

Day 9:

- methods to compute multiphase flow $\leadsto \frac{dp}{dx}$
- the drift flux model \leadsto
- Conversion from local to standard conditions using BO properties
- Exercise
- Pressure integration in multiphase flow
- Exercise
- final comments - end

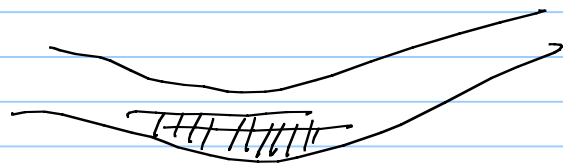
two approaches to study multiphase flow



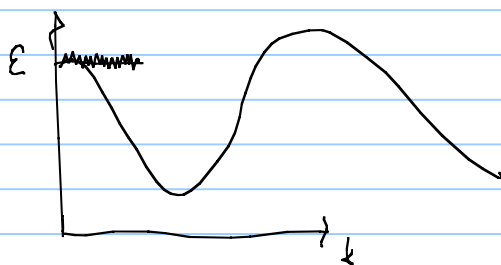
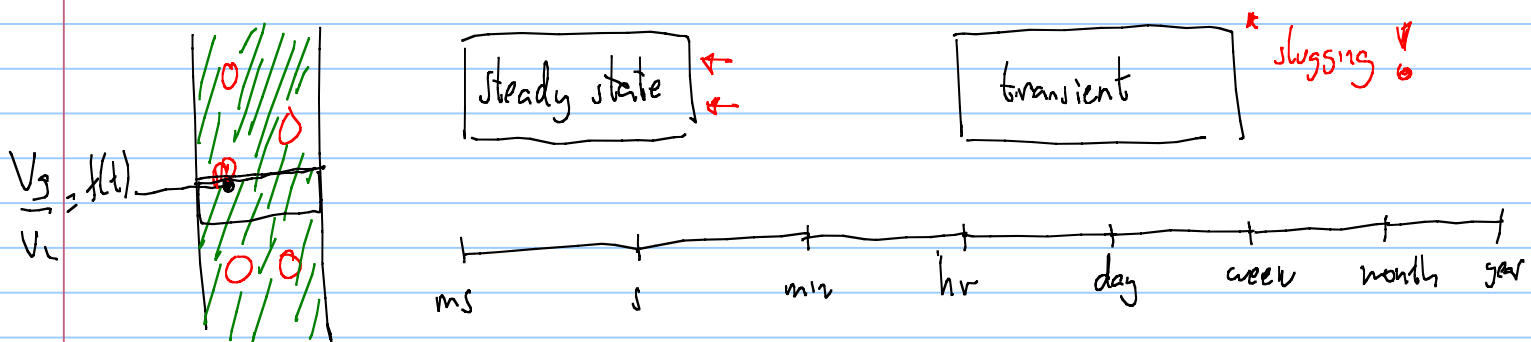
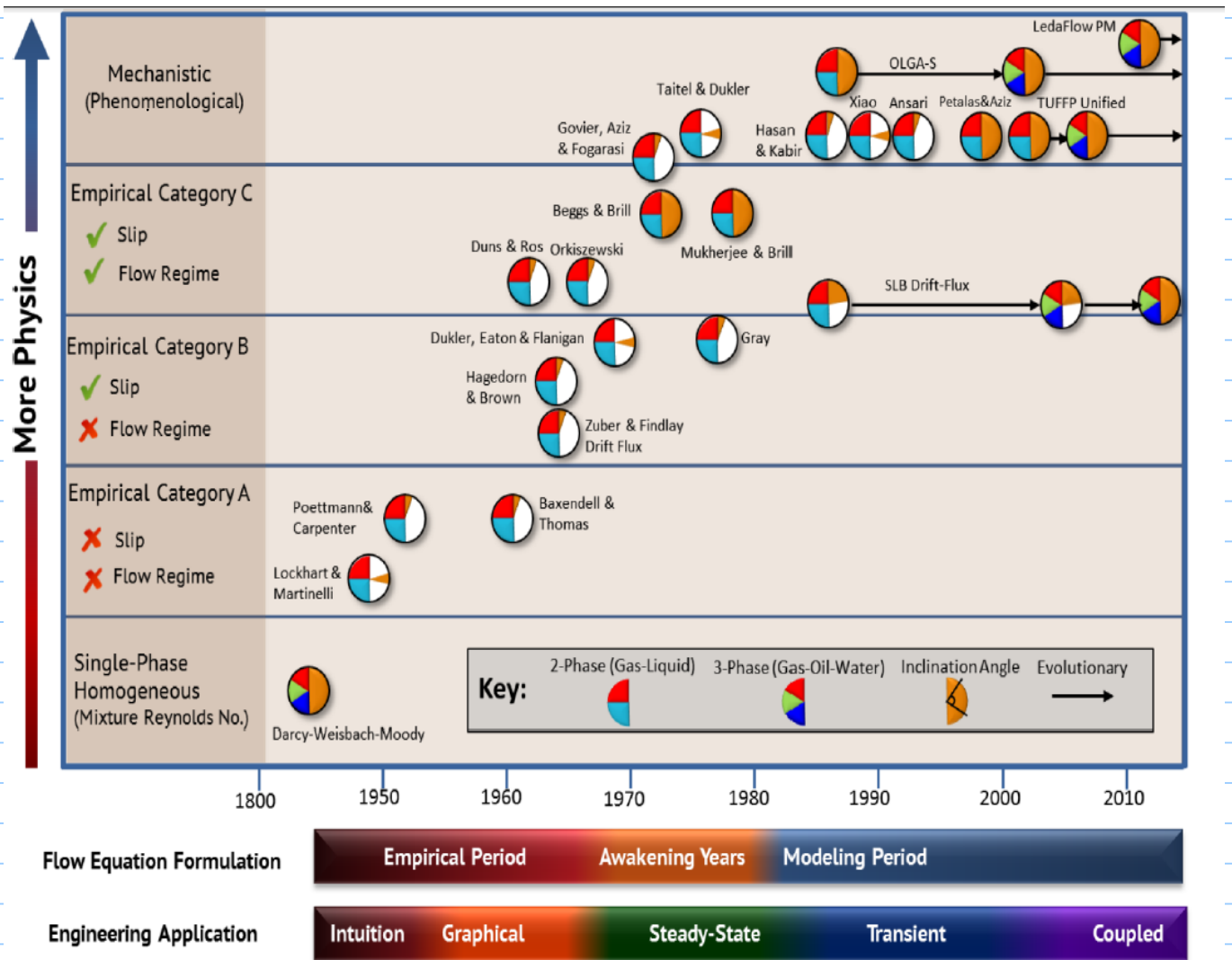
in this course we are interested mainly in $\frac{dp}{dx}$!
however, there are also some other problems:

~~underflow~~ TOP of line corrosion

- O+H emulsion
- liquid accumulation and slugging
- sand erosion
- hydrate and wax --
- etc



Shippen and Bailey (2012)



Drift flux model 1 momentum equation for mixture using average properties (similar to single phase)

$$\frac{dp}{dl} = -f_{TP} \cdot \rho_{TP} \cdot \frac{V_{TP}^2}{2\phi} - \rho_{TP} \cdot \sin \theta \cdot g - \rho_{TP} \cdot V_{TP} \cdot \frac{dV_{TP}}{dl}$$

slip
↓

$$\rho_{TP} = \epsilon \rho_g + (1-\epsilon) \rho_L$$

if λ_g is used then homogeneous model

$$V_{TP} = u_m = u_{SL} + u_{sg}$$

$\frac{dV_m}{dl} \approx 0 \rightarrow$ neglecting fluid expansion in segment and no change in cross section area.

$$\frac{dp}{dl} = -f_{TP} \cdot \rho_{TP} \cdot \frac{V_{TP}^2}{2\phi} - \rho_{TP} \sin \theta \cdot g$$

Pressure integration procedure for steady state multiphase flow in wellbores (T is given)

① • discretize the tubing in segments

② • start for point with pressure known (P_{wh})

• with P, T compute local rate of oil, gas and compute all properties

calculate B_o, B_g, ρ_o, ρ_g

and compute all properties $\left\{ \rho_o, \rho_g, \mu_o, \mu_g, \sigma_{og} \right\}$

• compute superficial velocities on that point $u_{so} = \frac{q_o}{A}$ $u_{sg} = \frac{q_g}{A}$

• compute pressure gradient at that point

$\frac{dp}{dl}$ $\begin{cases} \nearrow \text{drift flux} \\ \rightarrow \text{mechanistic model} \\ \searrow \text{empirical model} \\ \downarrow \text{multiphase export.} \end{cases}$

- integrate numerically the differential equation $\frac{dp}{dl} = C$ $P(l=0)$ known.
- use explicit integration method { euler
runge-kutta
- use an implicit integration method.

Warning!
Euler requires
very small
intervals

→ use Euler explicit.

$$P_1 = P_{wh} - \left. \frac{dp}{dl} \right|_{l=0} \cdot \Delta l$$

• P_1 . repeat from point ②

We need to compute q_o q_g from q_o q_g
using black oil tables

Single phase gas

$$q_g = B_g \cdot q_g^-$$

undersaturated oil

$$q_o = B_o \cdot q_o^-$$

assume $r_s = 0$

$$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \frac{B_g}{1-R_s \cdot r_s} & \frac{-B_g \cdot R_s}{1-R_s \cdot r_s} & 0 \\ \frac{-B_o \cdot r_s}{1-R_s \cdot r_s} & \frac{B_o}{1-R_s \cdot r_s} & 0 \\ 0 & 0 & B_w \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_g^- \\ q_o^- \\ q_w^- \end{bmatrix}$$

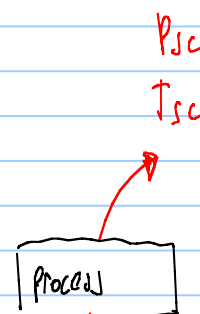
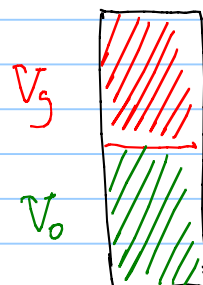
Local conditions calculated from standard
conditions traditional BO-approach

definition

$$B_o(p, T) = \frac{V_o(p, T)}{V_o^-}$$

$$B_g(p, T) = \frac{V_g(p, T)}{V_g^-}$$

@ p, T



$$R_s(p, T) = \frac{V_{\bar{o}}}{V_{\bar{o}_0}}$$

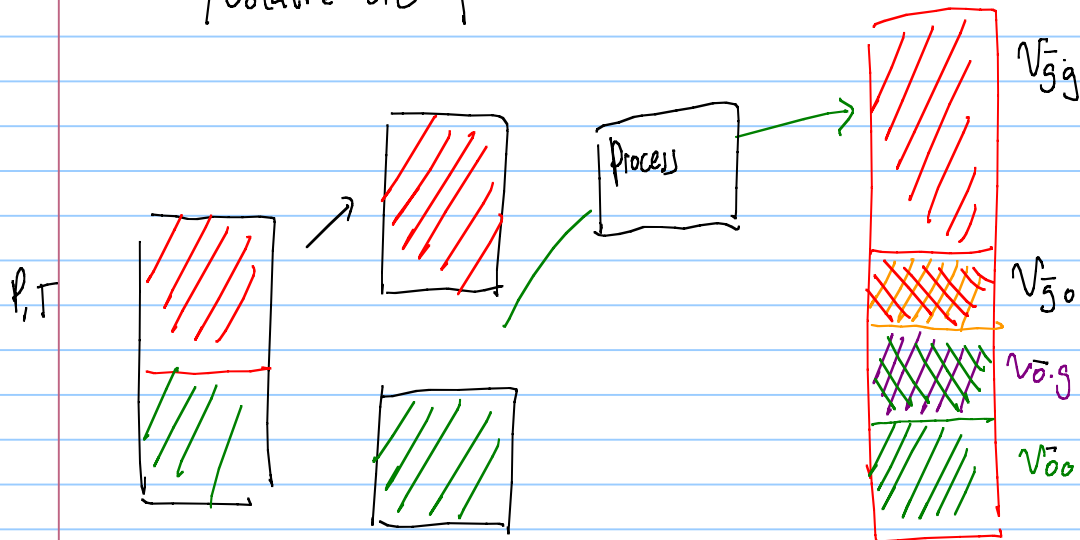
$$q_s = B_g q_{\bar{g}} - B_g R_s q_{\bar{o}}$$

$$q_o = B_o q_{\bar{o}}$$

if r_s is important keep track of where surface oil and surface gas is coming from

{ gas condensate
 wet gas
 volatile oil

T_{sc}, p_{sc}



$$r_s = \frac{V_{\bar{o}o}}{V_{\bar{g}g}}$$

$$R_s = \frac{V_{\bar{g}o}}{V_{\bar{o}o}}$$

BO Variable	Definition
Oil Volume Factor	$B_o(p, T) = \frac{V_o(p, T)}{V_{\bar{o}o}}$
Gas Volume Factor	$B_g(p, T) = \frac{V_g(p, T)}{V_{\bar{g}g}}$
Solution Gas Oil Ratio	$R_s(p, T) = \frac{V_{\bar{g}o}}{V_{\bar{o}o}}$
Solution Oil Gas ratio	$r_s(p, T) = \frac{V_{\bar{o}g}}{V_{\bar{g}g}}$

$$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \frac{B_g}{1-R_s \cdot r_s} & \frac{-B_g \cdot R_s}{1-R_s \cdot r_s} & 0 \\ \frac{-B_o \cdot r_s}{1-R_s \cdot r_s} & \frac{B_o}{1-R_s \cdot r_s} & 0 \\ 0 & 0 & B_w \end{bmatrix} \cdot \begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix} \quad \text{at } (p, T)$$

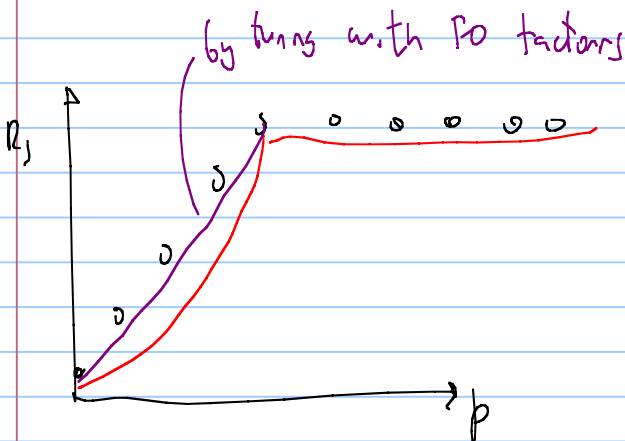
Local conditions calculated from standard conditions

qg [Sm ³ /d]	60000	qg [m ³ /d]	6.29E+02				
qo [Sm ³ /d]	500	qo [m ³ /d]	620.4				
Conversion matrix for							
p	Bo	Bg	Rs	rs			
[bara]	[m ³ /Sm ³]	[m ³ /Sm ³]	[Sm ³ /Sm ³]	[Sm ³ /Sm ³]	1.65E-02	-7.22E-01	
160	1.44	8.17E-03	105.22	3.92E-05	-2.10E-05	1.24E+00	
120	1.33	1.09E-02	72.47	2.40E-05			
80	1.24	1.65E-02	43.74	1.69E-05			

qg [Sm ³ /d]	60000	qg [m ³ /d]	6.29E+02				
qo [Sm ³ /d]	500	qo [m ³ /d]	621.2				
Conversion matrix for							
p	Bo	Bg	Rs	rs			
[bara]	[m ³ /Sm ³]	[m ³ /Sm ³]	[Sm ³ /Sm ³]	[Sm ³ /Sm ³]	1.65E-02	-7.21E-01	
160	1.44	8.17E-03	105.22	3.92E-05	0.00E+00	1.24E+00	
120	1.33	1.09E-02	72.47	2.40E-05			
80	1.24	1.65E-02	43.74	1.69E-05			

assuming $r_s = 0$

Class exercise: $\frac{dp}{dx}$ calculations in tubing



BO properties come from correlations

$$\left\{ p, T, \gamma_o, GOR, \gamma_g \right\}$$

tuning BO correlations $\left\{ FO1, FO2, FO3, FO4, FO5, FO6 \right\}$

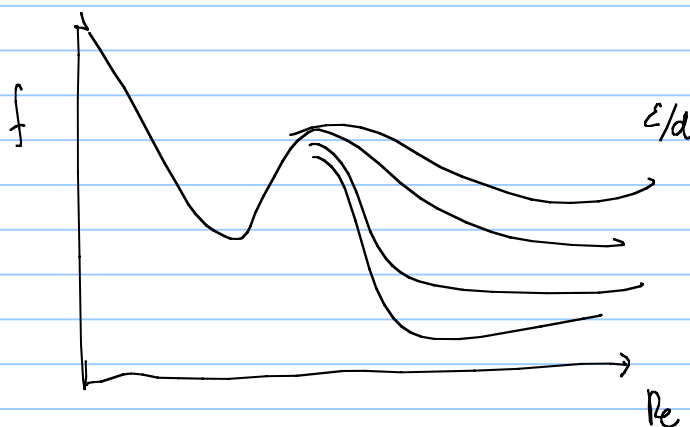
in field units

With known qd calculate pwf for $q_s = 1000$ stb/d

$$\frac{dp}{dx} = -f_m \cdot \sin \theta - \frac{1}{2} \frac{f_{ro}}{\phi} \frac{V_m^2}{\phi} f_m$$

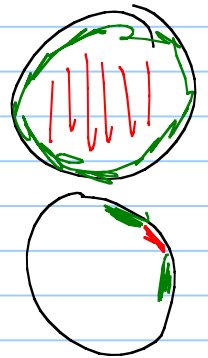
$$f_m = \epsilon f_g + (1-\epsilon) f_o$$

$$u_m = u_{so} + u_{sg}$$



$$\frac{u_m \cdot f_m}{V \cdot \rho \cdot \phi}$$

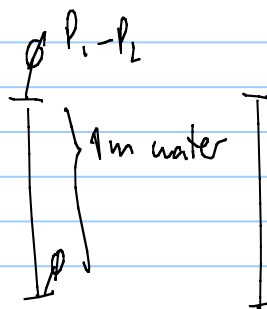
$$\mu_m = \epsilon (\mu_s) + (1-\epsilon) \mu_o$$



$$q_o = q_o \cdot B_o(p, r)$$

$$q_s = q_s B_g - q_o k_s B_g$$

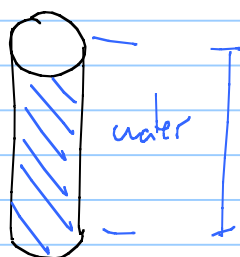
QC availability control



$$\Delta p = \rho \cdot g \cdot \Delta h = 1000 \cdot 10 \cdot 1 \text{ Pa}$$

$$10.000 \text{ Pa}$$

$$\approx 0.1 \text{ bar}$$



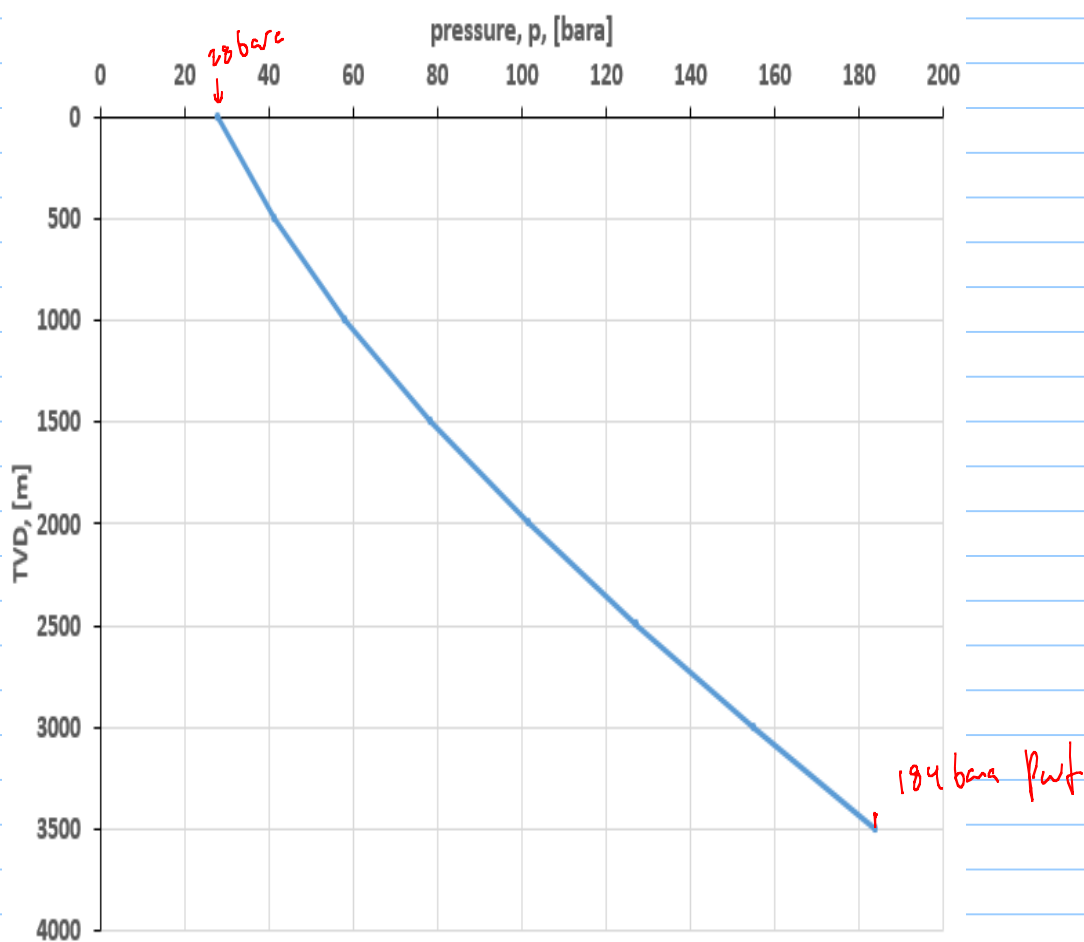
$$\Delta p = \rho_w \cdot \Delta L \cdot g < 10 \cdot 1 \cdot 1000 = 10000 \text{ Pa}$$

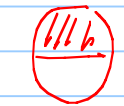
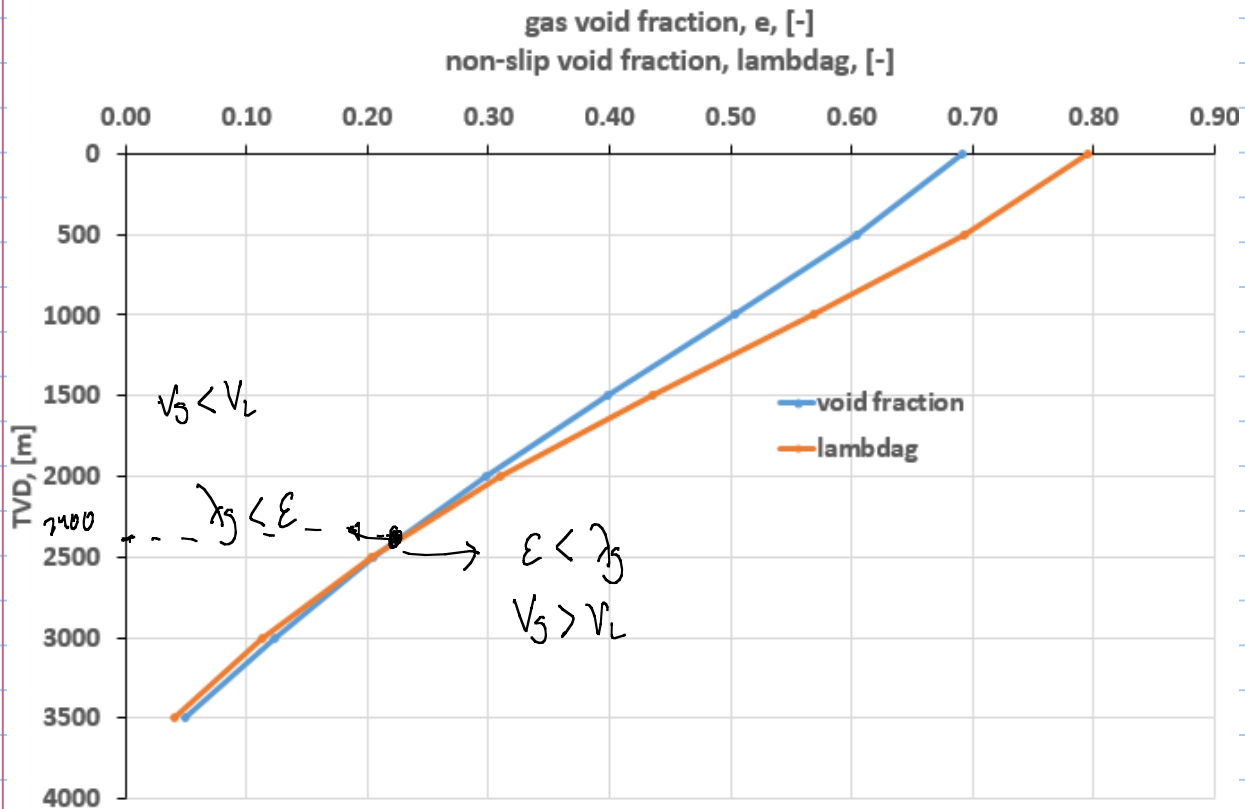
$$\frac{1 \text{ bar}}{10^5 \text{ Pa}} = 0.1 \text{ bar/m}$$



we are getting $0.0267 \frac{\text{bar}}{\text{m}} \approx 2.67\% \text{ water}$

Bg [m ³ /Sm ³]	deng [kg/m ³]	viscg [cp]	deno [kg/m ³]	viso [cp]	sigma_o_g [N/m]	qo [m ³ /d]	qg[m ³ /d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
3.229E-02	40.3	0.011	794.4	2.518	1.624E-02	1083.4	4.210E+03	0.710	2.757	0.80	0.69	0.0267
2.163E-02	60.1	0.012	770.6	1.295	1.279E-02	1137.2	2.560E+03	0.745	1.676	0.69	0.60	0.0335
1.528E-02	85.1	0.013	744.6	0.813	9.500E-03	1202.6	1.582E+03	0.788	1.036	0.57	0.50	0.0404
1.142E-02	113.8	0.015	717.5	0.575	6.668E-03	1279.3	9.867E+02	0.838	0.646	0.44	0.40	0.0467
9.100E-03	142.9	0.018	690.4	0.440	4.515E-03	1364.9	6.143E+02	0.894	0.402	0.31	0.30	0.0517
7.711E-03	168.6	0.020	664.8	0.356	3.044E-03	1455.7	3.701E+02	0.953	0.242	0.20	0.21	0.0552
6.867E-03	189.3	0.022	641.4	0.301	2.097E-03	1547.5	1.986E+02	1.014	0.130	0.11	0.12	0.0575
6.334E-03	205.2	0.024	620.5	0.263	1.493E-03	1637.0	6.988E+01	1.072	0.046	0.04	0.05	0.0589





$$\epsilon = \frac{A_g}{A}$$

slip

$$\lambda_g = \frac{q_g}{q_o + q_g}$$

assuming
 $v_l = v_g = v_m$

Home exercise • Case 2 $p_{wf} = 200 \text{ bara}$, $q_o = 1000 \text{ stb/d}$
 $p_{wh} ?$

$$P. < 100 + \frac{dp}{dx} \cdot \Delta x < 0$$



THANK YOU FOR YOUR ACTIVE PARTICIPATION!

--THE END--