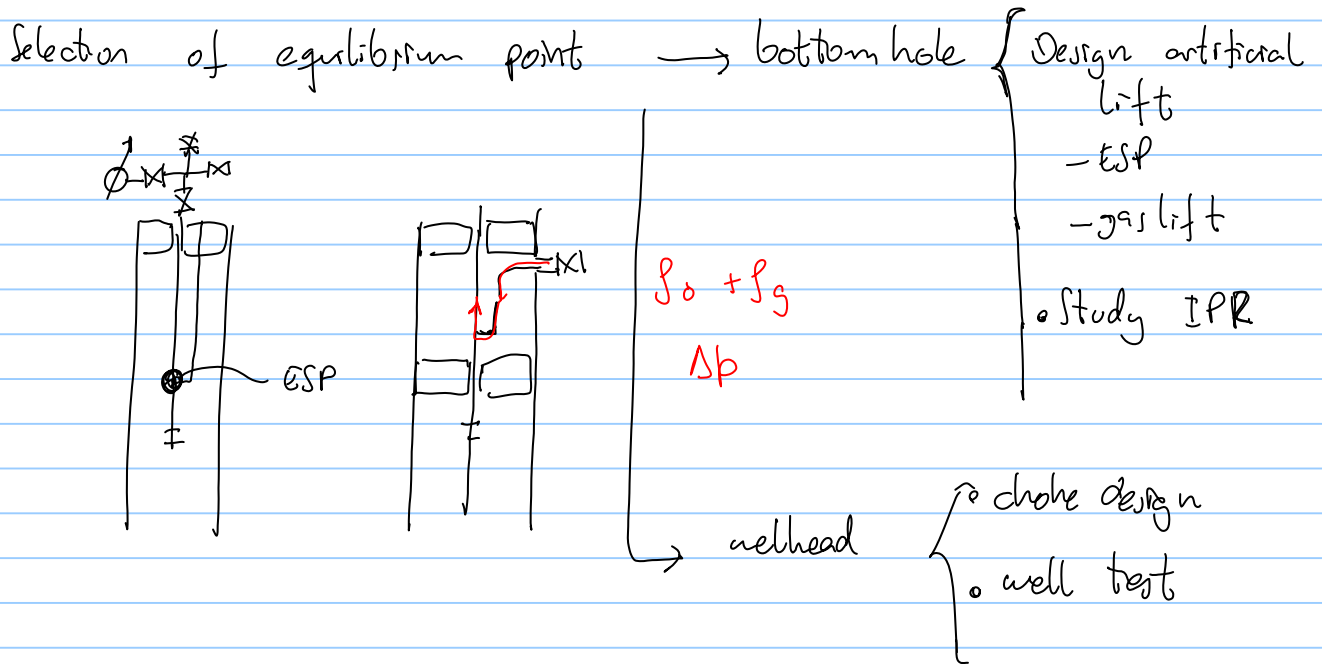


Offshore tree

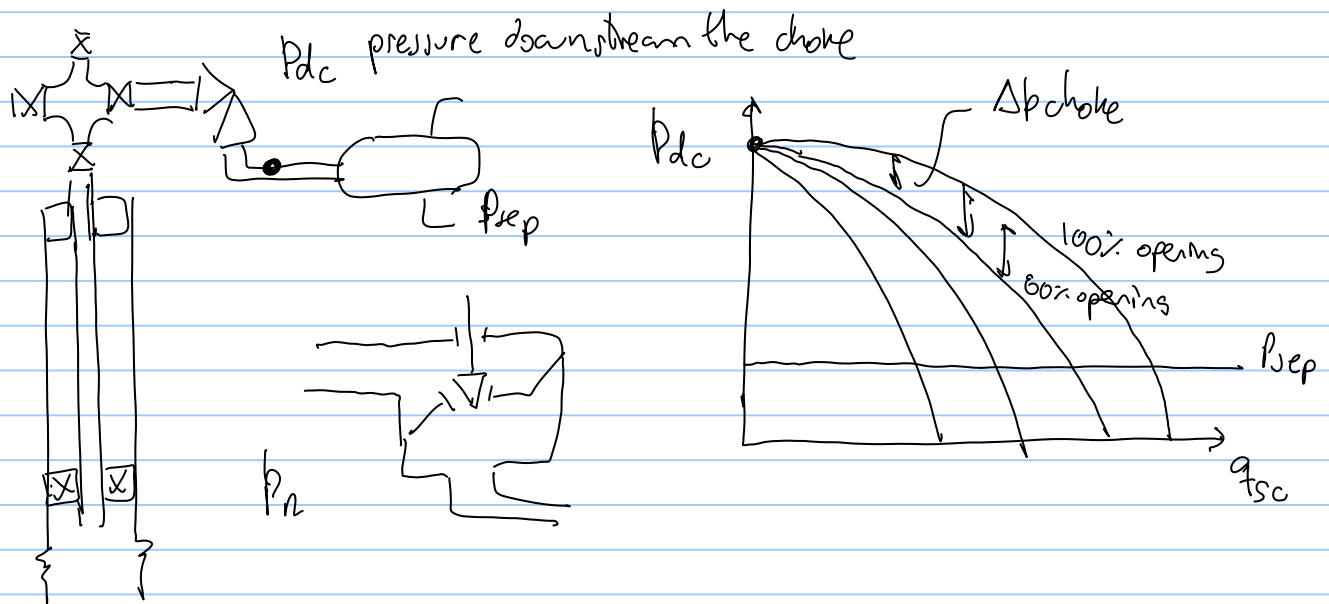


Subsea tree

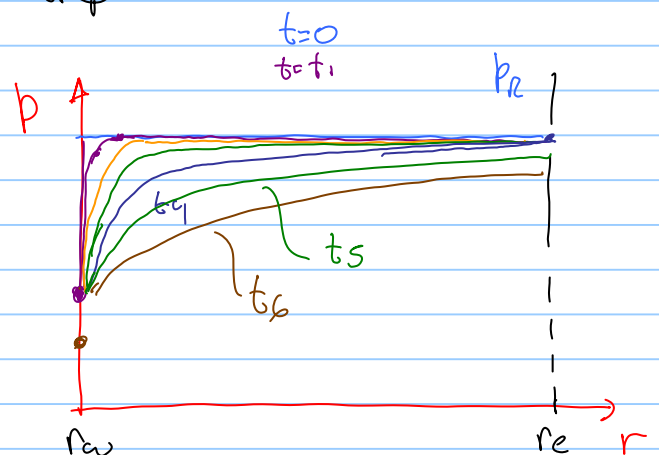
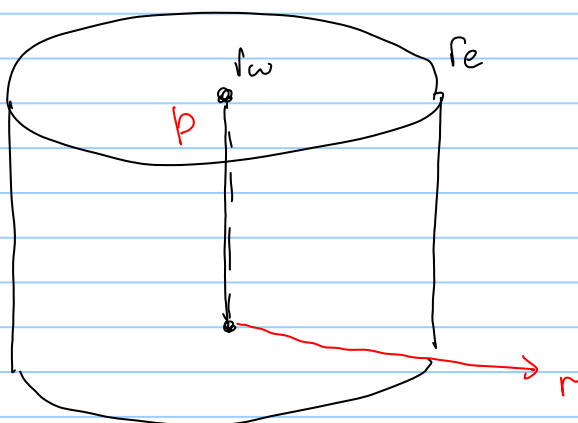




Flow equilibrium including the choke



• IPR Inflow performance relationship



$t_0 \rightarrow t_4$ transient, infinite acting

$t_4 \rightarrow$ forward \rightarrow pseudo steady state (PSS)

$t_0 \rightarrow t_1$ t_{ps} time to pseudo steady state

C_f, C_R, M, K } tight formation $\downarrow K \rightarrow t_{ps}$ is
in order
of months
years

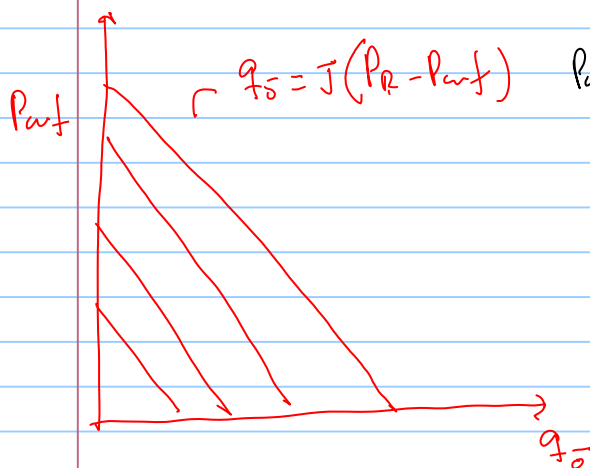
normal formation $\uparrow K \rightarrow t_{ps}$ hours
days

if t_{ps} is short { then I use IPR for PSS } * for this course

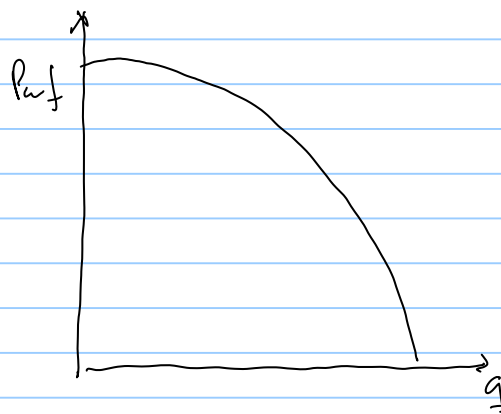
if t_{ps} is long { then use IPR for transient regime
 $IPR = f(t)$

pss IPRs

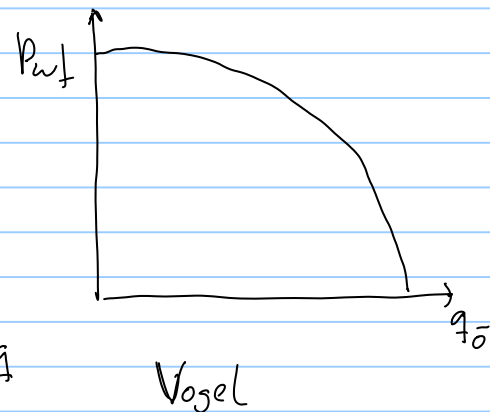
single phase
oil



GAS



Saturated oil
(mixture oil+gas)



$$q_o = C(P_R^2 - P_{wf}^2)^n$$

$$0.5 \leq n \leq 1$$

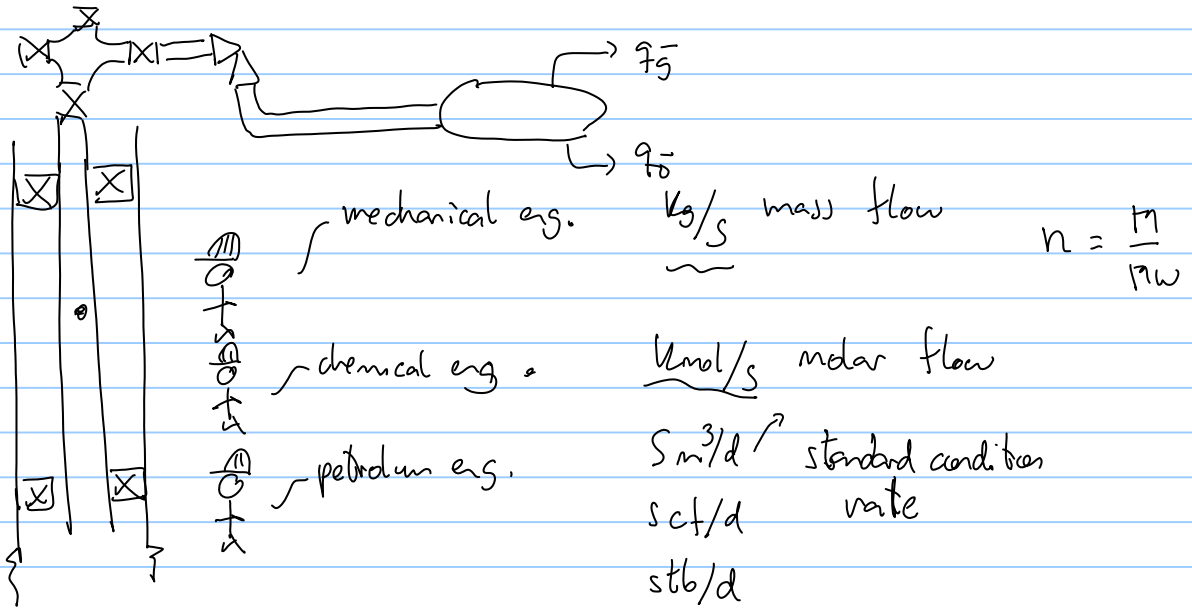
turbulent for low pressure $p \leq 100$ bara

$$m(p) = 2 \int_0^p \frac{p}{M^2} dp$$

for medium \rightarrow high pressure

$$q_o = C \left[m(P_R) - m(P_{wf}) \right]^n \quad \left\{ \begin{array}{l} \text{for the whole} \\ \text{pressure range} \end{array} \right.$$

$$\frac{q_o}{q_{o\max}} = 1 - 0.2 \frac{P_{wf}}{P_R} + 0.8 \frac{P_{wf}^2}{P_R^2}$$



$\Delta p \text{ friction} = V$



• Cave: dry gas

Ideal gas law

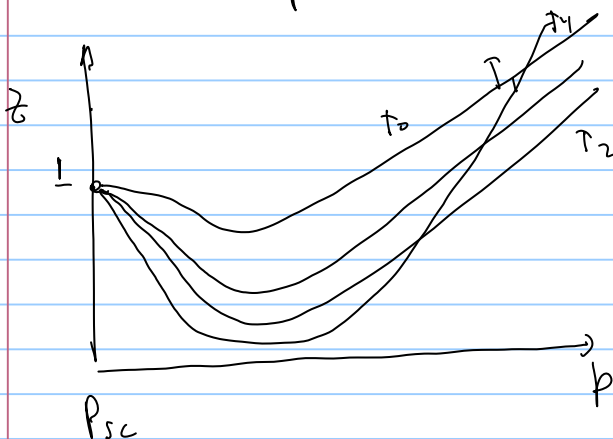
$\underline{pV = nRT}$

$0.7 \rightarrow 2$

Boyle \rightarrow Hooke $\frac{F}{x}$
Charles \rightarrow

insert a correcting factor in the ideal gas eq.

$pV = Z nRT$ ✓



Gay Lussac

Avogadro



Van der Waals corresponding states principles

pseudo-reduced pressure $p_r = \frac{p}{p_c} \sim p_{si}$

pseudo reduced T $T_r = \frac{T}{T_c}$ use absolute units only! $^{\circ}K, ^{\circ}R$

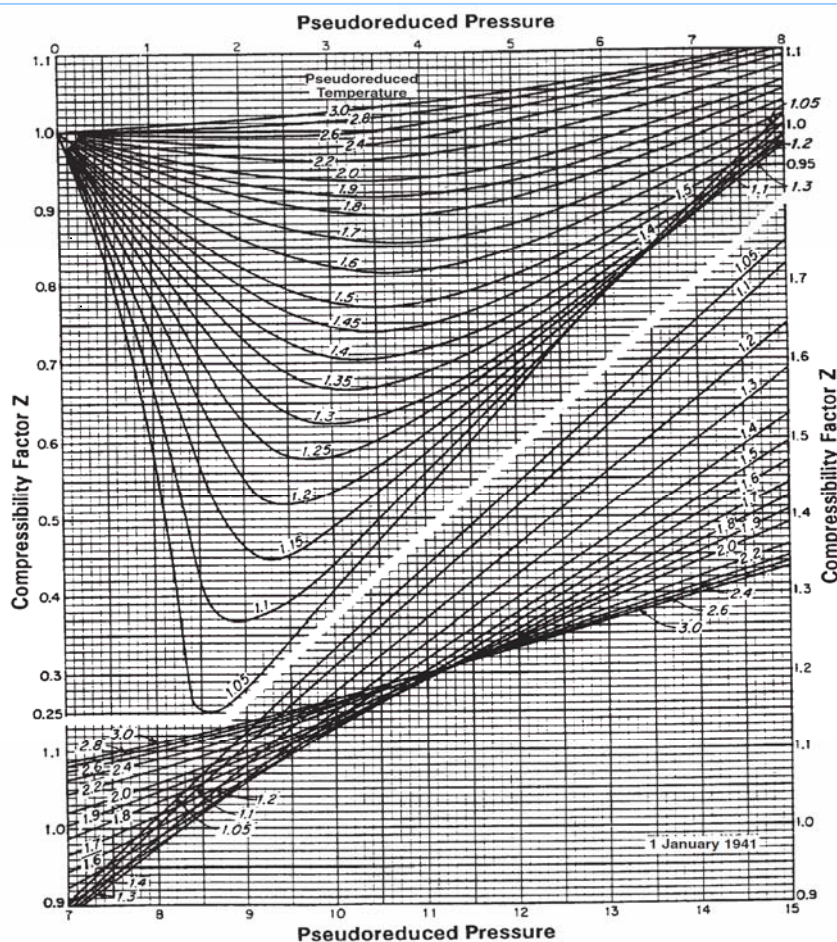
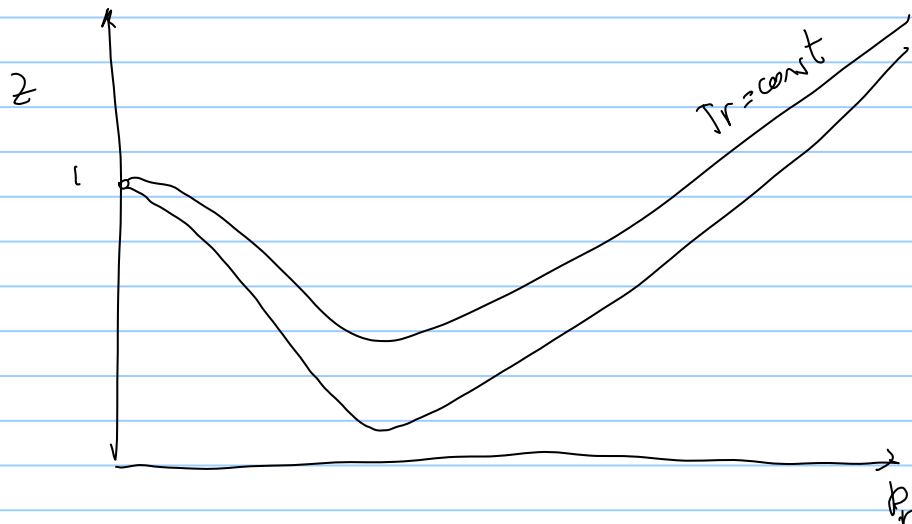


Fig. 3.6—Standing-Katz⁴ Z-factor chart.

$$p_r = 12$$

$$T_r = 1.4$$

Production gas is usually a mixture of several gases (components)

C_1	N_2	$y_i \sim$ mole fraction
C_2	CO_2	C_1 0.80
C_3	H_2S	C_2 0.10
C_4		C_3 0.03
C_5		\vdots
$C_7 +$		\vdots

$\Sigma \downarrow$

$$MW_{mix} = \sum_{i=1}^N y_i MW_i$$

components

Specific gravity of gas

$$\gamma_g = \frac{MW_{mix}}{MW_{air}} \rightarrow 28.97$$

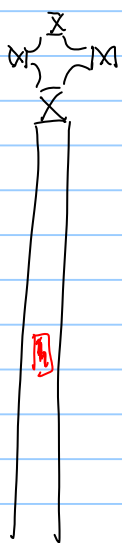
$$P_{cmix} = \sum_{i=1}^N y_i P_{ci}$$

$$T_{cmix} = \sum_{i=1}^N y_i T_{ci}$$

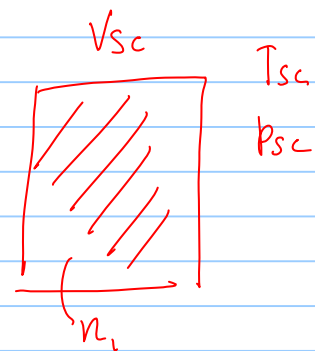
Correlation Sutton

$$P_{cmix} = f(\gamma_g)$$

$$T_{cmix} = f(\gamma_g)$$



n_i, P_i, T_i



$$PV = nRTz \quad \text{in S.C.}$$

$$P_{sc} V_{sc} = n R T_{sc} \quad z \approx 1$$

$$R = 8314.3 \frac{J}{Kmol \cdot K}$$

$$V_{sc} = n \cdot \frac{R T_{sc}}{P_{sc}}$$

$$V_{sc} = n \cdot 23.67 \frac{m^3}{Kmol}$$

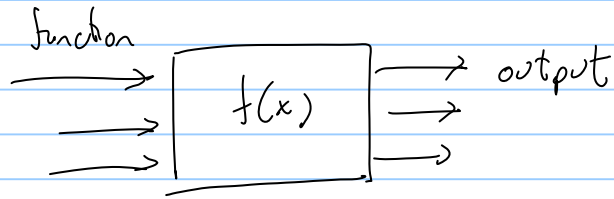
$$\hookrightarrow n \cdot 379.7 \frac{Scf}{Kmol}$$

Excel VBA

access the VBA module

alt + F11

function



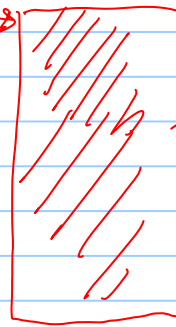
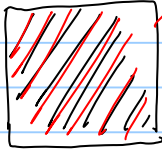
options → trust center → trust center options → macro settings
 ↳ enable all.

 $V @ P, T$

at any point in the production system

 $V_{sc} @ P_{sc}, T_{sc}$

no liquid is condensed



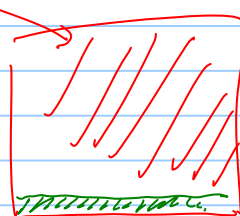
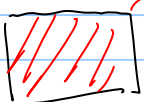
gas volume factor $B_{g,d} = \frac{V @ P, T}{V_{sc}} = \frac{\frac{n z R T}{P}}{\frac{n \cdot 1 \cdot R T_{sc}}{P_{sc}}} = \frac{z T (P_{sc})}{P (T_{sc})}$ ✓

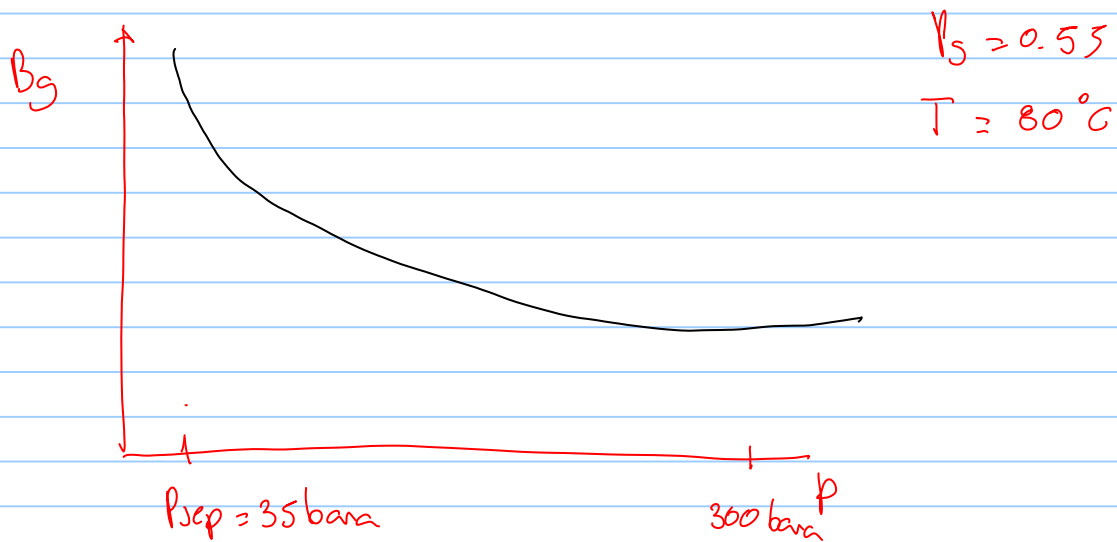
if $T = T_r$
 $P = P_r$

gas formation volume factor

if I give you $q_g @ sc$ find $q_g @ P, T$

$$q_g @ P, T = B_{g @ P, T} \cdot q_{sc}$$

 V, P, T  V_{sc}, P_{sc}, T_{sc} $B_{g,wet}$



* homework 2 :

plot B_g for

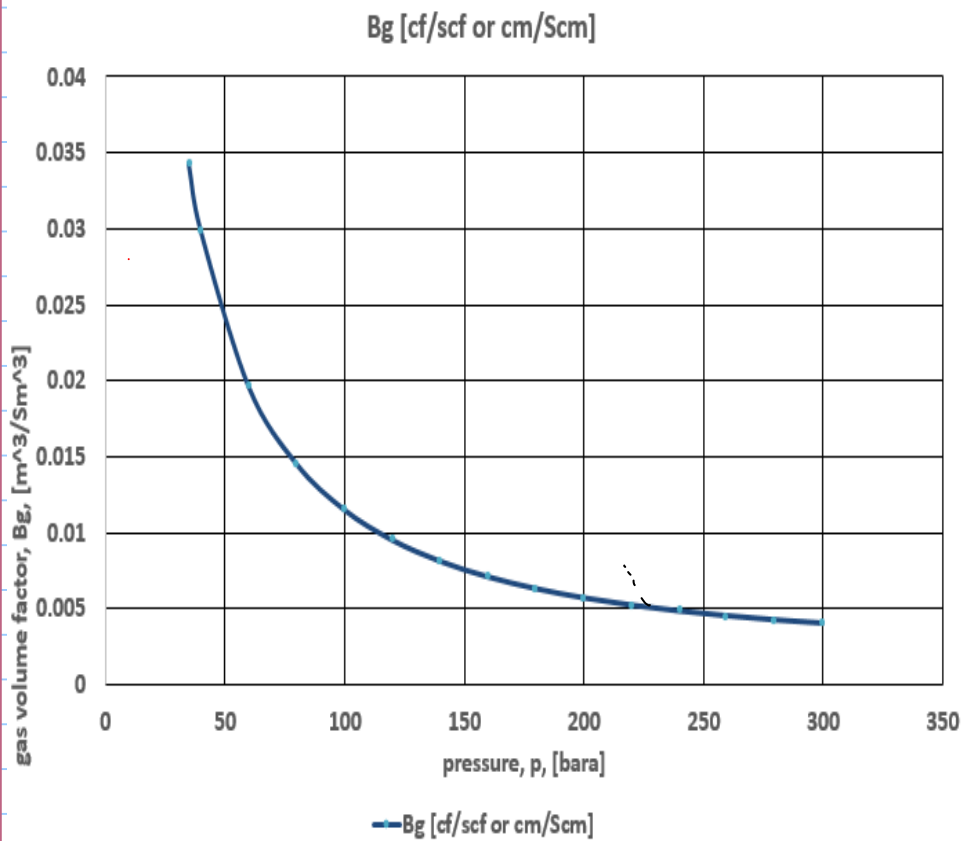
$$T = 120^\circ\text{C}$$

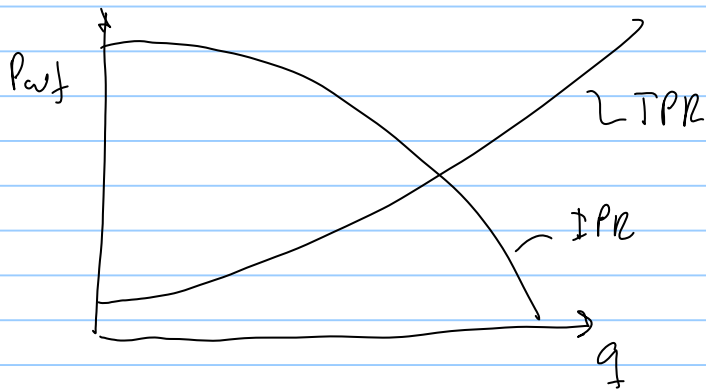
$$T = 60^\circ\text{C}$$

$$T = 80^\circ\text{C}$$

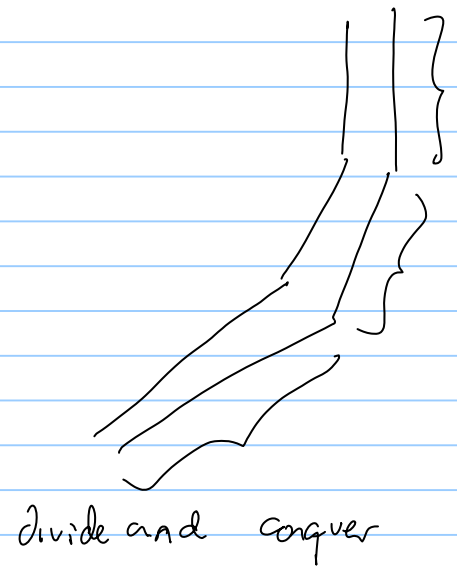
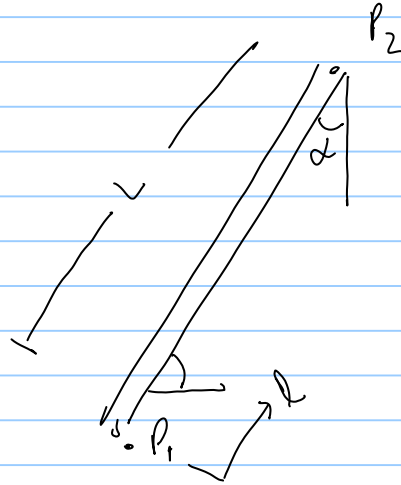
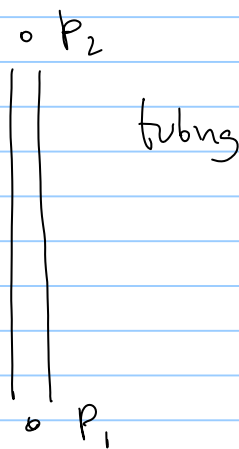
$$p \in [35 \text{ bara} - 300 \text{ bara}]$$

$$\gamma_s = 0.55$$





Development of Dry gas tubing equation \leadsto flowlines also applicable for



divide and conquer

$$\frac{dp}{dl} = \underbrace{-\rho \cdot g \cos \alpha}_{\text{hydrostatic}} - f \rho \frac{u^2}{2\phi}$$

$\downarrow \downarrow \downarrow$
 \downarrow
 P_2
 P_1
 α
 P_2
 P_1
 l

p changes along tubing

$$\rho = f(p, T)$$

$$\rho = \frac{p M_w}{z R T}$$

$$\frac{dp}{dl} = - \left(\frac{p M_w}{z R T} \right) g \cos \alpha - f \frac{p M_w}{z R T} \frac{u^2}{2\phi}$$

$$u = \frac{\dot{m}}{\rho \cdot A} \quad \text{kg/s}$$

$$A = \pi \cdot \phi^2 \cdot 0.25$$

$$\frac{dp}{dl} = - \frac{p M_w}{z R T} g \cos \alpha - f \frac{p M_w}{z R T} \frac{1}{2\phi} \frac{\dot{m}^2 (z R T)^2}{p^2 M_w^2 \pi^2 \phi^4}$$

$$u = \frac{\dot{m} \cdot z R T}{p M_w \cdot \pi \phi^2} \cdot \frac{4}{\pi \phi^2}$$

$$\frac{dp}{dl} = - \frac{p M_w}{z R T} g \cos \alpha - f \frac{\dot{m}^2 z R T^3}{\phi^5 \cdot \pi^2 p M_w}$$

Constant $B = \frac{\rho_w \cdot g \cos \alpha}{\bar{z} R \bar{T}}$

$$D = \frac{f \bar{z} R \bar{T} \delta \dot{m}^2}{\phi^5 \pi^2 \rho_w}$$

$$\frac{dp}{dl} = -B \cdot p - \frac{D}{p} \quad - \quad \frac{dp}{Bp + \frac{D}{p}} = dl$$

B and D are constant and equal to

$$B = \frac{\rho_w g \cos \alpha}{\bar{z} R \bar{T}}$$

$$\bar{z} = f(\bar{p}, \bar{T}) \quad \bar{T} = \frac{T_1 + T_2}{2}$$

$$D = \frac{\bar{f} \bar{z} R \bar{T} \delta \dot{m}^2}{\phi^5 \pi^2 \rho_w}$$

$$\int_1^2 \frac{dp}{Bp + \frac{D}{p}} = \int_1^2 -dl$$

$= -L$

$$\int_1^2 \frac{p dp}{Bp^2 + D}$$

change variable

$$U = Bp^2 + D$$

$$dU = 2p B dp$$

$$\frac{1}{2B} \int_1^2 \frac{dU}{U} = \frac{1}{2B} \ln \left(\frac{U_2}{U_1} \right)$$

return the variable change

$$\frac{1}{2B} \ln \left(\frac{Bp_2^2 + D}{Bp_1^2 + D} \right) = -L$$

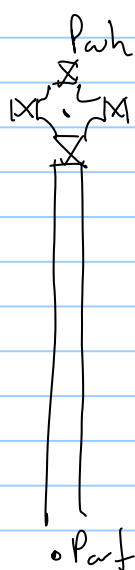


$$\frac{B p_2^2 + D}{B p_1^2 + D} = e^{-L 2B}$$

elevation factor
 $S = 2LB$

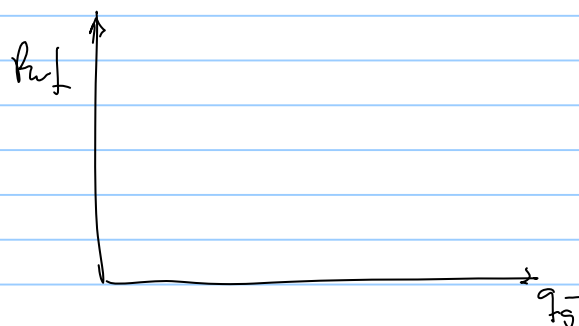
$$\frac{B p_2^2 + D}{B p_1^2 + D} = e^{-S}$$

$$p_1^2 = p_2^2 e^S + \left(\frac{B}{D}\right) e^S - 1$$



$$p_2 = P_{wh}$$

$$p_1 = P_{wf}$$



$$\frac{B}{D} = \frac{\dot{m}^2 \bar{z} R \bar{T} \theta \cdot f \bar{z} R \bar{T}}{M_w \pi^2 \phi^5 M_w g \cos \alpha}$$

$$\frac{B}{D} = \frac{q_{sc}^2 (\bar{z} R \bar{T})^2 \theta \cdot f}{\pi^2 \phi^5 \cos \alpha R^2 g} \left(\frac{p_{sc}}{T_{sc}}\right)^2$$

$$\dot{m} = \rho_{sc} \cdot q_{sc}$$

$$\downarrow$$

$$\frac{kg}{m^3} \cdot \frac{m^3}{d}$$

$$\rho_{sc} = \frac{p_{sc} \cdot M_w}{Z R T_{sc}}$$

$$p_1^2 = p_2^2 e^S + q_{sc}^2 \left(\text{constant} \right)$$

$$p_1^2 = p_2^2 e^S + \frac{q_{sc}^2}{C_r^2}$$

Tubing flow Equation-Dry gas

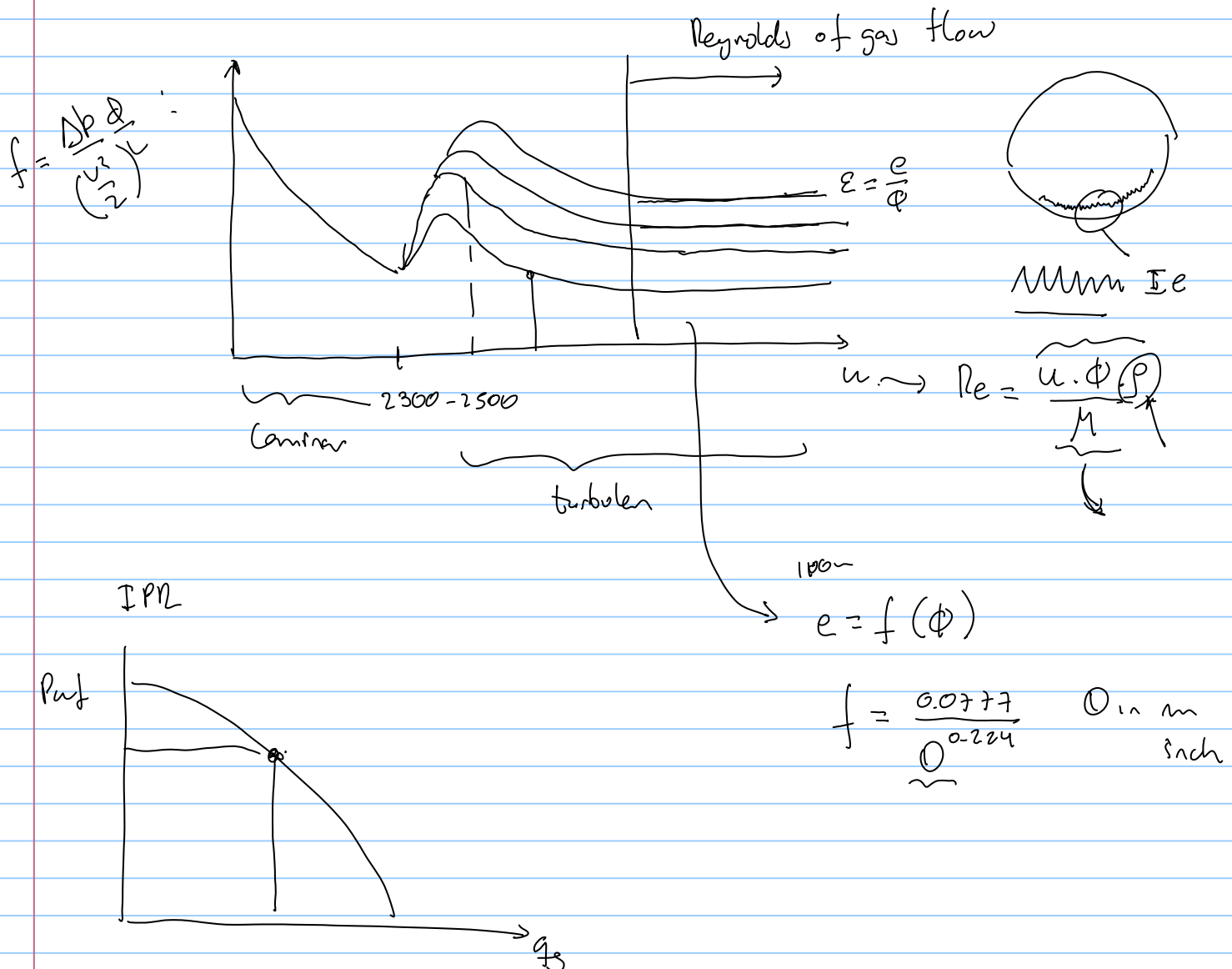
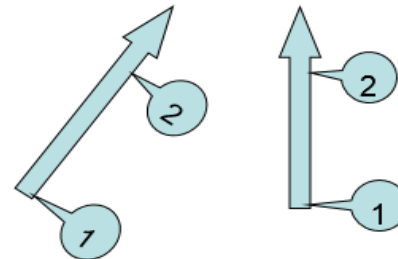
$$q_{sc} = \left(\frac{\pi}{4} \right) \left(\frac{R}{M_{air}} \right)^{0.5} \left(\frac{T_{sc}}{P_{sc}} \right) \left[\frac{D^5}{\gamma_g f_M Z_{av} T_{av} L} \right]^{0.5} \left(\frac{s e^s}{e^s - 1} \right)^{0.5} \left(\frac{p_1^2}{e^s} - p_2^2 \right)^{0.5}$$

$$\frac{s}{2} = \frac{M_g g}{Z_{av} R T_{av}} H = \frac{(28.97) \gamma_g g}{Z_{av} R T_{av}} H$$

$$q_{gsc} = C_T \left(\frac{p_1^2}{e^s} - p_2^2 \right)^{0.5}$$

$$p_{inlet} = p_1 = e^{s/2} \left(p_2^2 + \frac{q_g^2}{C_T^2} \right)^{0.5}$$

$$p_{wh} = p_2 = \left(\frac{p_1^2}{e^s} - \frac{q_g^2}{C_T^2} \right)^{0.5}$$



Check tubing calculator spreadsheet. Gas exercise.

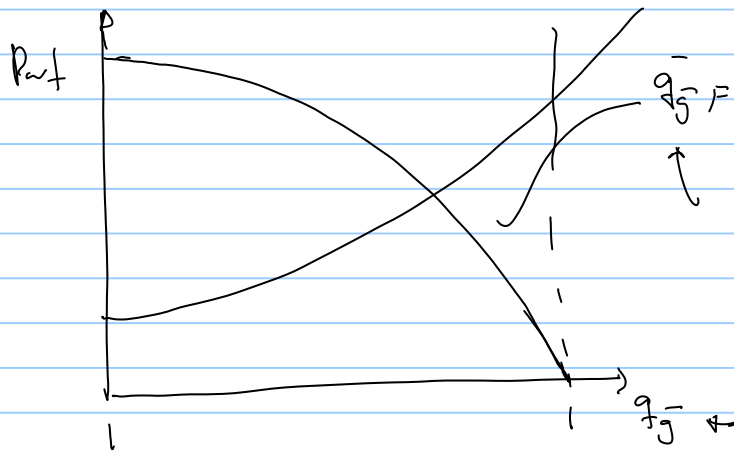
CALCULATION OF TUBING FLOW CONSTANT

height difference	[m]	3000
Internal Diameter	[m]	0.15
Gas gravity	[-]	0.55
Line length	[m]	3000
Inlet temperature	[K]	378
Outlet temperature	[K]	360
Inlet pressure	[bara]	303
Outlet pressure	[bara]	275
Ave. Temperature	[K]	369
Ave. Pressure	[bara]	289
Ave. Compressibility factor	[-]	0.981
Friction factor	[-]	0.012
Elevation Coeff, S		0.156
Tubing flow constant	[Sm ³ /bar]	3.58E+04

Day 3

http://folk.ntnu.no/stanko/Courses/POFE_UEM/2017/

• IPR - TPR equilibrium downhole

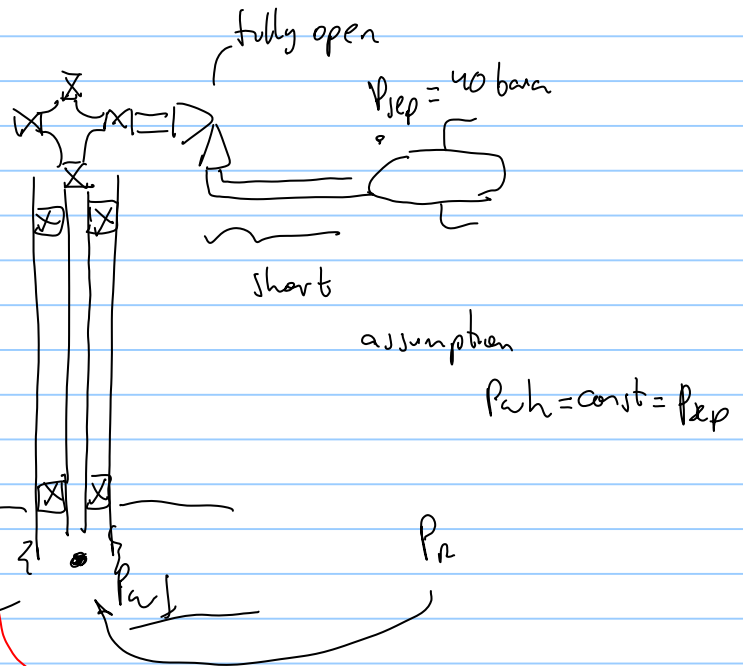


tubing equation $P_2^2 = \frac{P_1^2}{e^s} - \frac{q_g^2}{C_T^2}$ check

$$q_g = C_R (P_R^2 - P_{wf}^2)^n$$

Single_dry_gas_well_flow_equilibrium.xls - Compatibility Mode - Excel

Eva No 3		
p_R , Res pressure	304 bara	✓
C_R	104 Sm ³ /d/bar ²ⁿ	✓
n , exponent	0.9	✓
C_t , tubing	4.25E+04 Sm ³ /d/bar	✗
s , elevation	0.155	✗
C_{fl} , flowline	1.25E+05 4.00E+04 Sm ³ /d/bar	
p_{sep}	40 bara	



available independent variable

IPR	TPR
P_{wf}	P_{wf}
(bara)	(bara)
P_R	P_R
$P_2 = P_R - 20$	$P_2 = P_R - 20$
$P_3 = P_2 - 10$	$P_3 = P_2 - 10$
$P_4 = P_3 - 20$	$P_4 = P_3 - 20$

required

$$P_1 = e^{s/2} \left(P_2^2 + \frac{q_g^2}{C_T^2} \right)^{0.5}$$

$$P_2^2 = \frac{P_1^2}{e^s} - \frac{q_g^2}{C_T^2}$$

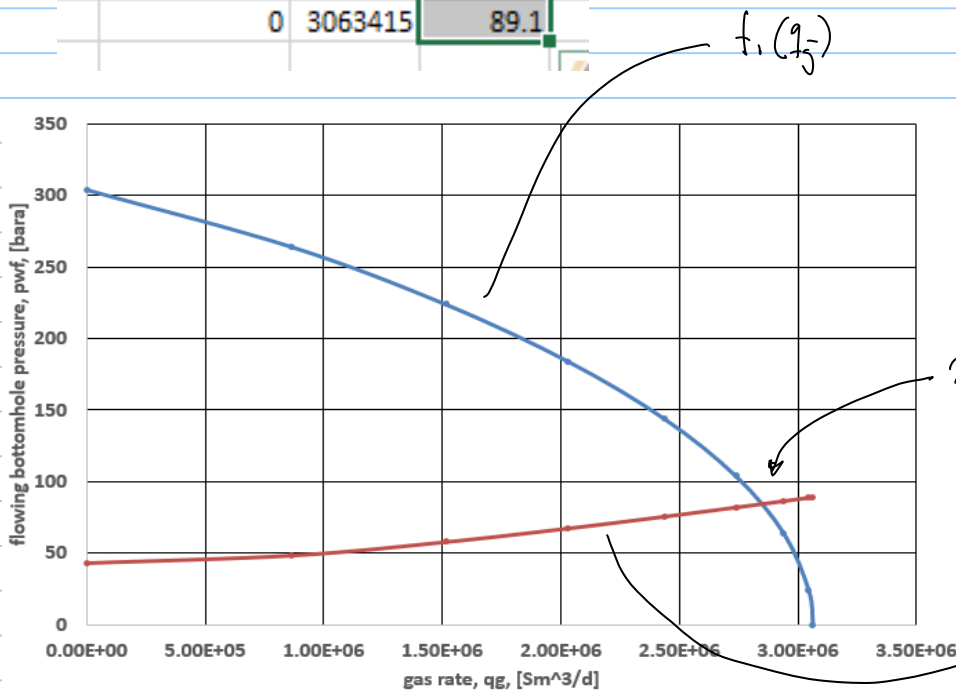
to freeze cells in excel (F4)

\$E\$4

pwf_avail [bara]	IPR qg	TPR pwf_req
	[sm ³ /d]	[bara]
(304)	0	43.2
264	866564	48.5
224	1514206	57.9
184	2031769	67.4
144	2437202	75.6
104	2738763	82.0
64	2940942	86.4
24	3046225	88.7
0	3063415	89.1

$p_{wf_avail} - p_{wf_req}$

304 - 43.2



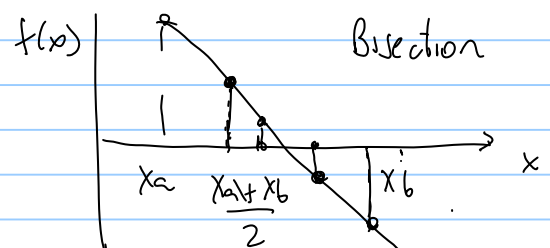
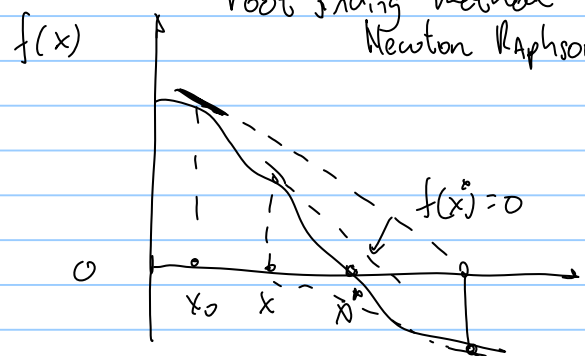
$\approx 2.8 \text{ E}6 \text{ Sm}^3/\text{d}$

find $q_{j_eq}^*$ that makes

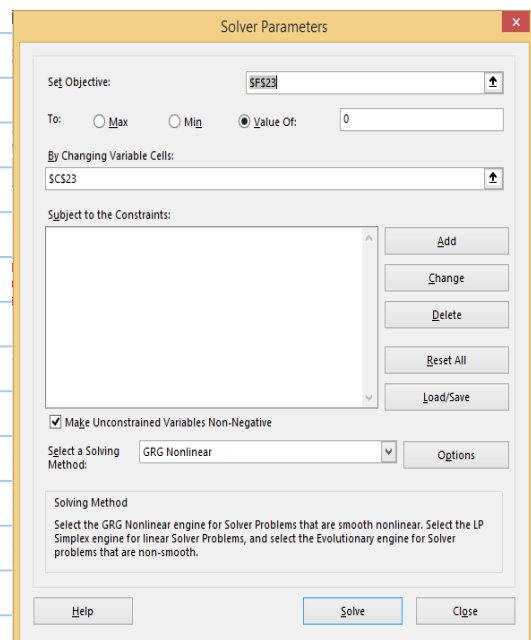
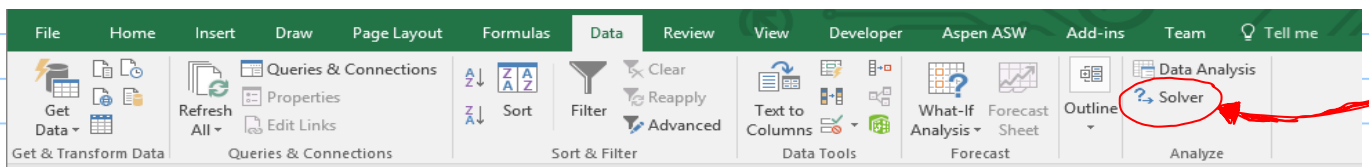
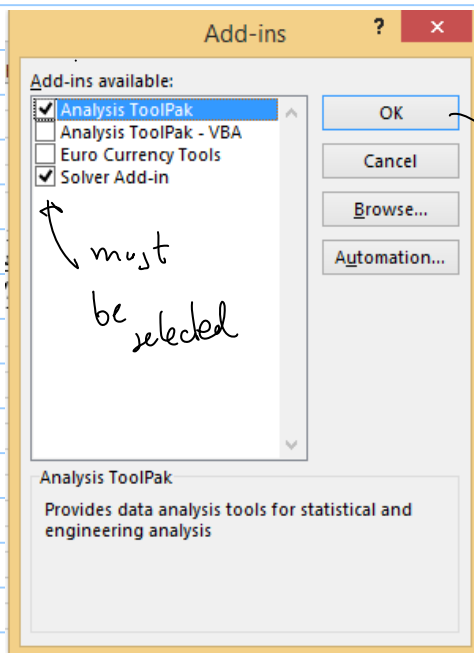
$$f_3(q_j) = f_1(q_j) - f_2(q_j)$$

$$f_1(q_j^*) = f_2(q_j^*)$$

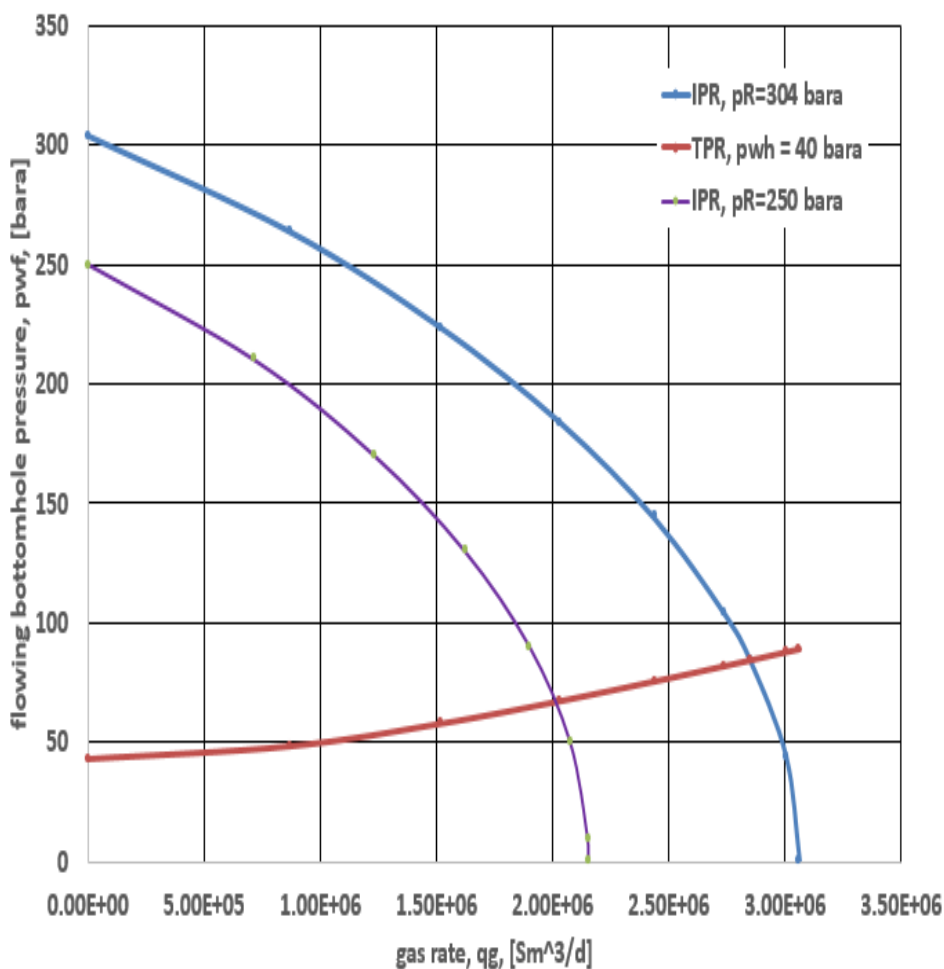
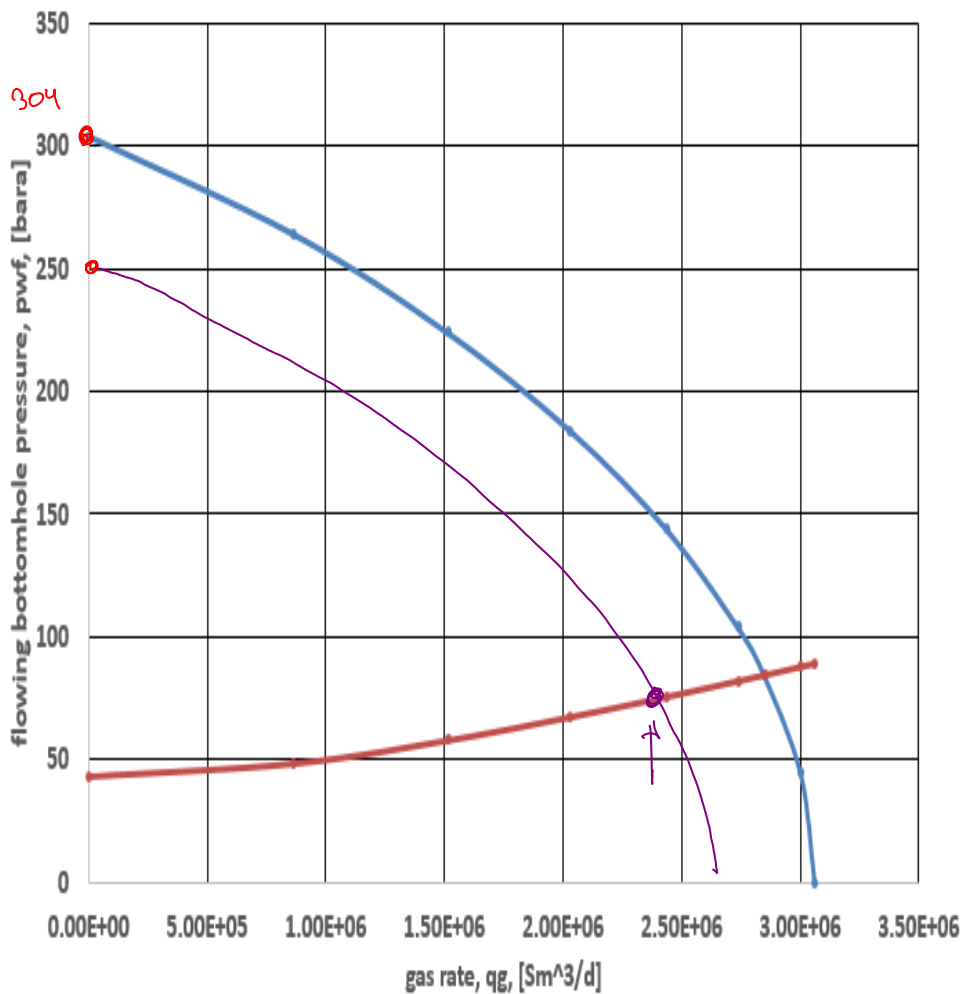
root finding method
Newton Raphson



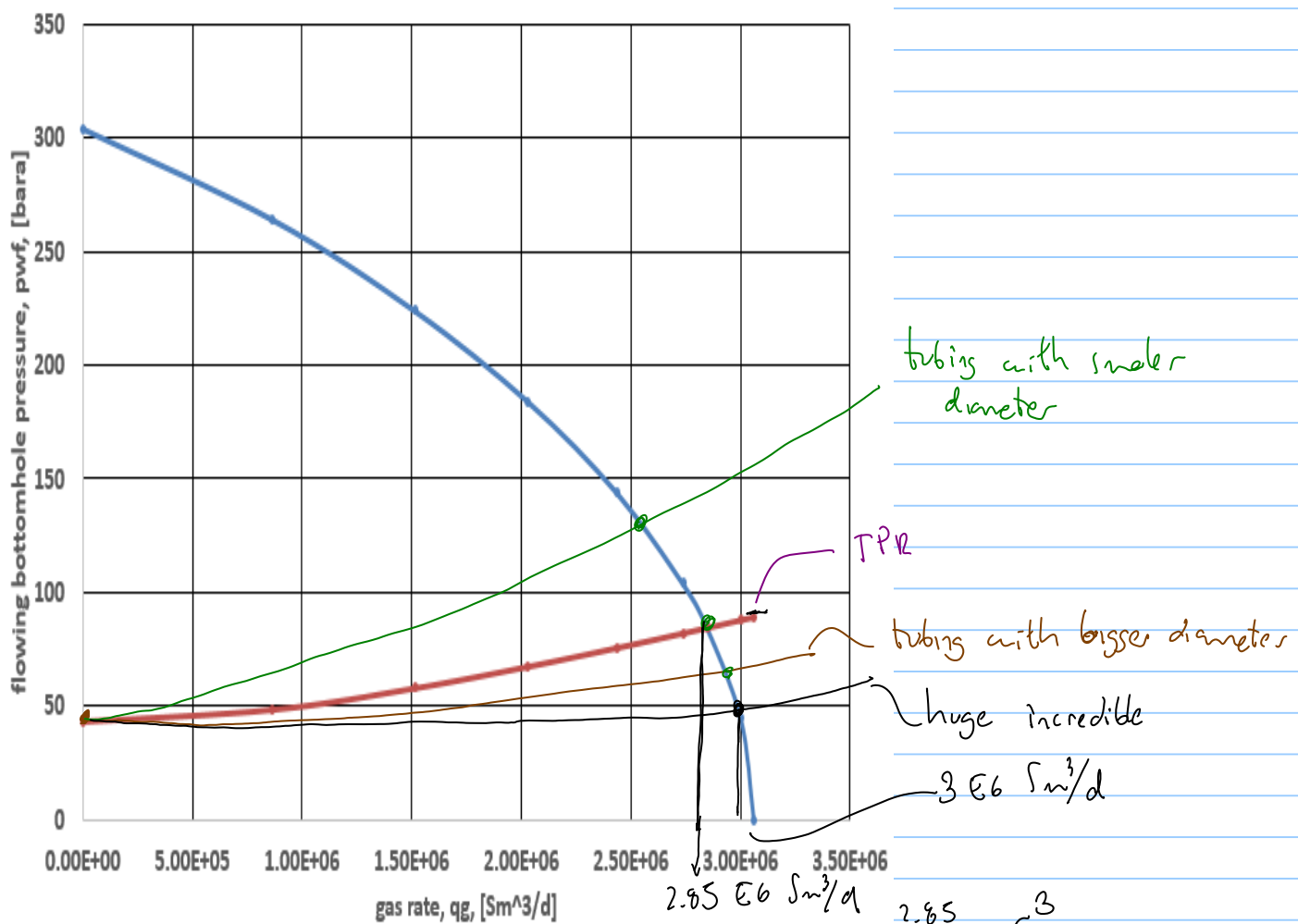
activate the solver in excel options, Add-in, go...



	IPR	TPR	
pwf_avail	qg	pwf_req	pwf_avail_pwf_requ
[bara]	[sm ³ /d]	[bara]	[bara]
304	0	43.2	260.8
264	866564	48.5	215.5
224	1514206	57.9	166.1
184	2031769	67.4	116.6
144	2437202	75.6	68.4
104	2738763	82.0	22.0
84.4	2850168	84.4	0.0
44.4	3004597	87.8	-43.4
0	3063415	89.1	-89.1



changing tubing size (ID).



$$\text{relative difference} = \left(\frac{q_{s, \text{orig}} + q_{s, \text{new}}}{q_{s, \text{original}}} \right) \times 100$$

2.85 3

Our well is NOT tubing-restricted

5% increase
in the rate of
well.

$$p_2^2 = \frac{p_1^2}{e^s} - \frac{q_s^2}{C_T^2}$$

$$C_T = \frac{\pi}{4} \left(\frac{R}{M_{arr}} \right)^{0.5} \frac{T_{sc}}{p_{sc}} \left[\frac{\phi^5}{p_g - f \cdot \bar{z} T L} \right]^{0.5} \left[\frac{s e^s}{e^s - 1} \right]^{0.5}$$

an increase in $\phi \rightarrow$ increase C_T

by increasing C_T we can address what would be the effect of a bigger or smaller diameter

if only diameter changes then

$$C_{T1} = f(\Phi_1)$$

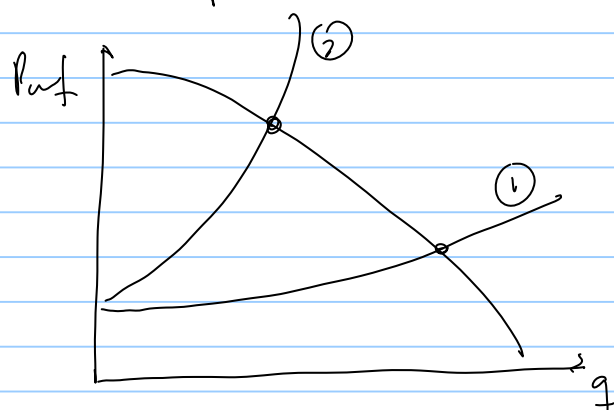
$$C_{T2} = f(\Phi_2)$$

$$\frac{C_{T1}}{C_{T2}} = \left(\frac{\Phi_1^5}{\Phi_2^5} \right)^{0.5} = \frac{\Phi_1^{2.5}}{\Phi_2^{2.5}}$$

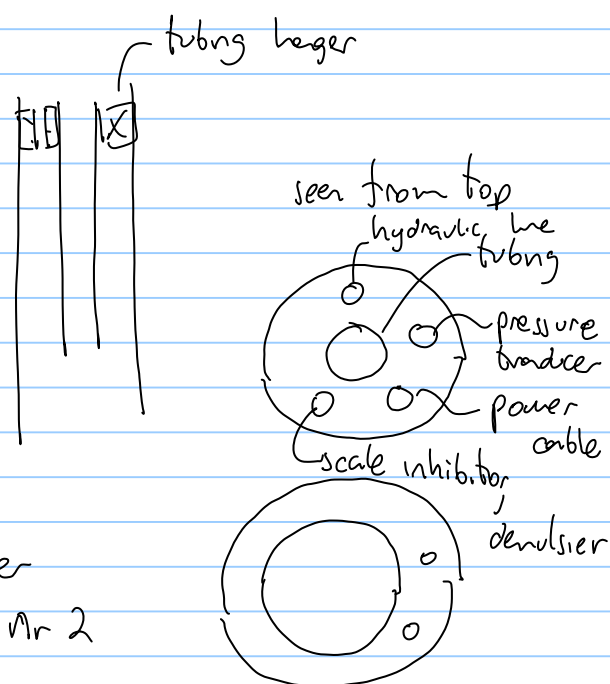
$$C_{T2} = C_{T1} \frac{\Phi_1^{2.5}}{(\Phi_2)^{2.5}}$$

tubing sizing.

- ID of production casing
- size and amounts of holes on the tubing hanger
- reduce Δp tubing and increase production



① is better than nr 2

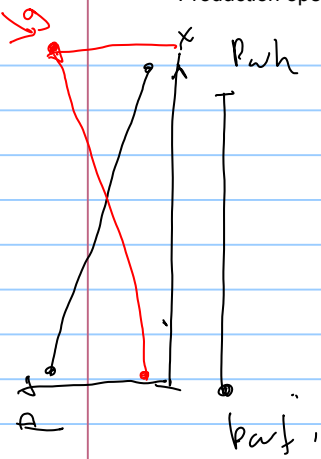


- Erosion

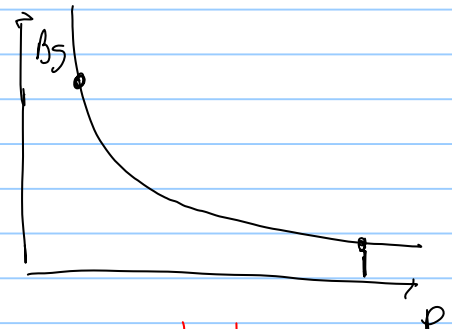
$$V_{\text{gas}} \leq V_{\text{erosional velocity}} \rightarrow \text{API 14 E}$$

$$\text{Limit } V_{\text{erosional}} = \frac{C}{\sqrt{\rho_m}} \quad \left\{ \begin{array}{l} 100 \\ 120 \end{array} \right.$$

[ft/s] [lb/ft³]



$$V_{gas} = \frac{q_g @ P, T}{A} = \frac{q_g - B_g(p, T)}{A}$$

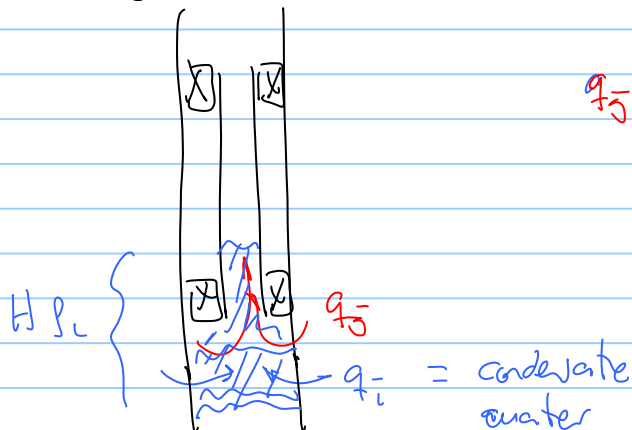


if wellhead $V_g \geq$ Verosional velocity \Rightarrow I might have erosion in tubing

• avoid liquid loading $V_g \leq$ Verosional velocity

$$V_{gas} \geq V_{turner}$$

• if producing with liquid then

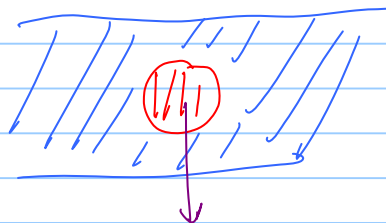
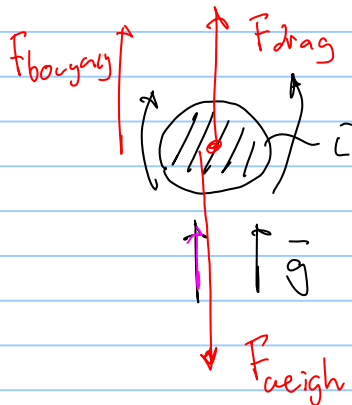


$q_g \uparrow$ such it can carry q_i

liquid accumulation in wellbore

\hookrightarrow liquid loading

Force balance on a droplet



$$F_{drag} = F_{weighth}$$

$$\frac{\pi \phi^2}{4}$$

$$\frac{1}{2} C_D \rho_g V_g^2 A_d = (\rho_i - \rho_g) g \cdot \frac{\pi \phi^3}{6}$$

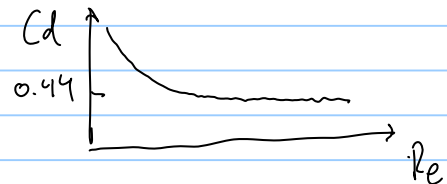
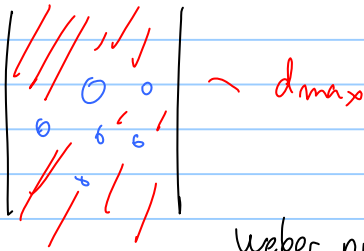
$$(\rho_i - \rho_g) g \cdot V_d$$

$$V = \sqrt{\frac{\phi \rho_g (P_L - P_g) \frac{1}{3}}{C_d \rho_g}} \quad (1)$$

$\rho_L \approx \text{const}$ $P_V = Z n R T$

Hiuze

C_d in the turbulent region is 0.44



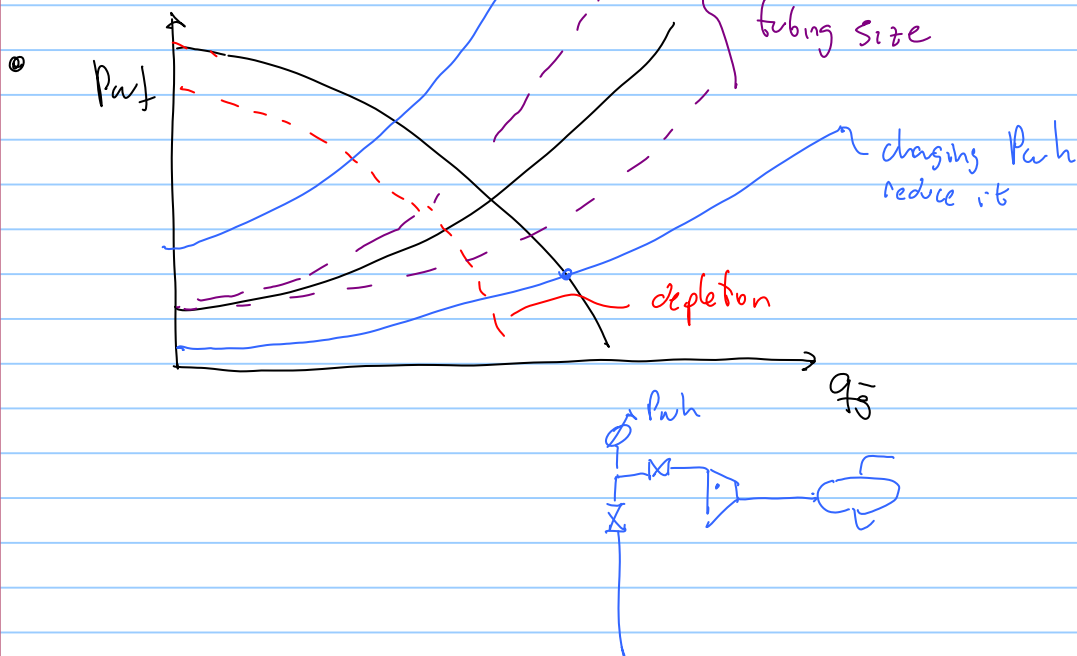
$$\text{Weber number} = \frac{(\rho_g u_g^2 \phi)}{\sigma}$$

when $Na \geq 30$ that gives you ϕ_{max}

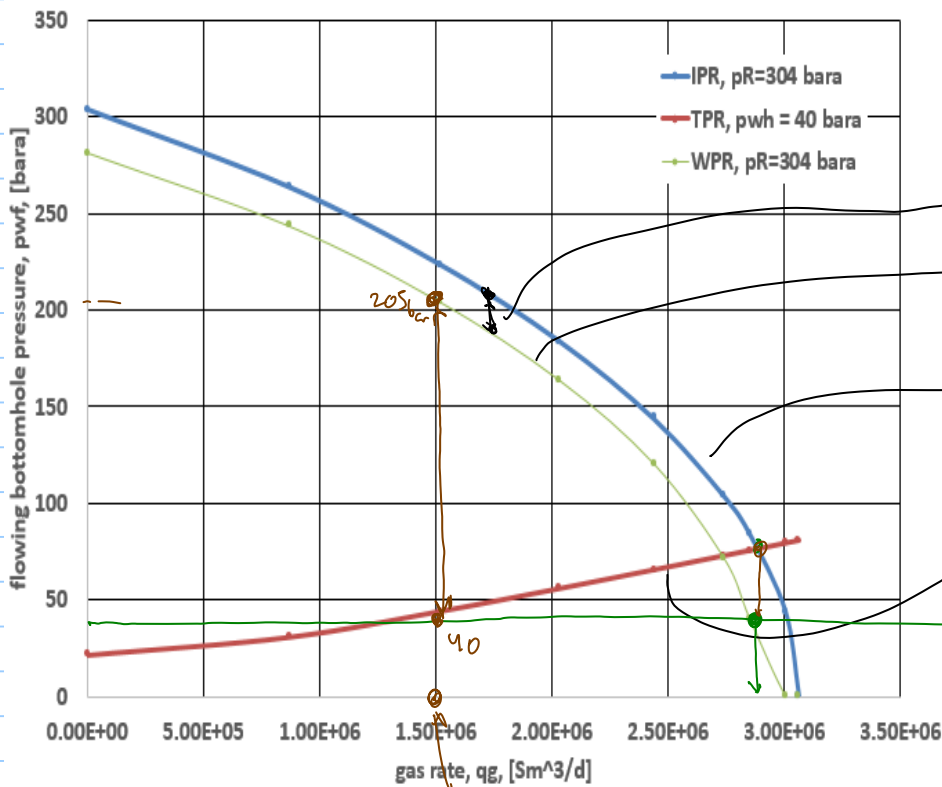
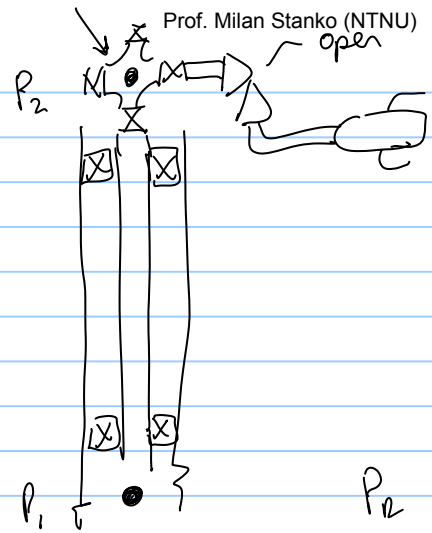
$$u_g^2 \rho_g \phi_{max} = 30 \quad (2) \quad \text{substitute eq (2) in (1)}$$

$$\text{Turner equation: } V = 5.46 \left[\frac{\sigma^{0.25} (P_L - P_g)^{0.25}}{\rho_g^{0.5}} \right]$$

$$\rho \propto \left[\frac{kg}{m^3} \right] \quad \sigma \propto \left[\frac{N}{m} \right]$$



Perform wellhead equilibrium



$$\Delta p_{chone} = 205 - 40 = 165 \text{ bar}$$

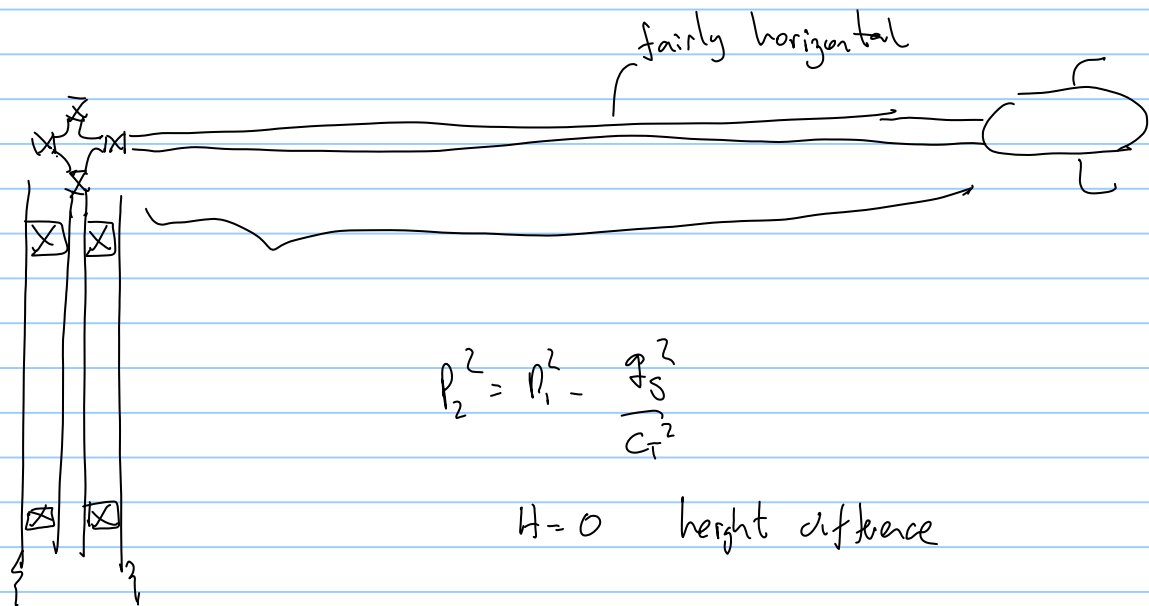
③ Homework.

Perform flow equilibrium for this class exercise but using the wellhead as the equilibrium point.

- Compute wPR
- $wPR - p_{sep}$
- find q_g such as $wPR - p_{sep} = 0$

find Δp chone such as $q_g = 2 \cdot 10^6 \text{ Sm}^3/\text{d}$

Day 4 :



Tubing flow Equation-Dry gas

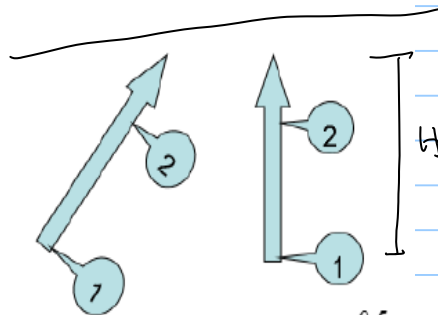
$$q_{sc} = \left(\frac{\pi}{4} \right) \left(\frac{R}{M_{air}} \right)^{0.5} \left(\frac{T_{sc}}{P_{sc}} \right) \left[\frac{D^5}{\gamma_g f_M Z_{av} T_{av} L} \right]^{0.5} \left(\frac{s e^s}{e^s - 1} \right)^{0.5} \left(\frac{p_1^2 - p_2^2}{e^s} \right)^{0.5}$$

$$Z_{av} = f(p_{av}, T_{av})$$

$$C_T = f(p_{av})$$

$$\frac{s}{2} = \frac{M_g g}{Z_{av} R T_{av}} H = \frac{(28.97) \gamma_g g}{Z_{av} R T_{av}} H$$

$$q_{gsc} = C_T \left(\frac{p_1^2}{e^s} - p_2^2 \right)^{0.5}$$



$$z_2 - z_1 = H$$

$$H=0$$

$$p_{inlet} = p_1 = e^{s/2} \left(p_2^2 + \frac{q_g^2}{C_T^2} \right)^{0.5}$$

$$p_{wh} = p_2 = \left(\frac{p_1^2}{e^s} - \frac{q_g^2}{C_T^2} \right)^{0.5}$$

$$i \mid H$$

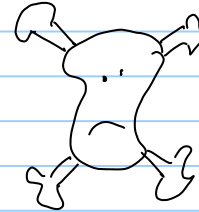
$$S=0$$

$$p_2^2 = \frac{p_1^2}{e^0} - \frac{q_g^2}{C_T^2}$$

$$C_T = \frac{\pi}{4} \left(\frac{R}{m_{air}} \right)^{0.5} \cdot \frac{T_s}{P_{sc}} \left[\frac{D^5}{\gamma_g f \bar{z} \bar{T} L} \right]^{0.5} \left[\frac{s \cdot e^s}{e^s - 1} \right]^{0.5}$$

$$\sqrt{p_1^2 - p_2^2} = q_g$$

$$\lim_{s \rightarrow 0} \left[\frac{s e^s}{e^s - 1} \right]^{0.5} = \frac{0}{0}$$



l'Hopital

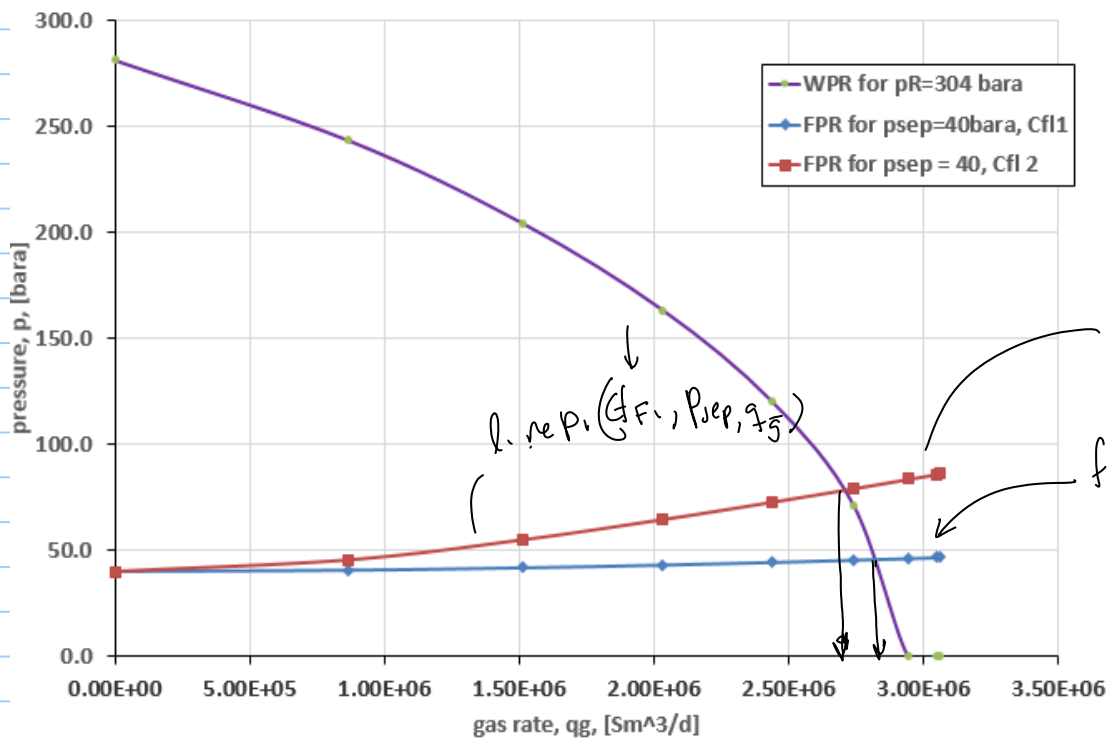
$$\lim_{s \rightarrow 0} \frac{\cancel{s}^1 \cancel{e^s}^1}{\cancel{e^s}^1 - 1^0} = 1$$

C_T for horizontal flowline is:

$$C_T = \frac{R}{q} \left(\frac{R}{m_{irr}} \right)^{0.5} \cdot \frac{T_s}{p_{sc}} \left[\frac{Q^5}{R_g \bar{z} \bar{T} L} \right]^{0.5}$$

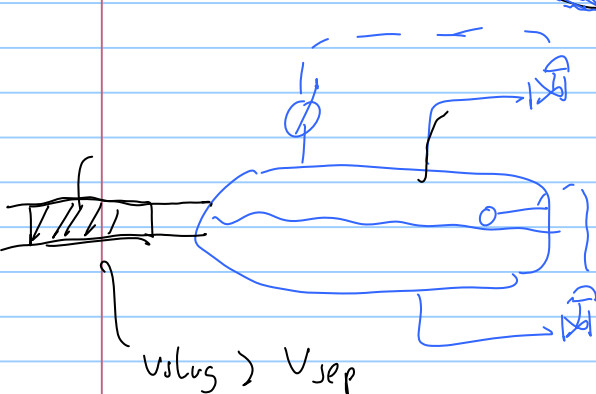
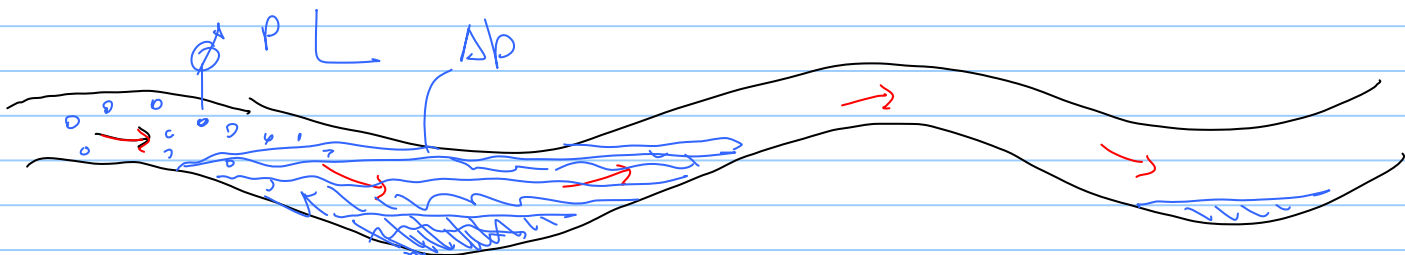
Class exercise \rightarrow flowline performance relationship
WPR vs FPR

	IPR	TPR			WPR	PPR for Cf1	PPR for Cf2
pwf_avail	qg	pwf_req	pwf_avail-pwf_req		pwh_avail	pwh_req	pwh_req
[bara]	[Sm ³ /d]	[bara]	[bara]		[bara]	[bara]	[bara]
304	0.00E+00	43.2	260.8		281.3	40.00	40.0
264	8.67E+05	48.5	215.5		243.5	40.60	45.5
224	1.51E+06	57.9	166.1		204.2	41.79	55.1
184	2.03E+06	67.4	116.6		163.4	43.18	64.7
144	2.44E+06	75.6	68.4		120.3	44.50	72.9
104	2.74E+06	82.0	22.0		71.5	45.61	79.3
64	2.94E+06	86.4	-22.4		#VALUE!	46.41	83.7
24	3.05E+06	88.7	-64.7		#VALUE!	46.84	86.0
0	3.06E+06	89.1	-89.1		#VALUE!	46.91	86.4



factors that affect the sizing of gas flowlines:

- Cost $\uparrow \Phi \uparrow$ Cost, \uparrow USD
- pressure drop \downarrow pressure drop $\uparrow \Phi$
- liquid transportation



- Corrosion
- Hydrates
- Slugging

intermittent production of batches of liquid and gas

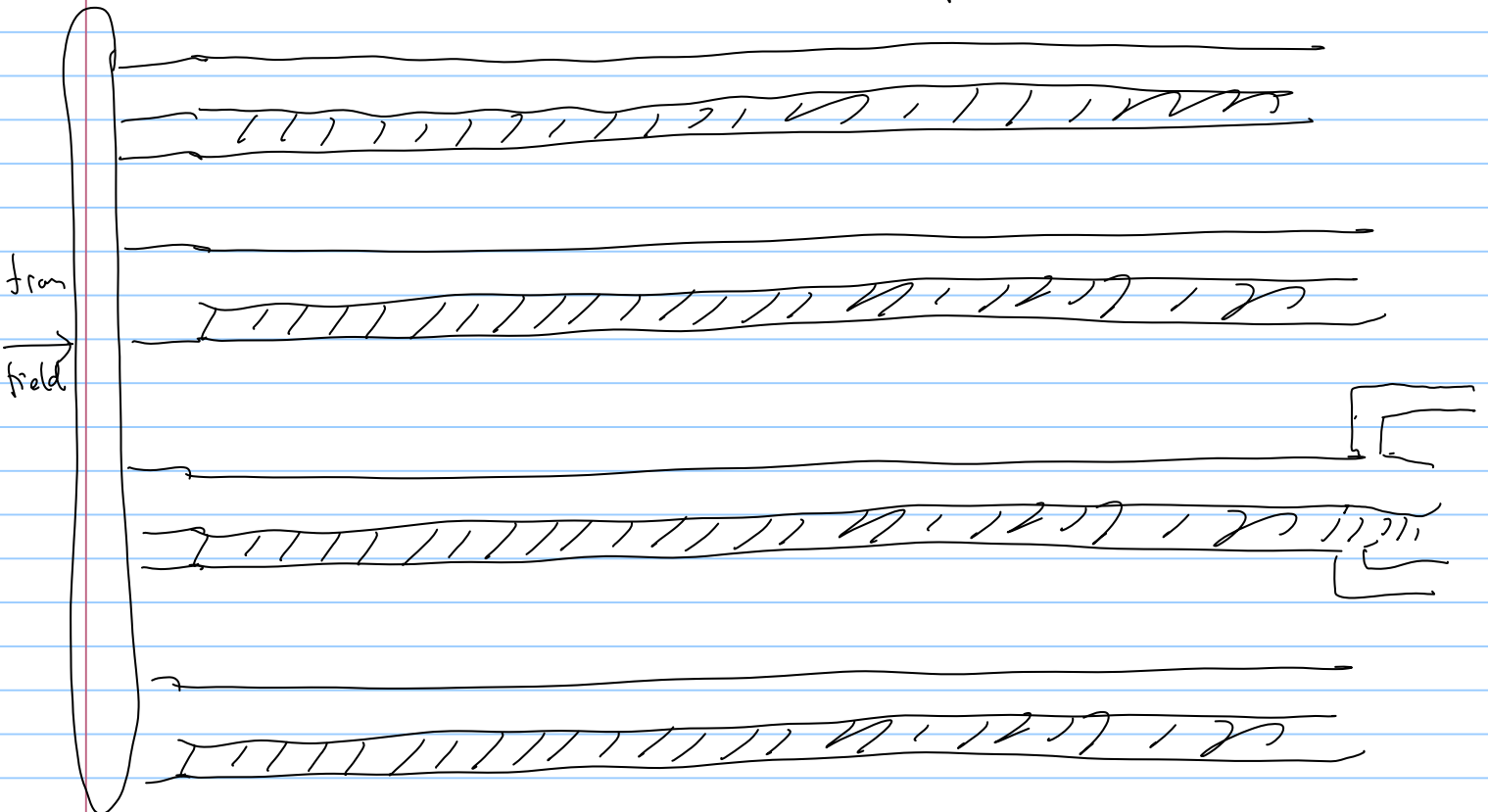
v_{gs} should be high enough to carry liquids and avoid accumulation.



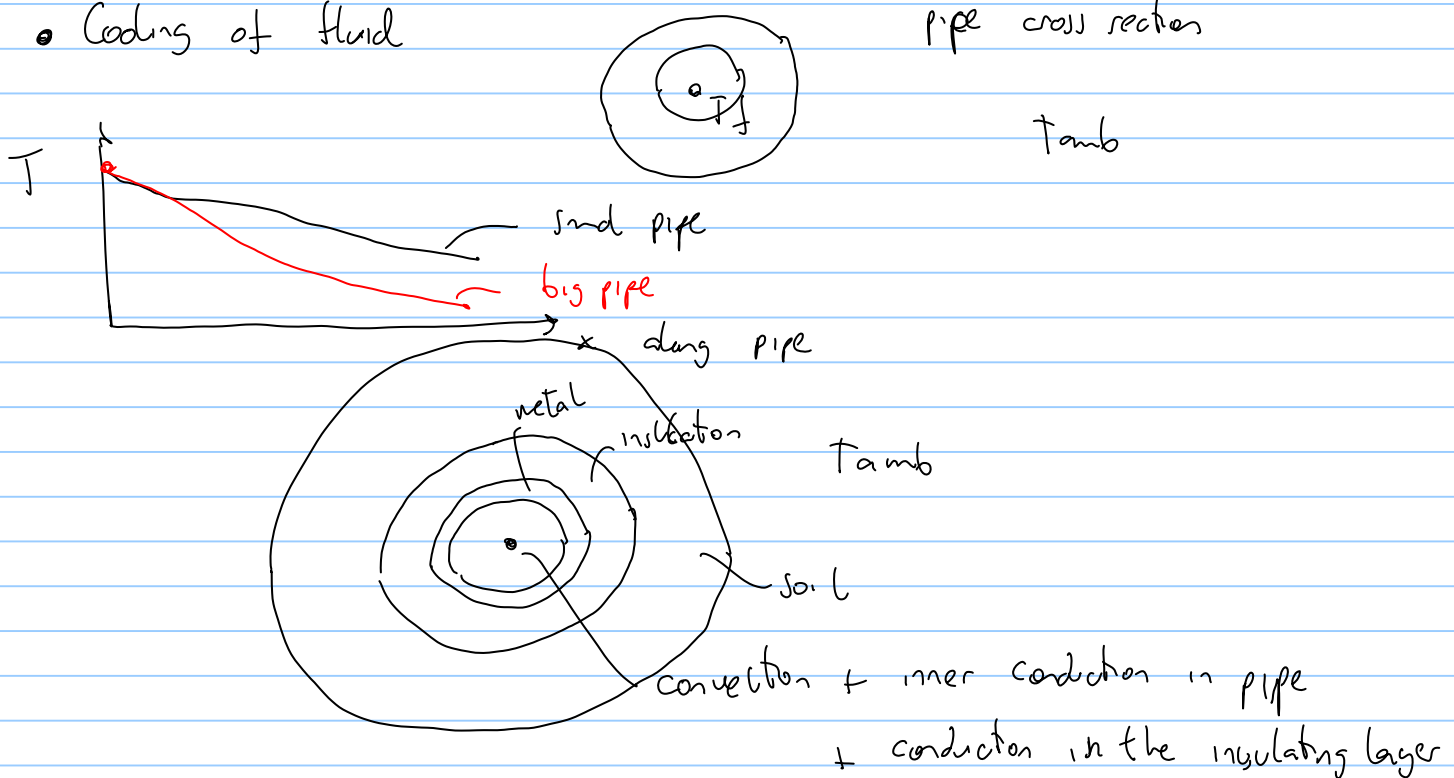
If slugging cannot be avoided

Snowwhite ~ subsea gas field
146 km from
cast
Block-2 Tanzania
Canal

slug catcher a multi pipe long separator



• Cooling of fluid



$$A \cdot U (T_{f, \text{ind}} - T_{\text{amb}}) = \dot{q}$$

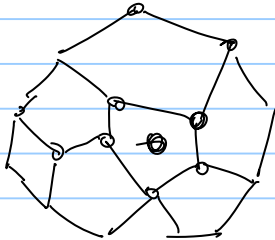
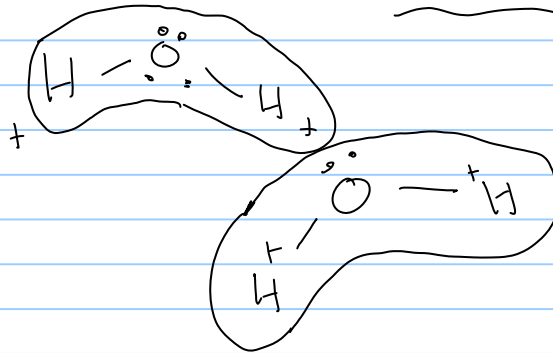
universal heat transfer coefficient
(includes conduction, convection, radiation)

lower temperature \rightarrow more condensation of liquid

Risk forming Hydrates

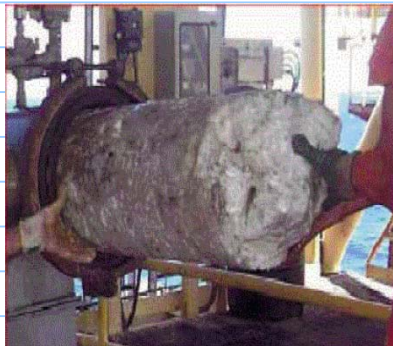
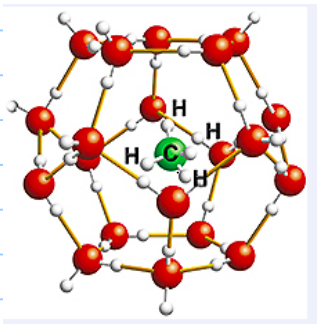
Short comment on Hydrates

at $p = 12 \text{ bara}$
 $T = 4^\circ \text{C}$



CH_4 CO_2
 C_2H_6 N_2
 C_3H_8
 C_4H_{10}

Ice with HC

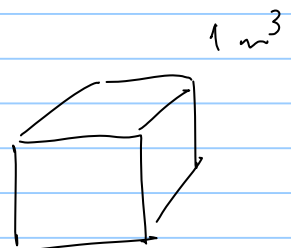


necessary conditions:

- low T
- liquid water (free water)
- small HC molecules

$$\underbrace{4 \text{ CH}_4}_{\sim} \underbrace{23 \text{ H}_2\text{O}}_{\sim} \sim \text{MW} = 478 \frac{\text{kg}}{\text{kmol}}$$

$$\rho \approx 900 \text{ kg/m}^3$$



How much methane in a 1 m^3 of hydrate methane clathrate

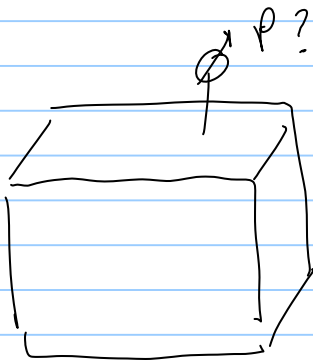
$$M_{\text{hyd}} = \rho \cdot V_{\text{ol}} = 900 \cdot 1 = 900 \text{ kg}$$

$$n = \frac{900 \text{ kg}}{478} = 1.88 \text{ kmol hydrate}$$

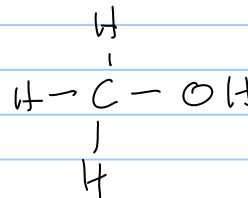
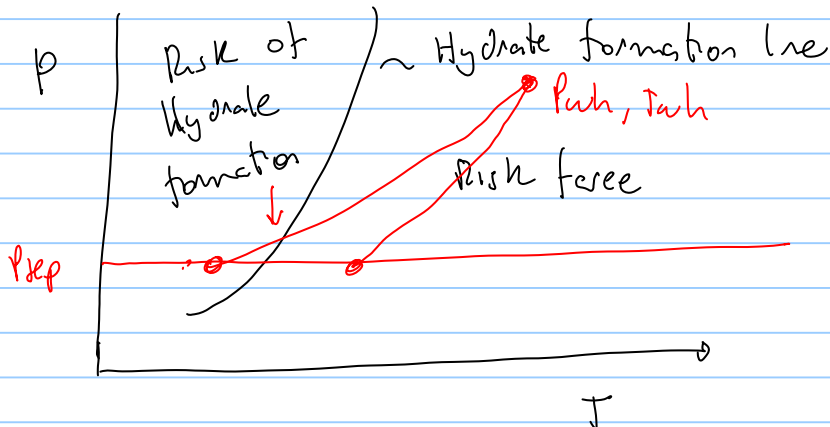
$$n_{\text{CH}_4} = 7.53 \text{ kmol}$$

$$V_{\text{sc}} = n \cdot 23.67$$

$$V_{\text{sc}} = 186 \text{ Sm}^3 \text{ of CH}_4$$



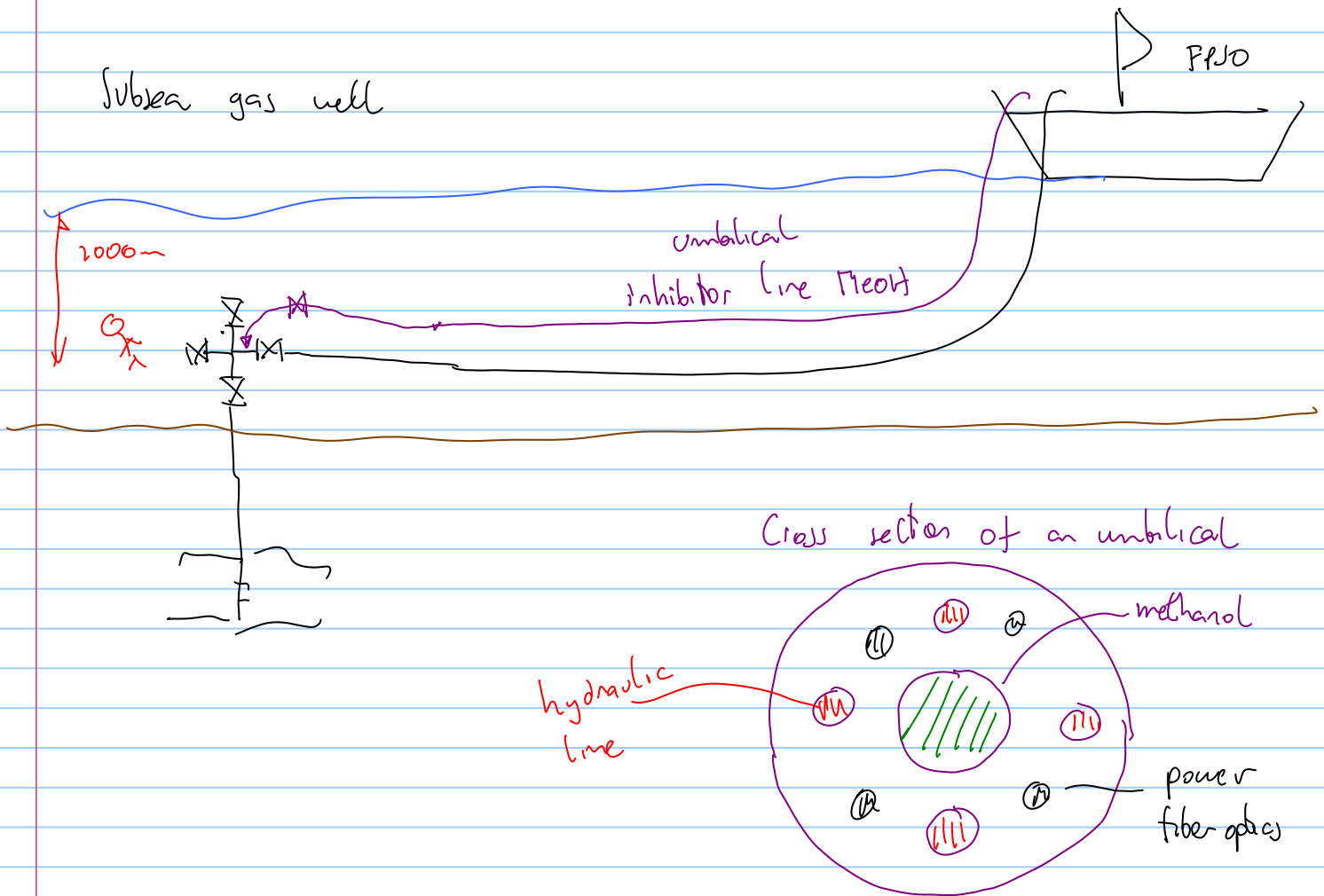
$$p = \frac{7.53 \cdot R \cdot T_{\text{sc}}}{1 \text{ m}^3} \approx 180 \text{ bara}$$



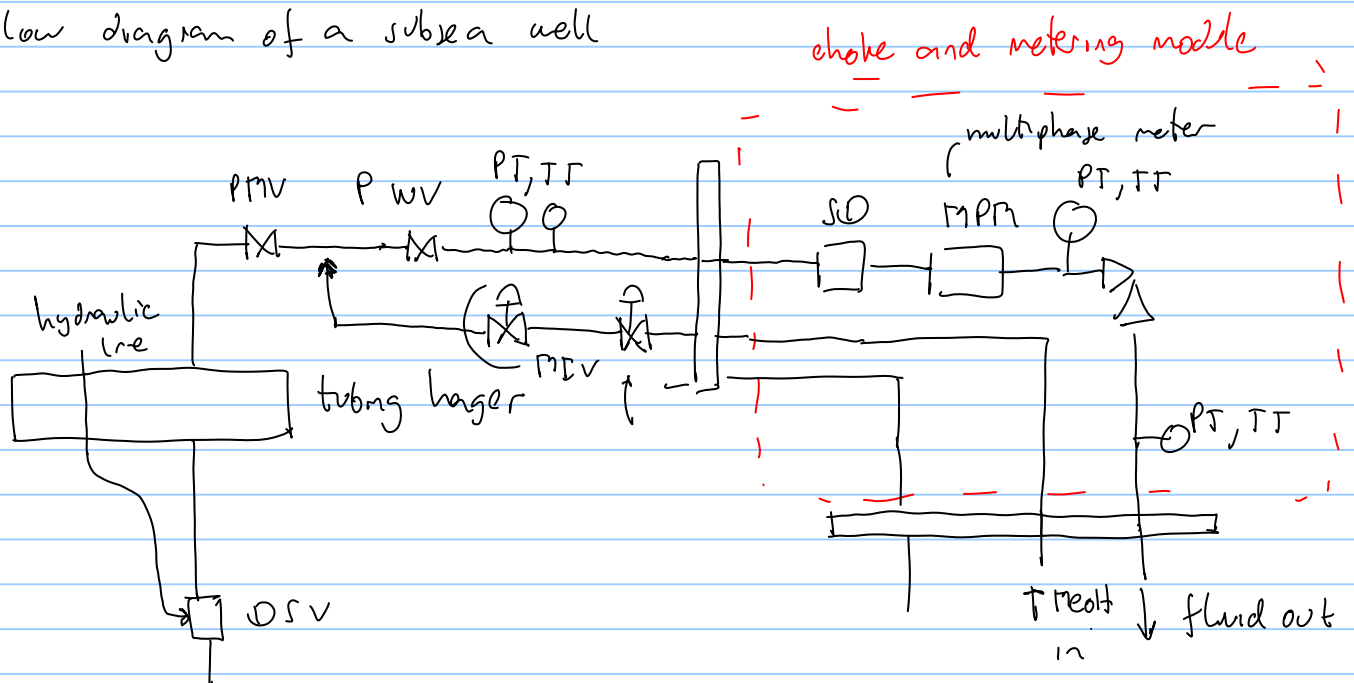
methanol
TEG
MEG

Using hydrate inhibitors to change the hydrate formation line





flow diagram of a subsea well



PwV production wing valve

PT pressure transducer

TT temperature transducer

SD .. Sand detection

PMV ... production master valve

MPV .. Methanol injection valve

- Better estimation of C_T

when using P_R as P_{in} to estimate P_{av} , $\frac{304+40}{2}$

we obtain $C_T = 42500 \text{ Sm}^3/\text{d}/\text{bar}$

$$S = 0.155$$

the equilibrium P_{wf} is actually 84.378 bar way different

than $P_R = 304 \text{ bar}$.

Using this $P_{av} = \frac{84.378 + 40}{2}$ we obtain $C_T = 41900 \text{ Sm}^3/\text{d}/\text{bar}$

$$S = 0.151$$

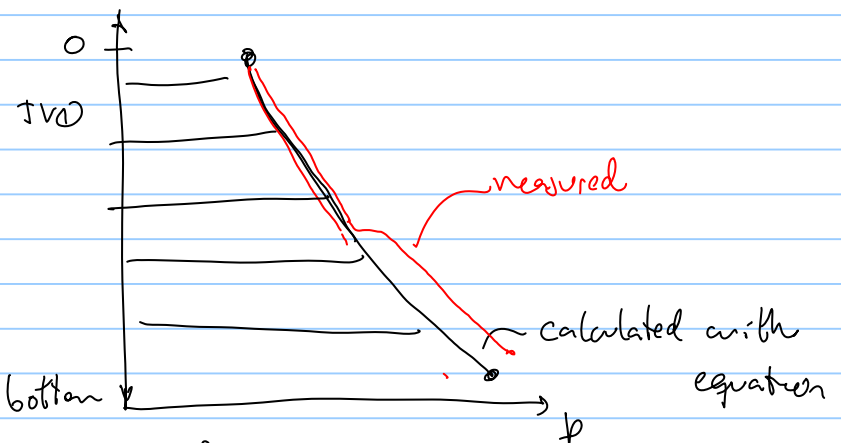
However the equilibrium point is not affected dramatically!

Pressure traverse
calculation

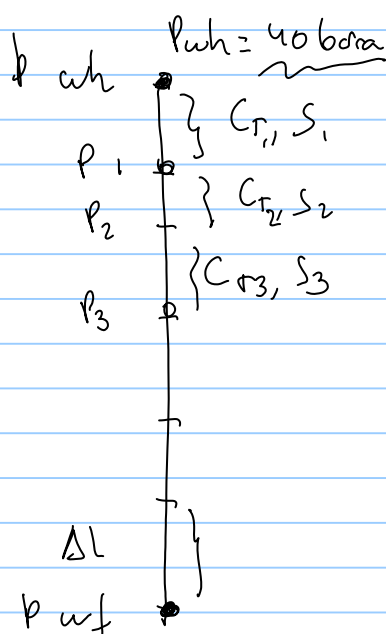
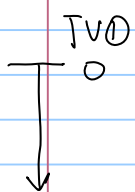
pressure distribution
along tubing

example .. traverse calculation

can help diagnose a well with scaling.



true vertical
depth



$$P_{wh} = 40 \text{ bar}$$

$$2.85 \text{ cm}^3/\text{d}$$

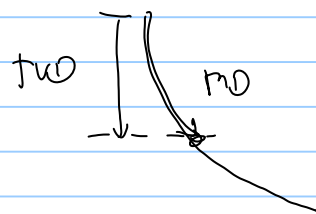
$$\left. \begin{matrix} P_1 \\ P_2 \end{matrix} \right\} C_{T1}, S_1$$

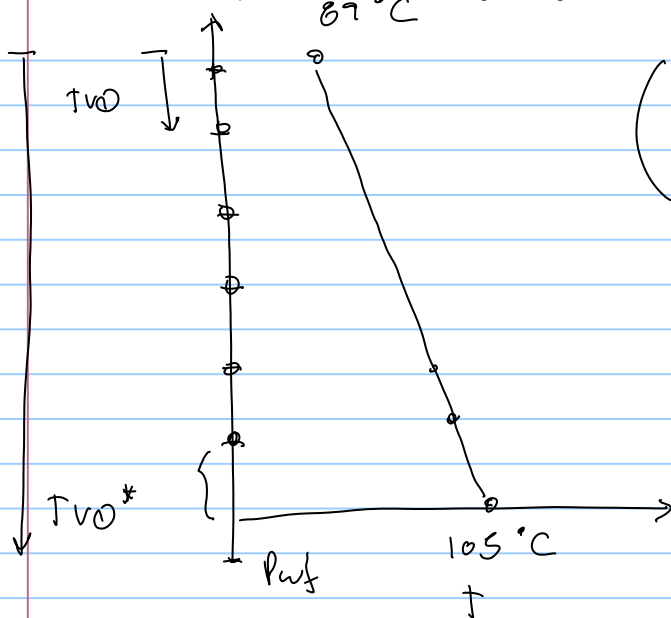
$$\left. \begin{matrix} P_2 \\ P_3 \end{matrix} \right\} C_{T2}, S_2$$

$$\left. \begin{matrix} P_3 \\ P_{wf} \end{matrix} \right\} C_{T3}, S_3$$

Calculate p distribution along tubing
for equilibrium rate

MO measured depth





$$\left(\frac{105^\circ\text{C} - 39}{TVD^* - 0} \right) = \left(\frac{105 - T}{TVD^* - TVD} \right)$$

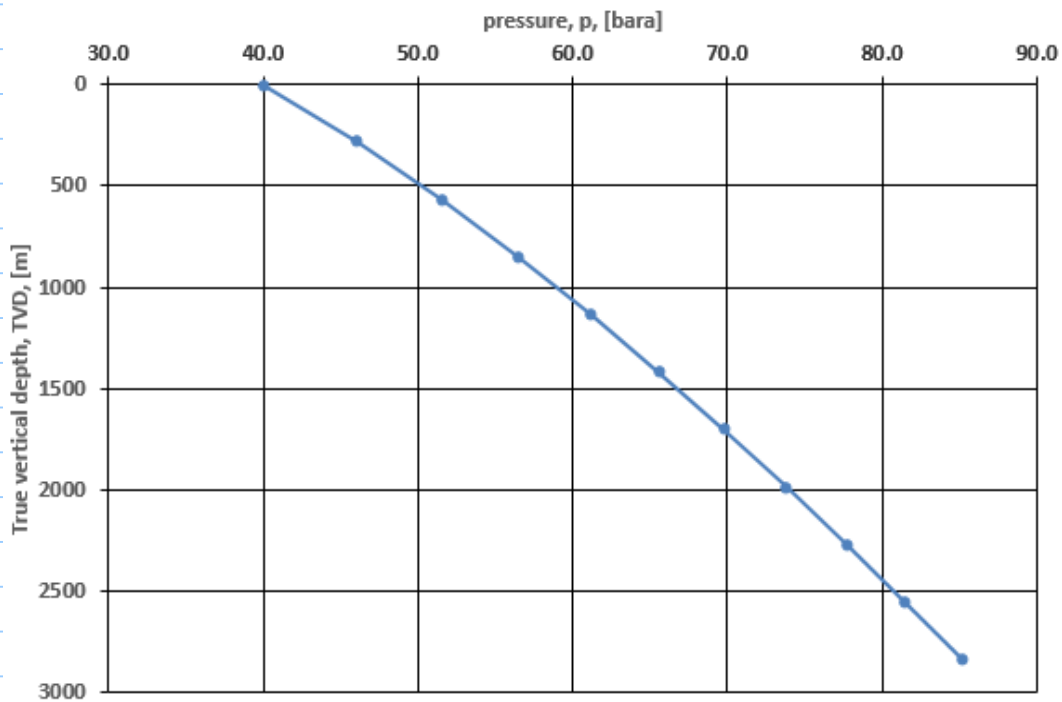
from equilibrium analysis $P_{wf_{eq}} = 84.378 \text{ bara}$

$P_{wh} = 40 \text{ bar}$

i assume $\Delta p = \text{const}$ for every section.

$$\Delta p = \frac{(P_{wf_{eq}} - P_{wh})}{N_{\text{section}}} = \frac{84.378 - 40}{10}$$

Tubing MD [m]	2837								
Tubing TVD [m]	2837								
Tubing ID [m]	0.157								
Tubing Cross section A [m ²]	0.019								
Wellhead pressure [bara]	40								
Gas gravity	0.55								
Twf [K]	378	104.85							
Twh [K]	360	86.85							
pR [bara]	304			DP	4.4378				
friction factor [-]	0.012								
qg [Sm ³ /d]	2.85E+06								
liquid density [kg/m ³]	8.97E+02								
TVD	T	Tav	passumed	Pav	Zav	S	Ct	p	
[m]	[K]	[K]	[bara]	[bara]	[-]	[-]	[Sm ³ /bar]	[bara]	
0	360.0	360.9	40.0	42.2	0.967	0.015	1.3E+05	40.0	
284	361.8	362.7	44.4	46.7	0.964	0.015	1.3E+05	46.0	
567	363.6	364.5	48.9	51.1	0.962	0.015	1.3E+05	51.5	
851	365.4	366.3	53.3	55.5	0.960	0.015	1.3E+05	56.5	
1135	367.2	368.1	57.8	60.0	0.959	0.015	1.3E+05	61.1	
1418	369.0	369.9	62.2	64.4	0.957	0.015	1.3E+05	65.6	
1702	370.8	371.7	66.6	68.8	0.956	0.015	1.3E+05	69.8	
1986	372.6	373.5	71.1	73.3	0.955	0.015	1.3E+05	73.8	
2269	374.4	375.3	75.5	77.7	0.954	0.015	1.3E+05	77.7	
2553	376.2	377.1	79.9	82.2	0.954	0.015	1.3E+05	81.4	
2837	378.0		84.4					85.1	



• Homework Solve this exercise by yourself at home

Find p vs TVD for the well studied in class ⚡
for $p_{wh} = 40 \text{ bara}$ and $q_g = 2.85 \text{ E6 Sm}^3/\text{d}$

Day 5:

- Class exercise. Calculate u_g vs TVD for the example we made yesterday

gas velocity

at p, T

TVD	T	p	z	B _g	q _g	u _g
[m]	[K]	[bara]	[-]			
0	360	40	z _{fact standing}	-		
⋮	⋮	⋮		-		
1	⋮	⋮		-		
⋮	⋮	⋮		-		
2834	300	85 bara		-		

$q_g \cdot B_g(p, T) = q_g$
 (at S.C)
 volume rate at local conditions

velocity of gas
 $u_g = \frac{q_g}{A} = \frac{q_g}{\pi \phi^2}$

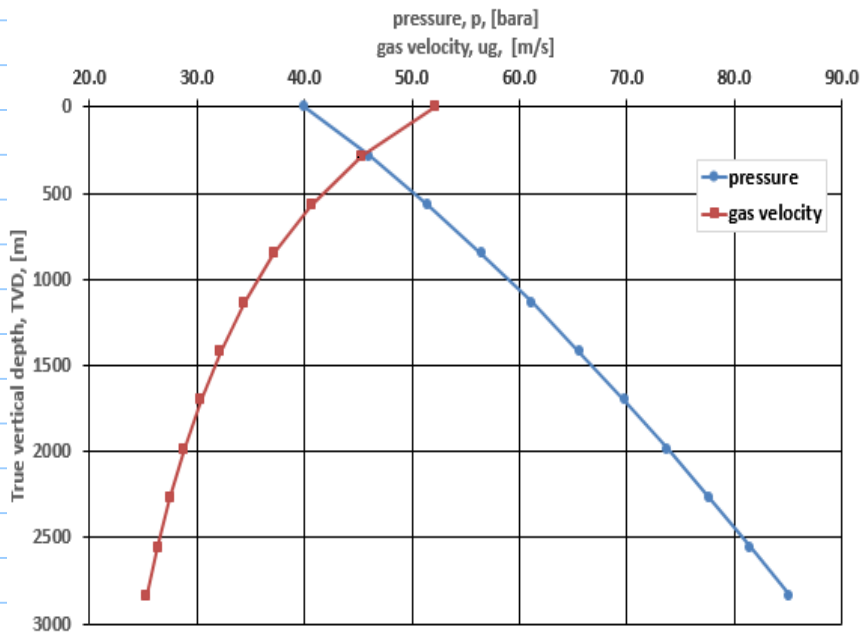
z

$\frac{m}{d} = \frac{24.3600 s}{\pi \phi^2}$

Tubing MD [m]	2837					
Tubing TVD [m]	2837					
Tubing ID [m]	0.157					
Tubing Cross section A [m ²]	0.019					
Wellhead pressure [bara]	40					
Gas gravity	0.55					
Twf [K]	378	104.85				
Twh [K]	360	86.85				
qg [Sm ³ /d]	2.85E+06					
liquid density [kg/m ³]	8.97E+02					
TVD	T	p	Z	B _g	qg _{local}	V _{local}
[m]	[K]	[bara]	[-]	[m ³ /Sm ³]	[m ³ /d]	[m/s]
0	360.0	40.0	0.9678	0.031	8.71E+04	52.1
284	361.8	46.0	0.9644	0.027	7.58E+04	45.3
567	363.6	51.5	0.9616	0.024	6.79E+04	40.6
851	365.4	56.5	0.9595	0.022	6.21E+04	37.1
1135	367.2	61.1	0.9577	0.020	5.75E+04	34.4
1418	369.0	65.6	0.9563	0.019	5.38E+04	32.2
1702	370.8	69.8	0.9552	0.018	5.08E+04	30.4
1986	372.6	73.8	0.9544	0.017	4.82E+04	28.8
2269	374.4	77.7	0.9537	0.016	4.60E+04	27.5
2553	376.2	81.4	0.9533	0.015	4.40E+04	26.3
2837	378.0	85.1	0.9531	0.015	4.23E+04	25.3

VBA func "z_{fact standing}"
 VBA func "B_g"
 $q_g \cdot B_g$
 $\frac{q_g}{A(24.3600)}$

checking if there is erosion in the tubing:

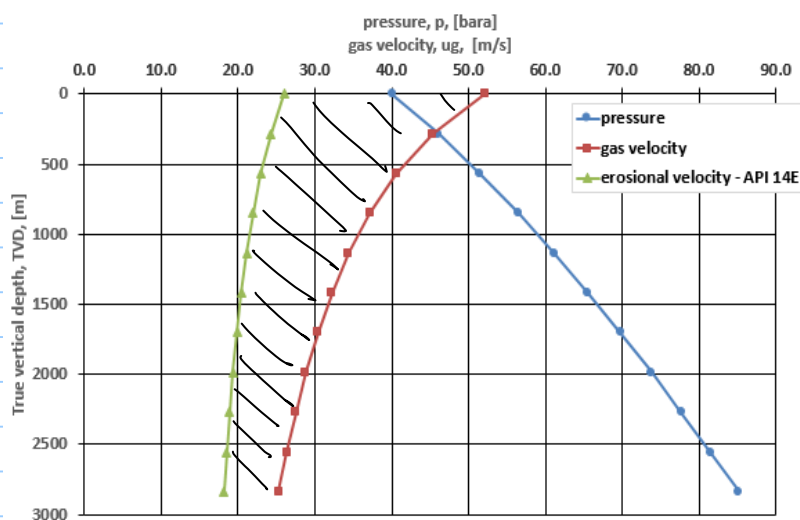


$$u_g \leq V_{\text{erosional}} = \frac{100}{\sqrt{\rho_g}}$$

↑
API

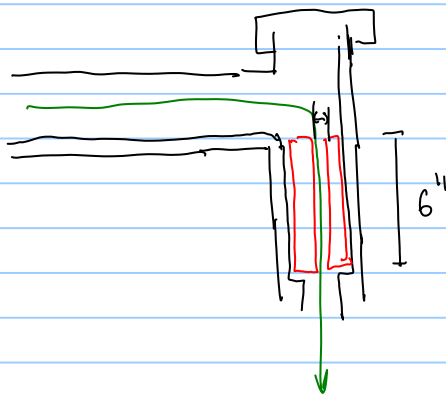
Tubing MD [m]	2837								
Tubing TVD [m]	2837								
Tubing ID [m]	0.157								
Tubing Cross section A [m ²]	0.019								
Wellhead pressure [bara]	40								
Gas gravity	0.55								
Twf [K]	378	104.85							
Twh [K]	360	86.85							
qg [Sm ³ /d]	2.85E+06								
liquid density [kg/m ³]	8.97E+02								
TVD	T	p	Z	Bg	qg_local	V_local	deng	Verosional	
[m]	[K]	[bara]	[-]	[m ³ /Sm ³]	[m ³ /d]	[m/s]	[kg/m ³]	[m/s]	
0	360.0	40.0	0.9678	0.031	8.71E+04	52.1	22.0	26.01	
284	361.8	46.0	0.9644	0.027	7.58E+04	45.3	25.3	24.26	
567	363.6	51.5	0.9616	0.024	6.79E+04	40.6	28.2	22.97	
851	365.4	56.5	0.9595	0.022	6.21E+04	37.1	30.9	21.96	
1135	367.2	61.1	0.9577	0.020	5.75E+04	34.4	33.3	21.13	
1418	369.0	65.6	0.9563	0.019	5.38E+04	32.2	35.6	20.45	
1702	370.8	69.8	0.9552	0.018	5.08E+04	30.4	37.7	19.86	
1986	372.6	73.8	0.9544	0.017	4.82E+04	28.8	39.8	19.35	
2269	374.4	77.7	0.9537	0.016	4.60E+04	27.5	41.7	18.89	
2553	376.2	81.4	0.9533	0.015	4.40E+04	26.3	43.5	18.49	
2837	378.0	85.1	0.9531	0.015	4.23E+04	25.3	45.3	18.13	

VBA funct deng (p , T , Z , B_g)
VBA function Verosional (ρ_g)

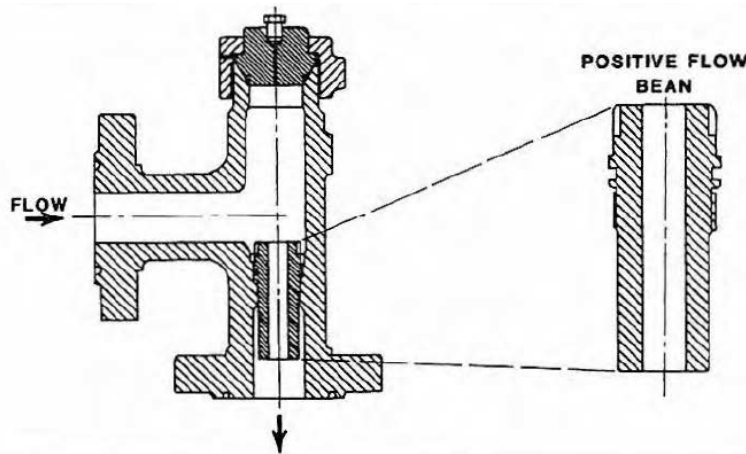
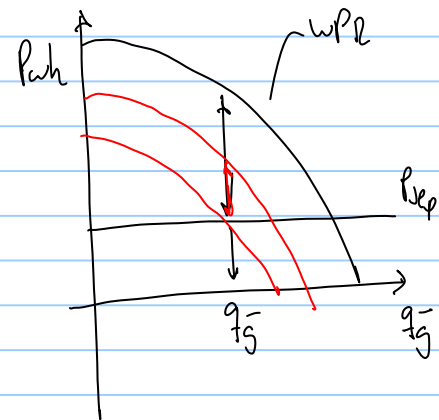


Production choke types $\left\{ \begin{array}{l} \text{Positive / bean choke} \\ \text{Adjustable} \end{array} \right.$

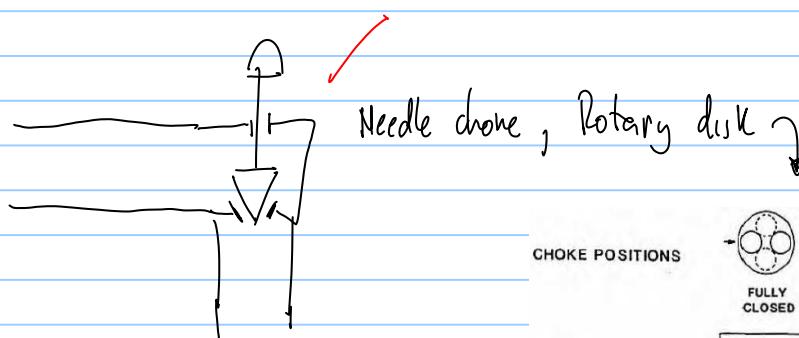
Bean



orifice size
 $\frac{1}{8}'' \sim 2''$
 Steps $\frac{1}{64}''$
 $\frac{8}{64}'' \rightarrow \frac{256}{64}''$

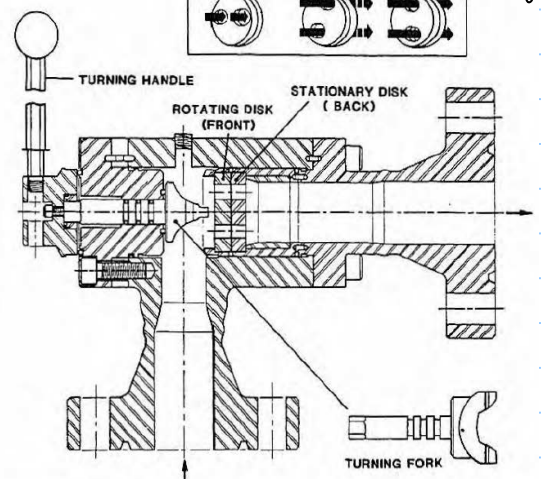
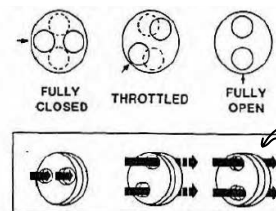


Adjustable



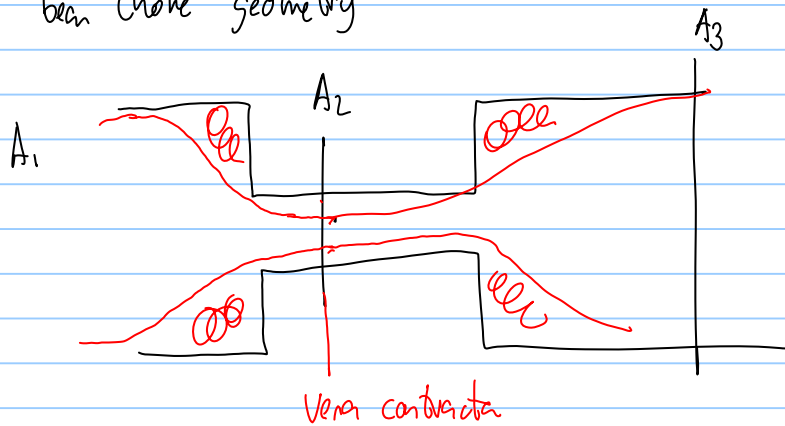
Needle choke, Rotary disk

CHOKE POSITIONS



Cage choke

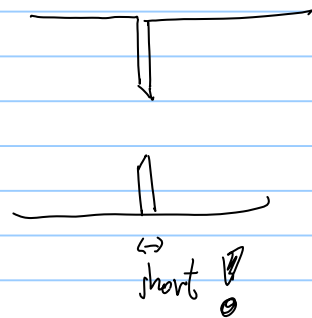
focusing on bean choke geometry



Different than

o

orifice for metering



main function of choke: dissipate a lot of energy in a concentrated place $\downarrow p$

control production

undesired consequences

p drops \rightarrow in oil you get gas vaporization

\rightarrow if gas you get liquid condensation

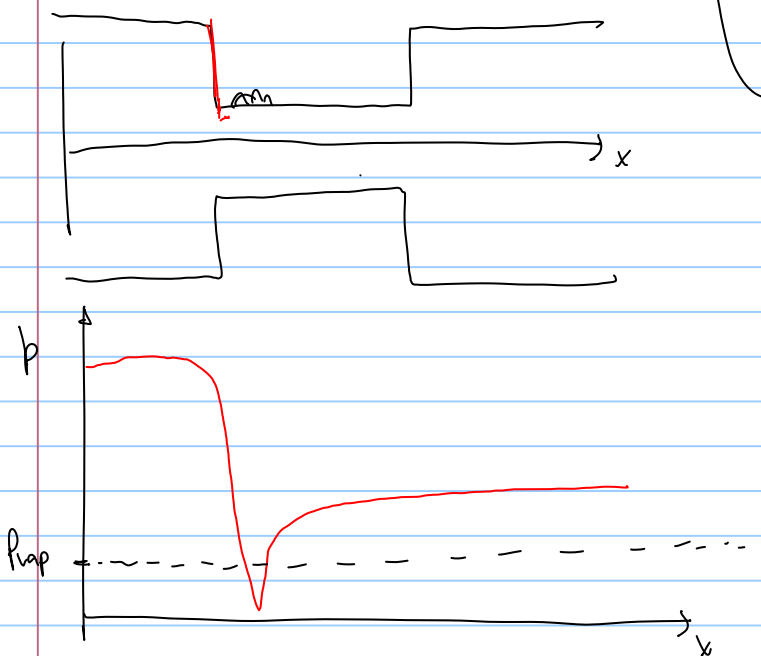
erosion \rightarrow due to high velocities in chokes
sand - worsens the problem.

cooling \rightarrow for gas Joule-Thomson effect

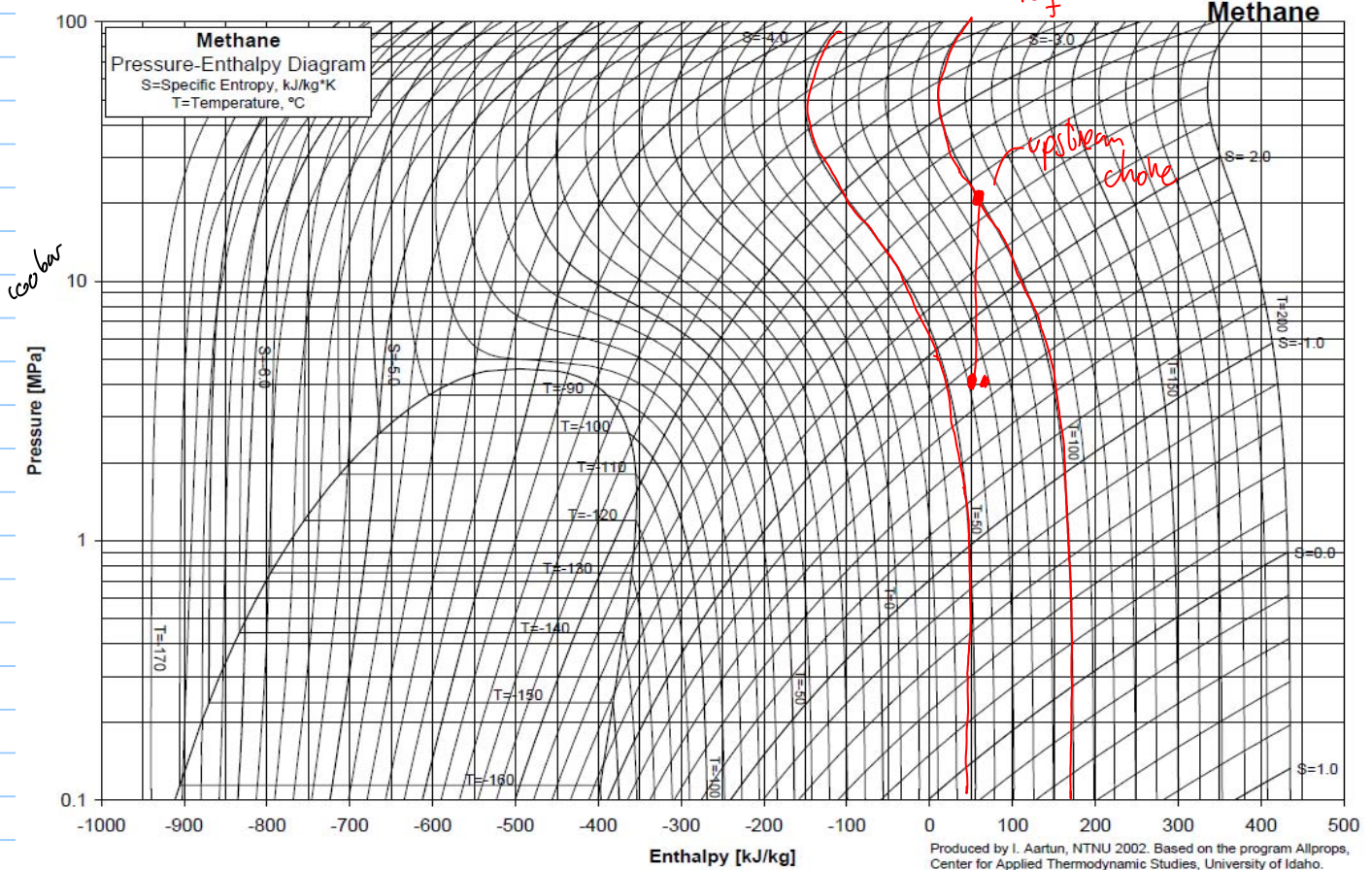
\rightarrow more liquid condensation

\rightarrow free water + low $T \rightarrow$ ice block!

cavitation.



p vs. h diagram



the process in the choke is isenthalpic (no \dot{q} or \dot{w} transferred out of the control volume)

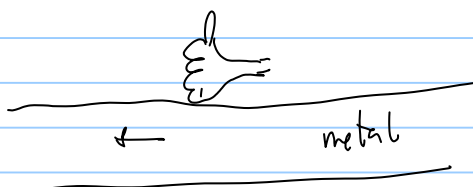
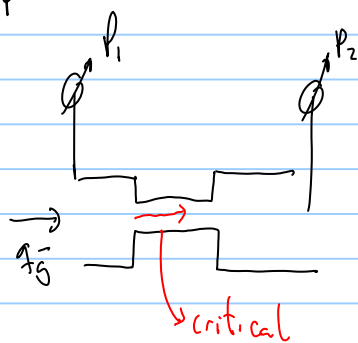
1st law of thermodynamics for open systems

$$\dot{m}(h_2 - h_1) = \dot{q} - \dot{w}$$

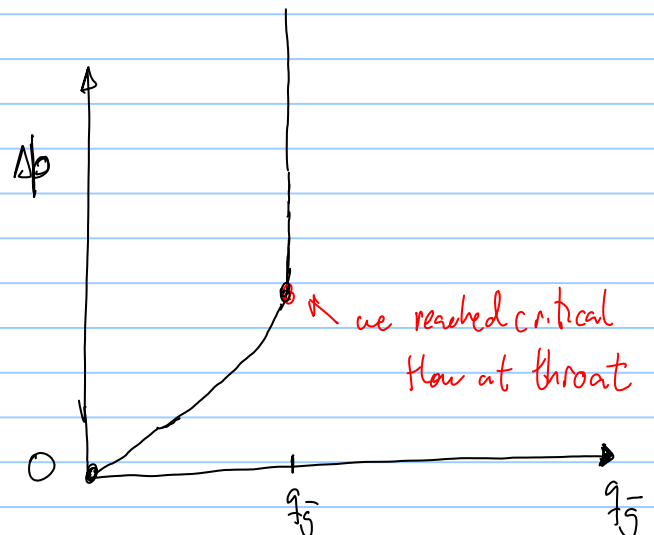
0 0

$h_2 = h_1$!

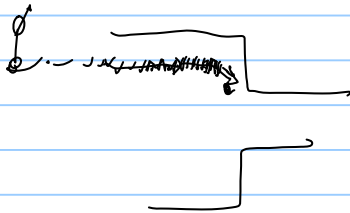
Experiment



P_1 is fixed



velocity of sound in a real gas $a = \sqrt{\frac{\gamma R T}{M_w}}$



$$\kappa = \frac{C_p}{C_v}$$

$$\kappa = 1.6$$

$$R = 8314.3$$

$$T = 80 + 273.15$$

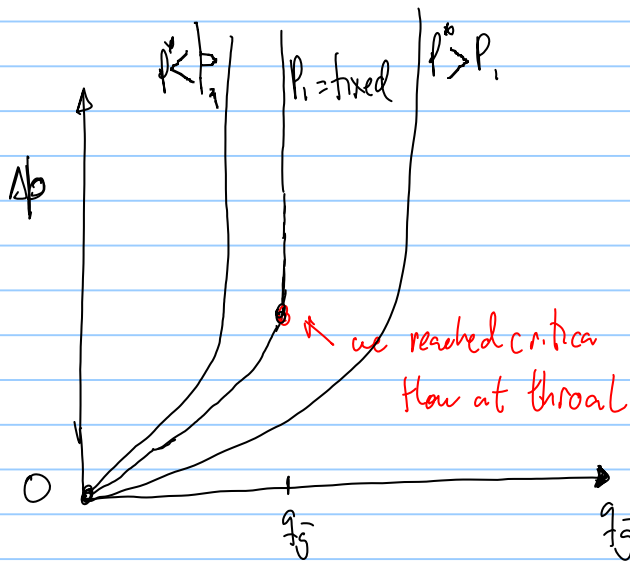
$$M = 1.6$$

as a rule of thumb critical pressure ratio 1)

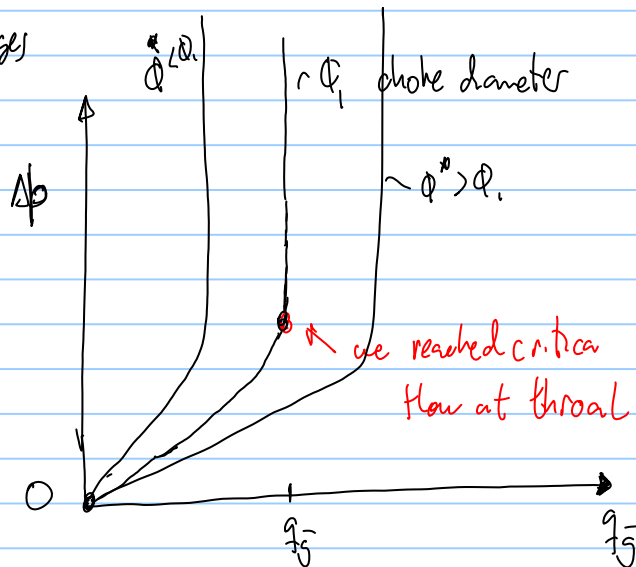
$$a \approx 542 \text{ m/s}$$

$$\frac{P_2}{P_1} \approx 0.5 \quad \checkmark$$

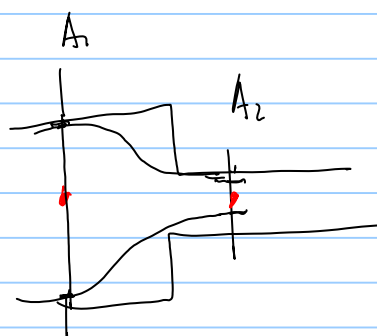
$$\text{in gas} \quad \left(\frac{P_2}{P_1} \right)_c = \left(\frac{2}{\kappa + 1} \right)^{\frac{\kappa}{\kappa - 1}}$$



if the orifice changes



Choke equation for liquid:



$$dp + \frac{\rho}{2} u du + f \frac{L}{D} \frac{u^2}{2} = 0$$

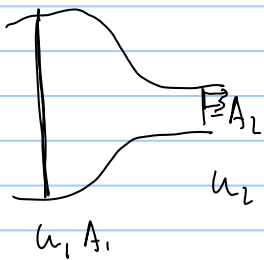
distance short no frictional losses

$$\int_1^2 \frac{dp}{\rho} + \int_1^2 u du = 0$$

$$p_2 - p_1 = \rho_L \cdot 0.5 \cdot (u_1^2 - u_2^2)$$

q_{sc} vs Δp

mass balance



$$u_2 \cdot A_2 = u_1 \cdot A_1$$

$$(u_1)^2 = \left(\frac{d_2^2}{d_1^2} u_2 \right)^2$$

$$\beta = \frac{d_2}{d_1}$$

$$A_1 = \frac{\pi \cdot d_1^2}{4}$$

$$A_2 = \frac{\pi \cdot d_2^2}{4}$$

$$u_1^2 = \beta^4 u_2^2$$

$$p_2 - p_1 = \rho_L \cdot 0.5 (\beta^4 u_2^2 - u_2^2)$$

$$\frac{\Delta p}{\rho_L (1 - \beta^4)} = u_2^2$$

$$u_2 = \sqrt{\frac{\Delta p \cdot 2}{\rho_L (1 - \beta^4)}}$$

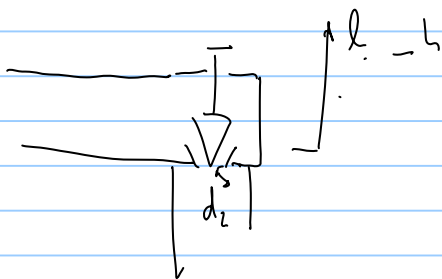
$$q_{sc} = u_2 \cdot A_2 \cdot C_d$$

↑ flow coefficient

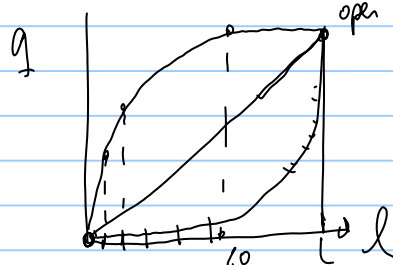
to account for
vena contraction

$$q = A_2 C_d \sqrt{\frac{\Delta p \cdot 2}{\rho_L (1 - \beta^4)}}$$

in adjustable chokes d_2 is changing



$$q = f(l) \cdot C_d \sqrt{\frac{\Delta p \cdot 2}{\rho}}$$



if Δp is fixed

Derivation of chone equation for gas \rightarrow orifice

$$\int_1^2 \frac{dp}{\rho} + \int_1^2 u du$$

$$\int_1^2 \frac{dp}{\rho} = 0.5 (u_1^2 - u_2^2)$$

$$u_2 \gg u_1 \quad u_2^2 \gg u_1^2$$

$$\rho = \frac{p M_w}{R T}$$

$$u_2^2 - u_1^2 \approx u_2^2$$

$$p v^n = \text{constant}$$

$$n = \kappa$$

adiabatic:

$$p \frac{1}{\rho^\kappa} = \text{constant} = C$$

$$\rho = \frac{p^{1/\kappa}}{C}$$

$$C^{1/\kappa} \int_1^2 \frac{dp}{p^{1/\kappa}} = C^{1/\kappa} \left(\frac{\kappa}{\kappa-1} \right) \left(p_2^{\frac{\kappa-1}{\kappa}} - p_1^{\frac{\kappa-1}{\kappa}} \right)$$

$$C^{1/\kappa} \frac{\kappa}{\kappa-1} p_1^{\frac{\kappa-1}{\kappa}} \left(\left(\frac{p_2}{p_1} \right)^{\frac{\kappa-1}{\kappa}} - 1 \right)$$

$$y = \frac{p_2}{p_1}$$

$$C^{1/\kappa} = \frac{p_1^{1/\kappa}}{\rho_1}$$

$$-u_2^2 = 2 \frac{p_1}{\rho_1} \frac{\kappa}{\kappa-1} \left(y^{\frac{\kappa-1}{\kappa}} - 1 \right) \quad \rho_1 = \frac{p_1 M_w}{R T_1}$$

$$u_2 = \sqrt{2 \frac{p_1}{\rho_1} \frac{R T_1}{M_w} \left(\frac{\kappa}{\kappa-1} \right) \left(y^{\frac{\kappa-1}{\kappa}} - 1 \right)}$$

$$u_2 = \frac{\dot{m}}{\rho_2 \cdot C_d A_2}$$

$$p \rho^\kappa = \text{Constant}$$

$$p_1 \rho_1^\kappa = p_2 \rho_2^\kappa$$

$$\rho_1 = \frac{p_1 M_w}{R T_1}$$

$$\rho_2 = \left(\frac{p_1}{p_2} \right)^{1/\kappa} \rho_1$$

$$\dot{m} = q_{sc} \cdot f_{sc}$$

$$\frac{p_{sc} M}{T_{sc} \sqrt{R}}$$

$$q_{sc} = c_d A_2 p_1 \left(\frac{T_{sc}}{p_{sc}} \right) \sqrt{2 \frac{R}{M_g Z_1 T_1}} \sqrt{\left(\frac{k}{k-1} \right) \left[(y)^{\frac{2}{k}} - (y)^{\frac{k+1}{k}} \right]}$$

$$A_2 = \frac{\pi d_2^2}{4}$$

if $y > y_c (\approx 0.5)$

then use this equation

if $y \leq y_c$

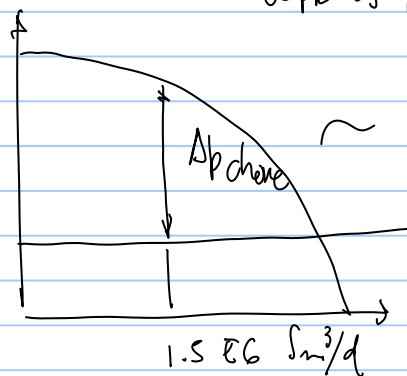
then use this equation with

$$y = y_c$$

Exercise

I performed

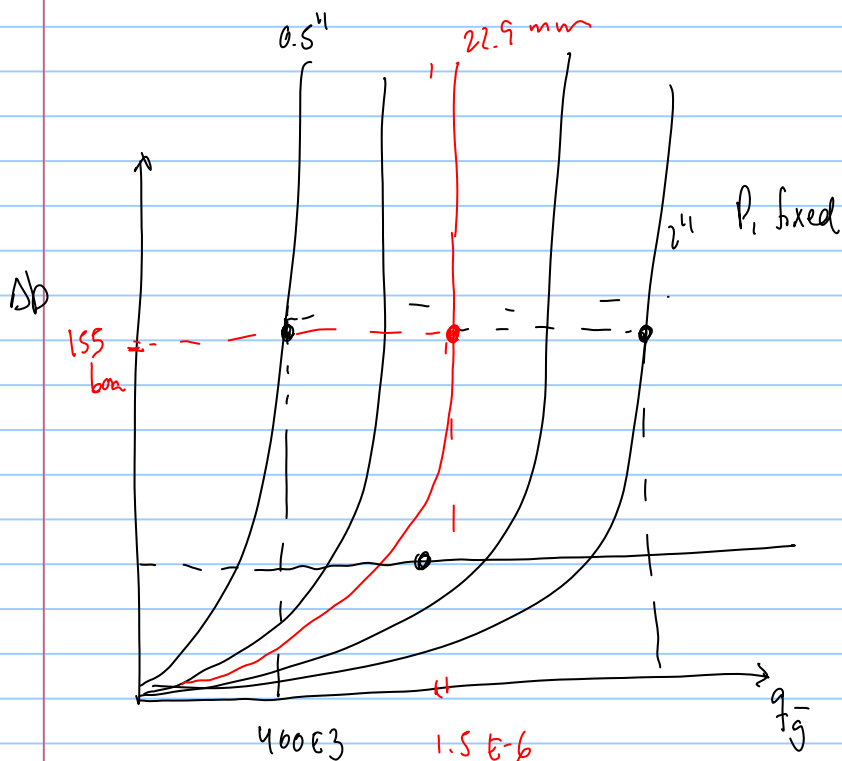
w PR vs P dep



$P_{h,avail} = 205 \text{ bara}$

$P_{h,reg} = 40 \text{ bara}$

$\Delta p = 155 \text{ bara}$



Excel Solver interface for the Choke Performance Equation - Dry Gas -Metric.

Set Objective: \$E\$16

To: ☐ Max ☐ Min ☒ Value Of: 1.5e6

By Changing Variable Cells: \$C\$13

Subject to the Constraints:

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Solve button is highlighted.

Choke Performance Equation - Dry Gas -Metric					
C _D Discharge Coefficient		0.865			
C _s		1.6259			
p _{sc} , Standard conditions pressure		101.325 kPa			
T _{sc} , Standard conditions temp		288.71 °K			
C _n		4.0075			
k, Adiabatic Constant		1.300			
Gas Gravity, Gamma		0.55	Air=1.0		
T _i , choke temperature		86.85 C		360 °K	
y _c Critical Ratio		0.546			
p ₂ , Downstream pressure		40 bara		4000 kPa	
d, ChokeDiameter		mm	12.5		
	p1	p1	y	Z1	q _{sc}
	bara	kPa	[]	[]	Sm ³ /d
	205.0	20500	0.195	0.9281	446.8E+3

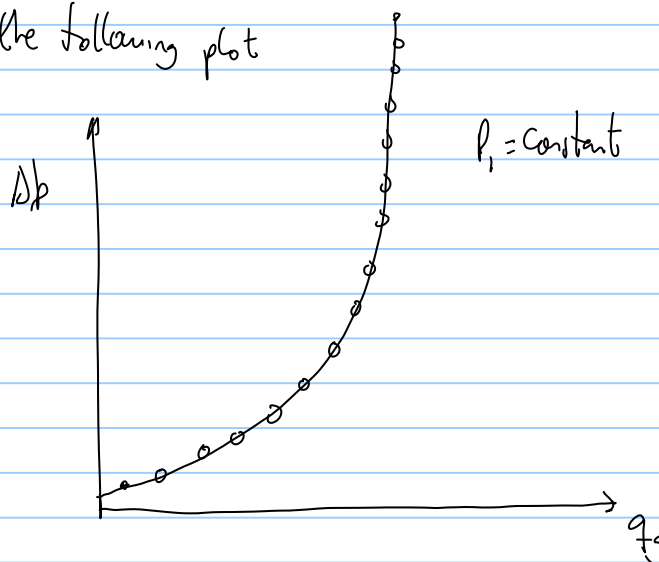
Home exercise

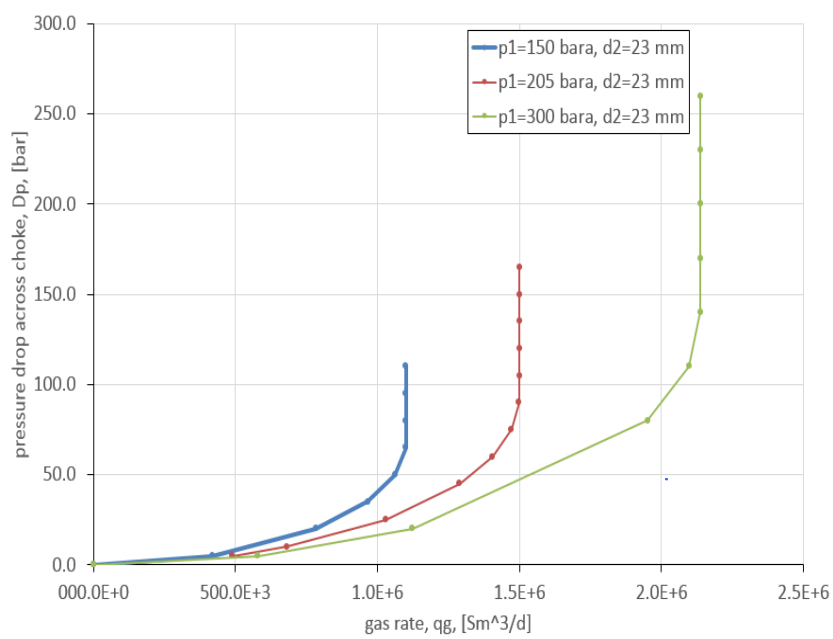
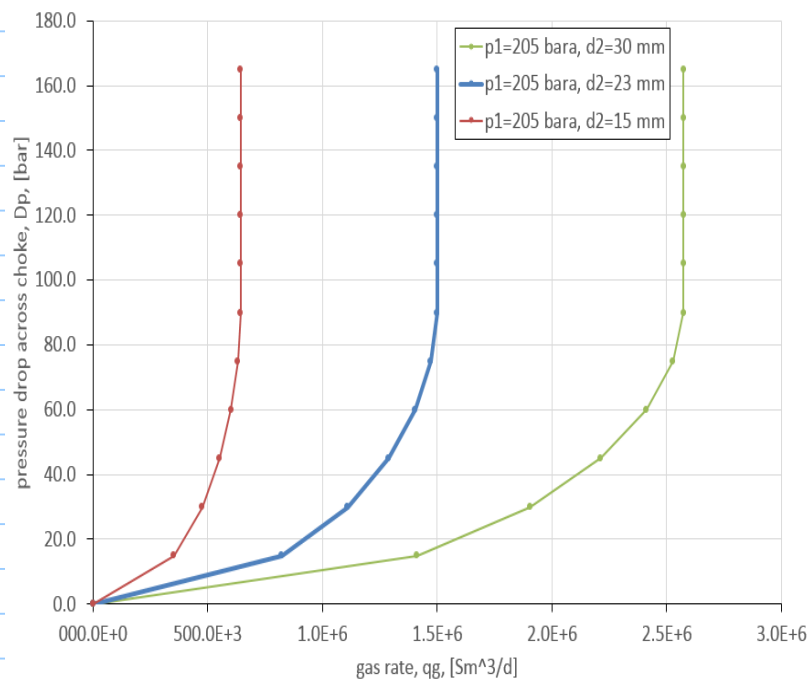
Using the same data shown before $P_1 = 205 \text{ bara}$

$P_2 = 40 \text{ bara}$

$d_2 = 22.9 \text{ mm}$

make the following plot

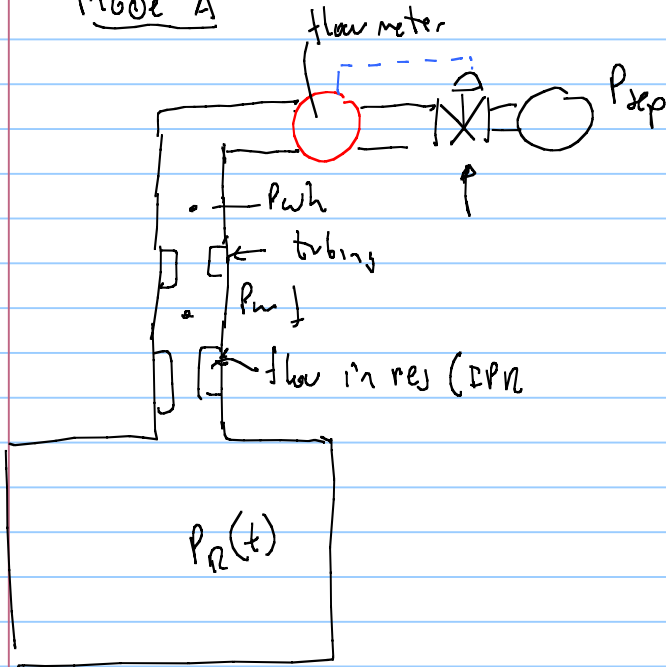




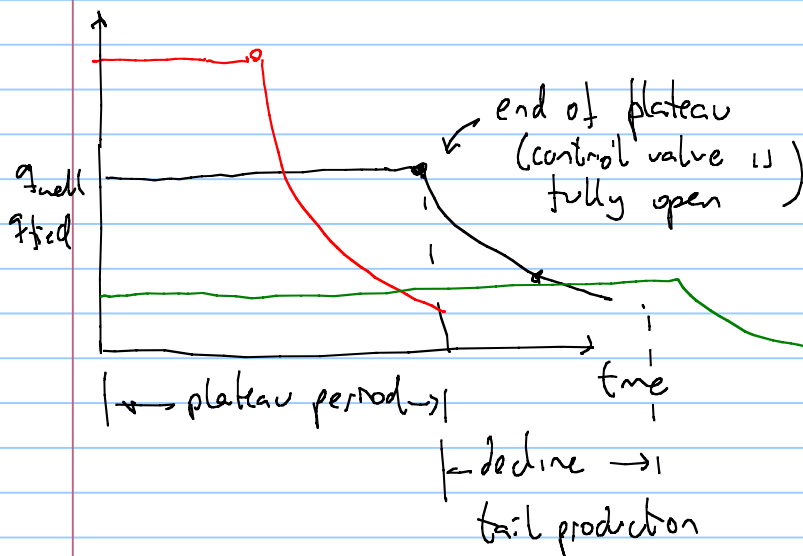
Week 2 - Day 6 :

Production modes of wells and field

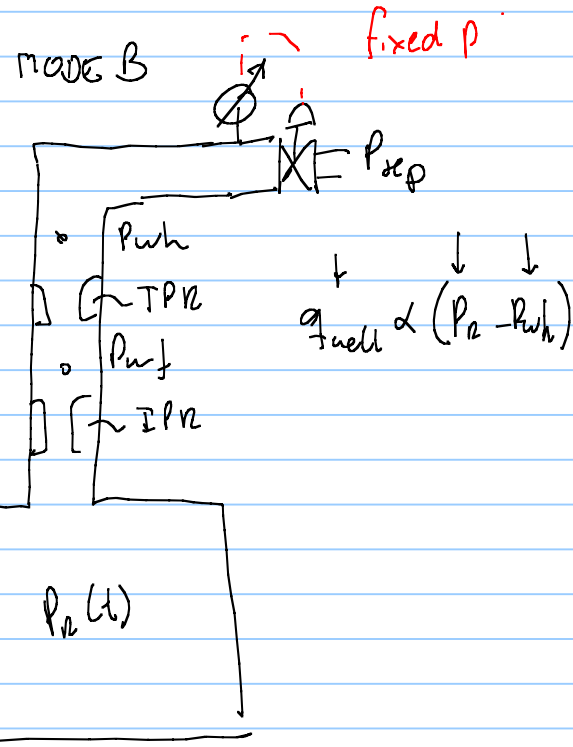
mode A



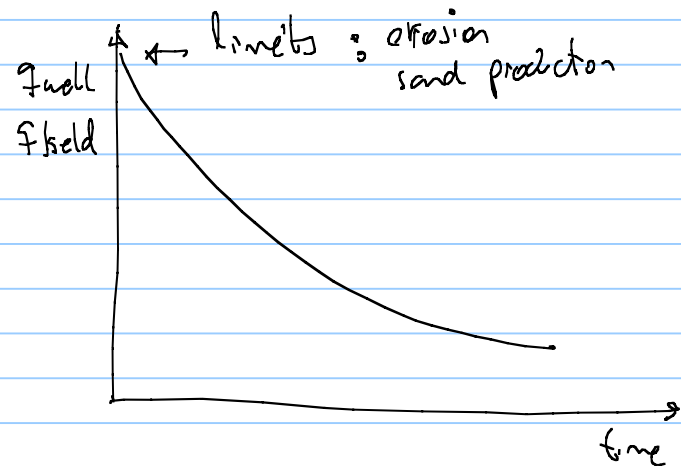
Constant production mode



- New development with standalone processing facilities
- Big size
- typically for gas (offshore), LNG predefined delivery contract
 ↓
 long term



constant pressure mode



- Produce as much as possible as early as possible
- Satellite fields (medium to small size)

the production is routed to existing processing facilities with spare capacity

Rule of thumb for defining plateau for oil reservoirs:

$$TRR = R.F. \cdot G$$

initial gas in place

R.F. = initial oil in place

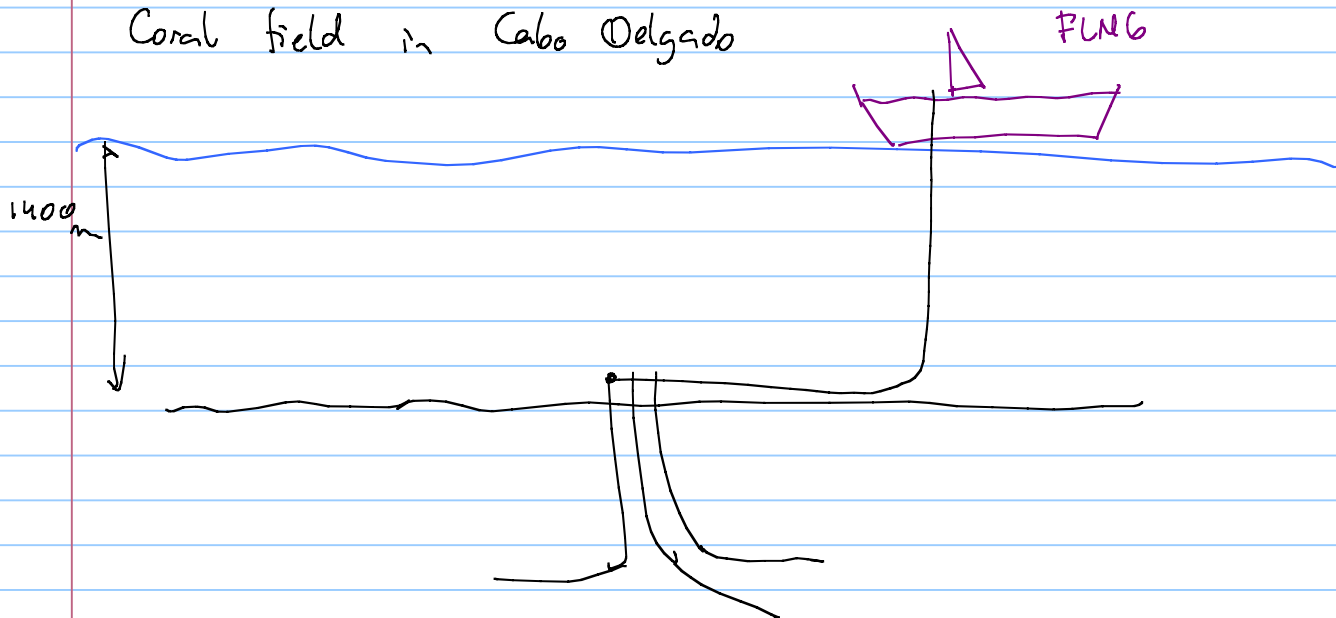
↑ recovery factor

$$q_{plateau} = \frac{0.1 \cdot TRR}{\text{Operational days per year}}$$

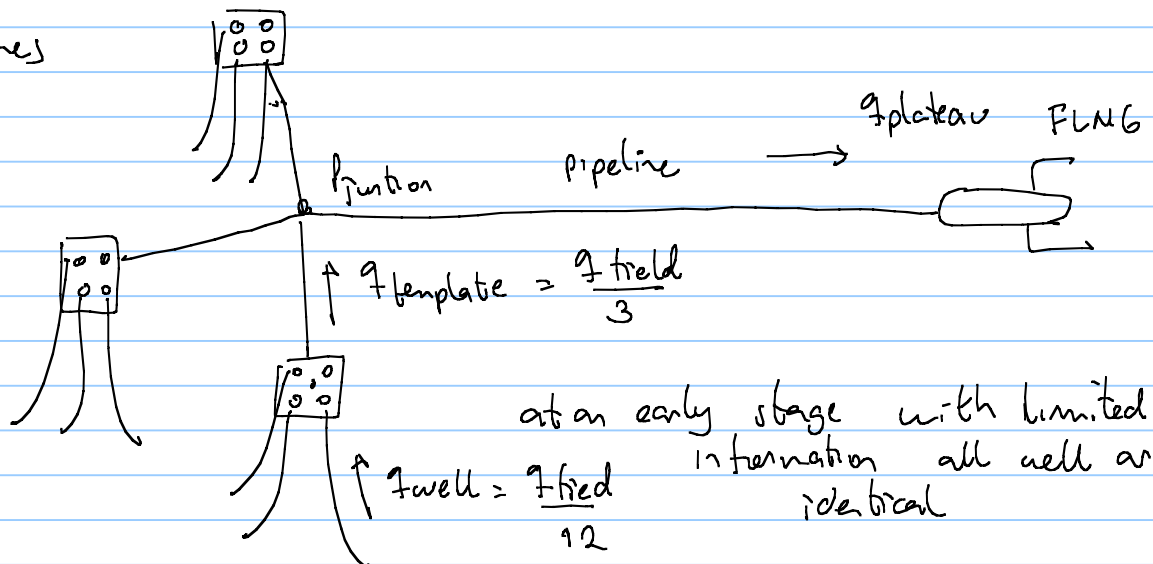
for gas

$$\text{yearly outtake} = (0.03 - 0.05) TRR$$

Coral field in Cabo Delgado

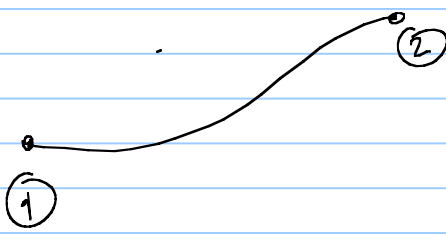


identical flowlines



at an early stage with limited information all wells are identical

o Pressure drop for liquids: undersaturated oil + water



$$z_1 + \cancel{\frac{1}{2} \frac{v_1^2}{g}} + \frac{p_1}{\rho g} = \frac{p_2}{\rho g} + \cancel{z_2 + \frac{1}{2} \frac{v_2^2}{g}} + f \frac{L}{D} \frac{v^2}{2g}$$

$$\text{ tubing } p_1 = f(p_2, q)$$

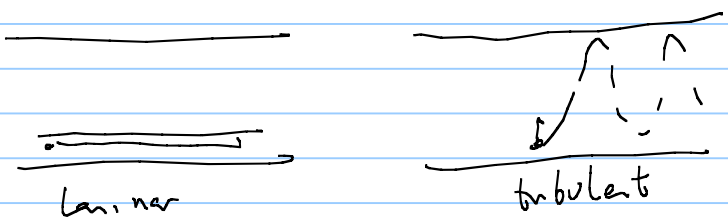
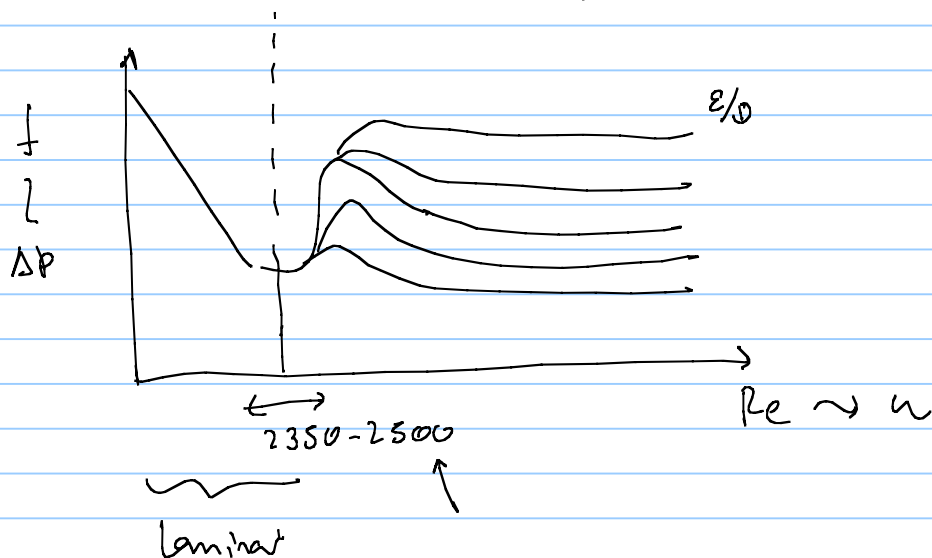
$$\text{ tubing } p_2 = f(p_1, q)$$

$$\text{ tubing } q = f(p_1, p_2)$$

$$q = V \cdot A = \frac{\pi \phi^2}{4} \cdot V$$

$$V = \left(\frac{q \cdot 4}{\pi \phi^2} \right)$$

$$V^2 = \frac{q^2 \cdot 16}{\pi^2 \phi^4}$$



$$f_{\text{laminar}} = \frac{64}{Re}$$

$$f_{\text{turbulent}} =$$

Colebrook-White equation

$$\text{implicit} \rightarrow \frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon}{3.7 D} + \frac{2.51}{Re \sqrt{f}} \right)$$

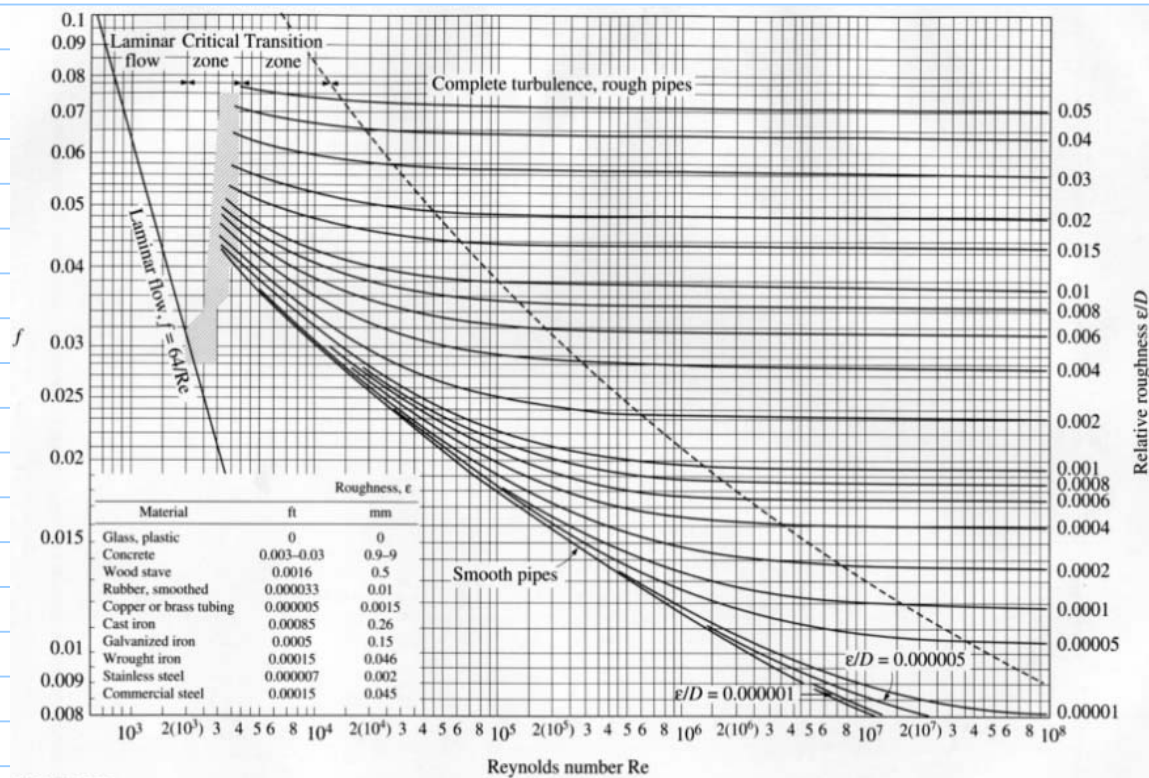


FIGURE A-27

The Moody chart for the friction factor for fully developed flow in circular tubes.

https://en.wikipedia.org/wiki/Darcy_friction_factor_formulae

all expressions for friction factor

Table of Colebrook equation approximations

Equation	Author	Year	Range	Ref
$f = 0.0055 \left[1 + \left(2 \times 10^4 \cdot \frac{\epsilon}{D} + \frac{10^6}{Re} \right)^{\frac{1}{4}} \right]$	Moody	1947	$Re = 4000 - 5 \cdot 10^8$ $\epsilon/D = 0 - 0.01$	
$f = 0.004 \left(\frac{\epsilon}{D} \right)^{0.3085} + 0.58 \left(\frac{\epsilon}{D} \right) + 88 \left(\frac{\epsilon}{D} \right)^{0.44} \cdot Re^{-0.9}$ where $\Psi = 1.62 \left(\frac{\epsilon}{D} \right)^{0.186}$	Wood	1966	$Re = 4000 - 5 \cdot 10^7$ $\epsilon/D = 0.00001 - 0.04$	
$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon/D}{3.715} + \frac{15}{Re} \right)$	Eck	1973		
$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon/D}{3.7} + \frac{5.74}{Re^{0.9}} \right)$	Swamee and Jain	1976	$Re = 5000 - 10^8$ $\epsilon/D = 0.000001 - 0.05$	
$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon/D}{3.71} + \left(\frac{7}{Re} \right)^{0.9} \right)$	Churchill	1973	Not specified	
$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon/D}{3.715} + \left(\frac{8.945}{Re} \right)^{0.9} \right)$	Jain	1976		
$f = 8 \left[\left(\frac{8}{Re} \right)^{12} + \frac{1}{(\Theta_1 + \Theta_2)^{12.8}} \right]^{\frac{1}{12}}$ where $\Theta_1 = \left[-2.457 \ln \left(\left(\frac{7}{Re} \right)^{0.9} + 0.27 \frac{\epsilon}{D} \right) \right]^{16}$ $\Theta_2 = \left(\frac{37530}{Re} \right)^{16}$	Churchill	1977		
$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{\epsilon/D}{3.7055} - \frac{5.0452}{Re} \log \left(\frac{1}{2.8257} \left(\frac{\epsilon}{D} \right)^{1.1089} + \frac{5.8606}{Re^{0.44492}} \right) \right]$	Chen	1979	$Re = 4000 - 4 \cdot 10^8$	
$\frac{1}{\sqrt{f}} = 1.8 \log \left[\frac{Re}{0.135 Re (\epsilon/D) + 6.5} \right]$	Round	1980		
$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon/D}{3.7} + \frac{4.518 \log \left(\frac{Re}{7} \right)}{Re \left(1 + \frac{Re^{0.09}}{29} (\epsilon/D)^{0.7} \right)} \right)$	Barr	1981		
$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{\epsilon/D}{3.7} - \frac{5.02}{Re} \log \left(\frac{\epsilon/D}{3.7} - \frac{5.02}{Re} \log \left(\frac{\epsilon/D}{3.7} + \frac{13}{Re} \right) \right) \right]$ or $\frac{1}{\sqrt{f}} = -2 \log \left[\frac{\epsilon/D}{3.7} - \frac{5.02}{Re} \log \left(\frac{\epsilon/D}{3.7} + \frac{13}{Re} \right) \right]$	Zigrang and Sylvester	1982		
$\frac{1}{\sqrt{f}} = -1.8 \log \left[\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \frac{6.9}{Re} \right]$	Haaland ^[2]	1983		
$\frac{1}{\sqrt{f}} = \Psi_1 - \frac{(\Psi_2 - \Psi_1)^2}{\Psi_2 - 2\Psi_1 + \Psi_1}$ or $\frac{1}{\sqrt{f}} = 4.781 - \frac{(\Psi_1 - 4.781)^2}{\Psi_2 - 2\Psi_1 + 4.781}$ where $\Psi_1 = -2 \log \left(\frac{\epsilon/D}{3.7} + \frac{12}{Re} \right)$	Bergides	1984		

$\Psi_1 = -2 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51 \Psi_2}{Re} \right)$ $\Psi_2 = -2 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51 \Psi_1}{Re} \right)$				
$A = 0.11 \left(\frac{68}{Re} + \epsilon \right)^{0.25}$ $\epsilon A \geq 0.018 \text{ then } f = A \text{ and } \epsilon A < 0.018 \text{ then } f = 0.0028 + 0.85A$	Tsai	1989		
$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon/D}{3.7} + \frac{95}{Re^{0.898}} - \frac{96.82}{Re} \right)$	Manandhi	1997	$Re = 4000 - 10^8$	$\epsilon/D = 0 - 0.06$
$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon/D}{3.7065} - \frac{5.0272}{Re} \log \left[\frac{\epsilon/D}{3.827} - \frac{4.657}{Re} \log \left(\left(\frac{\epsilon/D}{7.7918} \right)^{0.8941} + \left(\frac{5.3236}{208.816 + Re} \right)^{0.8941} \right) \right] \right)$	Monzon, Romeo, Ryo	2002		
$\frac{1}{\sqrt{f}} = 0.8686 \ln \left[\frac{0.4587 Re}{(S - 0.31)^{0.75-0.64S}} \right]$ <p>where</p> $S = 0.124 Re \frac{\epsilon}{D} + \ln(0.4587 Re)$	Goudar, Sonnad	2006		
$\frac{1}{\sqrt{f}} = 0.8686 \ln \left[\frac{0.4587 Re}{(S - 0.31)^{0.75-0.64S}} \right]$ <p>where</p> $S = 0.124 Re \frac{\epsilon}{D} + \ln(0.4587 Re)$	Vasanthan, Kouchakzadeh	2008		
$\frac{1}{\sqrt{f}} = \alpha - \frac{\alpha + 2 \log \left(\frac{Re}{B} \right)}{1 + \frac{Re}{B}}$ <p>where</p> $\alpha = \frac{0.744 \ln(Re) - 1.41}{1 + 1.32 \sqrt{\epsilon/D}}$ $B = \frac{\epsilon/D}{3.7} Re + 2.61 \alpha$	Buzzell	2008		
$f = \frac{6.4}{(\ln(Re) - \ln(1 + 0.01 Re \frac{\epsilon}{D} (1 + 10 \sqrt{\frac{\epsilon}{D}})))^{3.4}}$	Alici, Kargoz	2009		
$f = \frac{0.3479 - 0.0000947(7 - \log Re)^4}{\left(\log \left(\frac{\epsilon/D}{2.413} + \frac{1.888}{Re^{0.215}} \right) \right)^2}$	Elmaghrabi, Papadimitriou, Timopoulos	2010		
$f = 1.618 \left[\ln \left(0.234 e^{1.3007} - \frac{60.525}{Re^{1.1108}} + \frac{56.201}{Re^{0.0715}} \right) \right]^{-8}$	Fang	2011		
$f = \left[-2 \log \left(\frac{2.18 \beta}{Re} + \frac{\epsilon}{3.71} \right) \right]^{-8}, \beta = \ln \left(\frac{1.186}{\ln(1 + 1.186 \beta)} \right)$	Bric	2011		
$f = 1.325474505 \log_e (A - 0.8686068422 \beta \log_e (A - 0.8784893582 \beta \log_e (A + (1.6653680(35.2 \beta)^{1.8773492287})))^{-8}$ <p>where</p> $A = \frac{\epsilon/D}{3.7065}$ $B = \frac{2.5228}{Re}$	Alshar	2012		

exphat

Haaland

$$\frac{1}{\sqrt{f}} = -1.6 \log \left[\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \left(\frac{6.9}{Re} \right) \right]$$

Prof @ NTNU
~ 1983

$$f = \left[-1.6 \log \left[\left(\frac{\epsilon/D}{3.7} \right)^{1.11} + \left(\frac{6.9}{Re} \right) \right] \right]^{-2}$$

friction factor

Beware !

Darcy-Weisbach

Fanning (american)

$f = 4 \cdot f_{\text{FANNING}}$

$f_{\text{FANNING}} = \frac{\tau}{\frac{1}{2} \rho v^2}$

$f_{\text{Darcy}} = \frac{\tau}{\frac{1}{8} \rho v^2}$

Class exercise IPN-TPN equilibrium for undersaturated oil well (peregrino field, offshore Brazil).

[illegible]

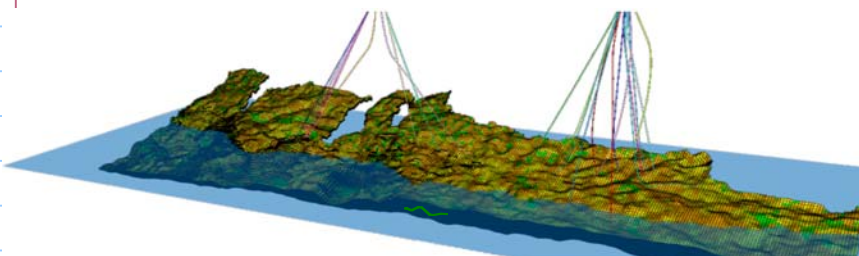
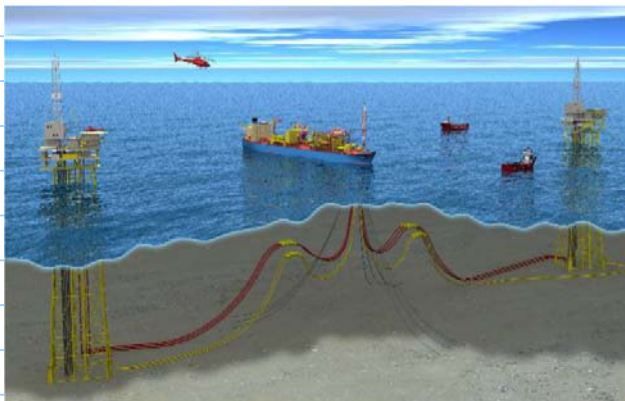
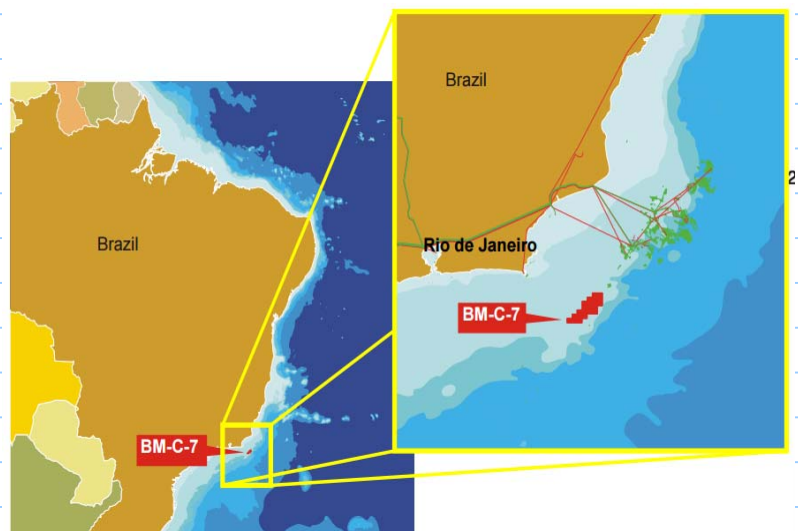
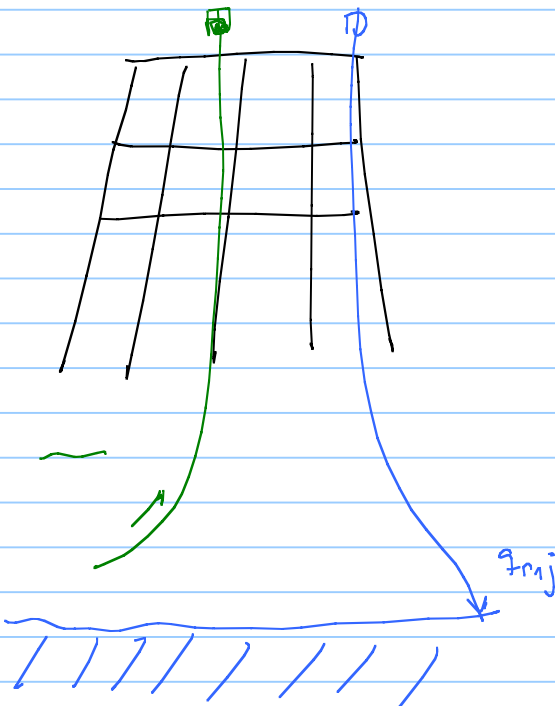
$$Q_o = J (P_e^{\leftarrow} - P_{wf})$$

~ pressure required at well bottom to flow against γ bore

Ø $P_{wh} = 7 \text{ bara}$

tubing P_1 ?

tubing P_2 ?

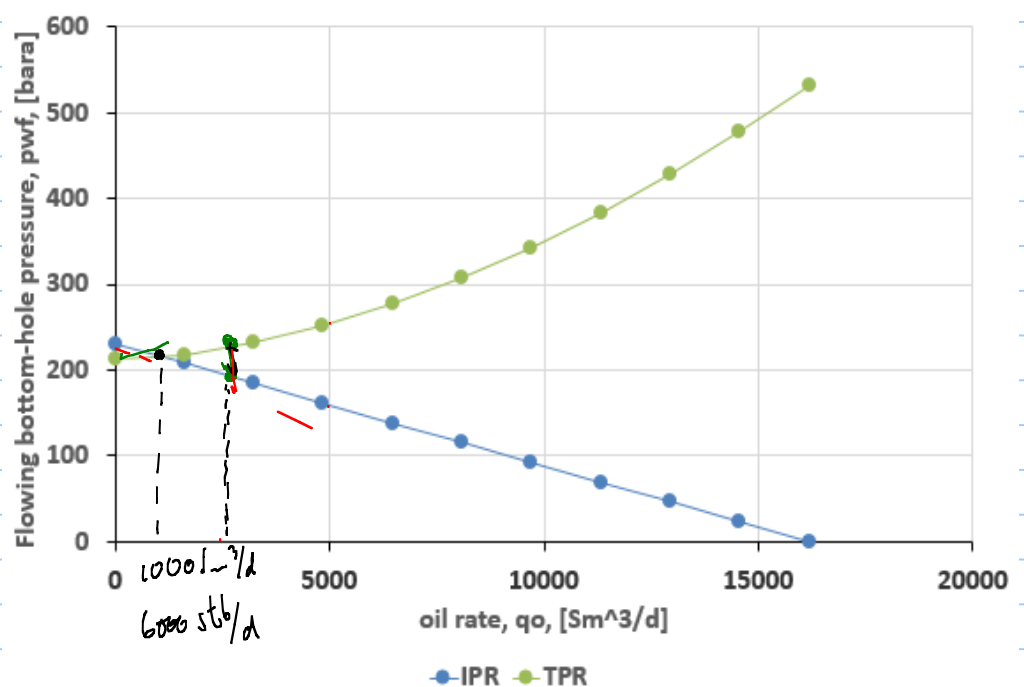


- LQ
- Drilling module
- Movable drill rig
- Electrical plant
- Wellheads
- Booster pumps
- Water Injection
- Uptime 99% bracket

pR	bara	231			
J, for oil flow	Sm ³ /d/bar	70			
L, tubing length	m	2340	IPR		TPR
d, tubing diameter	m	0.14	pwf	qo	pwf
e, tubing roughness	m	0.00010	[bara]	[Sm ³ /d]	[bara]
teta, tubing angle	[deg]	90	231	0	213
muo, oil viscosity	[Pa s]	0.1	208	1618	218
rhoo, oil density	[kg/m ³]	897	185	3236	233
pwh, wellhead pressure	[bara]	7	162	4853	252
			139	6471	277
			116	8089	307
			92	9707	343
			69	11325	383
			46	12942	428
			23	14560	477
			0	16178	531

3236 m^3/d $\Delta p_{\text{pump}} = 233 - 185$

$$\Delta p_{pm} = 48 \text{ bar}$$

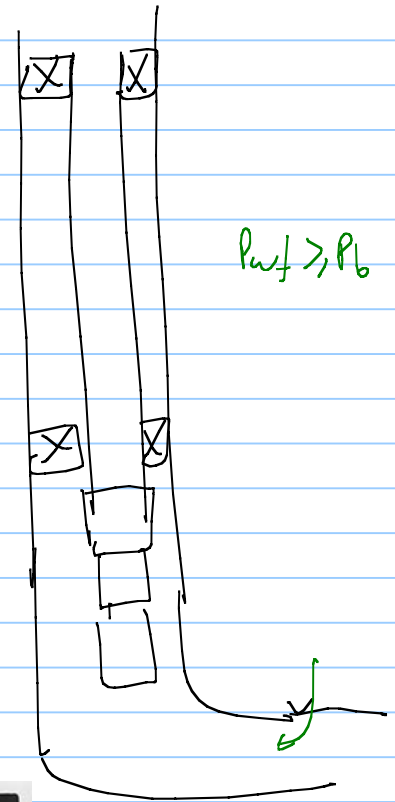
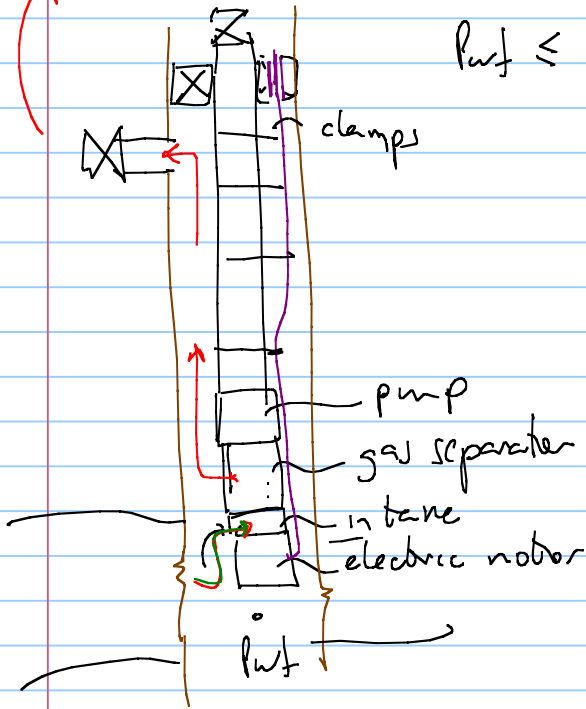


- Electric submersible pump (ESP) → as close as possible to formation
 - ↳ installed as vertical as possible
 - ↳ takes liquid ONLY
 - ↳ typical lifetime 10% GVF high toleran 2 years
 - ↳ gas volume fraction

producing gas through the annulus

$$P_{wf} \leq P_b(T_R)$$

bubble point pressure



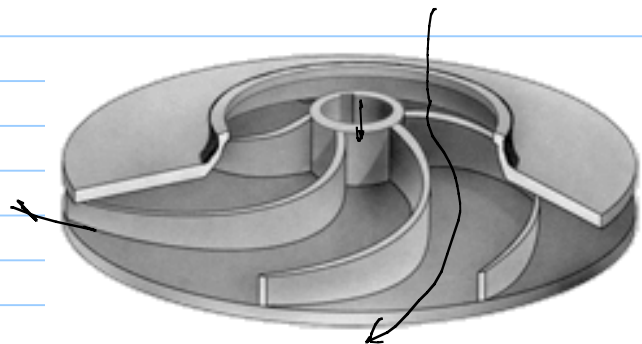
Armas

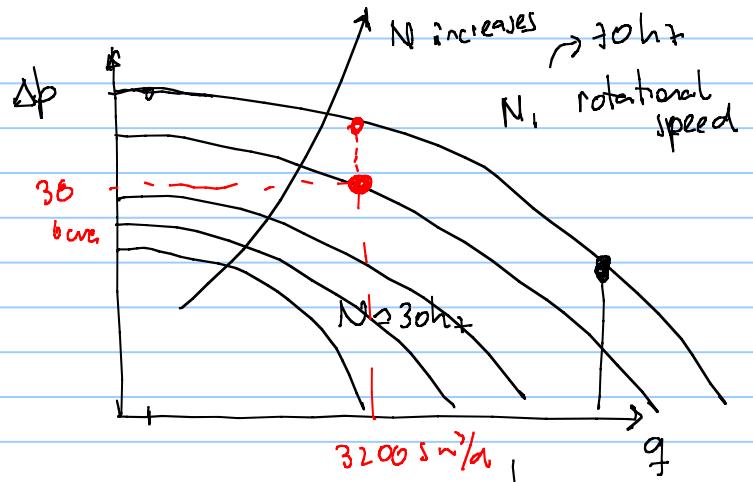
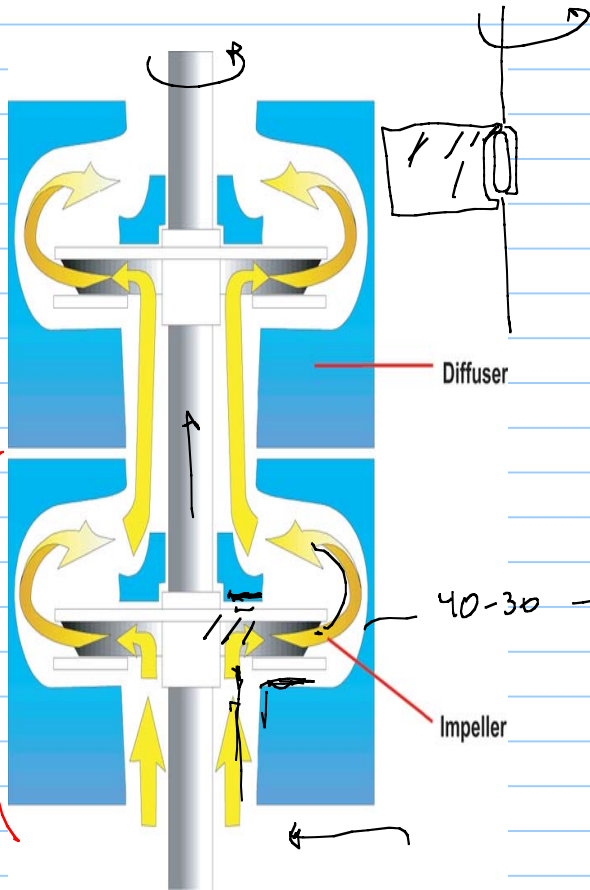
Aronutof ≈ 1930

Oklahoma

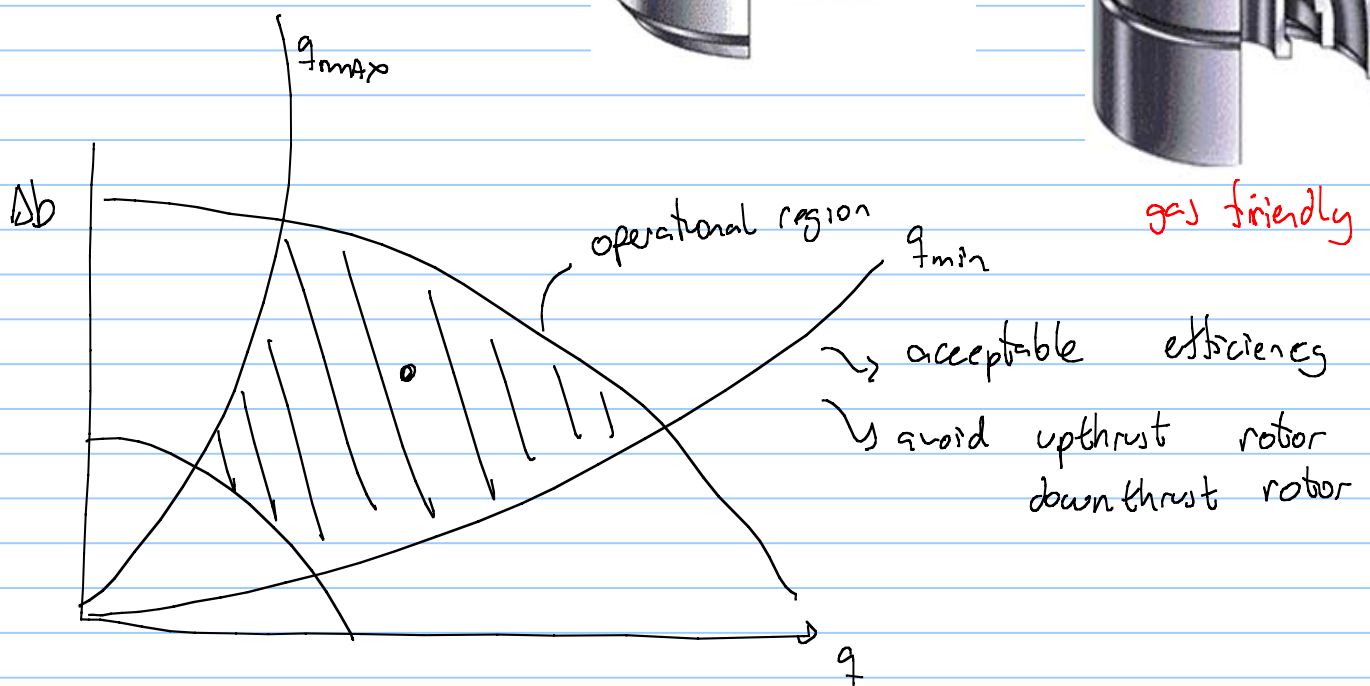
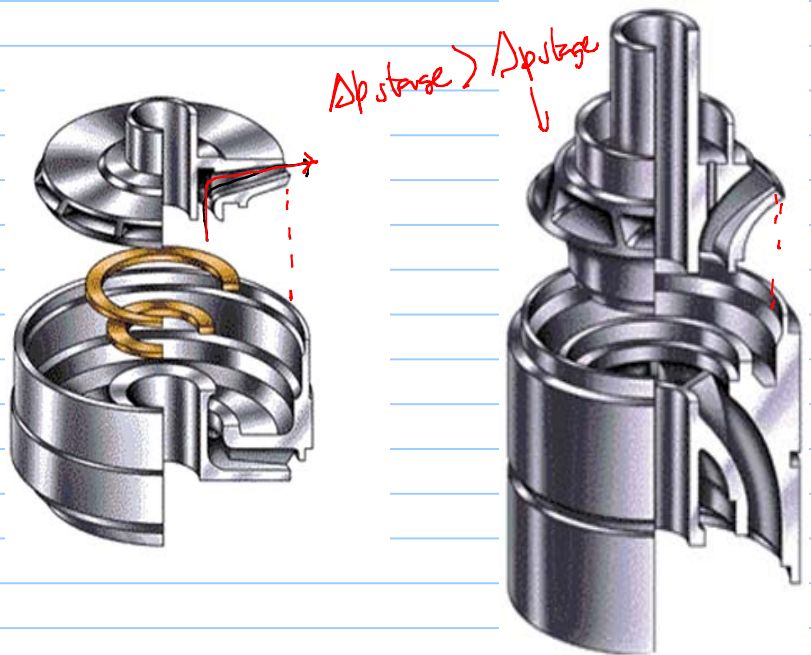
Reda pump

Schlumberger



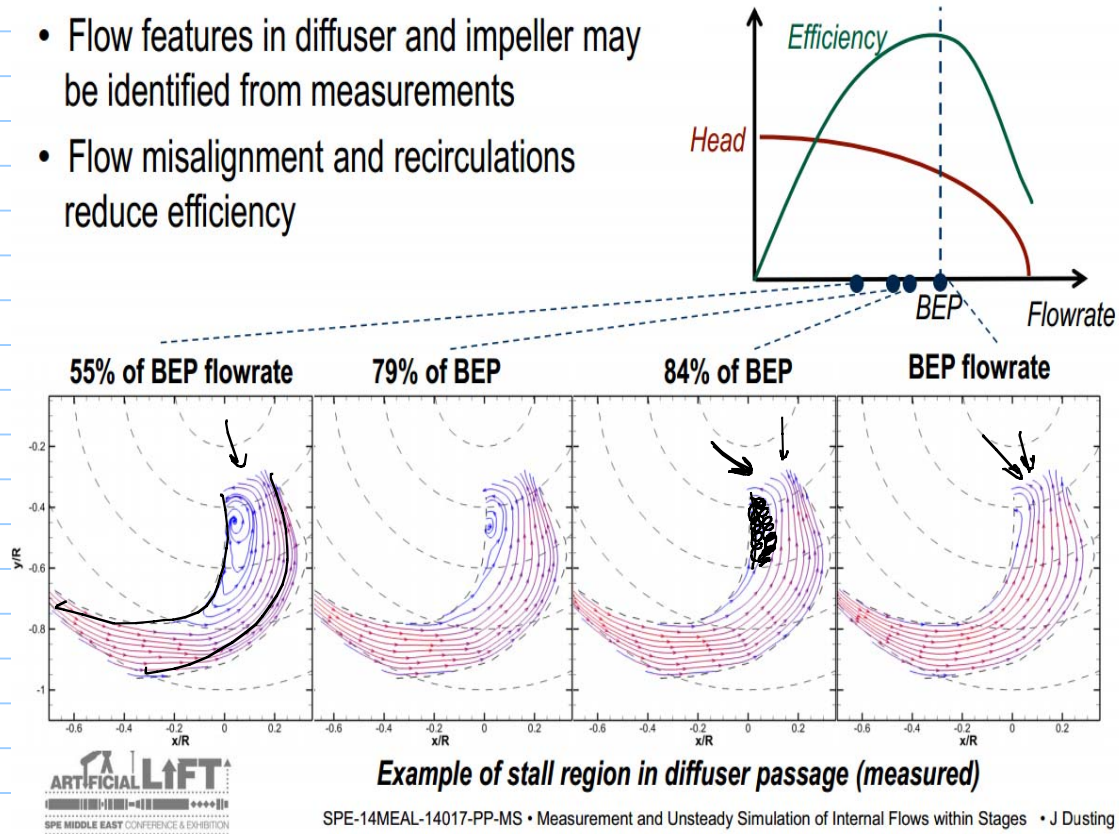


$$\text{Power} = q \Delta p$$

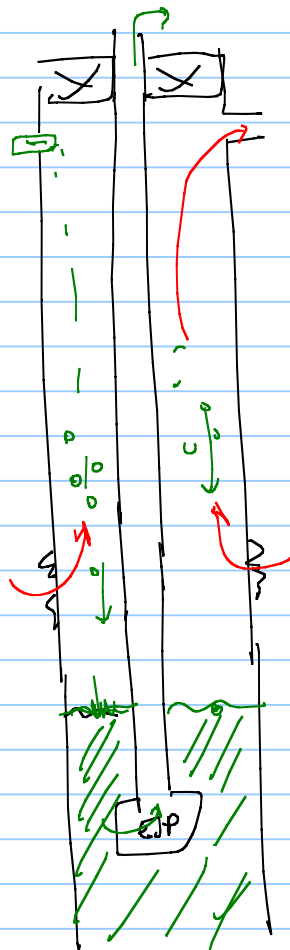


PIV measurement in a radial flow stage

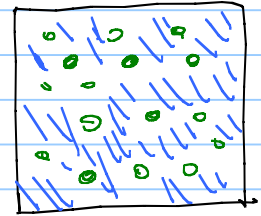
- Flow features in diffuser and impeller may be identified from measurements
- Flow misalignment and recirculations reduce efficiency



ESP are also used to drain liquid accumulated in gas wellbores



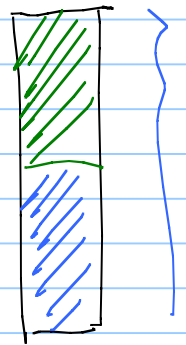
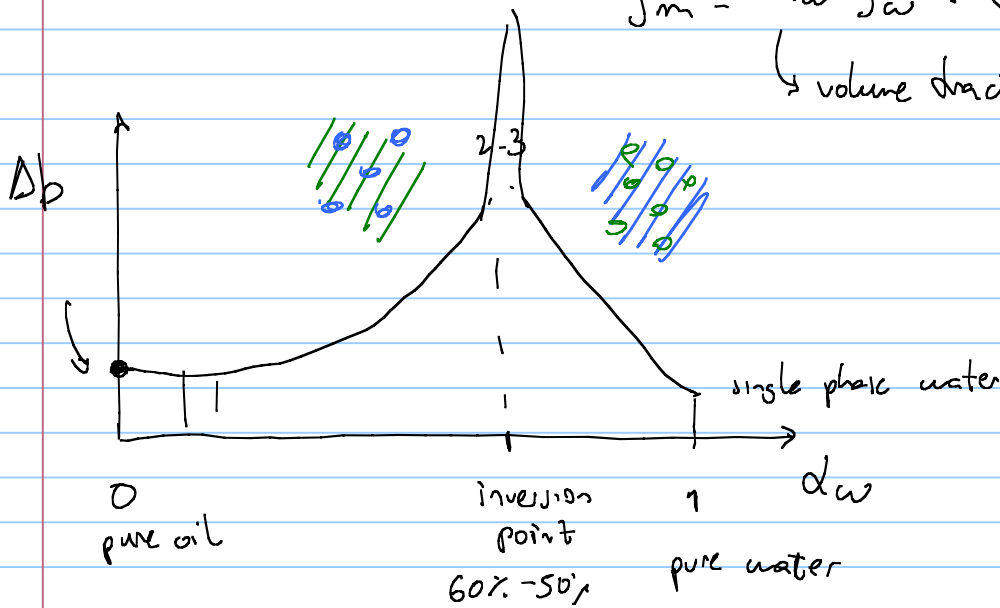
Pressure drop expressions for liquid are also useful to deal with
oil + water emulsions \rightarrow stable dispersion \rightarrow oil in water
water in oil



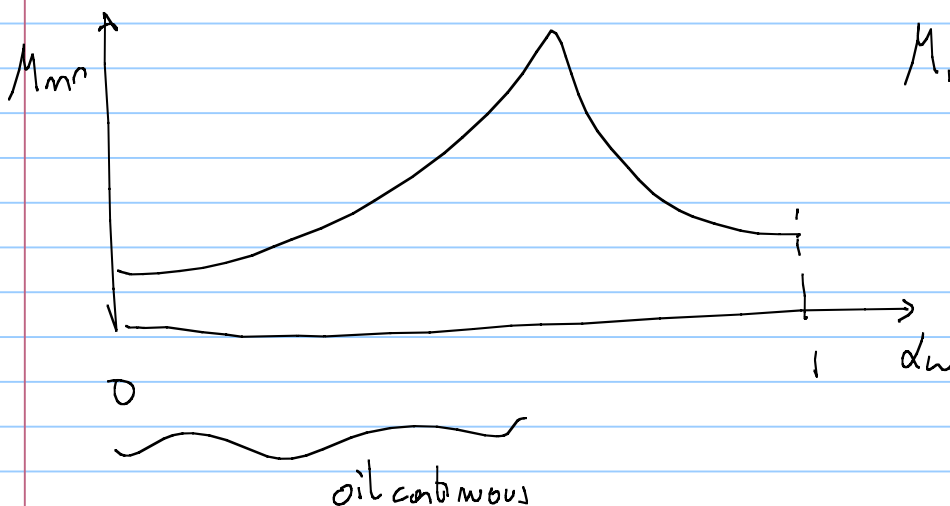
$$\frac{p_1}{\rho g} + z_1 + = \frac{p_2}{\rho g} + z_2 + \frac{f L}{D} \frac{V^2}{2g} \quad \leftarrow M_{\text{mixture}}$$

$$\rho_m = \alpha_w \rho_w + (1 - \alpha_w) \rho_o$$

\rightarrow volume fraction of water $\frac{q_w}{q_o + q_w} = \frac{10}{40} = 0.25$



oil continuous



$$\mu_m = \mu_o e^{\alpha_w}$$

water continuous

$$\mu_m = \mu_w e^{\alpha_o}$$

$$u_{fluid} = \frac{q}{\pi \phi^2} \leftarrow m^3/d \quad q_o \quad m^3/d$$



$$V_o(p, t)$$

$$b_g = \frac{q_g(p, t)}{q_o(p_{sc}, t_{sc})} \ll 1$$

$$V_g(p_{sc}, t_{sc})$$

$$V_o(p_{sc}, t_{sc})$$

$$GOR = \frac{\bar{V}_g}{\bar{V}_o}$$

$$B_o = \frac{q_o(p, t)}{q_o(p_{sc}, t_{sc})}$$

oil volume factor

formation volume factor

1.3-1.5

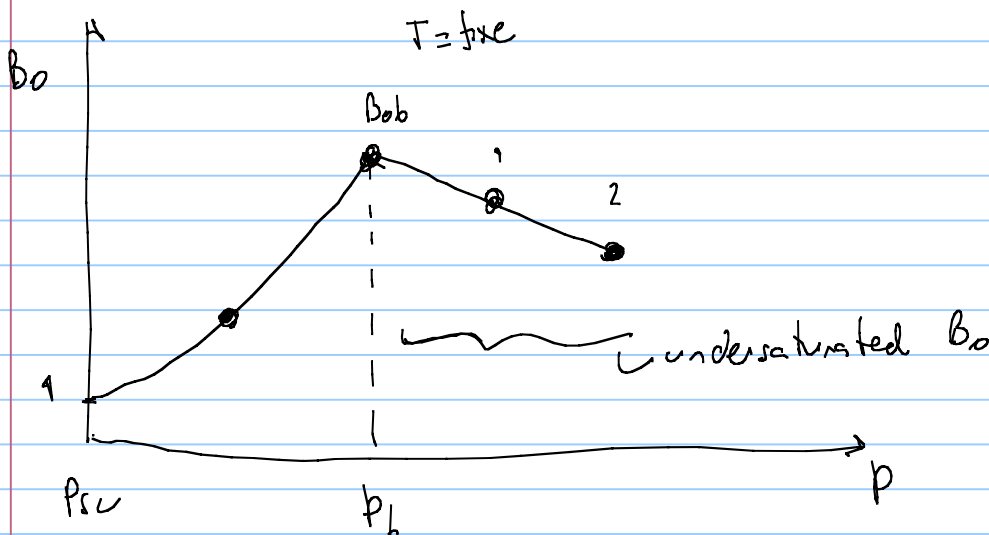
oil
"Normal" → 600-900 scf/stb

1. → 1.05

"low" → 5-150 scf/stb

1.7 → 2

"high" → 2500 →



oil compressibility

$$C_o = -\frac{1}{V_o} \frac{\partial V_o}{\partial p} \quad \left\{ \begin{array}{l} \text{thermodynamic} \\ \text{definition} \end{array} \right.$$

$$V_o = B_o \cdot V_{sc}$$

$$C_o = -\frac{1}{V_{sc} B_o} \frac{\partial (B_o V_{sc})}{\partial p}$$

$$C_o = - \frac{1}{B_o} \frac{\partial B_o}{\partial p}$$

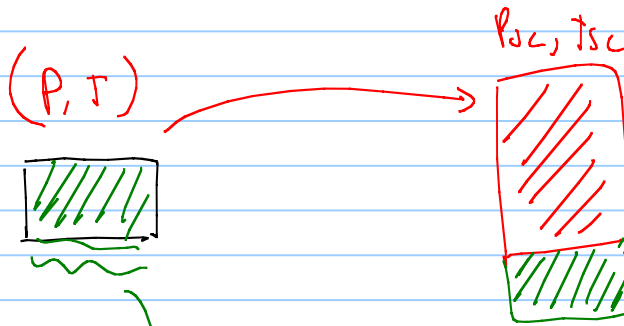
$$\int_{P_b}^{p > P_b} C_o \partial p = \int_{B_{ob}}^{B_o} - \frac{\partial B_o}{B_o}$$

check

$$B_o = B_{ob} e^{C_o(P_b - p)}$$

$$f(SG_g, SG_o, GOR, p, T)$$

Black oil correlations



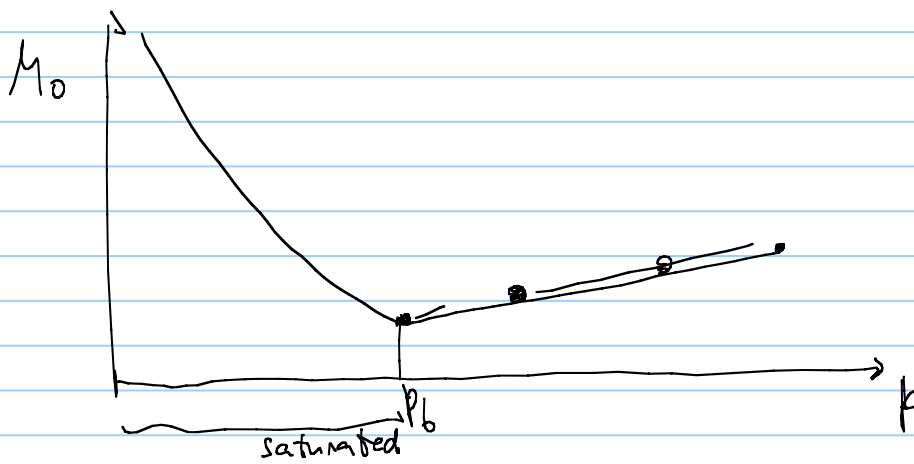
$$\rho_o = \frac{M_o}{V_o} = \frac{V_g \cdot \rho_g + V_o \cdot \rho_o}{V_o}$$

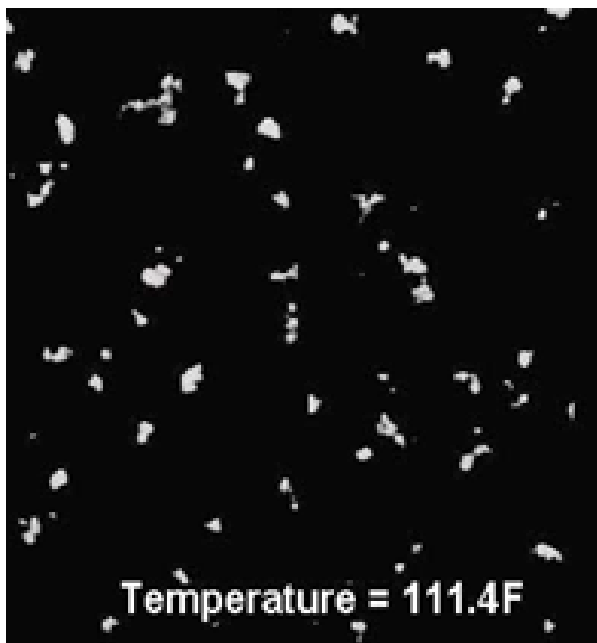
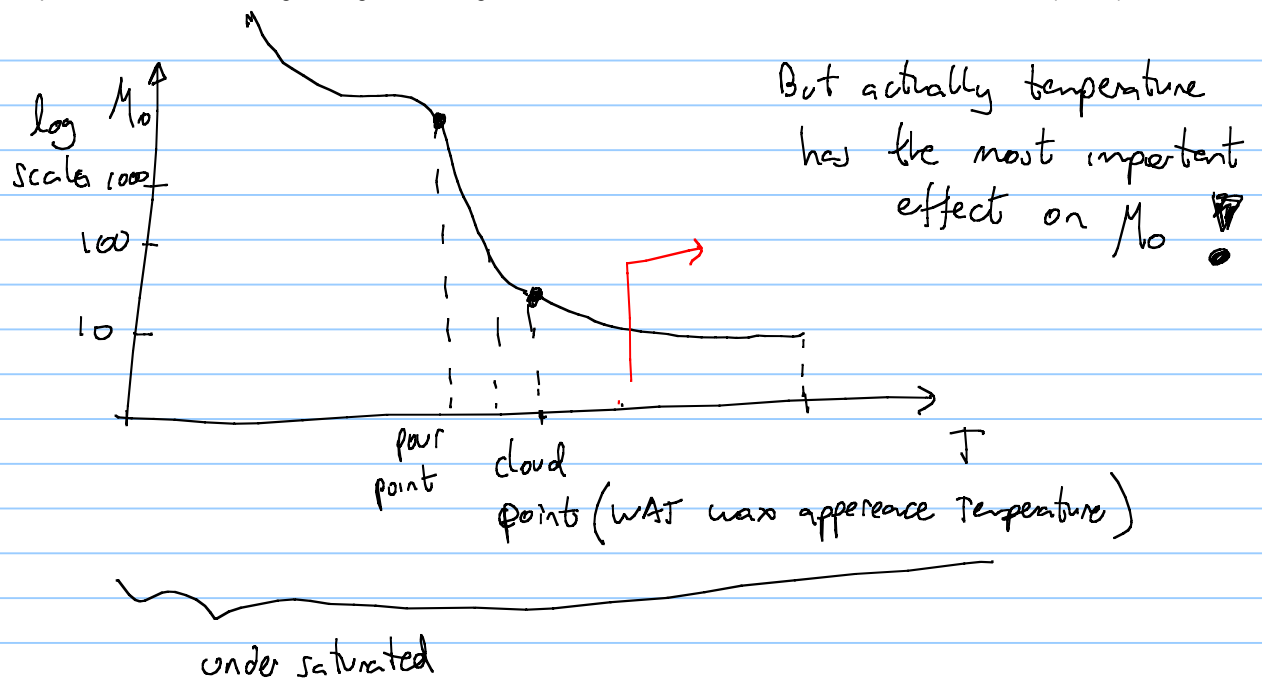
$$V_g = GOR \cdot V_o$$

$$Re = \frac{v \cdot \phi \cdot f}{\mu}$$

$$\rho_o = \frac{SG_g \cdot 1.224 \cdot GOR \cdot V_o + V_o \cdot SG_o \cdot 1000}{V_o}$$

$$\rho_o(p, T) = \frac{SG_g \cdot 1.224 \cdot GOR + SG_o \cdot 1000}{B_o}$$





cloud point

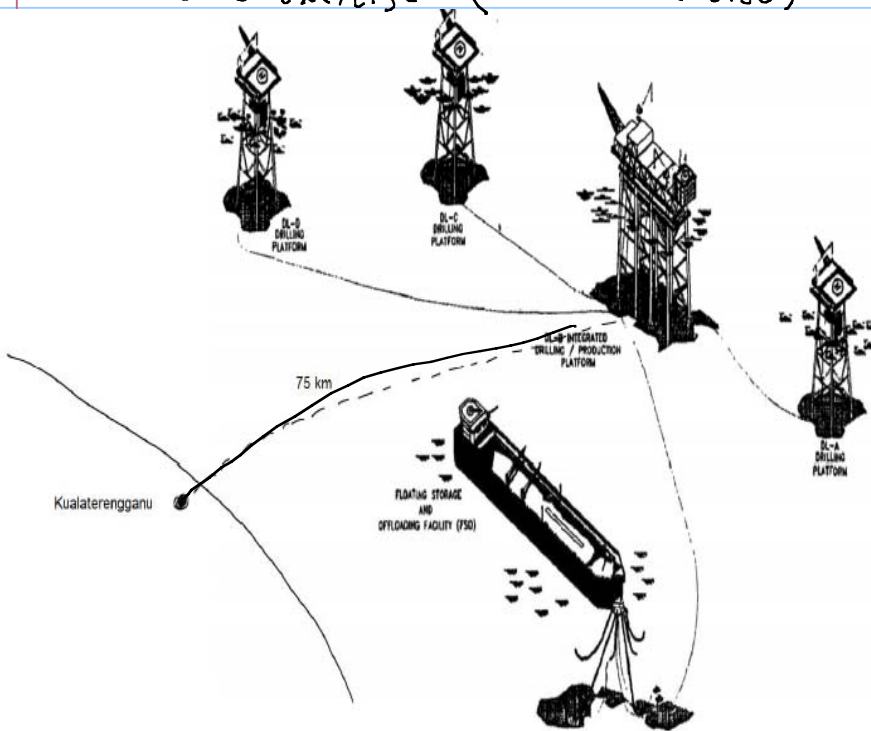


pour point (no flow)

Day 7:

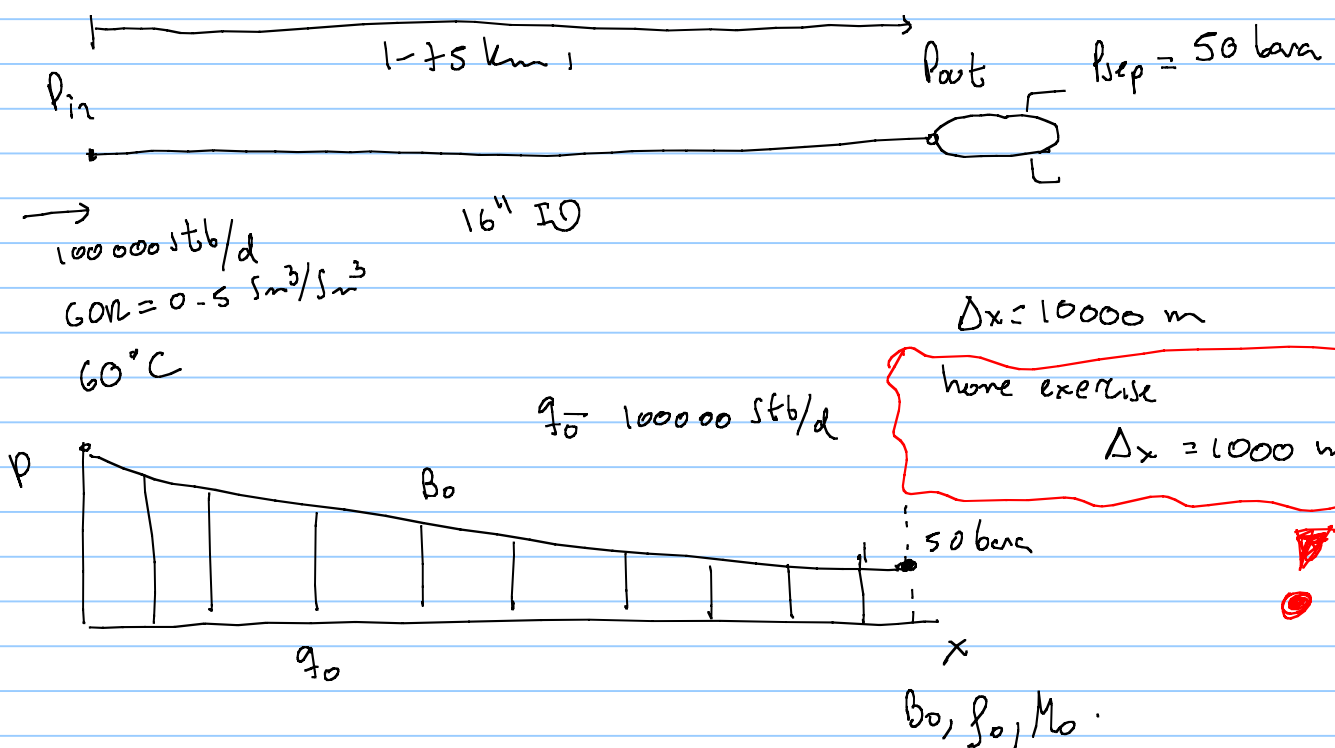
- exercise p and T drop in pipeline (± 5 km) \rightarrow oil with low GOR (Sm^3/Sm^3)
- Introduction to multiphase flow in wells (tubing)

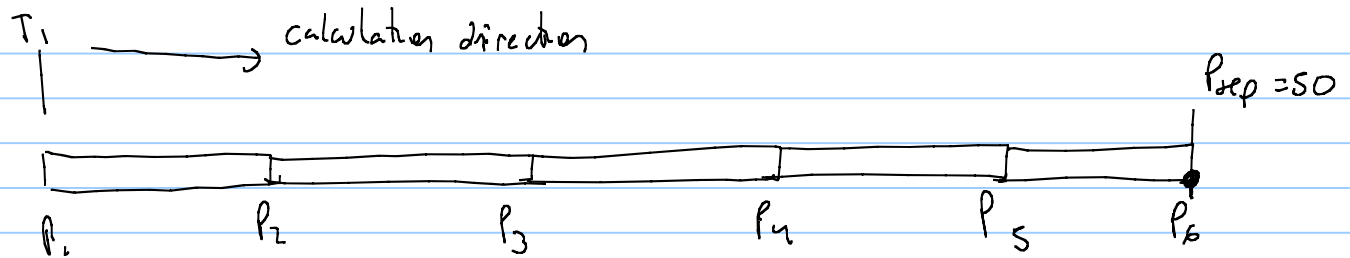
Class exercise (home exercise)



Oulang Malaysia

Kualaterengganu





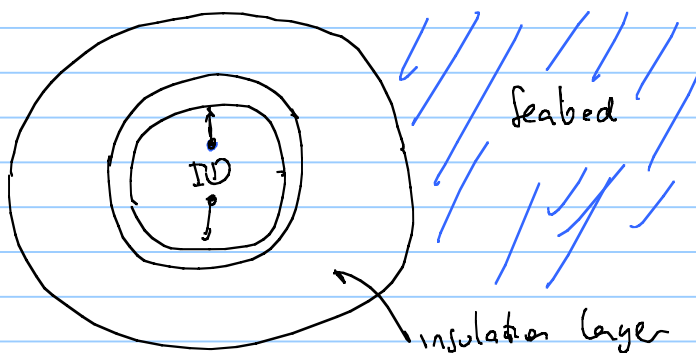
P calculation direction

$$P_5 = P_{sep} + \frac{\partial P}{\partial x} \cdot \Delta x$$

$$f(M_{06}, P_{06}, B_{06})$$

$f(t)$ and $f(B_0)$ are indicated as functions of the main function.

Cross section of pipe



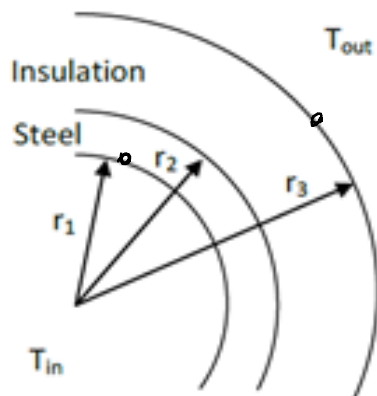
$$T = 14^\circ C$$

$$\textcircled{a} T = 20^\circ C \quad M_0 \approx 2000 \text{ cp}$$

too high

The overall heat transfer coefficient based on the pipe outer area is defined as:

$$\frac{1}{U} = \frac{r_{\text{pipe_inner}}}{h_{\text{out}} \cdot r_{\text{insulation_outer}}} + \frac{\ln \left(\frac{r_{\text{insulation_outer}}}{r_{\text{pipe_outer}}} \right) \cdot r_{\text{pipe_inner}}}{k_{\text{insulation}}} + \frac{\ln \left(\frac{r_{\text{pipe_outer}}}{r_{\text{pipe_inner}}} \right) \cdot r_{\text{pipe_inner}}}{k_{\text{pipe}}} + \frac{1}{h_{\text{inner}}}$$



$$T_{\text{fluid}} - T_1 = \frac{\dot{q}}{h}$$

$$(T_1 - T_2) = \frac{\dot{q}}{h}$$

$$(T_2 - T_3) = \frac{\dot{q}}{h}$$

$$(T_3 - T_{\text{out}}) = \frac{\dot{q}}{h}$$

$$\dot{q} = 2\pi r_3 \cdot L \cdot U (T_{\text{fluid}} - T_{\text{amb}})$$

Short derivation of Temperature equation for liquid flow in pipe

$$\frac{d\dot{q}}{dl} = \left[\frac{dh}{dl} + \left(\cancel{\frac{v dv}{dl}} \right) + \cancel{\frac{dz}{dl} \cdot g} \right] \dot{m}$$

$$\frac{d\dot{q}}{dl} = \frac{dh}{dl} \dot{m}$$

$$\text{for liquid } dh = C dT$$

$$d\dot{q} = dl \cdot 2\pi r_3 \cdot U \cdot (T - T_{amb}) \quad T_{fluid} = T$$

$$(T_{amb} - T) 2\pi r_3 U = \frac{dT}{dl} \cdot C \dot{m}$$

$$A = \frac{C \dot{m}}{2\pi r_3 U}$$

$$T_{amb} - T = \frac{dT}{dl} \frac{C \cdot \dot{m}}{2\pi r_3 U}$$

$$\frac{dT}{dl} + \frac{T - T_{amb}}{A} = 0 \quad \text{multiply by } e^{l/A}$$

$$\frac{dT}{dl} e^{l/A} + T e^{\frac{l}{A}} \frac{1}{A} = T_{amb} \frac{1}{A} e^{l/A}$$

$$\frac{d}{dl} (e^{l/A} \cdot T) = \frac{T_{amb}}{A} e^{l/A}$$

$$\int_0^l d(e^{l/A} \cdot T) = \int_0^l \frac{T_{amb}}{A} e^{l/A} dx$$

$$e^{l/A} T(l) - T(l=0) = \left(e^{l/A} T_{amb} - T_{amb} \right)$$

$$T(l) = (T(l=0) - T_{amb}) e^{-l/A} + T_{amb}$$

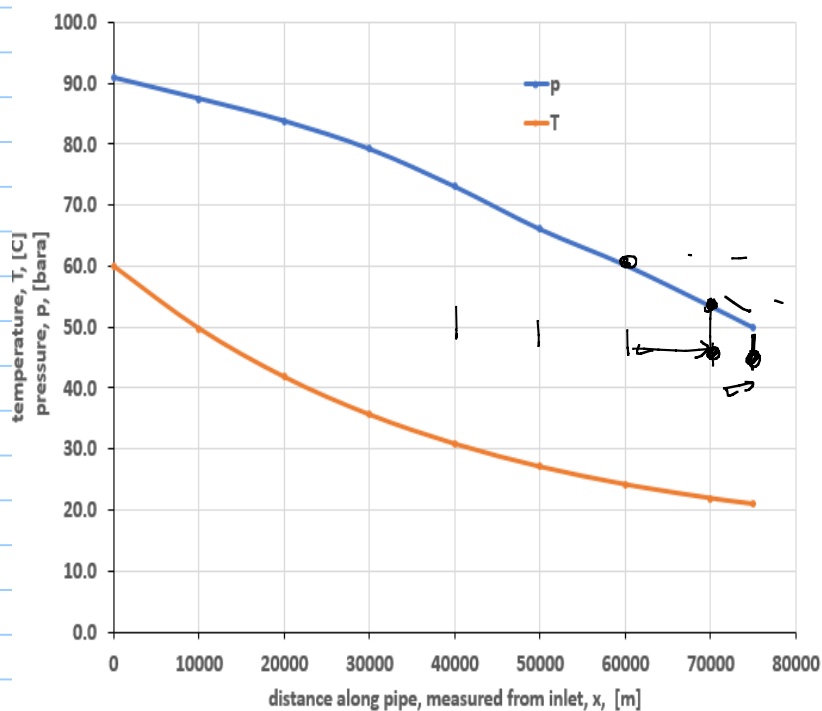
Function unburied_pipeline_TiL(Te, Ti0, L, A)

unburied_pipeline_TiL = Te + (Ti0 - Te) * Exp(-L / A)

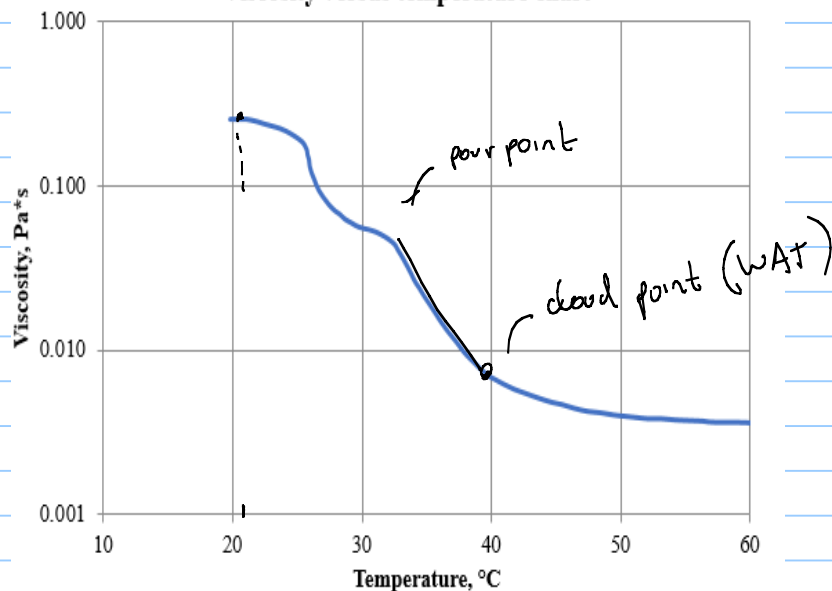
End Function

$$B_o = \frac{V_o}{V_o} \quad q_o = B_o \cdot q_o$$

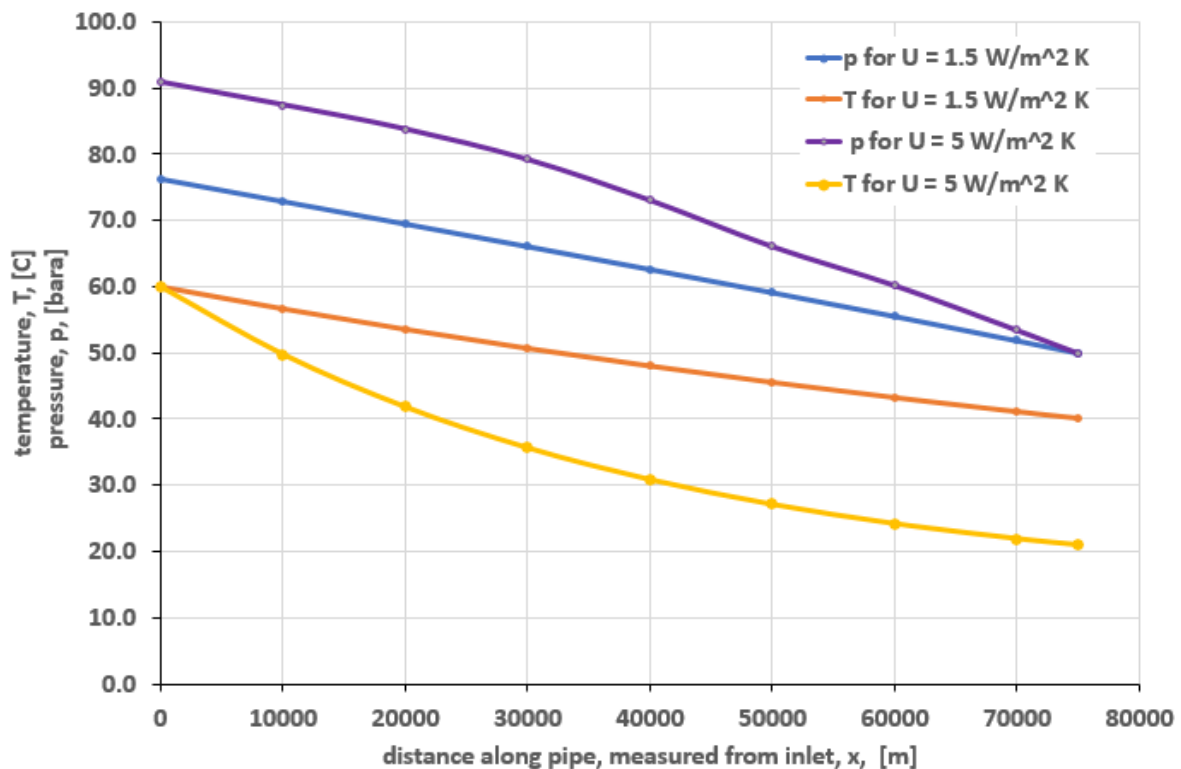
Distance from pipe inlet [m]	T [C]	Bo [m ³ /Sm ³]	deno [kg/m ³]	visco [Pa s]	qo [m ³ /d]	p [bara]
0	60.0	1.028	730.2	0.004	16338.4	90.9
10000	49.8	1.021	734.9	0.004	16235.9	87.5
20000	41.8	1.016	738.4	0.006	16158.4	83.8
30000	35.6	1.013	741.1	0.017	16099.6	79.2
40000	30.8	1.010	743.1	0.053	16054.9	73.1
50000	27.1	1.008	744.7	0.083	16020.7	66.1
60000	24.2	1.006	745.9	0.212	15994.6	60.2
70000	21.9	1.005	746.9	0.244	15974.4	53.5
75000	21.0	1.004	747.3	0.252	15966.2	50



Viscosity versus temperature chart



Comparing pipelines with different heat transfer coefficients (insulation)



saving in pump power
by insulating better
the pipe !

$$P = q \Delta p = (1510^5 \text{ kg}) \cdot \frac{15966.2 \text{ m}^3/\text{s}}{3600 \cdot 24}$$

$$P_{\text{hy}} = 0.28 \text{ MW}$$

$$P_{\text{actual}} = \frac{P_{\text{hydra}}}{\eta} = \frac{0.28}{0.7} =$$

$$P_{\text{actual}} = 0.4 \text{ MW}$$

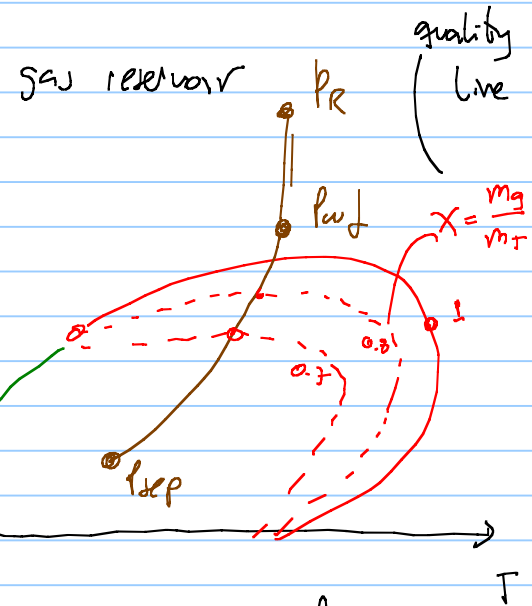
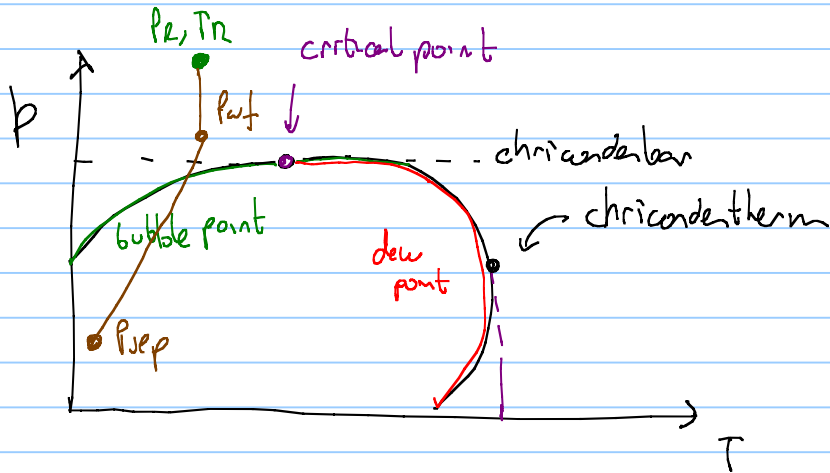
• Non-newtonian fluids { Some Oil + water emulsions
drilling muds

Not covered in this course!!

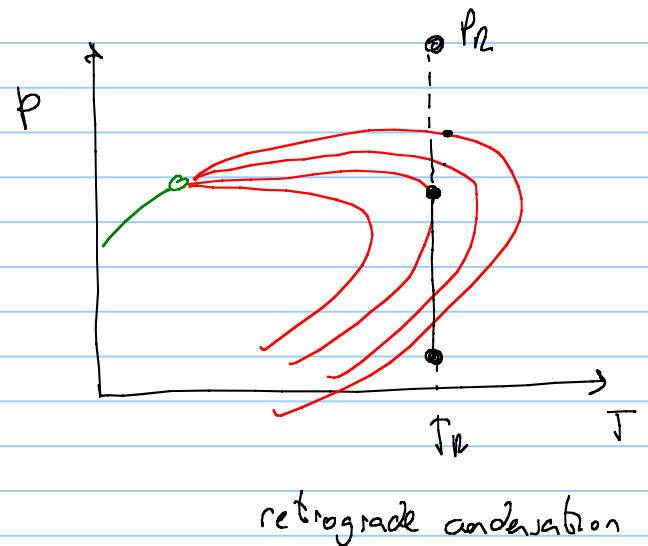
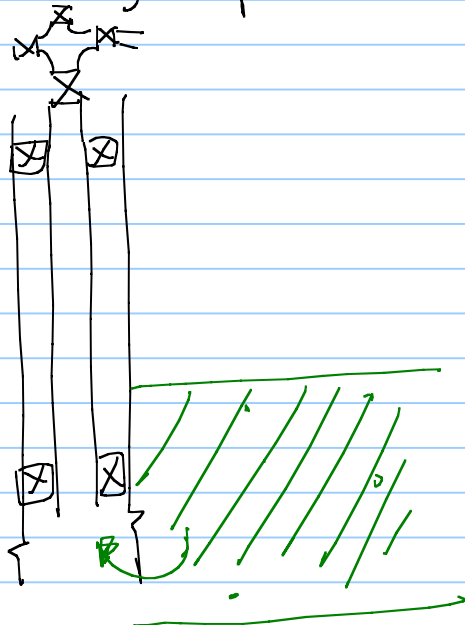
Multiphase flow in wellbores:

Phase diagram of reservoir fluids

oil reservoir composition z_i



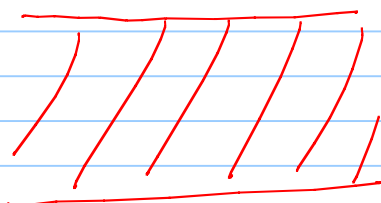
well flowing composition with time

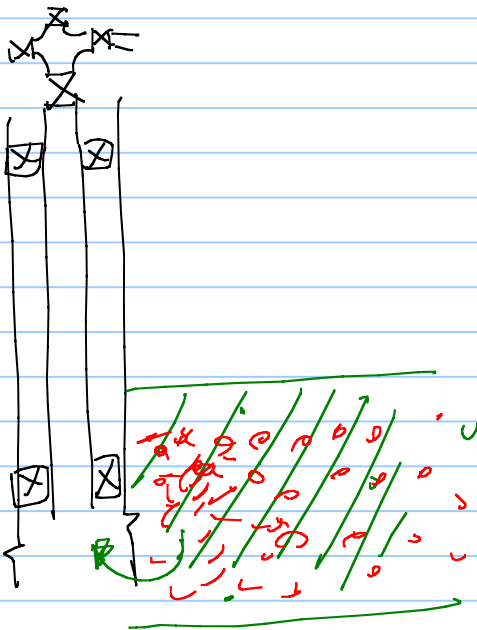


undersaturated oil $z_i = f(T)?$
 $P_{wf} > P_b(T_R)$

undersaturated gas z_i is constant

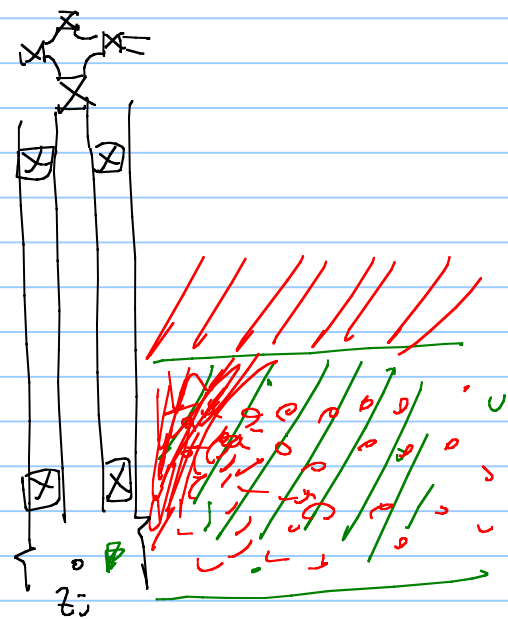
$P_{wf} > P_d(T_R)$





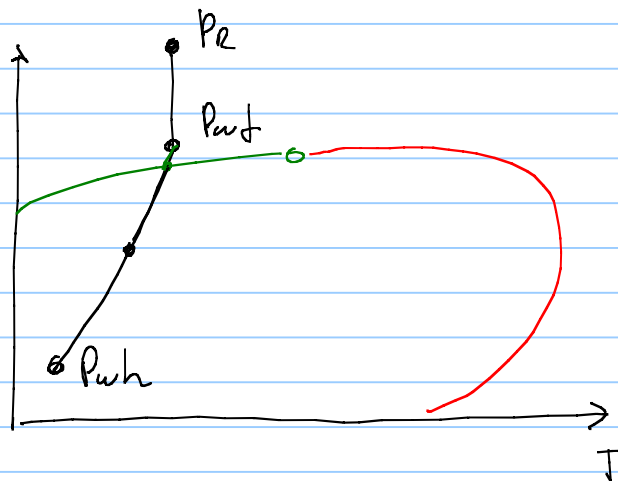
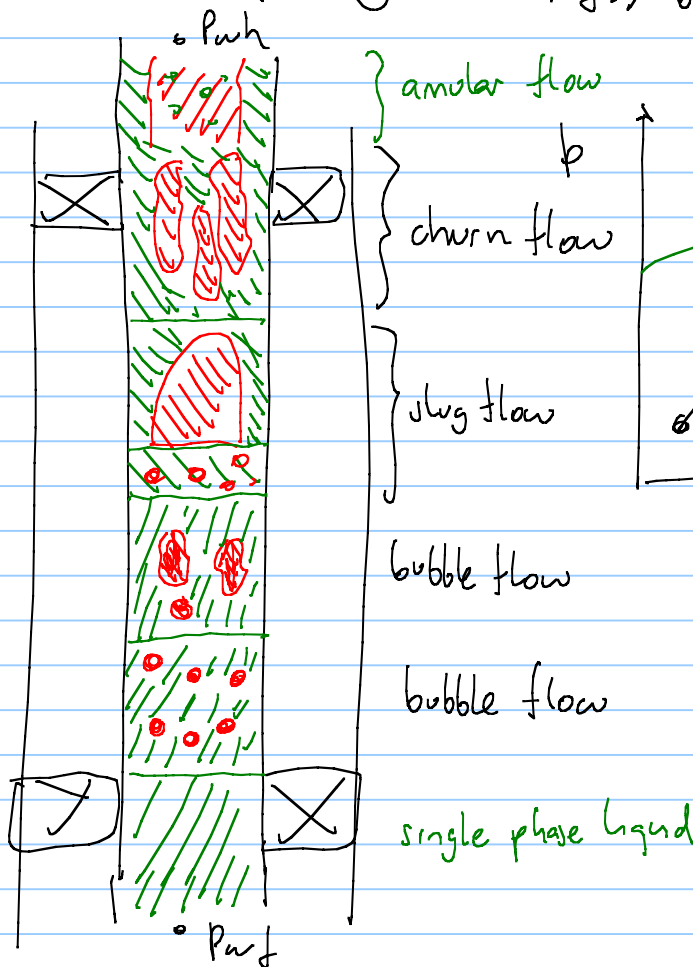
$S_g \uparrow$ with time

GOR also increases, z_u is $f(t)$

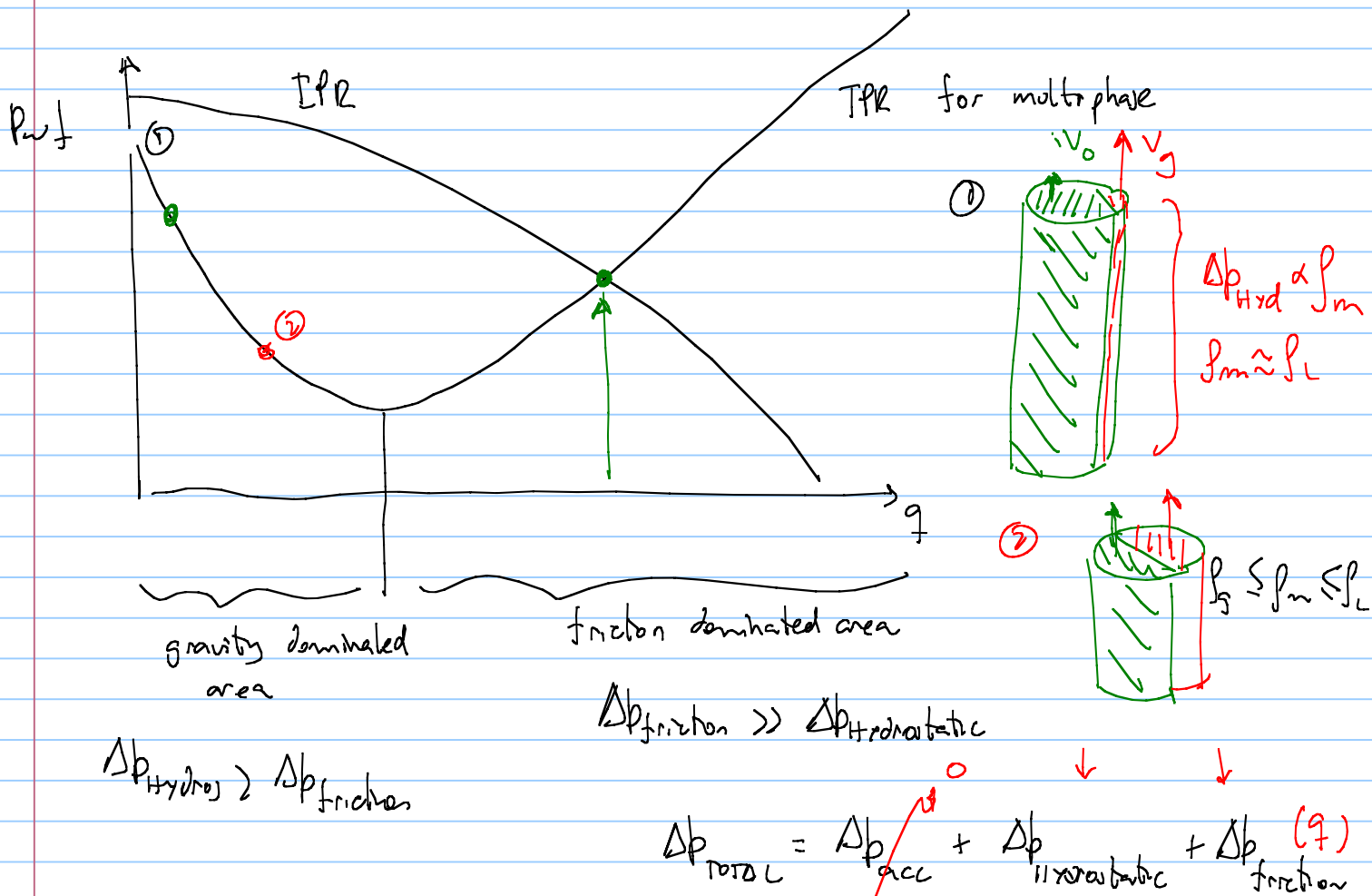
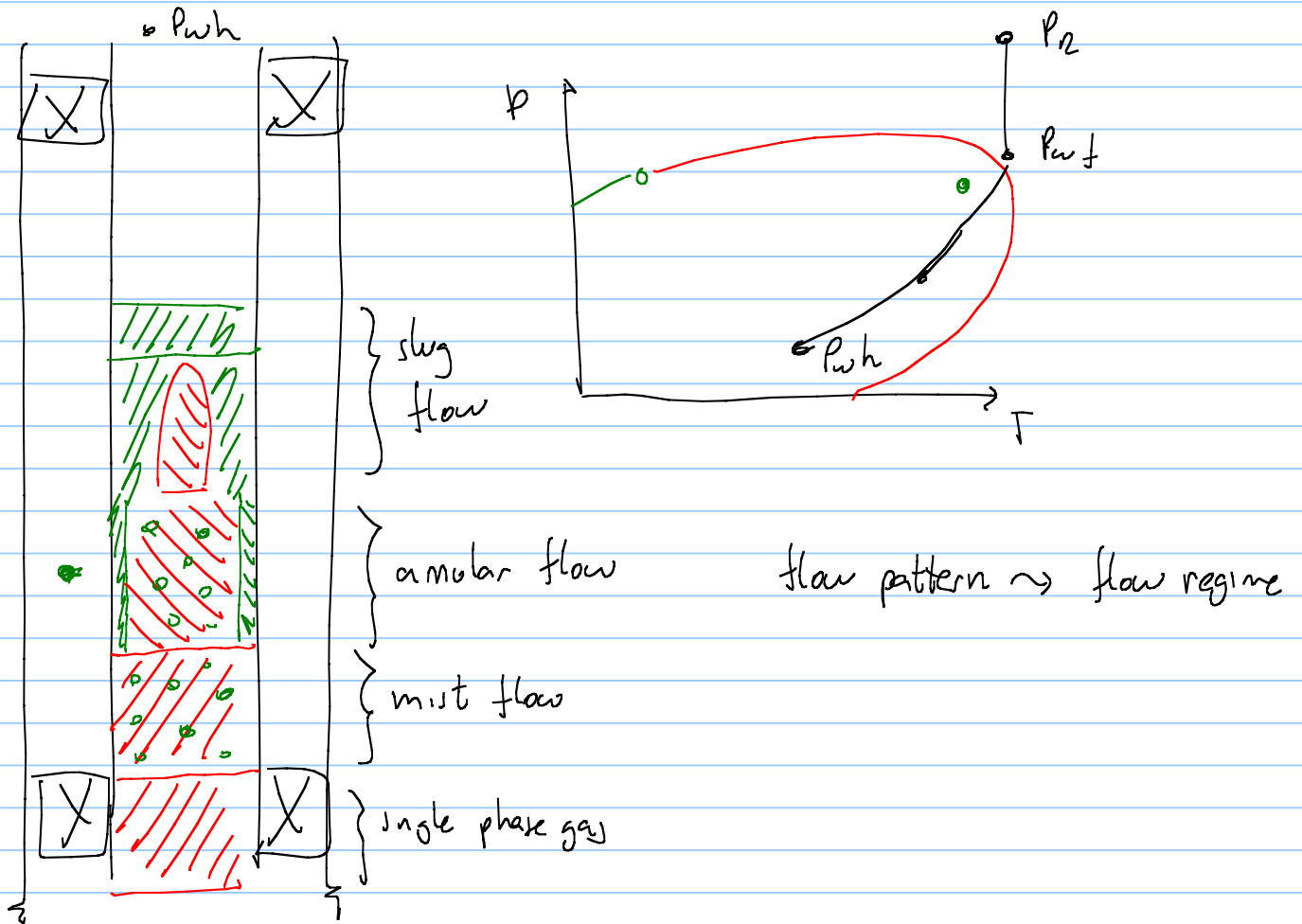


gas coning $z_i = f(t)$

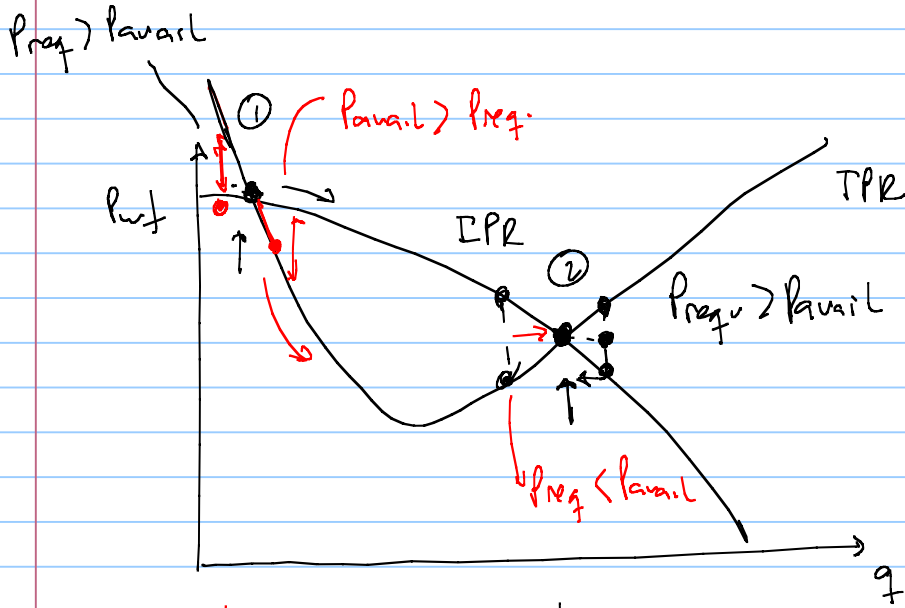
for wellbore production with $P_{wf} > P_b(I_{pr})$



for gas well



It is possible to have 2 intersections between TPR-IPR in two phase flow



unstable equilibrium

stable equilibrium

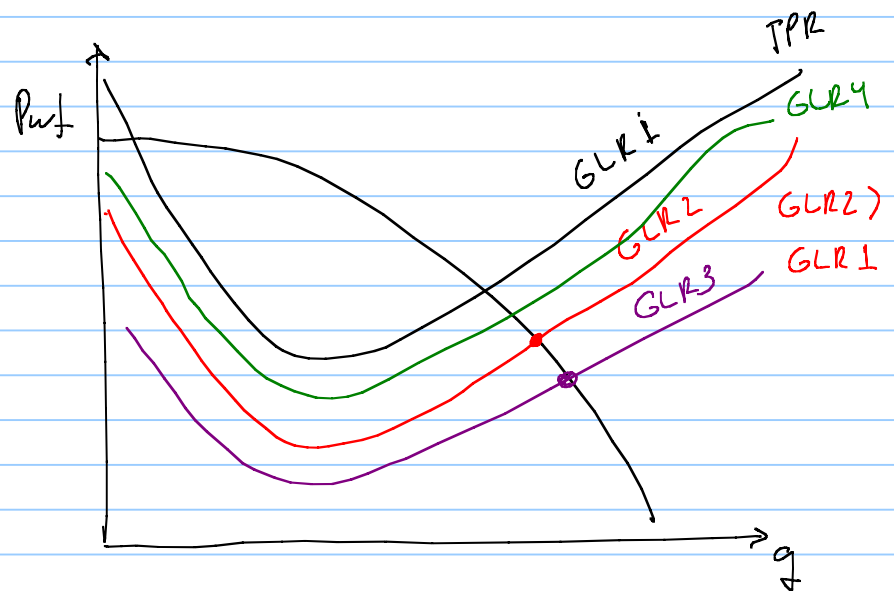


to move from unstable equilibrium to stable equilibrium we need sometimes flow induction methods.

Gilbert 1954 { IPR
TPR
Chone

GLR -- gas liquid ratio

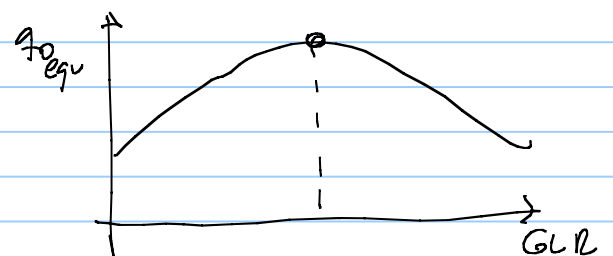
↑ GLR P_{mixture} ↓



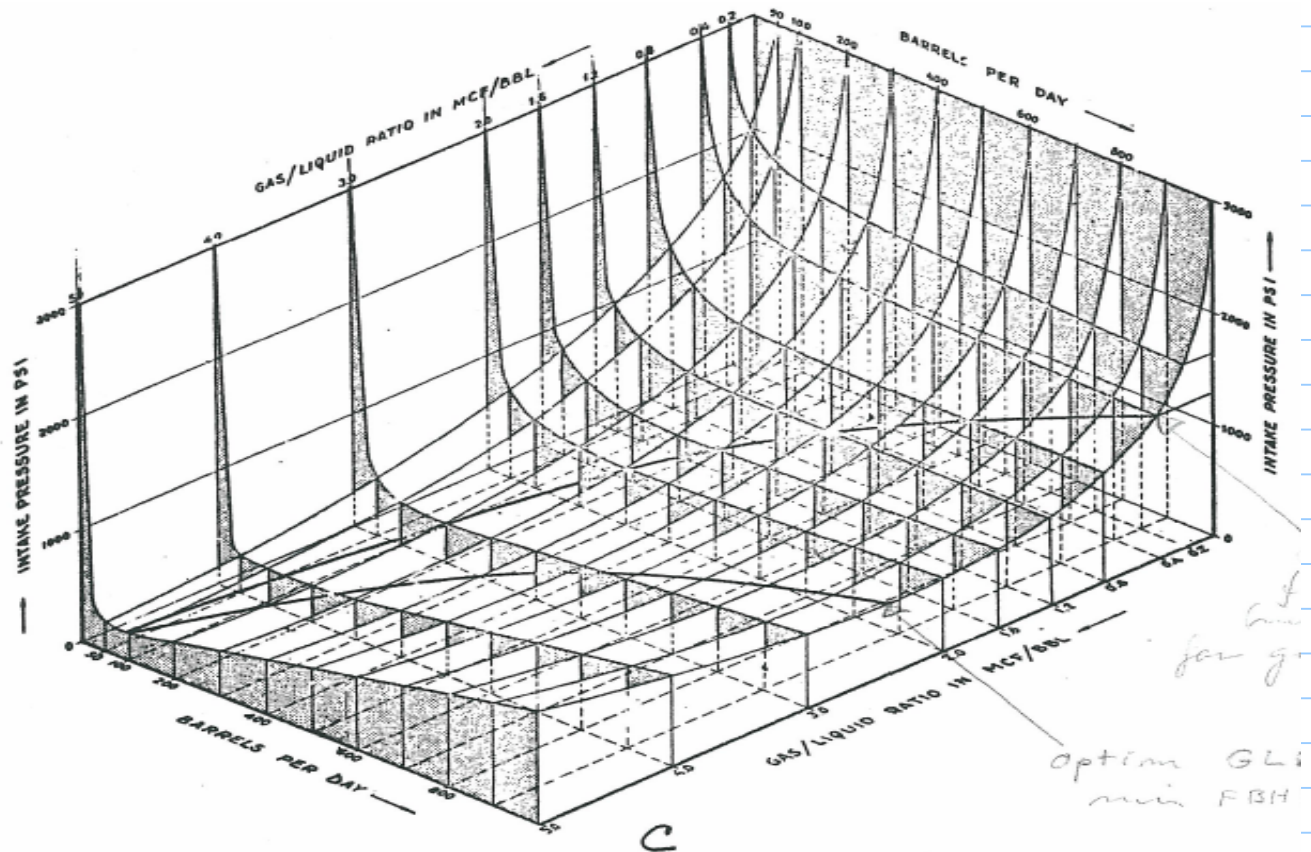
GLR₄ > GLR₃ > GLR₂ > GLR₁

ΔP_{hydrostatic} < ΔP_{friction}

change of trend in the equilibrium rate



Gilbert 1954



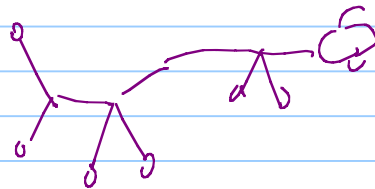
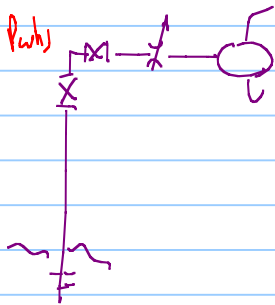
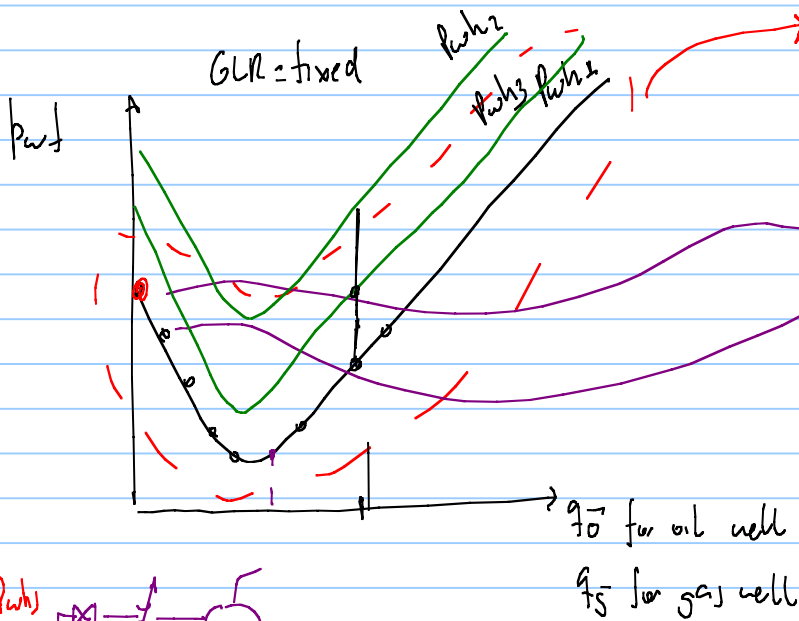
THE TWO-PHASE VERTICAL-LIFT FUNCTION
FOR 2.875-INCH TUBING SET AT 8000 FEET
(TUBING PRESSURE = ZERO PSI GAUGE)

FIGURE 6

Day 8

Multiphase flow :

- tubing tables
- pressure traverse curves \sim
- multiphase flow theory (flow patterns, maps, definitions)
- methods to study multiphase flow
- Conversion from s.c to local condition using BO properties.

 $P_{wh} = P_{wh1}$

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh2}$

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh3}$

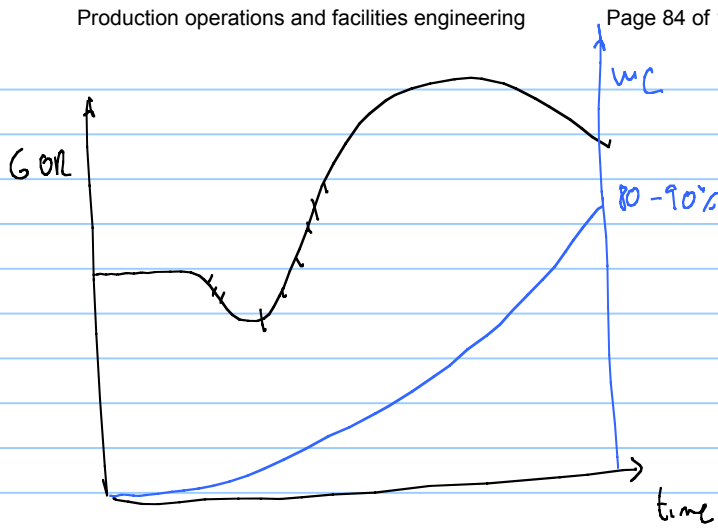
q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-

P_{wh} range for $\{ 200 \text{ bara} \rightarrow 30 \text{ bara} \}$

$\downarrow \quad \downarrow$

if i want

 $P_{wh} = 185$ $P_{wh} = 1400$ $P_{wh} = 30$



GOR and WC change during the life of the well

$P_{wh} = P_{wh1}$ GOR₁ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh2}$ GOR₁ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh3}$ GOR₁ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh1}$ GOR₂ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh2}$ GOR₂ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh3}$ GOR₂ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh1}$ GOR₃ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh2}$ GOR₃ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh3}$ GOR₃ WC₁

q_o	q_g	q_w	P_{wf}	J_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$w_h = P_{wh1} \quad GOR_1 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh2} \quad GOR_1 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh3} \quad GOR_1 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh1} \quad GOR_2 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh2} \quad GOR_2 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh3} \quad GOR_2 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh1} \quad GOR_3 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh2} \quad GOR_3 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

 $P_{wh} = P_{wh3} \quad GOR_3 \quad wC_2$

$q_{\bar{0}}$	$q_{\bar{g}}$	q_w	$P_{w\downarrow}$	I_{wh}
0	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-

$P_{wh} = P_{wh1} \quad GOR_1, wC_3$					$P_{wh} = P_{wh2} \quad GOR_1, wC_3$					$P_{wh} = P_{wh3} \quad GOR_1, wC_3$				
$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}	$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}	$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}
0					0					0				
-					-					-				

$P_{wh} = P_{wh1} \quad GOR_2, wC_3$					$P_{wh} = P_{wh2} \quad GOR_2, wC_3$					$P_{wh} = P_{wh3} \quad GOR_2, wC_3$				
$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}	$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}	$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}
0					0					0				
-					-					-				

$P_{wh} = P_{wh1} \quad GOR_3, wC_3$					$P_{wh} = P_{wh2} \quad GOR_3, wC_3$					$P_{wh} = P_{wh3} \quad GOR_3, wC_3$				
$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}	$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}	$q_{\bar{o}}$	$q_{\bar{g}}$	q_w	P_{wf}	J_{wh}
0					0					0				
-					-					-				

$$N_{\text{tables}} = N_{P_{wh}} \times N_{GOR_3} \times N_{wC_3} = 3 \times 3 \times 3 = 27$$

$$N_{\text{simulation per tables}} = 10 \text{ (rates } q_{\bar{o}})$$

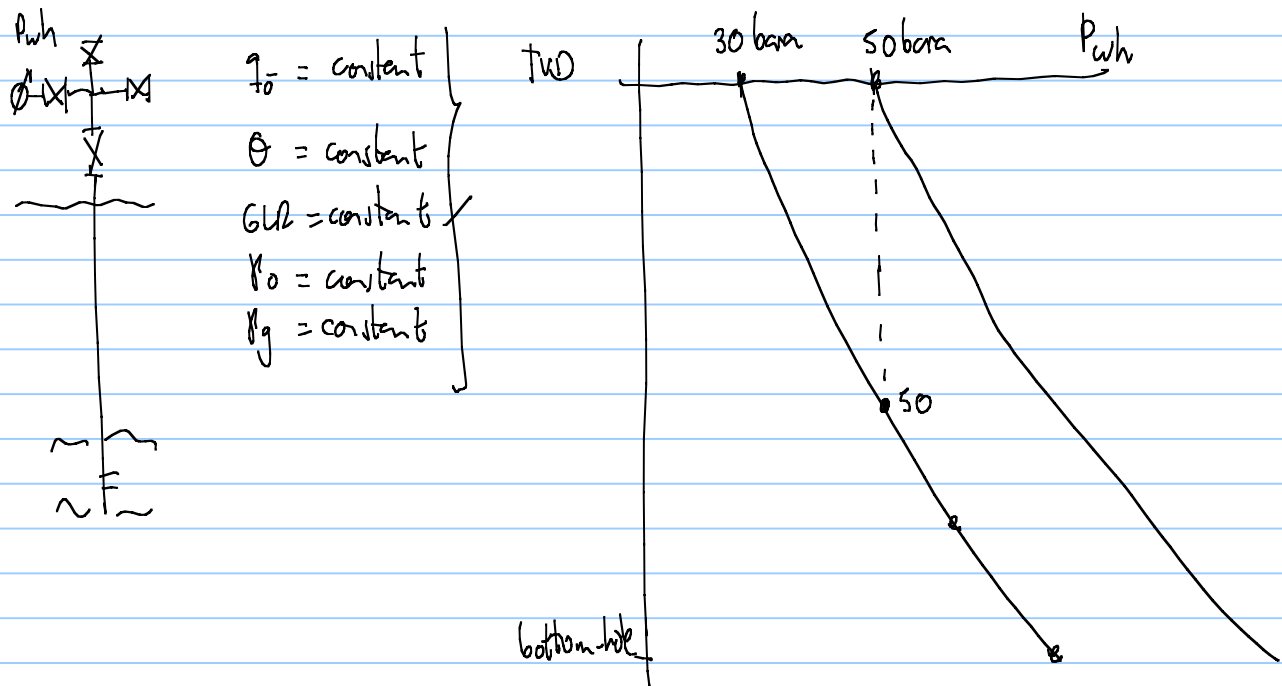
$$= 270 \text{ calculations of } \Delta p \text{ in tubing}$$

• Reservoir simulator uses tables

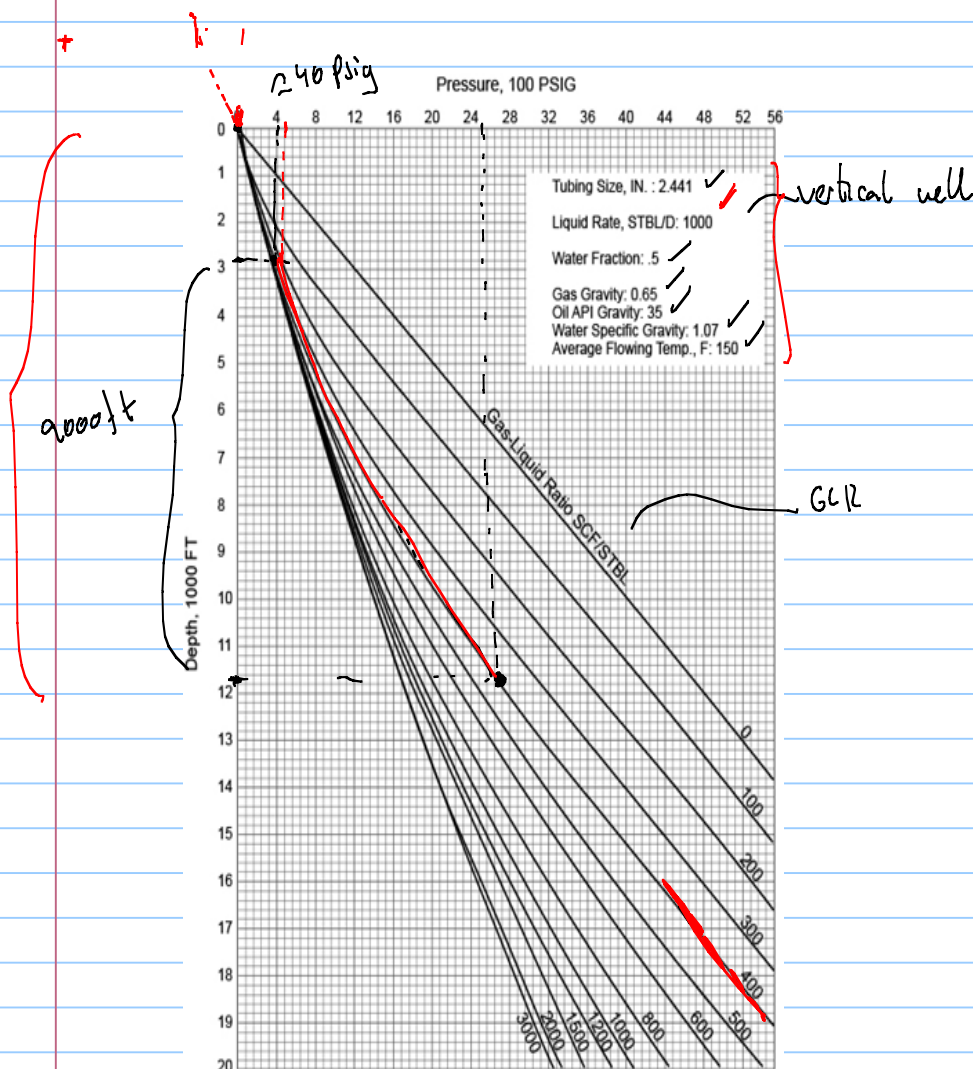
• GAP uses tables for wells
↳ commercial production simulator by PETEX

If you use tables use appropriate number of points to ensure a good interpolation!

- Pressure traverse curves: graphical method to solve pressure drop calculations in vertical oil wells



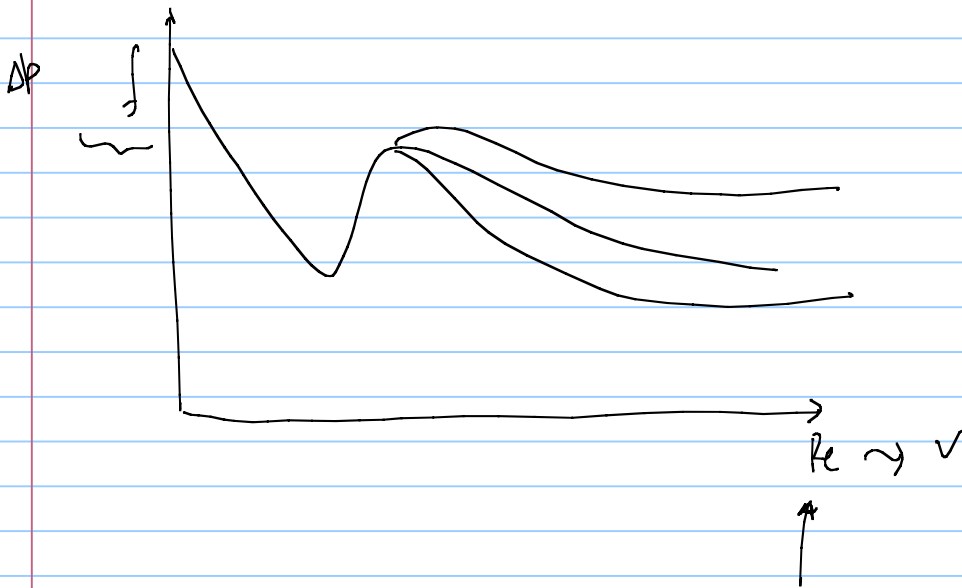
$$\frac{dp}{dz} = f(p) \sim$$



- Use the pressure traverses from the textbook to determine the Wellhead pressure for the following oil production well:

- Bottomhole flowing pressure – 2500 psi
- Well depth 9000 ft
- Tubing 2.441" ID (2 7/8")
- Total Flow rate 1000 bpd
- Water Cut 50%
- GLRp = 400 scf/bbl

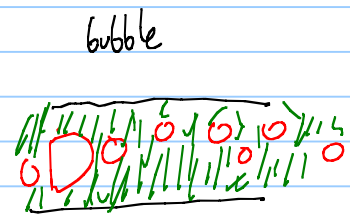
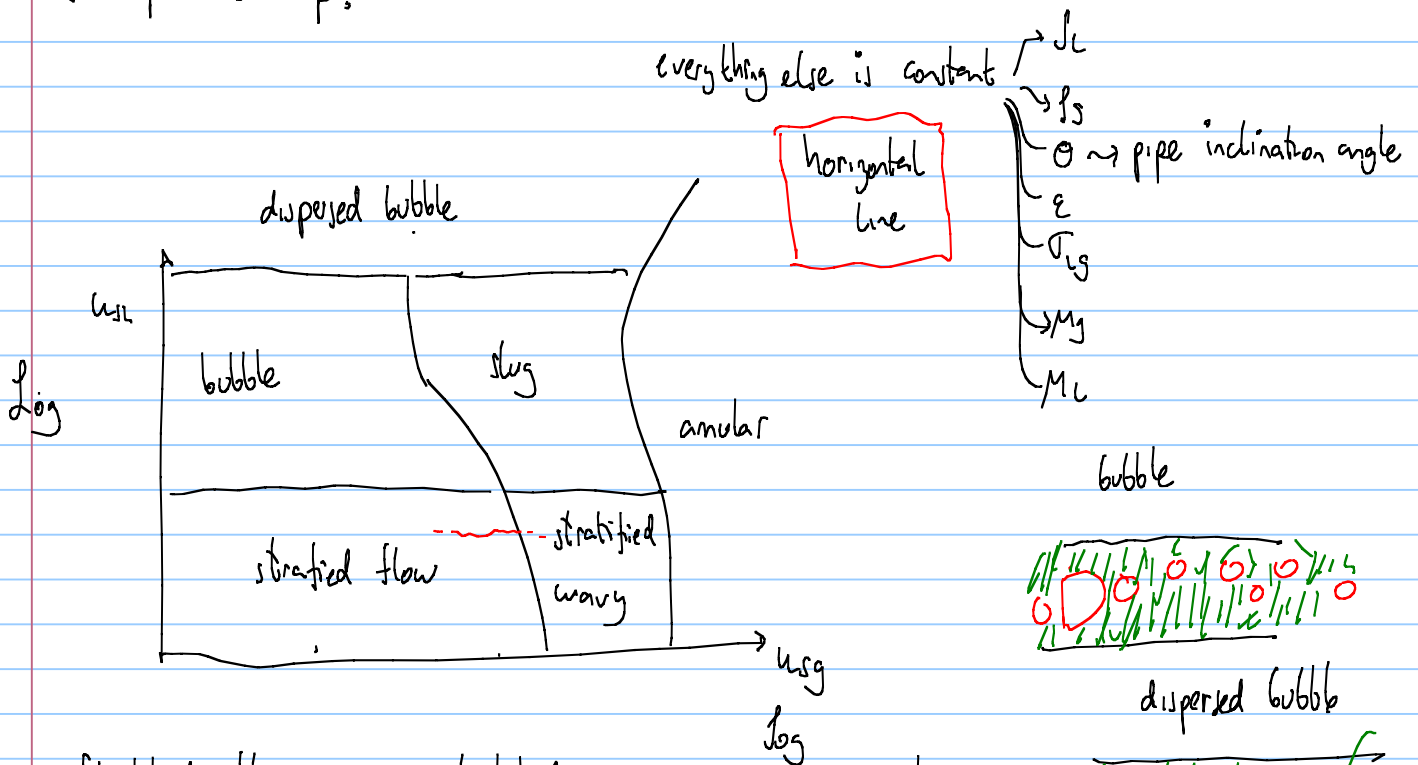
$$G'LR = \frac{q_{-}^{\vee}}{q_{\bar{0}} + q_{\bar{2}}}$$



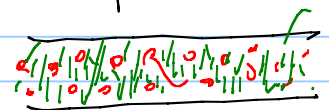
superficial velocity: $u_{sl} = \frac{q_L}{A}$ local volume rate @ p, T m/s
 A cross section area of pipe

$$u_{sg} = \frac{q_g}{A} \quad \text{m/s}$$

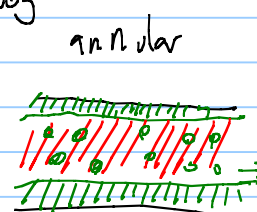
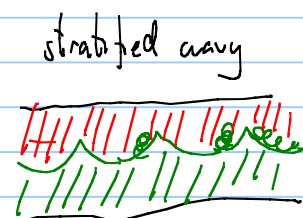
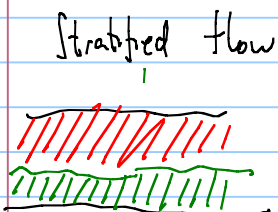
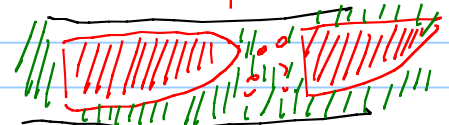
flow pattern map:



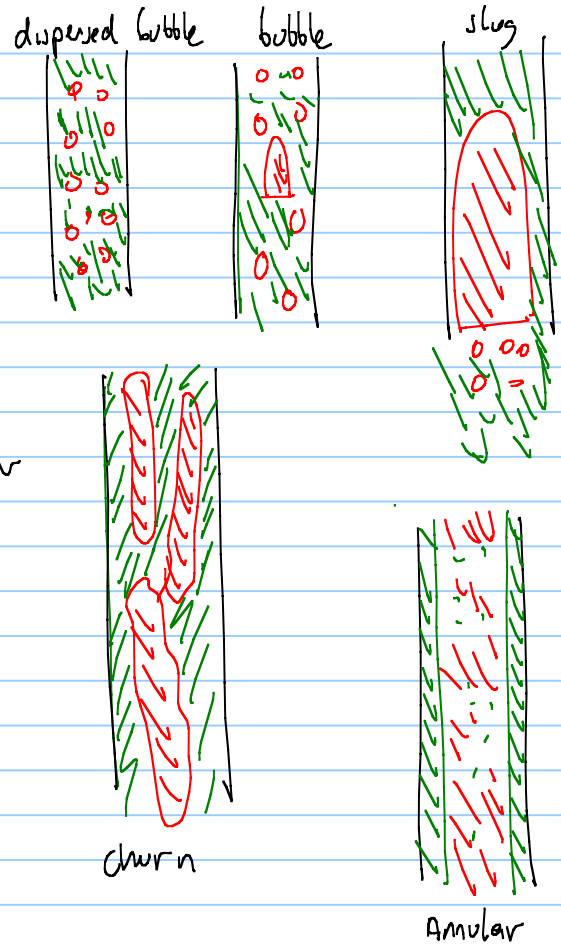
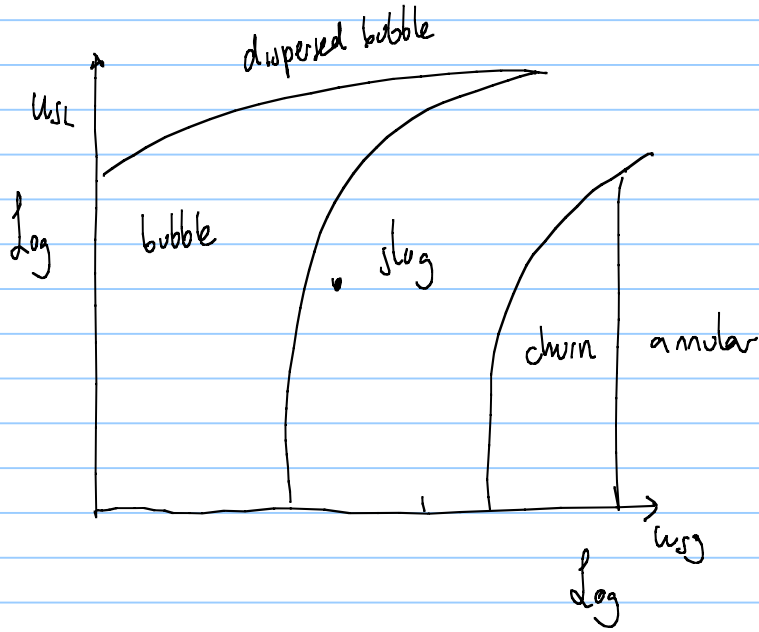
dispersed bubble



slug



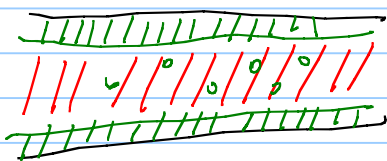
for vertical pipe



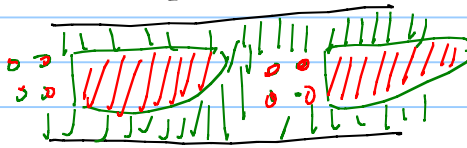
Re, We Weber number = $\frac{\rho \cdot V^2 \cdot d}{\sigma} \rightarrow$ inertia
 Re_s

Froude = $\frac{V^2}{gD}$ ✓

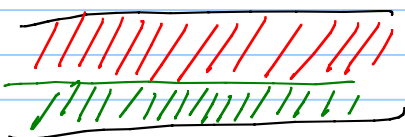
separated flow regimes
annular



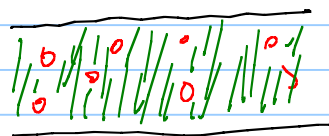
mixed flow regimes
slug



stratified



bubble



the transition and occurrence of flow patterns is dictated by an force equilibrium on the phase

segregating forces

gravitational acceleration
surface forces
viscosity

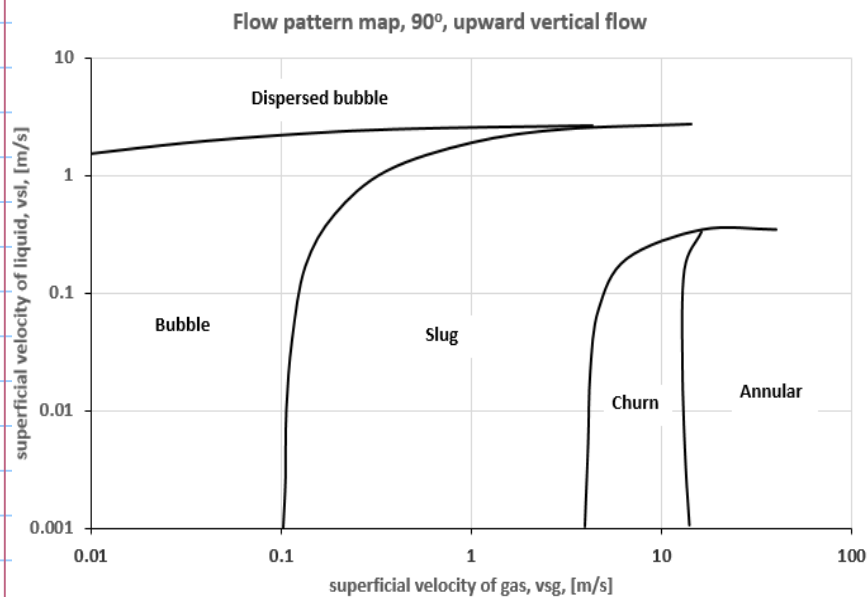
mixing forces

inertia V^2
viscosity

gradual transition
abrupt transition

class exercise to define flow pattern along tubing

Assumption: the map doesn't change along the tubing
it is not affected by changes in fluid properties

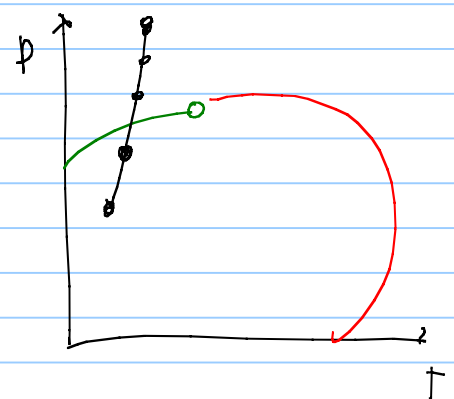
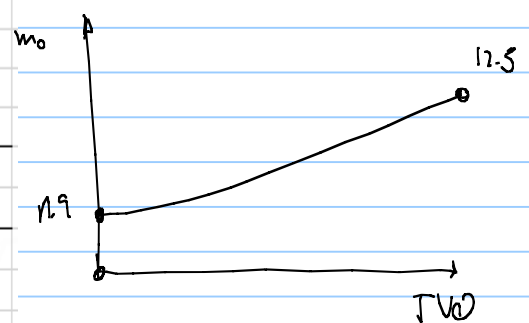


Vertical oil well

Tubing ID [m] 0.14

Cross section area [m²] 0.015

TVD [m]	m _o [kg/s]	m _g [kg/s]	ρ _{oil} [kg/m ³]	ρ _{gas} [kg/m ³]	q _o [m ³ /s]	q _g [m ³ /s]	v _{so} [m/s]	v _{sg} [m/s]
0	11.9	0.58	659.7	69.9				
610	12.1	0.37	630.0	89.3				
1219	12.5	0.04	591.8	114.3				
1829	12.5	0.00	577.7	-				
2438	12.5	0.00	568.6	-				
3048	12.5	0.00	561.5	-				

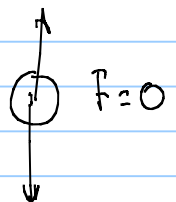
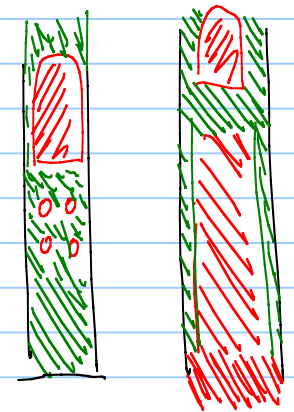
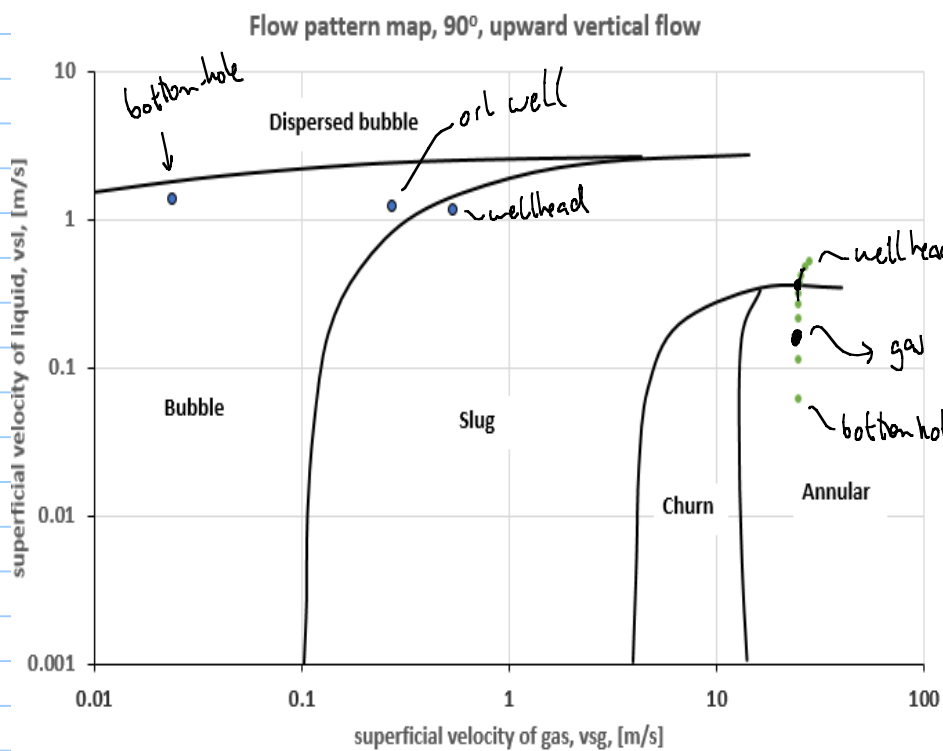


oil well

Vertical oil well								
Tubing ID [m]	0.14							
Cross section area [m ²]	0.015							
TVD [m]	m _o [kg/s]	m _g [kg/s]	ρ _{oil} [kg/m ³]	ρ _{gas} [kg/m ³]	q _o [m ³ /s]	q _g [m ³ /s]	v _{so} [m/s]	v _{sg} [m/s]
0	11.9	0.58	659.7	69.9	0.018	0.008	1.17	0.54
610	12.1	0.37	630.0	89.3	0.019	0.004	1.25	0.27
1219	12.5	0.04	591.8	114.3	0.021	0.000	1.37	0.02
1829	12.5	0.00	577.7	-	0.022	0.000	1.41	0.00
2438	12.5	0.00	568.6	-	0.022	0.000	1.43	0.00
3048	12.5	0.00	561.5	-	0.022	0.000	1.45	0.00

gas well

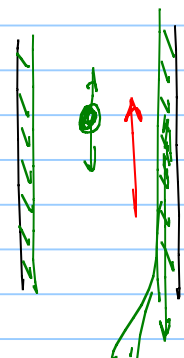
Vertical gas well								
Tubing ID [m]	0.157							
Cross section area [m ²]	0.019							
TVD	mw	mg	pw	p _{gas}	qw	qg	v _{sw}	v _{sg}
[m]	[kg/s]	[kg/s]	[kg/m ³]	[kg/m ³]	[m ³ /s]	[m ³ /s]	[m/s]	[m/s]
0	10.2	1.20E+01	997.0	22.0	0.010	0.545	0.53	28.17
284	9.2	1.30E+01	997.0	25.3	0.009	0.514	0.48	26.55
567	8.2	1.40E+01	997.0	28.2	0.008	0.496	0.42	25.63
851	7.2	1.50E+01	997.0	30.9	0.007	0.486	0.37	25.11
1135	6.2	1.60E+01	997.0	33.3	0.006	0.480	0.32	24.81
1418	5.2	1.70E+01	997.0	35.6	0.005	0.478	0.27	24.68
1702	4.2	1.80E+01	997.0	37.7	0.004	0.477	0.22	24.65
1986	3.2	1.90E+01	997.0	39.7	0.003	0.478	0.17	24.69
2269	2.2	2.00E+01	997.0	41.7	0.002	0.480	0.11	24.79
2553	1.2	2.10E+01	997.0	43.5	0.001	0.483	0.06	24.94
2837	0.0	2.22E+01	997.0	45.3	0.000	0.490	0.00	25.32

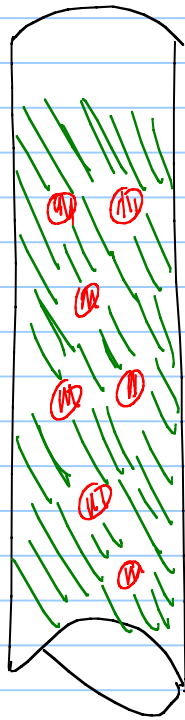


Short comment on liquid loading

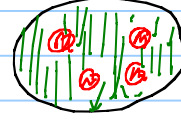
alternative explanation

wall slippage

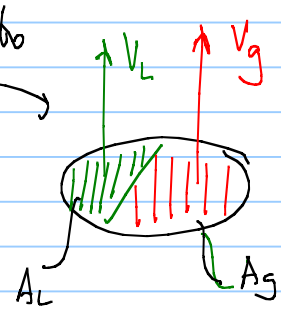
Transition from annular flow
to slug flow



1-D



equivalent to

homogeneous. $V_L = V_g$ non slip conditionnon homogeneous \rightarrow slip $V_L \neq V_g$

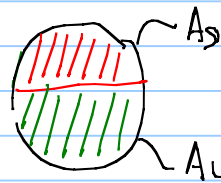
gas non-slip void fraction

$$\lambda_g = \frac{A_g}{A}$$

liquid no-slip holdup

$$\lambda_L = \frac{A_L}{A}$$

no-slip
 $V_g = V_L = V_m$



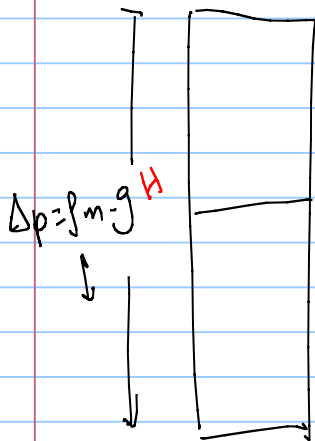
$$q_g = A_g \cdot V_m \quad (1)$$

$$q_L = A_L \cdot V_m \quad (2)$$

$$q_g + q_L = V_m (A_g + A_L)$$

$$V_m = \frac{q_g + q_L}{A} = u_{sg} + u_{sl}$$

$$\lambda_g, \lambda_L ? \text{ for } V_L = V_g = V_m$$

substitute $V_m = u_{sg} + u_{sl}$ in (1)

$$q_g = A_g \left(\frac{q_g + q_L}{A} \right) \Rightarrow \lambda_g = \frac{q_g}{q_g + q_L}$$

$$\lambda_L = \frac{q_L}{q_g + q_L}$$

$$\rho_m = \lambda_g \rho_g + \lambda_L \rho_L$$

But often $V_L \neq V_g$

void fraction

$$\epsilon = \frac{A_g}{A}$$

liquid holdup

$$H_L = \frac{A_L}{A}$$

 $V_g > V_m$ $\epsilon < \lambda_g$

$$q_L = \lambda_g \cdot A \cdot V_m = A \cdot \epsilon \cdot V_g$$

V_g V_L real gas and liquid velocities

S .. Slip ratio $\frac{V_g}{V_L}$ relative velocity
 $u_r = V_g - V_L$

$$\varepsilon = \frac{A_g}{A} \quad H_L = \frac{A_L}{A}$$

Drift velocity

$$\varepsilon + H_L = \frac{A_g + A_L}{A} = 1 \quad u_d = V_g - u_m$$

$$H_L = (1 - \varepsilon)$$

$$H_L = f(\theta, \phi, p, \rho_L, \rho_g, u_{sL}, u_{sg})$$

Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

Melkamu A. Woldesemayat, Afshin J. Ghajar *

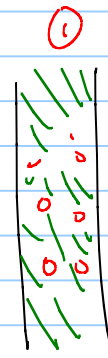
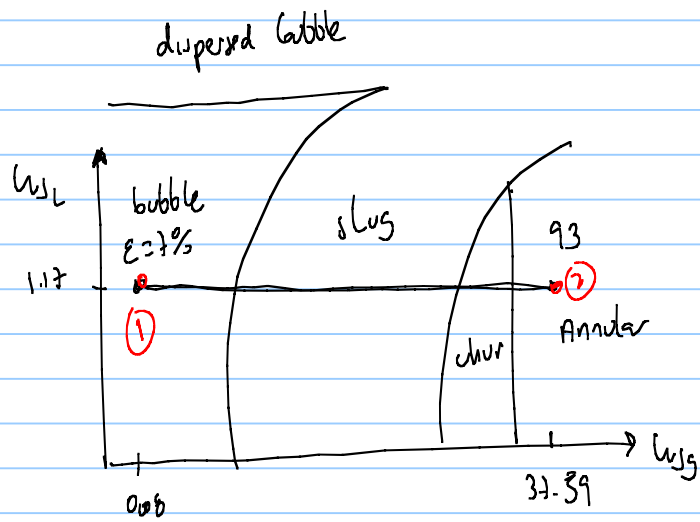
School of Mechanical and Aerospace Engineering, Oklahoma State University, Stillwater, OK 74078, USA

Received 1 June 2006; received in revised form 13 September 2006

$$\varepsilon = \frac{U_{SG}}{U_{SG} \left(1 + \left(\frac{U_{SL}}{U_{SG}} \right)^{0.1} \left(\frac{\rho_G}{\rho_L} \right)^{0.1} \right) + 2.9 \left[\frac{g D \sigma (1 + \cos \theta) (\rho_L - \rho_G)}{\rho_L^2} \right]^{0.25} (1.22 + 1.22 \sin \theta)^{\frac{P_{atm}}{P_{system}}}}$$

8 parameters.

p	[bara]	120												
denl	[kg/m^3]	659.7												
deng	[kg/m^3]	69.9												
D	[m]	0.14												
A	[m^2]	0.015												
teta	[deg]	90												
sigma_lg	[N/m]	0.07												
ql	qg	usl	usg	um	lambdag	Hg	Al	Ag	ul	ug	ur	S	ug-um	
[m^3/d]	[m^3/d]	[m/s]	[m/s]	[m/s]	[-]	[-]	[m^2]	[m^2]	[m/s]	[m/s]	[m/s]	[-]	[m/s]	
1561.5	100	1.17	0.08	1.25	0.06	0.07	0.014	0.001	1.26	1.06	-0.20	0.84	-0.19	
1561.5	712.325	1.17	0.54	1.71	0.31	0.29	0.011	0.004	1.65	1.85	0.20	1.12	0.14	
1561.5	1000	1.17	0.75	1.93	0.39	0.35	0.010	0.005	1.81	2.14	0.33	1.18	0.21	
1561.5	2500	1.17	1.88	3.05	0.62	0.54	0.007	0.008	2.55	3.48	0.93	1.37	0.43	
1561.5	5000	1.17	3.76	4.93	0.76	0.68	0.005	0.010	3.63	5.56	1.92	1.53	0.62	
1561.5	10000	1.17	7.52	8.69	0.86	0.79	0.003	0.012	5.55	9.54	3.99	1.72	0.84	
1561.5	25000	1.17	18.80	19.97	0.94	0.89	0.002	0.014	10.51	21.16	10.64	2.01	1.19	
1561.5	50000	1.17	37.59	38.77	0.97	0.93	0.001	0.014	17.71	40.26	22.55	2.27	1.50	



$\varepsilon = 7\%$



$\varepsilon = 93\%$

Homework

$$S \text{ vs } w_{sg}$$

$$S \text{ vs } H_L (1 - \varepsilon)$$

Day 9:

- methods to compute multiphase flow $\leadsto \frac{dp}{dx}$
- the drift flux model \leadsto
- Conversion from local to standard conditions using BO properties
- Exercise
- Pressure integration in multiphase flow
- Exercise
- final comments - end

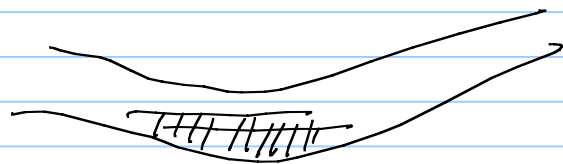
two approaches to study multiphase flow



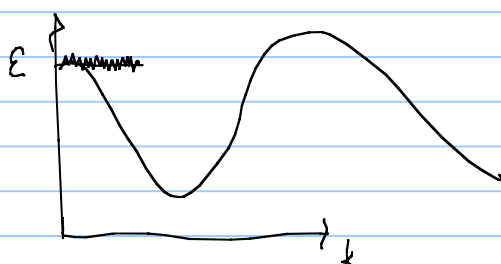
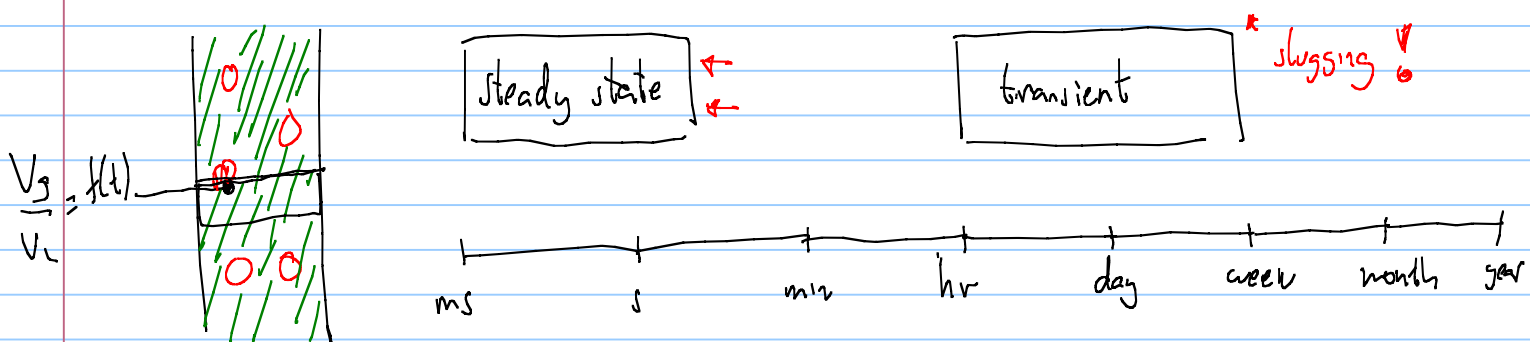
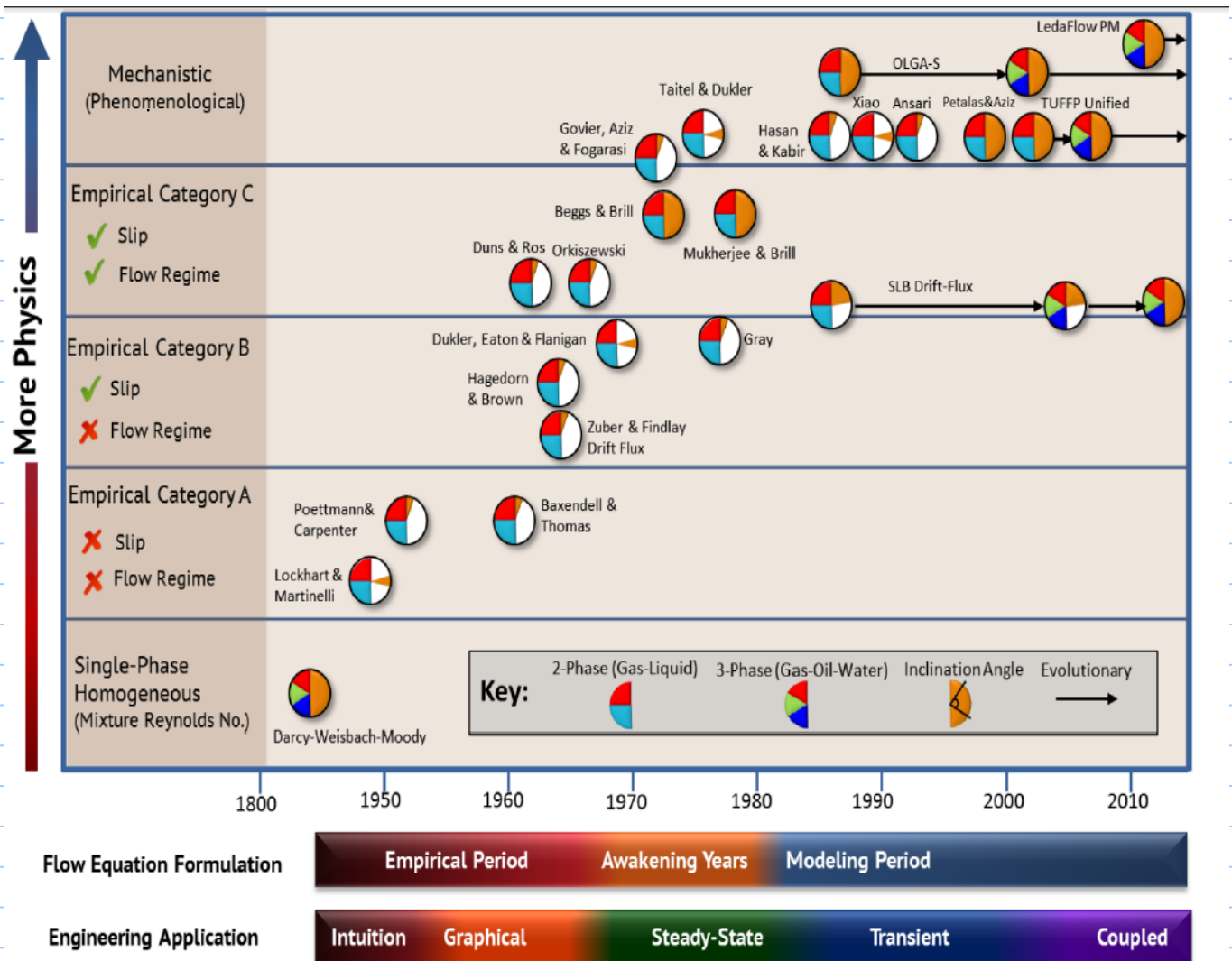
in this course we are interested mainly in $\frac{dp}{dx}$!
however, there are also some other problems:

~~liquid accumulation and slugging~~ TOP of line corrosion

- O+W emulsion
- liquid accumulation and slugging
- sand erosion
- hydrate and wax --
- etc



Shippen and Bailey (2012)



Drift flux model 1 momentum equation for mixture using average properties (similar to single phase)

$$\frac{dp}{dl} = -f_{TP} \cdot \rho_{TP} \cdot \frac{V_{TP}^2}{2\phi} - \rho_{TP} \cdot \sin \theta \cdot g - \rho_{TP} \cdot V_{TP} \cdot \frac{dV_{TP}}{dl}$$

$$\rho_{TP} = \varepsilon \rho_g + (1-\varepsilon) \rho_L$$

slip
↓

if λ_g is used then homogeneous model

$$V_{TP} = u_m = u_{SL} + u_{sg}$$

$\frac{dV_m}{dl} \approx 0 \rightarrow$ neglecting fluid expansion in segment and no change in cross section area.

$$\frac{dp}{dl} = -f_{TP} \cdot \rho_{TP} \cdot \frac{V_{TP}^2}{2\phi} - \rho_{TP} \sin \theta \cdot g$$

Pressure integration procedure for steady state multiphase flow in wellbores (T is given)

① • discretize the tubing in segments

② • start for point with pressure known (P_{wh})

• with P, T compute local rate of oil, gas and compute all properties

calculate B_o, B_g, ρ_o, ρ_g

and compute all properties $\left\{ \rho_o, \rho_g, \mu_o, \mu_g, \sigma_{og} \right\}$

• compute superficial velocities on that point $u_{so} = \frac{q_o}{A}$ $u_{sg} = \frac{q_g}{A}$

• compute pressure gradient at that point

$\frac{dp}{dl}$ $\begin{cases} \nearrow \text{drift flux} \\ \rightarrow \text{mechanistic model} \\ \searrow \text{empirical model} \\ \downarrow \text{multiphase export.} \end{cases}$

- integrate numerically the differential equation $\frac{dp}{dl} = C$ $P(l=0)$ known.
- use explicit integration method $\left\{ \begin{array}{l} \text{euler} \\ \text{runge-kutta} \end{array} \right.$
- use an implicit integration method.

Warning!
Euler requires
very small
intervals

→ use EULER
explicit.

$$P_1 = P_{wh} - \left. \frac{dp}{dl} \right|_{l=0} \cdot \Delta l$$

• P_1 . repeat from point ②

We need to compute q_o q_g from q_o q_g
using black oil tables

Single phase gas

$$q_g = B_g \cdot q_g^-$$

undersaturated oil

$$q_o = B_o \cdot q_o^-$$

assume $r_s = 0$

$$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \frac{B_g}{1-R_s \cdot r_s} & \frac{-B_g \cdot R_s}{1-R_s \cdot r_s} & 0 \\ -B_o \cdot r_s & B_o & 0 \\ 0 & 0 & B_w \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_g^- \\ q_o^- \\ q_w^- \end{bmatrix}$$

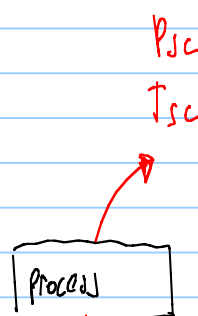
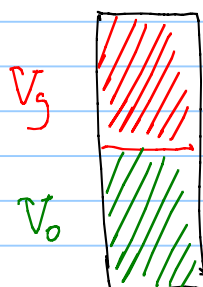
Local conditions calculated from standard
conditions traditional BO-approach

definition

$$B_o(p, T) = \frac{V_o(p, T)}{V_o^-}$$

$$B_g(p, T) = \frac{V_g(p, T)}{V_g^-}$$

@ p, T



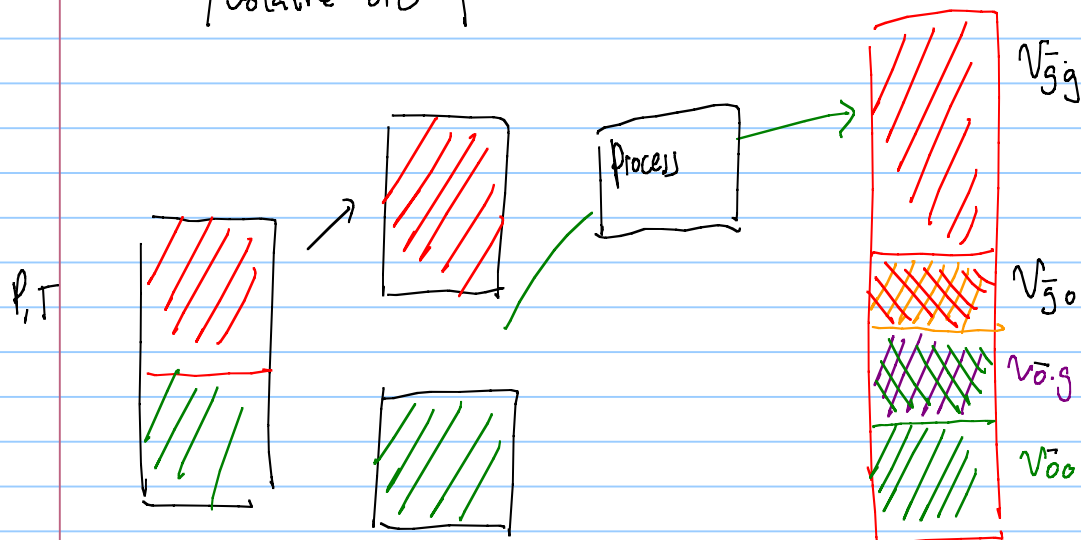
$$R_s(p, T) = \frac{V_{\bar{o}}}{V_{\bar{o}}}$$

$$q_s = B_g q_{\bar{g}} - B_g R_s q_{\bar{o}}$$

$$q_o = B_o q_{\bar{o}}$$

if r_s is important keep track of where surface oil and surface gas is coming from

$\left\{ \begin{array}{l} \text{gas condensate} \\ \text{wet gas} \\ \text{volatile oil} \end{array} \right\}$



$$r_s = \frac{V_{\bar{o}o}}{V_{\bar{g}g}}$$

$$R_s = \frac{V_{\bar{g}o}}{V_{\bar{o}o}}$$

BO Variable	Definition
Oil Volume Factor	$B_o(p, T) = \frac{V_o(p, T)}{V_{\bar{o}o}}$
Gas Volume Factor	$B_g(p, T) = \frac{V_g(p, T)}{V_{\bar{g}g}}$
Solution Gas Oil Ratio	$R_s(p, T) = \frac{V_{\bar{g}o}}{V_{\bar{o}o}}$
Solution Oil Gas ratio	$r_s(p, T) = \frac{V_{\bar{o}g}}{V_{\bar{g}g}}$

$$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \frac{B_g}{1-R_s \cdot r_s} & \frac{-B_g \cdot R_s}{1-R_s \cdot r_s} & 0 \\ \frac{-B_o \cdot r_s}{1-R_s \cdot r_s} & \frac{B_o}{1-R_s \cdot r_s} & 0 \\ 0 & 0 & B_w \end{bmatrix} \cdot \begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix} \quad \text{at } (p, T)$$

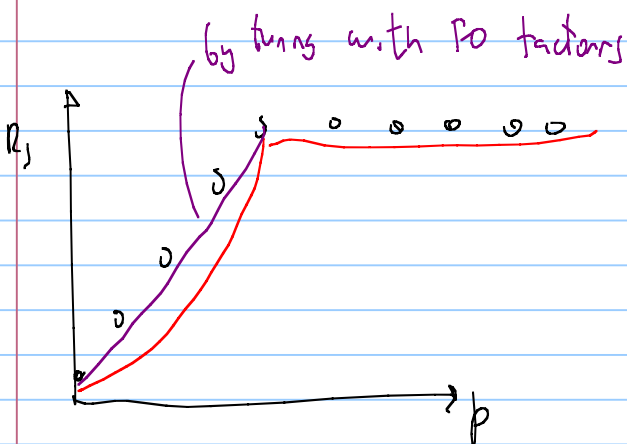
Local conditions calculated from standard conditions

qg [Sm ³ /d]	60000	qg [m ³ /d]	6.29E+02		
qo [Sm ³ /d]	500	qo [m ³ /d]	620.4		
Conversion matrix for					
p	Bo	Bg	Rs	rs	
[bara]	[m ³ /Sm ³]	[m ³ /Sm ³]	[Sm ³ /Sm ³]	[Sm ³ /Sm ³]	
160	1.44	8.17E-03	105.22	3.92E-05	1.65E-02 -7.22E-01
120	1.33	1.09E-02	72.47	2.40E-05	-2.10E-05 1.24E+00
80	1.24	1.65E-02	43.74	1.69E-05	

qg [Sm ³ /d]	60000	qg [m ³ /d]	6.29E+02		
qo [Sm ³ /d]	500	qo [m ³ /d]	621.2		
Conversion matrix for					
p	Bo	Bg	Rs	rs	
[bara]	[m ³ /Sm ³]	[m ³ /Sm ³]	[Sm ³ /Sm ³]	[Sm ³ /Sm ³]	
160	1.44	8.17E-03	105.22	3.92E-05	1.65E-02 -7.21E-01
120	1.33	1.09E-02	72.47	2.40E-05	0.00E+00 1.24E+00
80	1.24	1.65E-02	43.74	1.69E-05	

assuming $r_s = 0$

Class exercise: $\frac{dp}{dx}$ calculations in tubing



BO properties come from correlations

$$f \left(p, T, \gamma_o, GOR, \gamma_g \right)$$

tuning BO correlations $\left\{ \begin{array}{l} FO1, FO2, FO3, FO4, FO5 \\ FO6 \end{array} \right.$

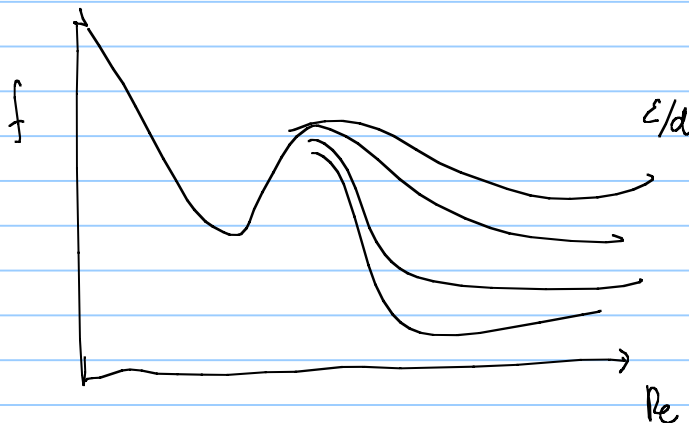
in field units

With known qd calculate pwf for $q_s = 1000$ stb/d

$$\frac{dp}{dx} = -\rho_m \cdot \sin \theta \cdot g - \frac{1}{2} \frac{f_{TP}}{\phi} \frac{V_m^2}{\rho_m}$$

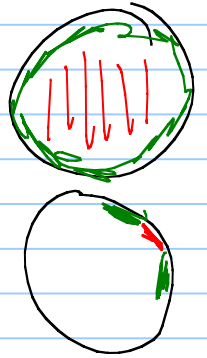
$$\rho_m = \epsilon \rho_g + (1-\epsilon) \rho_o$$

$$u_m = u_{so} + u_{sg}$$



$$\frac{u_m \cdot \rho_m \cdot \phi}{V \cdot \rho \cdot \phi}$$

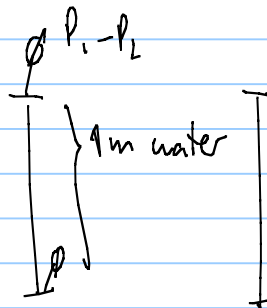
$$\mu_m = \epsilon(\mu_g) + (1-\epsilon)\mu_o$$



$$\gamma_o = \gamma_o \cdot B_o(p, T)$$

$$\gamma_g = \gamma_g \cdot B_g - \gamma_o \cdot k_s \cdot B_g$$

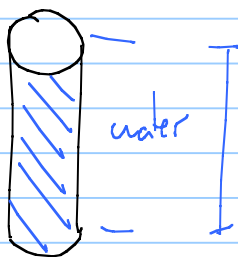
QC availability control



$$\Delta p = \rho \cdot g \cdot \Delta h = 1000 \cdot 10 \cdot 1 \text{ Pa}$$

$$10.000 \text{ Pa}$$

$$\approx 0.1 \text{ bar}$$



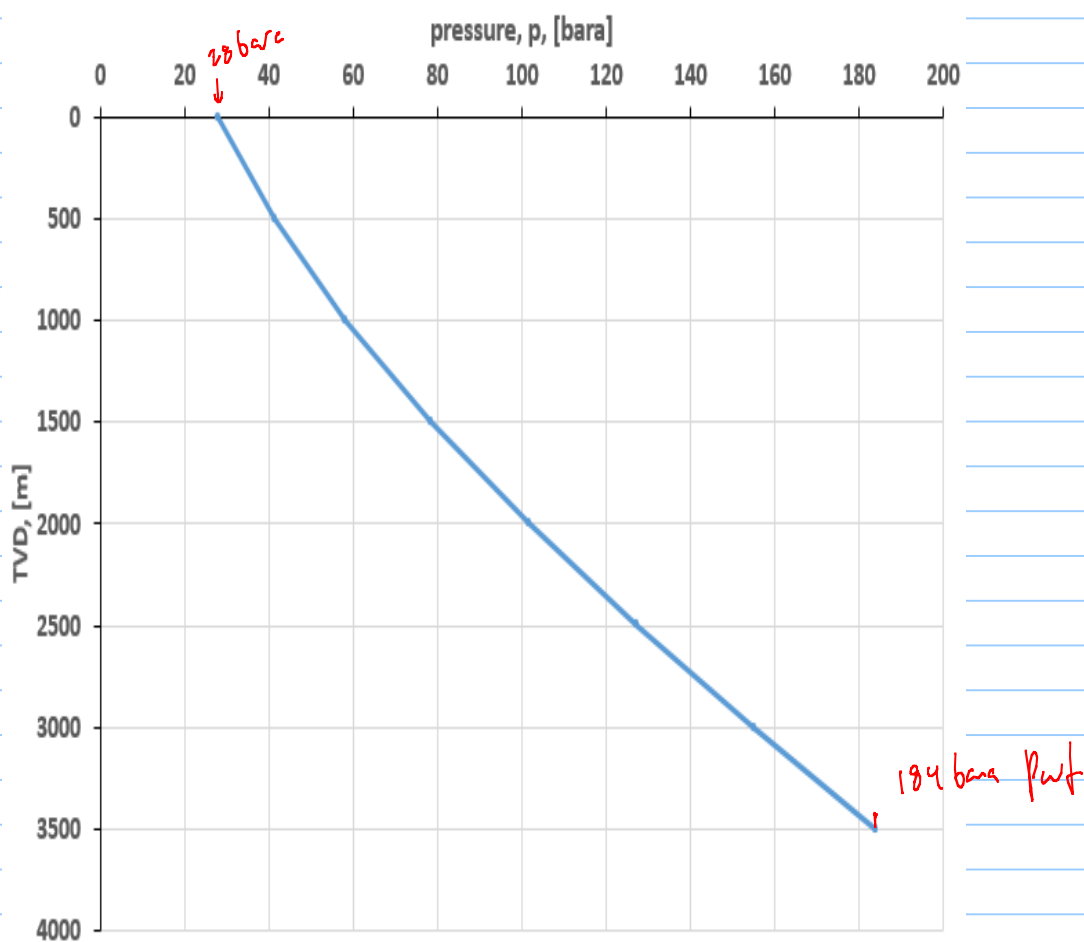
$$\Delta p = \rho_w \cdot \Delta L \cdot g < 10 \cdot 1 \cdot 1000 = 10000 \text{ Pa}$$

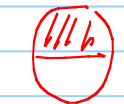
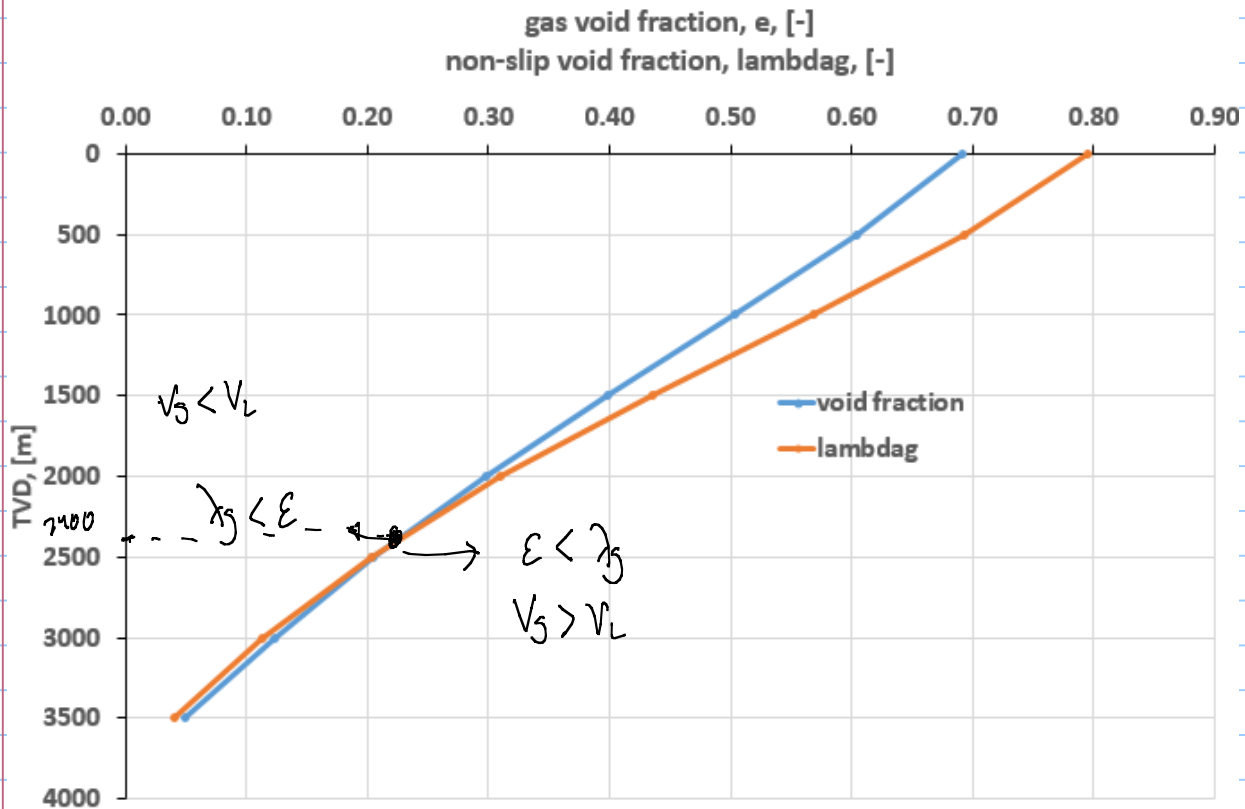
$$\frac{1 \text{ bar}}{10^5 \text{ Pa}} = 0.1 \text{ bar/m}$$



we are getting $0.0267 \frac{\text{bar}}{\text{m}} \approx 2.67\% \text{ water}$

Bg [m ³ /Sm ³]	deng [kg/m ³]	viscg [cp]	deno [kg/m ³]	viso [cp]	sigma_o_g [N/m]	qo [m ³ /d]	qg[m ³ /d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
3.229E-02	40.3	0.011	794.4	2.518	1.624E-02	1083.4	4.210E+03	0.710	2.757	0.80	0.69	0.0267
2.163E-02	60.1	0.012	770.6	1.295	1.279E-02	1137.2	2.560E+03	0.745	1.676	0.69	0.60	0.0335
1.528E-02	85.1	0.013	744.6	0.813	9.500E-03	1202.6	1.582E+03	0.788	1.036	0.57	0.50	0.0404
1.142E-02	113.8	0.015	717.5	0.575	6.668E-03	1279.3	9.867E+02	0.838	0.646	0.44	0.40	0.0467
9.100E-03	142.9	0.018	690.4	0.440	4.515E-03	1364.9	6.143E+02	0.894	0.402	0.31	0.30	0.0517
7.711E-03	168.6	0.020	664.8	0.356	3.044E-03	1455.7	3.701E+02	0.953	0.242	0.20	0.21	0.0552
6.867E-03	189.3	0.022	641.4	0.301	2.097E-03	1547.5	1.986E+02	1.014	0.130	0.11	0.12	0.0575
6.334E-03	205.2	0.024	620.5	0.263	1.493E-03	1637.0	6.988E+01	1.072	0.046	0.04	0.05	0.0589





$$\epsilon = \frac{A_g}{A}$$

slip

$$\lambda_g = \frac{q_g}{q_l + q_g}$$

assuming
 $v_l = v_g = v_m$

Home exercise • Case 2 $p_{wf} = 200 \text{ bara}$, $q_o = 1000 \text{ stb/d}$
 $p_{wh} ?$

$$P. < 100 + \frac{dp}{dx} \cdot \Delta x < 0$$



THANK YOU FOR YOUR ACTIVE PARTICIPATION!

--THE END--