PG8405 - Modeling and simulation of production systems and well construction

course plan

- Individual project
  - accounts for most of grade
  - deliverables:
    - report / paper format
    - all routines, computational files, simulation files
    - tentative: a powerpoint presentation
- common lectures on some topics
  - topics:
    - generalities about modeling in petroleum engineering
    - modeling of hydrocarbon production systems
      - well level
      - well + reservoir
      - component level
    - optimization

- model: numerical aproximation of a real physical system
  - ↳ equation

to make models I take into account usually first order effects.

models are derived from conservation principles

- make assumptions/simplifications
  - mass
  - momentum
  - energy conservation

steps:

- apply conservation principles to control envelope
- obtain equations in differential form
- discretize domain of interest
- discretize equations
- apply boundary conditions
- solving discretized equations in an iterative manner

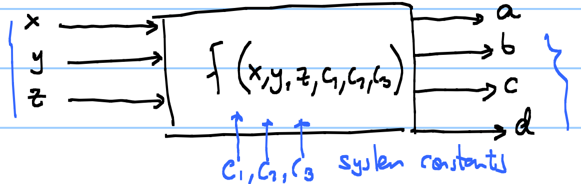
purpose

- Prediction of future performance to define best production control strategies
- to evaluate the dependance between parameters
  - ↳ sensitivity studies
  - ↳ correlations
- use to design a system
  - to troubleshoot a system
  - to optimize a system
- improve the understanding of systems

models can usually be visualized as function

black-box models → no access to model details

white-box models → full access to model details



garbage in → garbage out

changes in system/operating conditions ⇝ model input is not usually a determinist number but rather a range, collection of values

model constants/parameters " " " " " " " " " " "

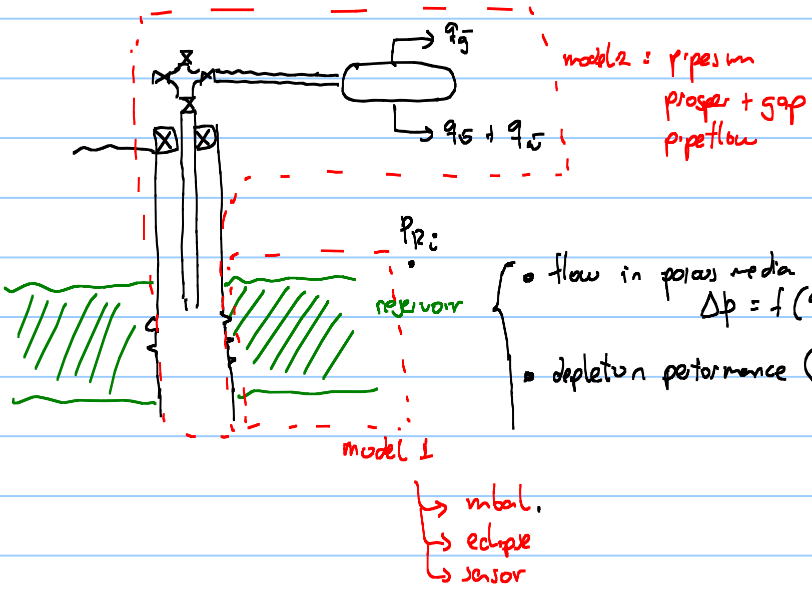
uncertainty

Performance of production system: (reservoir + well)

$\rightarrow q_{\bar{o}}(t)$?

$q_{\bar{w}}(t)$?

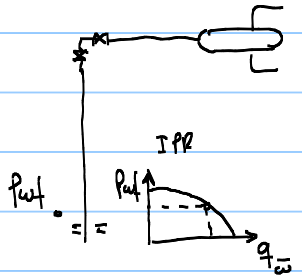
$q_{\bar{g}}(t)$?



- flow in porous media $\Delta p = f(q_{\bar{g}}, q_{\bar{o}}, q_{\bar{w}})$
- depletion performance (storability) change in time.

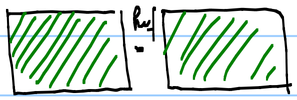
how do we couple these two models?

in pipesim



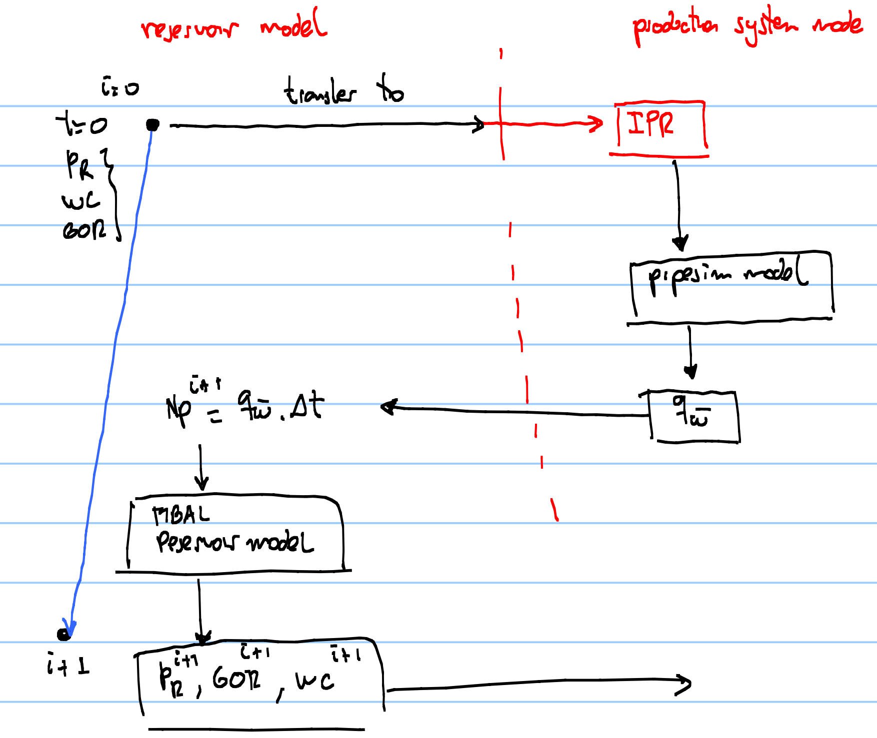
provides $q_{\bar{w}}$

requires $N_p = \int_0^t q_{\bar{o}}\,dt$
G_p
W_p

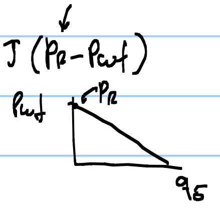


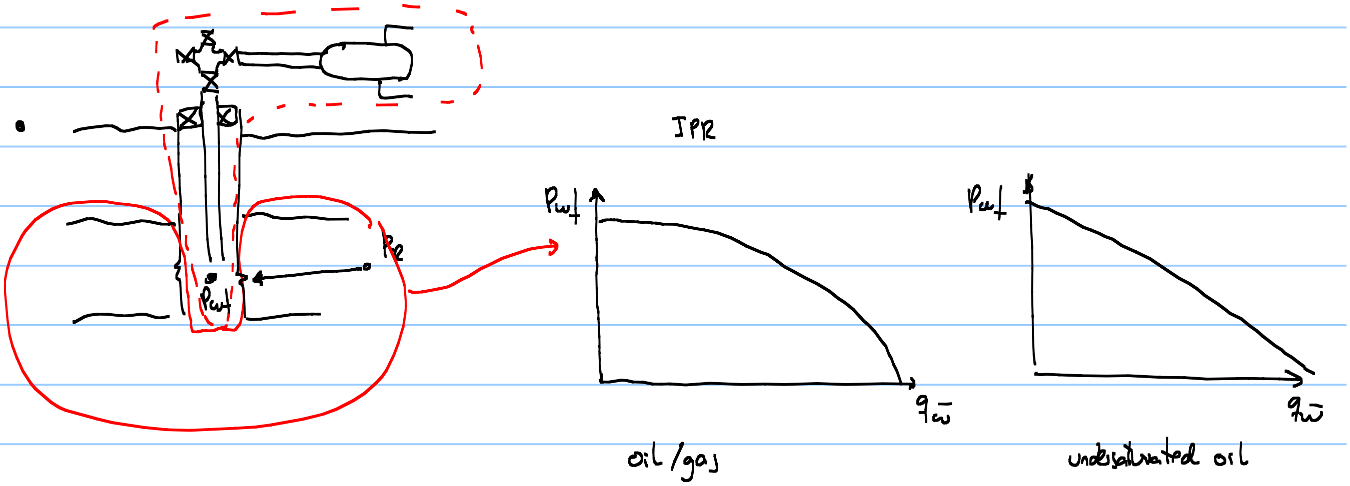
→ provides p_R
S_o, S_g, S_w

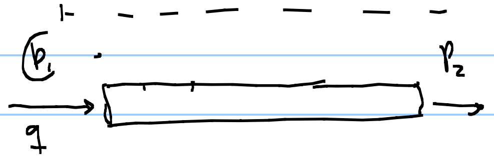
usually we can calculate GOR and WC



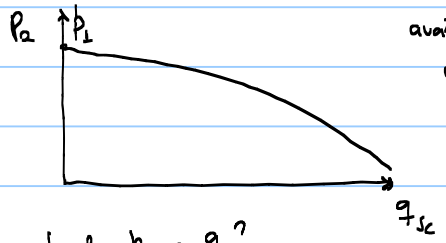
$q_s = J(P_R - P_{wf})$





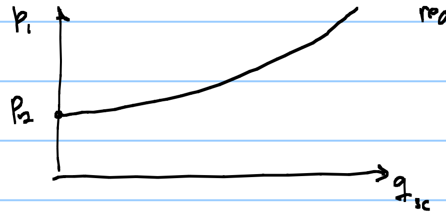


① P_1 is fixed, P_2 vs q?



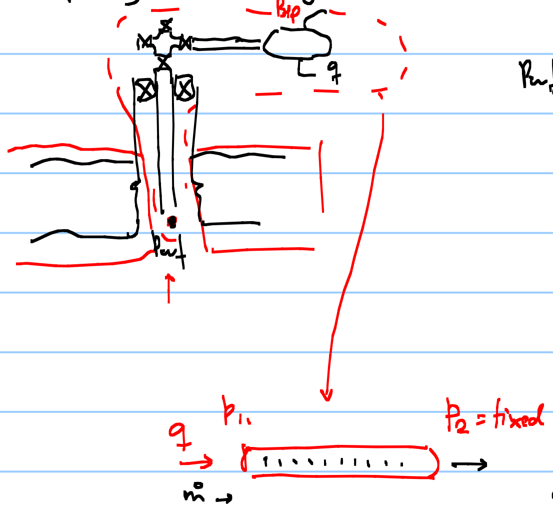
available pressure downstream of pipe
↳ co-current calculations

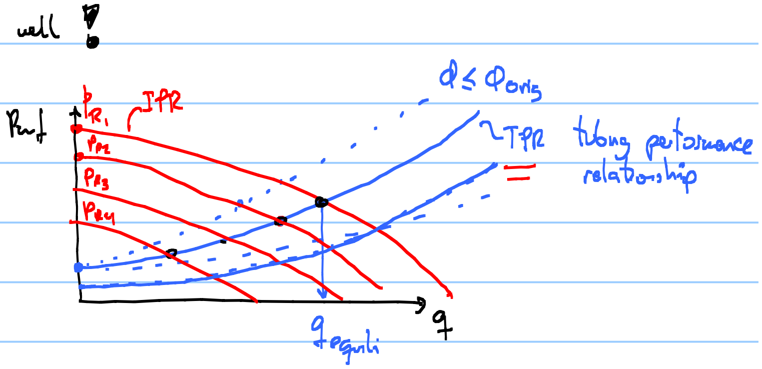
② P_2 is fixed, P_1 vs. q?

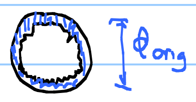


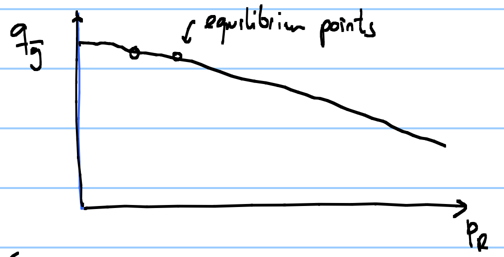
required pressure upstream the pipe
↳ counter current calculations

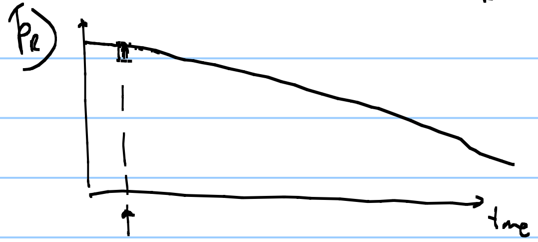
Find the operating rate of system reservoir + well !











• home/class exercise flow equilibrium for gas well

tubing

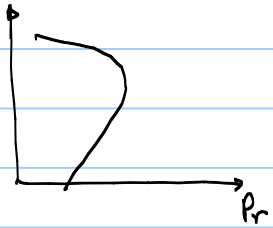
IPR

$u = \frac{Q}{A}$ ← $\pi \frac{D^2}{4}$

$Q = \frac{\dot{m}}{\rho}$

$Z = f(P_r, T_r)$ Z

$T_r = \frac{T}{T_c}$

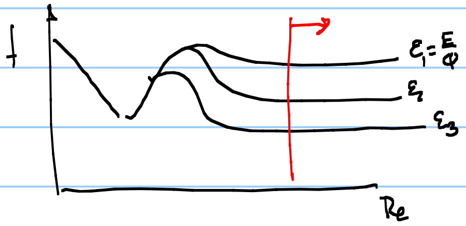


$T_c = f(M_g)$

$T_{wf}, T_{wh} \rightsquigarrow T_{av} = \frac{T_{wf} + T_{wh}}{2} \rightarrow T_r = \frac{T_{av}}{T_c}$

$P_{wf}, P_{wh} \rightsquigarrow P_{av} = \frac{P_{wf} + P_{wh}}{2} \rightarrow P_r = \frac{P_{av}}{P_c}$

$\rightarrow Z$



$q_{\bar g} = C_R \left(P_R^2 - P_{wf}^2\right)^n$ clear P_{wf} P_{wf} vs $q_{\bar g}$

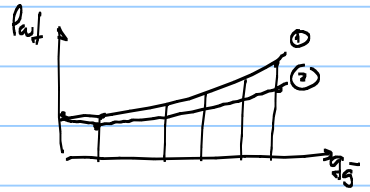
$$P_{wf} = \sqrt{\underbrace{P_R^2 - \left(\frac{q_{\bar g}}{C_R}\right)^{1/n}}_{(-)}} = f(q_{\bar g})$$

if $P_R^2 < \underbrace{\left(\frac{q_{\bar g}}{C_R}\right)^{1/n}}$ then $\sqrt{-} \rightarrow$ error!

```
try                          exception handling capabilities
    Pwf = f(qg)
if exception
    print messag "the rate tried is too high"
```

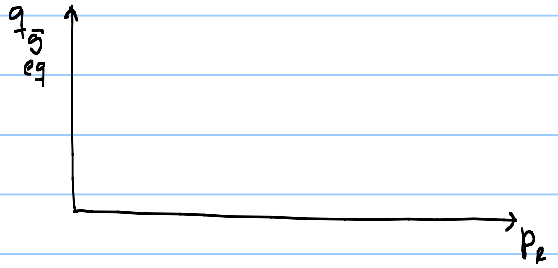
homework 1: Calculate TPR and compare → ① single C_T and S with $P_{in} = P_R$, $P_{out} = P_{sep}$

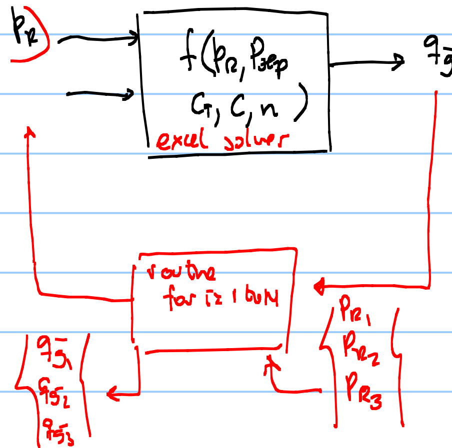
② several C_T and S with $P_{in} = P_{calc}$, $P_{out} = P_{sep}$



$$\varepsilon = \Sigma\, abs\, \frac{(P_{wf_①} - P_{wf_②})}{(P_{wf_②})} . 100$$

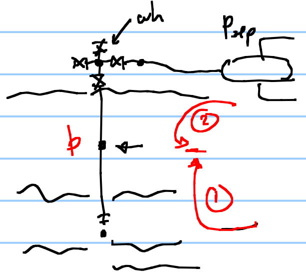
homework 2: Calculate $q_{\bar g_{eq}}$ for P_R = 304, 290, 280, 270 ..... 100 bar

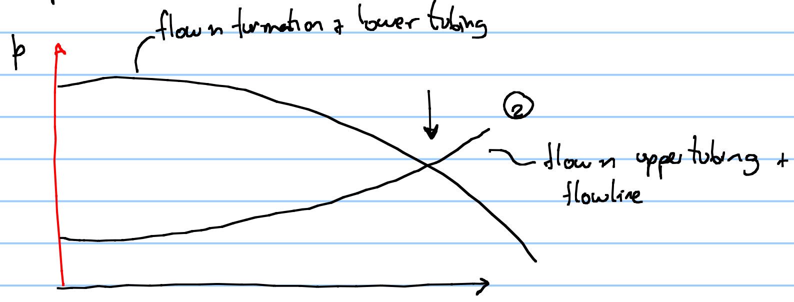




Day 2 20181010

- How to write and run and debug routines in excel VBA. / F8 - step by step
  F5 - Continue Running
- what happens if we move the equilibrium point





if equilibrium point is at wellhead than the available pressure curve it is called WPR
wellhead performance relationship

the tubing equation can be used for flowlines. for horizontal flowline $q_{\bar{g}} = C_{FL}\left(P_1^2 - P_2^2\right)^{0.5}$

↑ upstream ↳ downstream

$$p_{wh} = p_2 = \left(\frac{p_1^2}{e^s} - \frac{q_g^2}{C_T^2}\right)^{0.5}$$

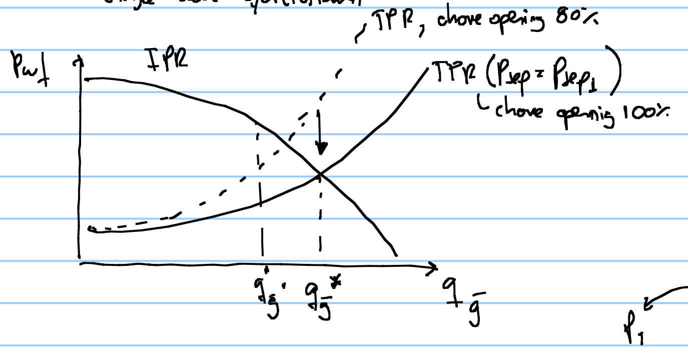
$$P_2 = \left(\frac{P_1^2}{1} - \frac{q_{\bar{g}}^2}{C_T^2}\right)^{0.5}$$

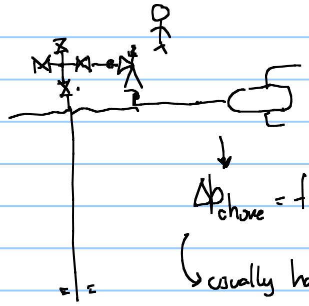
L'Hopital theorem

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}.$$

$$\lim_{s\to 0}\frac{s}{e^s - 1} = \lim_{s\to 0}\frac{1}{e^s} = 1$$

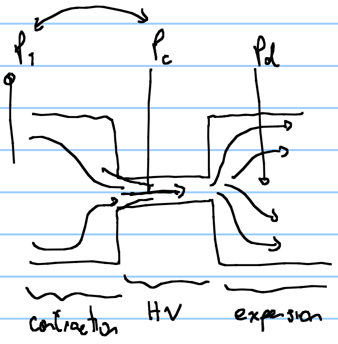
Single well equilibrium

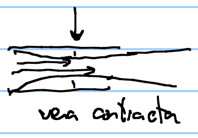




$\Delta p_{choke} = f(q_{\bar{g}}, \text{opening})$

↳ usually has a lot of uncertainty
for gas is OK
but for multiphase is ± 100%





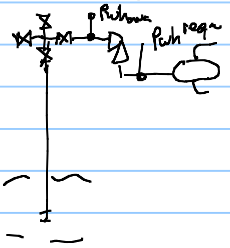
for liquid

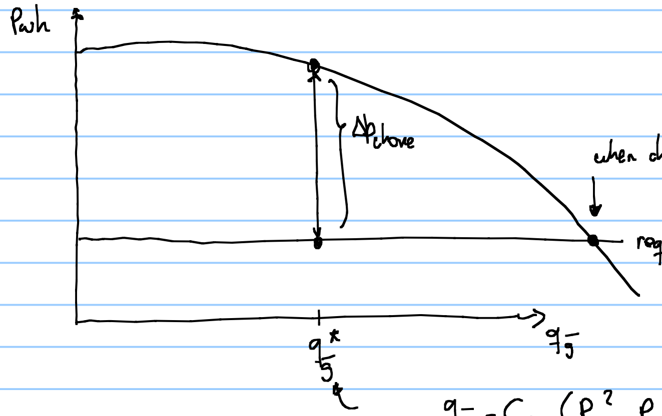
$$q_{\bar{o}} = \frac{A_2 \cdot C_d}{B_{o2}} \cdot \sqrt{\frac{2\cdot(p_2 - p_1)}{\rho\left(1-\beta^4\right)}}$$

for dry gas

$$q_{\bar{g}} = \frac{p_1 \cdot A_2 \cdot C_d \cdot T_{sc}}{p_{sc}} \cdot \sqrt{2 \cdot \frac{R}{Z_1 \cdot T_1 \cdot M_w} \cdot \frac{k}{k-1} \left(y^{\frac{2}{k}} - y^{\frac{k+1}{k}}\right)}$$

to overcome limitations in the choke equation, we place equilibrium point at wellhead

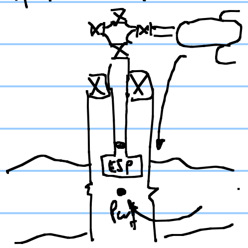


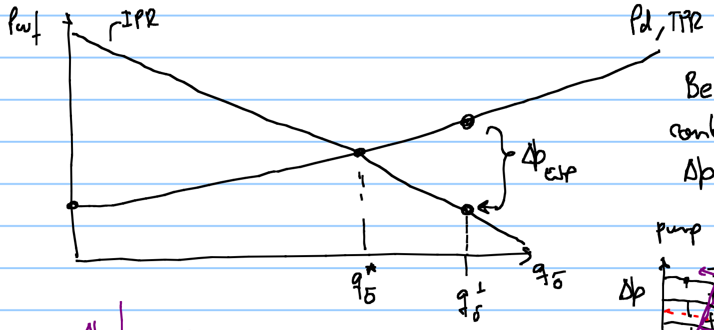


$q_g = C_R (P_R^2 - P_{wf}^2)^n$

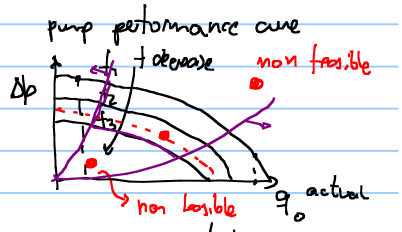
$$p_{wh} = p_2 = \left(\frac{p_1^2}{e^s} - \frac{q_g^2}{C_T^2} \right)^{0.5}$$

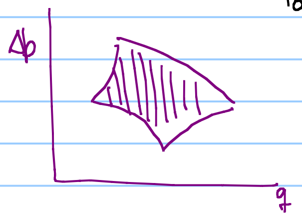
if an undersaturated oil well with ESP (electric submersible pump)





Be aware, not all combinations of q and Δp are feasible !





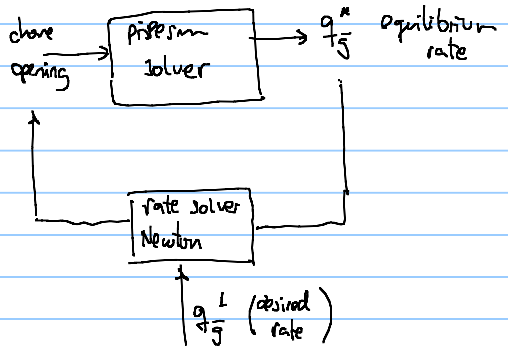
$f_{max} \simeq 70 hz$

$f_{min} \simeq 30 hz$

assuming $q_g = q_o$ actual

$P_{ave} = \frac{\Delta p \cdot q}{\eta}$

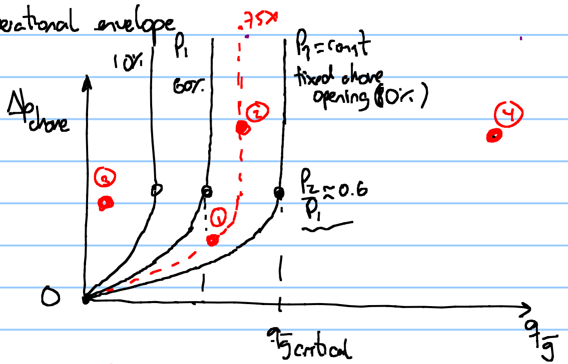
be aware ! in commercial software if i want a specific rate



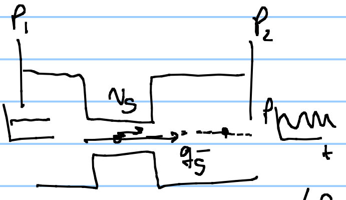
two solver levels

- equilibrium calculations
- choke opening calculation

Choke operational envelope



$$\Delta p = a\, q^2$$



$$\Delta p = P_1 - P_2$$

$$\Delta p = f(q_g)$$

sonic velocity 300 m/s

$\frac{P_2}{P_1} \simeq 0.6$ critical velocity in the throat is achieved!

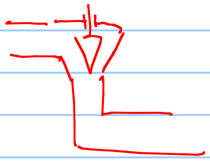
① choke operating subcritical range

② choke operating in critical range

③ choke too big

④ choke too small

$$\left.\frac{P_2}{P_1}\right|_c = 0.6$$



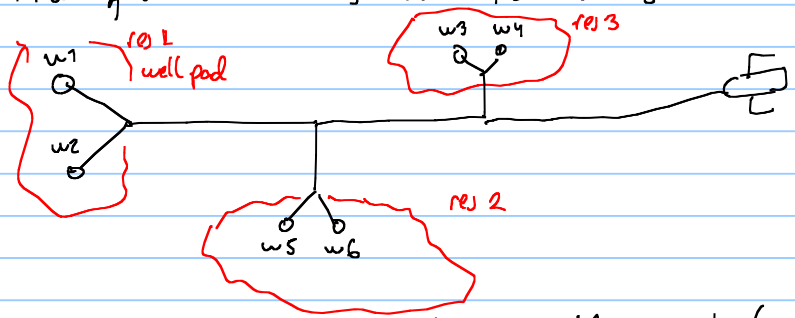
$$\Delta p_{critical} = P_1 - P_2 = \underline{P_1 - 0.6\,P_1}$$

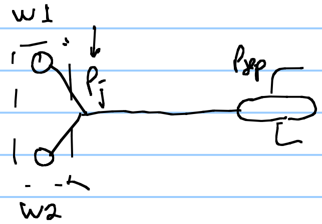
Homework

- Read choke equation development (single phase liquid) (dry gas)
- Read IPR for dry gas
- optional ! generate choke performance curve with excel file

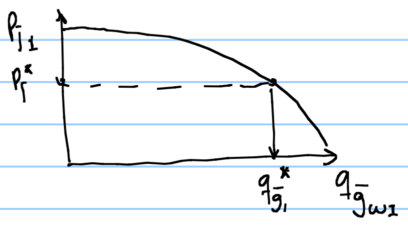
optional !
- solve network of two wells in excel

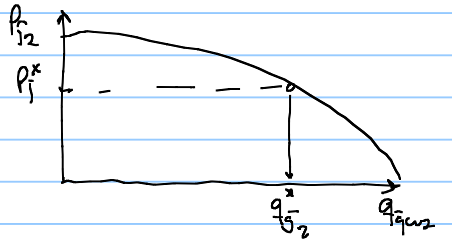
Production/flow equilibrium calculations for surface networks:

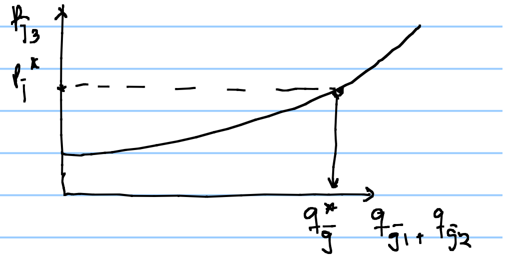




- define an equilibrium point (junction)
- calculate available and required pressure curves from boundaries
- intersect ! $P_{j\,avail\,1} = P_{j\,available\,2} = P_{j\,required}$



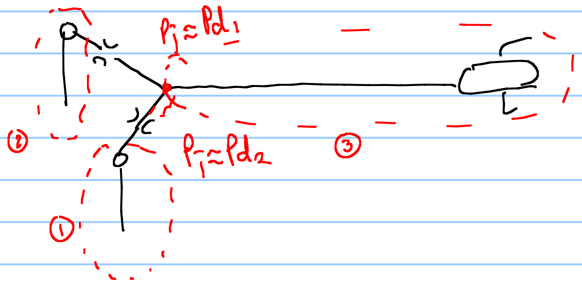




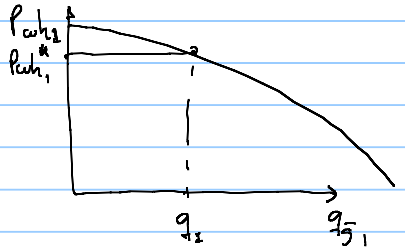
$q_g = q_{g_1} + q_{g_2}$?

$P_{j_1} = P_{j_2} = P_{j\,pipeline}$ momentum balance.

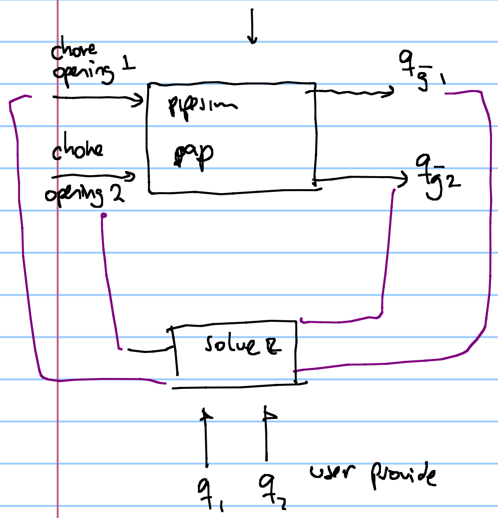
$$\boxed{q_g^* \overset{?}{=} q_{g_1}^* + q_{g_2}^*}$$ mass balance



q_1 and q_2 desired rates



in commercial software:

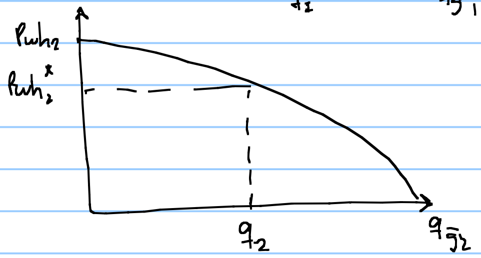


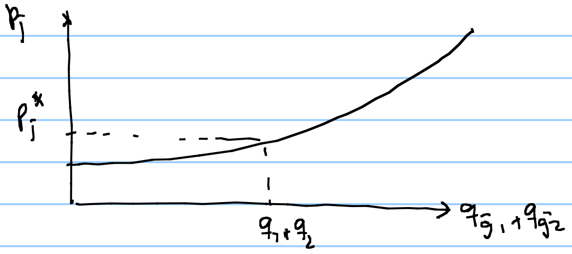
$\Delta p_{choke\,1} > 0$

$P_{wh1}^* - P_j^* > 0$

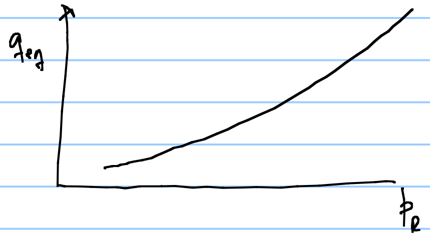
$\Delta p_{choke\,2} > 0$

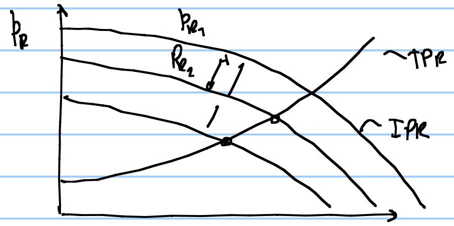
$P_{wh2}^* - P_j^* > 0$



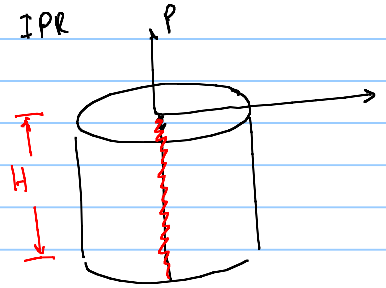


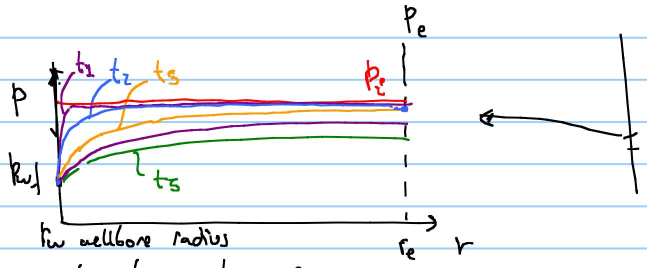
Class 3





Comment on IPR





0 – t_3 transient regime

IPR is a function of time
a function of p_i

{ p_e is constant, $t > t_3$ then steady state regime
p_e declines, $t > t_3$, the pseudo-steady states (PSS)

IPR is a function of
p_R (Average pressure in the drainage volume)
$\neq f(t)$

diffusivity equation for radial reservoir, homogeneous fluid and reservoir

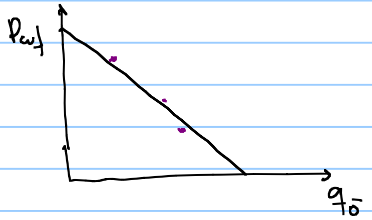
$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \underbrace{\frac{\phi \mu c}{k} \quad \frac{\partial p}{\partial t}} \rightsquigarrow \begin{array}{l} q_{\bar g} = C\left(p_R^2 - p_{wf}^2\right)^n \\ q_{\bar o} = J\left(p_R - p_{wf}\right) \end{array}$$

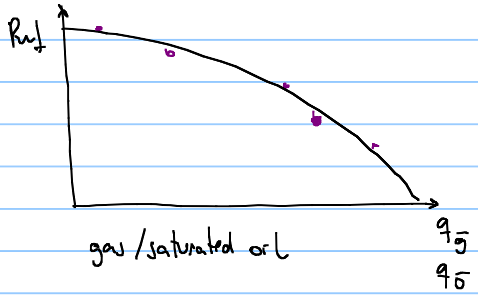
$$q_{sc} = U \int_{p_{wf}}^{\tilde{p}_R} F(p)\,dp$$

F(p) ← fluid properties, relative permeability

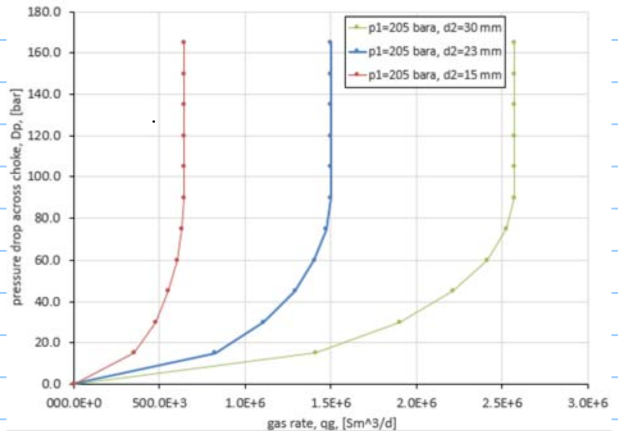
U ← depends on the geometry skin, formation properties

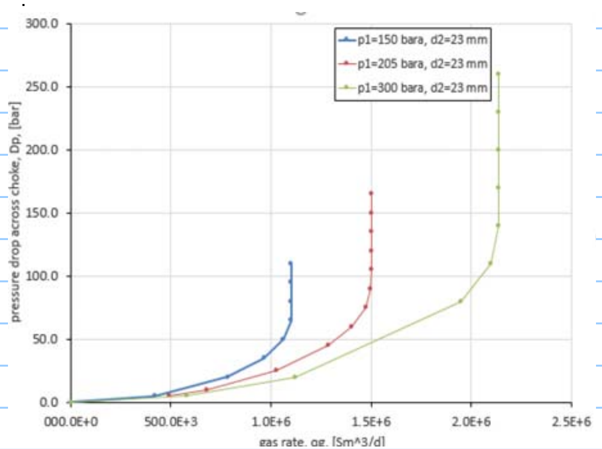
liquid, undersaturated oil



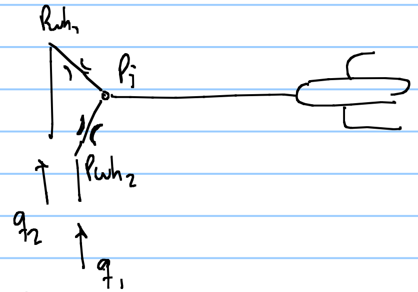


gas / saturated oil

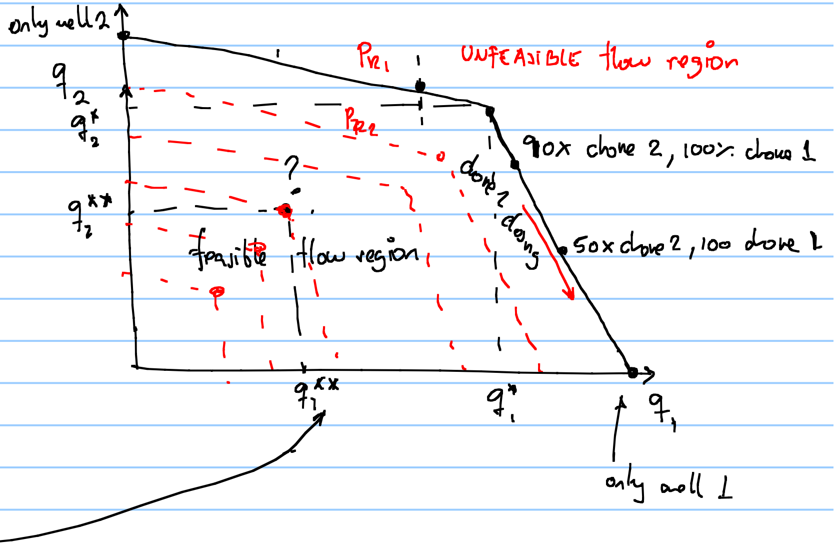




Network exercise



defined by reservoir management

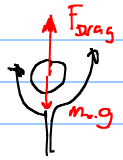


- exercise calculate this with the data from 2 wells network.

- exercise: solve pressure traverse calculations

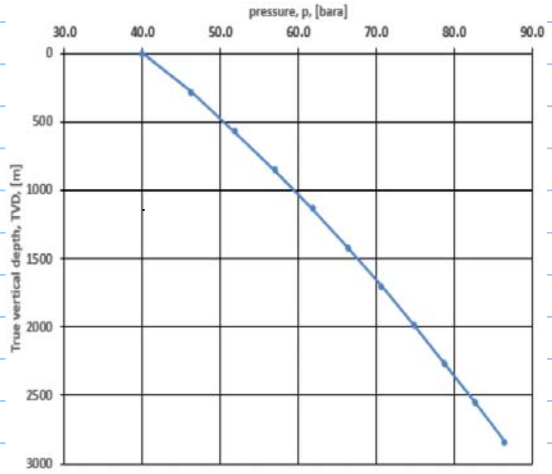
Verify liquid loading criteria

Turner

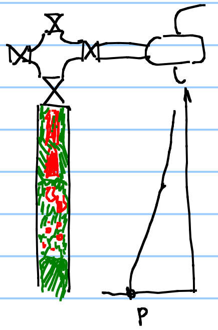


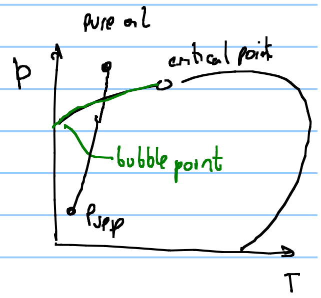
Erosion verification API 14E Erosion standard

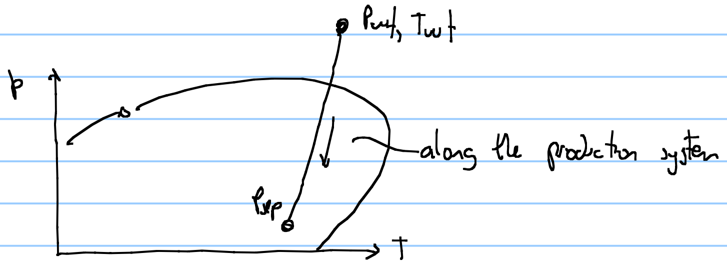
DNV O501 Erosion



- PVT model
  - Compositional (using Equation of state)
  - Black oil tables







EOS → SRK → Redlich Kwong Soave } cubic equations of state
PR → Peng Robinson

$$p = \frac{RT}{v-b} - \frac{a}{v(v+b)+b(v-b)} \quad \ldots\ldots\ldots\ldots\ldots \quad (4.19)$$

or, in terms of Z factor,

$$Z^3 - (1-B)Z^2 + \left(A - 3B^2 - 2B\right)Z - \left(AB - B^2 - B^3\right) = 0$$

applied for mixture → vapour
→ liquid
→ mixture → vapour
→ liquid

and $Z_c = 0.3074$. ………………………… (4.20)

The EOS constants are given by

$$a = \Omega_a^o \frac{R^2T_c^2}{p_c}\alpha, \quad \ldots\ldots\ldots\ldots \quad (4.21a)$$

where $\Omega_a^o = 0.45724$;

$$b = \Omega_b^o \frac{RT_c}{p_c}, \quad \ldots\ldots\ldots\ldots \quad (4.21b)$$

where $\Omega_b^o = 0.07780$;

$$\alpha = \left[1 + m\left(1 - \sqrt{T_r}\right)\right]^2; \quad \ldots\ldots\ldots\ldots \quad (4.21c)$$

and $m = 0.37464 + 1.54226\omega - 0.26992\omega^2$. …… (4.21d)

$$A = a\frac{p}{(RT)^2} = \Omega_a^o \frac{p_r}{T_r^2}\alpha(T_r),$$

where $\alpha(T_r) = T_r^{-0.5}$;

and $B = b\dfrac{p}{RT} = \Omega_b^o \dfrac{p_r}{T_r}$. …..

mixing rule

$$A = \sum_{i=1}^{N}\sum_{j=1}^{N} y_i y_j A_{ij},$$

$$B = \sum_{i=1}^{N} y_i B_i,$$

and $A_{ij} = \left(1 - k_{ij}\right)\sqrt{A_i A_j}$,

Binary interaction parameter (BIP)

	z_i
CH_4	
C_2	
C_3	
C_4	
C_5	
⋮	
C_nH_{2n+2}	

z_i ← molar fraction

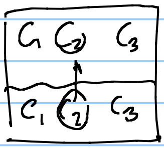
Gibbs energy / chemical potential

Fugacity expressions are given by

$$\ln\frac{f}{p} = \ln\phi = Z - 1 - \ln(Z-B) - \frac{A}{2\sqrt{2}B}\ln\left[\frac{Z + \left(1+\sqrt{2}\right)B}{Z - \left(1-\sqrt{2}\right)B}\right]$$

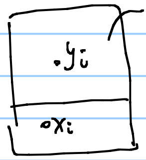
and

$$\ln\frac{f_i}{y_i p} = \ln\phi_i = \frac{B_i}{B}(Z-1) - \ln(Z-B) + \frac{A}{2\sqrt{2}B}\left(\frac{B_i}{B} - \frac{2}{A}\sum_{j=1}^{N} y_j A_{ij}\right)\ln\left[\frac{Z + \left(1+\sqrt{2}\right)B}{Z - \left(1-\sqrt{2}\right)B}\right],$$



$f_{Li} = f_{vi}$ component "i"

	z_i
C_1	0.8
C_{10}	0.2



y_{C_1}
$y_{C_{10}}$
x_{C_1}
$x_{C_{10}}$

$\left\{ \begin{array}{l} K_{C_1} = \\ K_{C_{10}} = \end{array} \right\}$

Assume
$K_i = \frac{y_i}{x_i}$ → $\boxed{\text{mass balance Rachford-Rice equation}}$ → $\boxed{F_v = \frac{n_v}{n_T} \quad y_i \quad x_i}$ → $\boxed{\text{PR EOS}}$

$K_i = \frac{y_i}{x_i}$

$\sum z_i = 1$

$\sum y_i = 1$

$\sum x_i = 1$

$z_i = F_v y_i + (1 - F_v) x_i.$

x_i

$Z_L^3 - (1 - B_L) Z_L^2 + (A - 3B^2 - 2B) Z_L - (AB - B^2 - B^3) = 0$

y_i

$Z_V^3 - (1 - B_V) Z_V^2 + (A_V - 3B_V^2 - 2B_V) Z_V - (A_V B_V - B_V^2 - B_V^3) = 0$

flash calculations

- $0 < f_v \leq 1$ ~> in mixture!
- $f_v \leq 0$ ~> in liquid
- $f_v \geq 1$ ~> in vapour

solve
$Z_L^* \quad Z_V^*$

↓

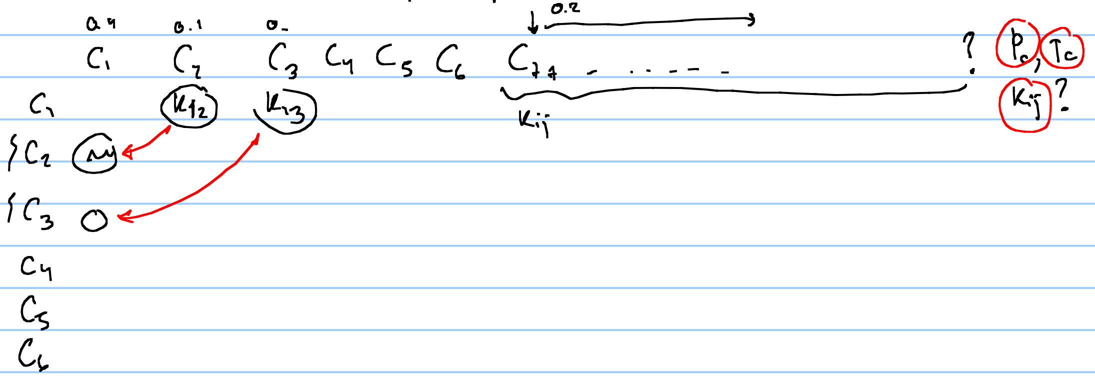
calculate fugacity

$f_{LC_1} \quad f_{LC_{10}}$
$\updownarrow \quad \updownarrow$
$f_{vC_1} \quad f_{vC_{10}}$

?

Class 4. EOS are not very predictive "out-of-the-box" ± 10% ≤ 20%

K_{ij} = BIPS   i can modify BIPS to improve predictability



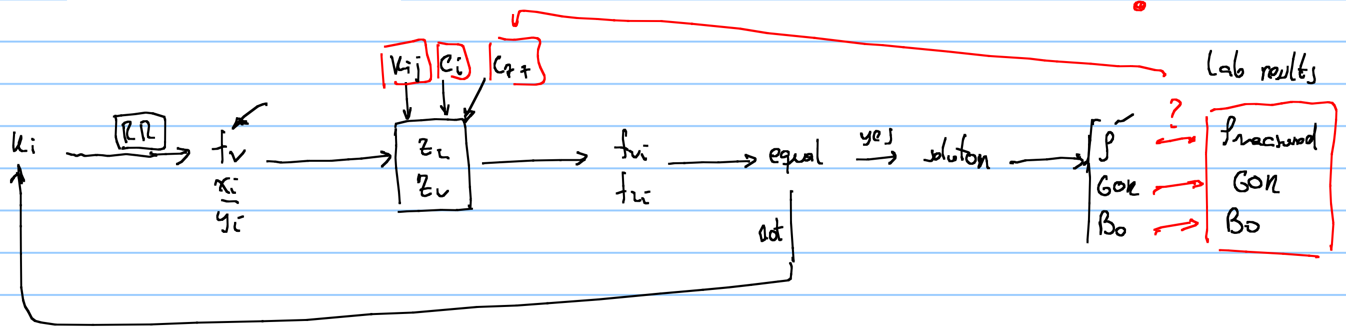
$$v_L = v_L^{EOS} - \sum_{i=1}^{N} x_i c_i$$

volume shift for heavy (pseudo components).

$$\text{nd } v_v = v_v^{EOS} - \sum_{i=1}^{N} y_i c_i,$$

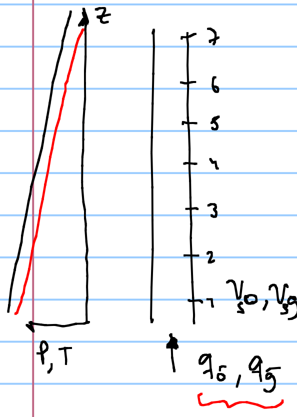
to improve predictability of EOS:
- change slightly properties of pseudo components
- change BIPS
- change volume shift

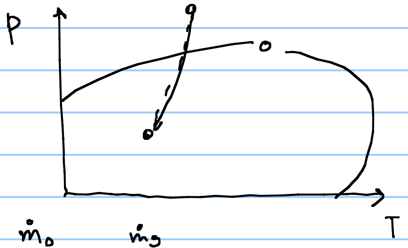
to match measured/lab data!

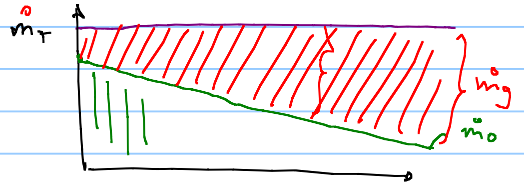


an EOS is:
- Equation being used ⟶ PR, SRK
- K_{ij}
- c_i (volume shift)
- properties of pseudo components

Class exercise







$$\frac{\dot m_o}{\rho_o} \qquad \frac{\dot m_g}{\rho_g}$$

$$v_{so}, v_{sg} = \frac{q_o}{A_T}, \frac{q_g}{A_T} \qquad \frac{\dot m_o(P,T)}{\rho_o(P,T)\cdot A_T}, \frac{\dot m_g(P,T)}{\rho_g(P,T)\cdot A_T}$$

F_v = molar basis

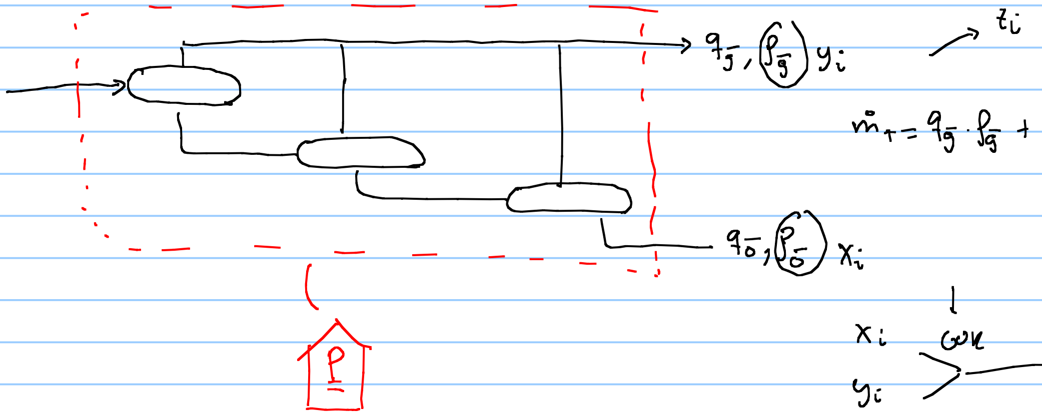
$$F_v = \frac{\dot n_g}{\dot n_g + \dot n_o}$$

$$\frac{m}{MW} = n$$

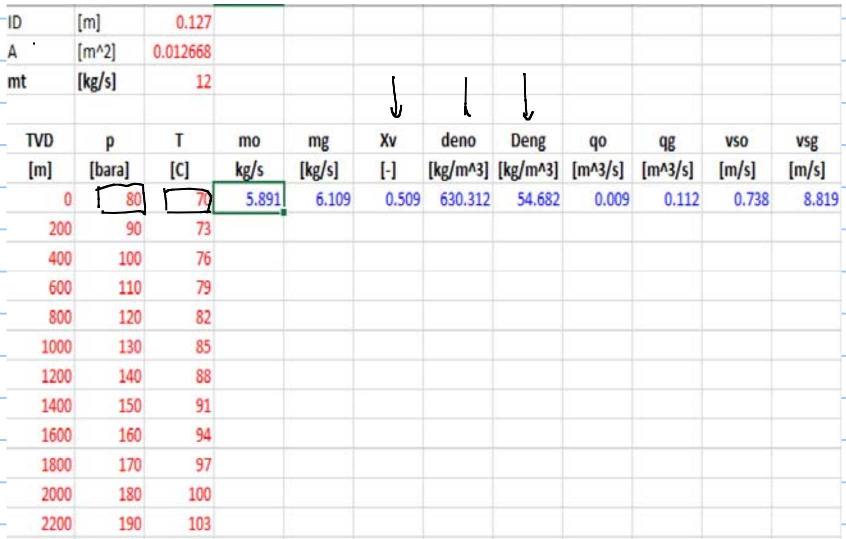
vapour mass fraction X_v

$$\dot m_g = \dot m_T \cdot X_v$$

$$\dot m_o = \dot m_T \cdot (1 - X_v)$$



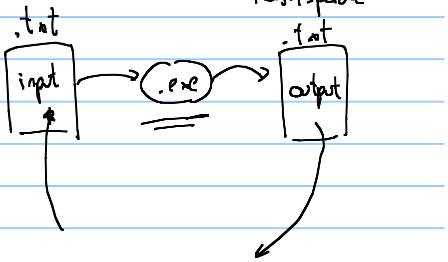
$\dot m_T = q_{\bar g}\cdot\rho_{\bar g} + q_{\bar o}\cdot\rho_{\bar o}$



ID	[m]	0.127									
A	[m^2]	0.012668									
mt	[kg/s]	12									
TVD	p	T	mo	mg	Xv	deno	Deng	qo	qg	vso	vsg
[m]	[bara]	[C]	kg/s	[kg/s]	[-]	[kg/m^3]	[kg/m^3]	[m^3/s]	[m^3/s]	[m/s]	[m/s]
0	80	70	5.891	6.109	0.509	630.312	54.682	0.009	0.112	0.738	8.819
200	90	73									
400	100	76									
600	110	79									
800	120	82									
1000	130	85									
1200	140	88									
1400	150	91									
1600	160	94									
1800	170	97									
2000	180	100									
2200	190	103									

home exercise retrieve properties from EOS automatically to populate excel table

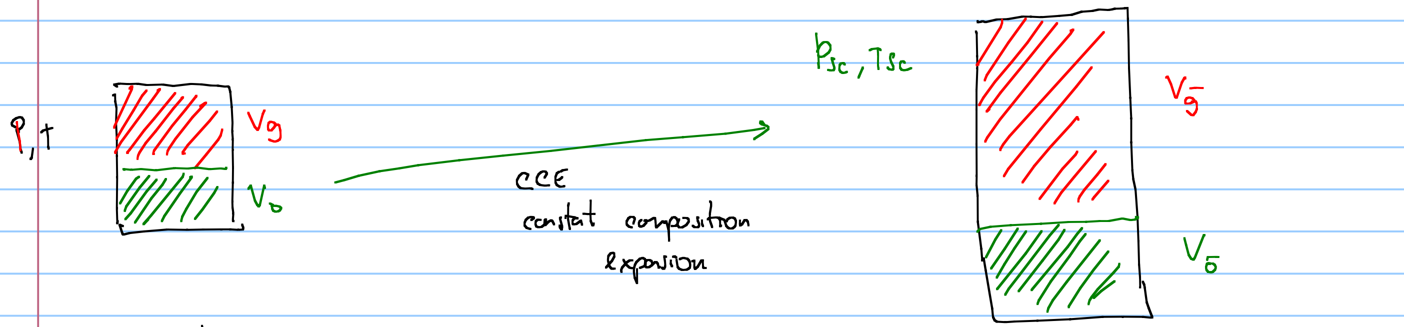
- HYSYS in VBA ← to install HYSYS on computer
- .exe utility Flash-pack



Black oil approach $\underbrace{B_o, R_s, B_g}_{\text{traditional BO}}$, r_s (R_v)

traditional BO: regular crudes (low GOR), dry gas

extended BO: volatile oil (↑API, ↑GOR) → condensation of oil from gas



traditional BO

oil volume factor $B_o = \frac{V_o(P,T)}{V_{\bar o}} \sim 1.1 \rightarrow 1.8$ if $P = P_R$, $T = T_R$ "formation volume factor"

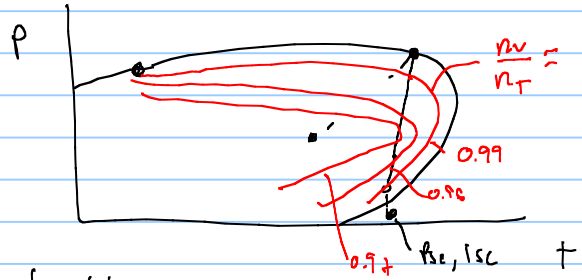
dead oil volatile oil

gas volume factor $B_g = \frac{V_g(P,T)}{V_{\bar g}} \sim 0.01$ 1E-2 – 1-E-03

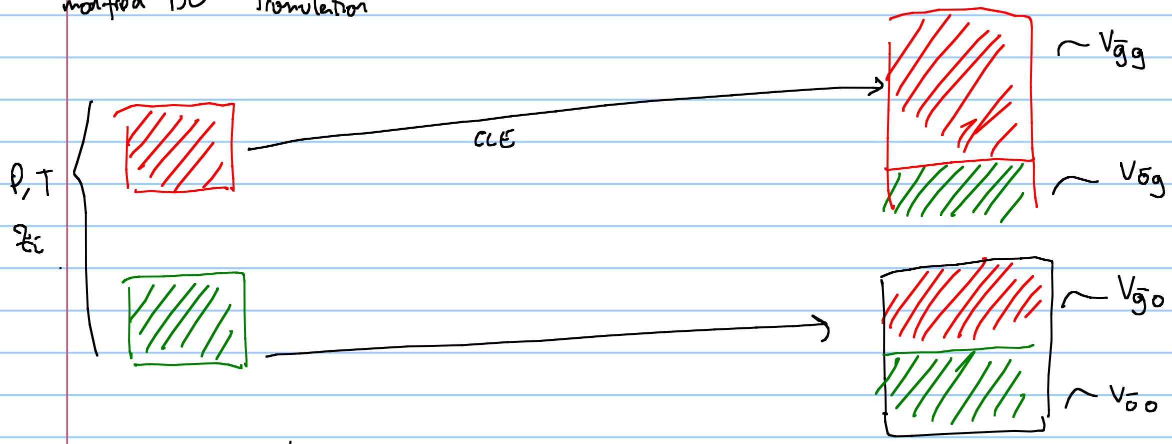
solution gas/oil ratio $R_s = \frac{V_{\bar g}}{V_{\bar o}} \sim \frac{Sm^3}{Sm^3}$ 20 → 2000

dead oil volatile /gas condensate

gas @ p^*, T^*



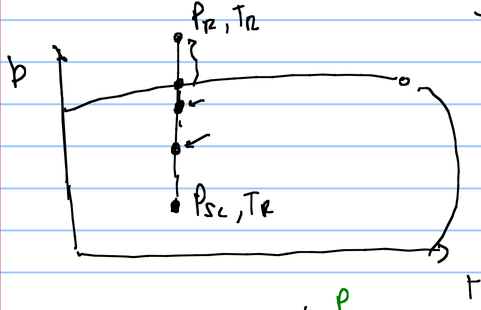
modified BO formulation

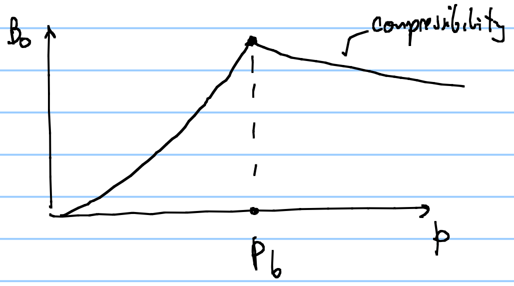


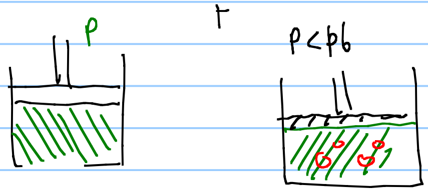
BO parameters

$$B_o(p,T) = \frac{V_o}{V_{\bar{o}o}} \qquad R_s = \frac{V_{\bar{g}o}}{V_{\bar{o}o}}$$

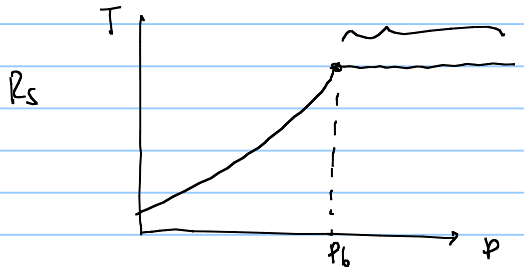
$$B_g(p,T) = \frac{V_g}{V_{\bar{g}g}} \qquad r_s = \frac{V_{\bar{o}g}}{V_{\bar{g}g}}$$

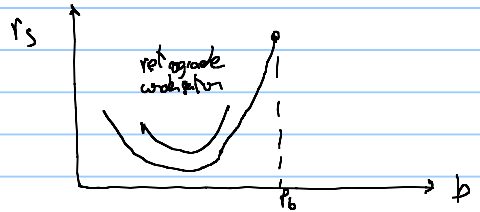


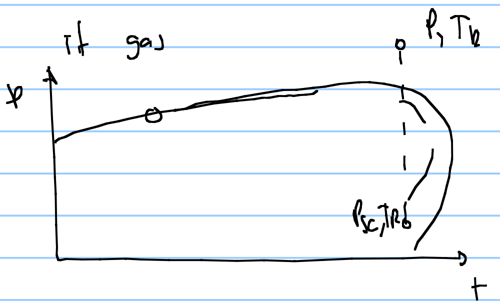


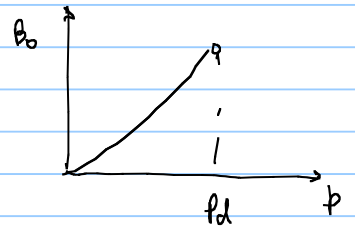


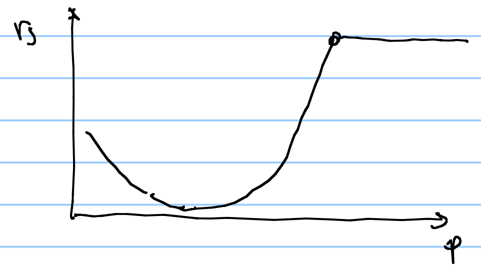
$$B_g = \frac{V_g\ (p,T)}{V_{\bar{g}g}}$$

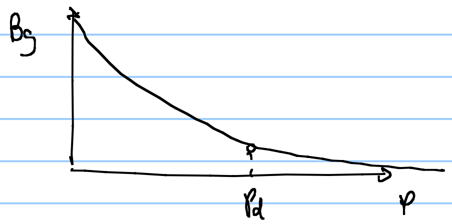












$$\overset{\checkmark}{q_{\bar o}} = q_{\bar o o} + q_{\bar o g} = \overset{\checkmark}{\frac{q_o}{B_o}} + r_s \overset{?}{q_{\bar g g}} \quad \frac{q_g}{q_{\bar g g}} = B_g$$

$$\downarrow$$

$$q_{\bar o} = \frac{q_o}{B_o(p,T)} + r_s(p,T)\frac{q_g}{B_g(p,T)}$$

$$q_{\bar g} = q_{\bar g o} + q_{\bar g g} = R_s \cdot q_{\bar o o} + \frac{q_g}{B_g} \qquad \frac{q_o}{q_{\bar o o}} = B_o$$

$$q_{\bar o o} = \frac{q_o}{B_o}$$

$$q_{\bar g} = \frac{R_s}{B_o} q_o + \frac{q_g}{B_g}$$

$$\begin{Bmatrix} \overset{\checkmark}{q_{\bar o}} \\ \overset{\checkmark}{q_{\bar g}} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{B_o} & \frac{r_s}{B_g} \\ \frac{R_s}{B_o} & \frac{1}{B_g} \end{Bmatrix}_{(p,T)} \cdot \begin{Bmatrix} \overset{?}{q_o} \\ \overset{?}{q_g} \end{Bmatrix}$$

$$\begin{Bmatrix} q_o \\ q_g \end{Bmatrix} = \begin{Bmatrix} \quad \end{Bmatrix} \begin{Bmatrix} q_{\bar o} \\ q_{\bar g} \end{Bmatrix}$$

$$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \frac{B_g}{1-R_s \cdot r_s} & \frac{-B_g \cdot R_s}{1-R_s \cdot r_s} & 0 \\ \frac{-B_o \cdot r_s}{1-R_s \cdot r_s} & \frac{B_o}{1-R_s \cdot r_s} & 0 \\ 0 & 0 & B_w \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_{\bar g} \\ q_{\bar o} \\ q_{\bar w} \end{bmatrix}$$

BO properties are generated → laboratory tests (not very common, unless reservoir) TR

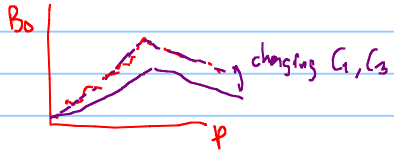
→ correlations (Standing, Beggs, Vasquez

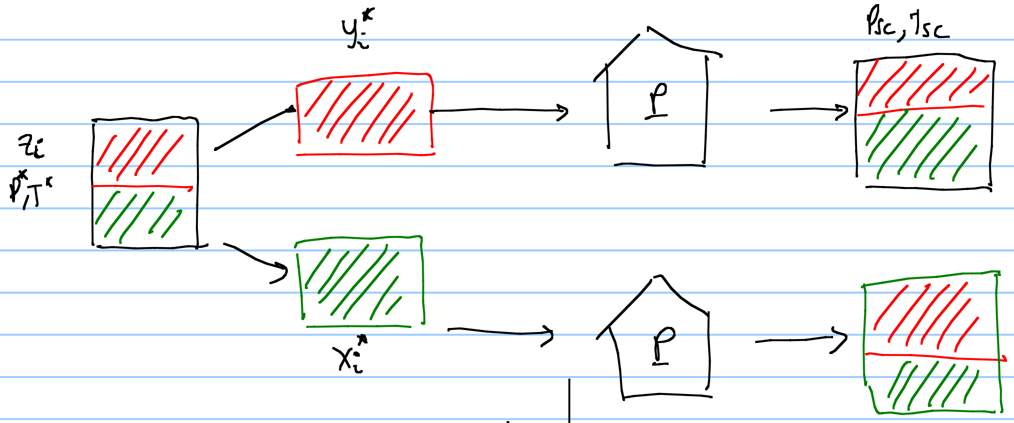
$B_g \approx$ $pV = zRT$

→ generate from EOS compositional model

as a function of
GOR → fv
$\rho_o \approx x_i$
$\rho_g \approx y_i$
z_i

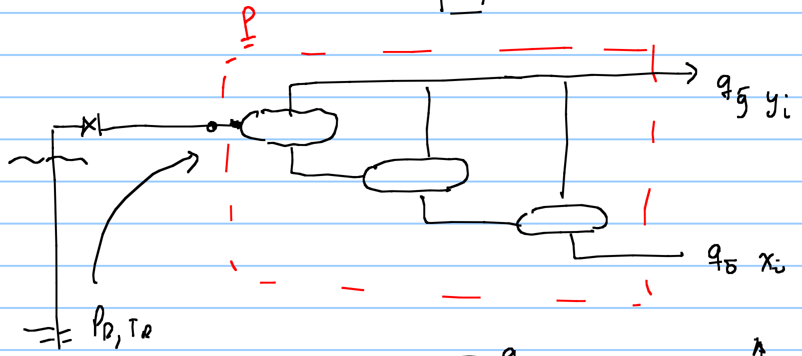
must be tuned to data

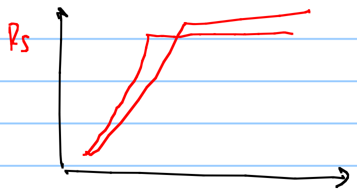


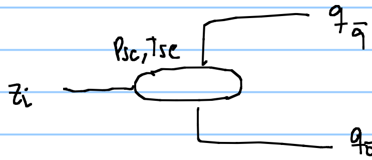


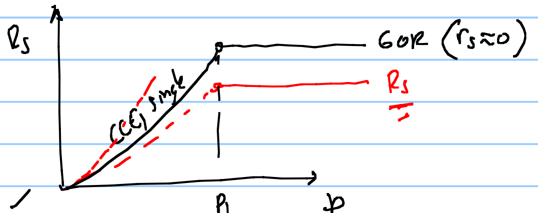
if P is CCE then $y_{i\,sc} = y_i^*$

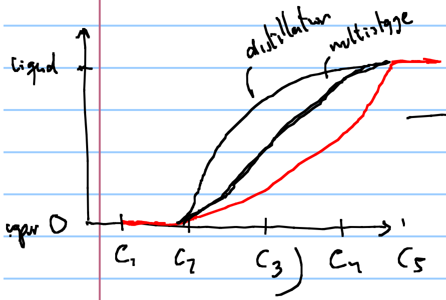
$x_{i\,sc} = x_i^*$

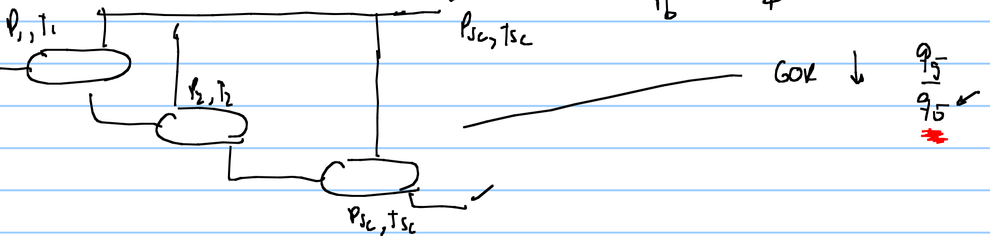


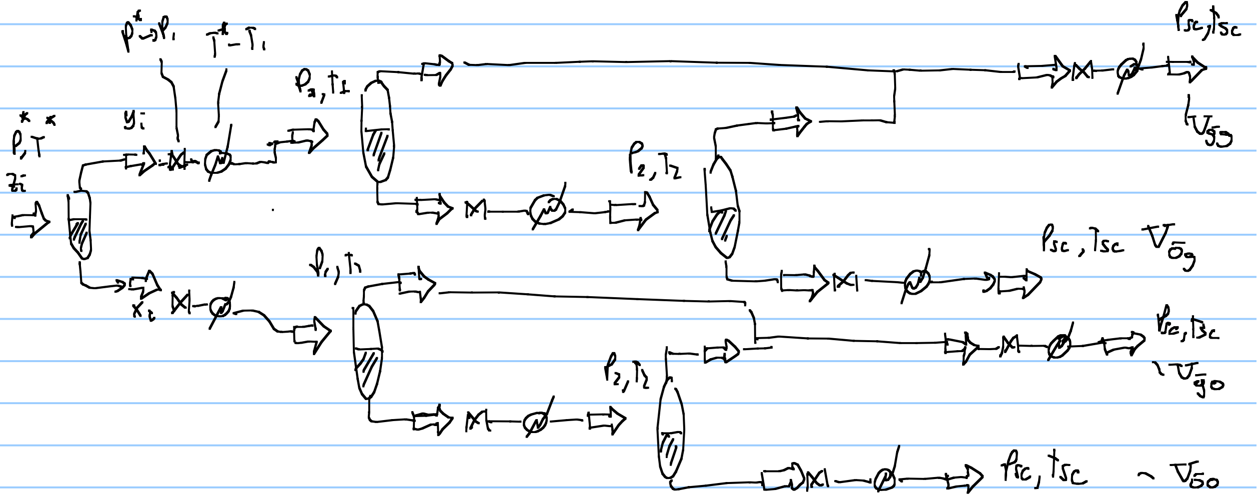










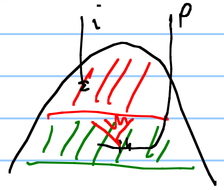


homework. Generate BO properties with the utility, P, T, z_i } details to be sent later !

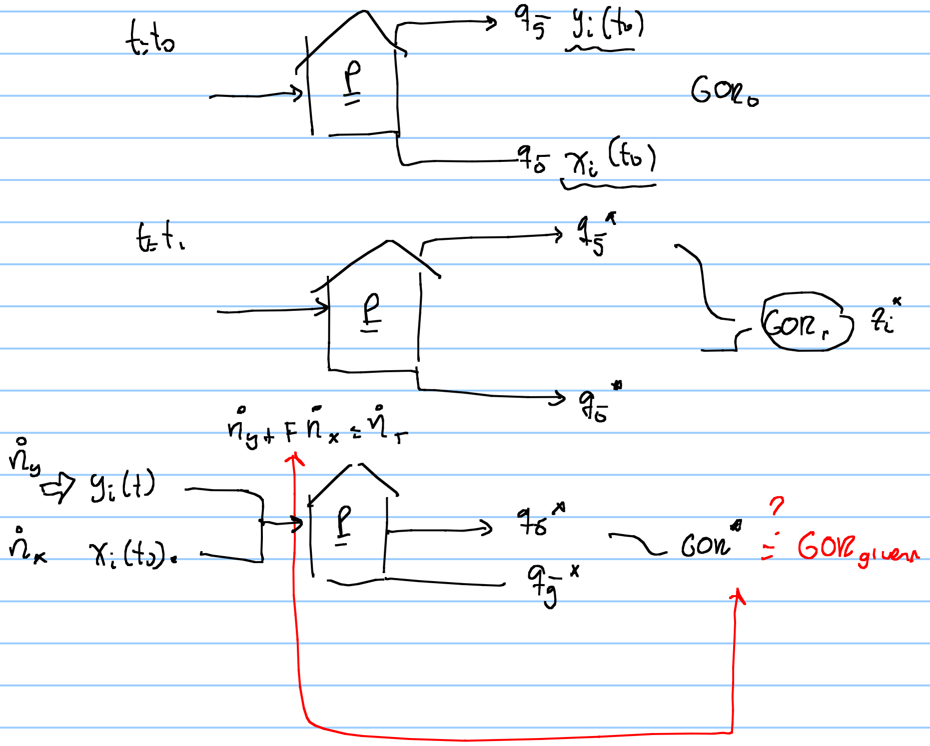
in time GOR changes !! z_i will change $\nearrow$ EOS use the new z_i !
$\searrow$ black oil

how to estimate numerically new z_i ?
$z_i(t)$

depends on the reservoir recovery process { gas injection



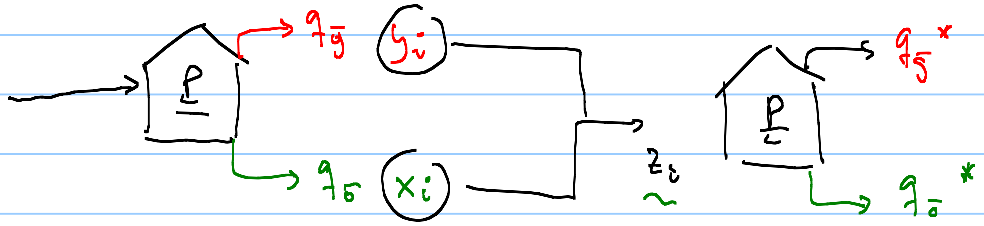
method commonly used recombination of separator fluids



http://www.ipt.ntnu.no/~stanko/files/Courses/PG8405

Class 5

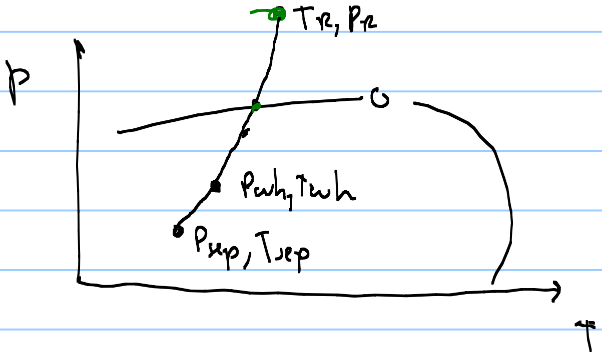
- **Class 1:** Generalities about modeling. Performance of production system: coupling between reservoir and well. Flow equilibrium for dry gas well. Excel VBA functions and routines.
- **Class 2:** Dry gas flowline equation. Choke and pumping design. Choke performance . Networks. Network solving in commercial software.
- **Class 3:** IPR. Feasible flow region of network. Pressure traverse in gas well. EOS and flash calculations. Introduction to Hysys.
- **Class 4:** tuning EOS. Calculating local volumes of oil and gas with compositional simulator. BO properties. BO properties generation. Recombination of separator fluids to match GOR.

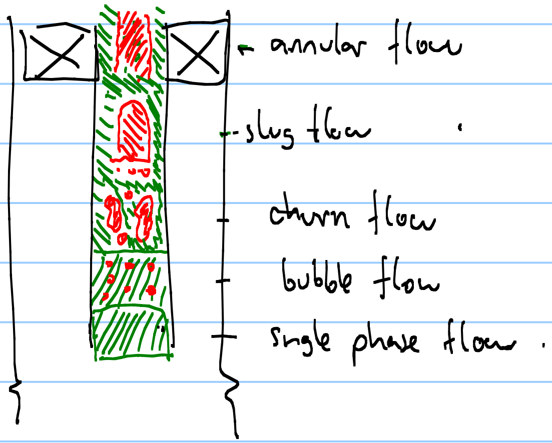


Class exercises:

- Compare the values of required flowing bottom-hole pressure calculated: 1) using a Ct with an average of reservoir and separator pressure versus 2) using a Ct with an average between the actual flowing-bottomhole pressure calculated and separator pressure (implicit).
- Create a vba macro to estimate the equilibrium point for several reservoir pressures.
- Read choke equation development for dry gas and liquid.
- Read IPR equation development for dry gas.
- **Optional:** generate choke performance curve with excel file.
- Solve flow equilibrium of 2-well network.
- Calculate feasible flow region for 2-well network varying choke DPs.
- Example of compositional calculations with Hysys.
- Calculate local rates of oil and gas at p,T using compositional simulator (Hysys or Flash-Pack)
- Generate BO properties for P, T with Flash-pack, Hysys.

multiphase flow in conduits (tubing, flowlines, pipelines)

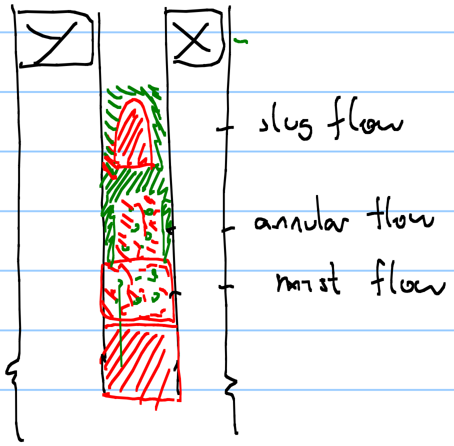




oil / gas configured in wellbore

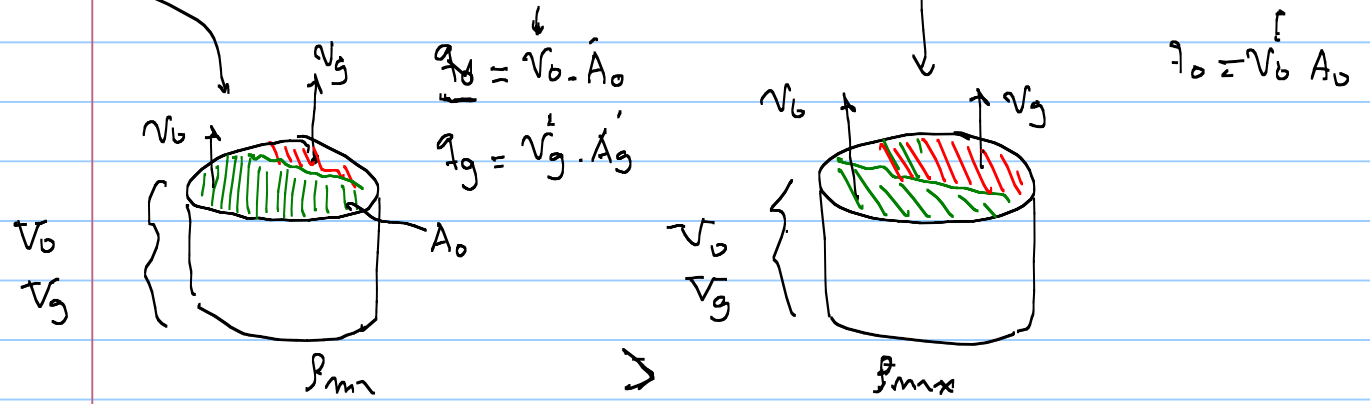
↳ flow pattern (pressure drop)

oil well



TPR tubing performance relationship

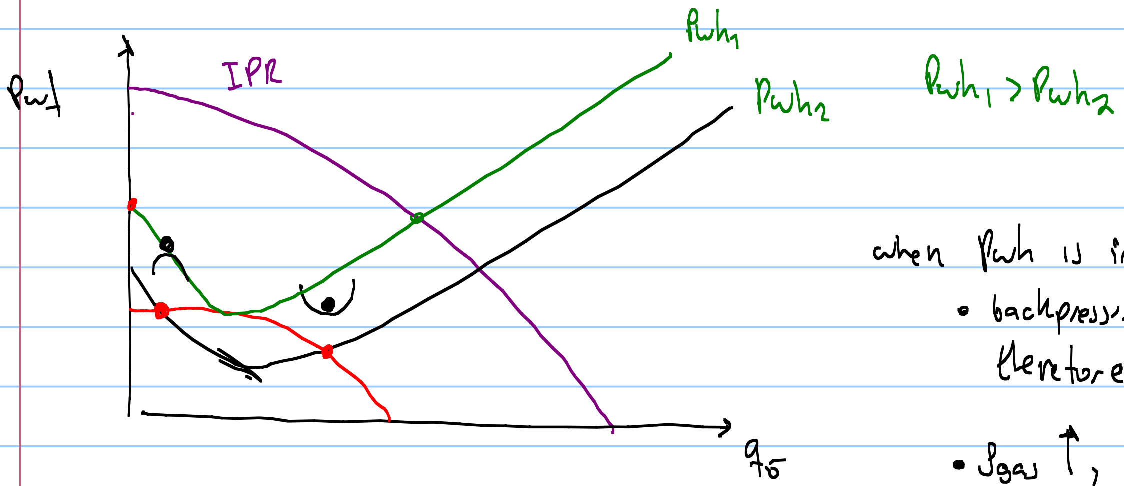
$$\Delta p = \underbrace{\Delta p_{Hydrostatic}}_{\rho_{mix}} + \underbrace{\Delta p_{friction}}_{v^2}$$



$q_o = \overset{\downarrow}{v_o} \cdot A_o$

$q_g = v_g \cdot A_g$

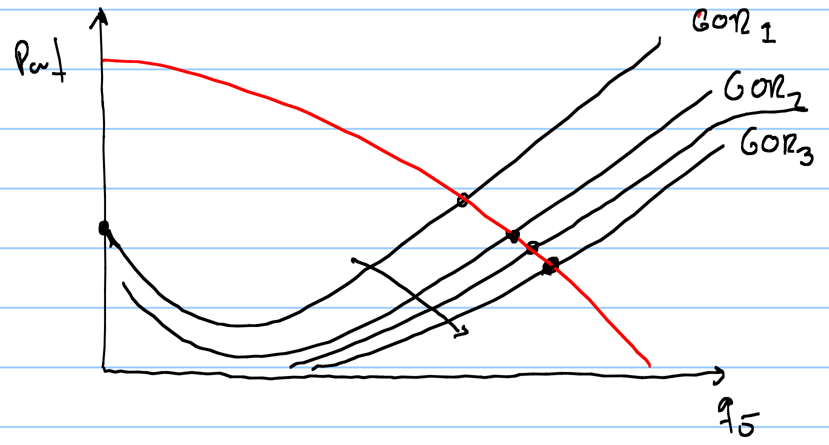
$q_o = \overset{\uparrow}{v_o} A_o$



when Pwh is increased

- backpressure is increased therefore $q_{o\,equ}$ ↓
- ρ_{gas} ↑, $\Delta p_{friction}$ ↓

$q_{o\,eq}$ ↑

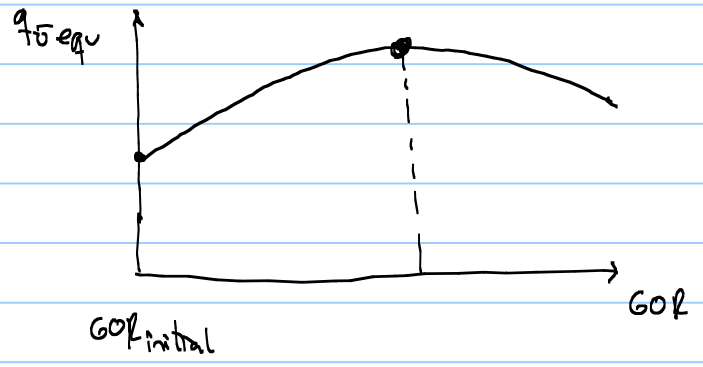


$GOR_2 > GOR_1$

↑GOR, ↓ ρ_{mix}, $\rho_{mix} \approx \rho_{gas}$

↓ $\Delta p_{hydrostatic}$

↑ $\Delta p_{frictional}$

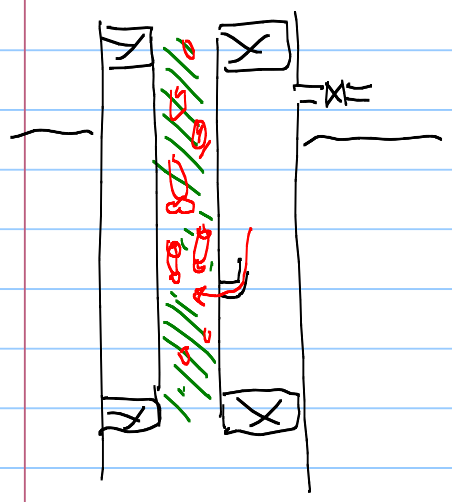


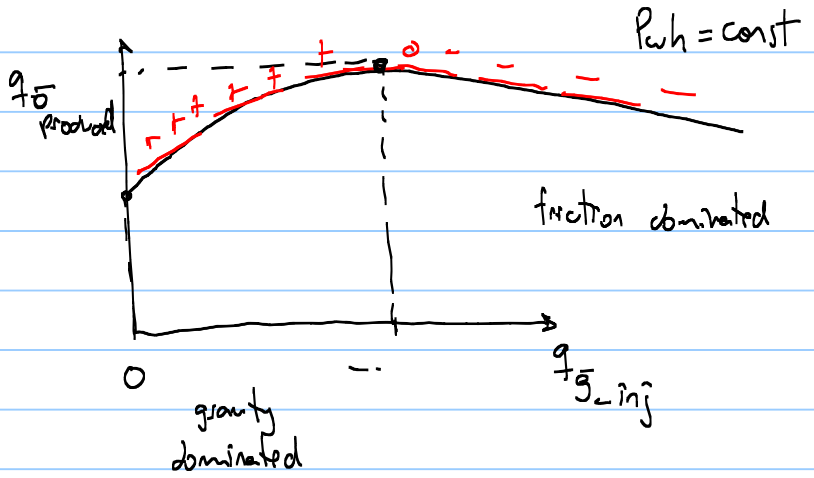
\ GOR changing due to → depletion

→ $q_{\bar{w}}$

$$GLR = \frac{q_{\bar{g}}}{q_{\bar{o}} + q_{\bar{w}}}$$

→ gas lift





$q_{\bar{o}} = f(q_{g\_inj})$ if funtion is concave

max $\dfrac{\partial f}{\partial q_{ginj}} = 0$

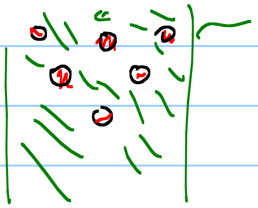
revenue function $Rev = q_{\bar{o}} \cdot P_o - \underset{\sim}{q_{g\_inj}} \cdot P_g$

max Rev. $\dfrac{\partial Rev}{\partial q_{g\_inj}} = \dfrac{\partial q_{\bar{o}}}{\partial q_{g\_inj}} \cdot P_o - P_g = 0$

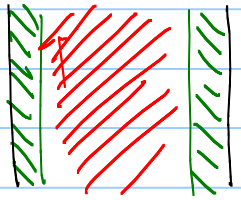
$$\frac{\partial q_{\bar{o}}}{\partial q_{ginj}} = \frac{P_g}{P_o}$$

//

theory about multiphase flow

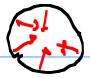


ρ_{mix}
V_{mix}
homogeneous model



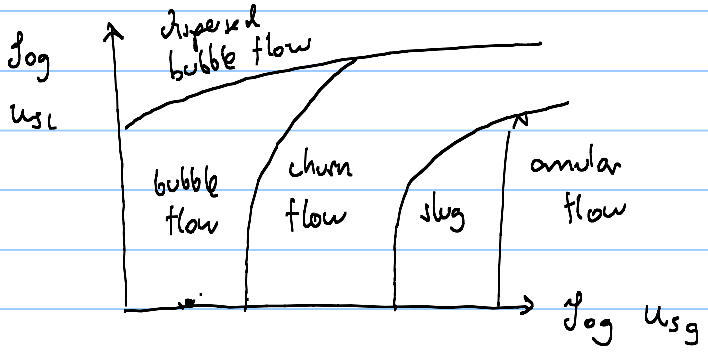
1 equ for each phase

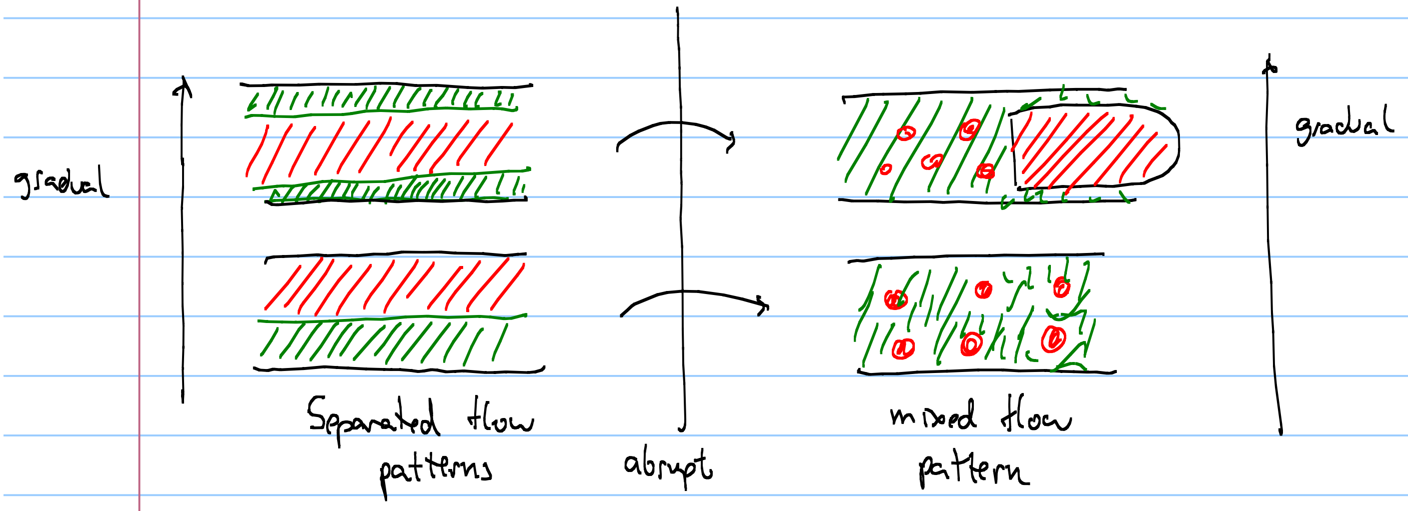
$v_g, v_L, \theta, ID, \varepsilon, \rho_L, \rho_g, \mu_L, \mu_g, \sigma_{Lg}$



$U_{sL} = \dfrac{q_L}{A}$

$U_{sg} = \dfrac{q_g}{A}$

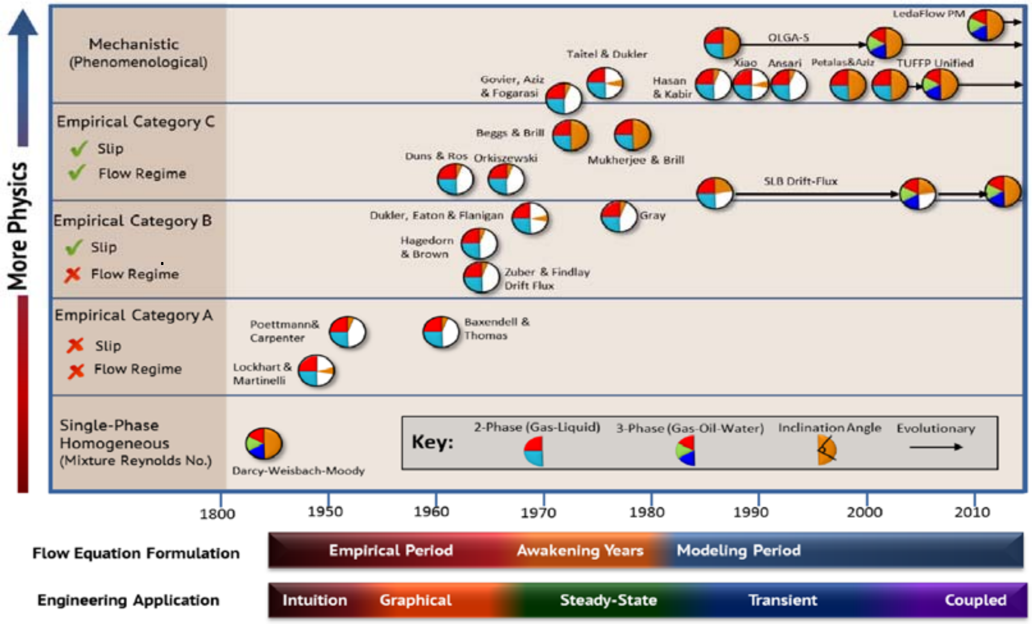




models :
- mechanistic :
  making momentum balance on phase to calculate
  - $\frac{dP}{dx}$
  - flow pattern transition
  - needs correlation → closure model
- mix → drift flux model

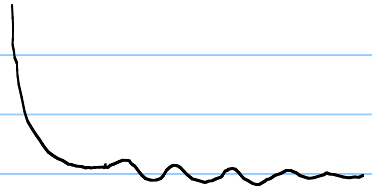
separating forces	mixing forces
• gravity	• inertia
• interfacial tension	• turbulence
• diffusion	
• centrifugal	

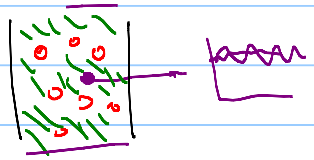
- correlation / empirical
  - lab / field measurements
    fure to a correlation.



- multiphase models for different purpose → $\frac{dP}{dx}$ $\frac{\partial}{\partial t} = 0$ steady-state

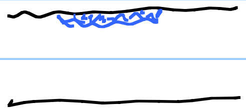
→ transient → slugging
severe slugging
liquid accumulation
start up
shut down



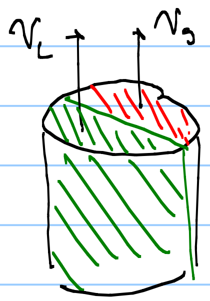


→ phase distribution
velocity distribution
etc.

- separator design
- tol corrosion
- emulsion oil + w



multiphase flow definitions



homogeneous model $V_g = V_L = V_{mix}$ no slip

$$\underline{V_{mix}} = \frac{q_L + q_g}{A} = u_{sL} + u_{sg}$$

$\frac{A_g}{A}$ $\frac{A_L}{A}$ if homogeneous flow

$$\lambda_g = \frac{A_g}{A} \quad \lambda_L = \frac{A_L}{A} \quad \lambda_g + \lambda_L = 1$$

$$q_L = V_L \cdot A_L$$

$$q_g = V_g \cdot A_g$$

for homogeneous flow:

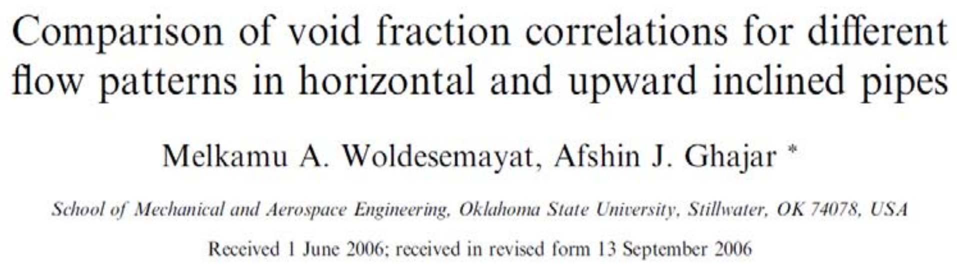
$$q_g = \overset{\checkmark}{A_g} \cdot V_m \qquad q_g = A_g \left(\frac{q_g + q_L}{A}\right) \Rightarrow \lambda_g = \frac{q_g}{q_g + q_L} \qquad \lambda_L = \frac{q_L}{q_L + q_g} \qquad \lambda_g = \frac{q_g}{q_L + q_g}$$

for most cases $V_g \neq V_L$ slip

Liquid holdup $H_L = \frac{A_L}{A}$ $\underline{\varepsilon = \frac{A_g}{A}}$ void fraction

$$H_L = f(u_{sL}, u_{sg}, \theta, ID, \rho_L, \rho_g, \mu_L, \mu_g - - -)$$

$$\varepsilon = \frac{U_{SG}}{U_{SG}\left(1+\left(\frac{U_{SL}}{U_{SG}}\right)^{\left(\frac{\rho_G}{\rho_L}\right)^{0.1}}\right)+2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L-\rho_G)}{\rho_L^2}\right]^{0.25}(1.22+1.22\sin\theta)^{\frac{p_{atm}}{p_{system}}}}$$



Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

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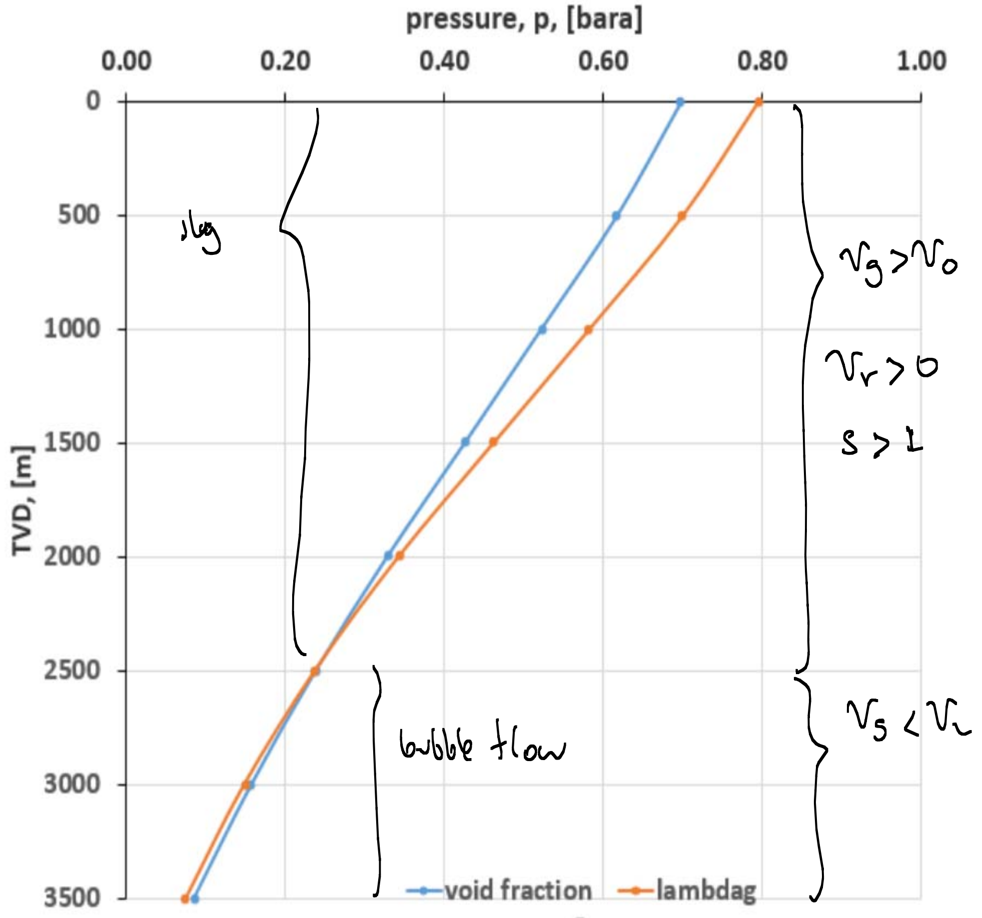
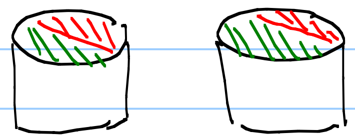


$\lambda_g > \varepsilon$??



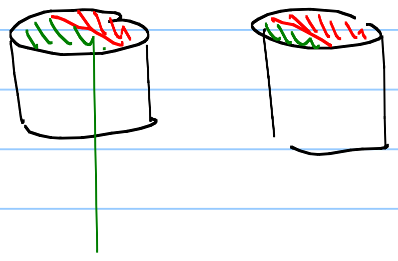
$q_o = A_o \cdot v_o$ q_o

$q_g = A_g \cdot v_g$ q_g

$v_r = v_g - v_L$

$S = \text{slip} = \frac{v_g}{v_L}$

$\lambda_g < \varepsilon$



$$\rho_{mix} = H_L \rho_L + (1-H_L)\rho_g$$

$$\rho_{mix} = \varepsilon \rho_g + (1-\varepsilon)\rho_L$$

drift flux model $\rightsquigarrow$ mixture equation to represent liquid + gas

$$\left.\frac{dp}{dx}\right|_{@P,T} = -\underset{\sim}{\rho_{mix}} \cdot g \cdot \sin\Theta - \rho_{mix} \cdot \frac{\overset{?}{f}\, V_m^{\overset{?}{2}}}{\phi \cdot 2}$$

neglecting acceleration

$\varepsilon \sim \rho_g (p,T)$

$\sim \rho_o (p,T)$

$\Big\} \; q_o (p,T$

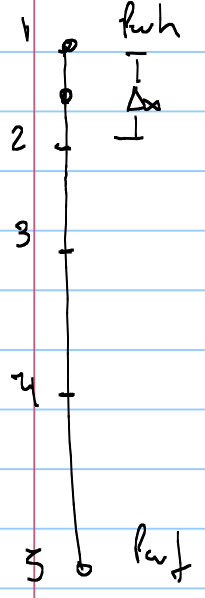
$q_g (p,T)$

f ... single phase flow but with Re_m

$$Re_m = \frac{\rho_{mx}\, \phi\, \tilde{V}_m}{\mu_m}$$

$$\mu_m = \mu_L (1-\varepsilon) + \varepsilon \mu_g$$

Integration procedure.



- subdivide the conduit (tubing, flowline) in segments
- start from boundary with know P, T (P_{wh})
- calculate local q_o, q_g
  properties ($\rho_o, \rho_g, \mu_o, \mu_g, \sigma_{og}$)
- calculate $\left.\frac{dp}{dx}\right|_{P,T}$ $\rightarrow$ correlation
  $\rightarrow$ drift flux
  $\rightarrow$ olga, leda
- Integration

$$P_2 = ?$$

Euler's method explicit

$$\left.\frac{dp}{dx}\right|_{P_1, T_1} = \frac{P_2 - P_1}{\Delta x}$$

$$P_2 = P_1 - \Delta x \frac{dp}{dx}$$

$$\left.\frac{dp}{dx}\right|_{\frac{P_1+P_2}{2},\ \frac{T_1+T_2}{2}} = \frac{P_2 - P_1}{\Delta x}$$

implicit

- Assume P_2 ←
- Calculate P_{av}.
- calculate $\left.\frac{dP}{dx}\right|_{P_{av}}$
- calculate $P_{2_{cal}} = P_1 - \left.\frac{dP}{dx}\right|_{av} \cdot \Delta x$

Is P_2 assumed = $P_{2\,calc}$? yes → not

↓

proceed to P_3

ODE $\frac{dP}{dx}$ = constant

P_1 =

T_1 =

x_2

x_3

x_4

$\frac{dP}{dx} = 0.02 \frac{bar}{m}$

hydrostatic colum of water

$\frac{dP}{dx} \simeq 0.1 \frac{bar}{m}$

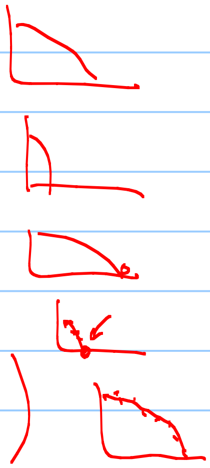
hydrostatic colum of air

$\frac{dP}{dx} \simeq 0.001 \frac{bar}{m}$

		BO table colum	3	4	5	6	8	10	7	9	11								
TVD [m]	T [C]	p[bara]	Rs [Sm^3/Sm^3]	rs [Sm^3/Sm^3]	Bo [m^3/Sm^3]	Bg [m^3/Sm^3]	deng [kg/m^3]	viscg [cp]	deno [kg/m^3]	viso [cp]	sigma_o_g [N/m	qo [m^3/d]	qg[m^3/d]	uso [m/s]	usg [m/s]	Flow pattern	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	0.0	728.8	1.8	1.15E-02	1174.3	4.566E+03	0.769	2.991		0.80	0.70	0.0260
500	63.0	41.0	33.7	1.25E-05	1.2	2.36E-02	56.4	0.0	711.0	1.1	9.23E-03	1224.4	2.862E+03	0.802	1.874		0.70	0.62	0.0312
1000	76.0	56.6	46.8	1.25E-05	1.3	1.65E-02	78.8	0.0	691.8	0.8	7.09E-03	1283.1	1.791E+03	0.840	1.173		0.58	0.52	0.0373
1500	89.0	75.2	62.2	1.31E-05	1.4	1.25E-02	105.5	0.0	671.3	0.6	5.17E-03	1352.5	1.161E+03	0.886	0.761		0.46	0.43	0.0428
2000	102.0	96.7	79.3	1.45E-05	1.4	9.82E-03	133.4	0.0	650.4	0.5	3.60E-03	1430.1	7.452E+02	0.937	0.488		0.34	0.33	0.0476
2500	115.0	120.5	97.4	1.68E-05	1.5	8.17E-03	159.9	0.0	629.9	0.4	2.47E-03	1514.4	4.721E+02	0.992	0.309		0.24	0.24	0.0512
3000	128.0	146.1	115.9	2.02E-05	1.6	7.12E-03	182.9	0.0	610.4	0.3	1.68E-03	1602.1	2.803E+02	1.049	0.184		0.15	0.16	0.0537
3500	141.0	172.9	133.8	2.46E-05	1.7	6.46E-03	201.6	0.0	592.5	0.3	1.18E-03	1690.3	1.380E+02	1.107	0.090		0.08	0.09	0.0552

Homework : flow equilibrium saturated oil well

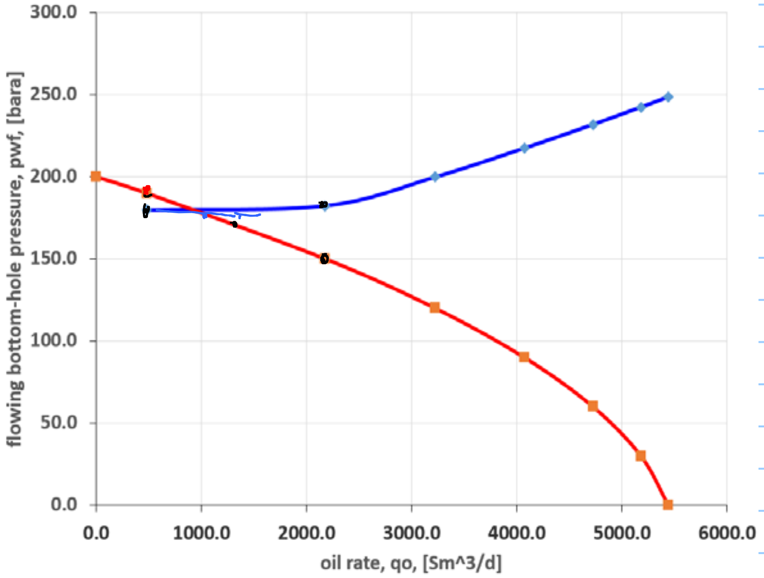
P_{wh}



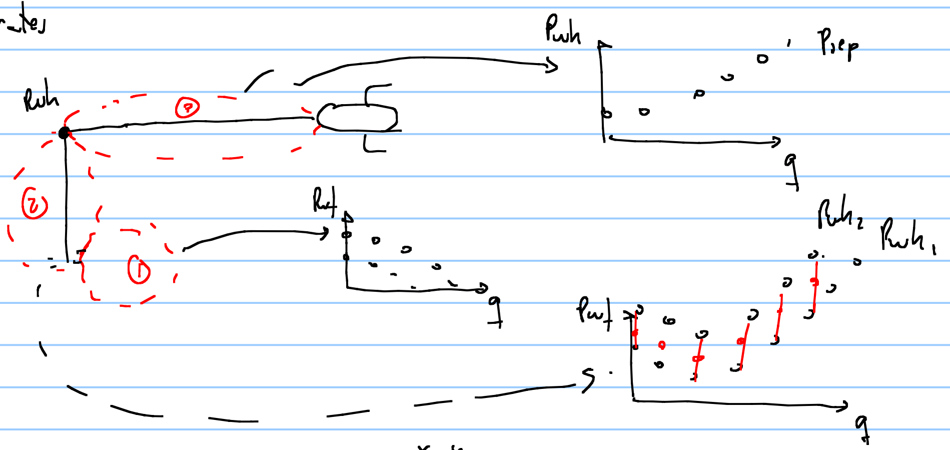
Vogel $q_o = q_{o\max}\left(1 - 0.2\frac{P_{wf}}{P_R} - 0.8\left(\frac{P_{wf}}{P_R}\right)^2\right)$

Class 6

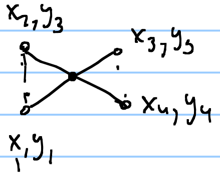
Solution of class exercise - flow equilibrium in a saturated oil well:



in commercial software, performance curves are often precomputed and later the collection of points is used to find operating rates

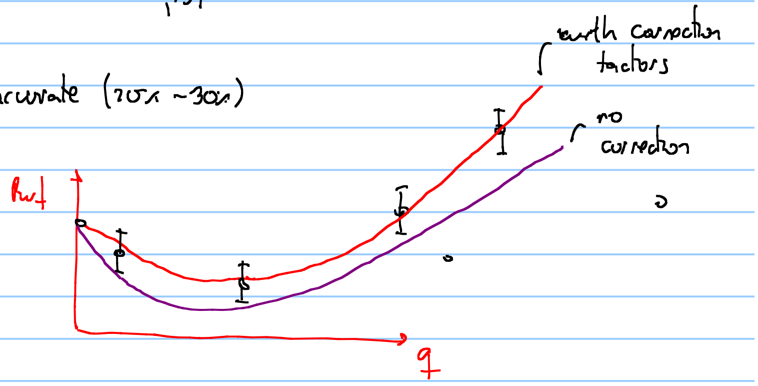


the insection is found by intersecting two straight lines

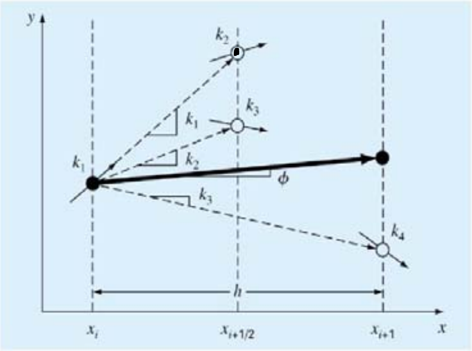


in most cases multiphase $\frac{dp}{dx}$ models are not 100% accurate (20% - 30%)

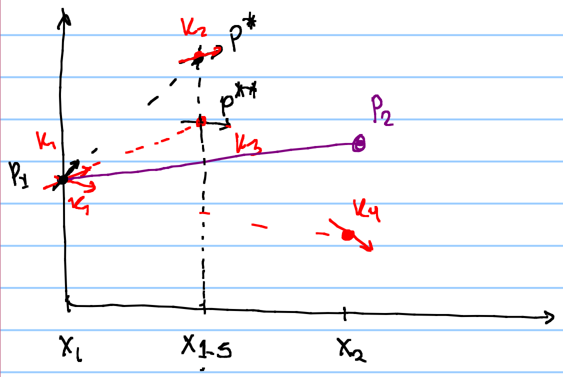
$$\left.\frac{dp}{dx}\right| = -\overset{C_1}{\underset{\sim}{\rho_{mix}}} \cdot g \cdot \sin\theta - \overset{C_2}{\rho_{mix}} \cdot \frac{f \cdot V_m^{2}}{\phi \cdot 2}$$



Classic 4th-order Runge-Kutta Method



l Methods 32 Prof. Jinbo Bi CSE, UConn



① $\left.\frac{dP}{dx}\right|_{P_1,x_1} = k_1$ Euler's

② $\frac{P^* - P_1}{\Delta x \cdot 0.5} = k_1 \rightarrow P^*$

③ @ $P^*, x_{1.5} \rightarrow \left.\frac{dP}{dx}\right|_{P^*, x_{1.5}} \rightarrow k_2$

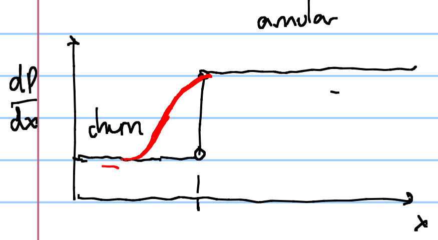
④ $\frac{P^{**} - P_1}{\Delta x \cdot 0.5} = k_2 \rightarrow P^{**}$

⑤ @ $P^{**}, x_{1.5} \rightarrow \left.\frac{dP}{dx}\right| = k_3$

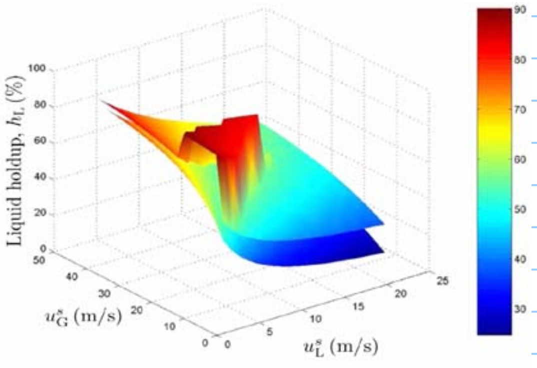
⑥ $\frac{P_2' - P_1}{\Delta x} = k_3 \rightarrow P_2'$

⑦ @ P_2', X_2 $\rightarrow \frac{dP}{dx} = k_4$

⑧ $y^*(t_0+h) = y^*(t_0) + \frac{k_1+2k_2+2k_3+k_4}{6}h \rightarrow P_2$

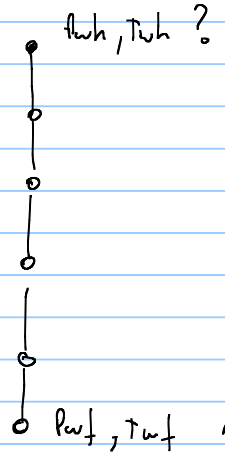


flow pattern based dp model looks like



what happens with integration of p, T ?

$\underbrace{\frac{dp}{dx}}$ momentum balance multiphase $\underbrace{\frac{dT}{dx}}$ energy balance (high GOR gas wells)



Pwf, Twf ~> $\frac{dp}{dx}$, $\frac{dT}{dx}$ →

$\frac{dp}{dx}$ is often a function of T

$$\frac{\partial p}{\partial x} = f(T, \ldots)$$

$$\frac{\partial T}{\partial x} = f(p, \ldots)$$ e.g Joule Thomson coefficient

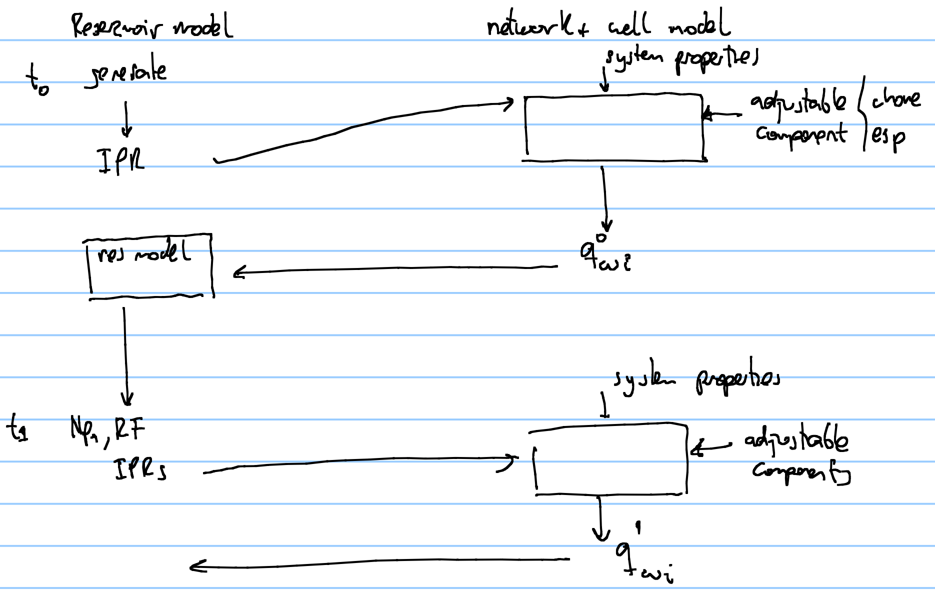
coupled system of differential equations

luckily $\frac{\partial p}{\partial x} = f(T)$ is weak

$\frac{\partial T}{\partial x} = f(p)$ is weak

---

field performance



coupling of reservoir and network models


SPE 65159

Linking reservoir and surface simulators: how to improve the coupled solutions

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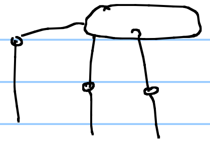
- undersaturated oil reservoir strong aquifer support → no multiphase flow in the formation during life of field
- field produced with ESPs

- well are standalone (no surface network)

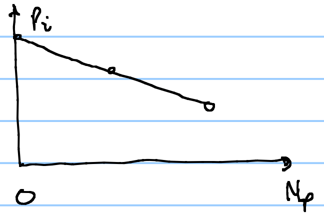


- well are identical

B_o, C_e, V, V_A

Reservoir model $P_R = P_i - A \cdot N_p$

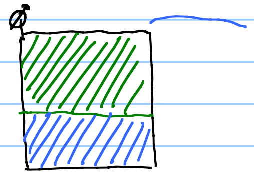
Material balance



initial oil in place

compressibility

$$\frac{1}{A} = \frac{\left(N \cdot B_{oi} \cdot c_o \cdot + N \cdot B_{oi} \cdot \frac{c_w \cdot S_w + c_f}{S_o} + V_a \cdot \phi_a \cdot (c_w + c_f) \cdot B_w \right)}{B_o(p)}$$



well model $q_o = J \cdot (P_R - P_{wf})$

ESP can operate down to a minimum bottomhole pressure $P_{suc} > P_b(T_R)$

$P_{wf min}$

$$q_{o max} = J\,(P_R - P_{wf min})$$

for "N_w" number of identical wells

$$q_{field} = N_w J\,(P_R - P_{wf min})$$

$$q_f = N_w\, J\,(P_i - A \cdot N_p - P_{wf min})$$

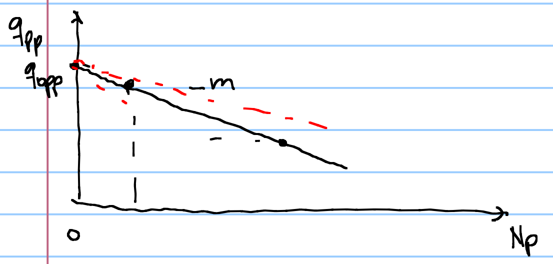
$$q_f = -N_w \cdot J \cdot A \cdot N_p + \underbrace{N \cdot J\,(P_i - P_{wf min})}_{\text{initial maximum production}}$$

$$\underline{q_f} = -N_w \cdot J \cdot A\; \underset{\sim}{N_p} + q_{of}$$

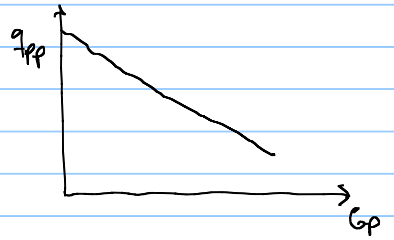
max field rate @ N_p

$q_f = q_{pp}$ field production potential

$q_{pp} = -m N_p + q_{opp}$ → field production potential @ initial reservoir pressure



for other cases • dry gas and network



production potential is calculated by,

@ a given N_p, run model to obtain maximum production → open choke

→ ESP, Perfmin, frequency

→ gas lift → optimization

→ multiphase booster → max frequency

$$\int_0^t q_f \cdot dt$$

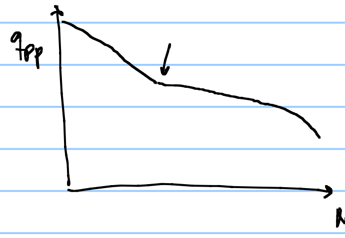
$$\left\{ q_{pp} = \underline{-m} N_p + q_{ppo} \right\}$$ — Perfmin, inital conditions

→ A → reservoir (size, structure, fluid properties)

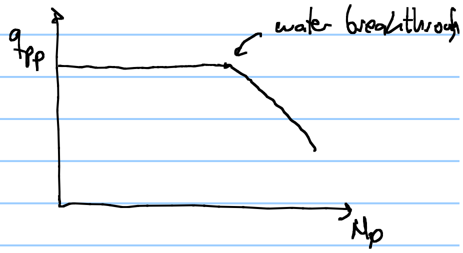
→ N_w → number wells

→ J → well productivity

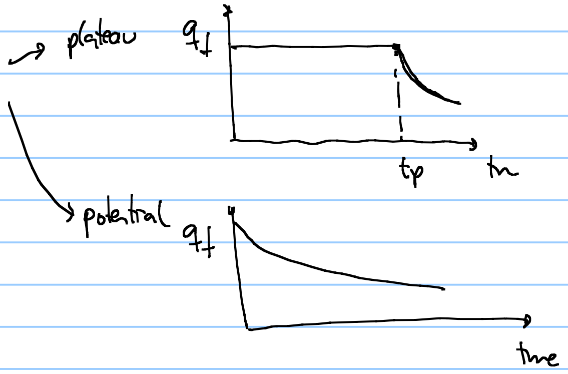
• saturated + undersaturated



• undersaturated + water injection $P_R \simeq$ constant



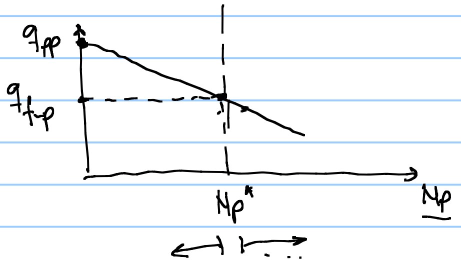
field are typically produced in two modes



$q_f = f(t)$?

$$q_f \begin{cases} q_{f-p} & \text{if } t \leq t_p \\ q_f = q_{pp} & \text{for } t > t_p \end{cases}$$

plateau will end when $q_{f-p} = q_{pp}$



for $N_p > N_p^*$ I cannot maintain desired plateau

for $N_p \leq N_p^*$ I can produce plateau

$N_p^*\ [Sm^3]$ $\quad \underline{q_{f-p}}\ [Sm^3/d]$ $\quad 0 \rightarrow N_p^*$

$$t_p = \frac{N_p^*}{q_{f-p}} \frac{[Sm^3]}{[Sm^3/d]} \frac{1}{\text{uptime}} \quad (1)$$

uptime ; number of operational days in a year $\left[\frac{\text{day}}{\text{year}}\right]$

$$q_{p-f} = q_{pp} = -m N_p^* + q_{ppo}$$

$$N_p^* = \left[\frac{q_{ppo} - q_{pf}}{m}\right] \quad (2)$$

$$t_p = \frac{q_{ppo} - q_{p-f}}{m \cdot q_{f-p}\ \text{uptime}} = \frac{1}{m} \frac{1}{\text{uptime}} \left(\frac{q_{ppo}}{q_{f-p}} - 1\right)$$

$$t_p = \frac{1}{N_w \cdot A \cdot J} \left[\frac{N_w J (P_R - P_{wf,min}) - 1}{q_{f-p}}\right]$$

$$t_p = \frac{1}{A \cdot J} \left[\frac{J (P_R - P_{wf,min})}{q_{fp}} - \frac{1}{N_w}\right]$$

for $t > t_p$

$$q_f = q_{pp} = -m \int_0^t q_f\, dt + q_{ppo}$$

$$q_f = -m \int_0^{t_p} q_{p-f}\, dt - m \int_{t_p}^{t} q_f\, dt + q_{ppo}$$

$$q_f = -m\ \underline{q_{p-f}}\, t_p - m \int_{t_p}^{t} q_f\, dt + q_{ppo}$$

$$q_f = -\frac{\cancel{m}}{\cancel{m}}(q_{ppo} - q_{pf}) - m \int_{t_p}^{t} q_f\, dt + q_{ppo}$$

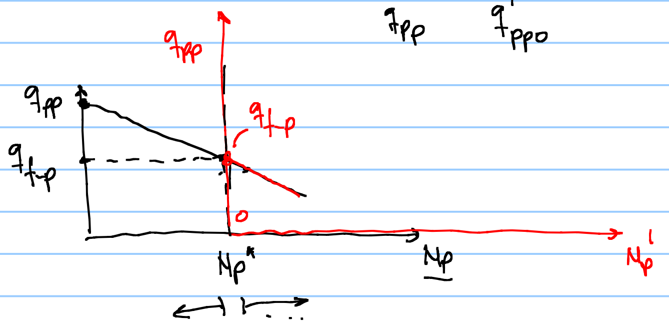
$$\underbrace{\underline{q_f}}_{q_{pp}} = \underbrace{q_{p-f}}_{q'_{ppo}} - \underline{m} \underbrace{\int_{t_p}^{t} \underline{q_f}\, dt}_{N_p'} \quad //$$

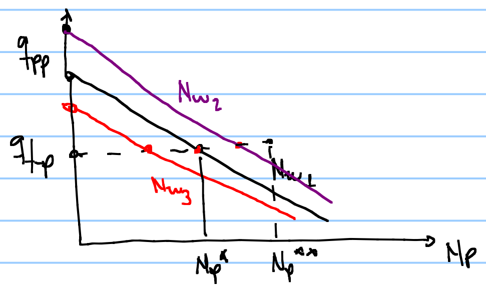
https://en.wikipedia.org/wiki/Integrating_factor

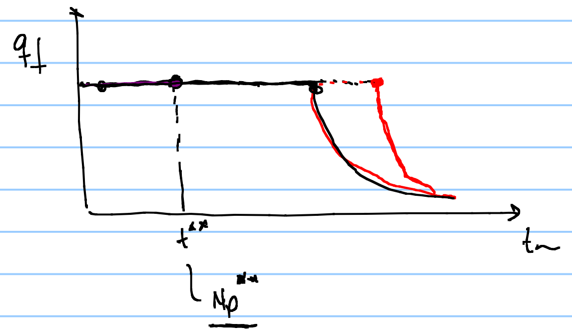
a solution is :

$$q_f = q_{f-p} \cdot e^{-\boxed{m}(t - t_p)}$$

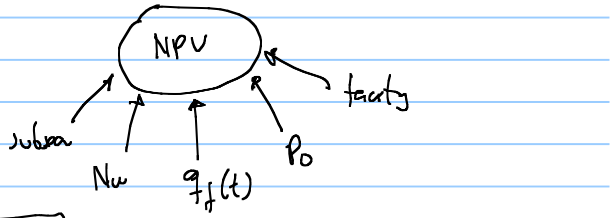
$m \rightarrow A \cdot J \cdot N_w$







in the field development process decision → NPV



time	expenditures	Revenue	Cash flow Revenue - Expenditure	DCF
0				
1				
2				
3				
4				
5				

$$PV = \frac{FV}{(1+i)^n}$$

n → year counter

i → interest rate

$$\overline{\sum DCF} = NPV$$

find q_{fp} such as NPV is maximum ↑q_{f-p} increase revenue ↑ ↑CAPEX $= f(q_{f-p})$?

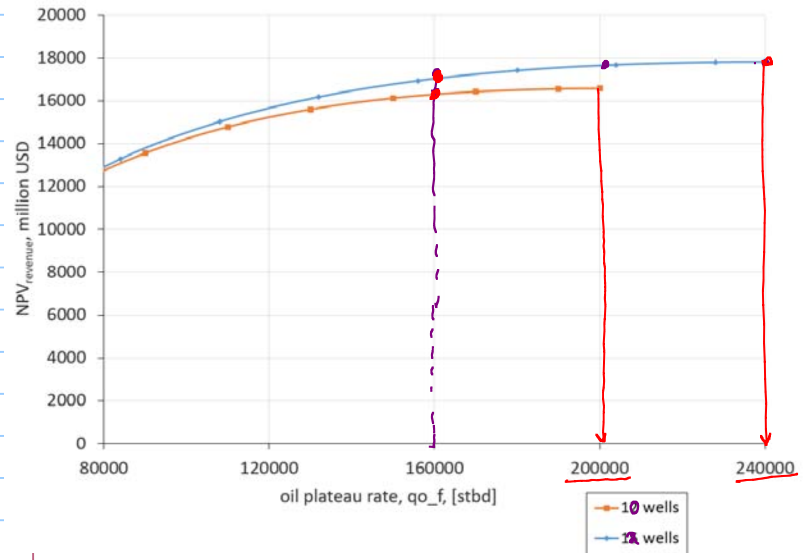
$$PV\ Revenue = \int_0^t \frac{P_o \cdot q_f}{e^{it}}\,dt$$

discrete discounting $\frac{1}{(1+i)^n}$

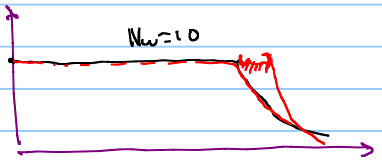
continuous discounting $\frac{1}{e^{it}}$

$$PV_{revenue} = P_o\left[\int_0^{t_p} \frac{q_{f-p}}{e^{it}}\cdot dt + \int_{t_p}^{t} \frac{q_{f-p}\, e^{-m(t-t_p)}}{e^{it}}\,dt\right]$$

$$PV_{revenue}(t) = q_{f-p}\cdot P_o\left[\frac{m+i - m\, e^{-\left(\frac{q_{fpo}}{q_{f-p}}-1\right)\frac{i}{m}} - i\, e^{-(m+i)t + \left(\frac{q_{fpo}}{q_{f-p}}-1\right)}}{i\,(m+i)}\right]$$



$$\frac{1}{e^{it}}$$



Class 7

$$NPV = \sum_{j=0}^{N_{years}} \frac{cashflow_j}{(1+i)^j} \qquad cashflow_j = \underline{Revenue_j} - \underline{CAPEX_j} - \underline{DRILLEX_j} - OPEX_j - \ldots$$

find $q_{plateau}$ such that NPV max   for revenue is max when $q_{plateau} = max$

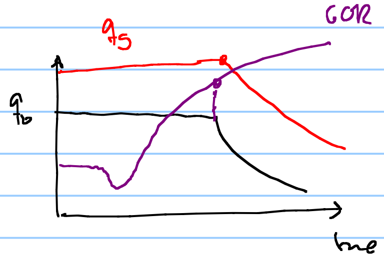
N_{well} $N_{well} = max$

$$\frac{\partial Revenue}{\partial q_{p-f}} = 0$$

Neglecting OPEX, all DRILLEX and CAPEX are invested in year "0"

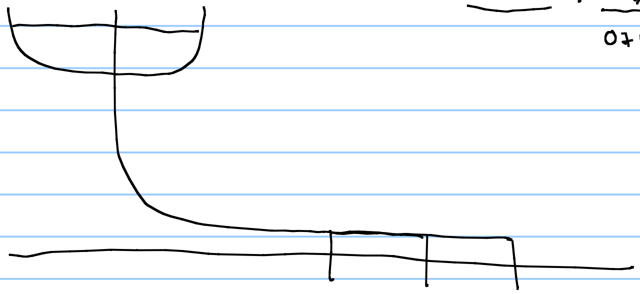
$$NPV(q_{pf}, N_w) = \underline{PV\,Revenue} - DRILLEX - \underline{CAPEX}$$

$DRILLEX = P_w \cdot N_w \neq f(q_{pf})$



$$CAPEX = \underbrace{CAPEX_{TOPSIDES}}_{\overset{!}{=} f(N_w)} + CAPEX_{SUBSEA} \to f(weight)$$

$$\underline{weight} = f(\underbrace{q_{max\,liq}}_{O+W} + \underline{q_{max\,gas}})$$



$$CAPEX_{TOPSIDES} = f(\underbrace{q_{max\,liquid}}_{q_{f-p}}, \underbrace{q_{max\,gas}}_{GOR \cdot q_{f-p}})$$

$$CAPEX_{topsides} = A + B\,q_{f-p} + C\,\underbrace{q_g}_{GOR\,q_{f-p}}$$

$(B+C)$

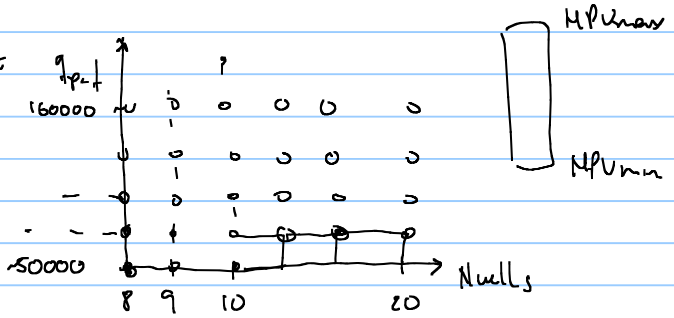
$$\frac{\partial NPV}{\partial q_{pf}} = \frac{\partial Revenue}{\partial q_{pf}} - \frac{\partial CAPEX_{topsides}}{\partial q_{pf}} = 0$$


$- \; (B+C)$

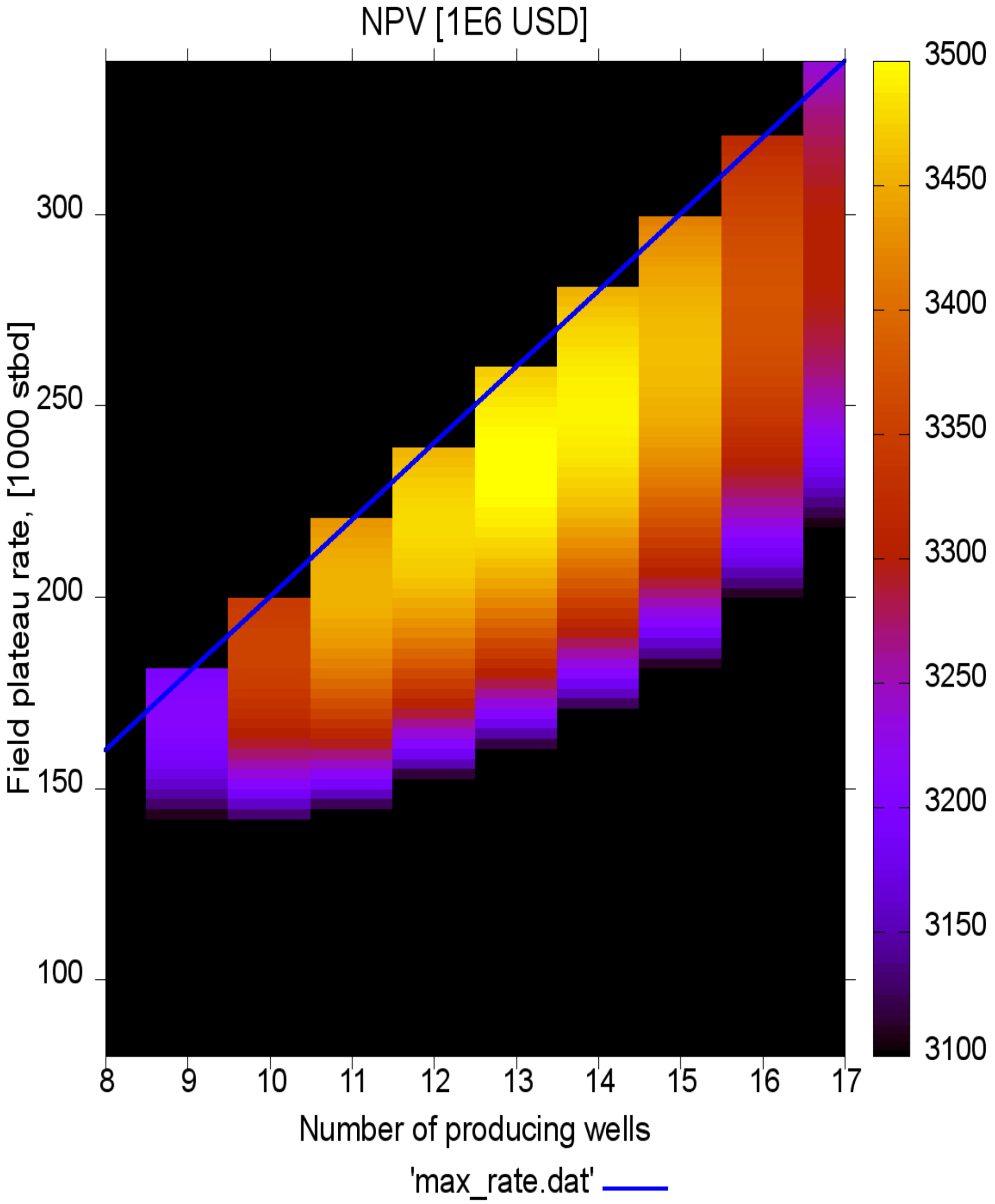
$$\frac{\partial Revenue}{\partial q_{pf}} = (B+C)$$

$$CAPEX_{subsea} = f(N_w, q_{pf}, N_{flowlines}, N_{templates})$$

to see effect of Nw   Color Plot



Using GNU plot:



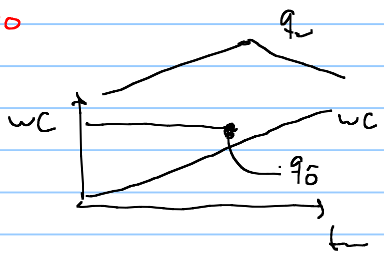
what happens when producing water?

$$CAPEX_{manifolds} = A + B\,q_{f-p} + C\,\underbrace{q_g}_{GOR\, q_{f-p}}$$

$$\downarrow$$

$$B\left(q_{wmax} + q_{p-f}\right)$$

$$wc = \frac{q_w}{q_o + q_w} \rightarrow q_w = \frac{wc}{1-wc}\, q_o$$



$$B\left(q_{p-f} + \frac{wc}{1-wc}\, q_{p-f}\right)$$

$$\downarrow$$

$$B\, q_{p-f}\left(1 + \frac{wc}{\underbrace{1-wc}}\right)$$

$$q_{p-f}\left(\frac{1}{1-wc}\right)$$

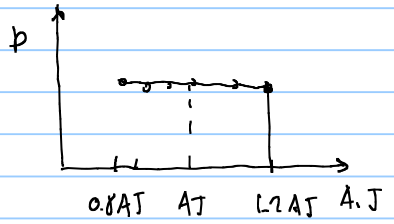
wc = 30%
constant

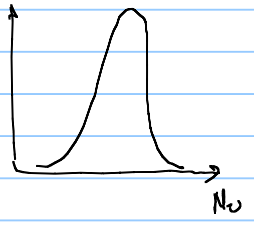
$$wc = f(\underbrace{N_p}) = C\_N_p$$

$$\text{at } t_p\ \boxed{wc^*} = C.N_p^*$$

$$q_{wmax} = \left(\frac{wc^*}{1-wc^*}\right) q_{p-f}.$$

Effect of uncertainty in the input (in this case well productivity index J, +-20%)





Monte Carlo sampling

$$\downarrow$$

$$AJ = AJ_{min} + Rand\left(AJ_{max} - AJ_{min}\right)$$

Homework: compute optimal qf_p and Nwells for variations of +-20% in J. Calculate the probability distribution of optimal qpf, Nwells and NPV

Last point: effect of a linear variation of oil price with time - descending, ascending.

END OF LECTURES