NTNU | Norwegian University of Science and Technology

OG621 – Oil and Gas Production wells

Spring Semester 2022

Information

- Lecturer: Assoc. Prof. Milan Stanko (Production Tech) (milan.stanko@ntnu.no).
- Prof. Adela Syikilili
- Assistant professor Beatrice Issara

Course scope

- Production performance of wells and gathering systems.
- Addresses the integrated production system, inflow, tubing and pipe flow, and technologies such as artificial lift
- Developing skills for planning, operating, monitoring, troubleshooting and controlling production of oil and gas production systems

Goals of the course

At the end of the course, the student should be able to:

- Perform common production engineering calculations
- Understand the fundamentals of petroleum production engineering
- Describe the main components of the production system, the most common well completions, artificial lift methods and configurations of production systems
- Describe, understand and explain the functionality of the main components of a production system
- Understand the factors and drivers involved in the planning and operation of oil and gas wells

Course content

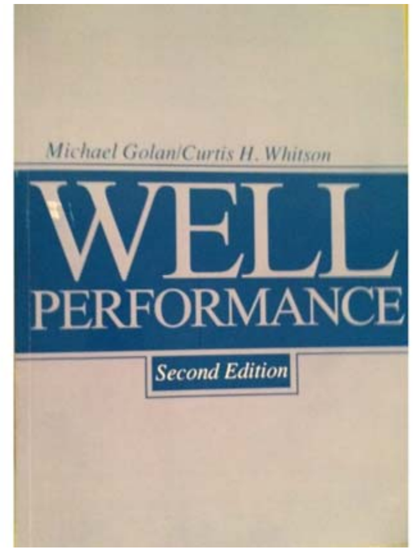
- Introduction (well layout, production engineering domain)
- Flow equilibrium
- Inflow performance relationship
  - Undersaturated Oil
    - Radial and horizontal wells
    - Water coning
  - Dry Gas
    - High velocity flow
  - Saturated oil
- Tubing performance
  - Dry Gas flow
  - Tubing size considerations
  - Multiphase flow of oil, gas and water
- Artificial lift
  - Gas lift
  - Electric submersible pump
- Temperature calculations in wellbore
- Choke performance

Reference material

- Milan's Compendium
- Book: Well performance (Golan and Whitson)
- Other relevant material, e.g. articles, Excel files, notes, links, will be provided or mentioned in the videos

Other

- Production wells compendium (Asheim)
- Book Nodal analysis of Oil and Gas production Systems, (Jansen)



Teaching

- Flipped classroom
  - Participants watch by themselves pre-recorded videos (ca 15-40 min) (on Youtube)
  - Live classes every week/2 weeks
    - Discussing further theory, exercises, tutorials on software, Q&A, advanced topics
    - In Zoom

How to watch the videos

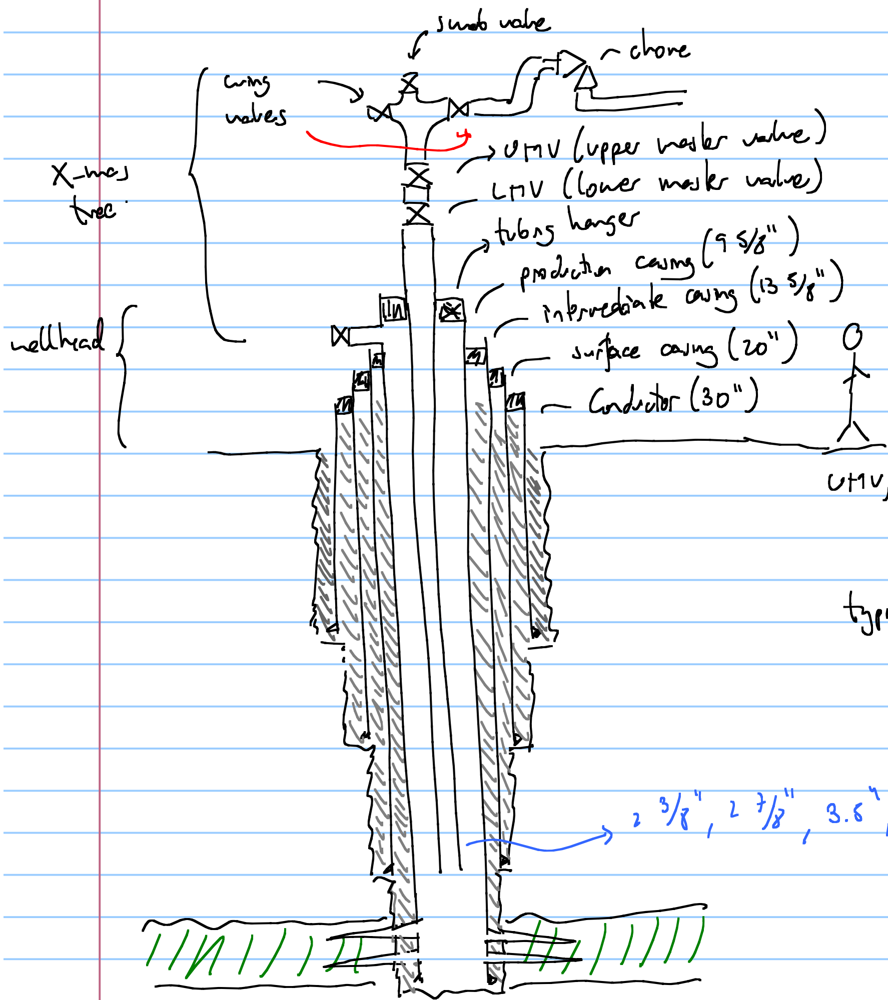
- Watch the entire video (can be watched at 1.5-2x speed)
- At certain time stamps (**or at the end of the video**), the videos might have embedded links to other relevant videos and material
- Pause when needed. Try to summarize what was presented with your own words. Take notes.
- DO THE EXERCISES BY YOURSELF
- Read the additional material provided, if any

Tools

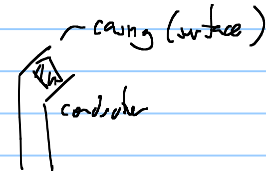
- Excel (VBA)
- Python (Jupyter Notebook) – using Google Colab – maybe?

Questions?

Video 01: Well layout and domain of production engineering



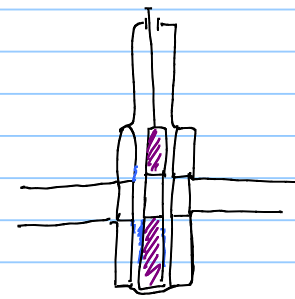
casing hanger



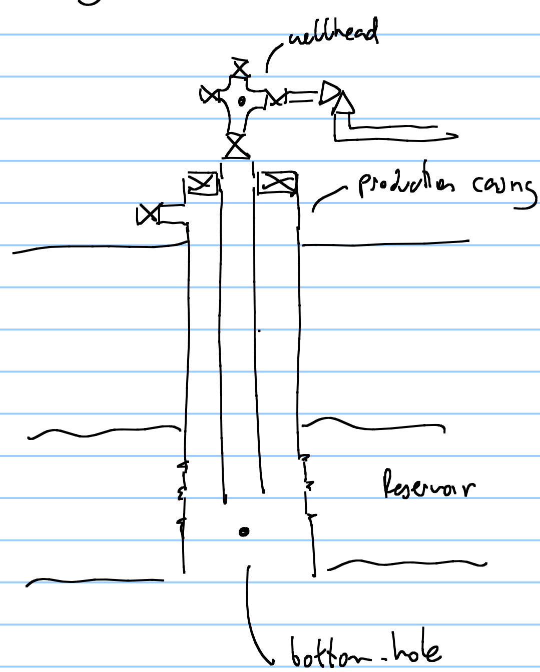
UMV, LMV, wing valves, swab valve on-off (fully open, fully closed)

typically gate valves

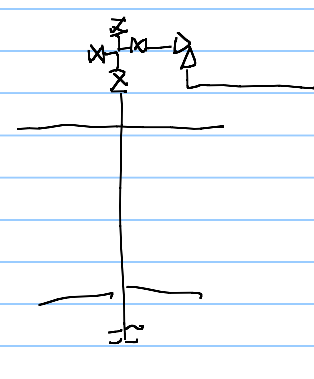
2 3/8", 2 7/8", 3.5", 4", 5", 7", 9"



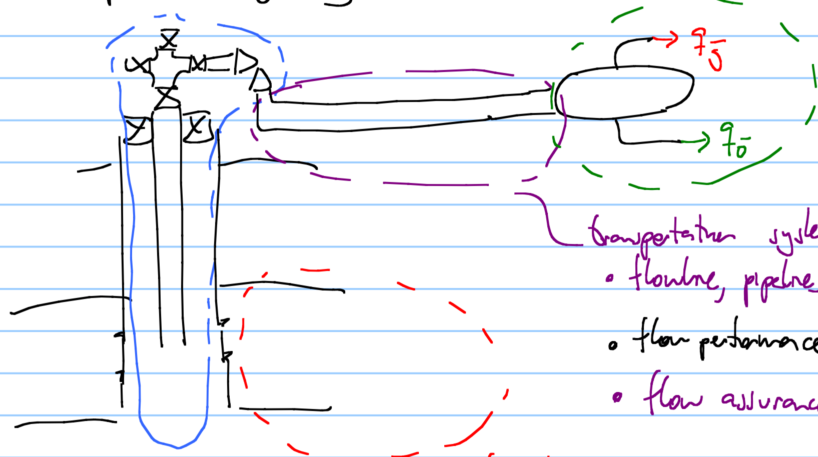
Drawing of a well



Line diagram



Domain of production engineering



processing facilities
- prepare reservoir stream to export (sales)

transportation system
- flowline, pipelines, network
- flow performance
- flow assurance

$\bar{o}$ standard conditions volumetric rate

stb/d

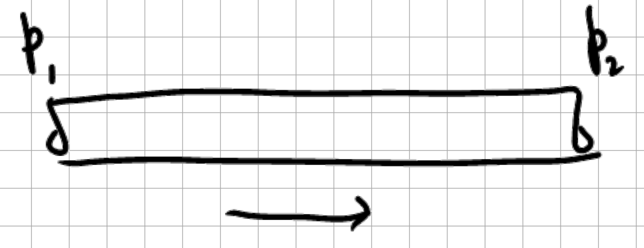
scf/d

Sm^3/d

wellbore
- flow performance
  Δp, ΔT, v
- upper and lower completion
- structural performance
- Artificial lift

near well bore formation
- performance
- fluid distribution
- interface with wellbore
- damage
- stimulation

flow equilibrium

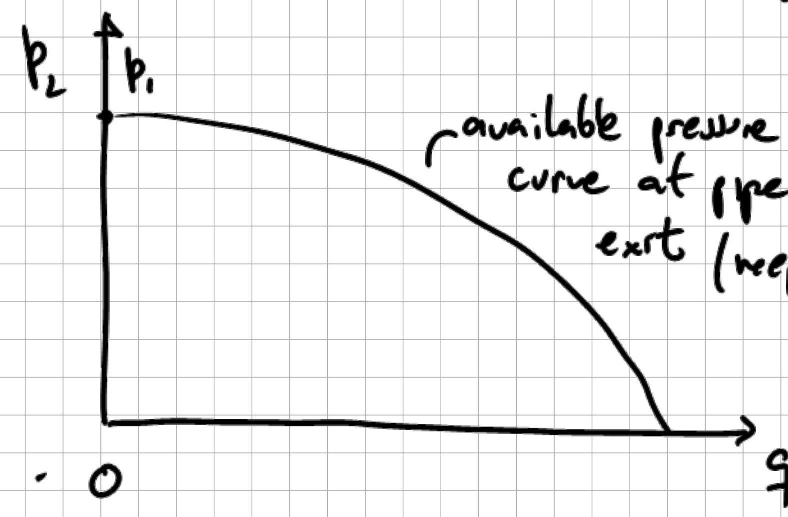


horizontal pipe

q volumetric rate circulating through pipe $[m^3/d]$

1) keep p_1 constant, vary q

$$p_2 = p_1 - \Delta p_{pipe}(q)$$

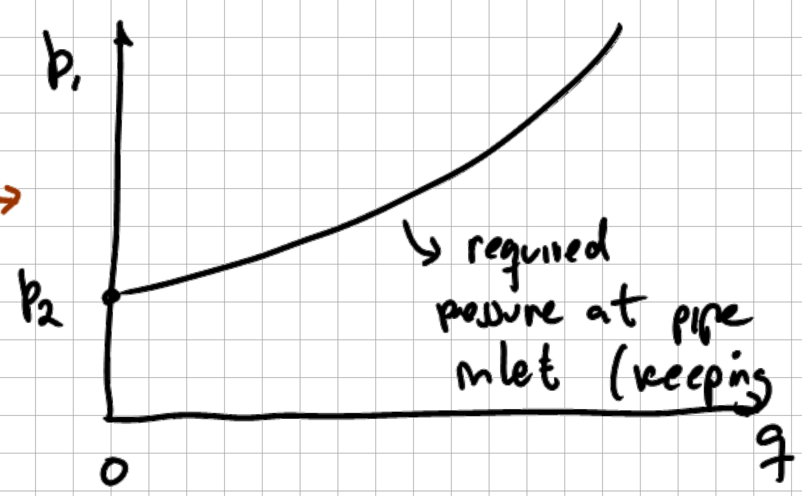


$q = 0$
$\Delta p_{pipe} = 0$

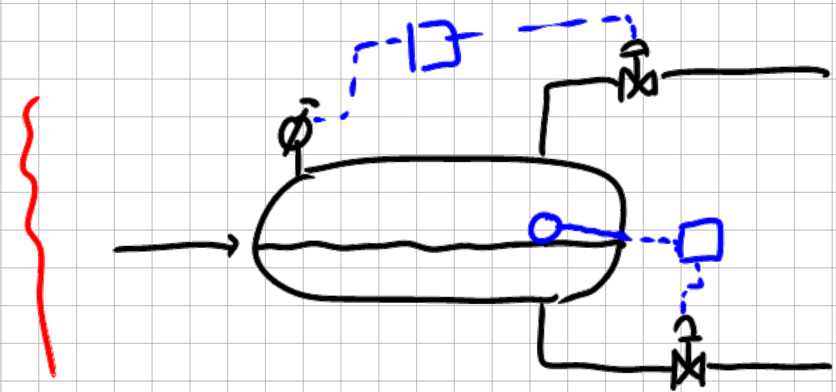
2) keep p_2 constant, vary q, compute p_1

$$p_1 = p_2 + \Delta p_{pipe}$$

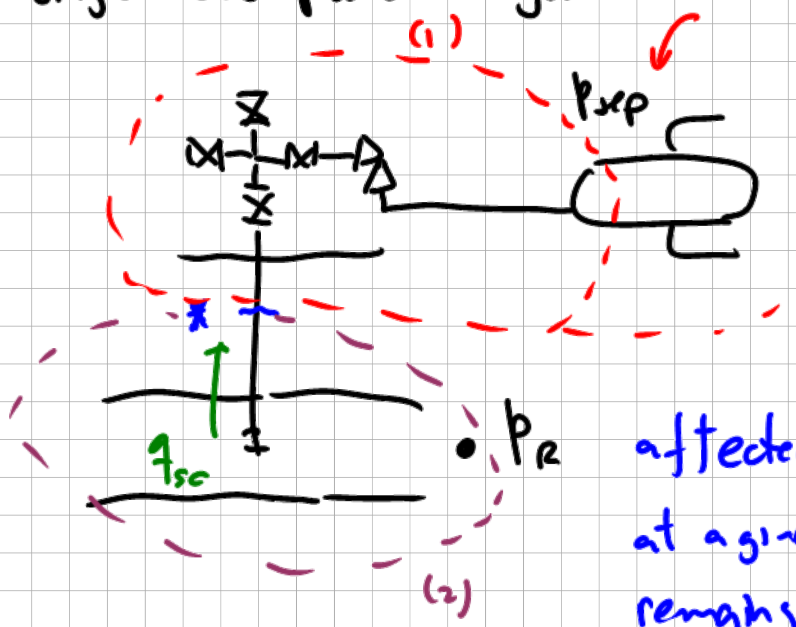
$q = 0$
$\Delta p_{pipe} = 0$
$q \neq 0$
$\Delta p_{pipe} =$
$\uparrow q \ \uparrow \Delta p_{pipe}$



3) provide $p_1, p_2 \rightarrow$ calculate q



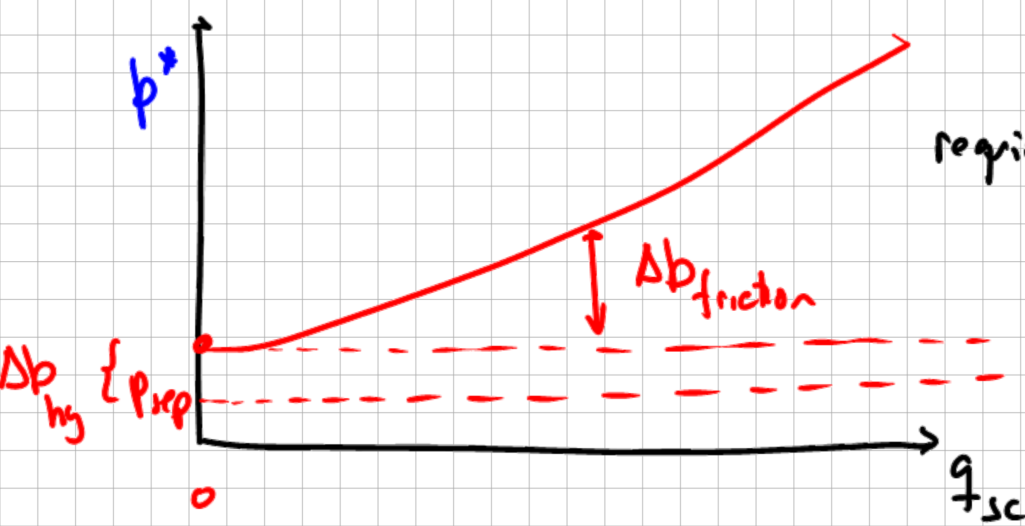
Single well production system



affected by depletion
at a given time "t"
remains fairly "constant"

system (1) resembles case 2

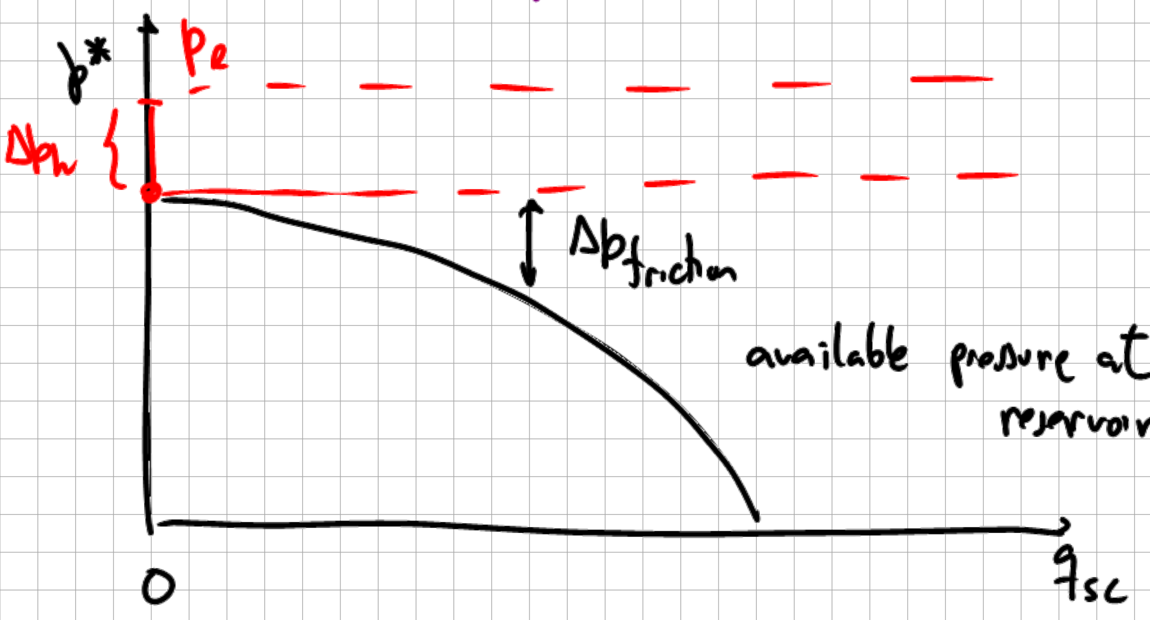
required pressure at p^* (seen from separator)



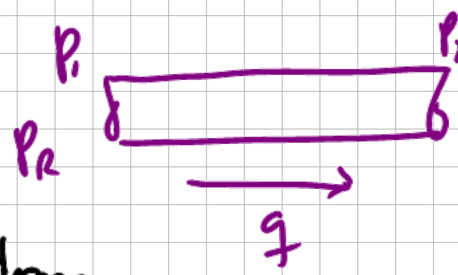
m^3/d affected by p, T
Sm^3/d unaffected by p, T, $p_{sc} = 1.01325$ bara
$T_{sc} = 15.56°C$

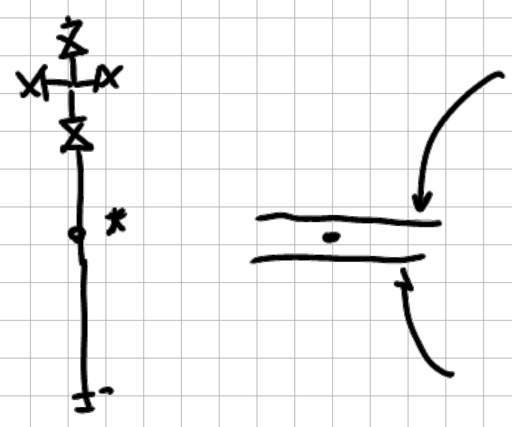
System (2) (formation and part of tubing) resembles case (1)

P_1 fixed, find P_2 by changing q



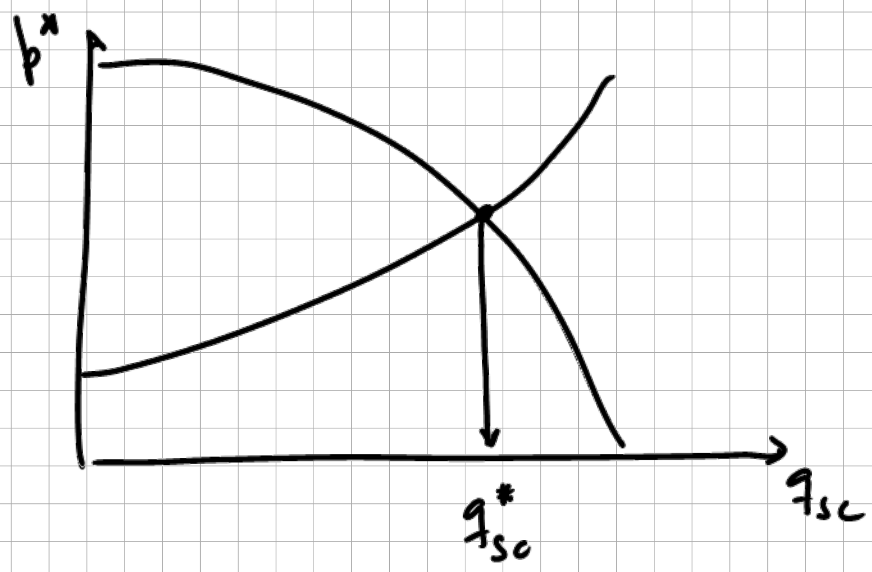
available pressure at * seen from reservoir



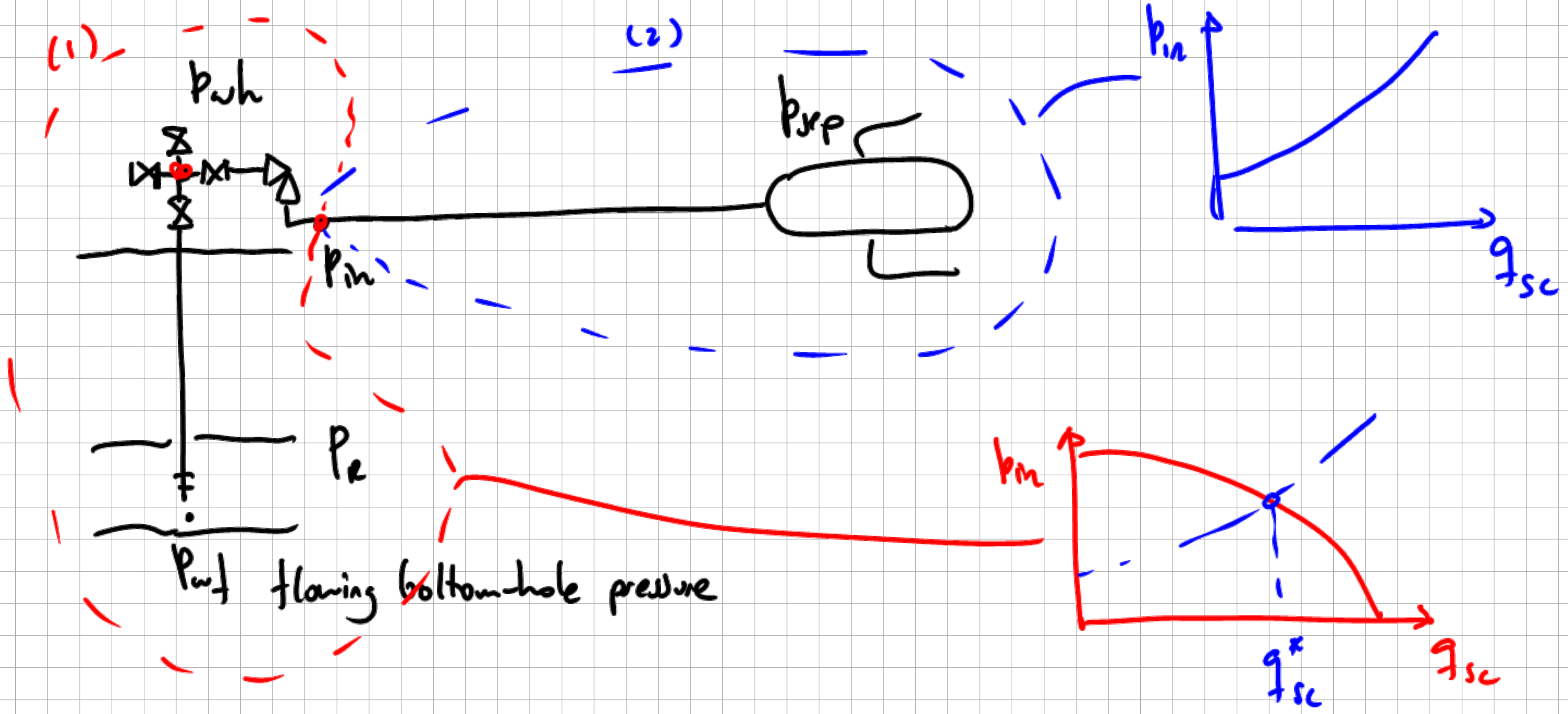


at p^* there should be only one pressure !

overimpose the two curves

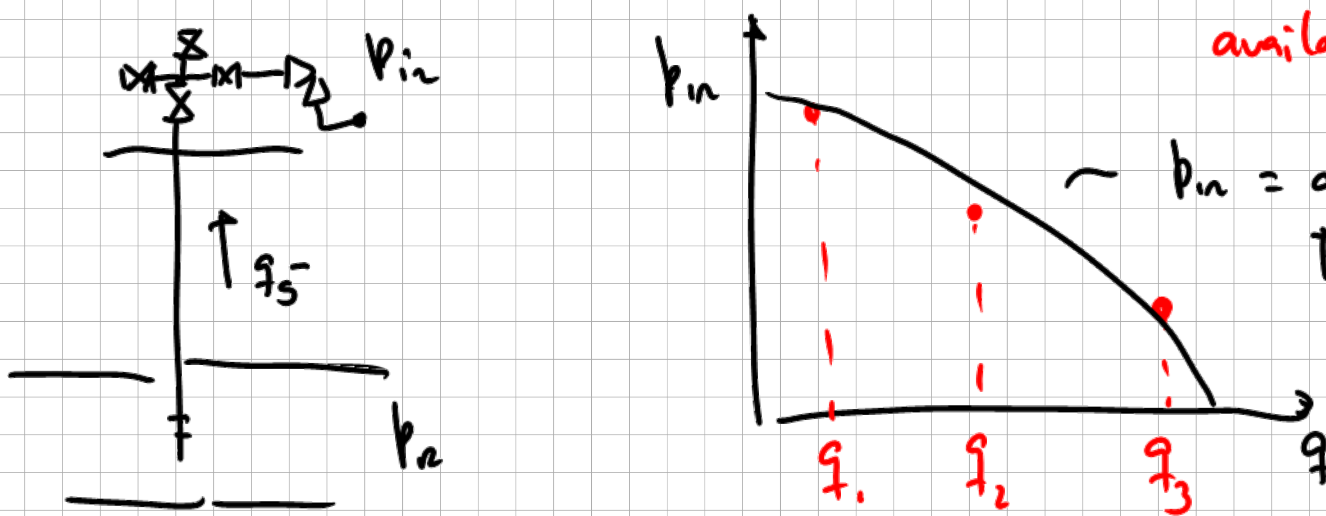


↳ equilibrium rate of the system when put together !



Example

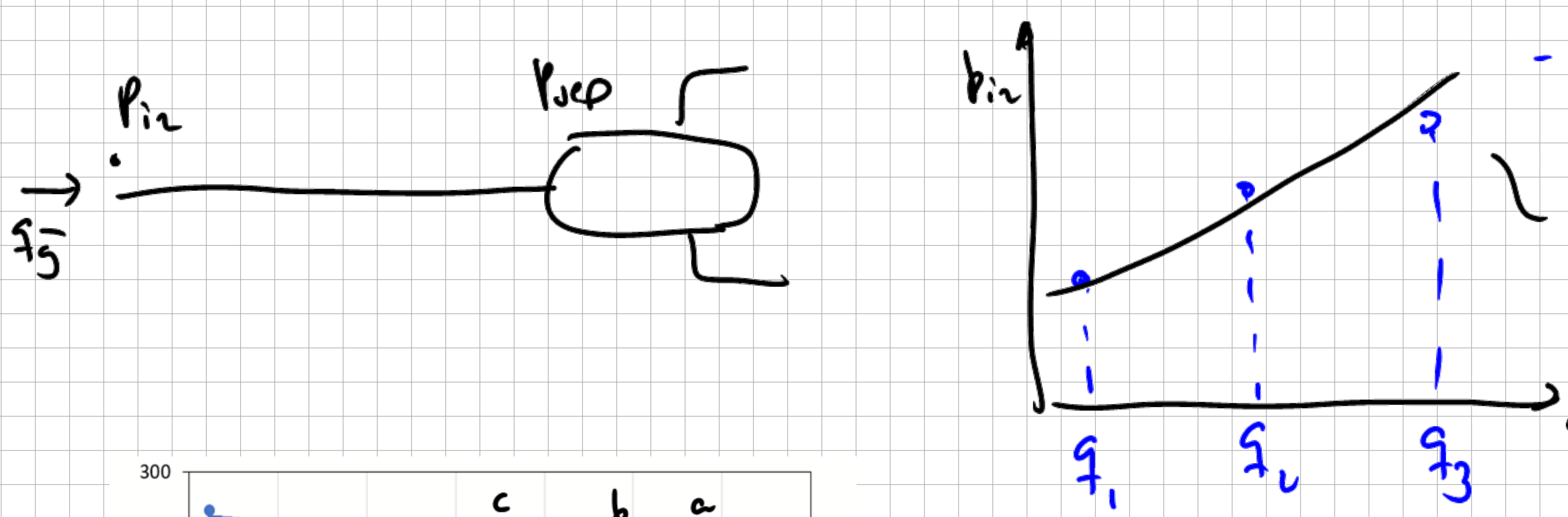
Dry gas production system



available pressure at choke discharge

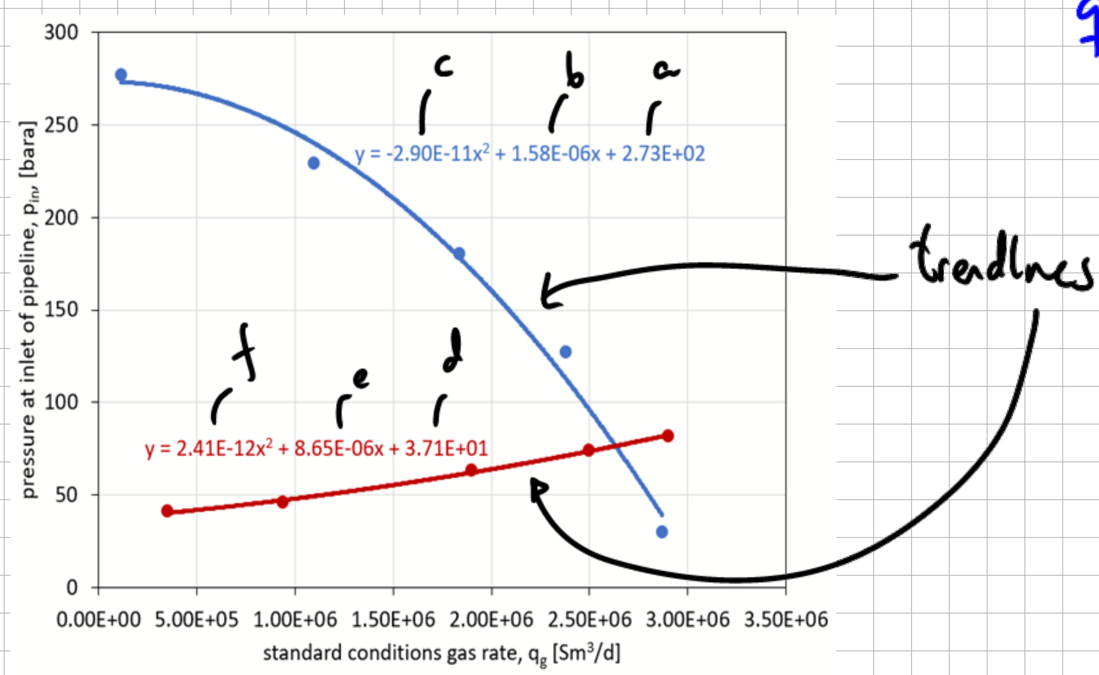
$\sim p_{in} = a + b\,q + c\,q^2$

q_g * at standard conditions



required pressure at pipeline inlet

$p_{in} = d + e\,q + f\,q^2$



pin (available)							
c	[bara/(Sm3/d)^2]	-2.90E-11					
b	[bara/Sm3/d]	1.58E-06					
a	[bara]	2.73E+02		qg	pin (av)	pin(req)	error
				[Sm3/d]	[bara]	[bara]	[bara]
pin(required)				2.63E+06	76.5	76.5	0.0
f	[bara/(Sm3/d)^2]	2.41E-12					
e	[bara/Sm3/d]	8.65E-06					
d	[bara]	3.71E+01					

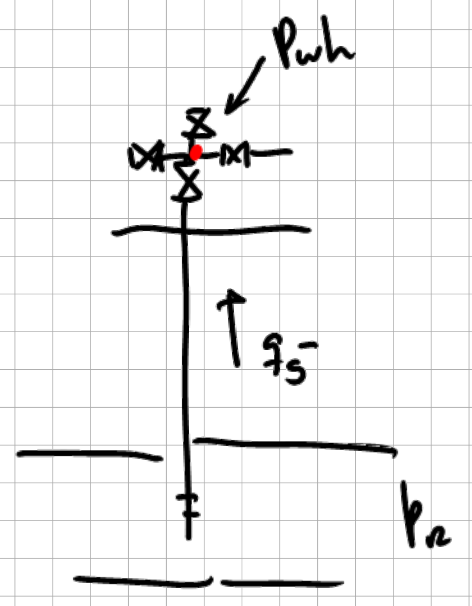
$p_{in} = 76.5$ bara

equilibrium gas rate

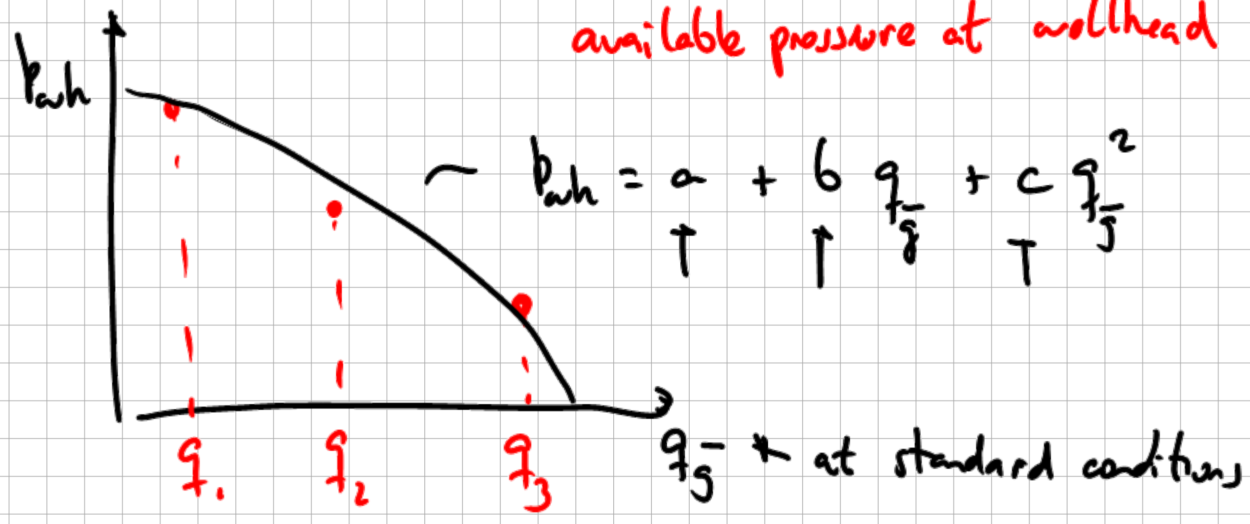
Video 4: Exercise on flow equilibrium

Example

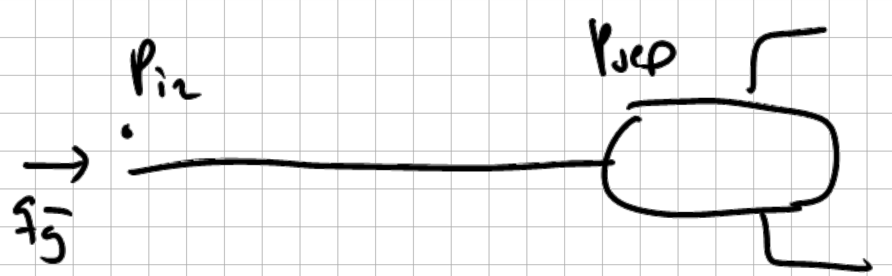
Dry gas production system



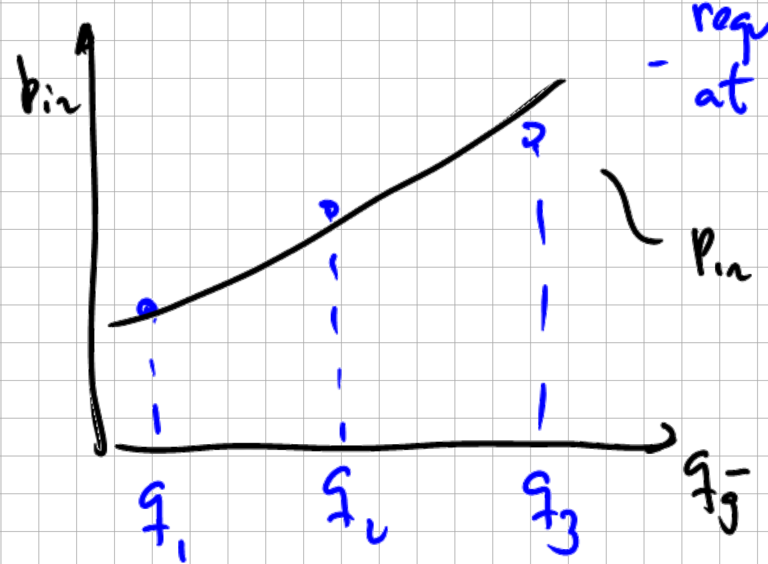
available pressure at wellhead



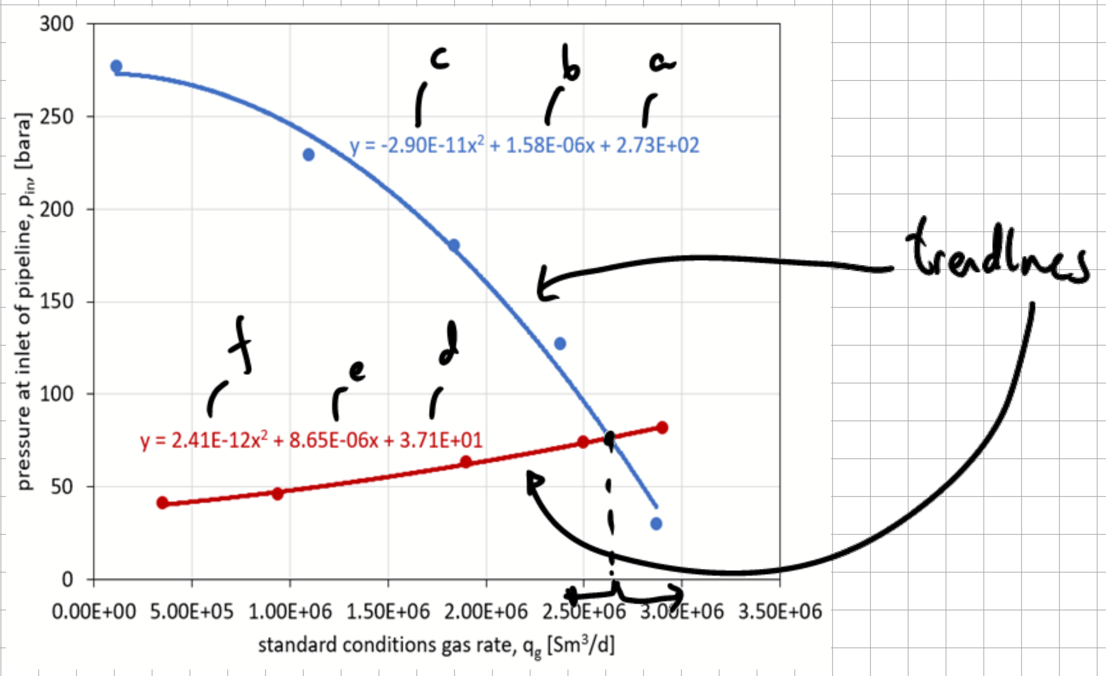
$$p_{wh} = a + b\, q_g + c\, q_g^2$$



required pressure at pipeline inlet



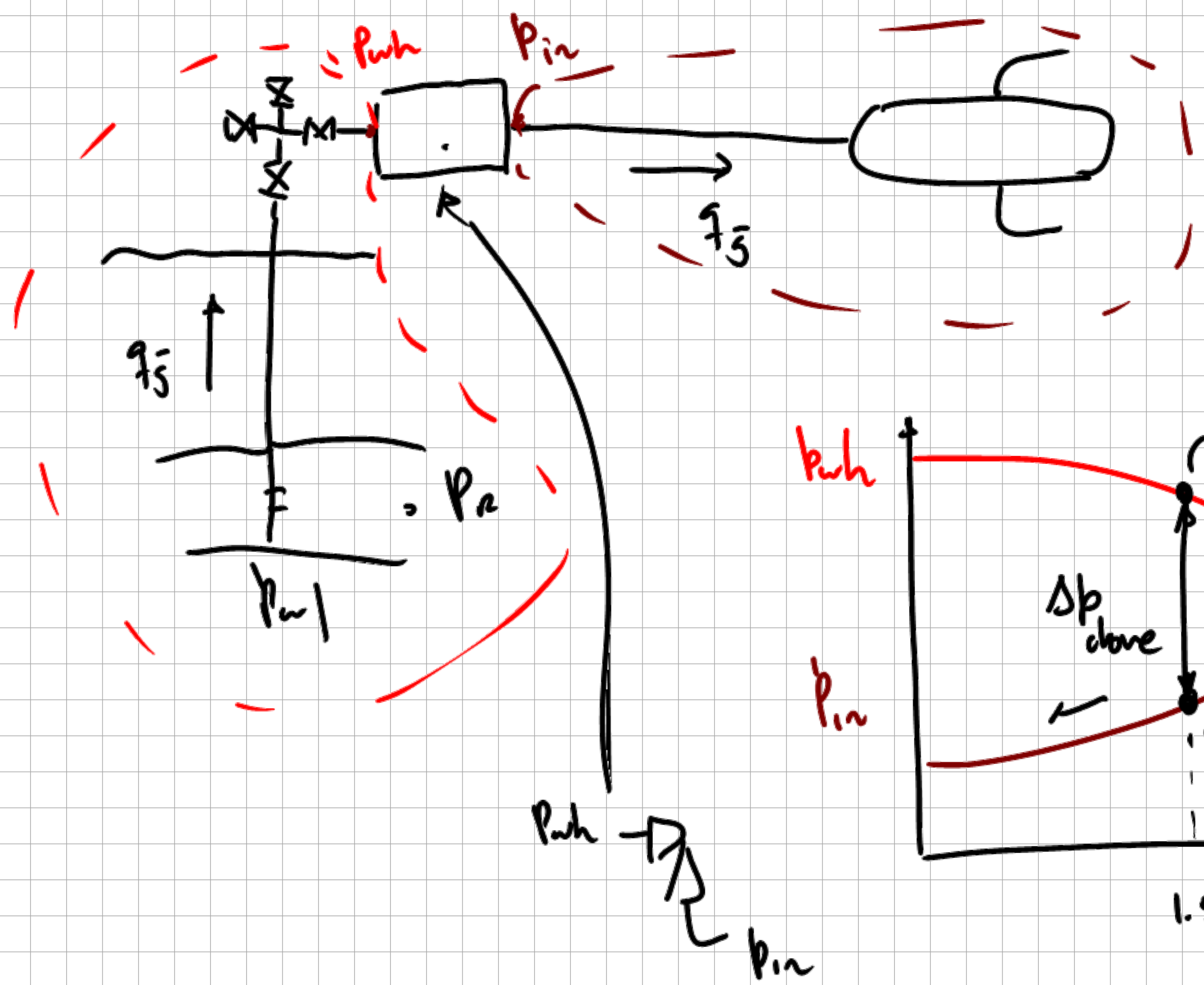
$$p_{in} = d + e\, q + f \cdot q^2$$

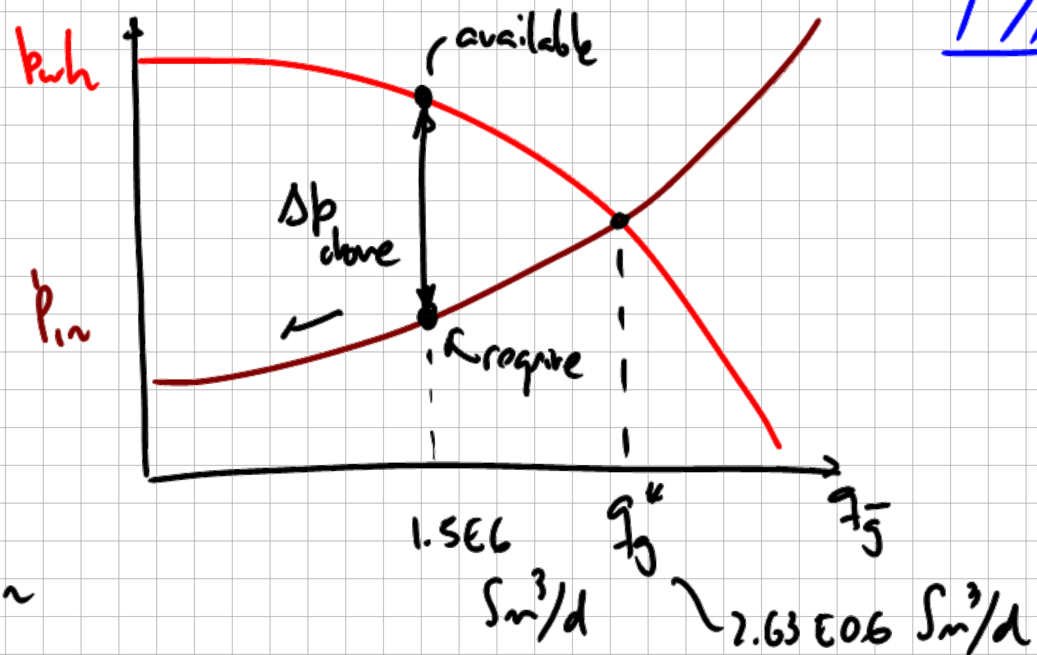


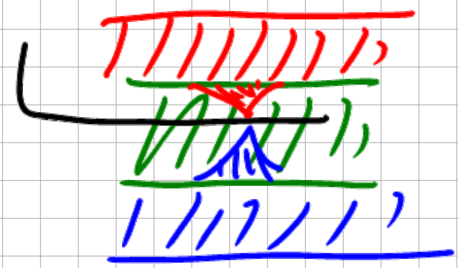
pin (available)							
c	[bara/(Sm3/d)^2]	-2.90E-11					
b	[bara/Sm3/d]	1.58E-06					
a	[bara]	2.73E+02		qg	pin (av)	pin(req)	error
				[Sm3/d]	[bara]	[bara]	[bara]
pin(required)				2.63E+06	76.5	76.5	0.0
f	[bara/(Sm3/d)^2]	2.41E-12					
e	[bara/Sm3/d]	8.65E-06					
d	[bara]	3.71E+01					

$p_{in} = 76.5$ bara

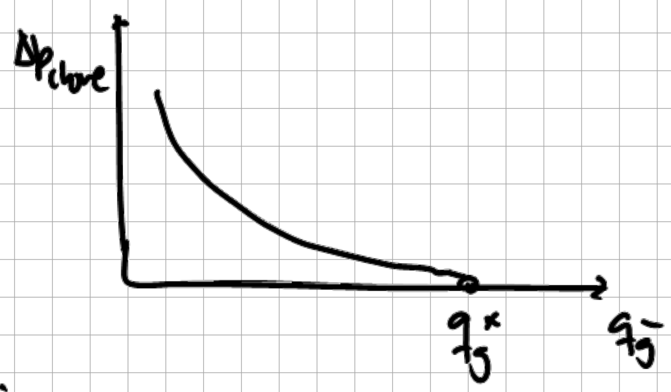
equilibrium gas rate



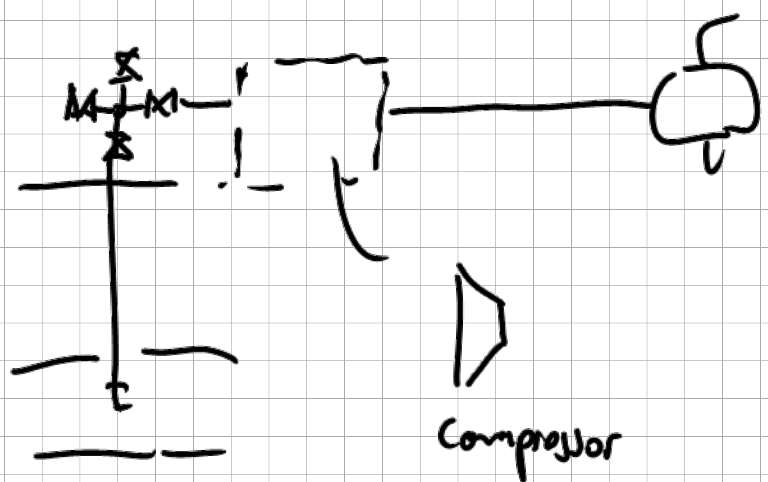


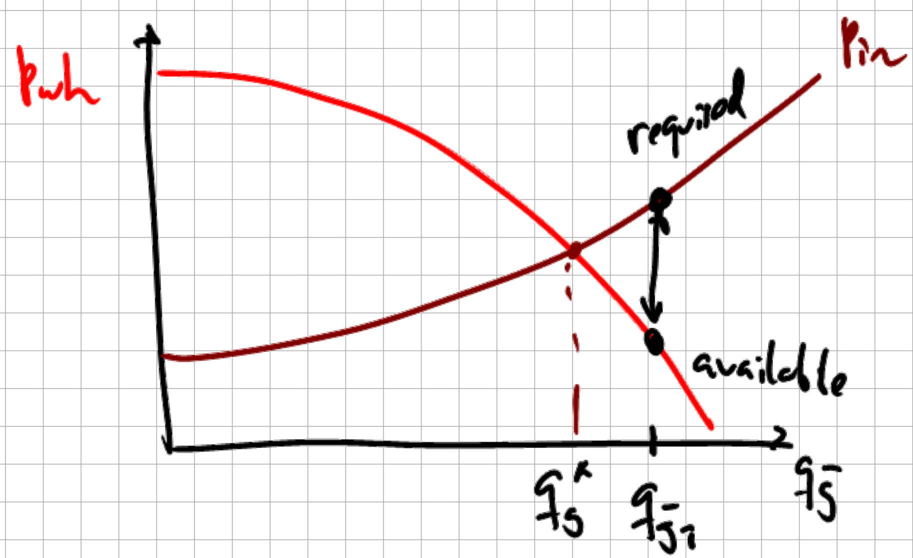


qg [Sm3/d]	p_{wh} (av) [bara]	p_{in} (req) [bara]	Deltap [bar]
1.50E+06	210.1	55.5	-154.6



what if i need a rate higher than equilibrium





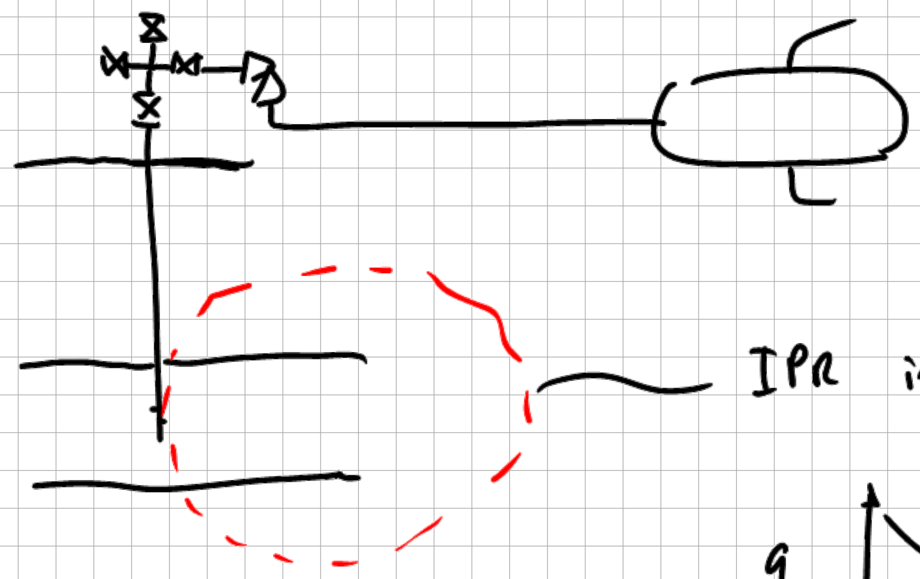
required > avail
energy must be
given to fluid

I desire to produce 2.8 E06 Sm3/d

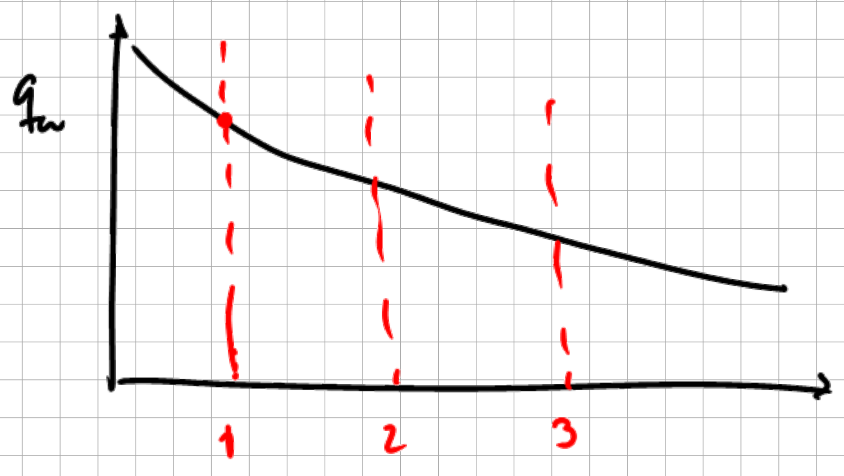
qg [Sm3/d]	p_{wh} (av) [bara]	p_{in} (req) [bara]	Deltap [bar]
2.80E+06	50.1	80.2	30.2

deltap to be provided by wellhead compressor

Video 5: Inflow performance relationship (intro)



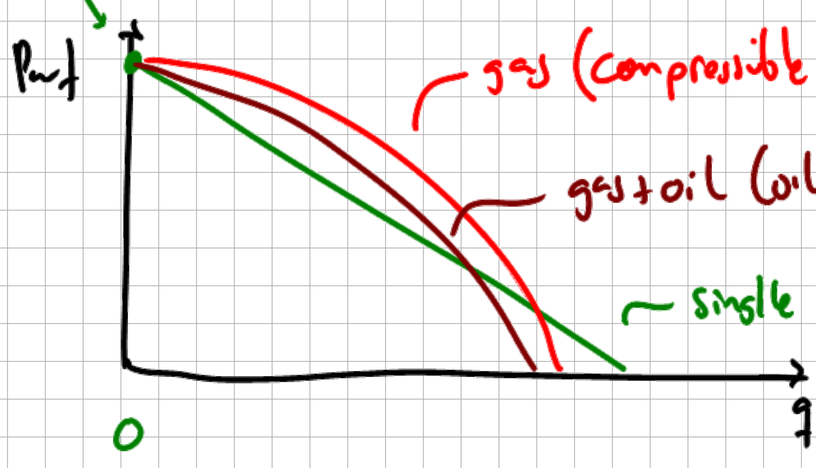
IPR inflow performance relationship



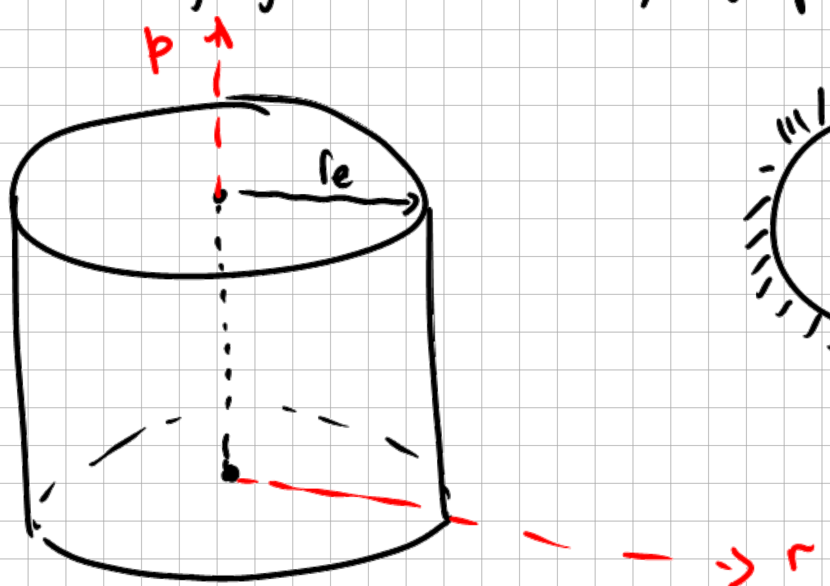
$q = f(p_{wf}, t)$

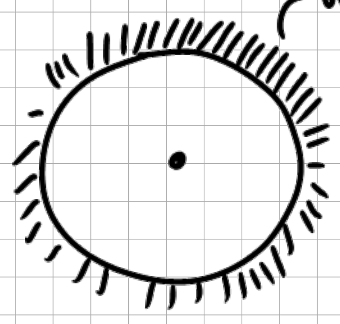
↳ instead of using t, i use p_R

at a given point in time:



vertical well, cylindrical reservoir, single phase fluid





at $t=0$

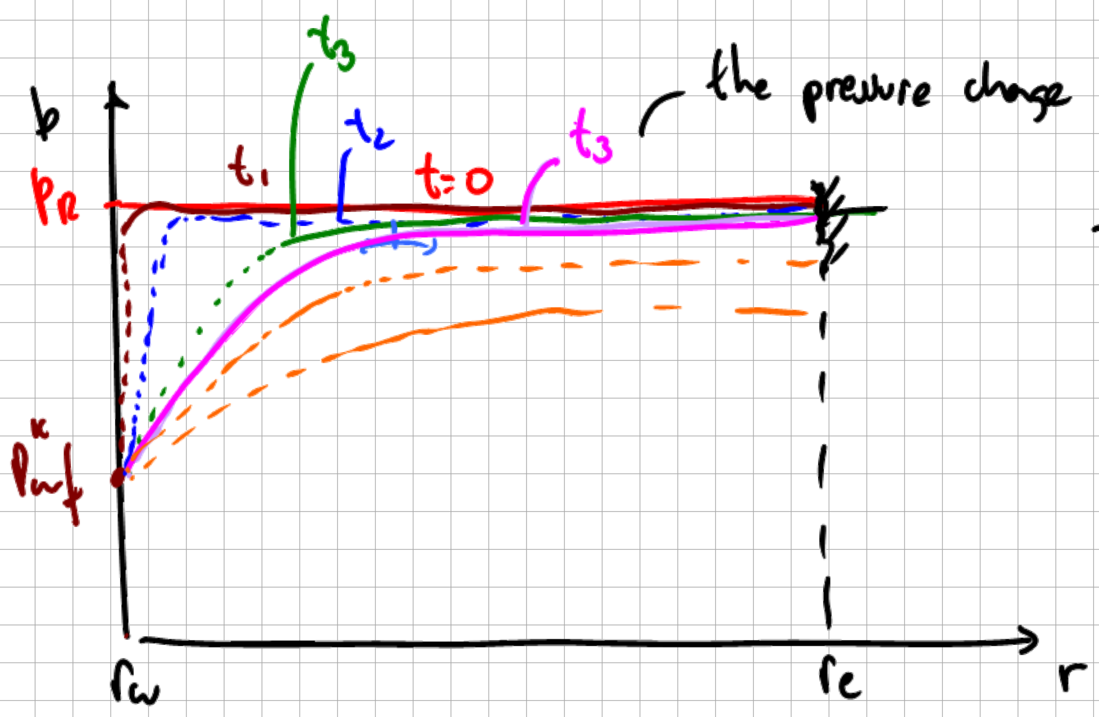
$q = 0$

$p(r_w) = p_R$

at $t>0$

$q \neq 0$

$p(r_w) = p_{wf}^*$



the pressure change reaches the boundary

$t=0 \rightarrow t=t_3$ infinite acting

t is important for IPR

(no boundary effects)

$t_3 \rightarrow$ steady state $P(r_e) = \text{const}$

pseudo steady state $q(r_e)=0$

(pss)

$$t_{pss} = \frac{281.3\ \phi . \mu . C_t . A}{k}$$

ϕ [-]

μ [cp] $\quad$ 1cp = 1E-3 Pa s

C_t [1/bar] $\quad C_t = C_f + C_{fluid}$ → gas, → oil

↳ $S_w C_w + S_o C_o + S_g C_g$

A [m²]

k [md]

Formation compressibility, c_f	[1/bar]	4.35E-05		
Oil compressibility at res conditions, C_o	[1/bar]	2.18E-04		
Gas compressibility at res conditions, C_g	[1/bar]	7.25E-03		
Total compressibility, assuming oil-filled pore, C_t	[1/bar]	2.61E-04		
Total compressibility, assuming gas-filled pore, C_t	[1/bar]	7.30E-03		
Oil viscosity at res conditions, μ_o	[cp]	0.5		
Gas viscosity at res conditions, μ_g	[cp]	1.20E-02		
Porosity, ϕ	[-]	0.3		
Outer radius of reservoir, r_e	[m]	600		
Top area of reservoir, A	[m2]	1130973.4		
		FLUID:	Gas	Oil
		k	t_{pss}	t_{pss}
		[md]	[hr]	[hr]
		0.01	835552.28	1245853.30
		0.1	83555.23	124585.33
		1	8355.52	12458.53
		10	835.55	1245.85
		1000	8.36	12.46
		10000	0.84	1.25

- if t_{pss} is short most of the production will occur in pss (ss) therefore t is not necessary to compute IPR
- if t_{pss} is large, most of the production will occur in IA, therefore it is important to consider time in IPR

IPRs are obtained
- empirical (field data) → "easier to use", → field data is needed
- Derived analytically (semi-analytically) from conservation equations.

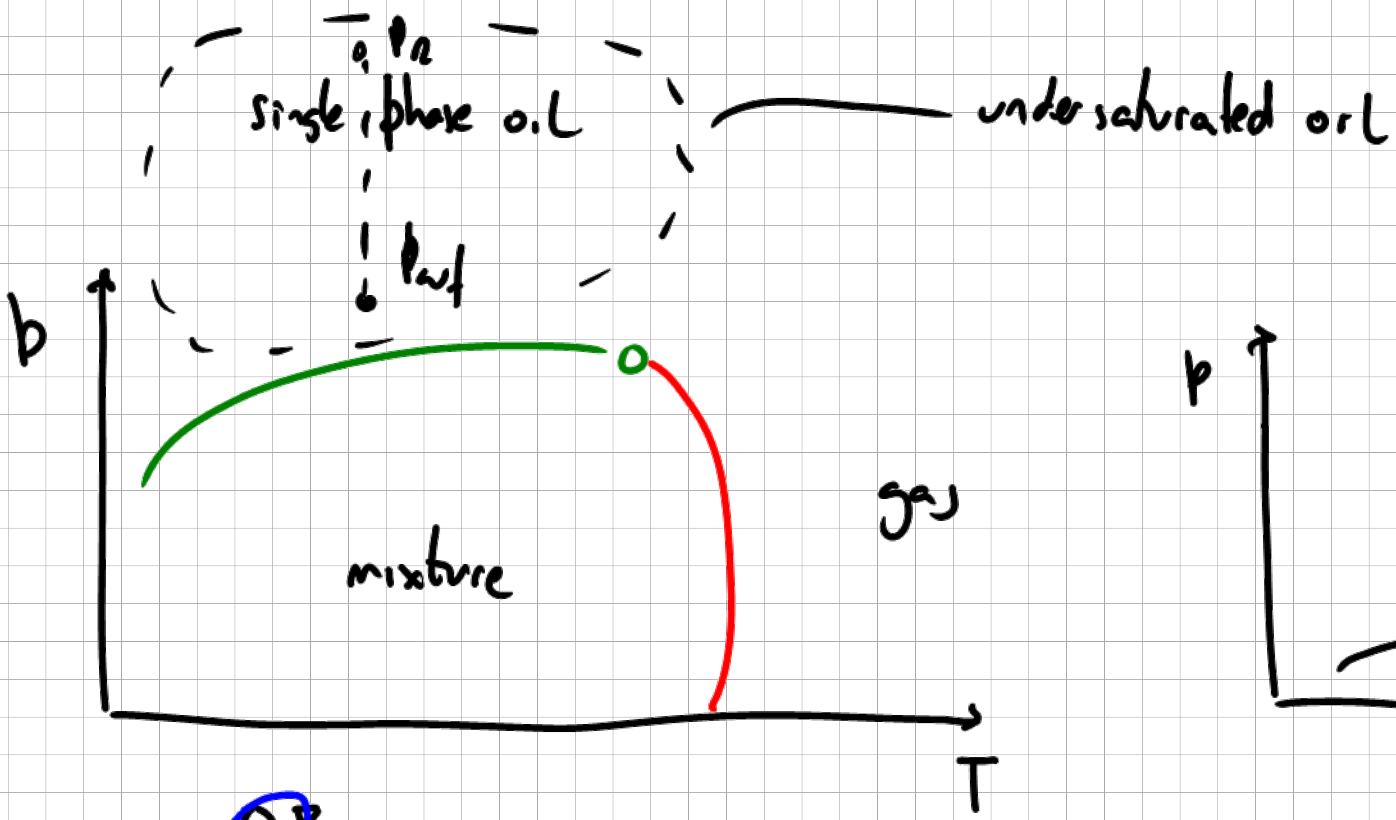
Conservation of mass + Darcy flow in a radial reservoir; single-phase fluid

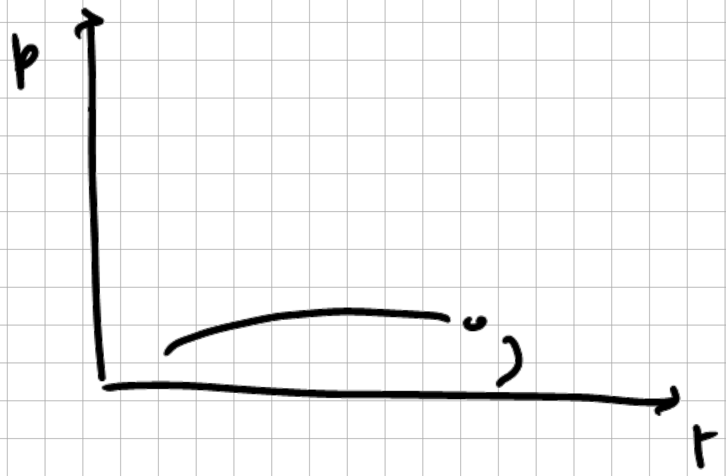
$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \alpha \frac{\partial p}{\partial t}$$

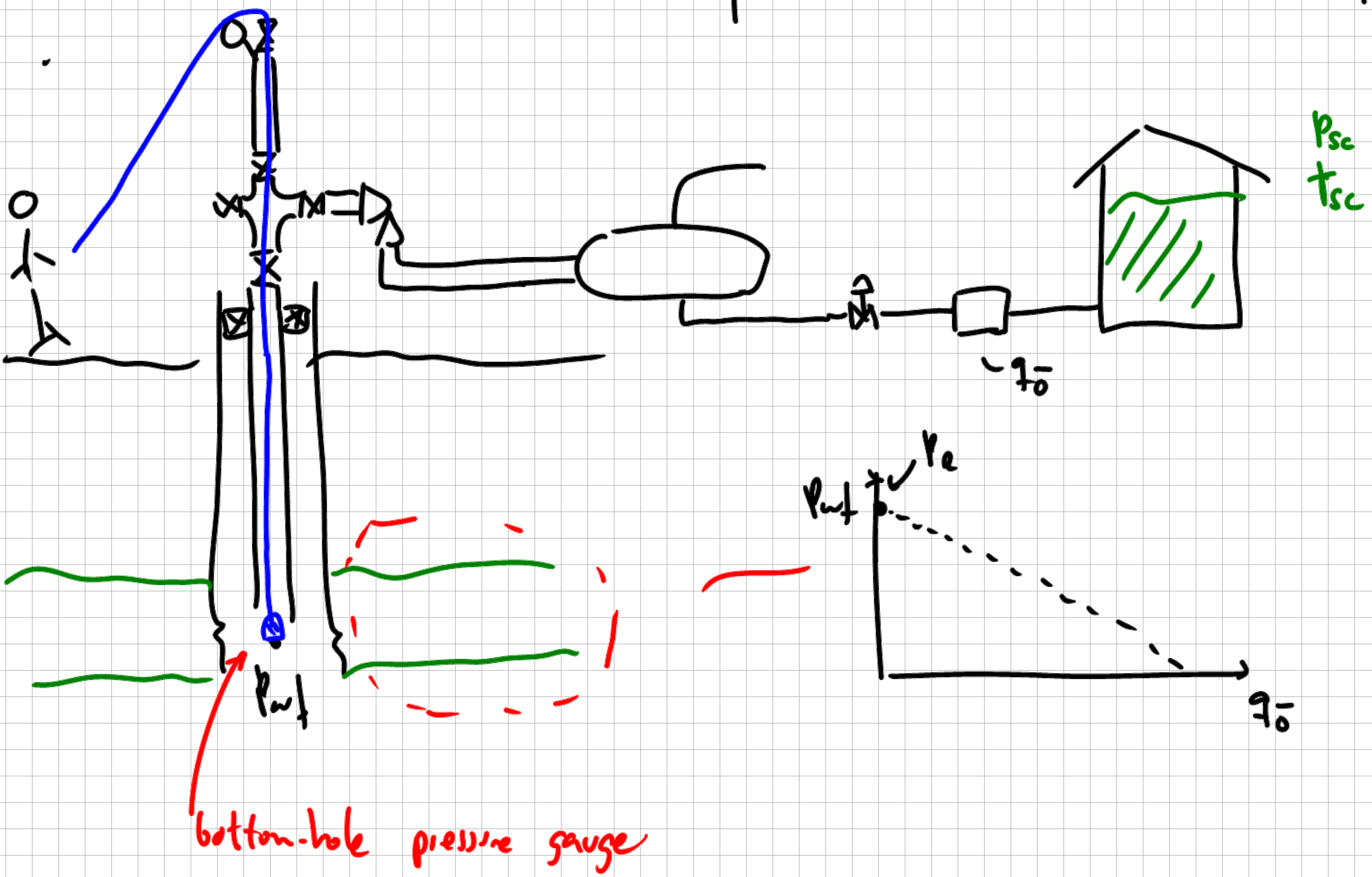
$$\alpha = \frac{\mu \phi c}{k}$$

- need alot of parameters about the reservoir
- Allow for well design and to estimate future performance
- always need adjustment with field data

Video 6: IPR for undersaturated oil (field test)







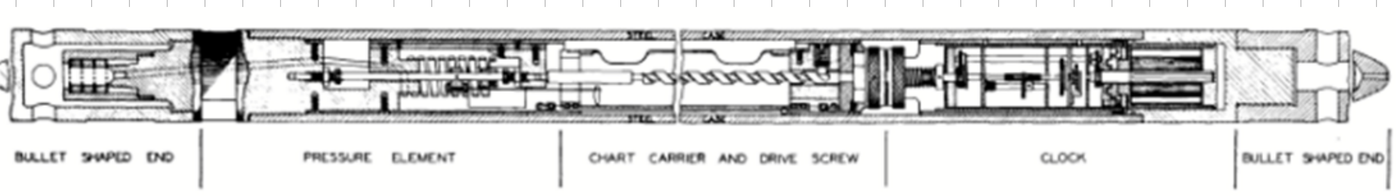


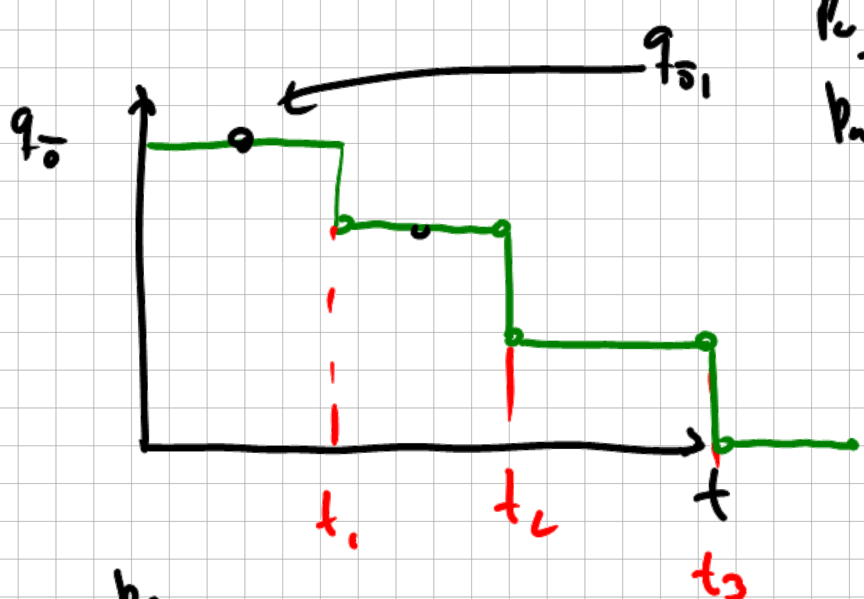
FIG. 1 CROSS-SECTION OF AMERADA PRESSURE GAGE.

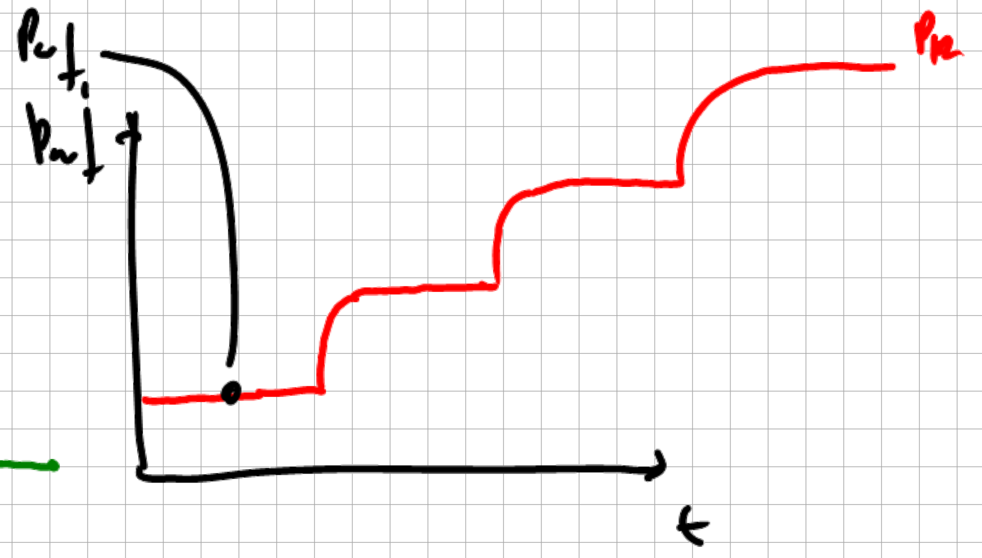
Bottom-hole Pressures in Oil Wells[1]

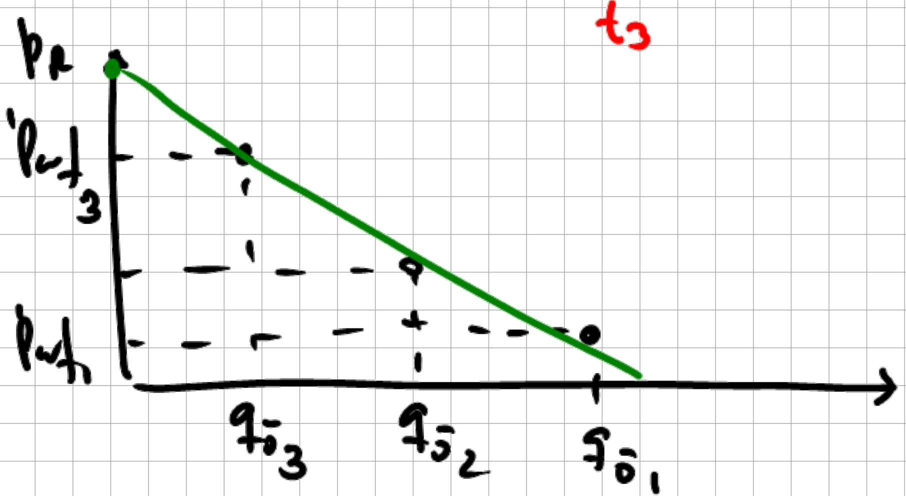
BY CHARLES V. MILLIKAN,[2] TULSA, OKLA. AND CARROLL V. SIDWELL,[3] SEMINOLE, OKLA.

(Tulsa Meeting, October, 1930)

THERE is nothing more important in petroleum engineering than a definite knowledge of the pressure at the bottom of an oil well at any existing operating condition, and the relation of this pressure to the pressure within the producing formation. A knowledge of bottom-hole pressures is fundamental in determining the most efficient methods of recovery and the most efficient lifting procedure, yet there is less information about these pressures than about any other part of the general problem of producing oil.



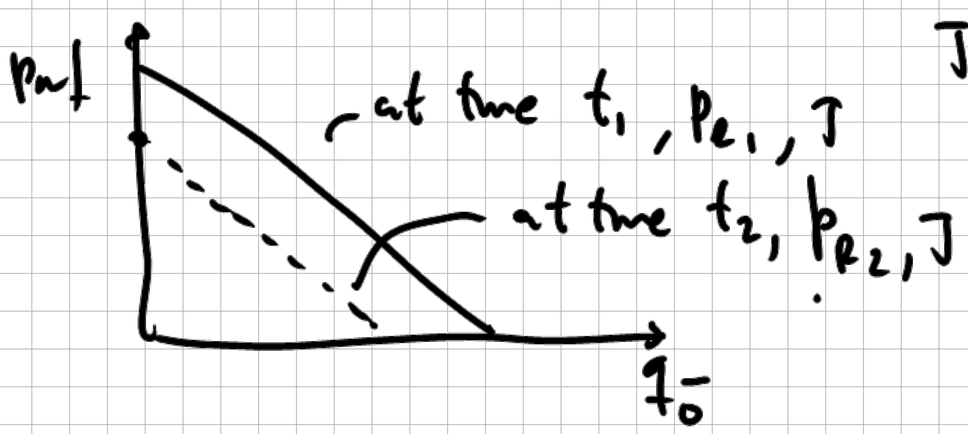




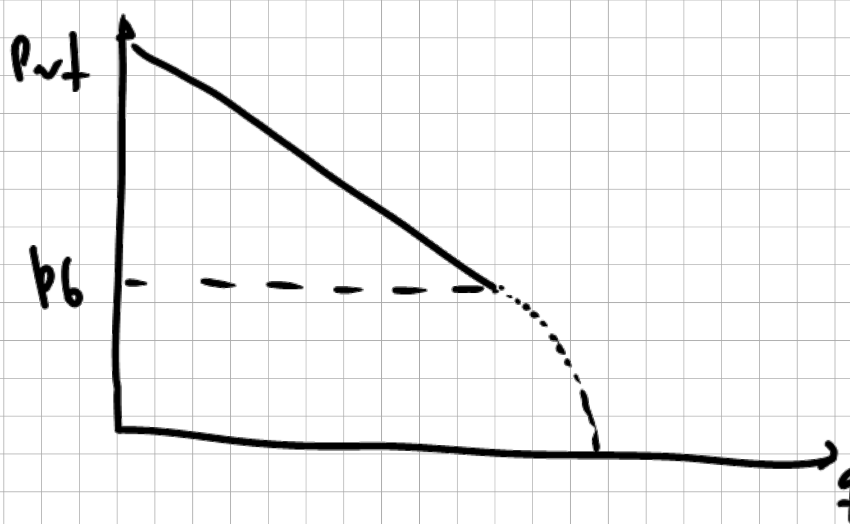
$$\underbrace{q_o}_{Sm^3/d} = J\,\overbrace{(p_R - p_{wf})}^{bara}$$

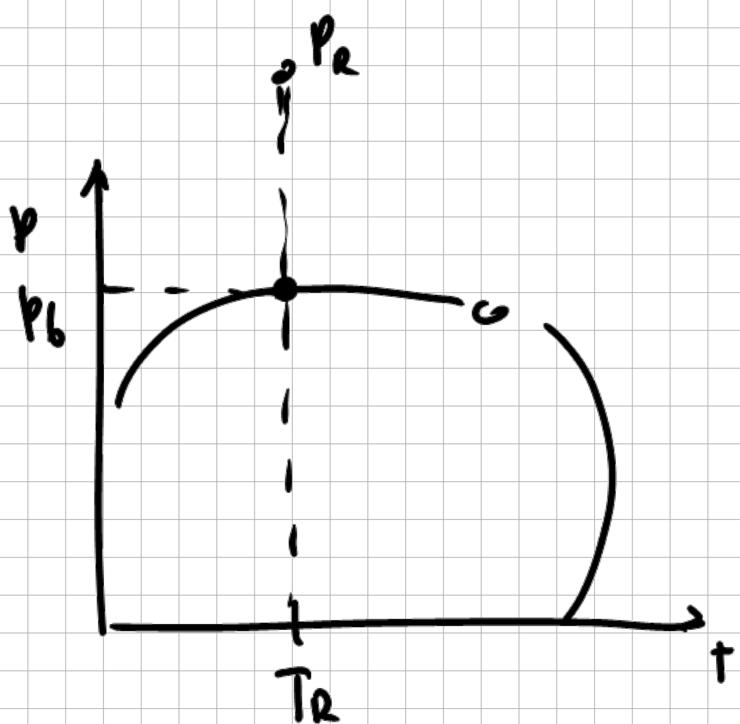
Productivity index

$$J = \left[\frac{Sm^3/d}{bara}\right]$$



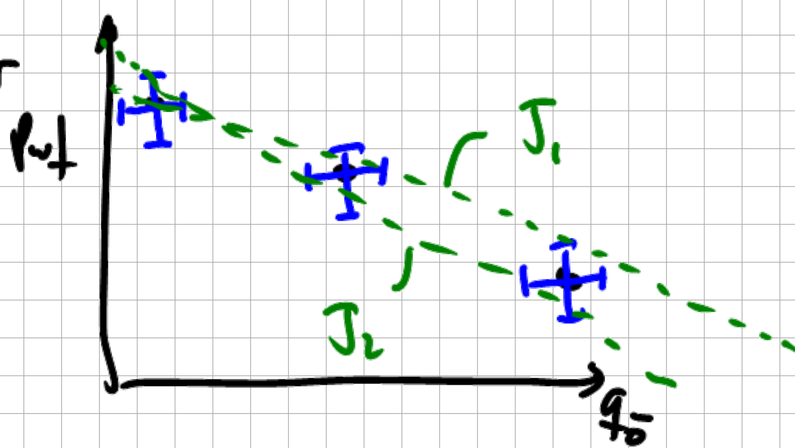
J usually doesn't change with depletion





- effect of measurement errors and variability

p = value + error



if one point is available, and reservoir pressure is known

$P_{wf,1}$, $q_{\bar{o},1}$

$$q_{\bar{o},1} = J\left(P_R - P_{wf,1}\right)$$

$$J = \frac{q_{\bar{o},1}}{\left(P_R - P_{wf,1}\right)}$$

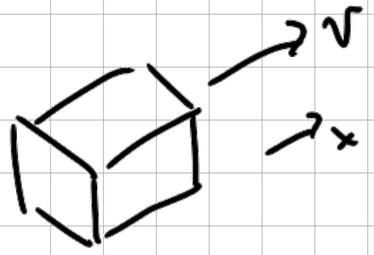
Darcy's law

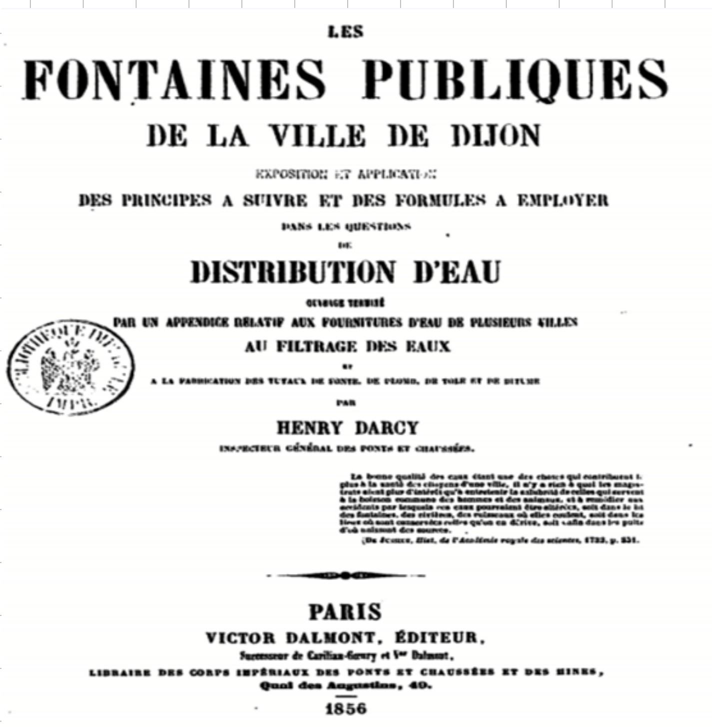
Dijon

1856 → Retired to Dijon

1858 → died

$$v = \frac{k}{\mu} \cdot \frac{dp}{dx}$$





LES

FONTAINES PUBLIQUES

DE LA VILLE DE DIJON

EXPOSITION ET APPLICATION

DES PRINCIPES A SUIVRE ET DES FORMULES A EMPLOYER

DANS LES QUESTIONS

DE

DISTRIBUTION D'EAU

PAR UN APPENDICE RELATIF AUX FOURNITURES D'EAU DE PLUSIEURS VILLES

AU FILTRAGE DES EAUX

HENRY DARCY

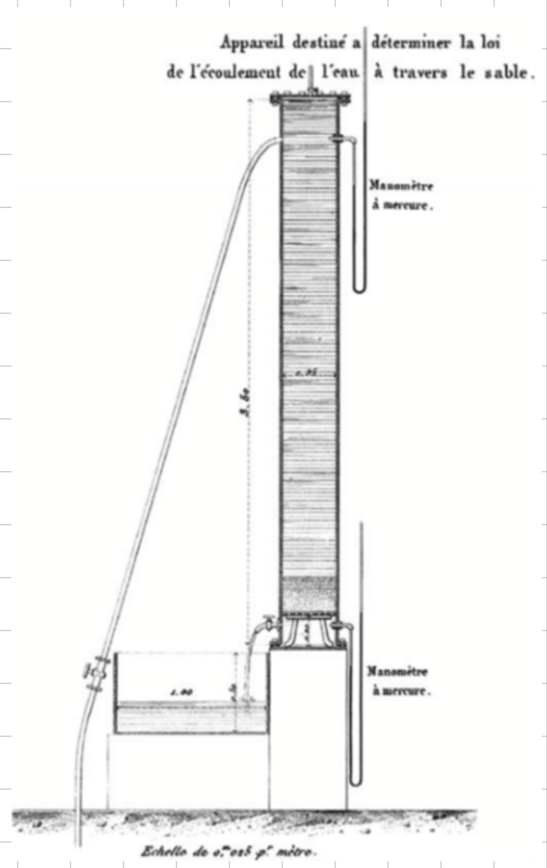
INSPECTEUR GÉNÉRAL DES PONTS ET CHAUSSÉES.

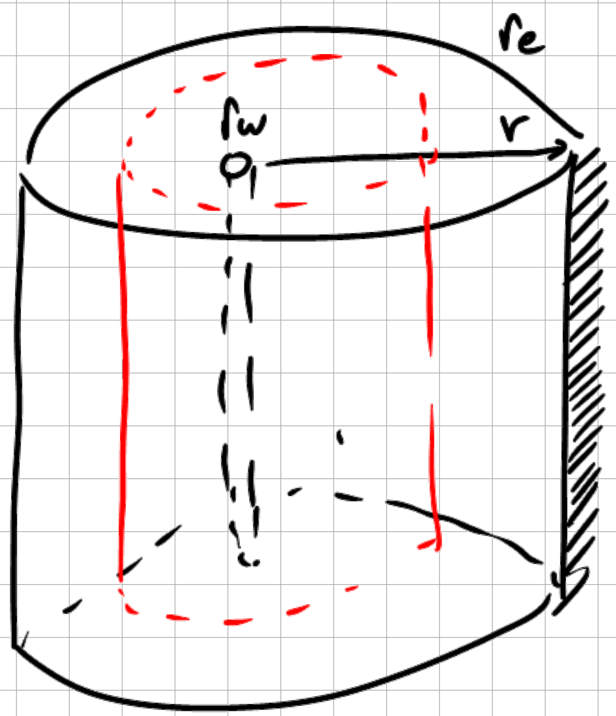
PARIS

VICTOR DALMONT, ÉDITEUR,

LIBRAIRE DES CORPS IMPÉRIAUX DES PONTS ET CHAUSSÉES ET DES MINES,

1856





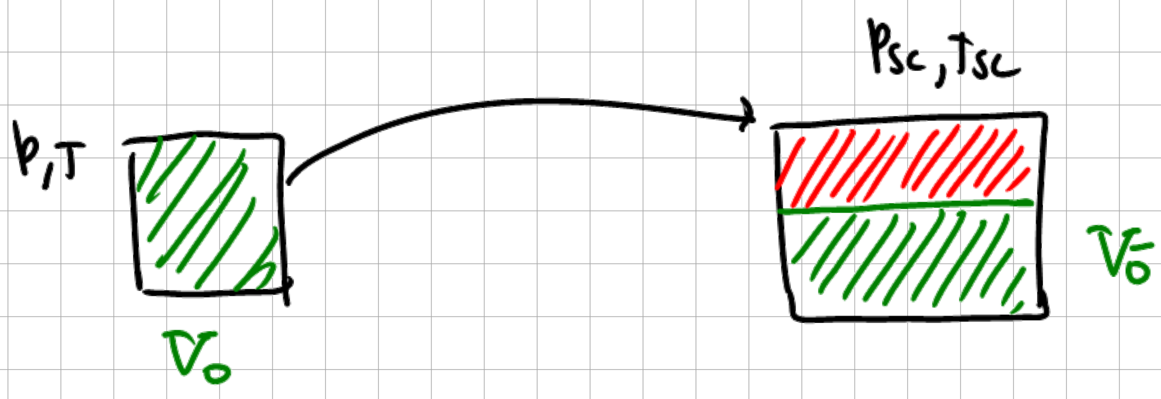
$$\left.\begin{array}{l} SS \\ PSS \end{array}\right\} \frac{\partial}{\partial t} = 0 \qquad v = \frac{k}{\mu} \frac{dp}{dr}$$

$$v \rightarrow \bar{q}_o$$

$$v = \frac{q_o}{A} \quad q_o \rightarrow [m^3/d] \ [m^3/s] \qquad \bar{q}_o \ ? \qquad \bar{q}_o \xrightarrow{?} q_o$$

$$A \rightarrow 2\pi r h$$

Black oil properties



oil volume factor

$$B_o(p,T) = \frac{V_o(p,T)}{\bar{V}_o} > 1$$

$$q_o = B_o(p,T) \cdot \bar{q}_o$$

$$B_o = [m^3/Sm^3]$$

$$v = \frac{\bar{q}_o \cdot B_o}{2\pi r h} = \underbrace{\frac{k}{\mu_o}}_{f(p,T)} \frac{\partial p}{dr}$$

$q_{\bar{o}} \stackrel{?}{=} f(r)$

$q_{\bar{o}} \neq f(r)$ } SS

$$\underbrace{\int_{r_w}^{r@p_R} \frac{dr}{r}}_{①} = \frac{2\pi k h}{q_{\bar{o}}} \underbrace{\int_{p_{wf}}^{p_R} \frac{dp}{B_o \mu_o}}_{②}$$

$$p_R = \frac{\int_V p\, dV}{V}$$

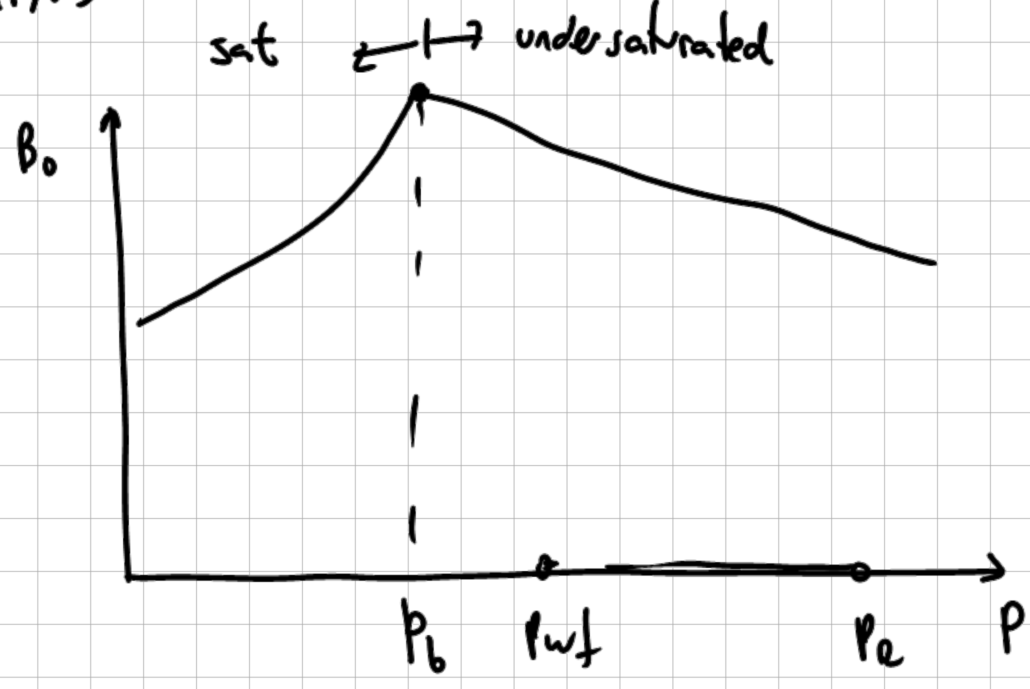
for SS, $r@p_R = 0.61\, r_e$

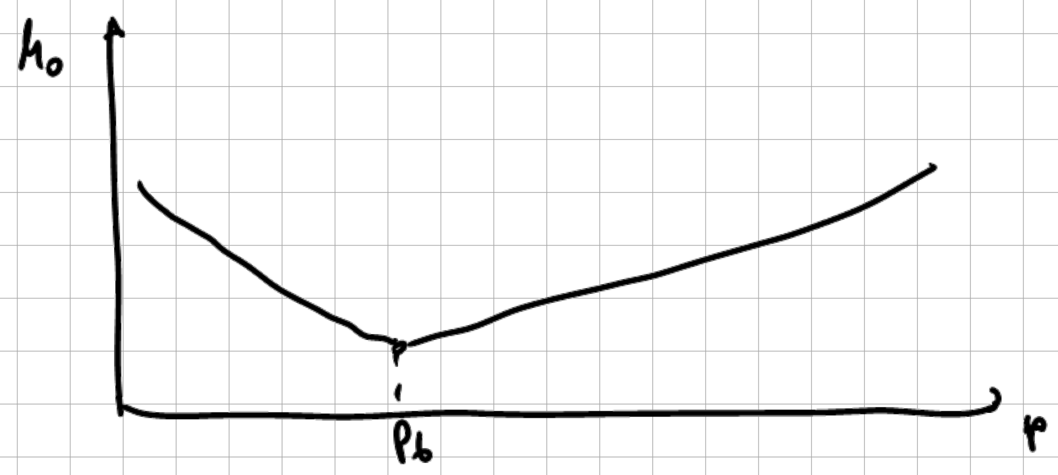
① $\ln\left(\frac{0.61\, r_e}{r_w}\right) = \ln(0.61) + \ln\left(\frac{r_e}{r_w}\right) = \ln\left(\frac{r_e}{r_w}\right) - 0.5$

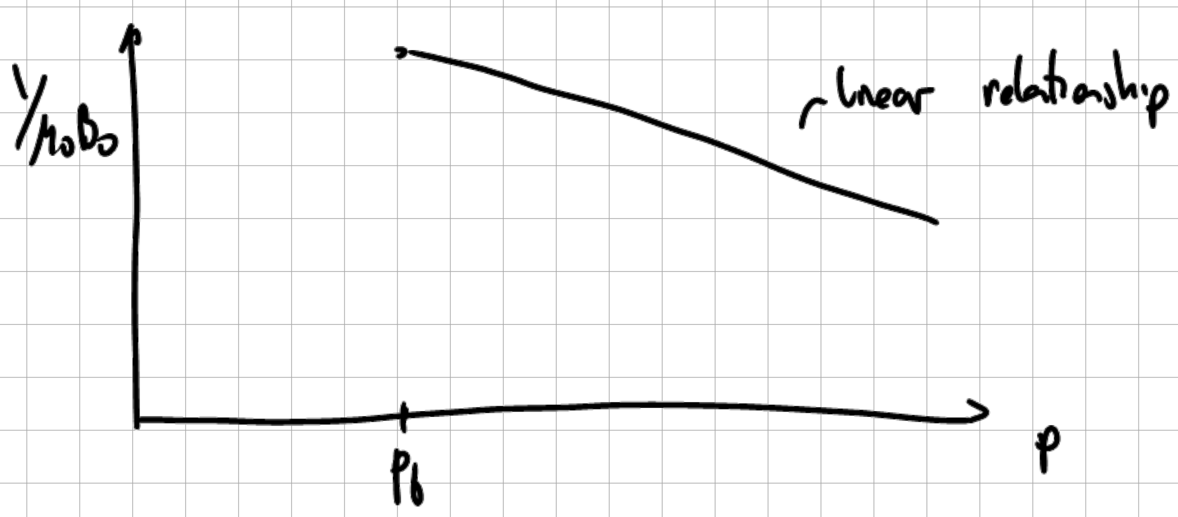
② $\int_{p_{wf}}^{p_R} \frac{1}{\mu_o B_o}\, dp$

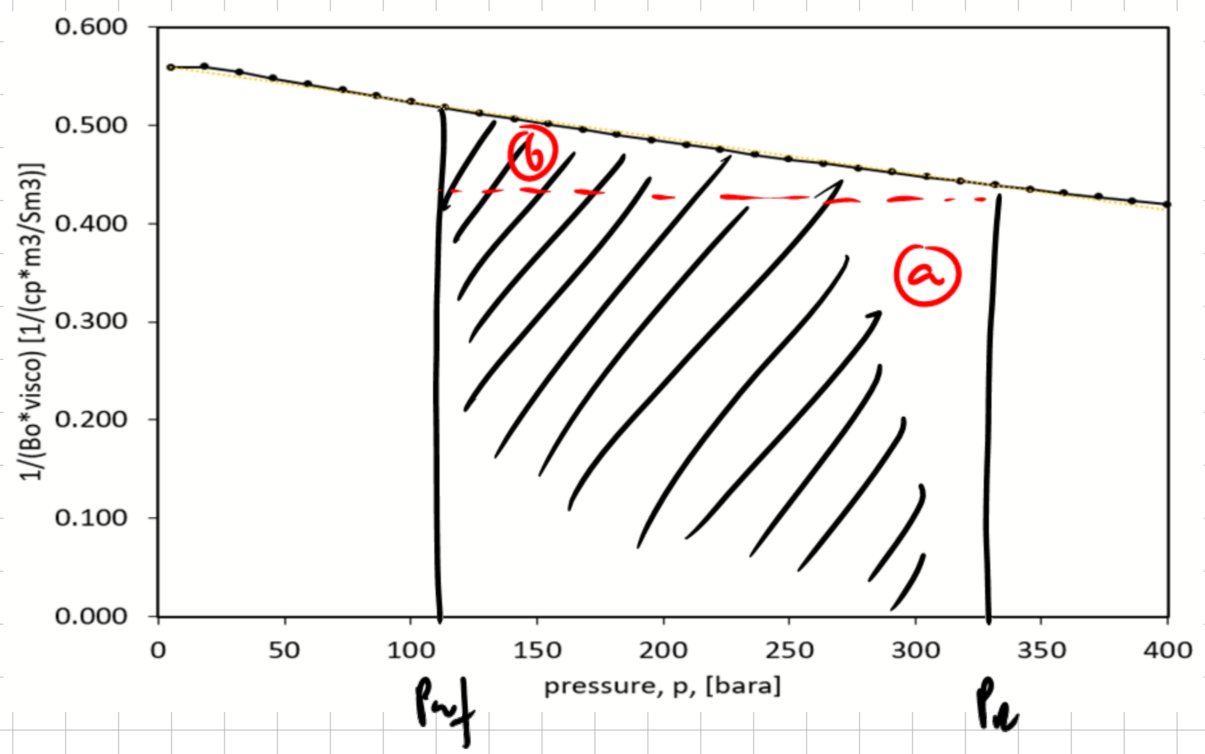
$\mu_o = f_1(p, T)$
$B_o = f_2(p, T)$

@ reservoir $T = \text{constant} = T_R$









$$\textcircled{2} \int_{P_{wf}}^{P_R} \frac{1}{\mu_o B_o} \cdot dp \approx (P_R - P_{wf}) \cdot \underbrace{\left[\left.\frac{1}{\mu_o B_o}\right|_{P_{wf}} + \left.\frac{1}{\mu_o B_o}\right|_{@ P_R} \right] 0.5}_{\left.\frac{1}{\mu_o B_o}\right|_{@ P_{av} = \frac{(P_R + P_{wf})}{2}}}$$

$$\textcircled{2} = \frac{(P_R - P_{wf})}{(\mu_o B_o)_{@ P_{av}}}$$

$$\ln\left(\frac{r_e}{r_w}\right) - 0.5 = \frac{2\pi k h}{q_o} \frac{(P_R - P_{wf})}{(\mu_o B_o)_{@ P_{av}}}$$

$$q_o = \frac{2\pi k h \quad (P_R - P_{wf})}{(\mu_o B_o)_{@ P_{av}} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.5 \right]}$$

J ≃ constant

$$q_{\bar{o}} = \frac{2\pi k h \, (p_e - p_{wf})}{(\mu_o B_o)_{@p_{av}} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.5\right]}$$

- steady-state
- vertical well, fully perforated
- undersaturated oil
- radial drainage volume
- homogeneous permeability

SI system

$$\left[Sm^3/s\right] = \frac{[m^2]\,[m]\,[Pa]}{[Pa\ s]\ [-]}$$

$$\left[Sm^3/d \ \frac{1d}{24h}\ \frac{1h}{3600\,s}\right] = \frac{\left[1\ md\ \frac{1\ d}{1000\ md}\ \frac{9.86E\text{-}13\ m^2}{1\ d}\right][m]\left[bara\ \frac{10^5\ Pa}{1\ bara}\right]}{\left[1\ cp\ \frac{10^{-3}\ Pa.s}{1\ cp}\right]}\ 2\pi$$

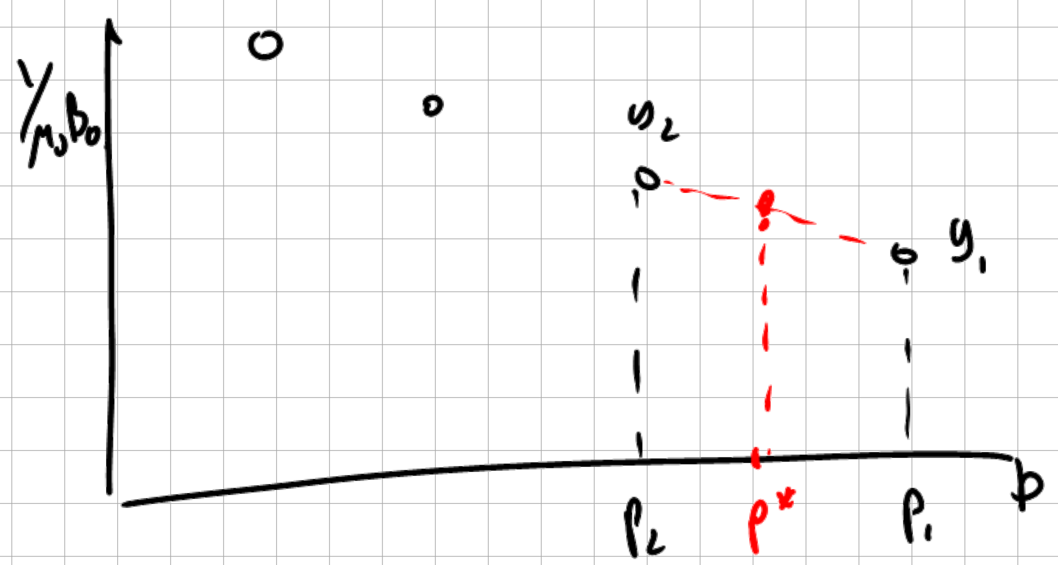
$$= \frac{1}{18.68}$$

$$q_{\bar{o}} = \frac{k h \, (p_e - p_{wf})}{(\mu_o B_o)\big|_{@p_{av}} \left(\ln\frac{r_e}{r_w} - 0.5\right)} \cdot \frac{1}{18.68}$$

J

p in bara
k in md
μ_o in cp
$q_{\bar{o}} = Sm^3/d$

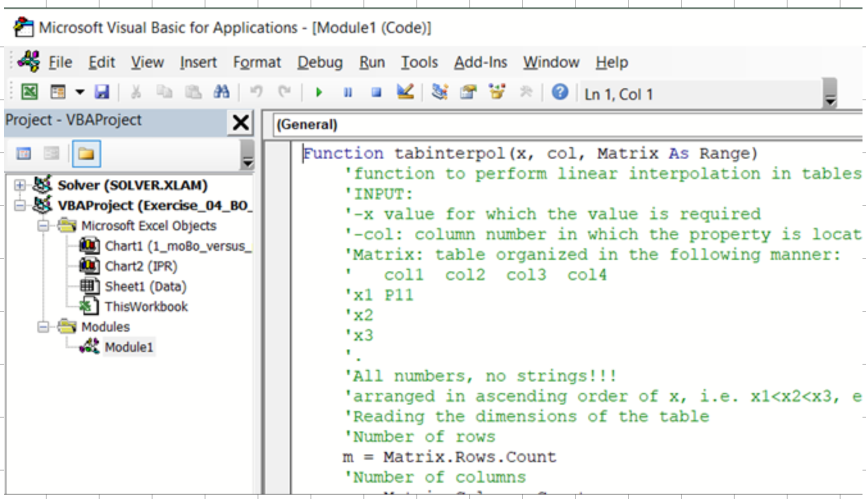
$$J = \frac{k h}{(\mu_o B_o)_{av} \left(\ln\left(\frac{r_e}{r_w}\right) - 0.5\right) \cdot 18.68}$$



$$\frac{y_2 - y_1}{p_2 - p_1} = \frac{y_2 - y^*}{p_2 - p^*} \qquad y = \frac{1}{\mu_o B_o}$$

VBA visual basic for application

while in excel press alt + F11

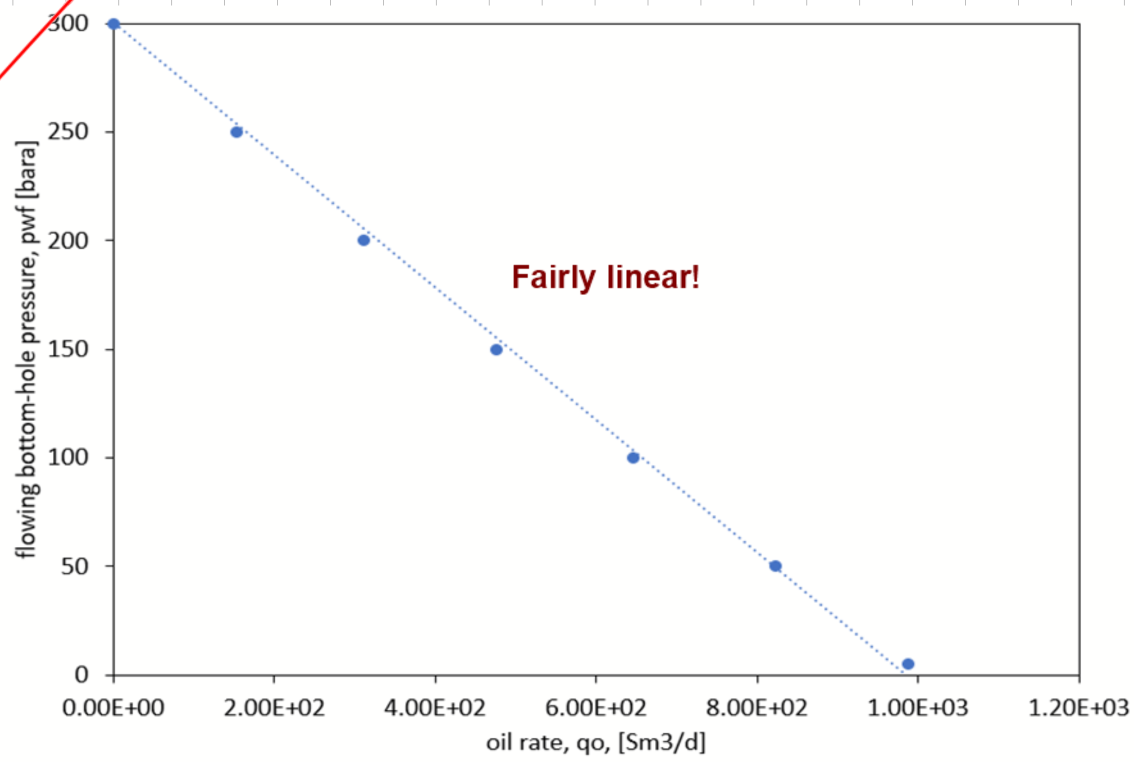


Use here the VBA function ProductivityIndex_J

use here the VBA function tabinterpol

r_e	[m]	500
r_w	[m]	0.1
k	[md]	10
h	[m]	100
p_R	[bara]	300

pwf [bara]	pav [bara]	1/(Bo visco) [1/(cp*m3/Sm3)]	J [Sm3/d/bar]	qo [Sm3/d]
300	300	0.449	3.0	0.00E+00
250	275	0.457	3.1	1.53E+02
200	250	0.465	3.1	3.11E+02
150	225	0.474	3.2	4.75E+02
100	200	0.483	3.2	6.46E+02
50	175	0.493	3.3	8.23E+02
5	153	0.502	3.3	9.88E+02

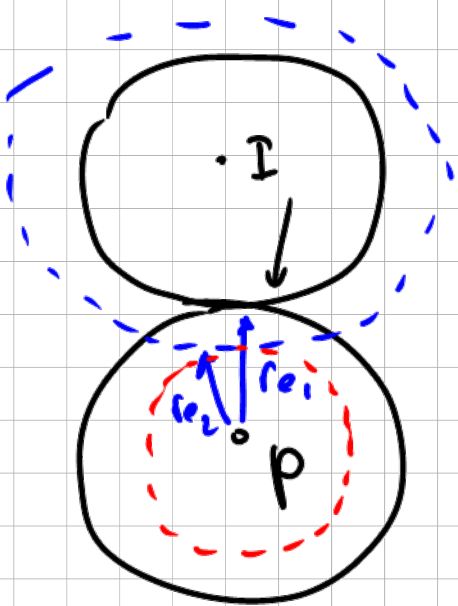


EFFECT OF THE BOUNDARY (re)

EFFECT OF THE REGIME (SS)

r_e	[m]	500
r_w	[m]	0.1
k	[md]	10
h	[m]	100
p_R	[bara]	300

$\ln\left(\frac{r_e}{r_w}\right) =$ 8.52E+00



$$J = \frac{kh}{18.68\,(\mu_o B_o)_{av} \cdot \left(\underbrace{\ln\frac{r_e}{r_w}}_{8.5} - \underbrace{0.5}_{10\%}\right)}$$

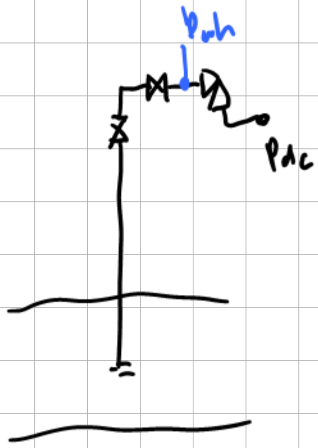
SS, boundary effect

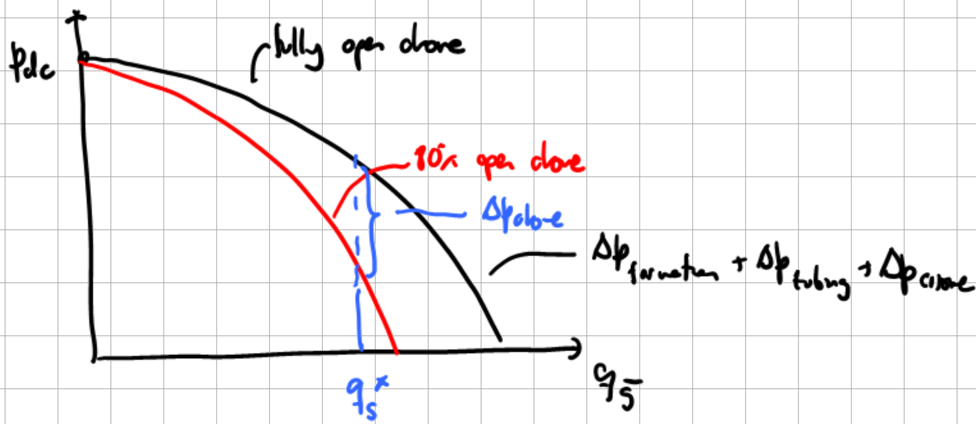
r_e	[m]	500
r_w	[m]	0.1
k	[md]	10
h	[m]	100
p_R	[bara]	300

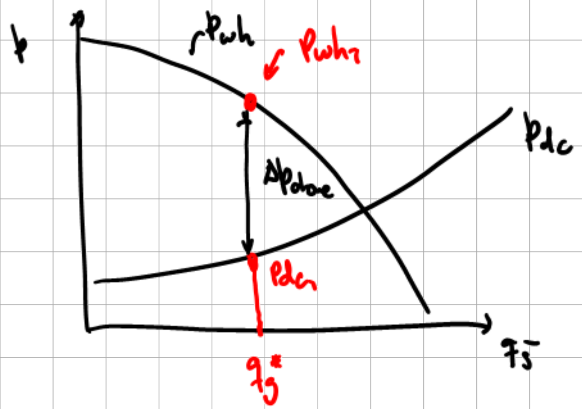
pwf [bara]	pav [bara]	1/(Bo visco) [1/(cp*m3/Sm3)]	J [Sm3/d/bar]	qo [Sm3/d]
300	300	0.449	3.0	0.00E+00
250	275	0.457	3.1	1.53E+02
200	250	0.465	3.1	3.11E+02
150	225	0.474	3.2	4.75E+02
100	200	0.483	3.2	6.46E+02
50	175	0.493	3.3	8.23E+02
5	153	0.502	3.3	9.88E+02

r_e	[m]	400
r_w	[m]	0.1
k	[md]	10
h	[m]	100
p_R	[bara]	300

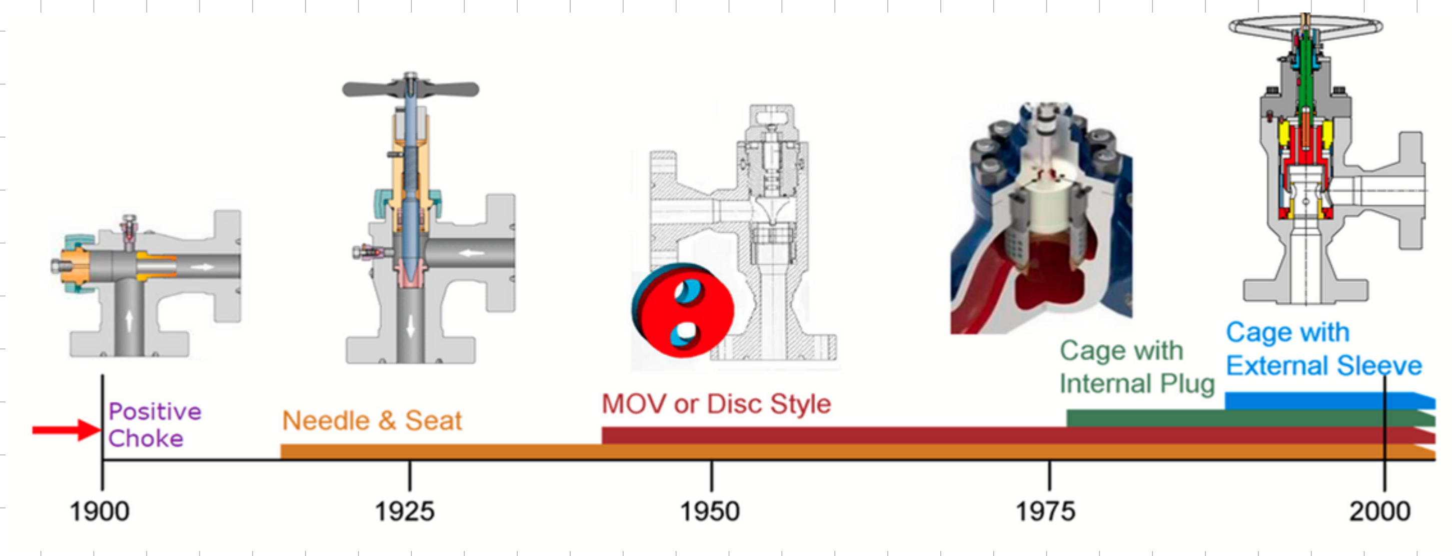
pwf [bara]	pav [bara]	1/(Bo visco) [1/(cp*m3/Sm3)]	J [Sm3/d/bar]	qo [Sm3/d]
300	300	0.449	3.1	0.00E+00
250	275	0.457	3.1	1.57E+02
200	250	0.465	3.2	3.20E+02
150	225	0.474	3.3	4.89E+02
100	200	0.483	3.3	6.64E+02
50	175	0.493	3.4	8.46E+02
5	153	0.502	3.4	1.02E+03

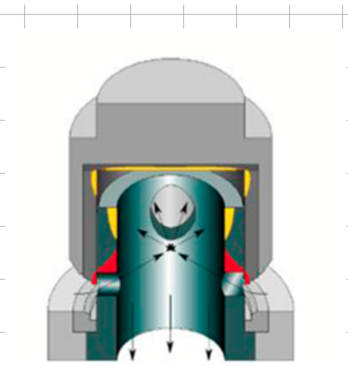






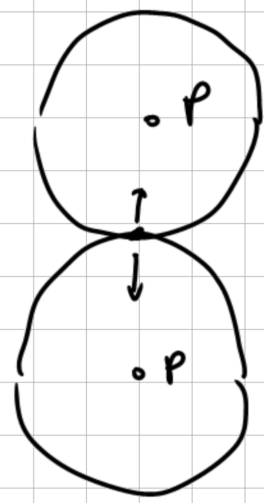
Short comment about wellhead choke equipment





$$q_{\bar{o}} = \frac{kh}{(\mu_o B_o)_{av}\left(\underbrace{\ln\left(\frac{r_e}{r_w}\right)}_{8}-0.5\right)\cdot 18.68}(p_R - p_{wf})$$

pseudo steady-state pss @ r_e → no-flow boundary



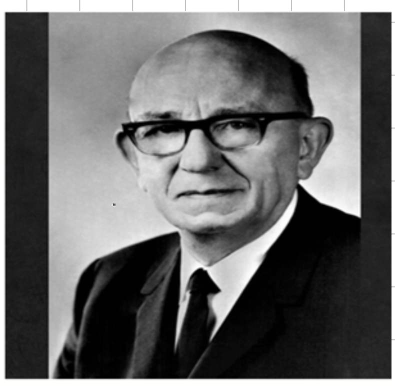
$$\int_{r_w}^{@p_e} \frac{dr}{r} = \frac{kh}{q_{\bar{o}}\cdot 18.68}\int_{p_{wf}}^{p_e}\frac{dp}{B_o\mu_o}$$

for pss r at which $p = p_R$ $r = 0.47\, r_e$

$$\ln\left(\frac{0.47\, r_e}{r_w}\right) = \ln\left(\frac{r_e}{r_w}\right) - 0.75$$

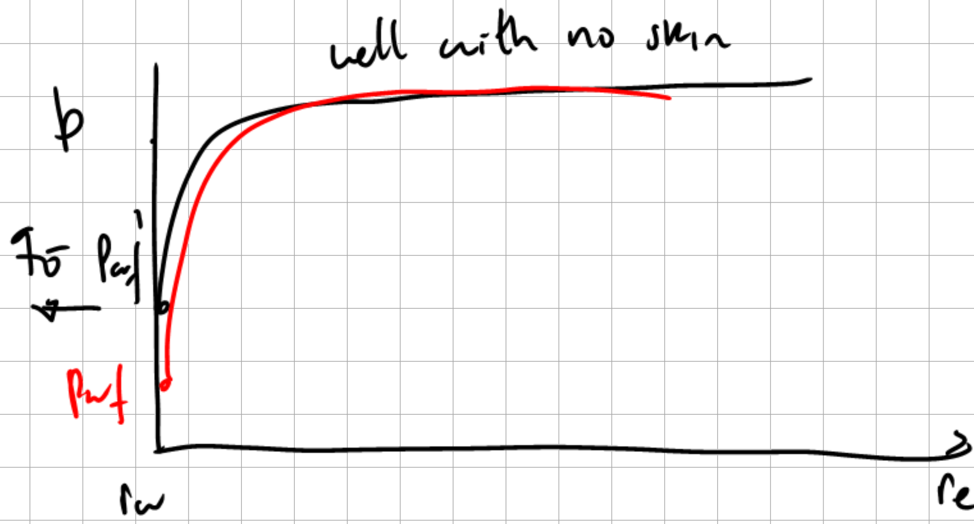
$$J = \frac{kh}{(\mu_o B_o)_{av}\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right] 18.68}$$

Skin factor 1953 → Hurst
↳ Antonius Van Everdingen (Dutch mining engineer) shell




THE SKIN EFFECT AND ITS INFLUENCE ON THE PRODUCTIVE CAPACITY OF A WELL

A. F. VAN EVERDINGEN, SHELL OIL CO., HOUSTON, TEXAS, MEMBER AIME



$$\underbrace{(p_{wf}'}_{\text{ideal}} - \underbrace{p_{wf})}_{\text{actual}} = \Delta p_{skin} = S\underbrace{\left[\frac{18.68\,(\mu_o B_o)_{av}\cdot q_{\bar{o}}}{kh}\right]}_{bar}$$

ideal ↓ no skin

actual: with skin

$$P_R - \cancel{P_{wf}'} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{k \cdot h} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 \right]$$

$$\cancel{P_{wf}'} - P_{wf} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{kh} S$$

$$P_R - P_{wf} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{kh} \left[\ln \frac{r_e}{r_w} - 0.75 + S \right]$$

$$J = \frac{kh}{18.68 \, (\mu_o B_o)_{av} \left[\underbrace{\ln \frac{r_e}{r_w}}_{\text{convergence}} \underbrace{- 0.75}_{\text{boundary}} + \underbrace{S}_{\text{skin (near wellbore non-idealities)}} \right]}$$

8

other definitions "equivalent" to skin

- flow efficiency $E_F = \frac{P_R - P_{wf}'}{P_R - P_{wf}} = \frac{\frac{\cancel{q_o}}{J'}}{\frac{\cancel{q_o}}{J}} = \frac{\ln\left(\frac{r_e}{r_w}\right) - 0.75}{\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S} \approx \frac{7}{7 + S}$

- apparent wellbore radius

$$\ln\left(\frac{r_e}{r_w}\right) + S = \ln\left(\frac{r_e}{r_{wa}}\right) \qquad r_{wa} = r_w e^{-S}$$

if $S > 0$ ⇝ damage (less than ideal) $(P_{wf}' - P_{wf}) > 0$

if $S = 0$ ⇝ ideal

if $S < 0$ ⇝ "stimulation" ⇝ productivity more than ideal

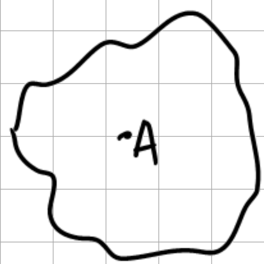
∘ Drainage area shape → 1965 Daniel Dietz

Determination of Average Reservoir Pressure From Build-Up Surveys

D. N. DIETZ
MEMBER AIME

KONINKLIJKE/SHELL LABORATORIUM
RIJSWIJK, THE NETHERLANDS

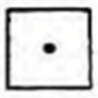
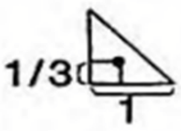
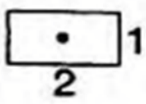
$$\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + \underbrace{S_A}_{\text{shape-area skin}}$$



$$A = \pi \cdot r_e^2 \qquad r_e = \sqrt{\frac{A}{\pi}}$$

← well performance (2nd edition) Golan, Whitson

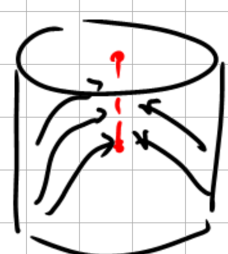
Table 2.4 Shape Factors for Nonradial Outer Boundary Geometries

Geometry	C_A	s_A	t_{DAcia}[a]	t_{DApss}[b]
	31.62	0.000	0.1	0.1
	30.88	0.012	0.09	0.1
	31.60	0.000	0.1	0.1
	27.6	0.068	0.09	0.2
 60°	27.1	0.077	0.09	0.2
 1/3, 1	21.9	0.184	0.08	0.4
 1, 2	21.84	0.185	0.025	0.3

∘ other cases that can be modeled (represented) with skin

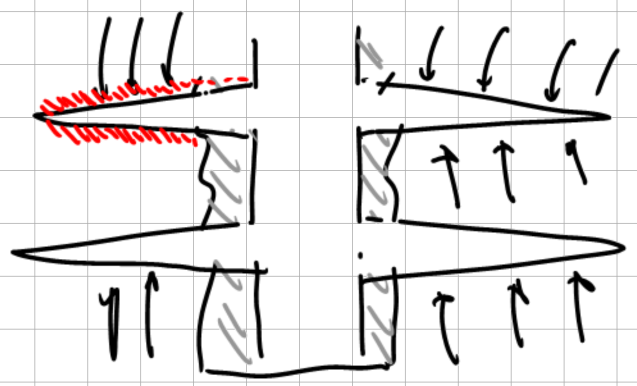
∘ formation damage (due to drilling) $S > 0$

∘ partial penetration



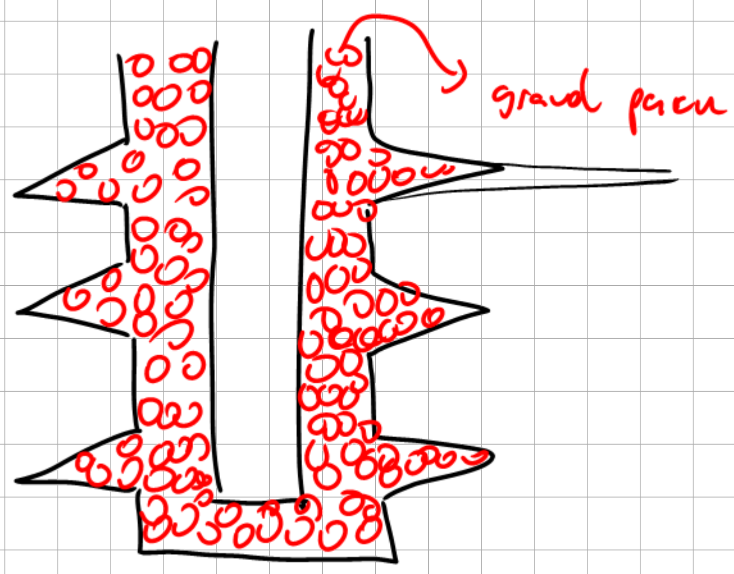
$k_v \ll k_H$ $S > 0$

- perforation



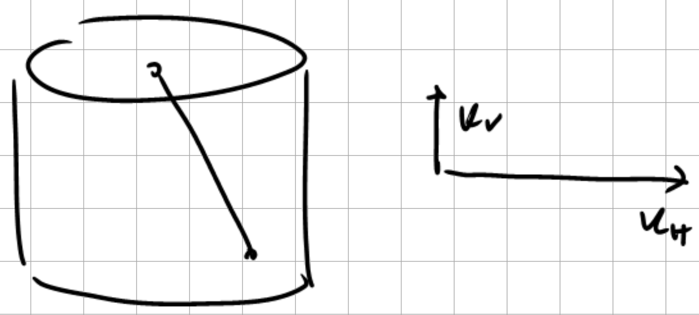
$s > 0$

- sand control (gravel pack)



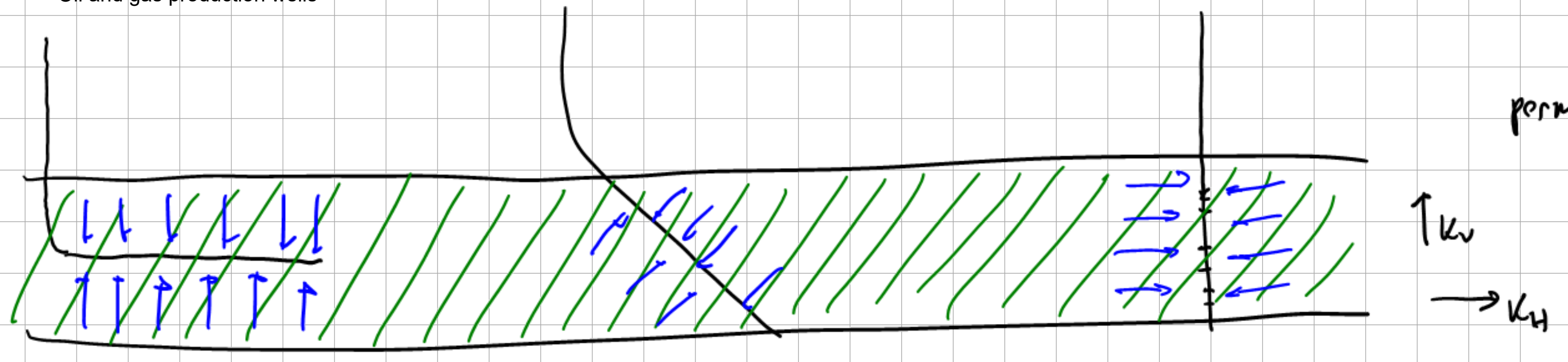
$s > 0$

- wellbore inclination
  (permeability anisotropy)



- acidizing (stimulation)
- fracking

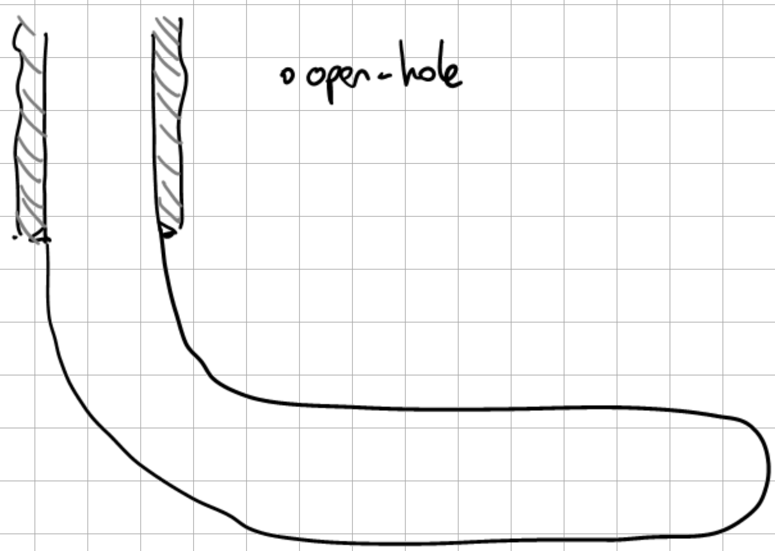
$s < 0$



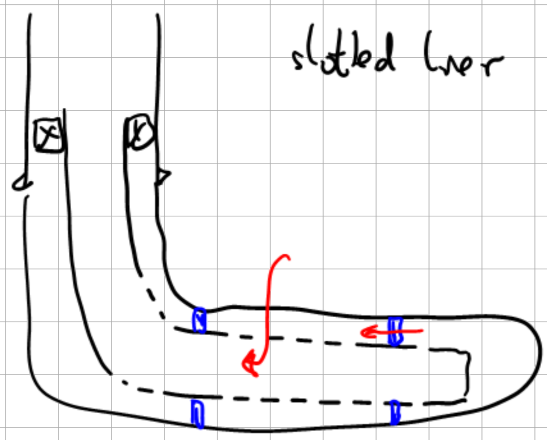
permeability anisotropy

$\frac{k_H}{k_v} \gg 1$ e.g. 10

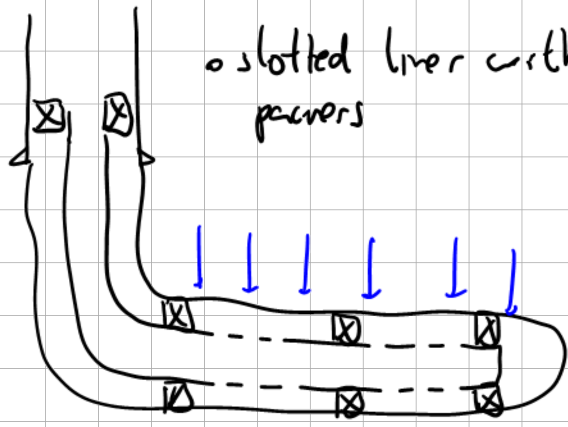
- open-hole



slotted liner

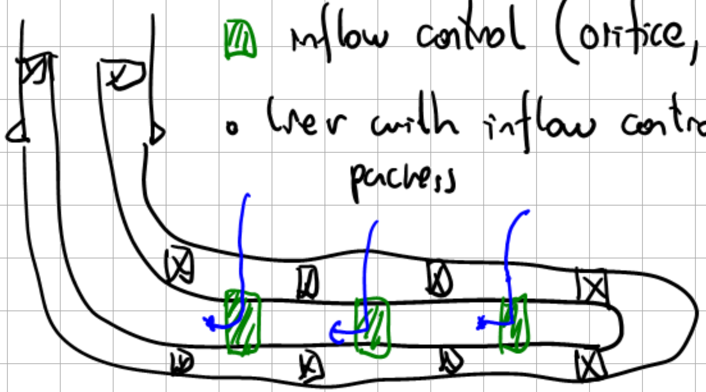


- slotted liner with packers



inflow control (orifice, control valve)

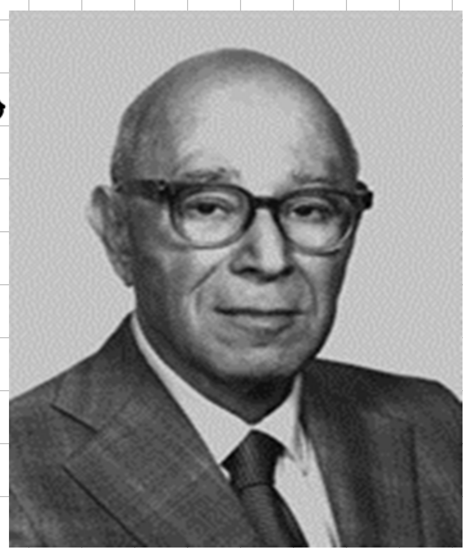
- liner with inflow control and packers



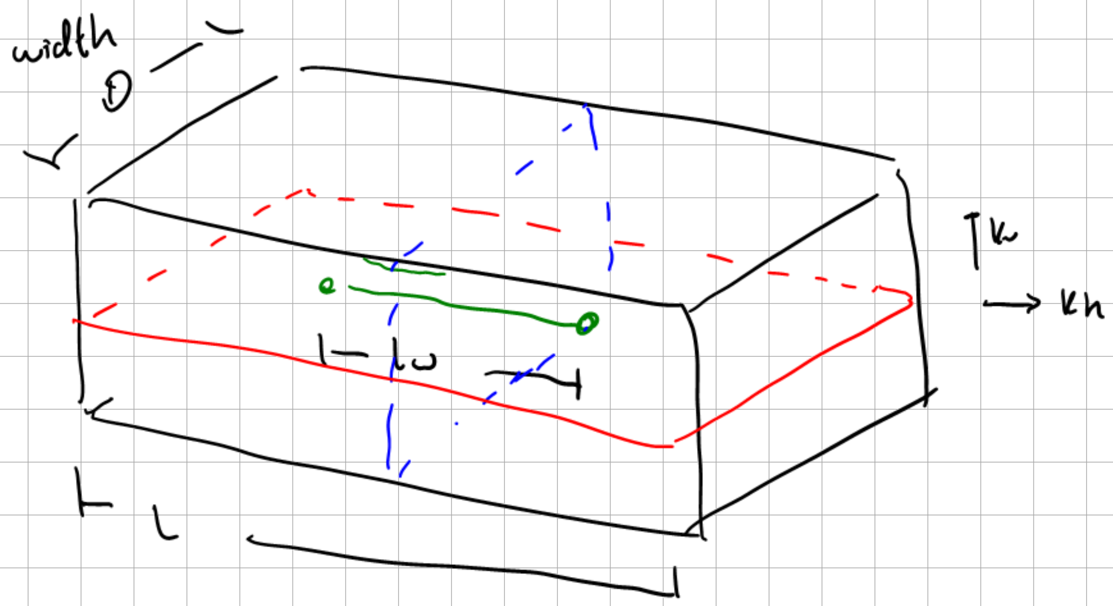
cemented and perforated liner

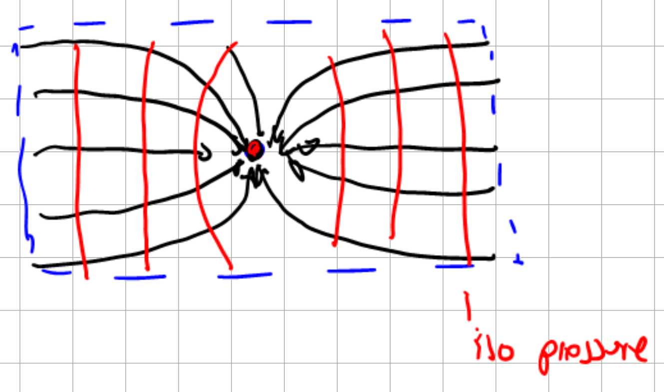
Morris

1937 - Muskat (1937) (Gulf oil)
1958 - Merkulov
1964 - Borisov
1974 - Gringarten (Imperial College), Ramey (Stanford)
1983 - Giger (IFP)
1988 - Joshi
1989 - Babuh, Odeh (Mobil)
1991 - Renard, Dupuy (IFP)
1994 - Buttler
Harald Asheim (NTNU)

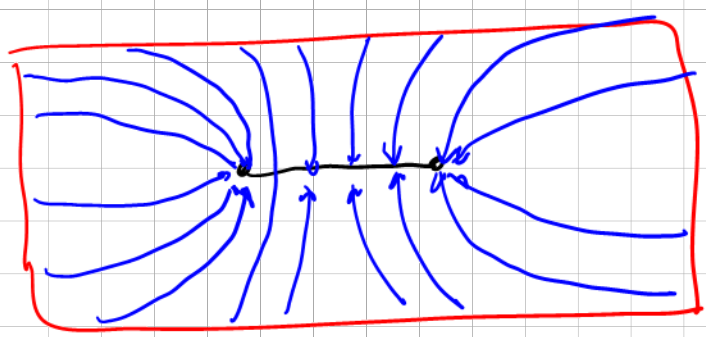


http://www.ipt.ntnu.no/~asheim/prodbr.html





$P = f(x)$

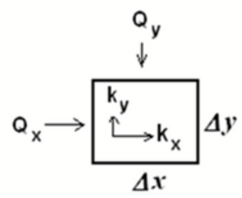


$\sim P = f(x)$

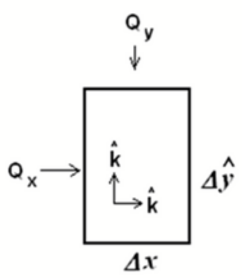
Superposition

Strategy to deal with permeability anisotropy

- Actual reservoir element



- Homogenous equivalent:



Permeability scaling: $\hat{k} = \sqrt{k_x k_y}$

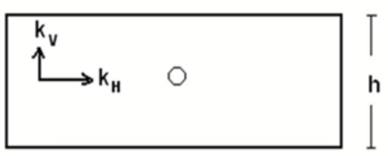
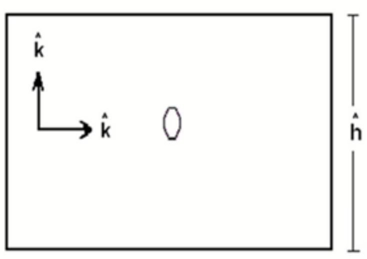


Figure 3.10 Anisotropic reservoir

$$k_H \frac{\partial^2 p}{\partial x^2} + k_V \frac{\partial^2 p}{\partial y^2} = \varphi \mu c \frac{\partial p}{\partial t}$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial\left(\sqrt{k_H / k_V}\, y\right)^2} = \frac{\varphi \mu c}{k_H} \frac{\partial p}{\partial t}$$



$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial \hat{y}^2} = 0$$

$$\hat{y} = (k_H / k_V)^{0.5} y$$

Height scaling: $\hat{h} = (k_H / k_V)^{0.5} h$

Harald Asheim's IPR equation

- undersaturated oil
- horizontal well
- pss
- scaling due to anisotropy

geometry related

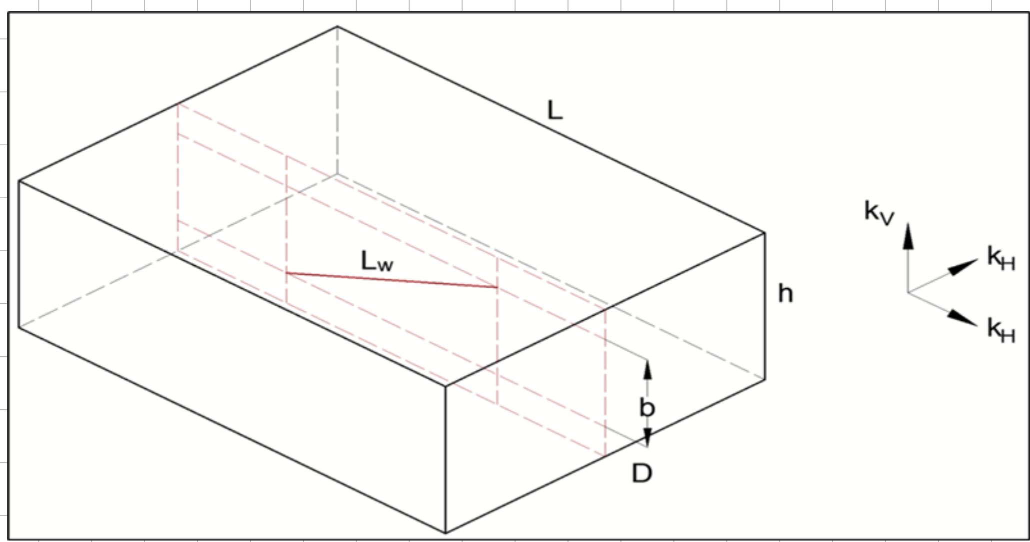
$$q_o = \frac{k_H \cdot h}{6.22 \cdot (\mu_o \cdot B_o)_{@p_{av}} \cdot \left[\frac{\pi \cdot D \cdot \hat{f}_a}{2 \cdot \hat{L}_w} + \frac{3 \cdot h \cdot \beta}{\bar{L}_w} \cdot \left(ln\left(\frac{h \cdot \beta}{2 \cdot \pi \cdot \hat{r}_w}\right) + s_b\right)\right]} [p_R - p_{wf}]$$

function of PVT, fluid properties

$$\beta = \sqrt{\frac{k_H}{k_V}}$$

$$\hat{L}_w = L_w \cdot \sqrt{1 - \left(\frac{b}{L_w}\right)^2 \cdot (\beta^2 - 1)}$$

$$\bar{L}_w = L_w \cdot \sqrt{1 - \left(\frac{b}{L_w}\right)^2}$$



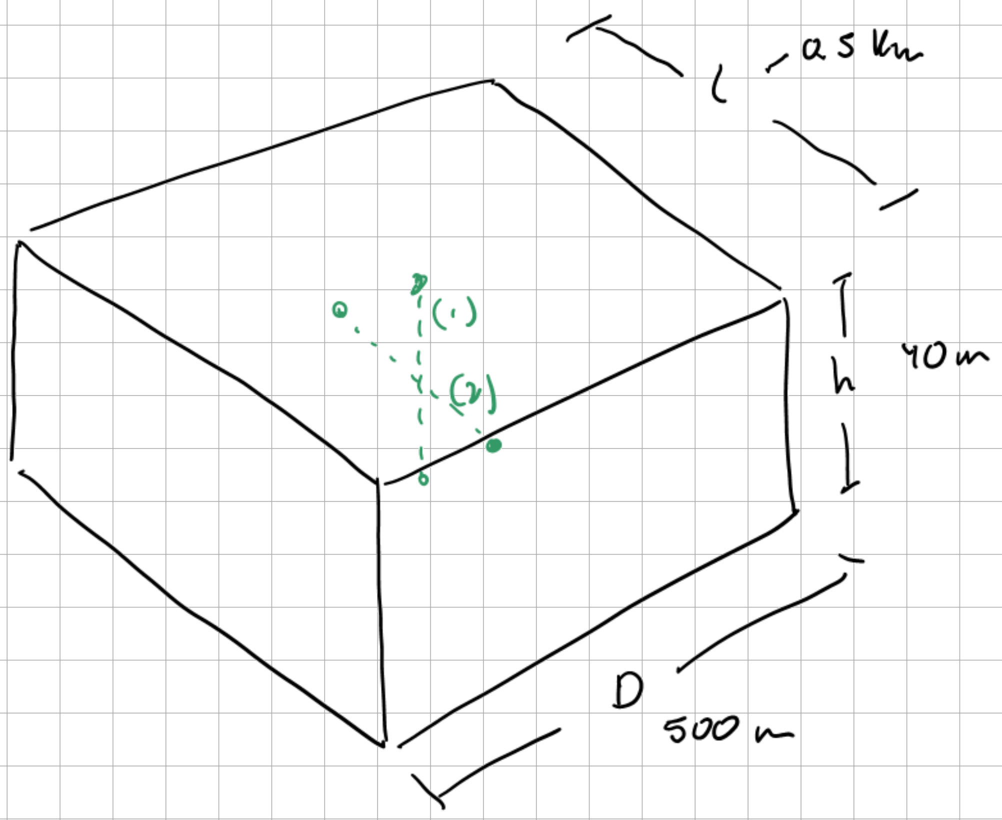
$$\boxed{\hat{r}_w = r_w \cdot \frac{1 + \sqrt{1 + (\beta^2 - 1) \cdot \left(\frac{\bar{L}_w}{L_w}\right)^2}}{2}}$$

well completed partially in the interval

$$\boxed{\hat{f}_a = \frac{\bar{L}_w}{L} \cdot \left[1 + \left(0.53 \cdot \left(\frac{L}{D}\right)^2 + 1.15 \cdot \left(\frac{L}{D}\right) + 0.164\right) \cdot \left(\frac{1 - \left(\frac{\bar{L}_w}{L}\right)}{0.45 + \left(\frac{\bar{L}_w}{L}\right)}\right)\right]}$$

$$\boxed{\hat{s}_b = \begin{Bmatrix} 0.69 \ \ if \ \ O(b) = O(h) \\ 0 \ \ if \ \ b = 0 \end{Bmatrix}}$$

skin factor, "shielding" effect from top and bottom walls



vertical well

$$\pi r_e^2 = A = 500 \cdot 500$$

$$r_e = \sqrt{\frac{500 \cdot 500}{\pi}}$$

Reservoir top area	[m2]	2.50E+05
Reservoir pressure, p_R	[bara]	300
Flowing bottom-hole pressure, p_{wf}	[bara]	200
p_{av}	[bara]	250
Oil viscosity, μ_o at average pressure	[cp]	1.877
Oil volume factor, B_o, at average pressure	[m3/Sm3]	1.144
Wellbore radius, r_w	[m]	0.15
Vertical well located in the center and perforated throughout		
External radius, r_e	[m]	282.1
Skin, s	[-]	0
Shape factor, s_A	[-]	0.012
Productivity Index, J	[Sm3/d/bar]	14.7
Horizontal well		
Wellbore length	[m]	500
Elevation difference between toe and heel, b (sign doesn't matter)	[m]	0
Productivity Index, J	[Sm3/d/bar]	63.2

```
Function J_vertical(k, h, Uo, Bo, re, rw, s, sa)
    'Productivity index for vertical well, undersaturated oil, pss, in Sm3/d/bar
    'k in md
    'h in m
    'Uo in cp
    'Bo in m^3/Sm^3
    'fa is shape factor
    J_vertical = (k * h) / (18.68 * Uo * Bo * (Log(re / rw) - 0.75 + s + sa))
    'Natural Log in Visual Basic is Log, not LN'
End Function
```

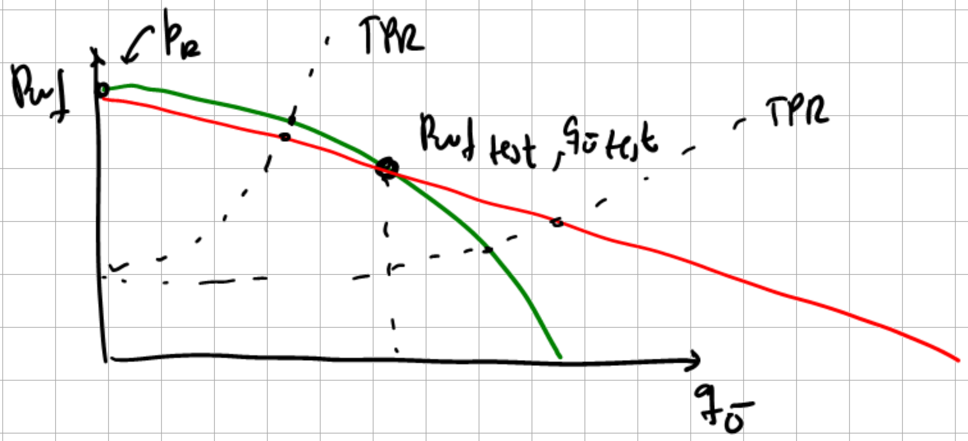
```
Function J_horizontal(L, D, h, b, Lw, kh, kv, Bo, viso, rw)
    'Productivity index for horizontal well, undersaturated oil, pss, in Sm3/d/bar
    'L Reservoir length along well direction [m]
    'D reservoir width [m]
    'h reservoir thickness [m]
    'Lw well length [m]
    'kh horizontal permeability [md]
    'kv vertical permeability [md]
    'Bo oil formation volume factor [m^3/Sm^3]
    'viso oil viscosity [cp]
    'rw well radius [m]
    'b, height difference betweeen heel and toe [m]
    Pi = Atn(1) * 4
    b = Abs(b)
    If b / h > 0.1 Then
        s_b = 0.69
    Else
        s_b = 0
    End If
    beta = (kh / kv) ^ 0.5
    Lw_hat = Lw * (1 + ((b / Lw) ^ 2) * (beta ^ 2 - 1)) ^ 0.5
    Lw_bar = Lw * (1 - (b / Lw) ^ 2) ^ 0.5
    rw_hat = 0.5 * rw * (1 + (1 + (beta ^ 2 - 1) * ((Lw_bar / Lw) ^ 2)) ^ 0.5)
    A1 = 0.53 * ((L / D) ^ 2) + 1.15 * (L / D) + 0.164
    A2 = (1 - (Lw_bar / L)) / (0.45 + (Lw_bar / L))
    fa = (Lw_bar / L) * (1 + A1 * A2)
    C1 = 3 * h * beta * (Log(beta * h / (2 * Pi * rw_hat)) + s_b) / Lw_hat
    C2 = (Pi * D * fa / (2 * Lw_bar))
    unit_conversion_constant = (9.869E-13 * 0.001) * 24 * 3600 * 100000 * 6 * Pi / (0.001)
    J_horizontal = unit_conversion_constant * kh * h / (viso * Bo * (C1 + C2))
End Function
```

$q_{\bar{o}} = J\,(p_R - p_{wf})$ ⟶ Undersaturated oil

geometry ⟶

fluid-properties ⟶ $(\mu_o B_o)_{@p_{av}}$

for gas, saturated oil using a linear IPR is not adequate !

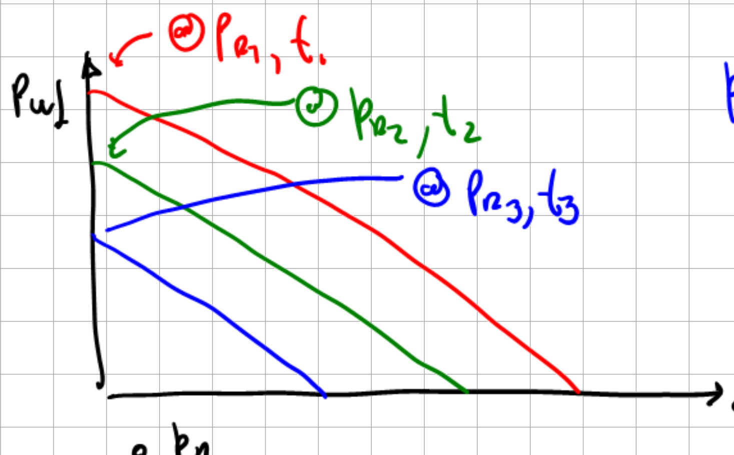


changes to IPR due to depletion

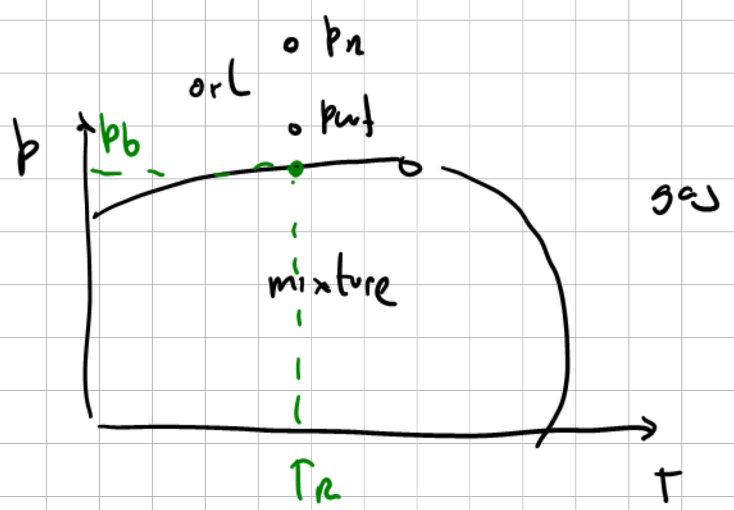
p_R → goes down

$J \sim (\mu_o B_o)_{@p_{av}}$, r_e, psd, ss, s , otherwise J remains fairly constant with time

$p_{av} = (p_R + p_{wf}) \cdot 0.5$



$p_{R1} > p_{R2} > p_{R3}$

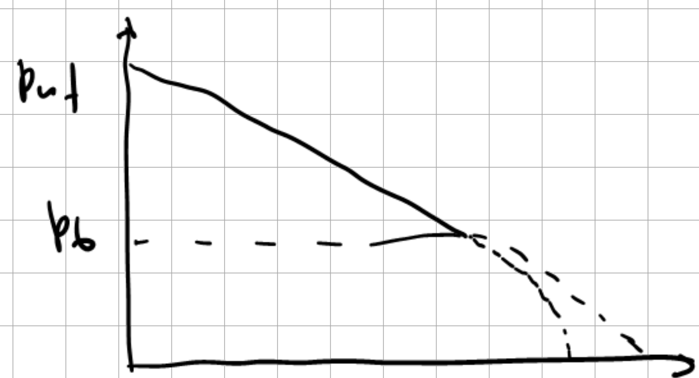


linear IPR valid if $p_{wf} > p_b$

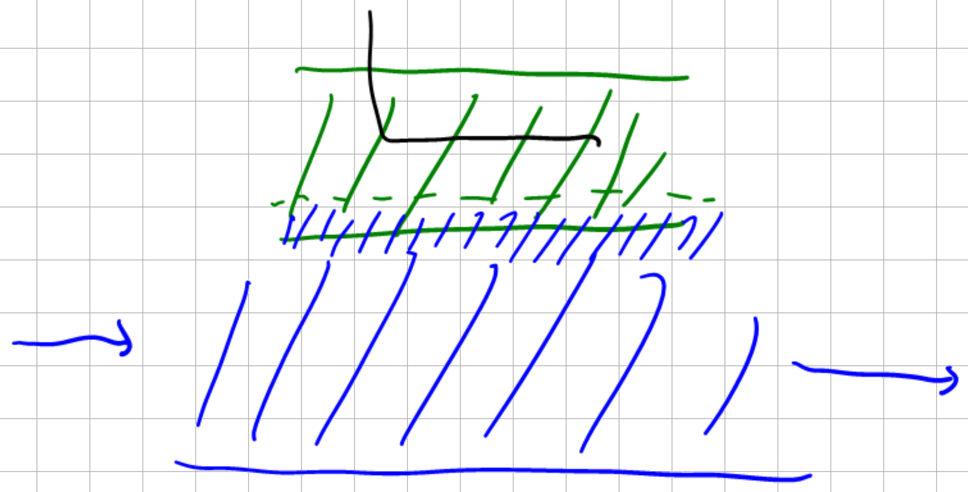
$p_e > p_b$

water injection

strong hydrodynamic aquifer



no aquifer, no injection, undersaturated oil reservoir



flow of water + oil (undersaturated) → Exhibits a linear IPR

liquid

incompressible

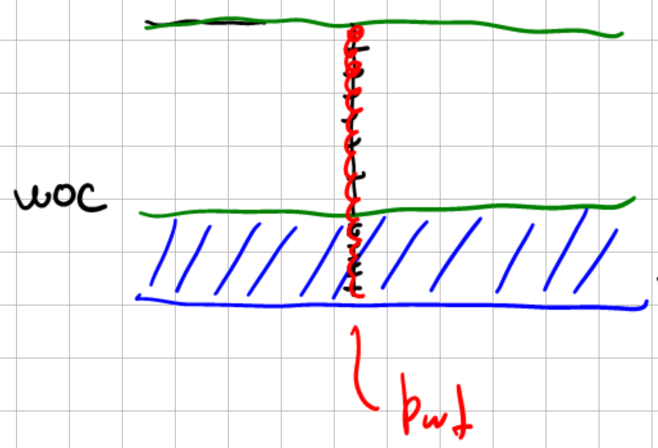
$$J_L = \frac{(q_o + q_w)}{(P_R - P_{wf})}$$

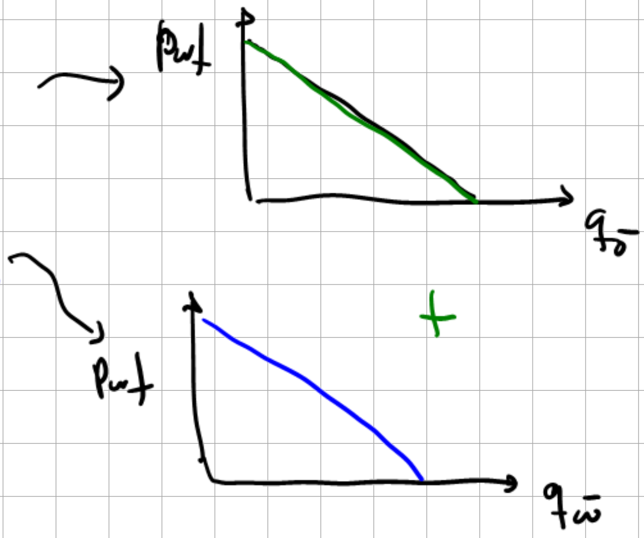
$$q_L = J_L (P_R - P_{wf})$$

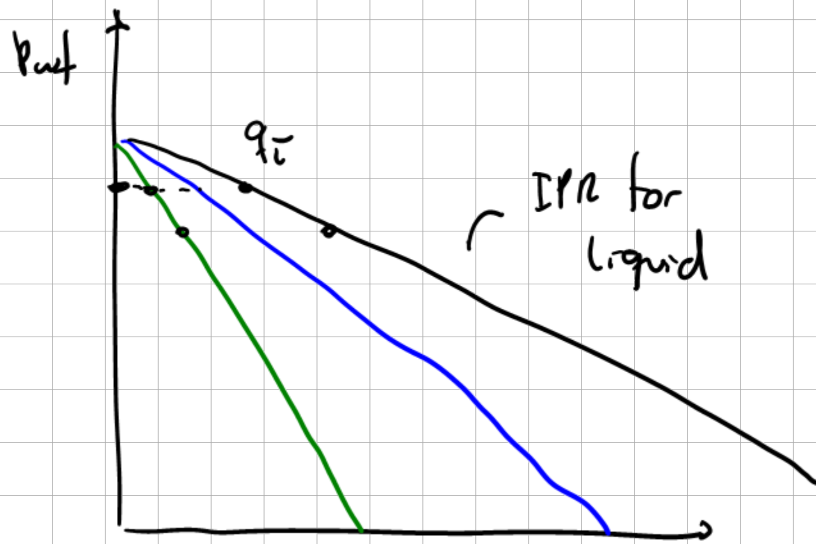
$$q_o = q_L (1 - w_c)$$

$$w_c = \frac{q_w}{q_o + q_w}$$

$$q_w = q_L (w_c)$$

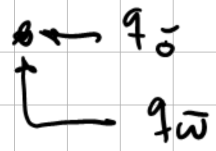


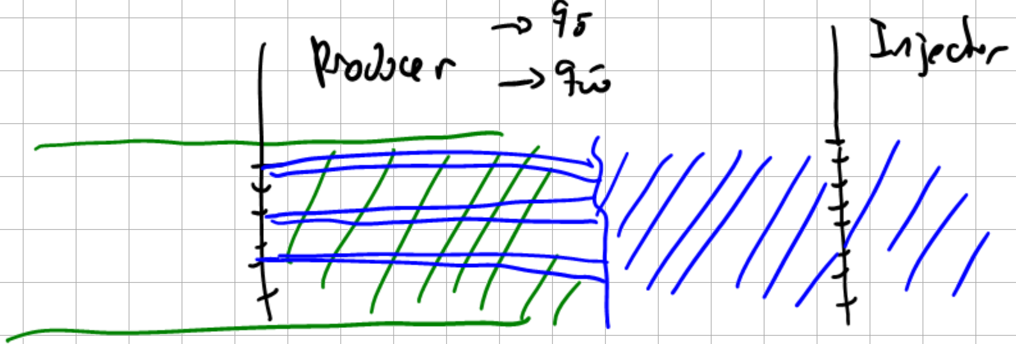


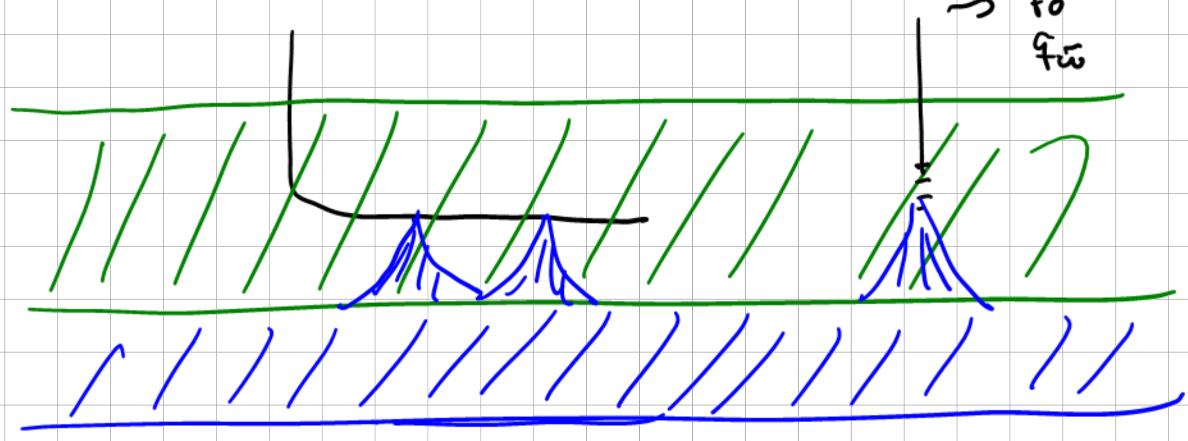


mobility $\lambda_o = \frac{k_{ro}}{\mu_o}$ $\lambda_w = \frac{k_{rw}}{\mu_w}$

$\lambda_o < \lambda_w$





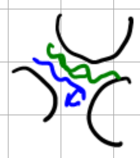


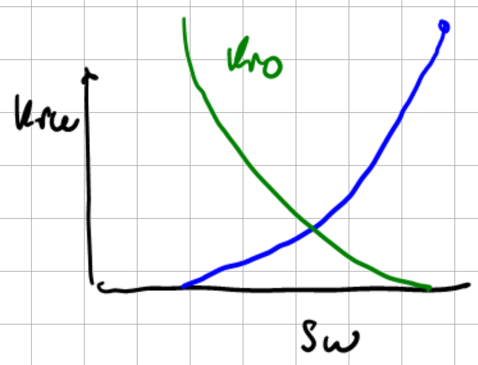
water coning - cusping

$J\ f(K)$ when two phases flowing

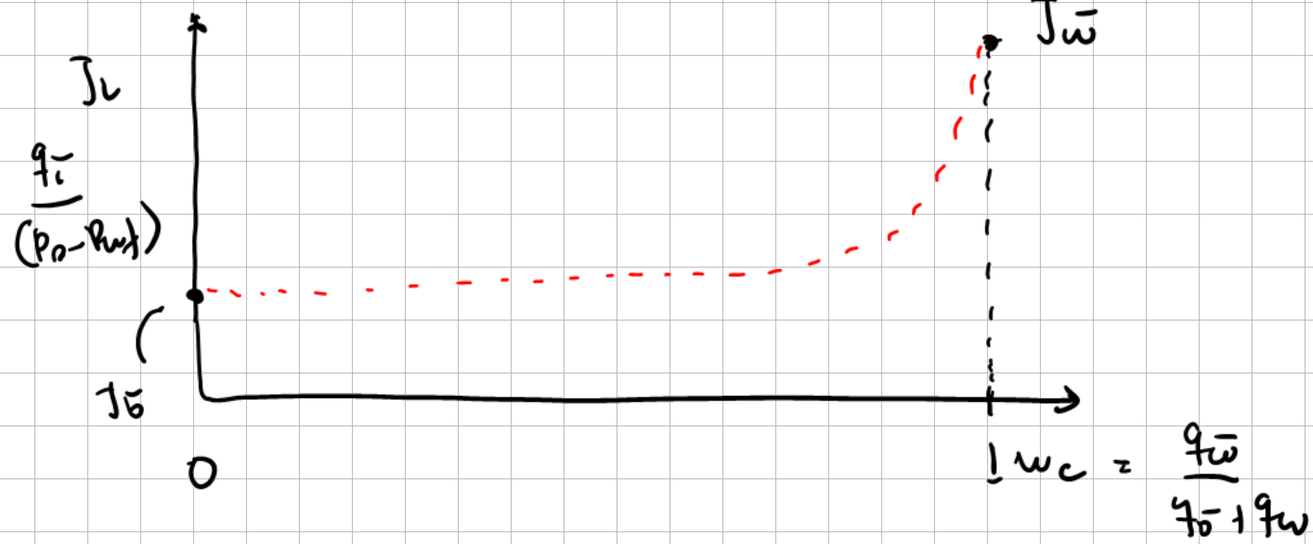
$K\ k_{ro}$

k_{rw}





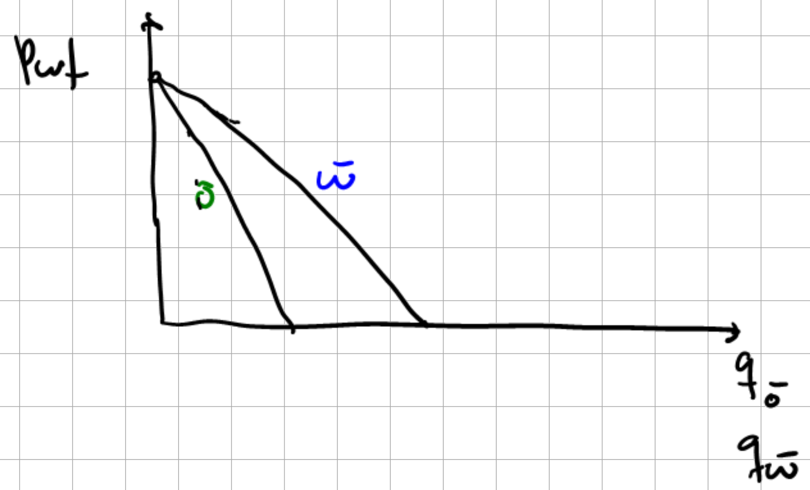
$S_w = \dfrac{V_w}{V_T}$ in the pore

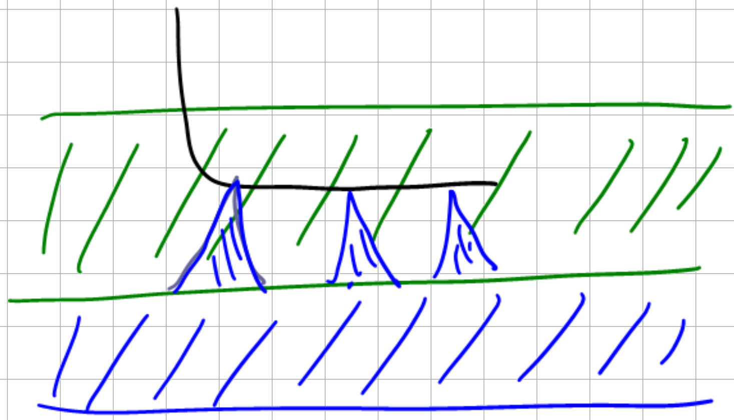


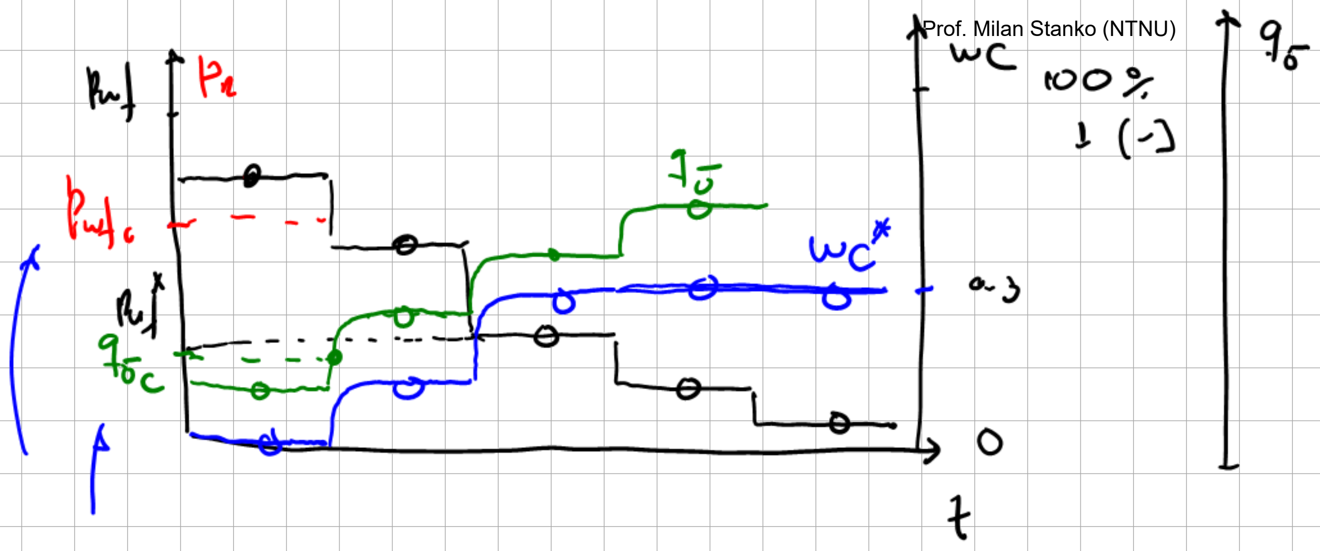
$$q_l = J_l\,(p_R - P_{wf})$$

$$q_o = q_l \cdot (1 - w_c)$$

$$q_w = q_l\,(w_c)$$

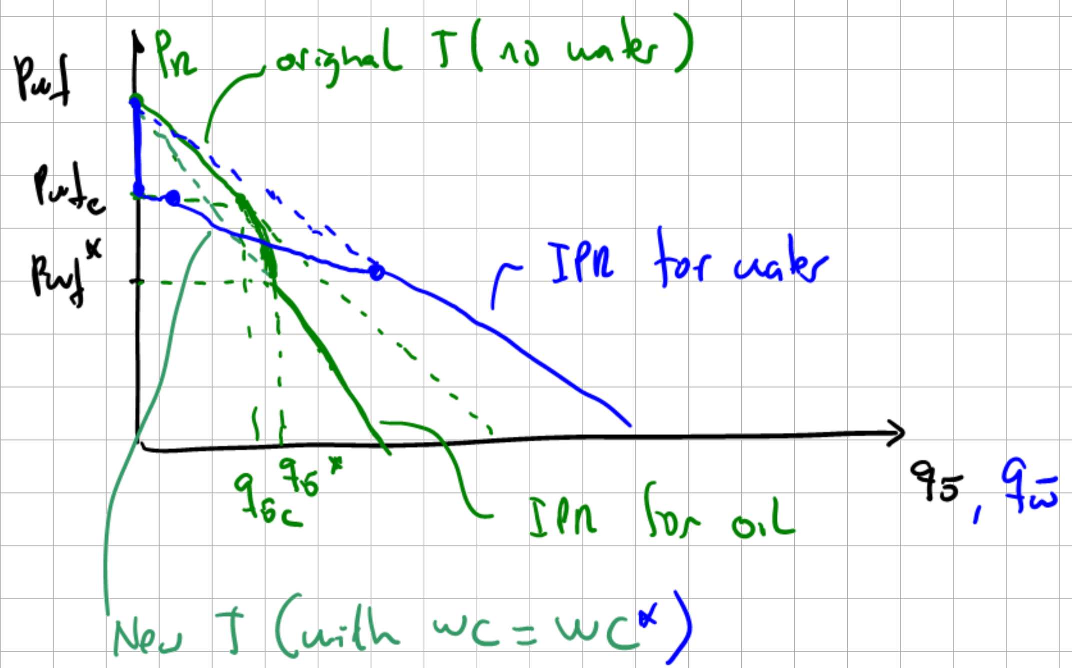






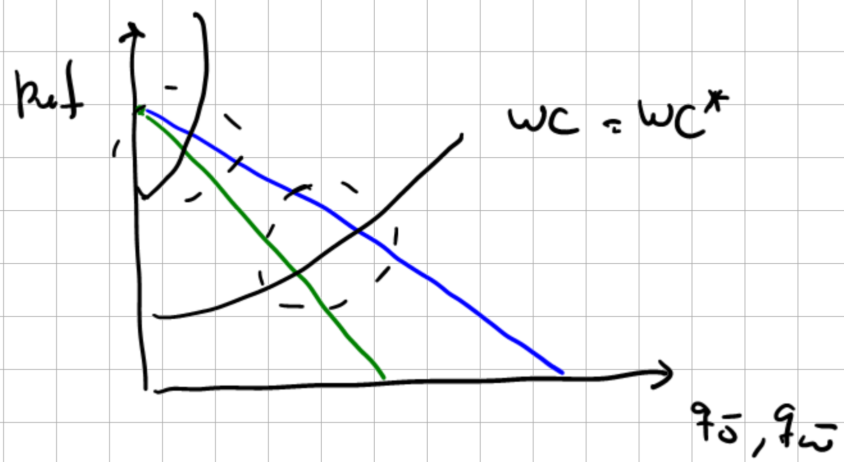
$q_{\bar o} < q_{\bar o c}$ $p_{wf c} < p_{wf}$ → no coning

$q_{\bar o} \geq q_{\bar o c}$ $p_{wf_c} \geq p_{wf}$ → coning



in many cases $q_{\bar o c}$ is very small

$q_{\bar o}^*$ is very small



Example: one model for water coning from the literature:

Experimental Investigation of Cresting and Critical Flow Rate of Horizontal Wells

Tove Aulie, Evert Grødal, Harald Asheim, Norwegian Inst. of Technology, and Piet Oudeman, Koninklijke/Shell E&P Laboratorium

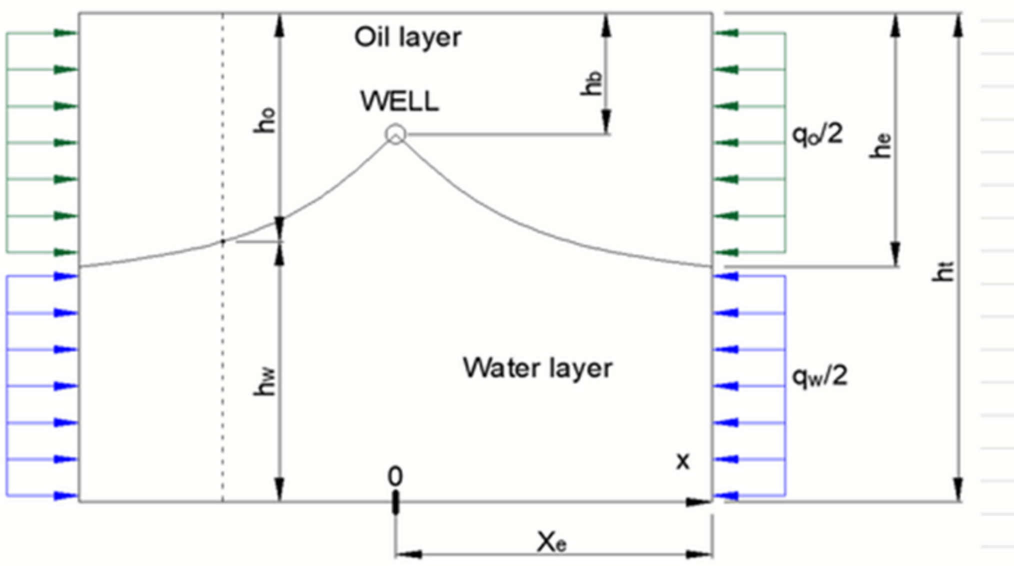
ABSTRACT

Cresting towards horizontal wells with bottom water drive and edge water drive has been experimentally investigated using a

$$h_o(x)^2 - h_e^2 + \frac{q_o}{\lambda_o \Delta\rho g D}\left(\frac{D}{2} - x\right)^2 = 0 \qquad (1)$$

Prof. Milan Stanko (NTNU)		
Total layer height, ht (oil plus water)	[m]	50
Initial water layer height, hw (ht-he)	[m]	10
Initial oil layer height, ho (he)	[m]	40
Horizontal distance from well to outer boundary, xe	[m]	300
Vertical distance between well and top of reservoir, hb	[m]	5
Horizontal permeability, k	[md]	100
Oil viscosity	[cp]	1.0
water viscosity	[cp]	0.6
Oil mobility	[md/cp]	100.0
water mobility	[md/cp]	166.7
Oil density	[kg/m^3]	800
Water density	[kg/m^3]	1024
Oil Bo	[m3/Sm3]	1.0
Water Bw	[m3/Sm3]	1.0
Well length, L, [m]	[m]	500
Critical oil flow to start producing water, qoc (ho=hb@x=0)	[Sm3/d]	49.19
Mobility ratio M (o/w)	[-]	0.6
upper limit of f (qw/qo)	[-]	0.42
upper limit for WC	[%]	29.4
deltaf (qw/qo) - for plotting	[-]	0.014

f (qw/qo)	WC	qo/qoc
[-]	[%]	[-]
0.00	0	1.0
0.01	1	1.0



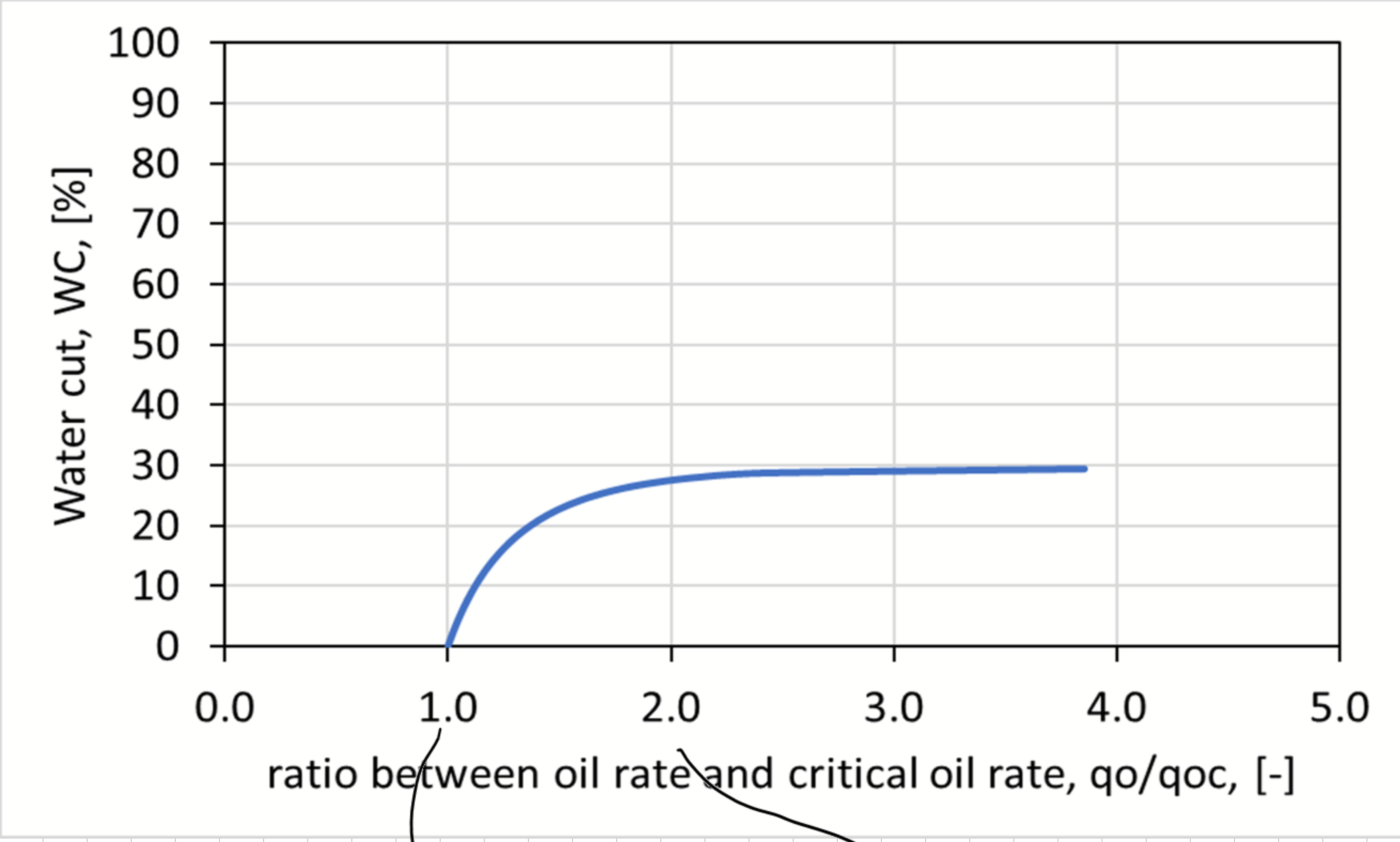
ASSUMPTIONS:

*Steady state flow, the oil and water volumetric flows in their layers

*Dupuis-Forchheim assumption: the flow towards the well is primari

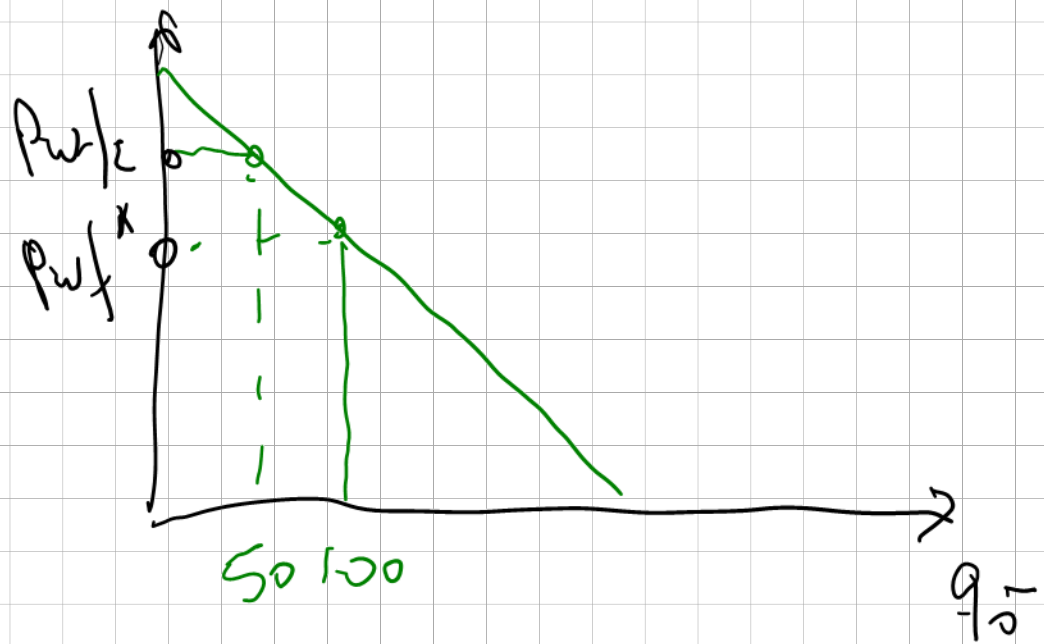
*Capillary pressure is neglected

$$q_{\bar o c} = \frac{(\rho_w - \rho_o)\cdot g \cdot \lambda_o \cdot \left(h_e^2 - h_b^2\right)\cdot L}{x_e \cdot B_o}$$

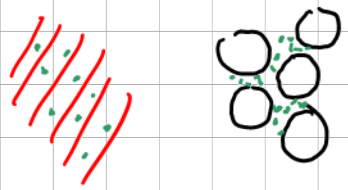


Ca 50 Sm^3/d Ca. 100 Sm^3/d

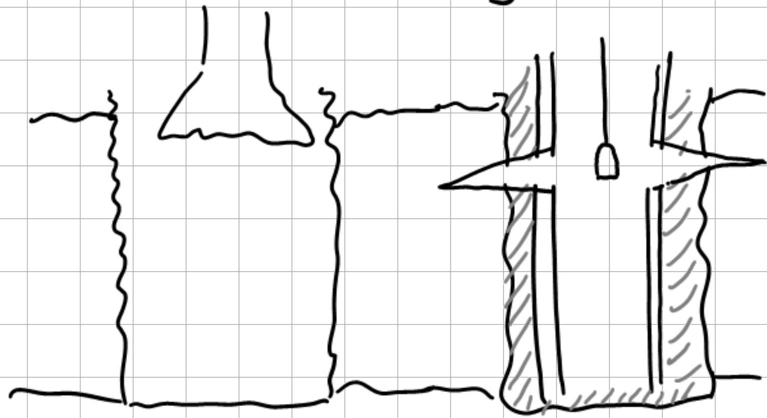
p_{wf}



formation damage



cementing and perforating



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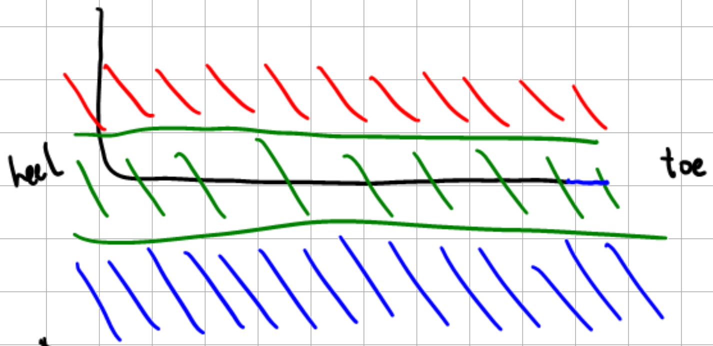
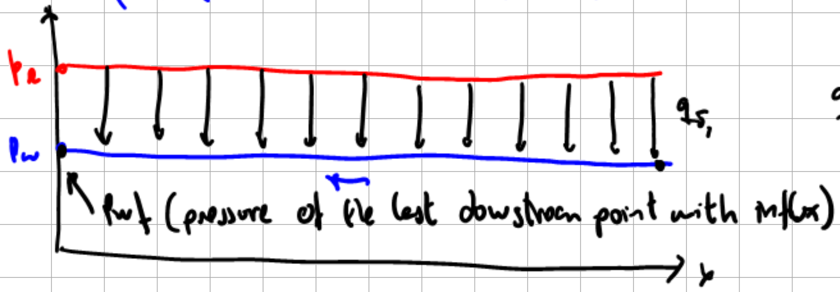
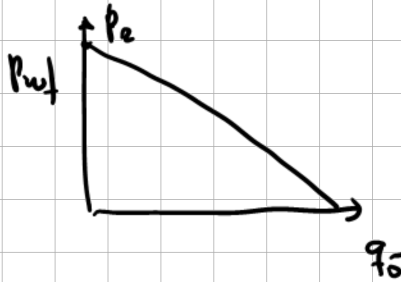
variation of pressure along wellbore in horizontal wells



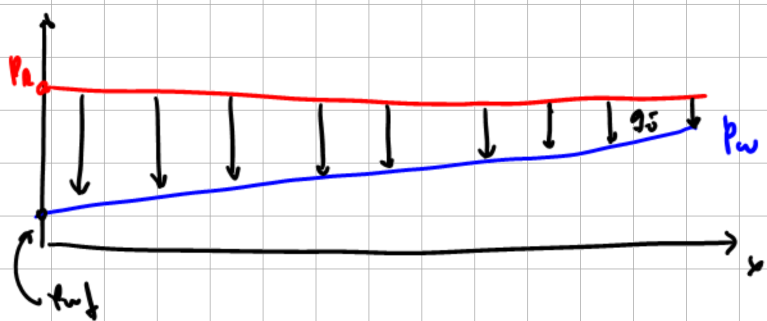


$$q_{\bar{o}} \propto (p_R - p_w)$$

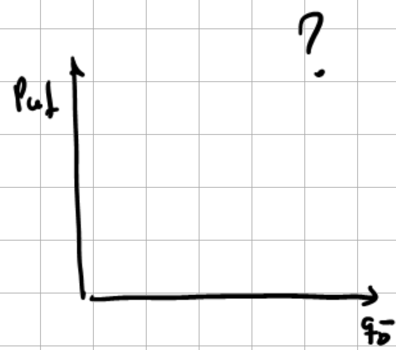
IPR for this case

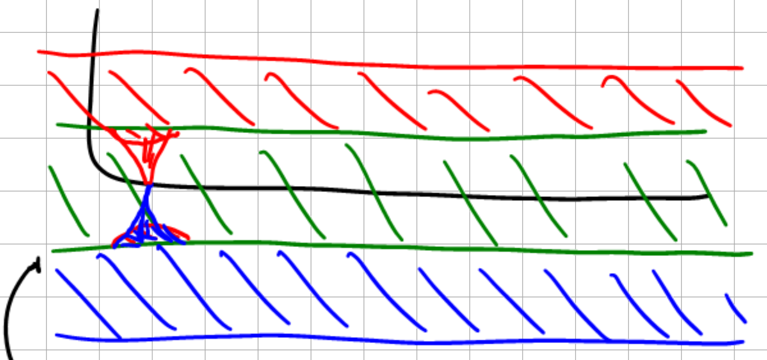


$$q_{\bar{o},well} = \int_{x_{toe}}^{x_{heel}} q_{\bar{o},rel} \cdot dx$$

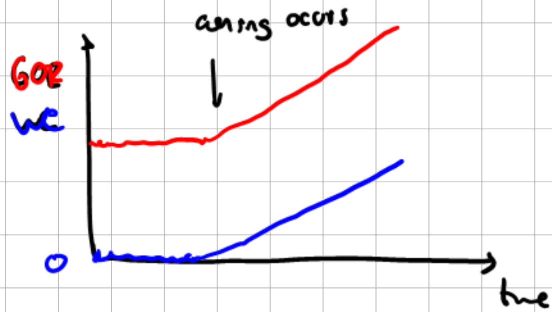


$$q_{\bar{o}} = \alpha (p_R - p_w)$$



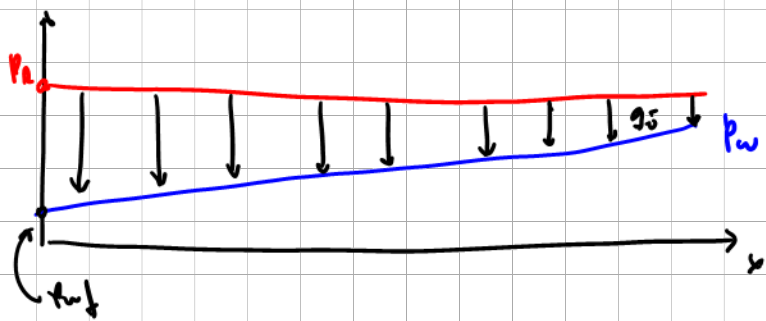


gas/water coning, due to low pressure @ heel



$$q_g = GOR \cdot q_o$$

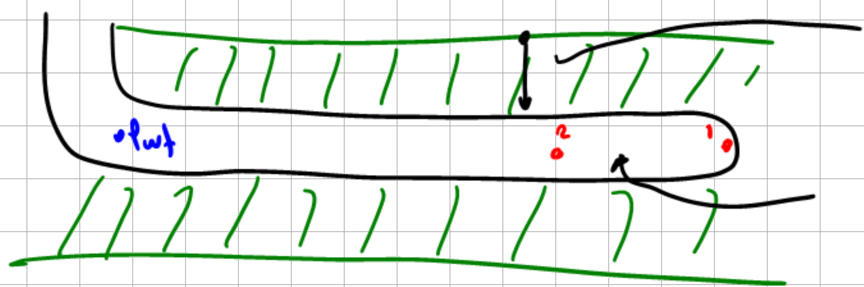
$$q_w = \frac{WC}{1-WC}\, q_o$$



$$q_o = \alpha\,(p_R - p_w)$$

?

probably is linear $\quad q_{o_w} = J\,(p_R - p_{wf})$



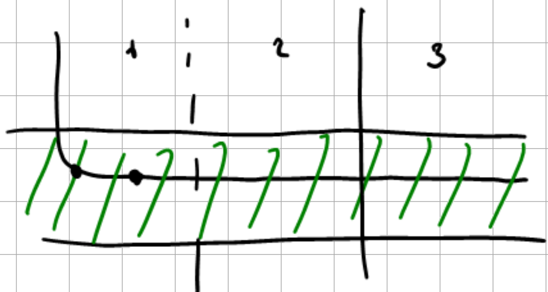
$$\frac{P_1}{\rho g} + \cancel{z_1} + \cancel{\frac{v_1^2}{2g}} = \frac{P_2}{\rho g} + \cancel{z_2} + \cancel{\frac{v_2^2}{2g}} + \underbrace{f\frac{L}{\Phi}\frac{v^2}{2g}}_{\text{friction losses}}$$

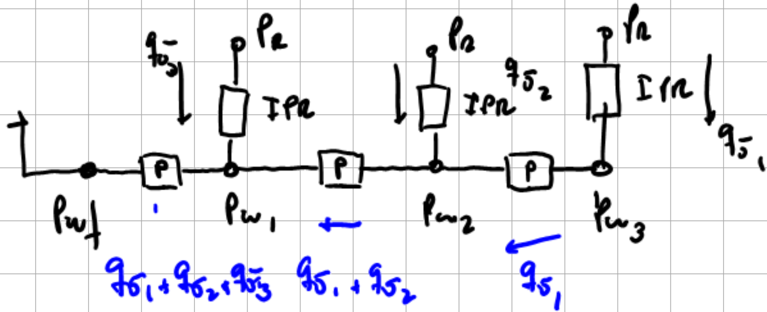
$1 \circ \longrightarrow \circ\, 2$

$$(P_1 - P_2) = \rho f \frac{L}{\Phi}\frac{v^2}{2}$$

$$\frac{dP}{dx} = \rho\,\frac{f}{\Phi}\,\frac{v^2}{2} \qquad v^2 = \left(\frac{q_{o_w} B_o}{A}\right)^2$$

$$\frac{dP}{dx} = \rho\,\frac{f\, 8\,(q_{o_w}(x)\, B_o)^2}{\pi^2 \Phi^5} \qquad A = \frac{\pi \Phi^2}{4}$$





	N EQ	N unknowns
$q_{o1} = J_1 (P_R - P_{w1})$	1	2
$q_{o2} = J_2 (P_R - P_{w2})$	1	2
$q_{o3} = J_3 (P_R - P_{w3})$	1	2
$P_{w3} - P_{w2} = \rho \frac{f\,8}{\pi^2 \phi^5} (q_{o1} \cdot B_{o1})^2$	1	0
$P_{w2} - P_{w1} = \rho \frac{f\,8}{\pi^2 \phi^5} (q_{o1} B_{o1} + q_{o2} B_{o2})^2$	1	0
$P_{w1} - P_{wf} = \rho \frac{f\,8}{\pi^2 \phi^5} (q_{o1} B_{o1} + q_{o2} B_{o2} + q_{o3} B_{o3})^2$	1	1
	6	7

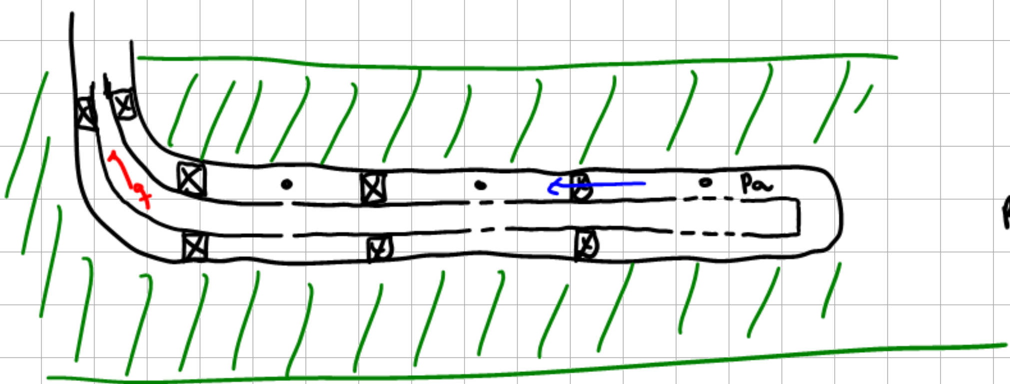
$q_o \rightarrow P_{wf}$

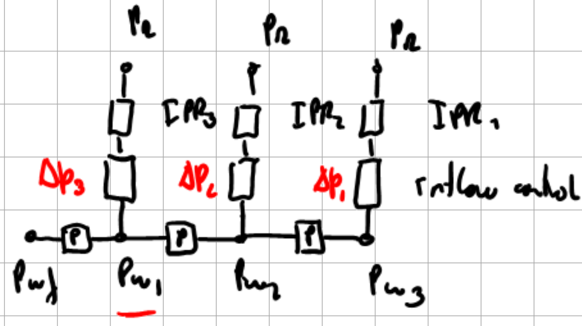
$P_{wf} \rightarrow q_o$

provide P_{wf}

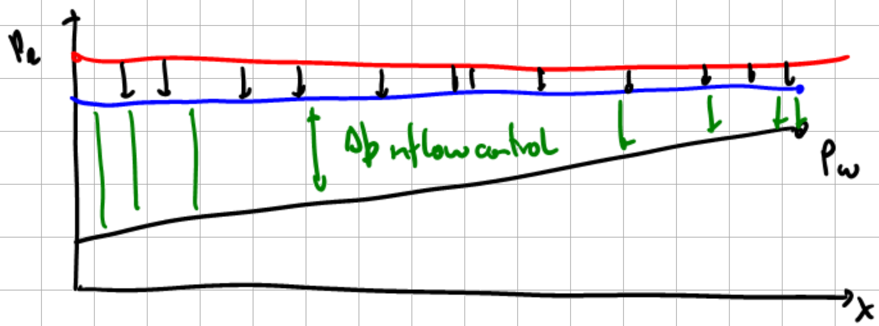
provide $q_o = q_{o1} + q_{o2} + q_{o3}$

Inflow control

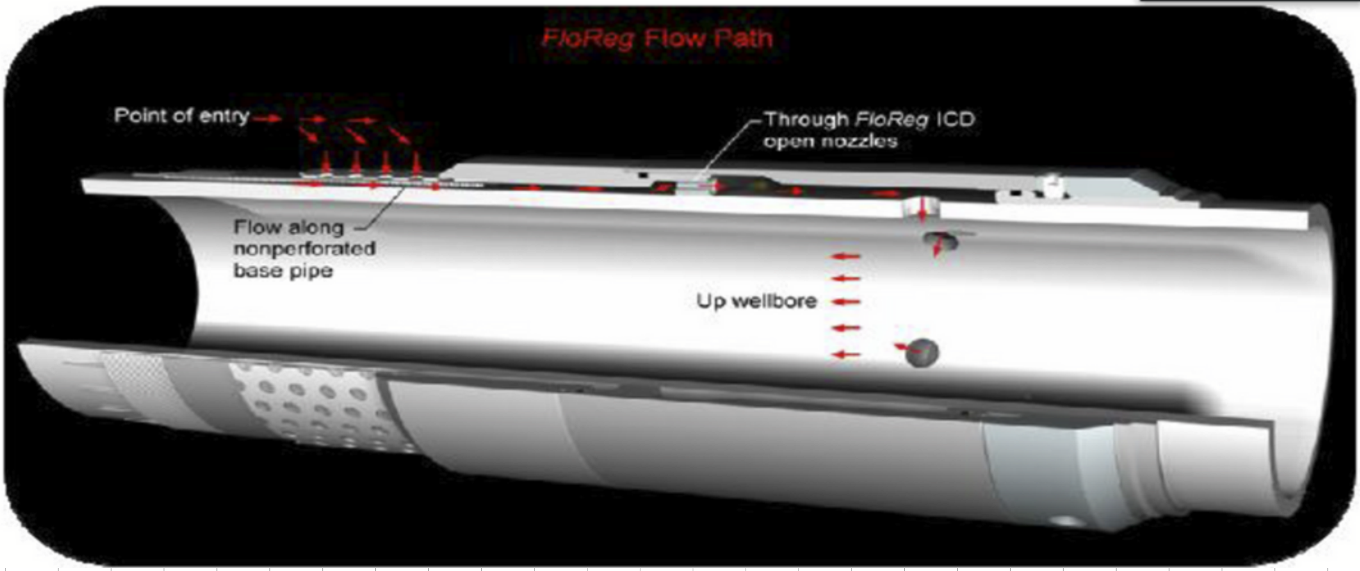




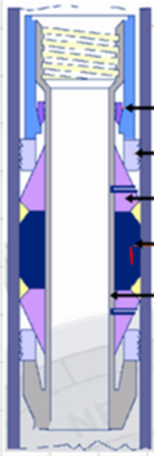
$\Delta p_3 > \Delta p_1$ to compensate



ICD: inflow control device



As the packer sets, the inner mandrel moves up, driving the cone underneath the slips, pushing them into the casing wall. The sealing element is compressed & extruded to the casing wall.

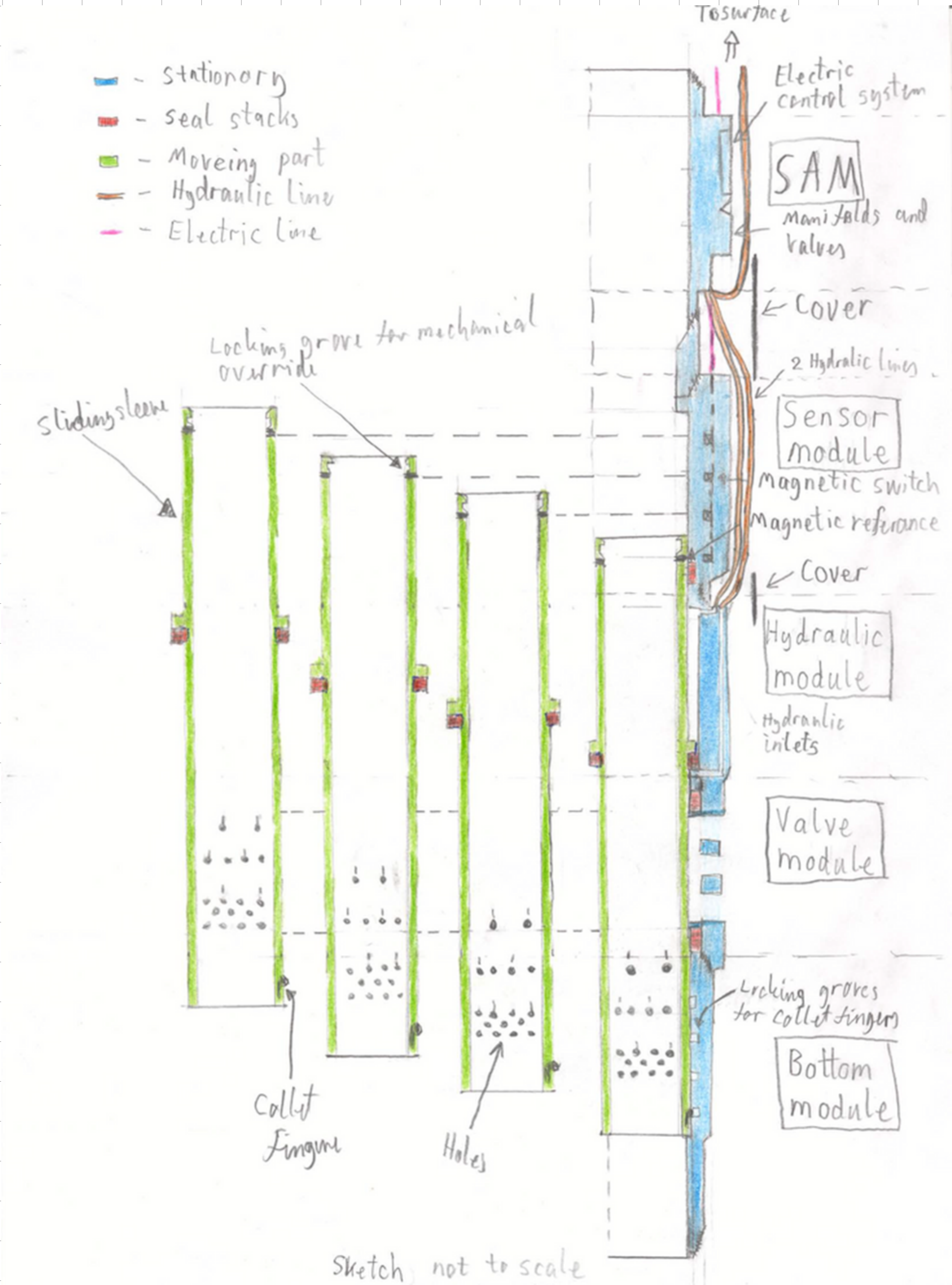


Ability to effectively set a packer depends on having a clean, non corroded set point and reaching the set point without fouling the slips or failing other components.

Hydro-Set, Hydro-Release

inflow control technologies

ICV : inflow control valve



AICV autonomous inflow control valve device

WC ↑ GOR ↑

U. S. DEPARTMENT OF THE INTERIOR
HAROLD L. ICKES, Secretary

BUREAU OF MINES
John W. Finch, Director

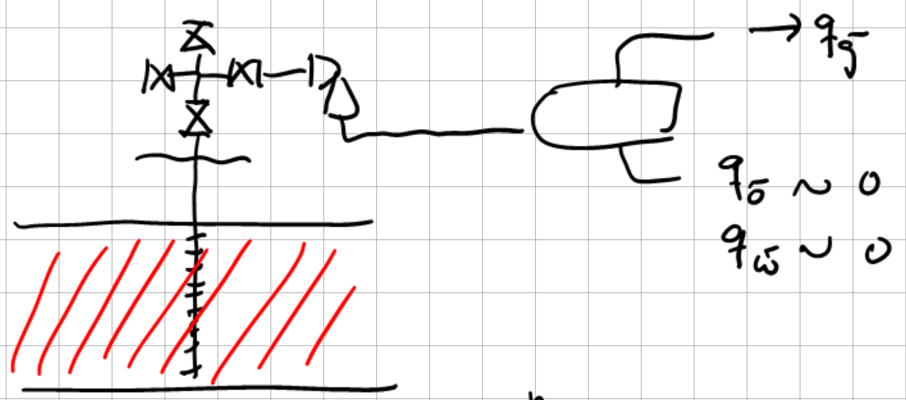
(1935)

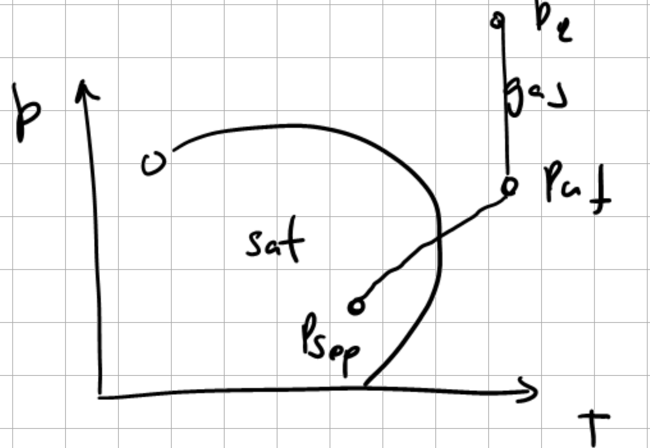
Monograph 7

Back-Pressure Data on Natural-Gas Wells and Their Application to Production Practices

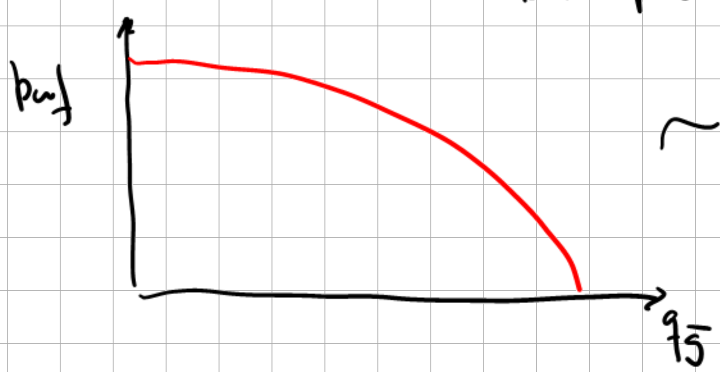
By

E. L. RAWLINS AND M. A. SCHELLHARDT





Back-pressure equation



empirically, measured in the field

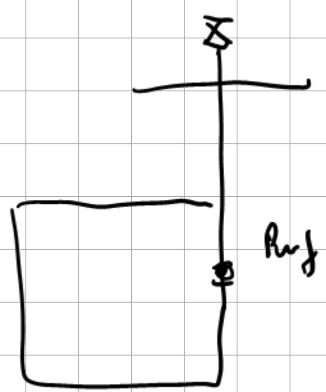
$$\sim \quad q_g = C\left(p_R^2 - p_{wf}^2\right)^n \rightarrow 0.5 \leq n \leq 1$$

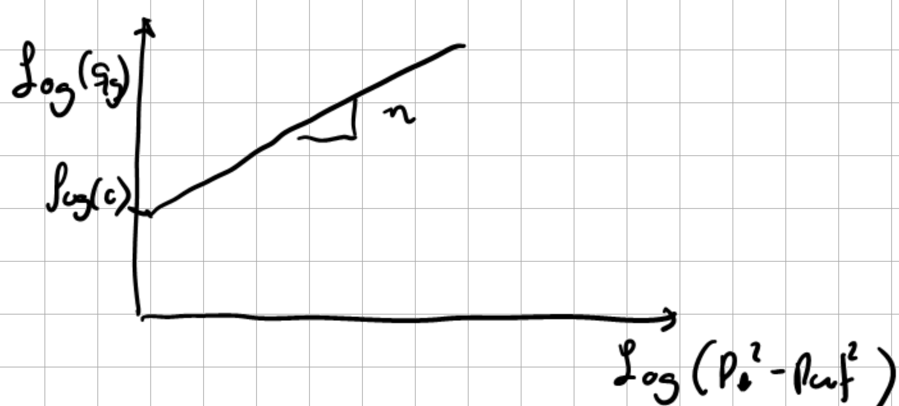
$$\frac{q_g}{C} = \left(p_R^2 - p_{wf}^2\right)^n \qquad \log$$

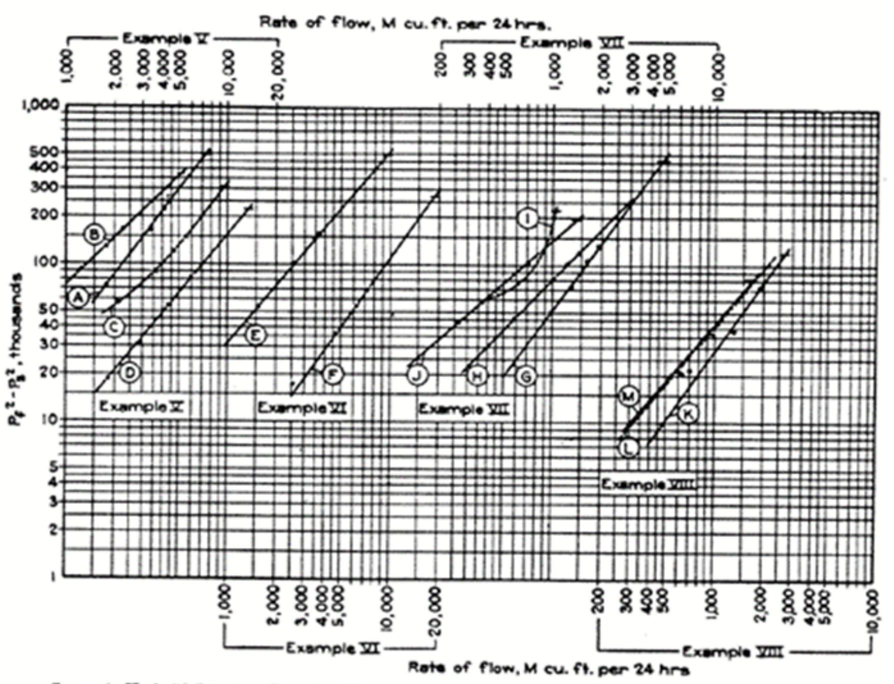
$$\log\left(\frac{q_g}{C}\right) = \log\left(\left(p_R^2 - p_{wf}^2\right)^n\right)$$

$$\log(q_g) - \log(C) = n \cdot \log\left(p_R^2 - p_{wf}^2\right)$$

$$\log(q_g) = \log(C) + n \log\left(p_R^2 - p_{wf}^2\right)$$



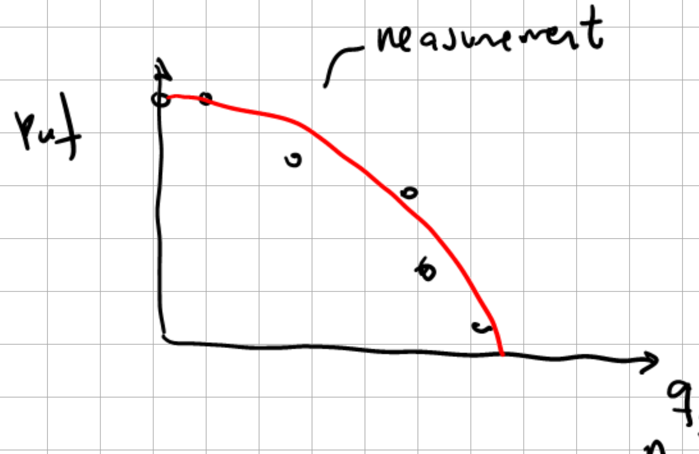




Example V, A, Well tested Dec. 13, 1929, P_f = 706
B, " " Mar. 21, 1930, " = 610
C, " " Dec. 4, 1930, " = 558
D, " " Dec. 30, 1931, " = 524
Example VI, E, " " Dec. --, 1929, " = 708
F, " " Dec. --, 1931, " = 524
Example VII, G, Well tested Dec. 14, 1929, P_f = 687
H, " " Mar 22, 1930, " = 504
I, " " Dec. 3, 1930, " = 477
J, " " Dec. 28, 1931, " = 445
Example VIII, K, " " Aug. 12, 1929, " = 349
L, " " Aug. 30, 1930, " = 320
M, " " Aug. 4, 1932, " = 290

P_f = Shut-in formation pressure, lb. per sq. in. absolute

FIGURE 24.—Variation in delivery capacities of gas wells at different times in their productive lives, examples V, VI, VII, and VIII



$$q_g = C\left(p_R^2 - p_{wf}^2\right)^n$$

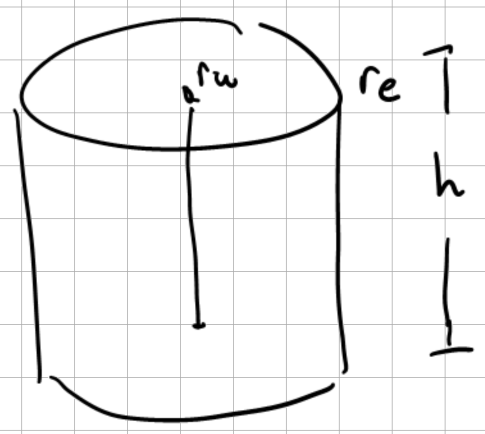
in Excel change C, n to match data

analytical derivation of Dry gas IPR

$q_{\bar{g}}$

$$v = \frac{k}{\mu_g}\frac{dp}{dr}$$

$$\frac{q_g}{2\pi r h} = \frac{k}{\mu_g}\frac{dp}{dr}$$

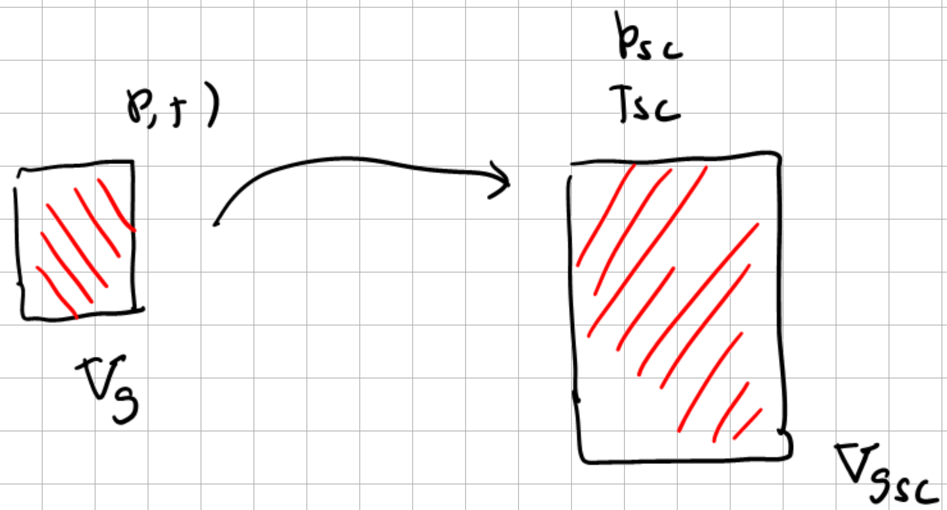


$$q_g = f(q_{\bar{g}})$$

$$B_g = \frac{q_g}{q_{\bar{g}}}$$

$$q_g = q_{\bar{g}} \cdot B_g$$

$$\int_{r_w}^{r_{@pR}} \frac{q_{\bar{g}}}{r} = \int_{p_{wf}}^{p_R} \frac{2\pi k h}{B_g \mu_g}\frac{dp}{dr}$$



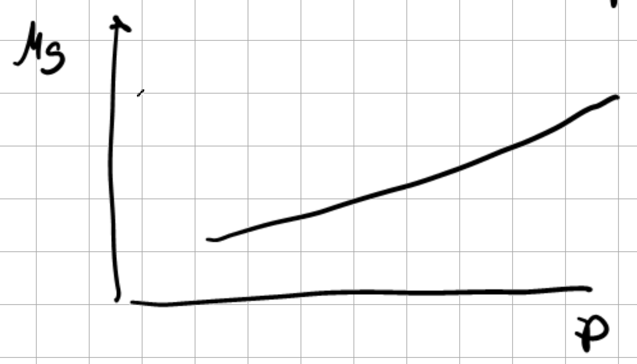
$$B_g = \frac{V_g}{V_{\bar{g}}}$$

gas volume factor

$$B_g = f(p,T)$$

$B_g \sim$ 1E-2 → 1E-3

for PSS
$$q_{\bar{g}} = \frac{2\pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)} \int_{p_{wf}}^{p_R} \frac{1}{B_g \mu_g} dp \rightarrow ?$$



mass conservation

$$q_{\bar{g}} \cdot \rho_{\bar{g}} = q_g \rho_g$$

$$B_g = \frac{q_g}{q_{\bar{g}}} = \frac{\rho_{\bar{g}}}{\rho_g}$$

Real gas equation

$$\frac{p}{\rho} = zRT \qquad R = \frac{R_u}{MW}$$

gas deviation factor

deviation from ideal gas

$$\rho = \frac{p}{zRT}$$

Boyle (English)

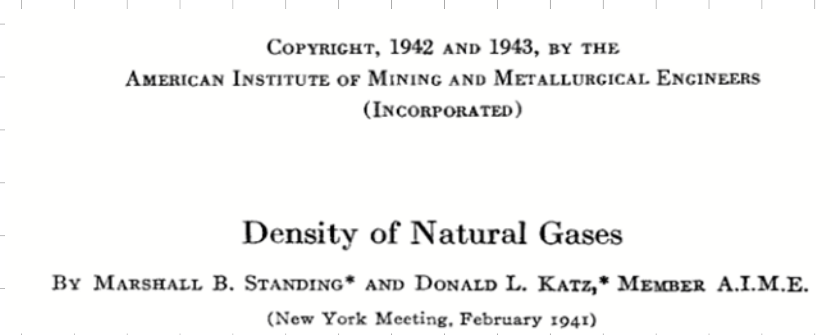
Hooke (English)

Charles (French)

Gay-Lussac (French)

Avogadro (Italian)

$z \sim$



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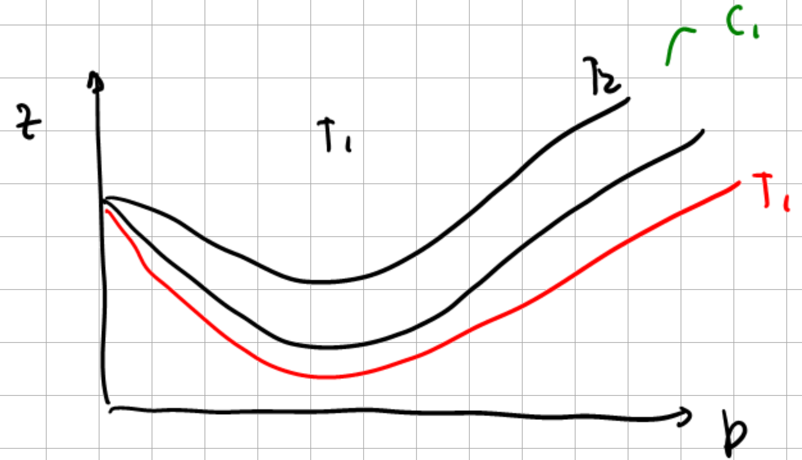
(INCORPORATED)

Density of Natural Gases

BY MARSHALL B. STANDING* AND DONALD L. KATZ,* MEMBER A.I.M.E.

(New York Meeting, February 1941)



$$T_r = \frac{T}{T_c} \qquad p_r = \frac{p}{p_c}$$

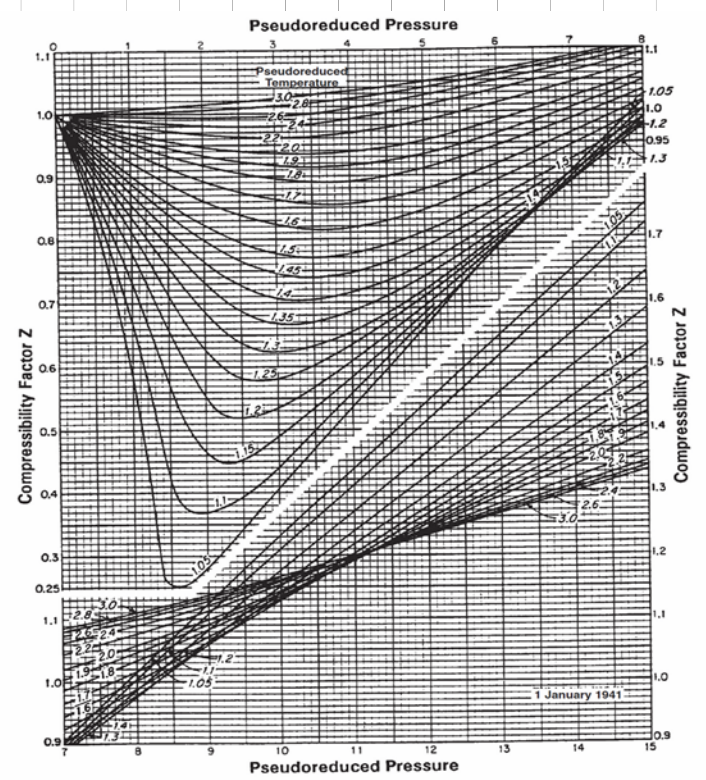
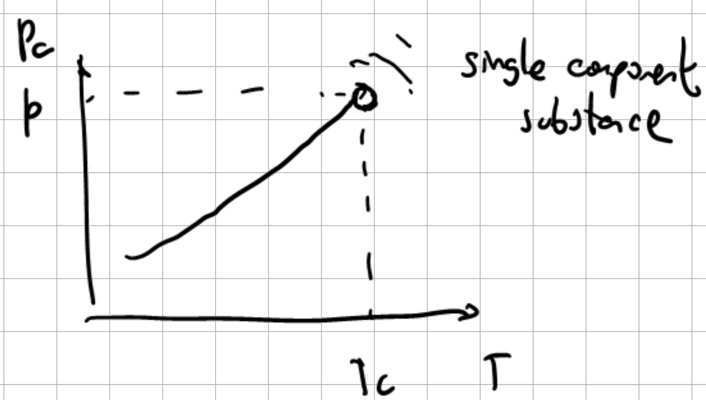


Fig. 3.6—Standing-Katz[4] Z-factor chart.

T_c, p_c

$T_c, p_c = f(MW)$



Gas composition

Component	mole fraction
C_1	z_{c_1}
C_2	z_{c_2}
C_3	z_{c_3}
⋮	

$$\sum_{1}^{N} z_i = 1$$

$$MW_{mix} = \sum_{1}^{N} Mw_i z_i \qquad T_c, p_c = f(MW_{mix}) \rightsquigarrow \text{Sutton}$$

$$B_g = \frac{\rho_{\bar g}}{\rho_g} \qquad \rho_{\bar g} = \frac{p_{sc}}{\underbrace{\cancel{z_{sc}}}_{1} R\, T_{sc}} \qquad \rho_g = \frac{p}{z\, R\, T}$$

$$B_g = \frac{p_{sc}}{T_{sc}} \cdot \frac{z\, T}{p}$$

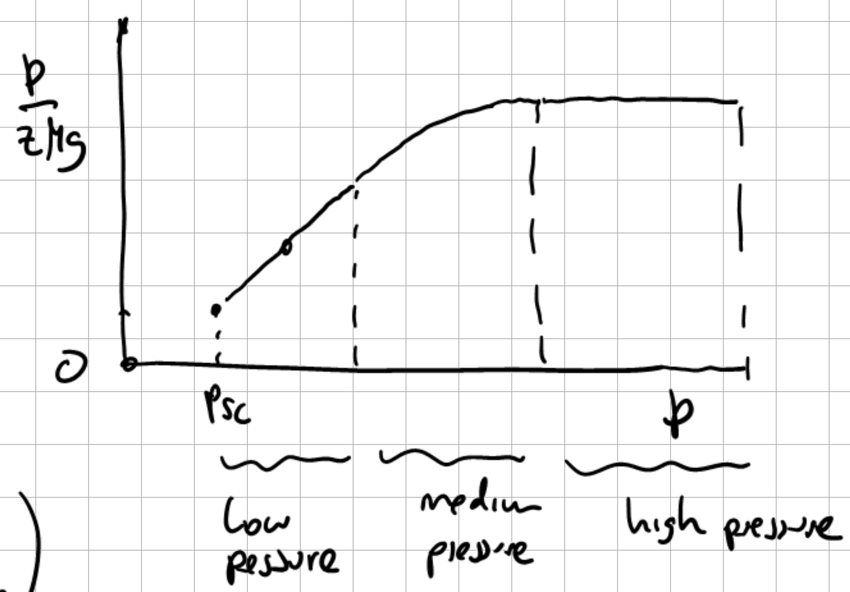
p_{sc} 1.01325 bara

T_{sc} 15.56 °C

$T = T_R$

$$q_{\bar g} = \frac{2\pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)} \int_{p_{wf}}^{p_R} \frac{1}{\mu_g B_g}\, dp$$

$$q_{\bar g} = \frac{2\pi k h}{\ln\left(\frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \underbrace{\int_{p_{wf}}^{p_e} \frac{p}{z \mu_g}\, dp}$$



low pressure (p_{wf}, p_R)

$$q_g = \frac{2\pi k h}{\left(\ln \frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R}\, 0.5 \left[p_R^2 - p_{wf}^2\right] \left(\frac{1}{z \mu_g}\right)_{@ p_R}$$

in the low pressure region $\left(\frac{1}{\mu_g z}\right) = \text{const}$

$$q_{\bar g} = C\left(p_R^2 - p_{wf}^2\right)^n \qquad \text{if } n = 1 \text{ it's a match!}$$

high pressure

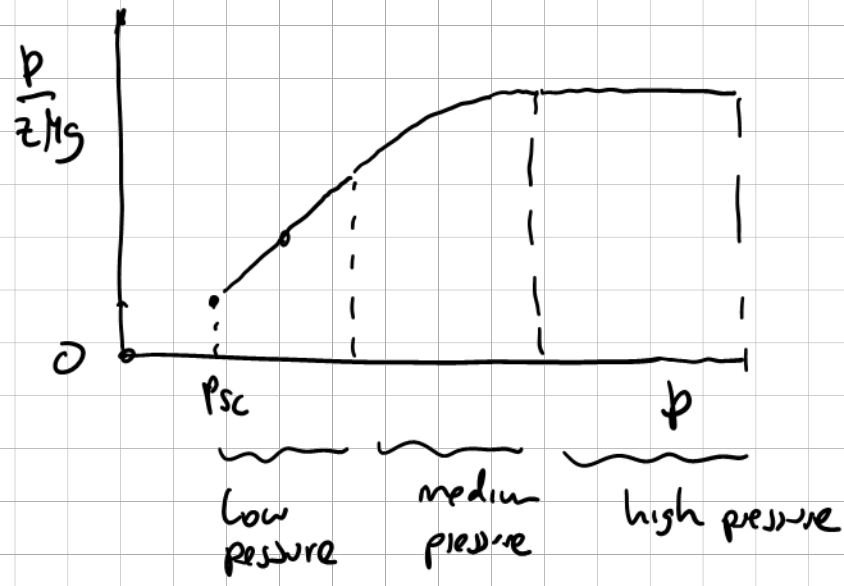
$$q_{\bar g} = \frac{2\pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \underbrace{\left(\frac{p}{\mu B_g}\right)_{@}}_{\text{avg pressure}} \int_{p_{wf}}^{p_R} dp = \underbrace{\frac{2\pi k h}{\left(\ln \frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \frac{p_R}{(\mu_g z)_{@ p_R}}}_{\text{constant}} (p_R - p_{wf})$$

similar to undersaturated oil IPR

$$q_{\bar o} = J\,(p_R - p_{wf})$$

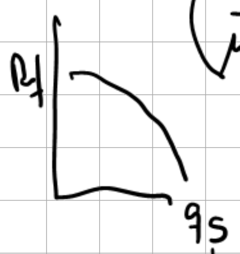
doesn't resemble $q_{\bar g} = C\left(p_R^2 - p_{wf}^2\right)^n$

$$q_g = \frac{2\pi k h}{\ln\left(\frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \underbrace{\int_{p_{wf}}^{p_R} \frac{p}{z \mu_g} dp}$$



LP

$$\left(\frac{1}{\mu_g z}\right)_{@p_R} \frac{(p_R^2 - p_{wf}^2)}{2}$$



?

HP

$$\frac{p_R}{(\mu_g z)_{@p_R}} (p_R - p_{wf})$$

p [bara] $\quad q_g = [Sm^3/d] \quad p_{sc} = 1.01325$ bara

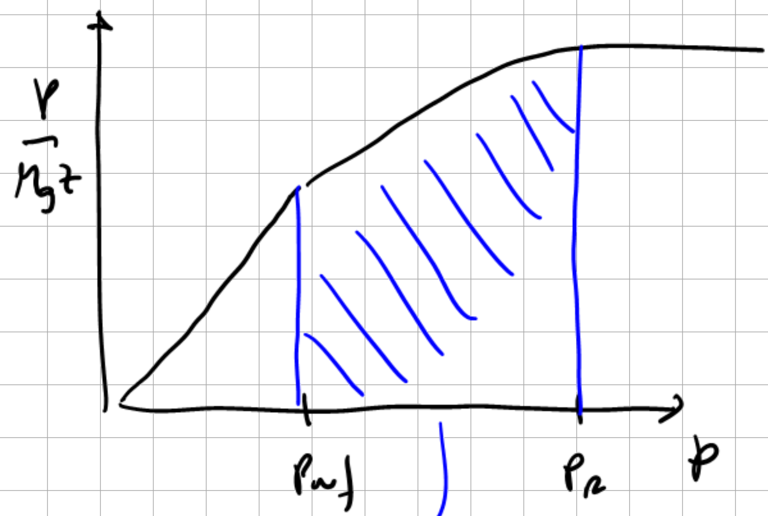
T [K] $\quad \mu_g = [cp] \quad T_{sc} = 15.56$ °C

k [md]

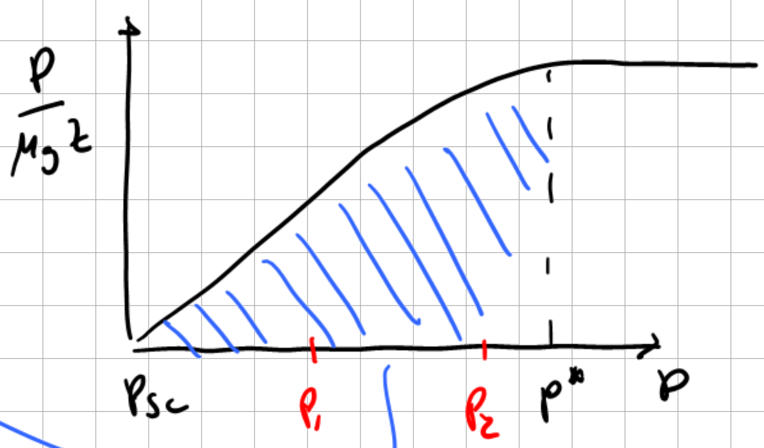
excluding the 2 !

$$q_g = \frac{k h \, 7.63}{\left(\ln\frac{r_e}{r_w} - 0.75 + S\right) T_R} \underbrace{2 \int_{p_{wf}}^{p_R} \frac{p}{\mu_g z} dp}$$

$$(\underbrace{\bigcirc}_{p_R} - \underbrace{\bigcirc}_{p_{wf}})$$

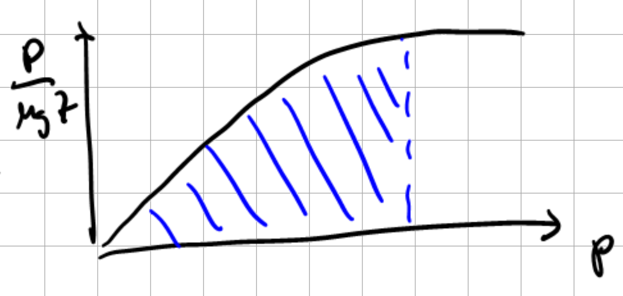


$$m(p^*) = 2 \int_{0 \to p_{sc}}^{p^*} \frac{p}{\mu_g z} dp$$



$$2 \int_{p_1}^{p_2} \frac{p}{\mu_g z} dp = 2 \int_{p_{sc}}^{p_2} \frac{p}{\mu_g z} dp - 2 \int_{p_{sc}}^{p_1} \frac{p}{\mu_g z} dp = m(p_2) - m(p_1)$$

=



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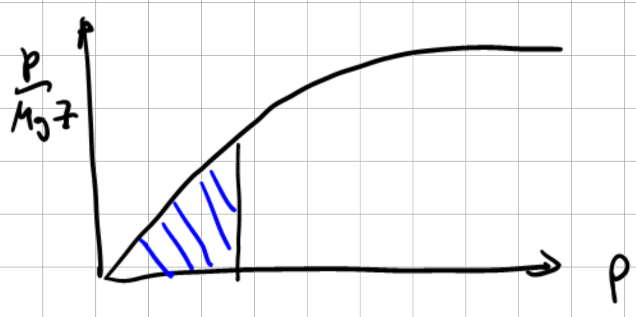
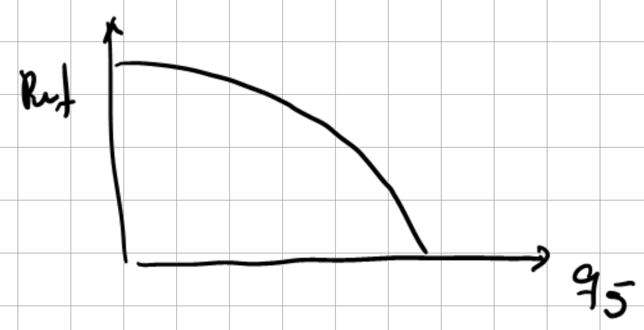


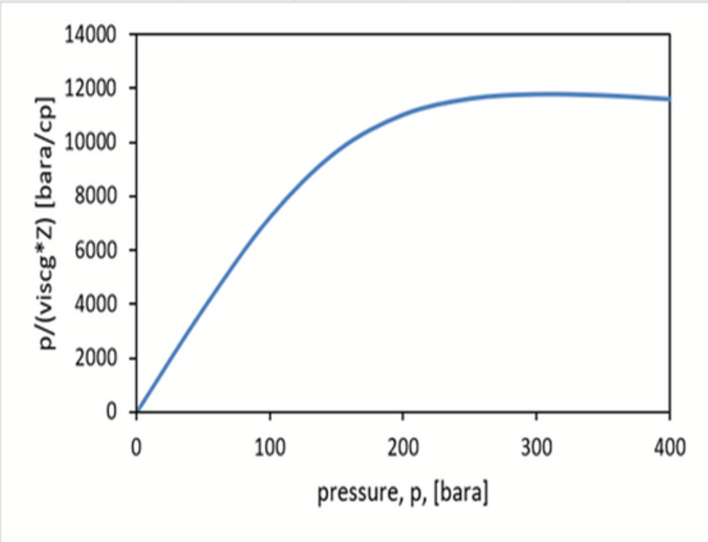
IM og

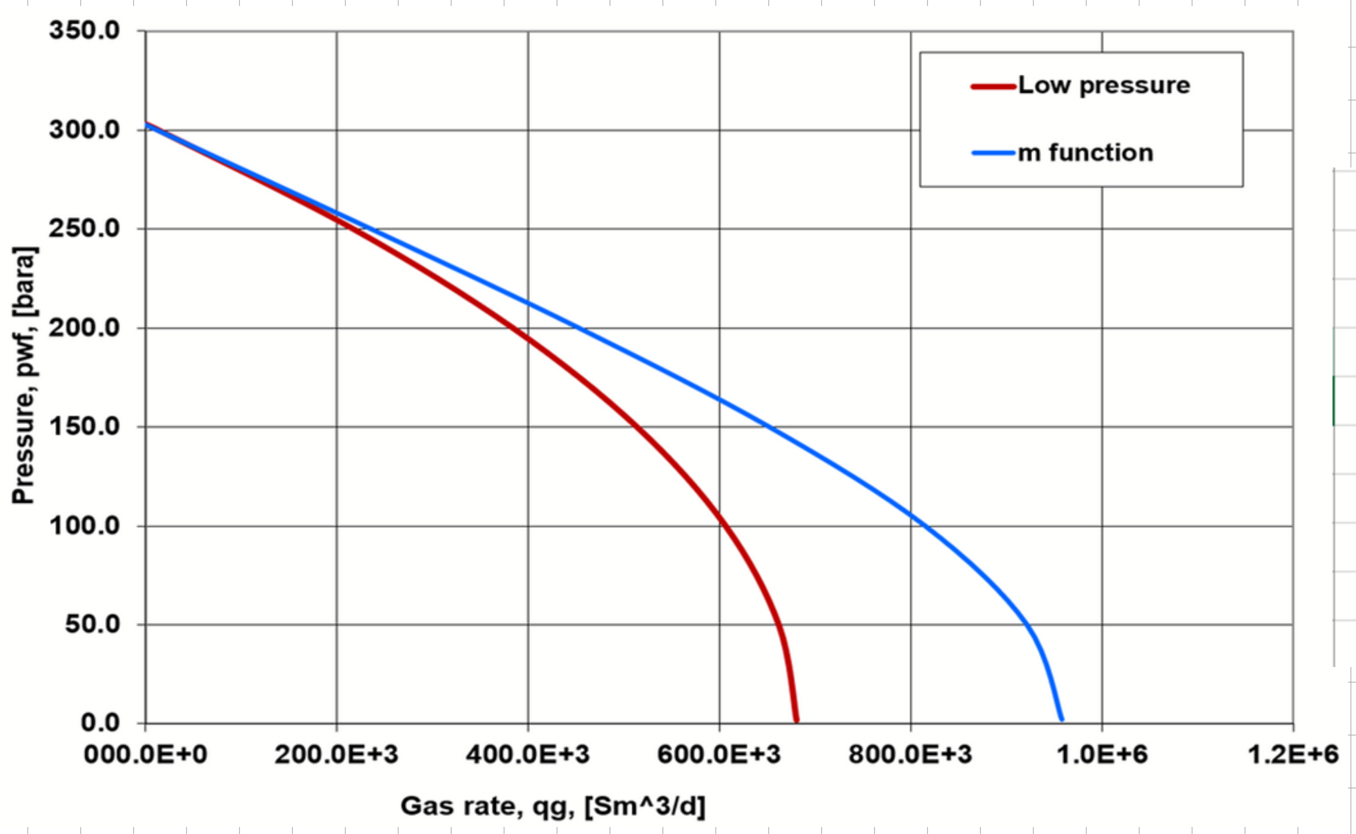
$$q_s = \frac{7.63\, kh}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right)} \frac{1}{T_R} \left[m(p_R) - m(p_{wf})\right] \quad ?$$



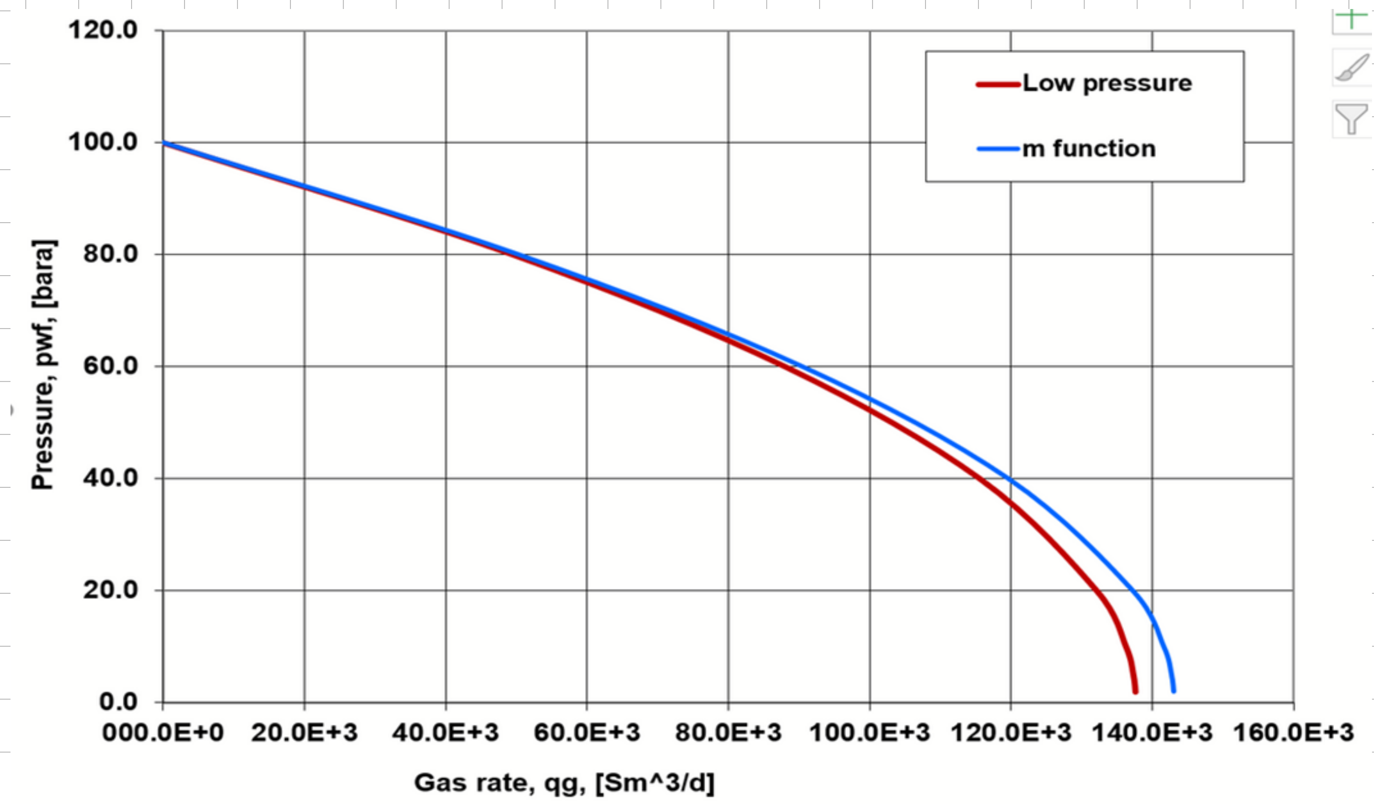
Well Data		
p_R	[bara]	400
T_R	[C]	105
T_R	[K]	378
z_R	[-]	1.069
$deng_R$	[kg/m3]	276
$Viscosity_R$	[cp]	0.032

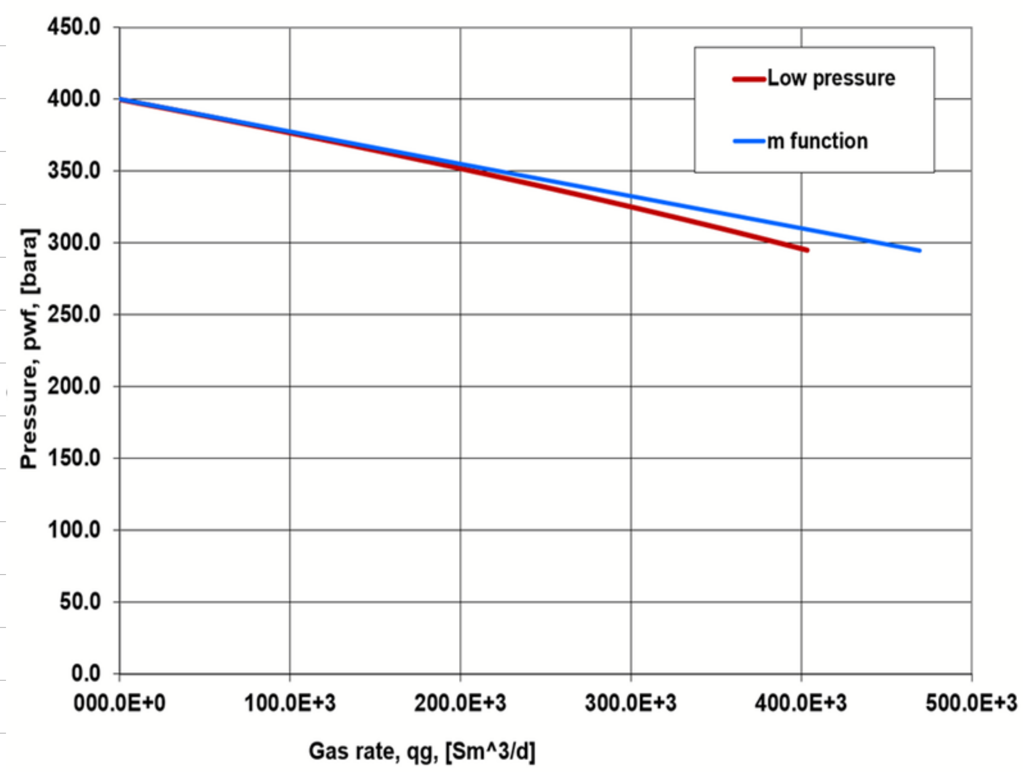
p [bara]	Z [-]	deng [kg/m3]	viscg [cp]	p/viscg*Z [bara/cp]
2	0.997	1.5	0.013	152
50	0.932	39.6	0.014	3825
100	0.882	83.6	0.016	7224
150	0.859	128.8	0.018	9662
200	0.867	170.2	0.021	11029
250	0.899	205.1	0.024	11622
300	0.948	233.6	0.027	11788
350	1.006	256.8	0.030	11745
400	1.069	276.1	0.032	11607





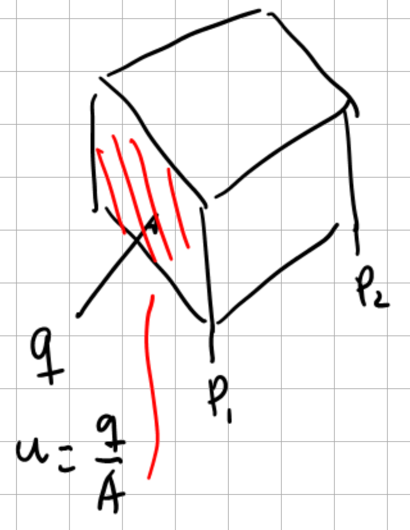
pwf bara	LP - qg [Sm3/d]	m(p) [bara2/cp]	qg [Sm3/d]
303.0	000.0E+0	5026555	000.0E+0
300.0	13.4E+3	4955852	13.5E+3
250.0	217.4E+3	3783006	237.1E+3
200.0	384.3E+3	2645766	453.9E+3
150.0	514.1E+3	1603483	652.6E+3
100.0	606.8E+3	750299.5	815.2E+3
50.0	662.5E+3	191885.9	921.7E+3
2.0	681.0E+3	226.2864	958.2E+3

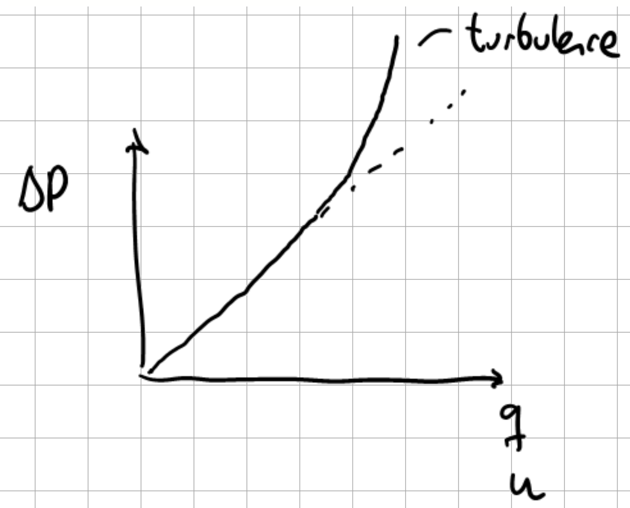


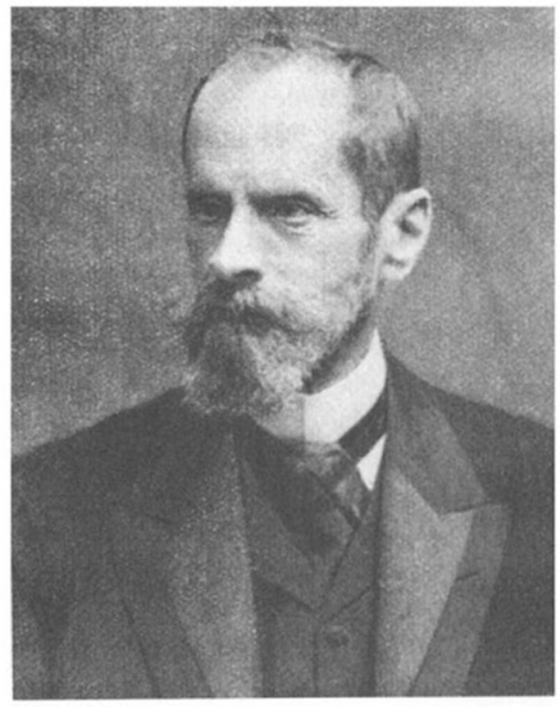


Low pressure
m function
Pressure, pwf, [bara]
Gas rate, qg, [Sm^3/d]
450.0
400.0
350.0
300.0
250.0
200.0
150.0
100.0
50.0
0.0
000.0E+0
100.0E+3
200.0E+3
300.0E+3
400.0E+3
500.0E+3

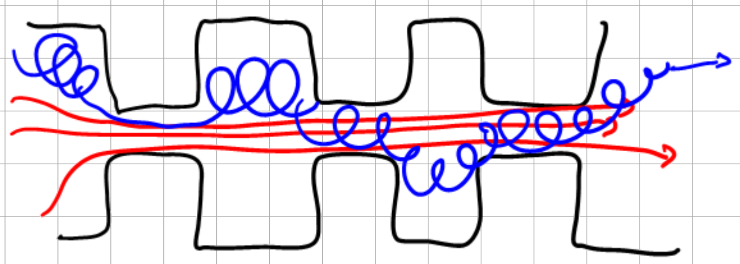
[19] P. Forchheimer. *Wasserbewegung durch Boden.* Zeitschrift des Vereines Deutscher Ingenieuer, 45 edition, 1901.






Professor Philipp Forchheimer.

not valid for medium-high rate gas wells, high rate sat oil



$$\Delta P = A \cdot u + B u^2$$

$$\Delta P = A u \left(1 + \frac{B}{A} u\right)$$

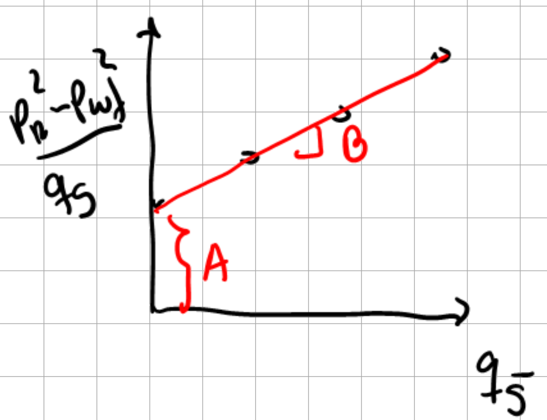
Use of Short Term Multiple Rate Flow Tests To Predict Performance of Wells Having Turbulence (1976)

Lloyd G. Jones and E. M. Blount, Mobil Research and Development Corp., and O. H. Glaze, Mobil Oil Corp., Members SPE-AIME

LP dry gas wells

$$\left(P_R^2 - P_{wf}^2\right) = A\, q_{\bar{g}} + B\, q_{\bar{g}}^2$$

$$\frac{\left(P_R^2 - P_{wf}^2\right)}{q_{\bar{g}}} = A + B\, q_{\bar{g}}$$



quadratic equation

$$a x^2 + b x + c = 0$$

$$q_{\bar{g}} = \frac{7.63\, k h}{T_R\, M_g\, z} \frac{\left(P_R^2 - P_{wf}^2\right)}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right)}$$

$$\left(P_R^2 - P_{wf}^2\right) = \frac{T_R\, M_g\, z}{7.63\, k h}\left[\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right) q_{\bar{g}}\right] + B\, q_{\bar{g}}^2$$

rate dependent skin

$$B = D\, \frac{z\, M_g\, T_R}{7.63 \cdot k\, h}$$

$$\left(P_R^2 - P_{wf}^2\right) \frac{7.63\, k h}{T_R\, M_g\, z} = \left(\ln \frac{r_e}{r_w} - 0.75 + S\right) q_{\bar{g}} + D\, q_{\bar{g}}^2$$

$$\frac{(p_R^2 - p_{wf}^2)}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + \underbrace{D\, q_{\bar{g}}}_{\text{rate-dependent skin}}\right)} \frac{7.63\, kh}{T_R\, \mu_g\, z} = q_{\bar{g}}$$

$$D = \left[\frac{d}{Sm^3}\right] \rightarrow D = f(?)$$

alternative approach

USBM (1935)

$$q_{\bar{g}} = C\left(p_R^2 - p_{wf}^2\right)^n \qquad n? \begin{cases} 1 & \text{laminar (Darcy flow)} \\ 0.5 & \text{turbulent (HVF)} \end{cases}$$

$$q_{\bar{g}} = \underline{C\ \left(p_R^2 - p_{wf}^2\right)^{n}}$$ — accounts for turbulent flow

$$C = \frac{(7.63\ kh)^n}{(T\mu_g Z)^n D^{1-n}[\ln(r_e/r_w) - 0.75 + s]^{2n-1}}.$$

extends to other IPRs

$$q_{\bar{g}} = C\left[m(p_R) - m(p_{wf})\right]^n$$

$$C = \frac{(7.63\ \ kh)^n}{T^n D^{1-n}[\ln(r_e/r_w) - 0.75 + s]^{2n-1}}.$$

$$q_g = \frac{2\pi k h}{\ln\left(\frac{r_e}{r_w} - 0.75\right)} \; \frac{7.63}{T_R} \underbrace{\int_{P_{wf}}^{P_e} \frac{p}{z\,\mu_g}\,dp}$$

LP

$$\left(\frac{1}{\mu_g z}\right)_{@P_R} \frac{\left(P_R^2 - P_{wf}^2\right)}{2}$$

$$\frac{\left(m(P_e) - m(P_{wf})\right)}{2}$$

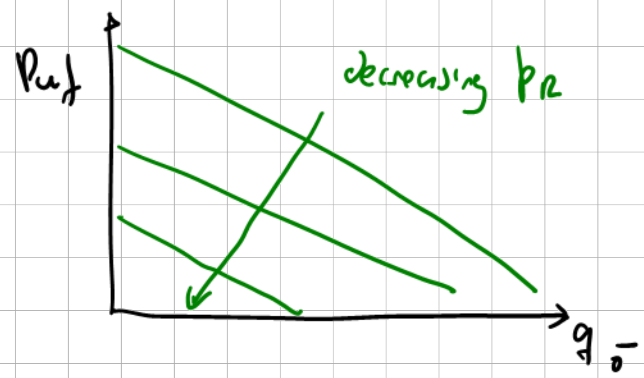
HP

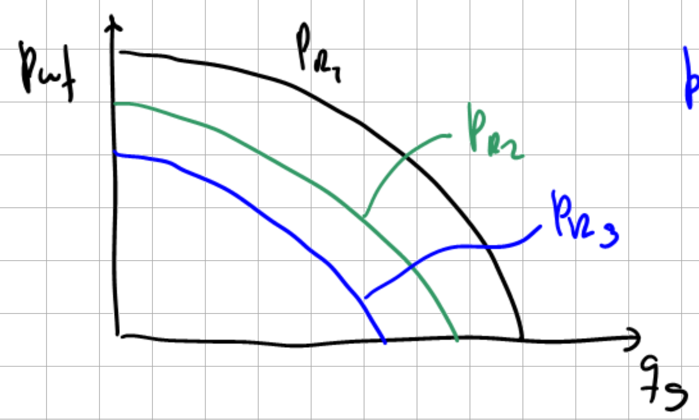
$$\frac{P_R}{(\mu_g z)_{@P_R}} \left(P_R - P_{wf}\right)$$

$$q_o = \underset{\sim}{J}\left(P_R - P_{wf}\right)$$

geometry $\rightarrow$

fluid properties $\rightarrow$ $(\mu_o B_o)_{@P_{av}}$



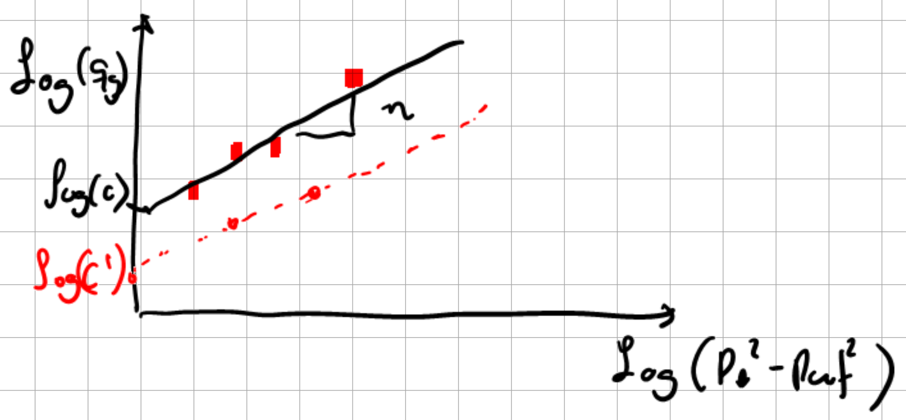


$P_{R_1} > P_{R_2} > P_{R_3}$

Diagnostic of well productivity issues

$$q_g = C\left(P_R^2 - P_{wf}^2\right)^n$$

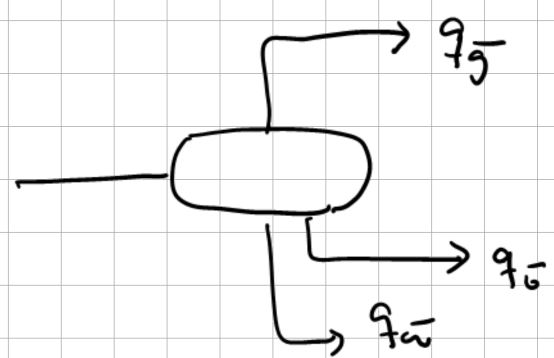
$$\log(q_g) = \log(C) + n \log\left(P_R^2 - P_{wf}^2\right)$$



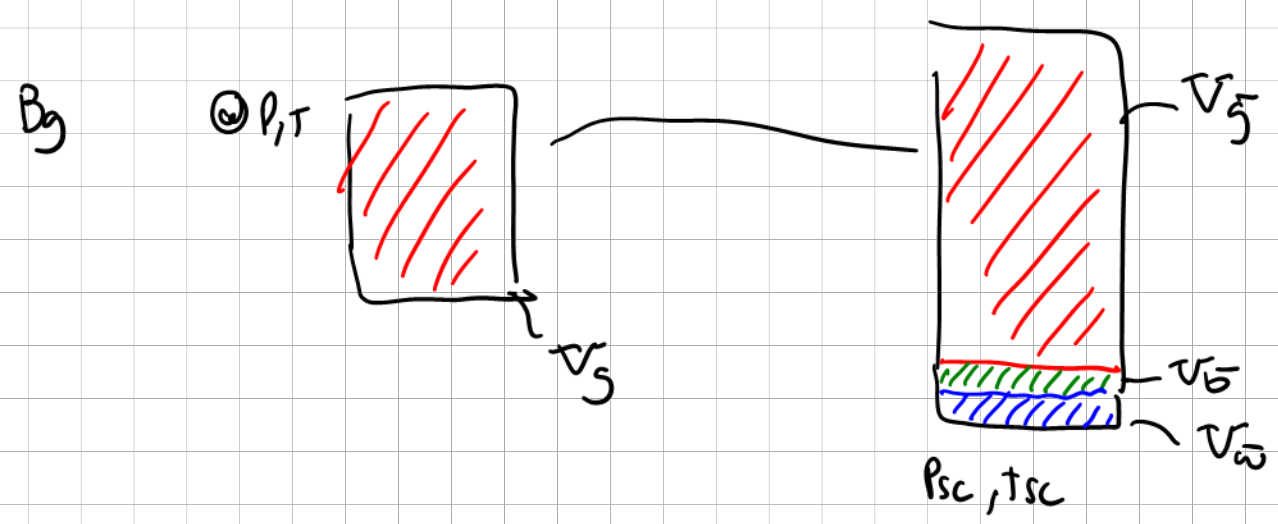
- Gas wells produce condensate q_o, water q_w

$$CGR = \frac{q_o}{q_g}$$

$$WGR = \frac{q_w}{q_g}$$



- the same IPR equations can be used as long as flow $P_R \rightarrow P_{wf}$ is single phase



$$B_{gd} = \frac{V_g}{V_{\bar{g}}}$$

$$B_{gw} = \frac{V_g}{V_{\bar{g}} + V_{\bar{o}} + V_{\bar{w}}}$$

$\frac{P}{\rho} = zRT$ → assumes single-phase gas

$$\int_{P_{wf}}^{P_R} \frac{P}{z\,\mu_g}\,dp$$ → neglects the dropout of condensate / water

If there is dropout of condensate and water → generate a table of B_{gd} {e.g. EOS}

$$\int_{P_{wf}}^{P_R} \frac{1}{\mu_g B_{gd}}\,dp$$ ← integrate B_{gd} directly

---

Expressions are available for horizontal wells:

SPE 99712

Generalized Horizontal Well Inflow Relationships for Liquid, Gas, or Two-Phase Flow

R. Kamkom and D. Zhu, Texas A&M U.

base pressure. The IPR equation for horizontal gas wells in term of the real gas pseudo-pressure is

$$q_g = \frac{kL\left(m(\bar{p}) - m(p_{wf})\right)}{1424T\left(\ln\left[\frac{hI_{ani}}{r_w(I_{ani}+1)}\right] + \frac{\pi y_b}{hI_{ani}} - 1.224 + s\right)} \qquad (13)$$

$$I_{ani} = \sqrt{\frac{k_H}{k_V}}$$

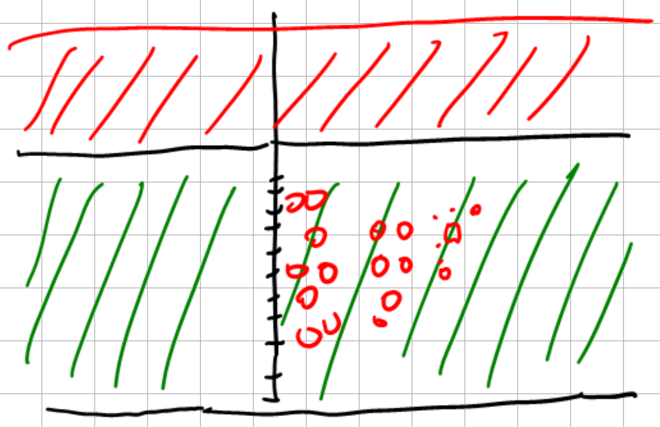
IPR:

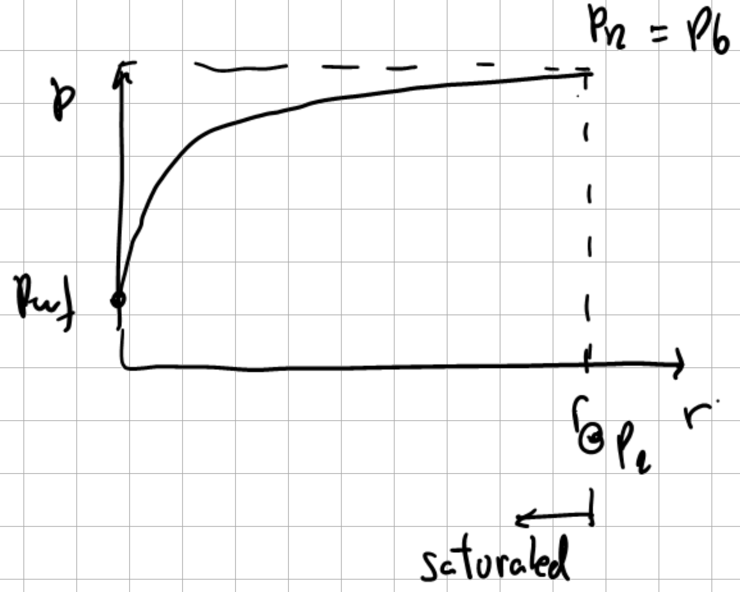
$$q_g = \underbrace{C}_{\text{geometry}} \underbrace{\int_{P_{wf}}^{P_R} F(p)\,dp}_{\text{function of fluid properties}}$$

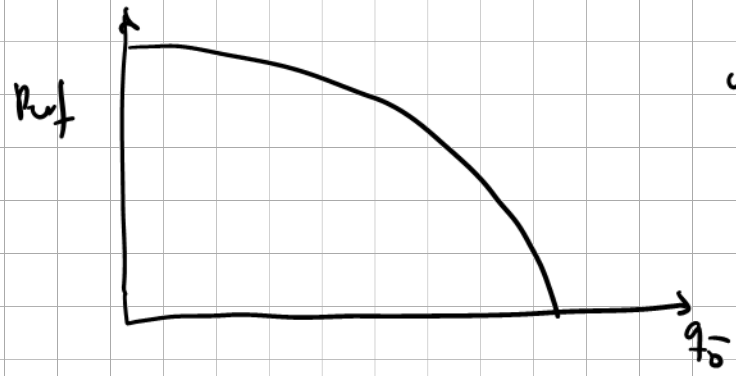
Saturated oil

$p_R = p_b @ T_R$

solution-gas drive







what equation?

$\sim q_{\bar{o}} = C(p_R^2 - p_{wf}^2)$

$q_{\bar{o}} = J(p_R - p_{wf})$

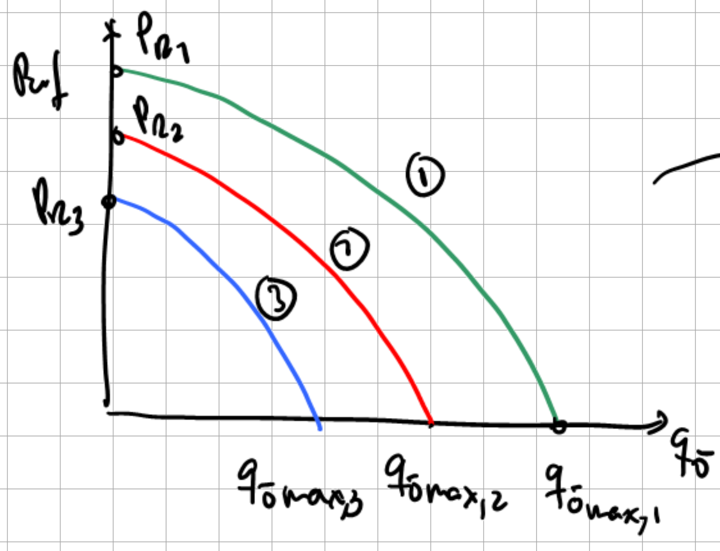
?

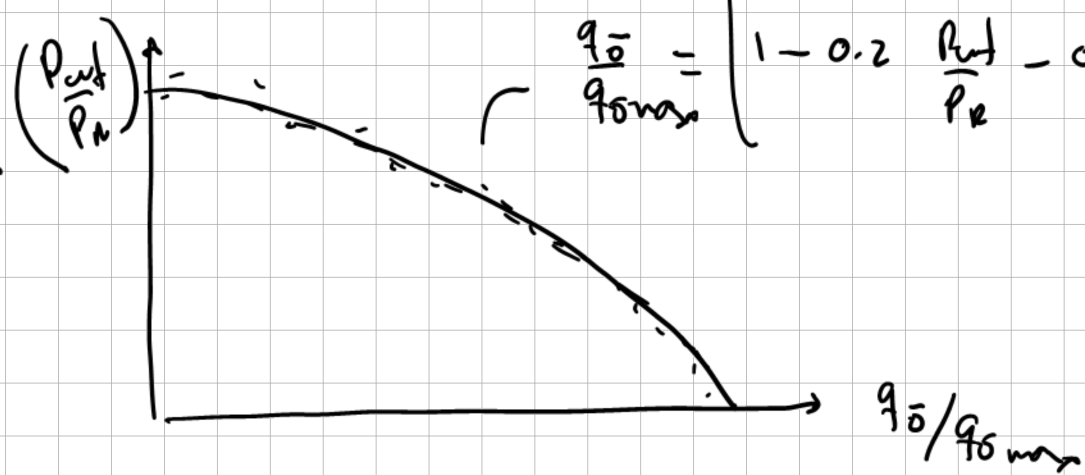
Reservoir simulation → 1968 Vogel

Inflow Performance Relationships for Solution-Gas Drive Wells

J. V. VOGEL
MEMBER AIME

SHELL OIL CO.
BAKERSFIELD, CALIF.





$$\frac{q_{\bar{o}}}{q_{\bar{o}max}} = \left[1 - 0.2\frac{p_{wf}}{p_R} - 0.8\left(\frac{p_{wf}}{p_R}\right)^2\right]$$

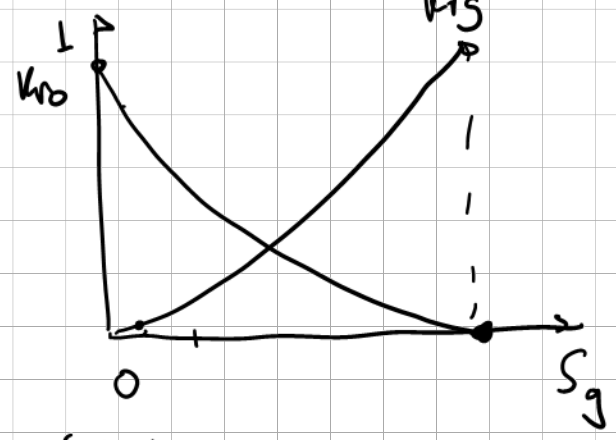
$q_{\bar{o}}$	p_{wf}
☐	☐
☐	☐
☐	☐

→

$\frac{q_{\bar{o}}}{q_{\bar{o}max}}$	$\frac{p_{wf}}{p_R}$

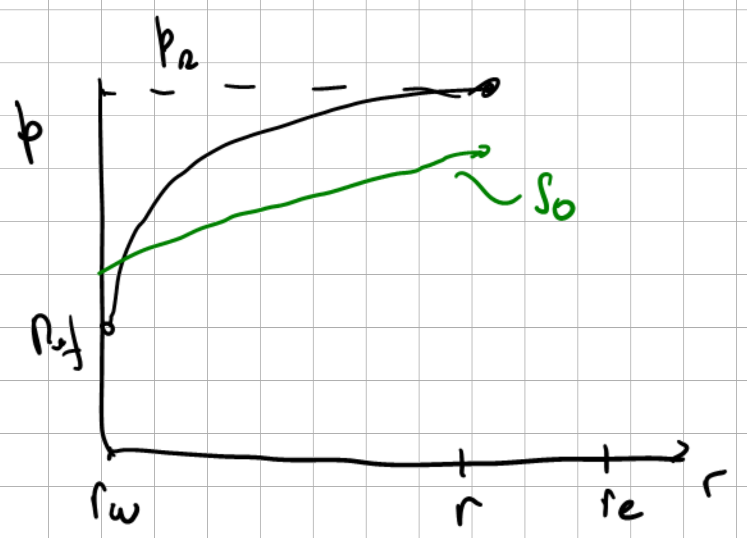
Analytical derivation

$$q_o = \frac{h\ k}{1868\left(\ln\frac{r_e}{r_w} - 0.75 + s\right)} \int_{p_{wf}}^{p_R} \frac{k_{ro}}{\mu_o B_o}\,dp$$



effective permeability $(k \cdot k_{ro})$

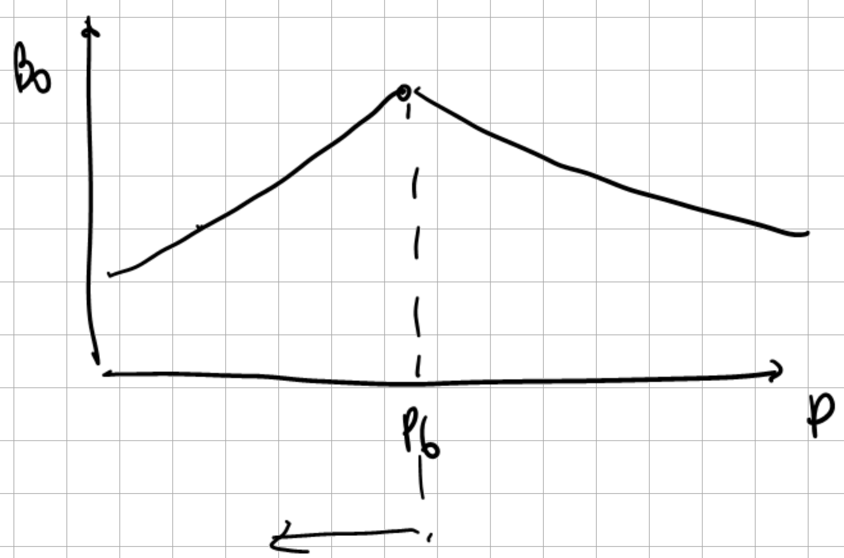
$k_{ro} = f(S_o)$

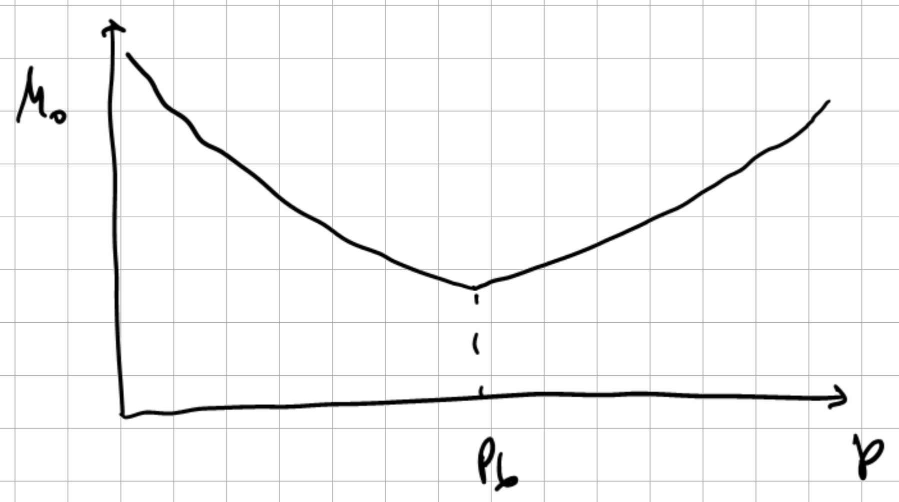


$S_o = f(p) \longrightarrow k_{ro} = f(p)$

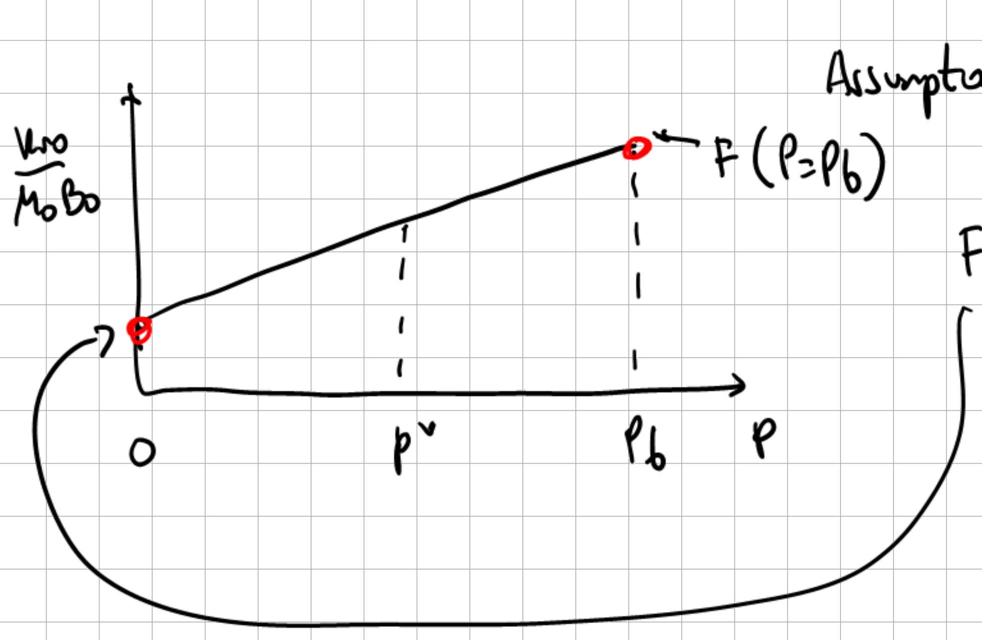
$$\int_{p_{wf}}^{p_R} \frac{k_{ro}}{\mu_o B_o}\,dp$$

$k_{ro} = f(S_o) = f(p)$





Assumption



$F(p=0)$

? $p \geq p_b$ single phase oil

$k_{ro} = 1$

$$\int_{p_{wf}}^{p_R} F(p)\,dp$$

$$F(p) = F(p=0) + \frac{F(p_b) - F(p=0)}{(p_b - 0)}(p^* - 0)$$

$$\int_{p_{wf}}^{p_R} (a + bx)\,dx$$

$$F(p) = F(p=0) + [F(p_R) - F(p=0)] \cdot \frac{p}{p_R} \qquad \text{EQ. 2-16}$$

Therefore, the solution of the pressure function integral will have a linear term in addition to the quadratic term:

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0)\cdot(p_R - p_{wf}) + [F(p_R) - F(p=0)] \cdot \frac{1}{p_R \cdot 2}({p_R}^2 - {p_{wf}}^2) \qquad \text{EQ. 2-17}$$

Expanding terms:

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0)\cdot p_R - F(p=0)\cdot p_{wf} + [F(p_R) - F(p=0)] \cdot \frac{1}{p_R \cdot 2}({p_R}^2 - {p_{wf}}^2) \qquad \text{EQ. 2-18}$$

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0)\cdot p_R - F(p=0)\cdot p_{wf} + F(p_R)\cdot\frac{p_R}{2} - F(p_R)\cdot\frac{{p_{wf}}^2}{p_R \cdot 2} - F(p=0)\cdot\frac{p_R}{2} + F(p=0)\cdot\frac{{p_{wf}}^2}{p_R\cdot 2} \qquad \text{EQ. 2-19}$$

Grouping terms by pressure:

$$\int_{p_{wf}}^{p_R} F(p)dp = [F(p=0) + F(p_R)]\cdot\frac{p_R}{2} - F(p=0)\cdot p_{wf} - \frac{[F(p_R) - F(p=0)]}{2}\cdot\frac{{p_{wf}}^2}{p_R} \qquad \text{EQ. 2-20}$$

Dividing by $[F(p=0) + F(p_R)]\cdot\frac{p_R}{2}$

$$\frac{2}{[F(p=0)+F(p_R)]\cdot p_R}\cdot\int_{p_{wf}}^{p_R} F(p)dp = 1 - \frac{F(p=0)\cdot 2}{[F(p=0)+F(p_R)]}\cdot\frac{p_{wf}}{p_R} - \frac{[F(p_R) - F(p=0)]}{[F(p=0)+F(p_R)]}\cdot\left(\frac{p_{wf}}{p_R}\right)^2 \qquad \text{EQ. 2-21}$$

Defining a variable "V"

$$V = \frac{F(p=0)\cdot 2}{[F(p=0)+F(p_R)]} \qquad \text{EQ. 2-22}$$

Therefore:

$$1 - V = \frac{F(p_R) - F(p=0)}{[F(p=0)+F(p_R)]} \qquad \text{EQ. 2-23}$$

Substituting back in the integral of the pressure function:

$$\frac{2}{[F(p=0)+F(p_R)]\cdot p_R}\cdot\int_{p_{wf}}^{p_R} F(p)dp = 1 - V\cdot\frac{p_{wf}}{p_R} - (1-V)\cdot\left(\frac{p_{wf}}{p_R}\right)^2 \qquad \text{EQ. 2-24}$$

Substituting Eq. 2-24 back in the IPR equation:

$$q_{\ddot{o}} = \frac{k\cdot h\cdot[F(p=0)+F(p_R)]\cdot p_R}{18.68\cdot\left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)\cdot 2}\left[1 - V\cdot\frac{p_{wf}}{p_R} - (1-V)\cdot\left(\frac{p_{wf}}{p_R}\right)^2\right] \qquad \text{EQ. 2-25}$$

Making $q_{\ddot{o},max}$:

$$q_{\ddot{o},max} = \frac{k\cdot h\cdot[F(p=0)+F(p_R)]\cdot p_R}{18.68\cdot\left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)\cdot 2} \qquad \text{EQ. 2-26}$$

The following expression is obtained:

$$q_o = q_{o,max}\left[1 - V\cdot\frac{p_{wf}}{p_R} - (1-V)\cdot\left(\frac{p_{wf}}{p_R}\right)^2\right]$$

Vogel says V = 0.2 →

Vogel found this same equation using data points generated with reservoir simulator, with V = 0.2.

Using Eq. 2-22, and assuming V = 0.2, $F(p=0)$ is then:

$$F(p=0) = \frac{F(p_R)}{9} \qquad \text{EQ. 2-28}$$

Eq. 2-26 can then be further simplified:

$$q_{o,max} = \frac{k\cdot h\cdot\left[\frac{10}{9}\cdot F(p_R)\right]\cdot p_R}{18.68\cdot\left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)\cdot 2} = \frac{k\cdot h\cdot\left[\left(\frac{k_{ro}}{\mu_o\cdot B_o}\right)_{@pR}\right]\cdot p_R}{18.68\cdot\left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)\cdot 1.8} = \frac{J}{1.8}\cdot pR \qquad \text{EQ. 2-29}$$

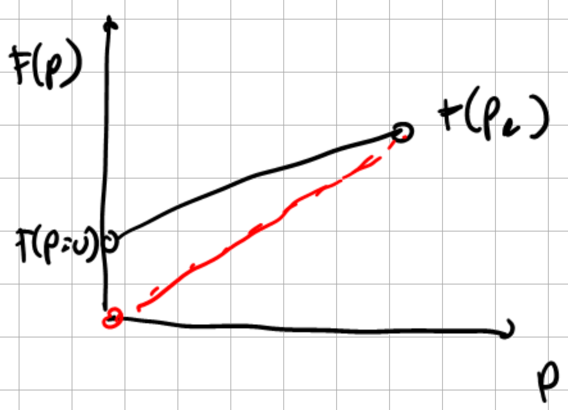
The Isochronal Testing of Oil Wells

By

M. J. Fetkovich, Member AIME, Phillips Petroleum Co.

1973

Fetkovich proposed



$v = 0$

$$\frac{q_o}{q_{o\,max}} = \left[1 - \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

$$q_{o,max} = \frac{k \cdot h \cdot [F(p_R)] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o}\right)_{@p_R}\right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{J}{2} \cdot pR$$

$$q_o = \frac{J}{2} p_R \left(1 - \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

$$q_o = \frac{J}{2} \left[p_R - \frac{(p_{wf})^2}{p_R}\right]$$

$$q_o = \frac{J}{2 p_R} \left[p_R^2 - p_{wf}^2\right] \rightsquigarrow q_g = C\left(p_R^2 - p_{wf}^2\right)^n$$

Comments to gas IPR derivations

LP

$$\int a\,p\,dp \simeq \int x\,dx = \frac{x^2}{2}$$

$$\frac{p^2}{2}$$

HP

$$\int a\,dp \simeq \int dx = x$$

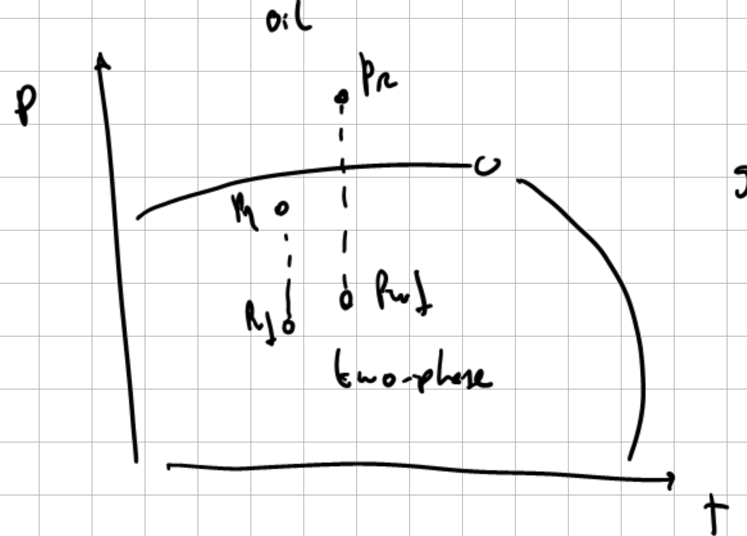
High velocity flow (Forchheimer)

$$a x^2 + b \cdot x + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(\;)\, q_g^2 + (\;)\, q_g + (p_R^2 - p_{wf}^2) = 0$$

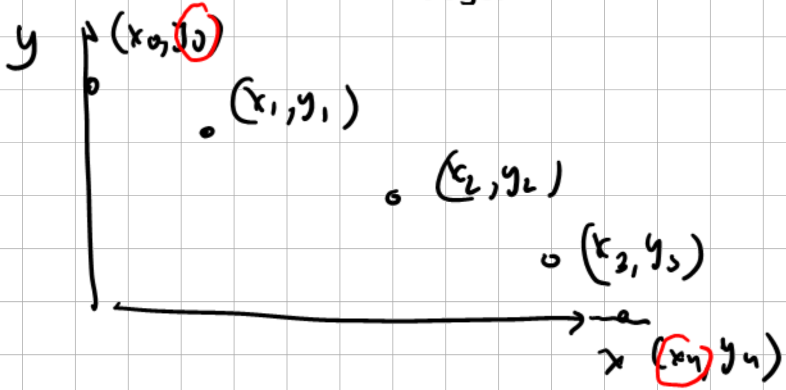
$$7.63 \cdot k\, h$$

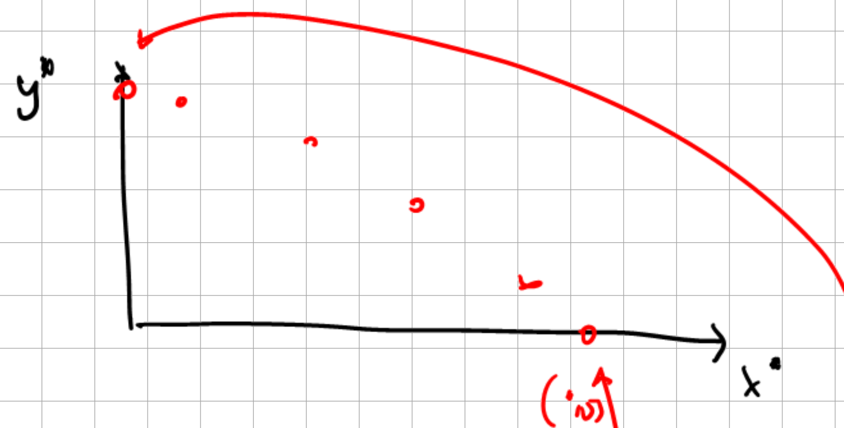
$$(p_R^2 - p_{wf}^2)\,\frac{7.63\, k h}{T_R\, \mu_g\, Z} = \left(\ln \frac{r_e}{r_w} - 0.75 + S\right) q_g + D\, q_g^2$$



Vogel

y vs x





$\max(y) = y_0$

$\max(x) = x_4$

x^*	y^*
$\frac{x_0}{x_4} = 0$	$\frac{y_0}{y_0} = 1$
$\frac{x_1}{x_4}$	$\frac{y_1}{y_0}$
$\frac{x_2}{x_4}$	$\frac{y_2}{y_0}$
$\frac{x_3}{x_4}$	$\frac{y_3}{y_0}$
$1 = \frac{x_4}{x_4}$	$\frac{y_4}{y_0} = 0$

Fetkovich IPR for saturated oil = gas IPR

$$q_g = C\left(p_r^2 - p_{wf}^2\right)^n \quad n = 1 \longrightarrow q_o = C\left(p_r^2 - p_{wf}^2\right)$$

$p_{wf} = 0$: $q_{o\,max} = C p_r^2$

$$q_o = \frac{C}{p_r^2}\left(1 - \left(\frac{p_{wf}}{p_r}\right)^2\right)$$

$$\frac{q_o}{q_{o\,max}} = 1 - \frac{p_{wf}^2}{p_r^2}$$

$$q_o = \frac{C}{p_r^2}\left(1 - V\frac{p_{wf}}{p_r} - (1-V)\left(\frac{p_{wf}}{p_r}\right)^2\right)$$

Quiz 2 - OG621, 2021

1. What are the units of C and n in the backpressure equation?

$$q_g = C\left(p_R^2 - p_{wf}^2\right)^{n} \qquad n, C\ ?$$

n is unitless

$$C = \frac{q_g}{\underbrace{\left(p_R^2 - p_{wf}^2\right)}_{bara^2}{}^{n}} \longrightarrow \frac{\left(Sm^3/d\right)}{\left(bara^2\right)^n} = \left[\frac{Sm^3/d}{bar^{2n}}\right]$$

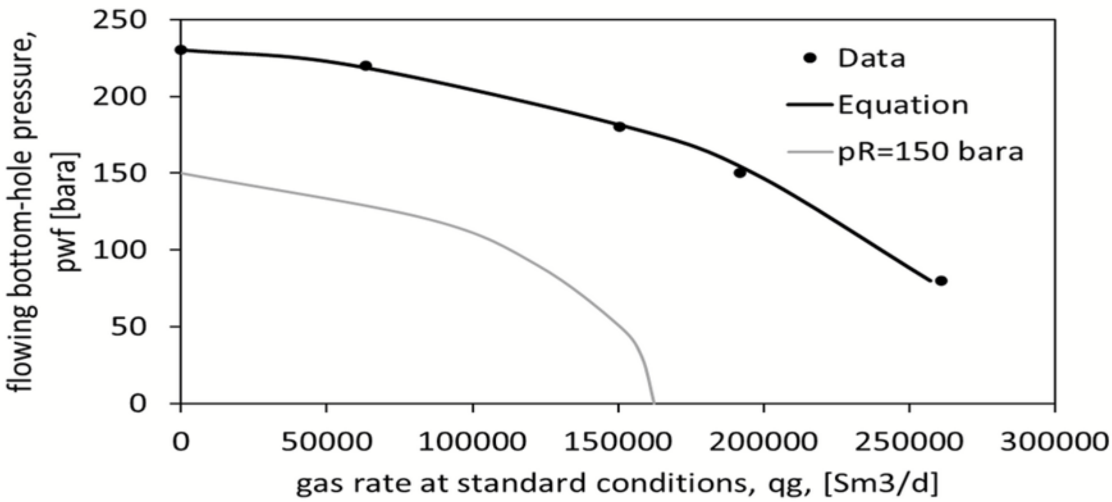
$0.5 \le n \le 1$

turbulent Darcy

$$\frac{Scf/d}{psia^{2n}}$$

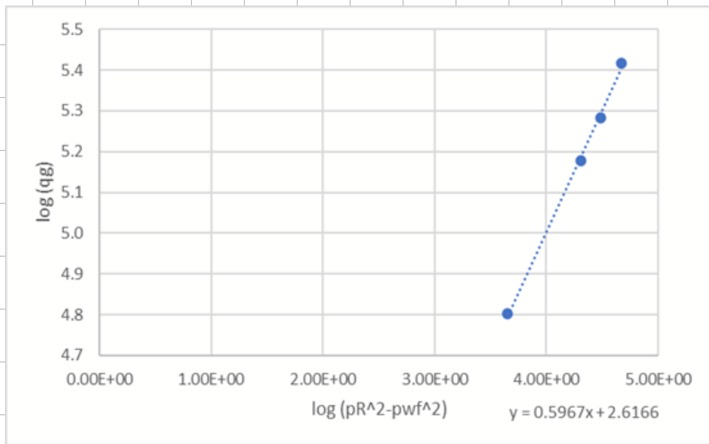
2. Using the backpressure equation, obtain a C and n that match the measured values.

C	[Sm3/d/bar^2n]	281.3		
n	[-]	0.63		
Test nr.	**q**	**pwf**	**q_calc**	**q_diff^2**
	[sm3/d]	**[bara]**	**[sm3/d]**	
Shut-in	0	230	0	
1	63400	220	58422.97	2.48E+07
2	150413	180	152868.4	6.03E+06
3	191787	150	196274.8	2.01E+07
4	260765	80	257009.6	1.41E+07
			SUM=	6.50E+07



C	[Sm3/d/bar^2n]	281.3
n	[-]	0.63

Test nr.	q [sm3/d]	pwf [bara]	q_calc [sm3/d]	q_diff^2	log(qg)	log(pR^2-pwf^2)
Shut-in	0	230	0			
1	63400	220	58422.97	2.48E+07	4.8020893	3.65E+00
2	150413	180	152868.4	6.03E+06	5.1772854	4.31E+00
3	191787	150	196274.8	2.01E+07	5.2828192	4.48E+00
4	260765	80	257009.6	1.41E+07	5.4162493	4.67E+00
			SUM=	6.50E+07		



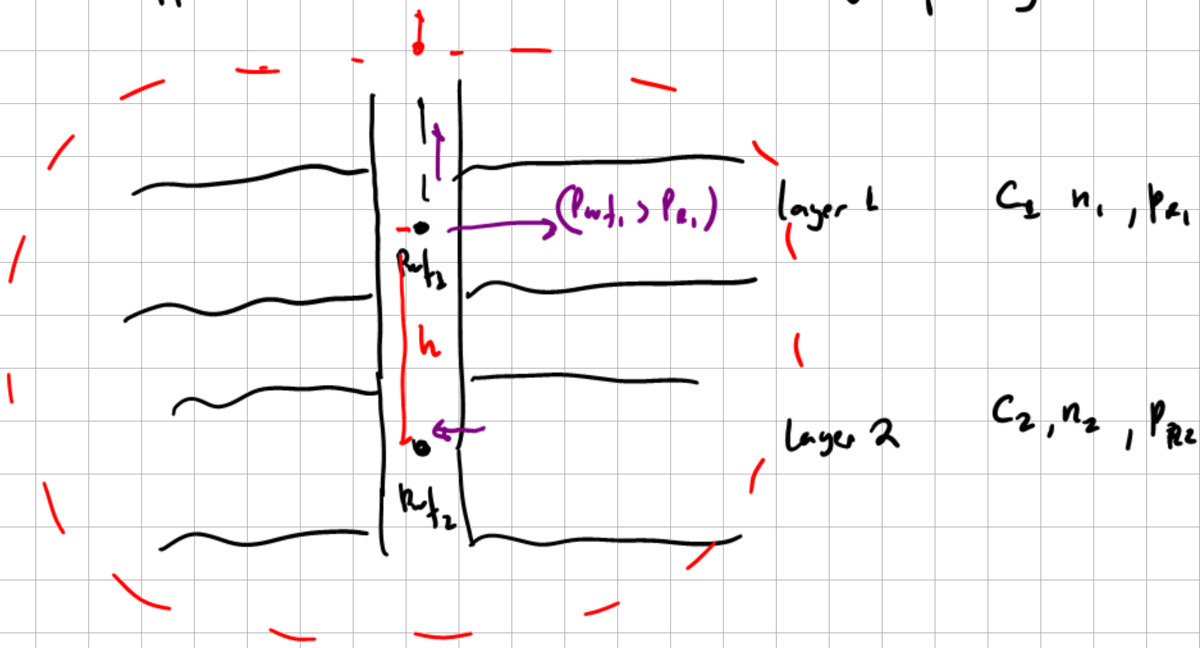
$$\log_{10}(C) = 2.6166$$

$$C = 10^{2.6166} \simeq 458 \ \frac{Sm^3/d}{bara^{2.n}}$$

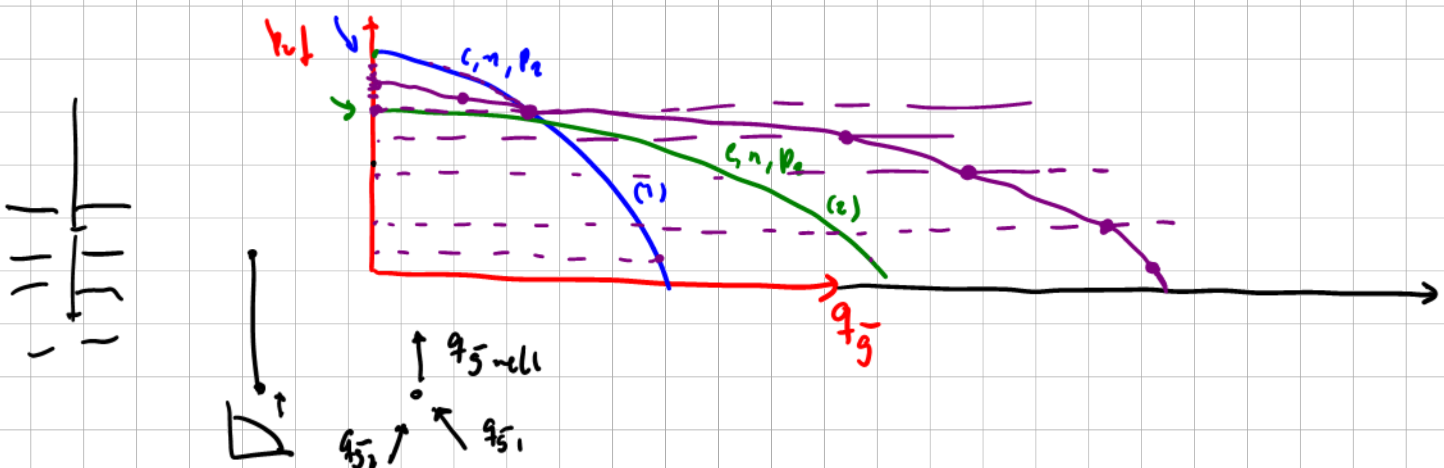
$$n = 0.59$$

Similar to the values obtained with tuning, but poorer fit

what happens when there is more than 1 layer producing to a well bore



h is small, such that $P_{wf_1} \approx P_{wf_2}$



$P_{wf} = 80$ bara two layers (1) $C = 880$ $Sm^3/d/bara^{2n}$

$n = 0.6$

$P_R = 130$ bara

$P_{wf_1} = P_{wf_2}$

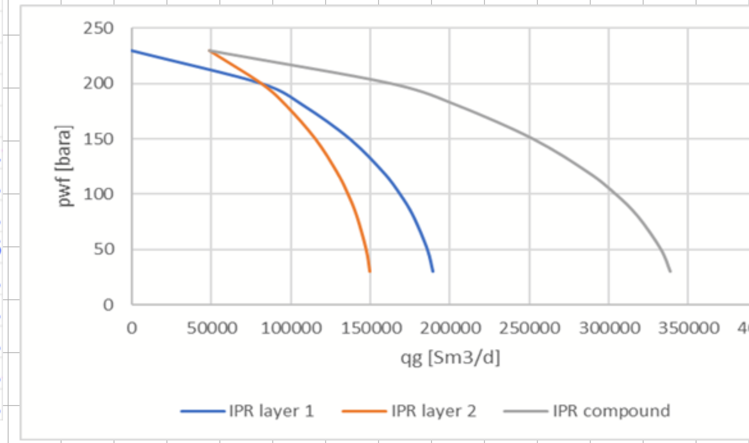
(2) $C = 200$ $Sm^3/d/bara^{2n}$

$n = 0.8$

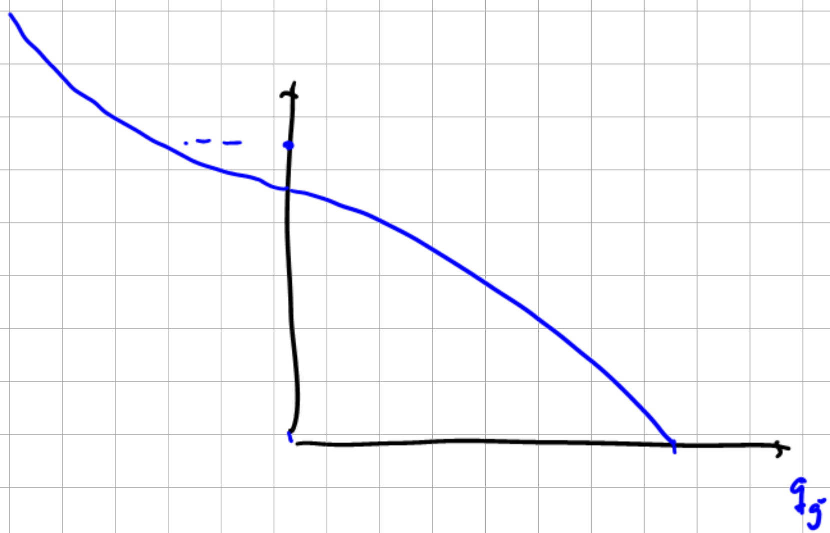
$P_R = 250$ bara

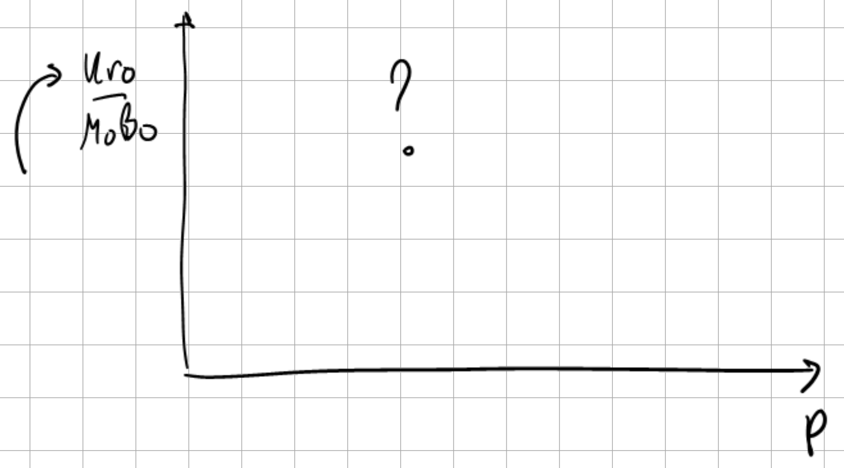
Calculate production from the well

Layer 1							
pR	[bara]	230					
C	[Sm3/d/bara^2n]	280					
n	[-]	0.6					
				pwf (pwf1 =pwf2)	qg1	qg2	qg well
Layer 2				[bara]	[Sm3/d]	[Sm3/d]	[Sm3/d]
pR	[bara]	250		230	0	49022.19	49022.1
C	[Sm3/d/bara^2n]	200		200	81943	81722.1	163665.
n	[-]	0.6		180	108195.8	97312.78	205508.
				150	137051	115416	25246
				120	157919	128919	286838.
				100	168512.6	135870.6	304383.
				80	176860.1	141386.4	318246.
				50	185616.4	147204.4	332820.
				30	189129.8	149547.2	338677.



Comment about well Injection performance





(1941) Evinger and Muskat

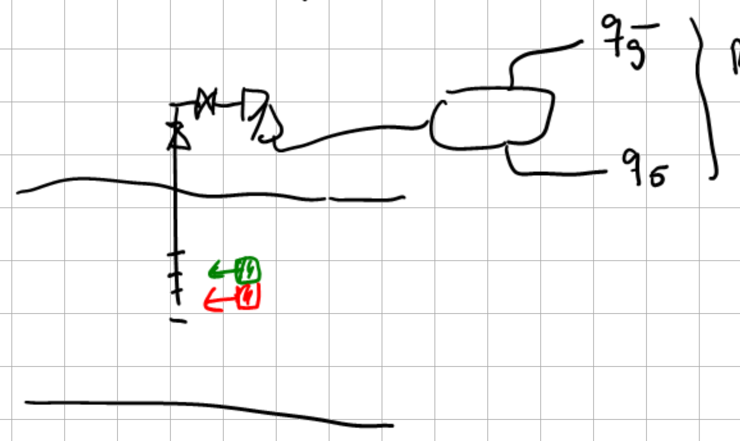
Calculation of Theoretical Productivity Factor

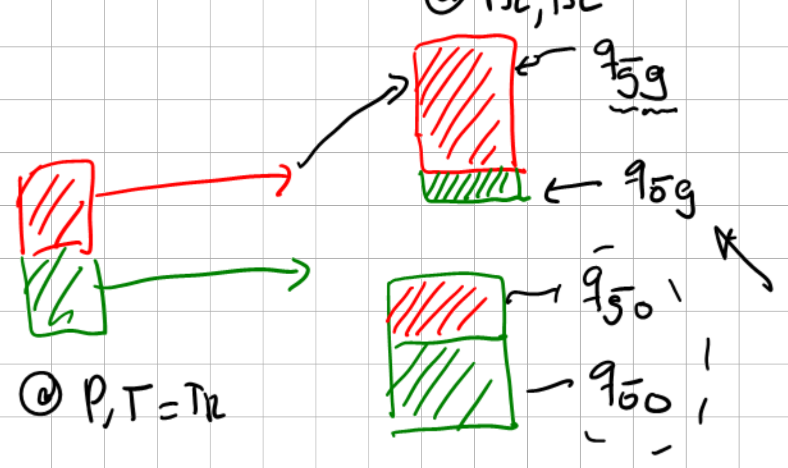
By H. H. Evinger* and M. Muskat*

(New York Meeting, February 1941)

$k_{ro} = f(p)$

$$\frac{q_{\bar g}}{q_{\bar o}} \qquad GOR = R_p = \frac{q_{\bar g}}{q_{\bar o}} = \frac{q_{\bar g g} + q_{\bar g o}}{q_{\bar o o}} \approx \frac{q_{\bar g g}}{q_{\bar o o}} + \frac{q_{\bar g o}}{q_{\bar o o}} = \frac{q_{\bar g g}}{q_{\bar o o}} + R_s(p)$$





$$q_{\bar g} = q_{\bar g g} + q_{\bar g o}$$

$$q_{\bar o} = q_{\bar o g} + q_{\bar o o}$$

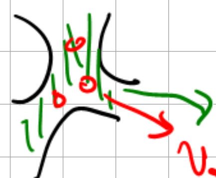
very small

$$q_{\bar o g} << q_{\bar o o}$$

$$q_{\bar o} = q_{\bar o o}$$

$$R_s = \frac{q_{\bar g o}}{q_{\bar o o}} \qquad R_s(p) \text{ solution gas oil ratio}$$

$$\frac{q_{\bar g g}}{q_{\bar o o}} = \frac{q_g}{B_g}\frac{B_o}{q_o} = \frac{k_{rg}}{k_{ro}}\frac{\mu_o B_o}{\mu_g B_g}$$

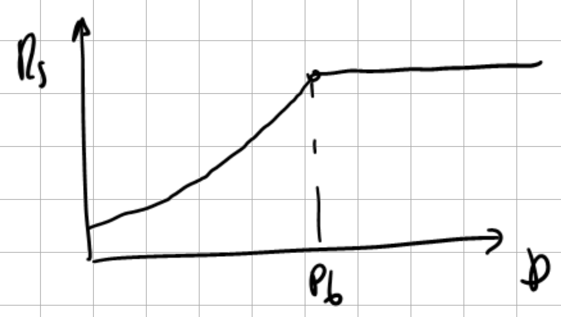


$$v_o = \frac{q_o}{A} \qquad v_g = \frac{q_g}{A}$$

$$\frac{q_g}{q_o} = \frac{k_{rg}}{\mu_g}\frac{\mu_o}{k_{ro}}$$

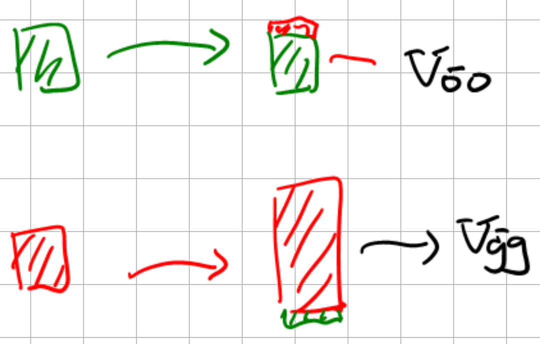
$$v_g = \frac{k_{rg} k}{\mu_g}\frac{dP}{dx}$$

$$v_o = \frac{k_{ro} k}{\mu_o}\frac{dP}{dx}$$



$$\frac{V_o}{V_{\bar o}} = B_o = \frac{q_o}{q_{\bar o o}}$$

$$\frac{V_g}{V_{\bar g}} = B_g = \frac{q_g}{q_{\bar g g}}$$

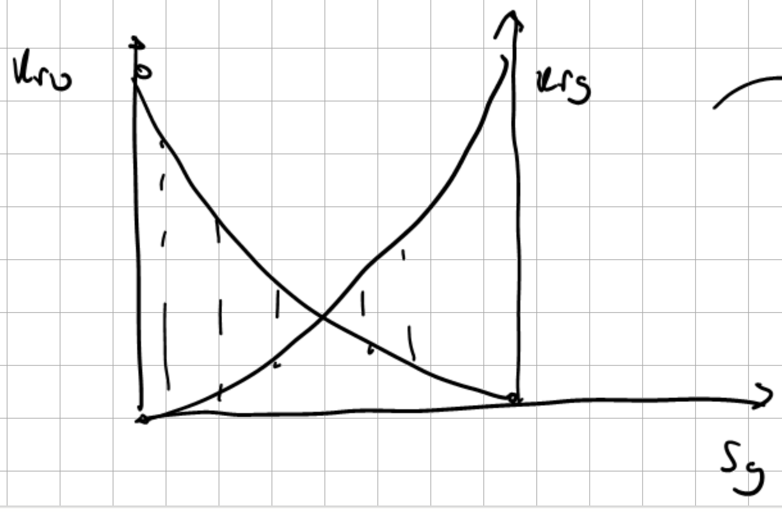


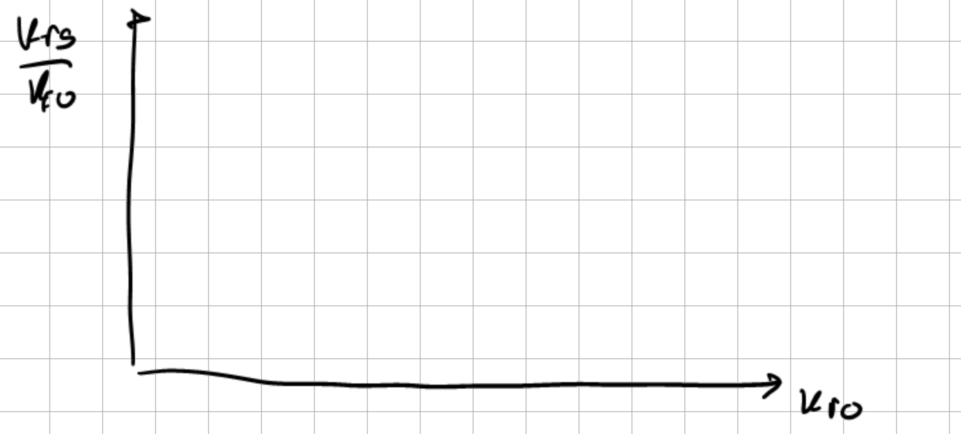
$$R_p = \frac{k_{rg}}{k_{ro}}\underbrace{\frac{\mu_o B_o}{\mu_g B_g}}_{} + R_s(p) \qquad f(p)$$

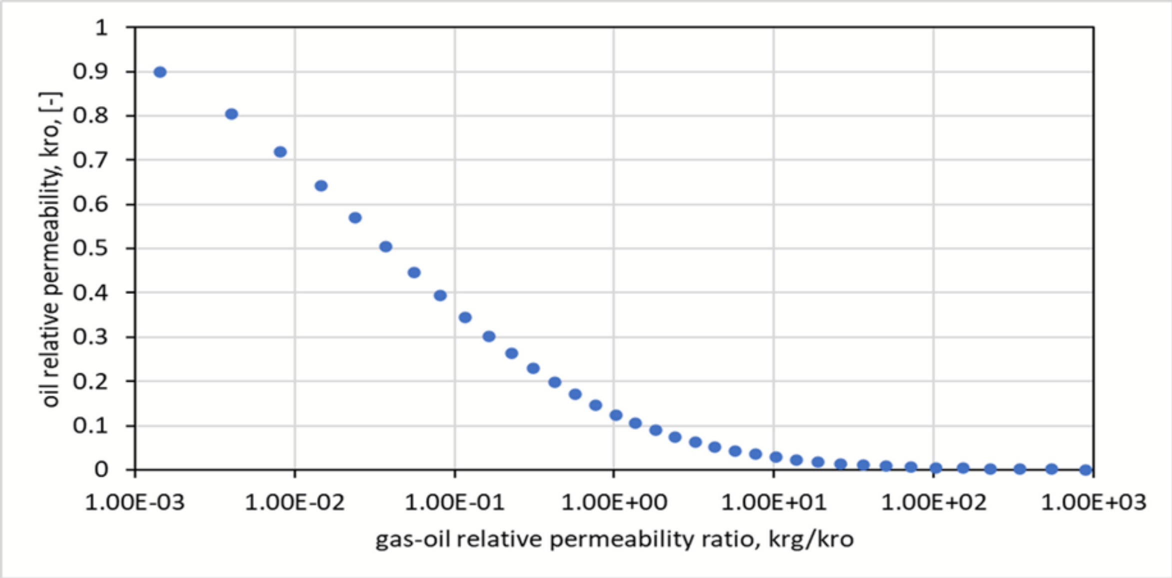
measured

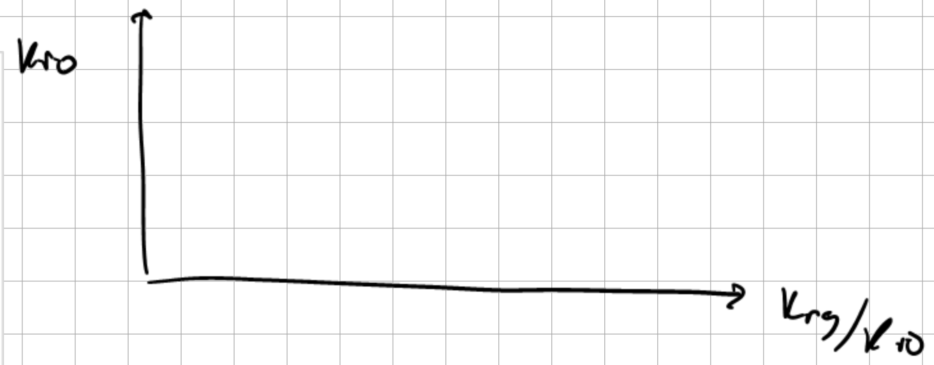
$$\frac{k_{rg}}{k_{ro}} = \frac{\mu_g B_g}{\mu_o B_o}(R_p - R_s) \qquad (1) \qquad f(p)$$

But we need k_{ro}, NOT $\frac{k_{rg}}{k_{ro}}$







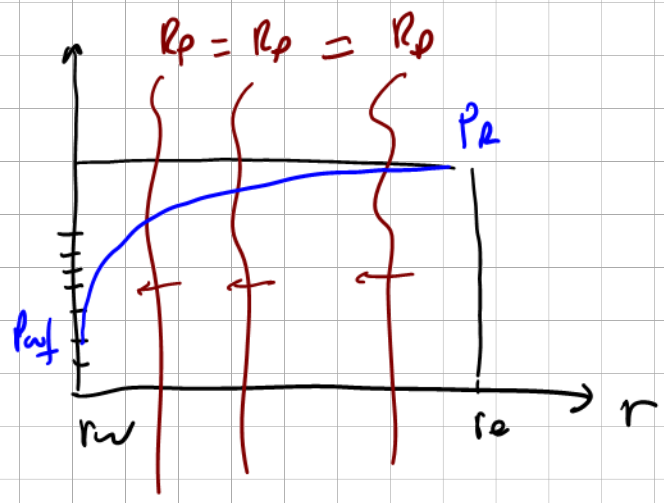


(2)

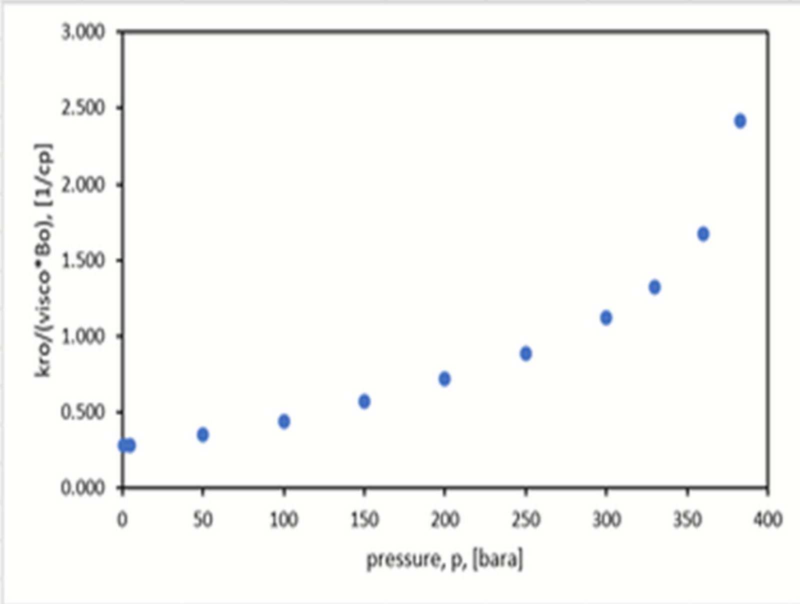
1) from (1) at p ~> $\frac{k_{rg}}{k_{ro}}$

2) from (2) with $\frac{k_{rg}}{k_{ro}}$ ~> k_{ro}

3) k_{ro} f(p)



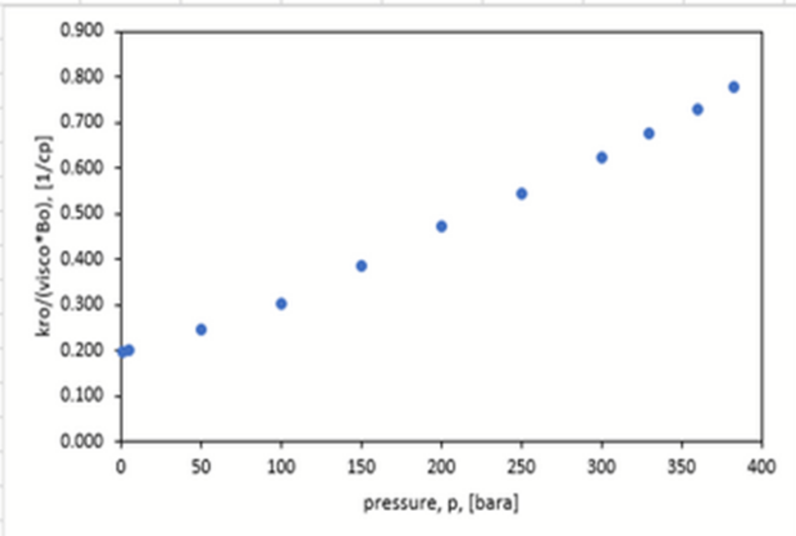
Rp	[Sm3/Sm3]	300							
COLUMN	4	6	3	5	2				
p	Rs	viscg	Bg	Visco	Bo	krg/kro	kro	kro/(visco*Bo)	So
[bara]	[Sm3/Sm3]	[cp]	[m3/Sm3]	[cp]	[m3/Sm3]	[-]	[-]	[1/cp]	[-]
1	0.5	1.12E-02	2.66E-02	1.4	1.2	5.54E-02	4.49E-01	0.277	0.60
5	2.6	1.14E-02	2.59E-02	1.4	1.2	5.50E-02	4.50E-01	0.282	0.60
50	25.5	1.32E-02	1.81E-02	1.1	1.2	4.96E-02	4.67E-01	0.352	0.60
100	48.8	1.58E-02	1.22E-02	0.9	1.2	4.46E-02	4.82E-01	0.443	0.60
150	76.4	1.82E-02	8.20E-03	0.7	1.3	3.81E-02	5.03E-01	0.573	0.61
200	107.2	2.14E-02	6.25E-03	0.5	1.4	3.65E-02	5.09E-01	0.722	0.61
250	143.9	2.51E-02	5.29E-03	0.4	1.5	3.59E-02	5.12E-01	0.888	0.61
300	187.9	2.90E-02	4.69E-03	0.3	1.6	3.23E-02	5.29E-01	1.121	0.62
330	220.4	3.15E-02	4.45E-03	0.3	1.7	2.65E-02	5.57E-01	1.322	0.62
360	258.3	3.41E-02	4.26E-03	0.2	1.8	1.61E-02	6.29E-01	1.672	0.64
383	292.6	3.60E-02	4.14E-03	0.2	1.9	3.19E-03	8.34E-01	2.414	0.67



NOT LINEAR!!!

For higher GOR

Rp	[Sm3/Sm3]	800							
COLUMN	4	6	3	5	2				
p	Rs	viscg	Bg	Visco	Bo	krg/kro	kro	kro/(visco*Bo)	So
[bara]	[Sm3/Sm3]	[cp]	[m3/Sm3]	[cp]	[m3/Sm3]	[-]	[-]	[1/cp]	[-]
1	0.5	1.12E-02	2.66E-02	1.4	1.2	1.48E-01	3.17E-01	0.196	0.56
5	2.6	1.14E-02	2.59E-02	1.4	1.2	1.47E-01	3.17E-01	0.199	0.56
50	25.5	1.32E-02	1.81E-02	1.1	1.2	1.40E-01	3.24E-01	0.244	0.56
100	48.8	1.58E-02	1.22E-02	0.9	1.2	1.33E-01	3.30E-01	0.304	0.56
150	76.4	1.82E-02	8.20E-03	0.7	1.3	1.23E-01	3.40E-01	0.387	0.56
200	107.2	2.14E-02	6.25E-03	0.5	1.4	1.31E-01	3.32E-01	0.471	0.56
250	143.9	2.51E-02	5.29E-03	0.4	1.5	1.51E-01	3.14E-01	0.545	0.56
300	187.9	2.90E-02	4.69E-03	0.3	1.6	1.77E-01	2.95E-01	0.624	0.55
330	220.4	3.15E-02	4.45E-03	0.3	1.7	1.93E-01	2.85E-01	0.675	0.54
360	258.3	3.41E-02	4.26E-03	0.2	1.8	2.09E-01	2.75E-01	0.730	0.54
383	292.6	3.60E-02	4.14E-03	0.2	1.9	2.19E-01	2.69E-01	0.778	0.54



LINEAR??

$$q_{\bar{o}} = \frac{k\cdot h}{18.68\cdot\left(ln\left(\frac{r_e}{r_w}\right)-0.75+s\right)}\underbrace{\int_{p_{wf}}^{p_R}\frac{k_{ro}}{\mu_o\cdot B_o}dp}_{\left[m(p_R)-m(p_{wf})\right]} =$$

$$\int_0^{p_{wf}}\frac{k_{ro}}{\mu_o B_o}dp = m(p_{wf})$$

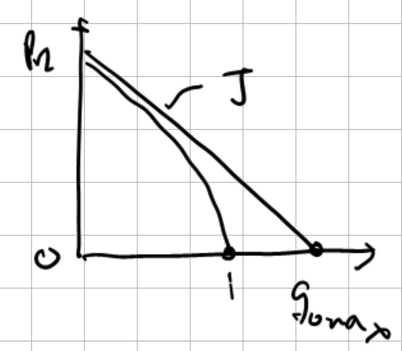
Not the same gas m function !

$$\int \frac{p\,dp}{z\mu_g}$$

Vogel (V=0.2)

$$q_{\bar{o}} = q_{\bar{o},max}\left[1 - V\cdot\frac{p_{wf}}{p_R} - (1-V)\cdot\left(\frac{p_{wf}}{p_R}\right)^2\right]$$

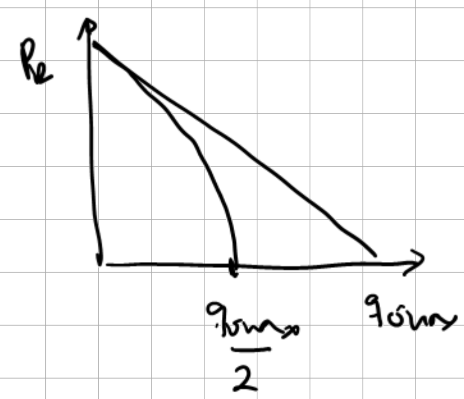
$$\frac{\overbrace{k\cdot h\cdot\left[\left(\frac{k_{ro}}{\mu_o\cdot B_o}\right)_{@pR}\right]}^{J}\cdot(p_R-0)}{18.68\cdot\left(ln\left(\frac{r_e}{r}\right)-0.75+s\right)\cdot 1.8} = \frac{J}{1.8}\cdot pR \rightarrow q_{\bar{o}max}$$



Fetkovich

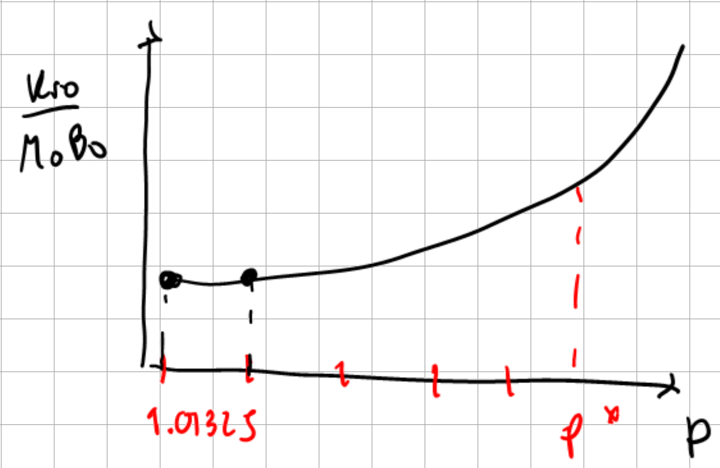
$$q_{\bar{o}} = q_{\bar{o},max}\left[1 - \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

$$\frac{k\cdot h\cdot\left[\left(\frac{k_{ro}}{\mu_o\cdot B_o}\right)_{@pR}\right]\cdot p_R}{18.68\cdot\left(ln\left(\frac{r_e}{r_w}\right)-0.75+s\right)\cdot 2} = \frac{J}{2}\cdot pR$$

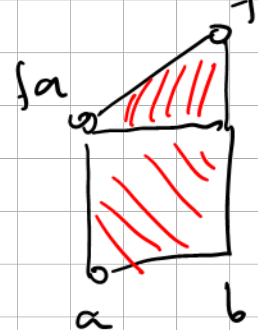


oil m function in VBA:

```
Function m_function_oil(p, col, Matrix As Range)
    'p in bara
    'n number of intervals to use in the integration
    p1 = 1.01325
    p2 = p
    If p2 - p1 > 40 Then
        n = Round((p2 - p1) / 10, 0)
    Else
        n = 10
    End If
    DP = (p2 - p1) / n
    pj = p1
    Sum = 0
    fa = tabinterpol(pj, col, Matrix)
    For J = 1 To n
        pj = pj + DP
        fb = tabinterpol(pj, col, Matrix)
        Sum = Sum + (DP * (fa + fb)) * 0.5
        fa = fb
    Next
    m_function_oil = Sum
End Function
```

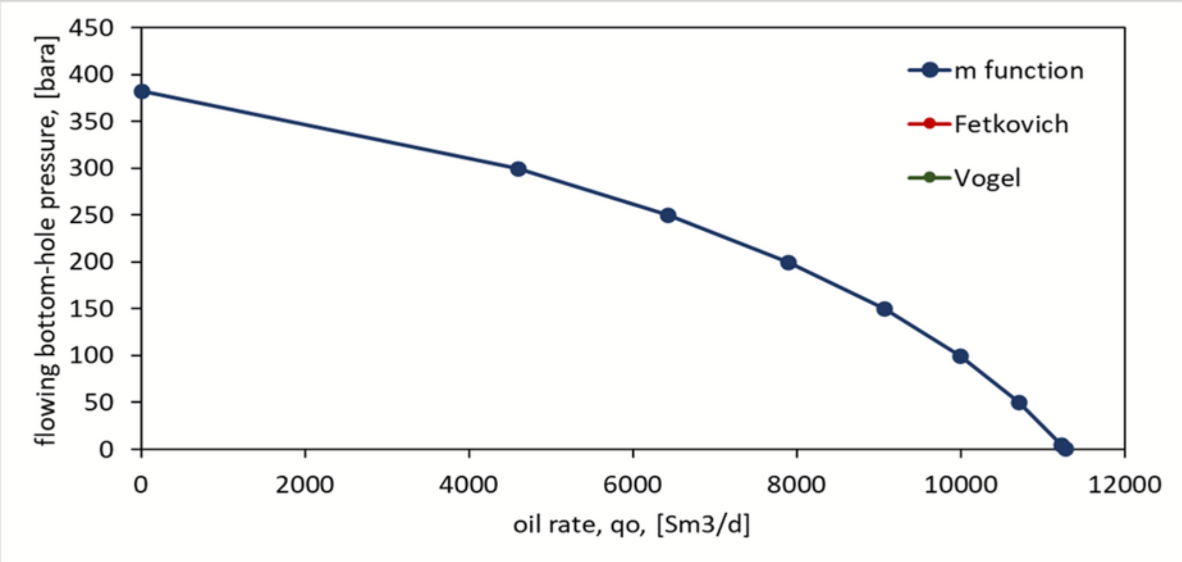


fb (from interpolation on table)

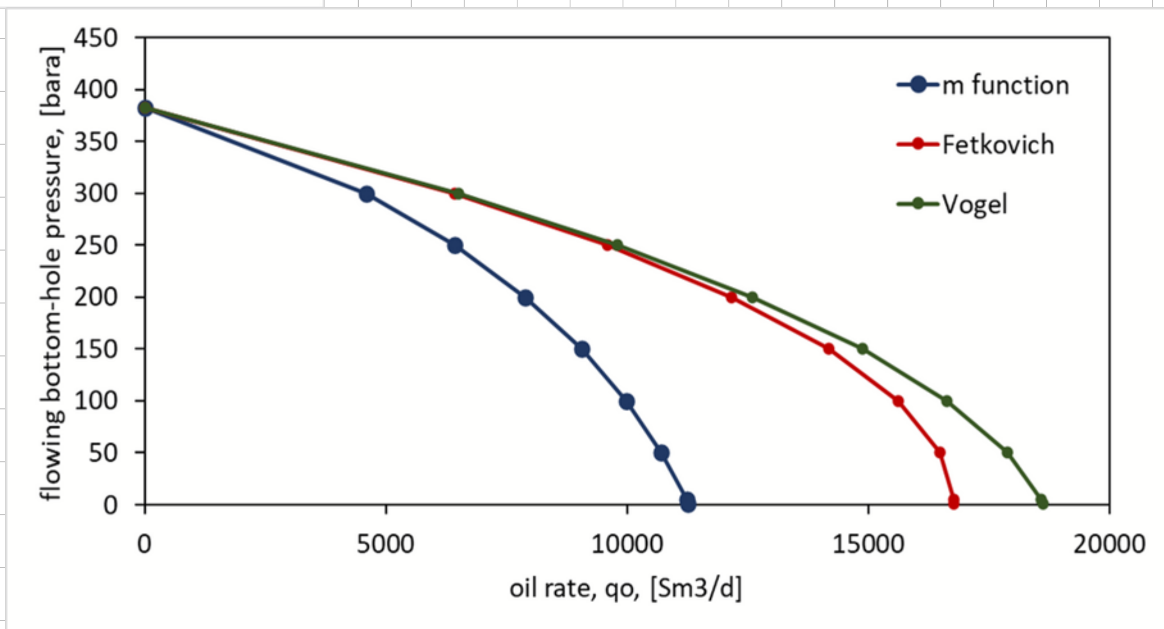


$$Area = (fa + fb)\cdot\frac{(b-a)}{2}$$

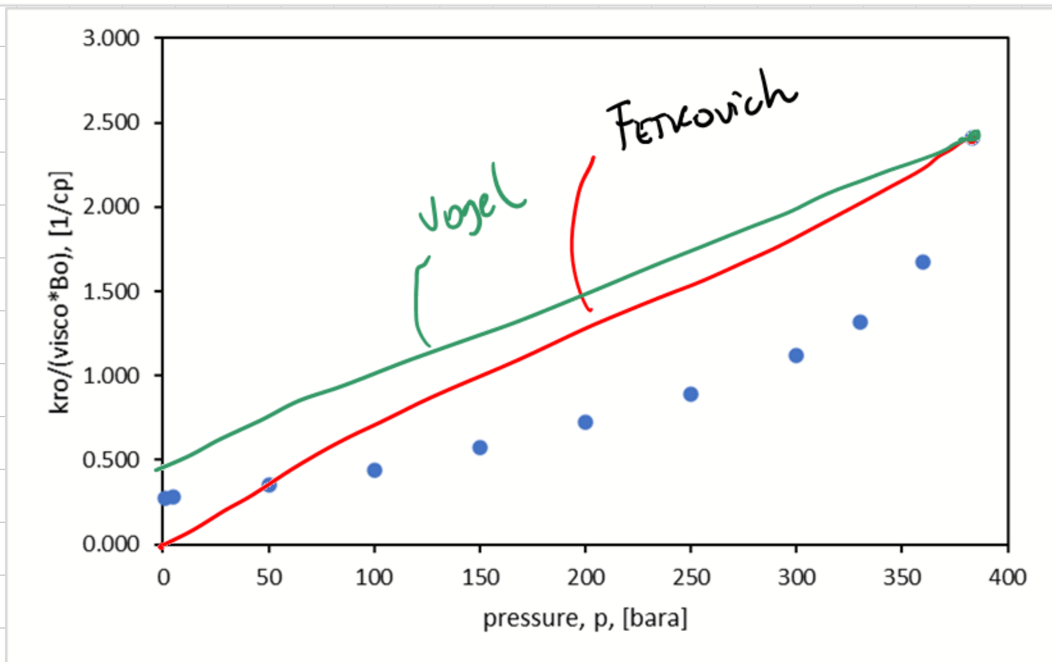
```
Function ipr_sat_oil_qo_m_function(k, h, re, rw, s, mpR, mPwf)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'mpR, m function at reservoir pressure [bara/cp]
    'mpwf, m functino at flowing bottom-hole pressure [bara/cp]
    ipr_sat_oil_qo_m_function = k * h * (mpR - mPwf) / ((Log(re / rw) - 0.75 + s) * 18.68)
End Function
```



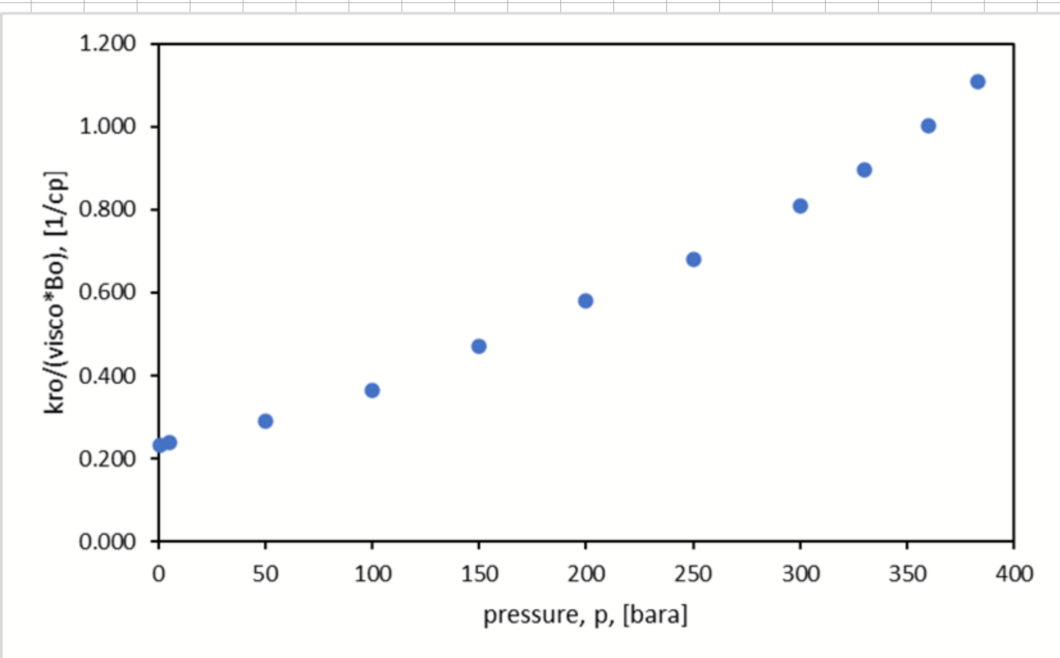

```
Function ipr_sat_oil_qo_Fetkovich(k, h, re, rw, s, pR, Pwf, Bo_pR, visco_pR, kro_pR)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'pR, reservoir pressure [bara]
    'pwf, flowing bottom-hole pressure [bara]
    'kro_pR oil relative permeability at reservoir pressure, [-]
    'visco_pR oil viscosity at reservoir pressure [cp]
    'Bo_pR oil formation volume factor at reservoir pressure, [m3/Sm3]
    J = k * h * (kro_pR / (visco_pR * Bo_pR)) / ((Log(re / rw) - 0.75 + s) * 18.68)
    qomax = J * (pR - 0) / 2
    ipr_sat_oil_qo_Fetkovich = qomax * (1 - (Pwf / pR) ^ 2)
End Function
```




```
Function ipr_sat_oil_qo_Vogel(k, h, re, rw, s, pR, Pwf, Bo_pR, visco_pR, kro_pR)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'pR, reservoir pressure [bara]
    'pwf, flowing bottom-hole pressure [bara]
    'kro_pR oil relative permeability at reservoir pressure, [-]
    'visco_pR oil viscosity at reservoir pressure [cp]
    'Bo_pR oil formation volume factor at reservoir pressure, [m3/Sm3]
    J = k * h * (kro_pR / (visco_pR * Bo_pR)) / ((Log(re / rw) - 0.75 + s) * 18.68)
    qomax = J * (pR - 0) / 1.8
    ipr_sat_oil_qo_Vogel = qomax * (1 - 0.2 * (Pwf / pR) - 0.8 * (Pwf / pR) ^ 2)
End Function
```



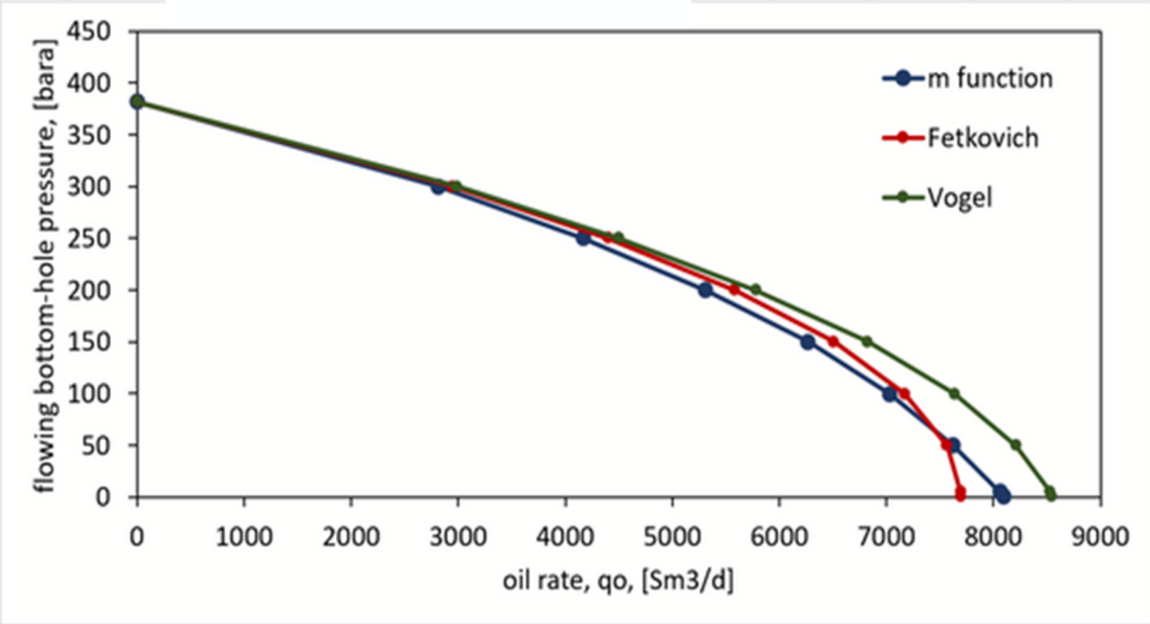
Changing the GOR (Rp) manually to 500 Sm3/Sm3

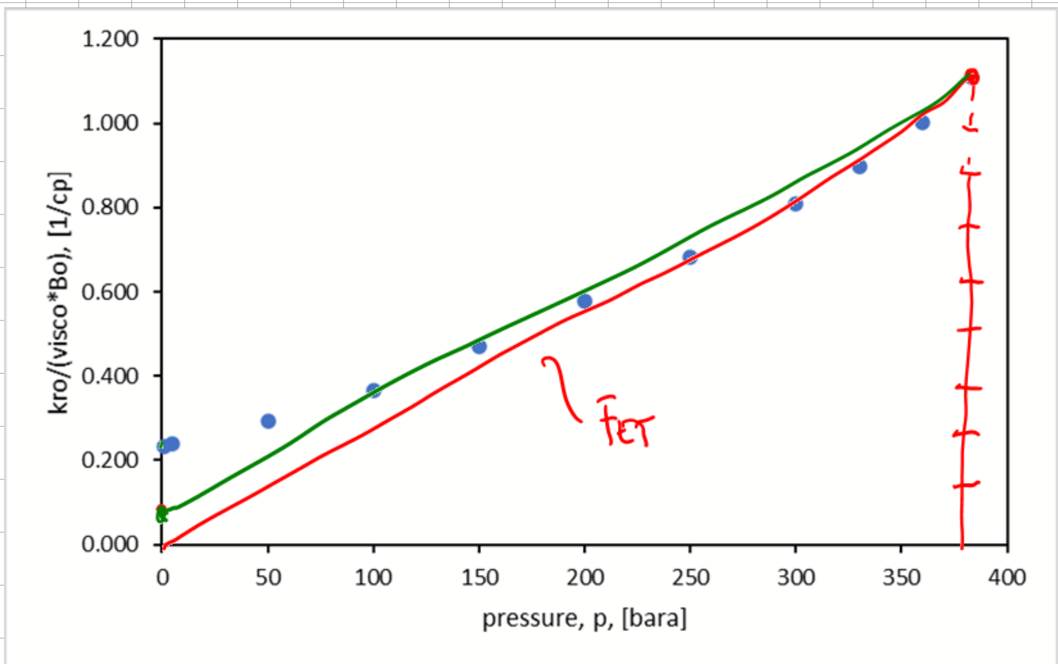


MORE LINEAR....

THE PREDICTION IS BETTER

pwf [bara]	m(p) [bara/cp]	qo [Sm3/d]	qo - Fet [Sm3/d]	qo -Vogel [Sm3/d]	qo - Fet-v2 [Sm3/d]	qo -Vogel-v2 [Sm3/d]
382	222.6	0	0	0		
300	145.3	2810	2950	2990		
250	108.0	4166	4401	4503		
200	76.5	5312	5588	5782		
150	50.2	6268	6511	6826		
100	29.3	7028	7170	7637		
50	12.9	7625	7566	8212		
5	0.9	8059	7697	8530		
1	0.0	8094	7698	8549		





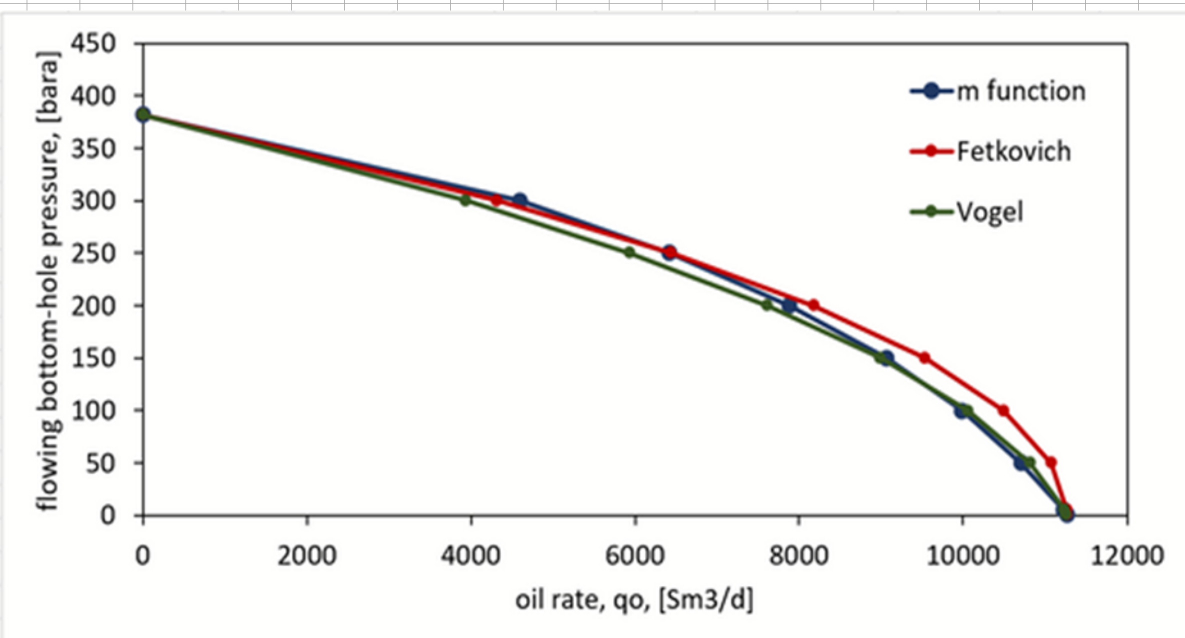
WE CHANGE THE RP BACK TO 300...

But, what if we use the q_{omax} from the m function equation? (11268 Sm^3/d)

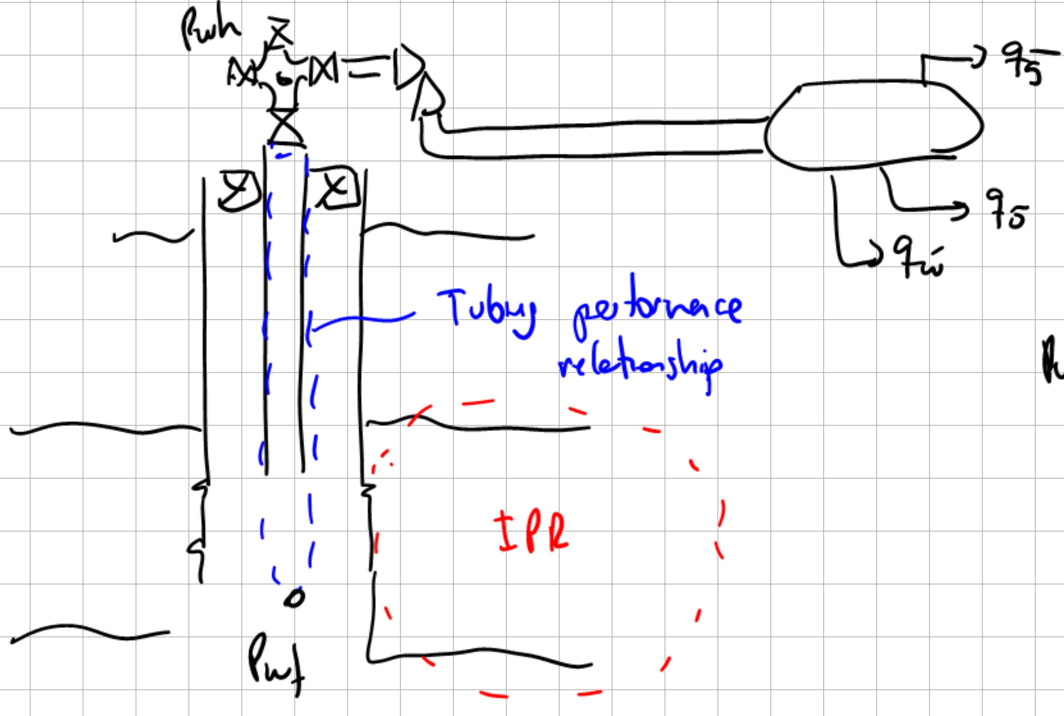
$$q_o = q_{omax}\left(1-\left(\frac{P_{wf}}{P_R}\right)^2\right)$$

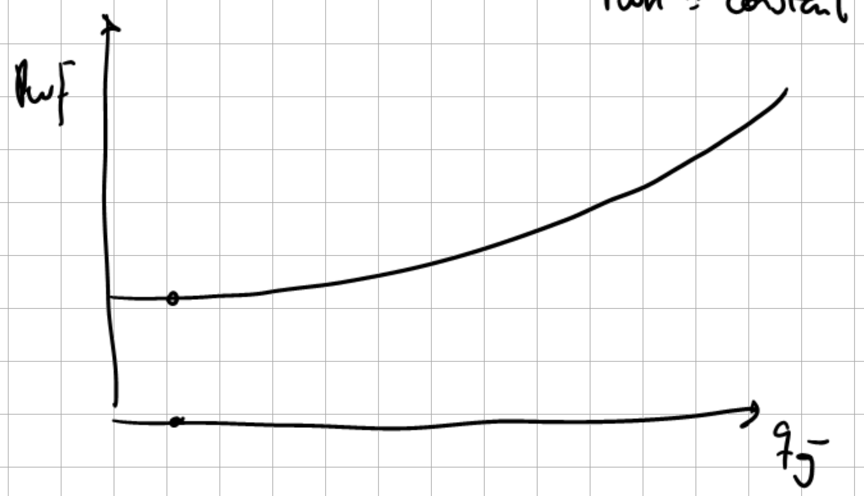
$$q_o = q_{omax}\left(1-0.2\left(\frac{P_{wf}}{P_R}\right)-0.8\left(\frac{P_{wf}}{P_R}\right)^2\right)$$

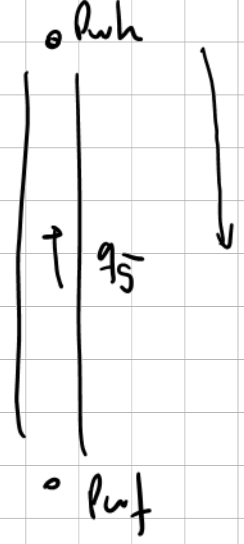
pwf [bara]	m(p) [bara/cp]	qo [Sm3/d]	qo - Fet [Sm3/d]	qo -Vogel [Sm3/d]	qo - Fet-v2 [Sm3/d]	qo -Vogel-v2 [Sm3/d]
382	309.9	0	0	0	0	0
300	183.5	4596	6425	6511	4318	3938
250	133.3	6421	9584	9807	6442	5932
200	93.1	7884	12169	12592	8179	7617
150	60.7	9061	14180	14867	9530	8993
100	35.3	9985	15616	16631	10495	10060
50	15.4	10708	16477	17884	11075	10818
5	1.1	11227	16762	18576	11266	11236
1	0.0	11268	16765	18617	11267	11261



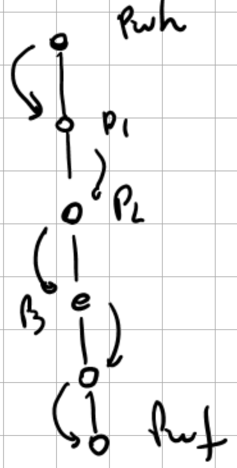
the prediction is improved considerably!

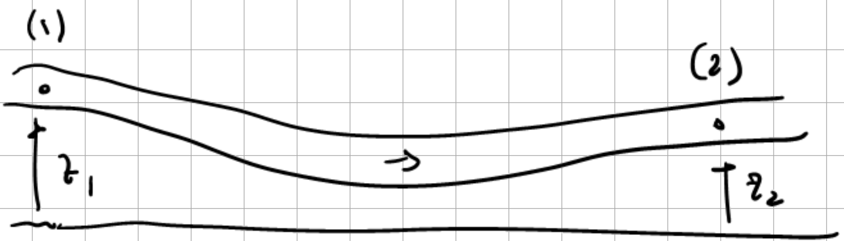






Consider current to calculate Pwf (pressure integration)





Head → $h_1 = h_2$ (no energy losses) $\qquad h = z + \frac{p}{\rho g} + \frac{v^2}{2g}$ $\qquad$ incompressible

$h_1 = h_2 + \Delta h_{losses}$ $\qquad \Delta h_{losses} = \underbrace{\Delta h_{accessories}}_{\text{valves, restrictions}} + \Delta h_{friction}$ $\qquad \Delta h_{friction} = f \frac{L}{\phi} \frac{v^2}{2g}$

Moody friction factor

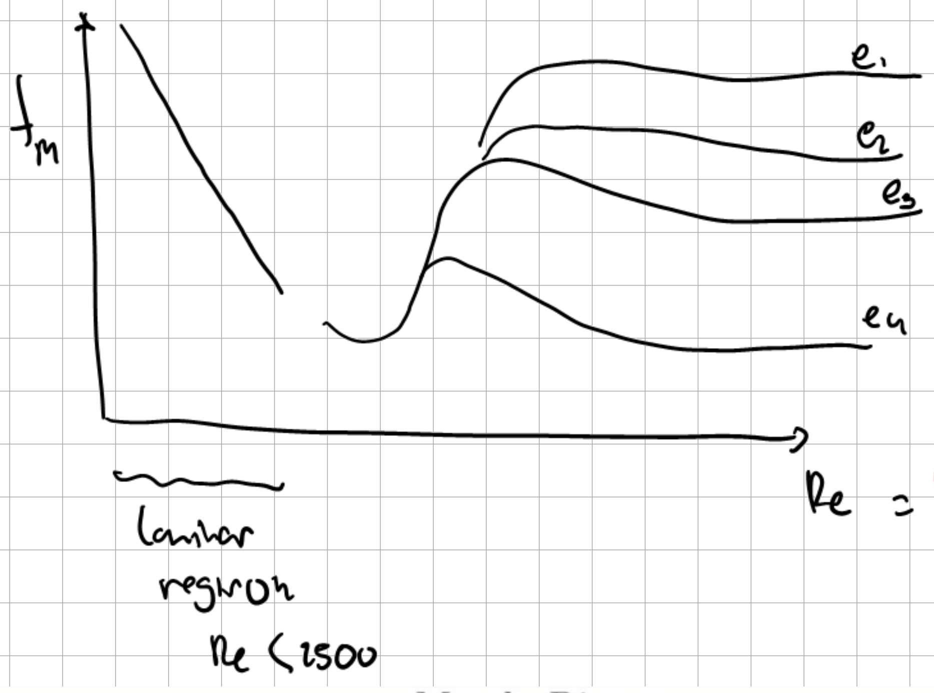
$$h_1 = z_1 + \frac{p_1}{\rho g} + \cancel{\frac{v_1^2}{2g}} = z_2 + \frac{p_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + f \frac{L}{\phi} \frac{v^2}{2g}$$

its incompressible, so $v_1 = v_2 = v$, ϕ = constant

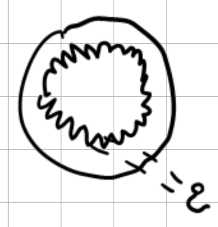
$$\Delta p = (p_1 - p_2) = \rho g \cdot \left((z_2 - z_1) + f \frac{L}{\phi} \frac{\rho g}{2g} v^2 \right)$$

$$(p_1 - p_2) = \underbrace{\Delta z \cdot \rho g}_{\text{hydrostatic}} + \underbrace{f \frac{L}{\phi} \rho \frac{v^2}{2}}_{\text{friction}}$$

for gas $\rho = f(p, T)$

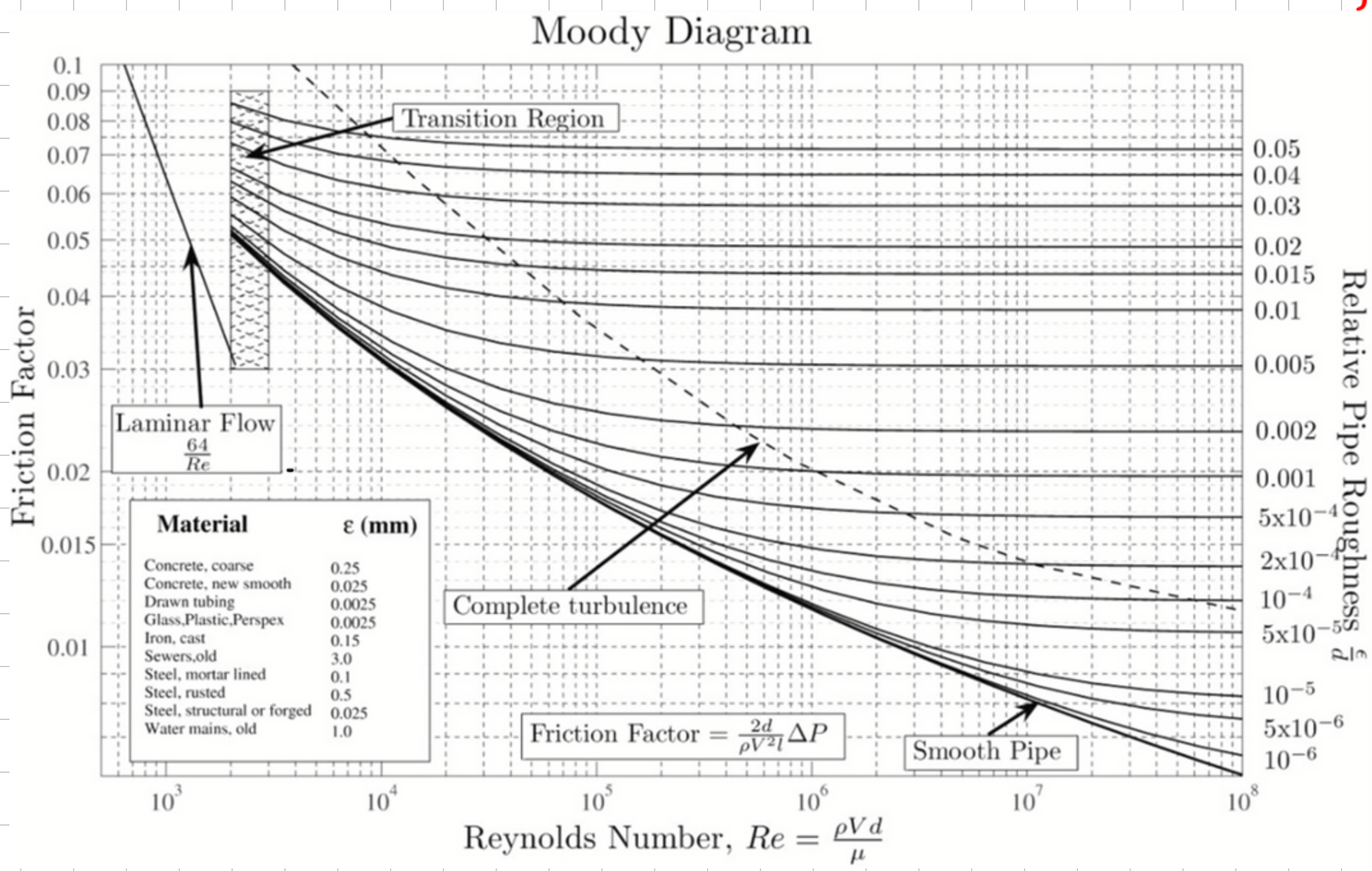


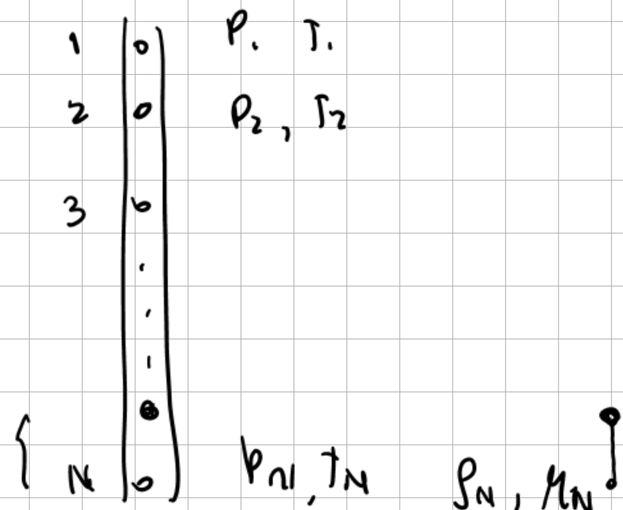
ε/D dimensionless roughness



$$Re = \frac{\rho D v}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

$\mu \rightarrow f(p,T)$





$N \rightarrow N-1$, gas properties don't change much

valid if ΔL is small

ΔT, Δp is small

$$v_g = \frac{q_g}{A} = \frac{q_{\bar{g}} B_g(p,T)}{A}$$

$$B_g = \frac{q_g}{q_{\bar{g}}}$$

$$q_{\bar{g}} \cdot \rho_{\bar{g}} = q_g \cdot \rho_g$$

$$B_g = \frac{q_g}{q_{\bar{g}}} = \frac{\rho_{\bar{g}}}{\rho_g} = \frac{zT}{p}\left(\frac{p_{sc}}{T_{sc}}\right)$$

$$\rho_{\bar{g}} = \frac{p_{sc}}{R z_{sc} T_{sc}}$$

m^3/d $\frac{1\,d}{24\,h}$ $\frac{1\,h}{3600\,s}$

$$\rho_g = \frac{p}{zRT}$$

qg [Sm3/d]	2.85E+06
Gas gravity	0.7
Tubing ID [m]	0.157
Tubing cross section area [m2]	0.019
Tubing roughness [m]	1.50E-06

TVD [m]	p [bara]	T [C]	Z [-]	deng [kg/m3]	Bg [m3/Sm3]	viscg [cp]	qg [m3/d]	vg [m/s]	p-calc [bara]
0	40	87	0.948	28.6	3.00E-02	1.36E-02	8.54E+04	51.03	
284	46	89	0.942	32.9	2.60E-02	1.38E-02	7.41E+04	44.28	
567	51	90	0.938	36.8	2.32E-02	1.39E-02	6.63E+04	39.61	
851	56	92	0.934	40.4	2.12E-02	1.41E-02	6.04E+04	36.14	
1135	61	94	0.930	43.6	1.96E-02	1.43E-02	5.59E+04	33.43	
1418	66	96	0.928	46.7	1.83E-02	1.45E-02	5.23E+04	31.25	
1702	70	98	0.926	49.5	1.73E-02	1.46E-02	4.92E+04	29.44	
1986	74	99	0.924	52.2	1.64E-02	1.48E-02	4.67E+04	27.92	
2269	78	101	0.922	54.8	1.56E-02	1.49E-02	4.45E+04	26.61	
2553	81	103	0.921	57.3	1.49E-02	1.51E-02	4.26E+04	25.47	
2837	85	105	0.920	59.6	1.44E-02	1.52E-02	4.09E+04	24.46	

2837 - 2553

• p_{N-1} • p_{wf} ?

$$p_{N-1} = p_{wf} - \rho g \Delta z - f \frac{L}{D} \rho \frac{v^2}{2}$$

using conditions at p_{wf} !

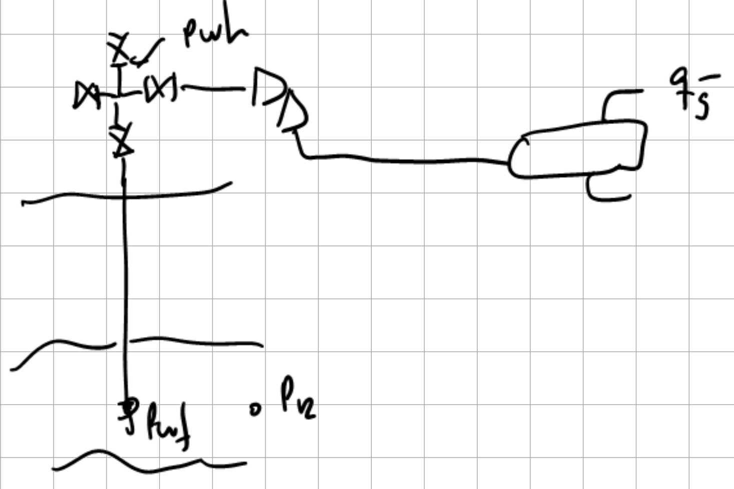
```
Function Pout(qt1, ID, den, visc, Length, teta, pin, roughness)
    'Function that give pressure available at the oulet of a pipe with a flow qt and inlet pressure pressure pin
    'Calculation made for liquid single phase flow
    'Takes in data in SI
    'qt flow [m^3/d]
    'ID inner diameter of pipe [m]
    'den density of fluid, [kg/m^3]
    'visc viscosity of fluid, [Pa s]
    'Length, pipe length, [m]
    'teta inclination angle of pipe with respect to horizontal [°]
    'pin, discharge pressure required, [bara]
    'roughness of pipe [m]

    'Gravitational acceleration g, [m/s(2]
    g = 9.81
    'Pi number
    Pi = 4 * Atn(1)
    qt = qt1 / (3600# * 24#)  '[m^3/s]

    'Calculating area and velocity
    Area = Pi * (ID ^ 2) / 4
    v = qt / Area

    Presscalc1 = pin - (Length * Sin(teta * Pi / 180) * den * g / 100000#) - (ffactor(den, visc, ID, roughness, v) * Length * (v ^ 2) * den / (ID * 200000#))
    Pout = Presscalc1

End Function
```

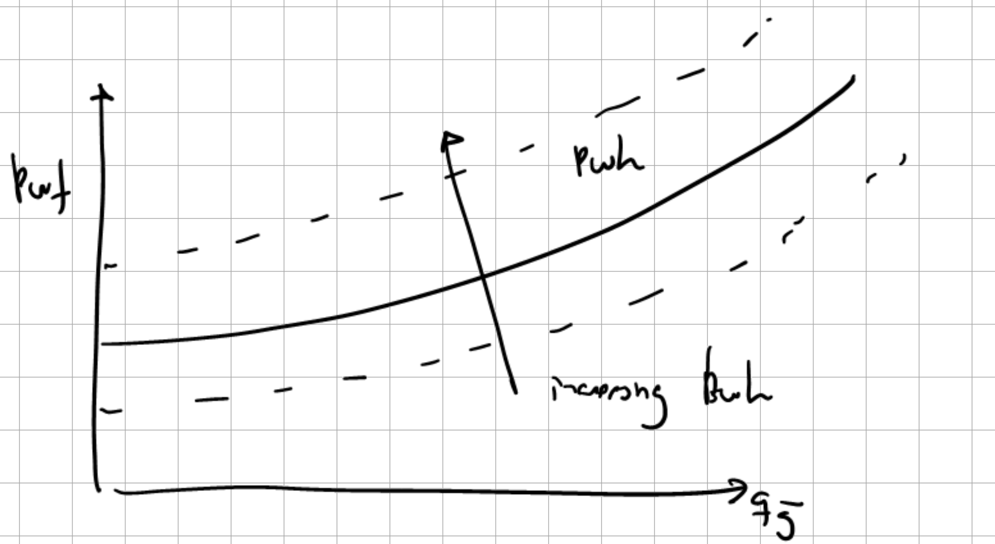
change for m^3/d to m^3/s

dividing by 1E5 to take from Pa to bara

TVD [m]	p [bara]	T [C]	Z [-]	deng [kg/m3]	Bg [m3/Sm3]	viscg [cp]	qg [m3/d]	vg [m/s]	p-calc [bara]
0	40	87	0.948	28.6	3.00E-02	1.36E-02	8.54E+04	51.03	35.55
284	46	89	0.942	32.9	2.60E-02	1.38E-02	7.41E+04	44.28	41.50
567	51	90	0.938	36.8	2.32E-02	1.39E-02	6.63E+04	39.61	47.03
851	56	92	0.934	40.4	2.12E-02	1.41E-02	6.04E+04	36.14	52.27
1135	61	94	0.930	43.6	1.96E-02	1.43E-02	5.59E+04	33.43	57.29
1418	66	96	0.928	46.7	1.83E-02	1.45E-02	5.23E+04	31.25	62.15
1702	70	98	0.926	49.5	1.73E-02	1.46E-02	4.92E+04	29.44	66.88
1986	74	99	0.924	52.2	1.64E-02	1.48E-02	4.67E+04	27.92	71.52
2269	78	101	0.922	54.8	1.56E-02	1.49E-02	4.45E+04	26.61	76.08
2553	81	103	0.921	57.3	1.49E-02	1.51E-02	4.26E+04	25.47	80.59
2837	85	105	0.920	59.6	1.44E-02	1.52E-02	4.09E+04	24.46	85.04

surprisingly close !





$$q_{\bar{s}} = C_T \cdot \left(\frac{P_{wf}^2}{e^S} - P_{wh}^2\right)^{0.5}$$

tubing coefficient, tubing elevation coefficient

for same P_{wf}, P_{wh}:

- $C_T \uparrow$ $q_s \uparrow$
- $S \uparrow$ $q_s \downarrow$

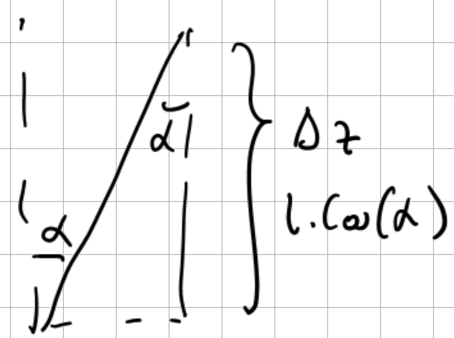
$$u_g = \frac{q_g}{A} = \frac{\dot{m}_g}{\rho_g \cdot A} \qquad \rho_g = \frac{p}{z\,R\,T} \rightarrow \frac{R_0}{MW}$$

$$u_g^2 = \left(\frac{\dot{m}_g}{\rho_g A}\right)^2$$

$f(v, \rho, \mu)$

$$-\frac{dp}{dl} = \frac{p \cdot M_g}{Z \cdot R \cdot T} \cdot g \cdot \cos(\alpha) + \frac{8 \cdot f_M \cdot \dot{m}^2}{\pi^2 \cdot D^5} \cdot \frac{Z \cdot R \cdot T}{p \cdot M_g}$$

$f(p)$ "R_0"



$$S = 2 \cdot L \cdot C_a = 2 \cdot \frac{M_g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot g \cdot \cos(\alpha)$$

$$q_{sc} = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left[\left(p_{wf}^2 - p_t^2 \cdot e^S\right) \cdot \left(\frac{S}{e^S - 1}\right)\right]^{0.5}$$

By defining the tubing constant C_T:

$$C_T = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left(\frac{S \cdot e^S}{e^S - 1}\right)^{0.5} \qquad \text{EQ. A-24}$$

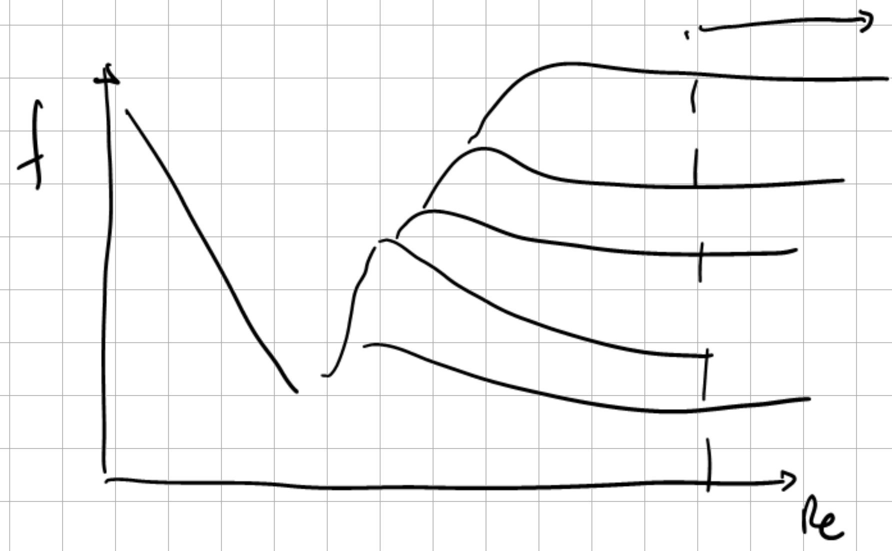
This yields:

$$q_{sc} = C_T \cdot \left[\left(\frac{p_{wf}^2}{e^S} - p_t^2 \cdot\right)\right]^{0.5} \qquad \text{EQ. A-25}$$

→ to calculate C_T, S, a seed P_{wf}, P_{wh}, T_{wf}, T_{wh} are required!

Z_{av}, T_{av}

Luckily the effect of seed is modest



fully turbulent regime → $f = f(e/D)$

$Re = \frac{v \phi \rho}{\mu}$

ρ is low

v is high

μ is high

Re is very high

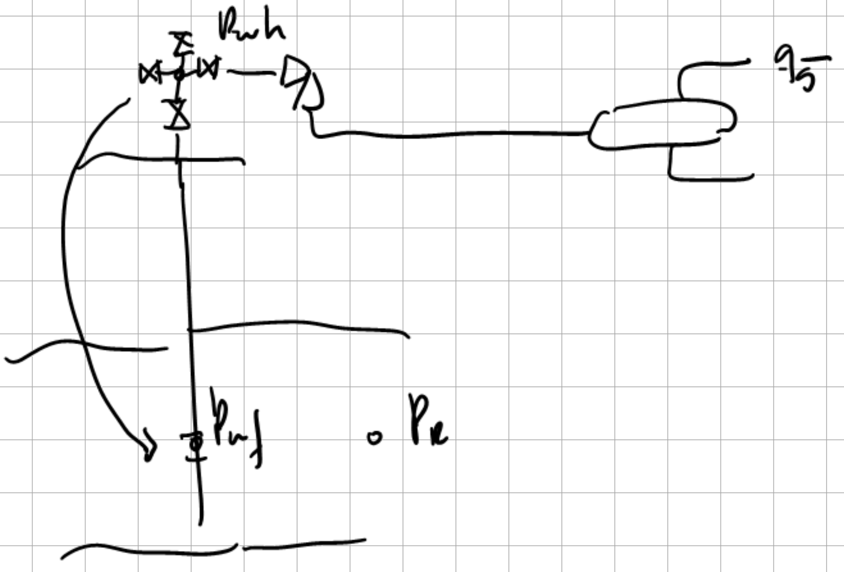
$e = f(\phi)$ Smith (1950)

↳ $f = f(\phi)$

DETERMINING FRICTION FACTORS FOR MEASURING PRODUCTIVITY OF GAS WELLS

R. V. SMITH, U. S. BUREAU OF MINES, BARTLESVILLE, OKLAHOMA

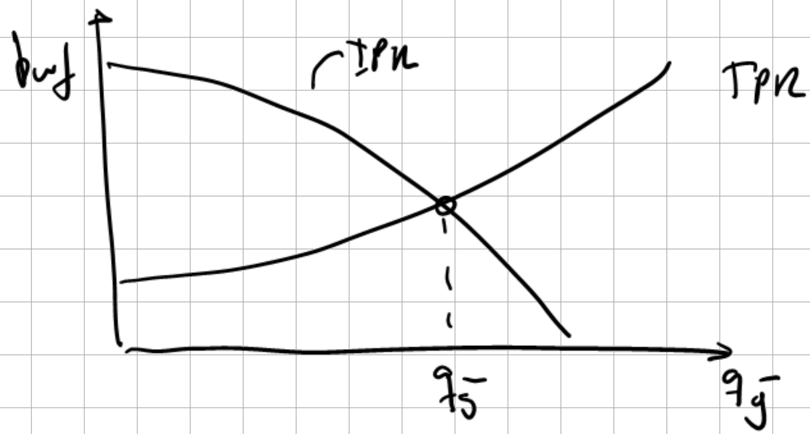
$$f_M = \frac{0.01748}{D^{0.224} \cdot \left(|1\,m| \cdot \left|\frac{39.37\,in}{1\,m}\right|\right)^{0.224}} = \frac{0.0077}{D^{0.224}}$$



1) neglecting flowline → $p_{wh} = p_{sep}$

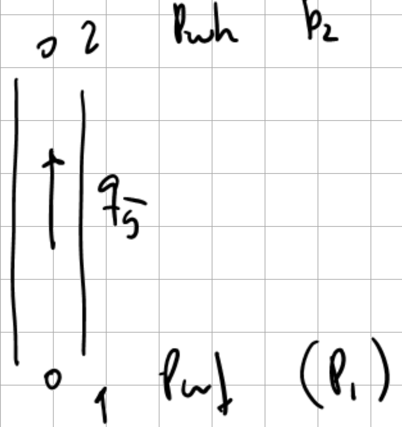
fully open choke

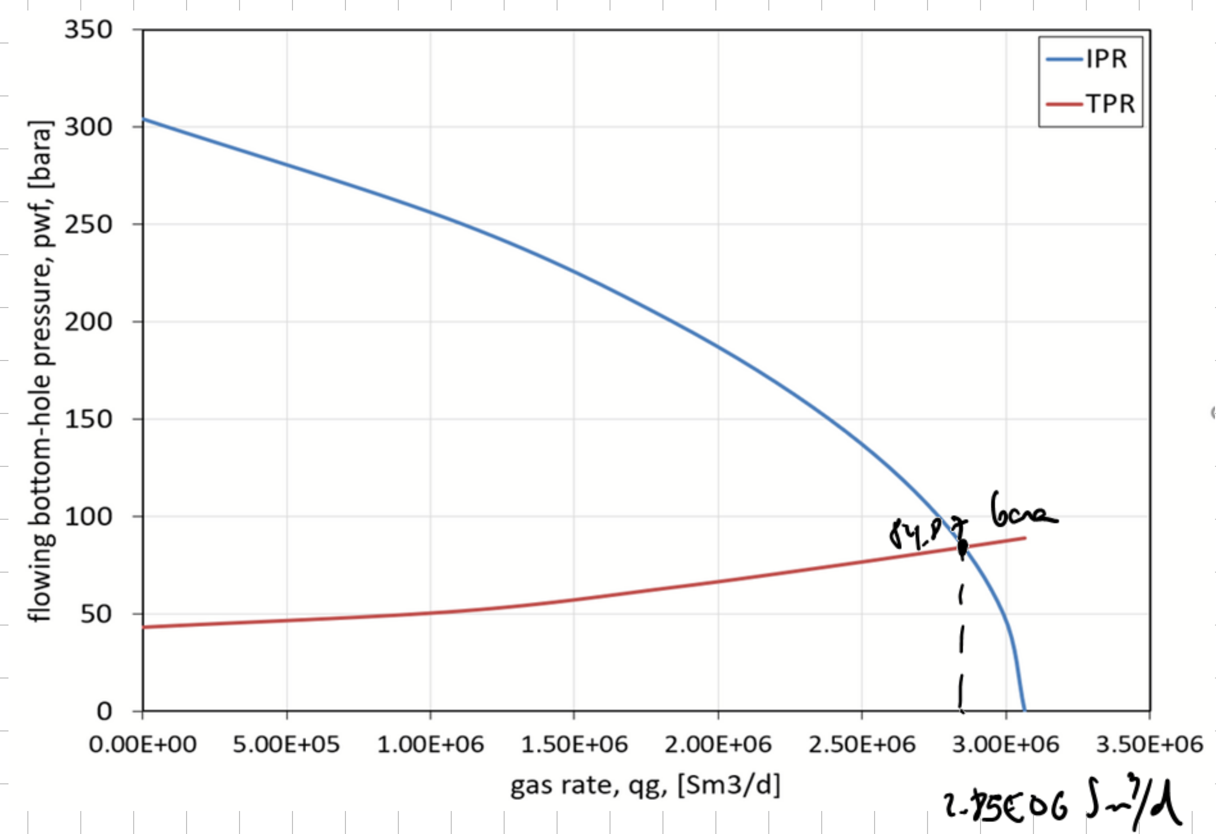
• select an equilibrium point → bottom-hole

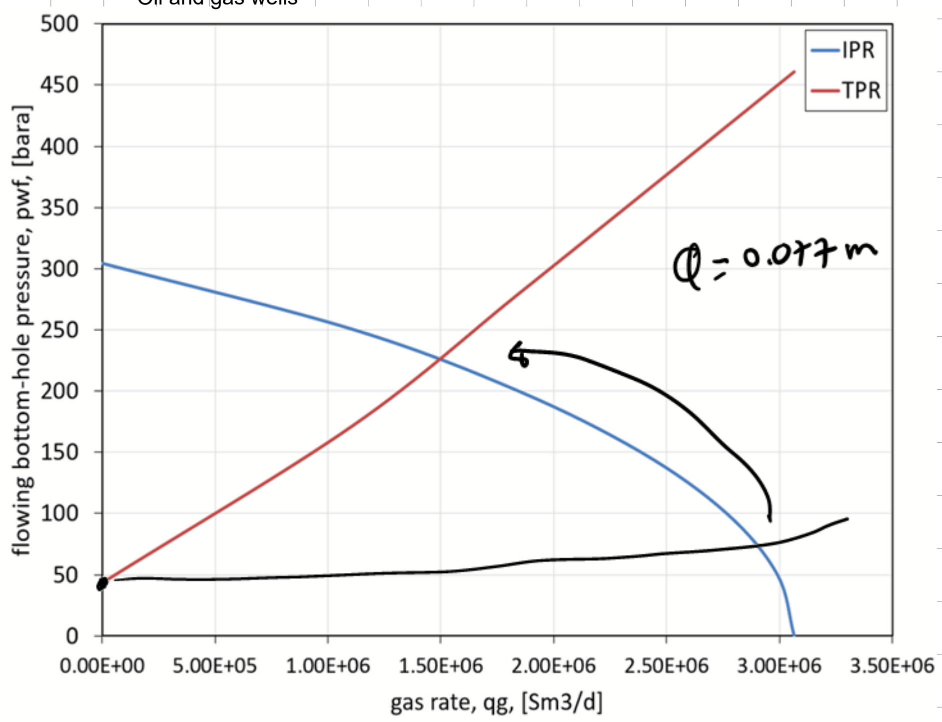


$$q_g = C_R \left(p_R^2 - p_{wf}^2\right)^{n}$$

$$q_g = C_T\left(\frac{p_{wf}^2}{e^S} - p_{wh}^2\right)^{0.5} \rightarrow p_{wf} = \sqrt{\left(p_{wh}^2 + \left(\frac{q_g}{C_T}\right)^2\right)e^S}$$

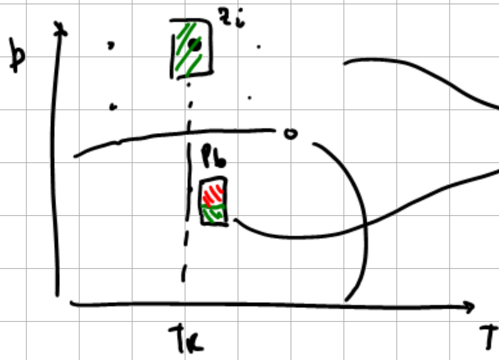


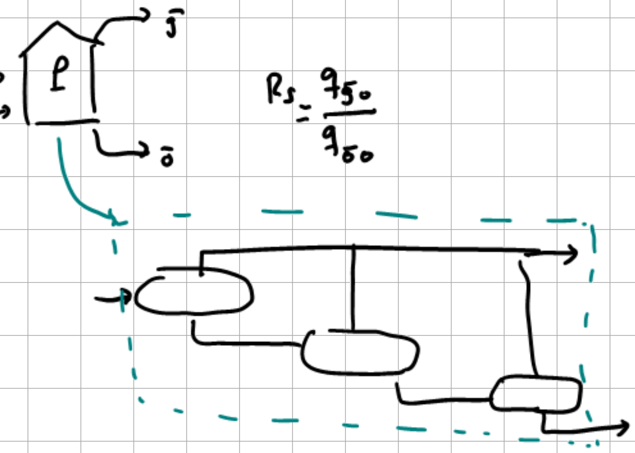




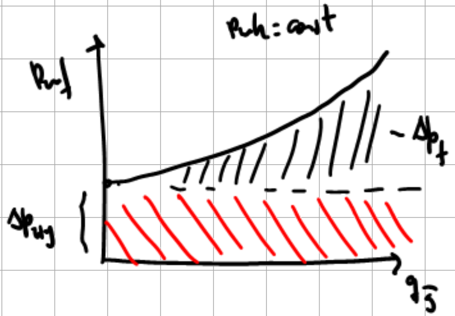
effect of tubing size

$\phi \downarrow \rightarrow G \downarrow \rightarrow q_{g}^{*} \downarrow$

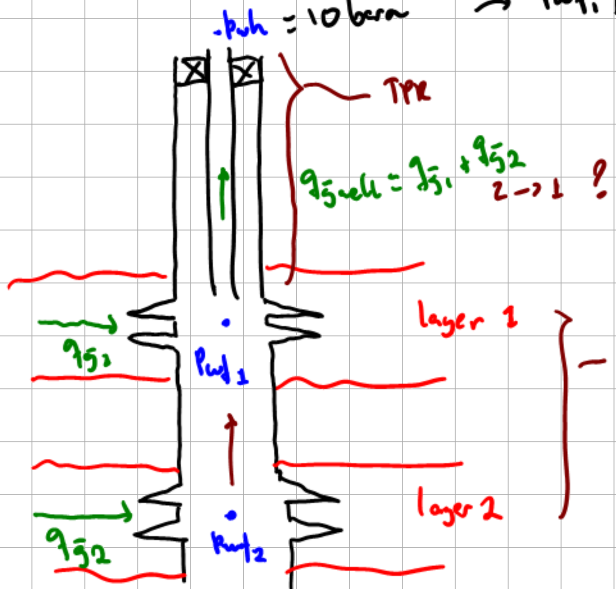




$$R_s = \frac{q_{go}}{q_{go}}$$



$p_{wh} = 10$ bara $\rightarrow p_{wf_1}, p_{wf_2}, q_{g1}, q_{g2}$



$$q_{sc} = C_T \cdot \left[\left(\frac{p_{wf}^2}{e^S} - p_t^2 \cdot\right)\right]^{0.5}$$

— CPR casing performance relationship

$$q_g = C_c \left[\frac{p_{wf_2}^2}{e^{S_c}} - p_{wf_1}^2\right]^{0.5}$$

$$C_c > C_T \;\; ?$$

$$Q_c > Q_T$$

$$C_T = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left(\frac{S \cdot e^S}{e^S - 1}\right)^{0.5}$$

$D \uparrow \rightarrow C \uparrow$

$$S = 2 \cdot \frac{M_g \cdot g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot \cos(\alpha) = 2 \cdot \frac{28.97 \cdot \gamma_g \cdot g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot \cos(\alpha)$$

$S_T > S_c$

$L_T > L_c$

		eq	new unknowns
TPR	$q_{well} = C_T \left(\frac{P_{wf_1}^2}{e^S} - P_{wh}^2 \right)^{0.5}$	(1)	2
IPR 1	$q_{g_1} = C_{R_1} \left(P_{R_1}^2 - P_{wf_1}^2 \right)^{n_1}$	1	1
CPR	$q_{g_2} = C_c \left(\frac{P_{wf_2}^2}{e^{S_c}} - P_{wf_1}^2 \right)^{0.5}$	1	2
IPR$_2$	$q_{g_2} = C_{R_2} \left(P_{R_2}^2 - P_{wf_2}^2 \right)^{n_2}$	1	0
mass conservation	$q_{well} = q_{g_1} + q_{g_2}$	1	0
		5	5 ✓

$P_{wf_1} = 150$ bara

$\xrightarrow{300.000}$ ↑ 230000

pwh [bara]	pwf1 [bara]	qwell [Sm3/d]	q1 [Sm3/d]	q2 [Sm3/d]
10	150.0	2.34E+05	3.23E+05	-8.94E+04

Oil and gas wells, Prof. Milan Stanko (NTNU)	pR [bara]	C [Sm3/bar2n]	n
Layer 1	200	2010	0.52
Layer 2	250	1150	0.54

Inflow performance relationship (IPR)

$$q_g = c(p_R^2 - p_{wf}^2)^n$$

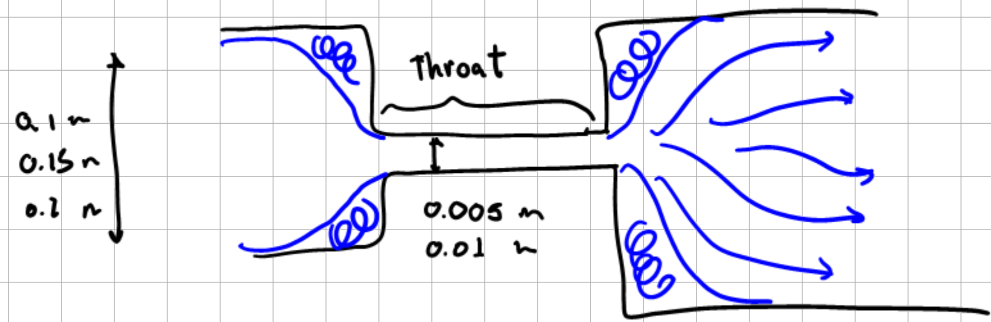
	Cf [Sm3/d/bar]	s
Tubing	2.09E+03	0.58
Casing between 1-2	4.00E+03	0.02

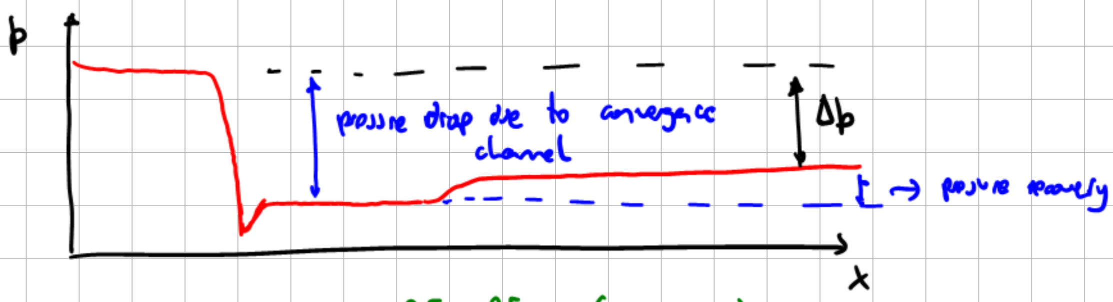
Tubing performance relationship (TPR)

$$p_{wh} = p_2 = \left(\frac{p_1^2}{e^s} - \frac{q_g^2}{C_T^2}\right)^{0.5}$$

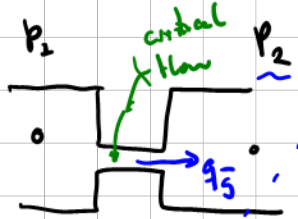
pwh [bara]	pwf1 [bara]	qwell [Sm3/d]	q1 [Sm3/d]	q2 [Sm3/d]	pwf2 (from TPR) [bara]	pwf2 (from IPR) [bara]	error [bar]
10	197.5	3.08E+05	7.21E+04	2.36E+05	208.1	208.1	0.0

Production choke





Dry gas

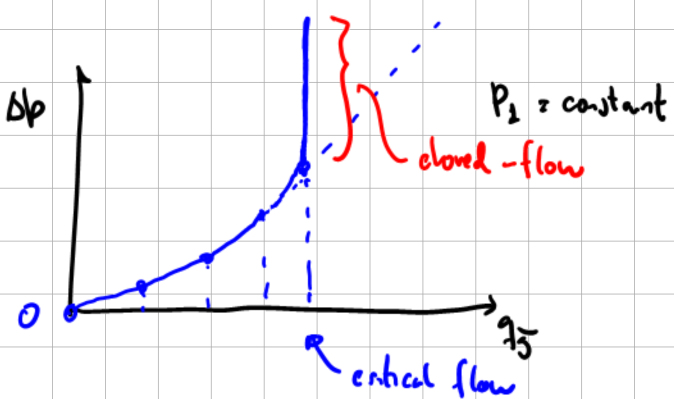


$\rightarrow v_g = v_{sound}$ (fluid, p,T)

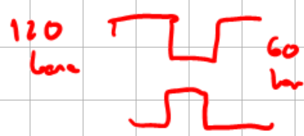
↳ 300 m/s (air)

Mach number $M = 1$

$M = \frac{v_g}{v_{sound}}$



$\frac{p_2^{out}}{p_1^{in}} = 0.5$ (0.4 - 0.6)



- flow performance $f(p_{in}, p_{out}, q_s, \phi_{throat})$
- temperature performance

B. Choke Equations

Undersaturated oil flow

Based on a frictionless flow contraction from an upstream point 1 to a downstream point 2.

The single-phase Bernoulli equation for steady state frictionless flow along a streamline, neglecting elevation changes, is:

$$\frac{dp}{\rho} + V \cdot dV = 0 \qquad \text{Eq. B-1}$$

Where:

p	Pressure
ρ	Density
V	Velocity

$p_{throat} \approx p_2$ $\quad \Delta p_{recovery} \approx 0$

Undersaturated oil

$$q_{\bar{o}} = \frac{A_2 \cdot C_d}{B_{o,@2}} \cdot \sqrt{\frac{2 \cdot (p_2 - p_1)}{\rho \cdot (1 - \beta^4)}}$$

dry gas

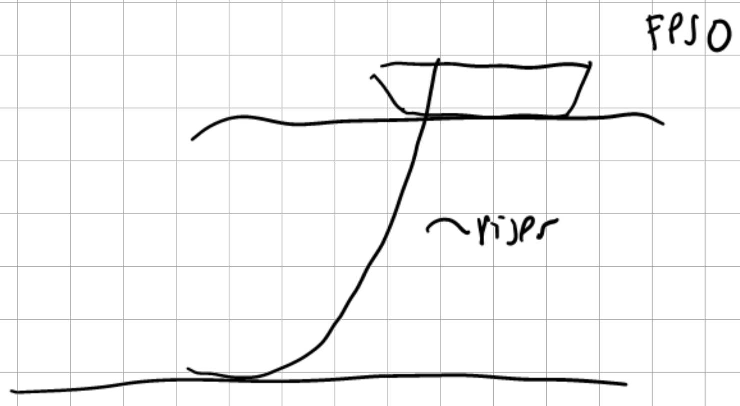
$$q_{\bar{g}} = \frac{p_1 \cdot T_{sc} \cdot A_2 \cdot C_d}{p_{sc}} \cdot \sqrt{2 \cdot \frac{R}{Z_1 \cdot T_1 \cdot M_w} \cdot \frac{k}{k-1} \cdot \left(y^{\frac{2}{k}} - y^{\frac{k+1}{k}}\right)}$$

$\frac{p_2}{p_1} \rightarrow y_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$ — adiabatic exponent of gas $\quad k = \frac{C_p}{C_v}$

$k = 1.3$

$\frac{p_2}{p_1} > y_c$ $\qquad$ $\frac{p_2}{p_1} < y_c$

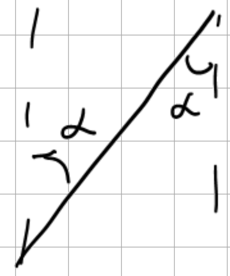
$$q_{\bar{g}} = \frac{p_1 \cdot T_{sc} \cdot A_2 \cdot C_d}{p_{sc}} \cdot \sqrt{2 \cdot \frac{R}{Z_1 \cdot T_1 \cdot M_w} \cdot \frac{k}{k-1} \cdot \left(y_c^{\frac{2}{k}} - y_c^{\frac{k+1}{k}}\right)}$$



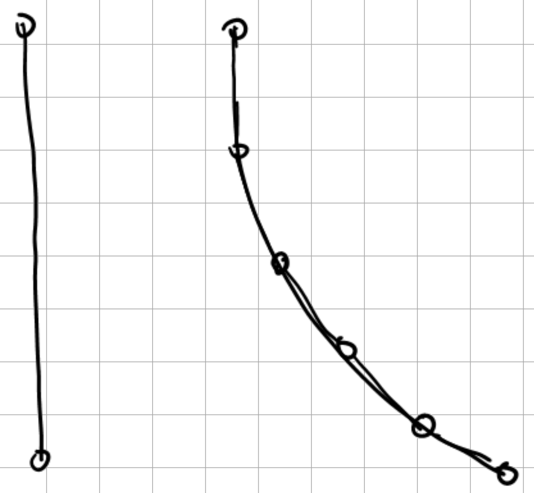
C_{PL}, C_{FL}, C_{aver} $\qquad p_1 \rightarrow p_2$

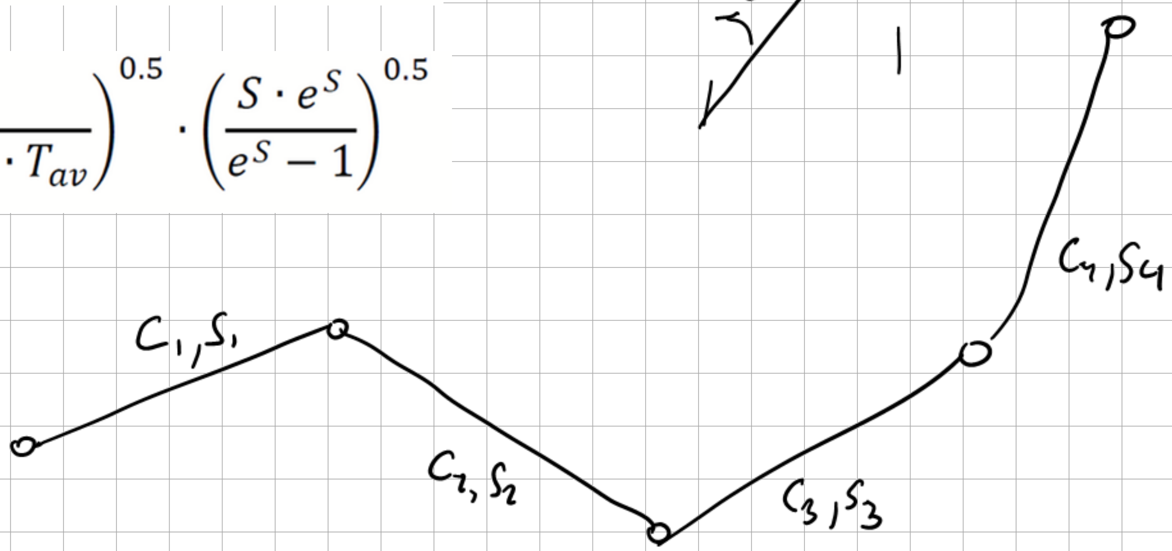
$$q_{sc} = C_T \cdot \left[\left(\frac{p_{wf}^2}{e^S} - p_t^2 \cdot \right) \right]^{0.5}$$

$$S = 2 \cdot L \cdot C_a = 2 \cdot \frac{M_g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot g \cdot \cos(\alpha)$$



$$C_T = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left(\frac{S \cdot e^S}{e^S - 1}\right)^{0.5}$$





Special case of horizontal pipe $\quad S=0$

however when calculating C $\rightarrow \dfrac{S \cdot e^S}{(e^S - 1)}$ $\qquad S=0 \qquad \dfrac{0 \cdot 1}{(1-1)^{\to 0}} = \dfrac{0}{0}$

use L'Hopital rule

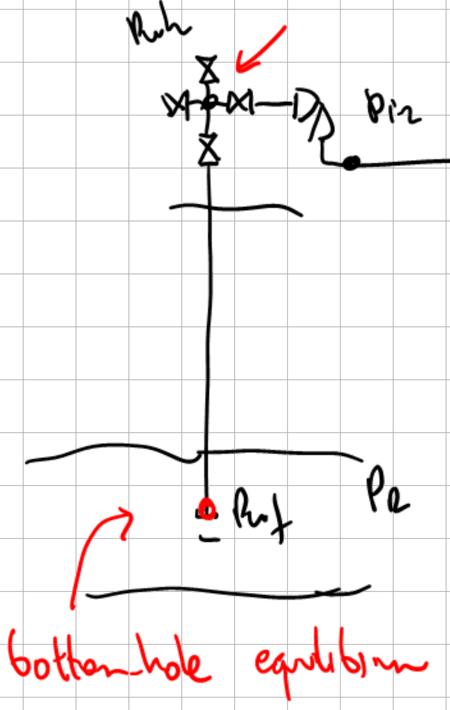
$$\lim_{x \to 0} \frac{f(x)}{g(x)} \rightarrow \text{undefined}$$

$$\lim_{x \to 0} \frac{f'(x)}{g'(x)} \quad \frac{(e^S + S e^S)}{(e^S)} \overset{S \to 0}{=} \frac{1+0}{1} = 1$$

in horizontal pipe $\quad \dfrac{S e^S}{(e^S - 1)} = 1$

if $\phi \uparrow \rightarrow C_{FL} \uparrow \rightarrow$ lower Δp (or higher rate)

if $L \uparrow \rightarrow C_{FL} \downarrow \rightarrow$ higher Δp (or lower rate)

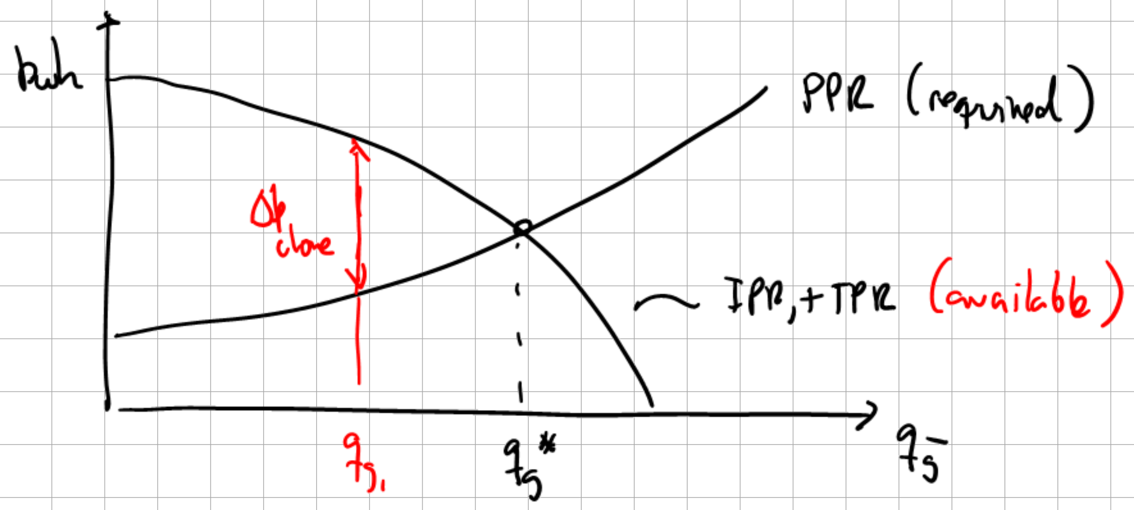


$\Delta p_{choke} = (P_{wh} - P_{in})$

when $\Delta p_{choke} = 0$

$P_{wh} = P_{in}$

to do equilibrium with flowline



p_R, Res pressure	304	bara	
C_R	104	Sm3/d/bar^2n	
n, exponent	0.9		
C_t, tubing	4.25E+04	$Sm^3/d/bar$	
s, elevation	0.155		
C_{fl}, flowline	1.25E+05	4.00E+04	$Sm^3/d/bar$
p_{sep}	40	bara	

$$q_g = c(p_R^2 - p_{wf}^2)^n$$

Tubing performance relationship (TPR)

$$p_{wh} = p_2 = \left(\frac{p_1^2}{e^s} - \frac{q_g^2}{C_T^2}\right)^{0.5}$$

Flowline performance relationship (FPR)

$$\boxed{q_{\bar{g}} = C_{FL} \cdot \left(p_{in}^{\ 2} - p_{out}^{\ 2}\right)}$$

assumed

IPR-qg

pwf [bara]	qg-IPR [Sm3/d]	pwf-TPR [Sm3/d]	WPR [Sm3/d]	PPR - 1 [bara]	PPR - 2 [bara]
304	0.00E+00	43			
250	1.11E+06	52			
200	1.84E+06	64			
150	2.38E+06	74			
100	2.76E+06	82			
50	2.99E+06	87			
0	3.06E+06	89			
84.378937	2.85E+06	84			

wellhead performance relationship

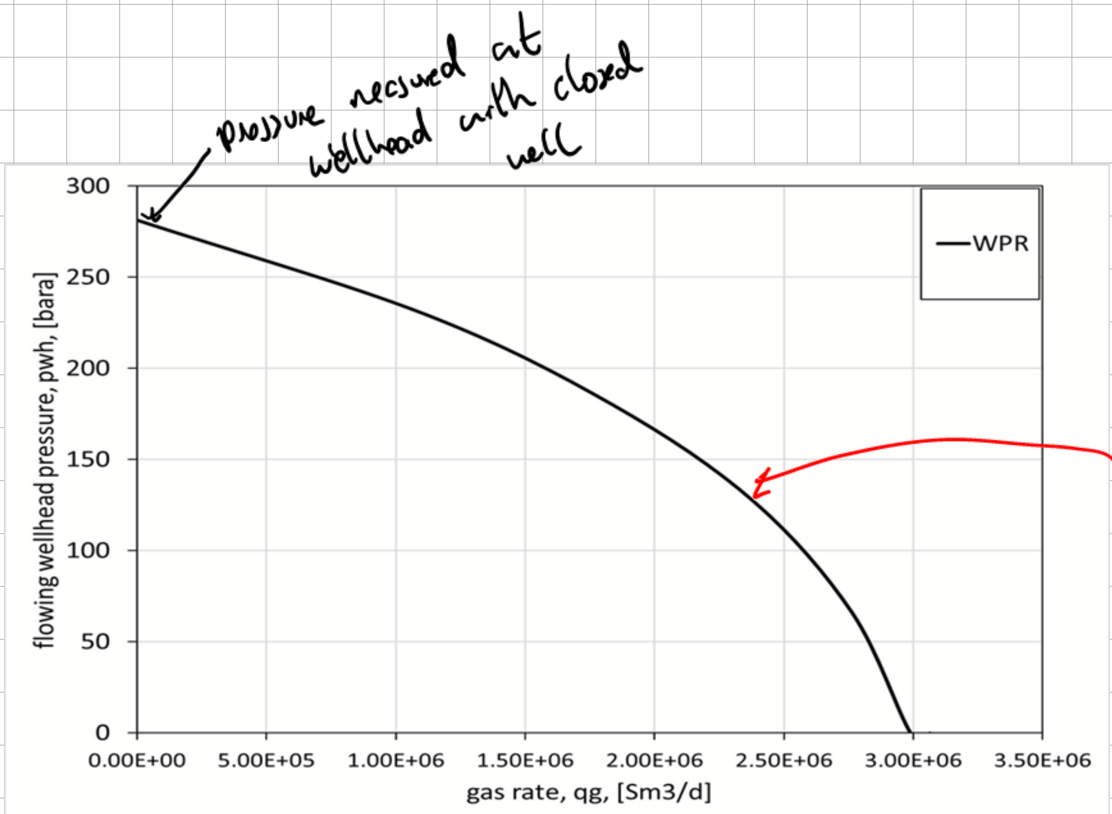
pwf → pwh
TPR

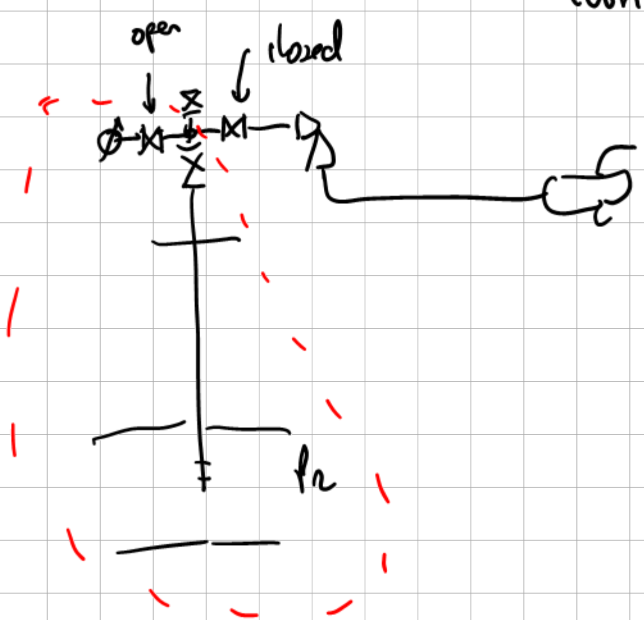
required, with TPR, assuming Pwh = 40 bara

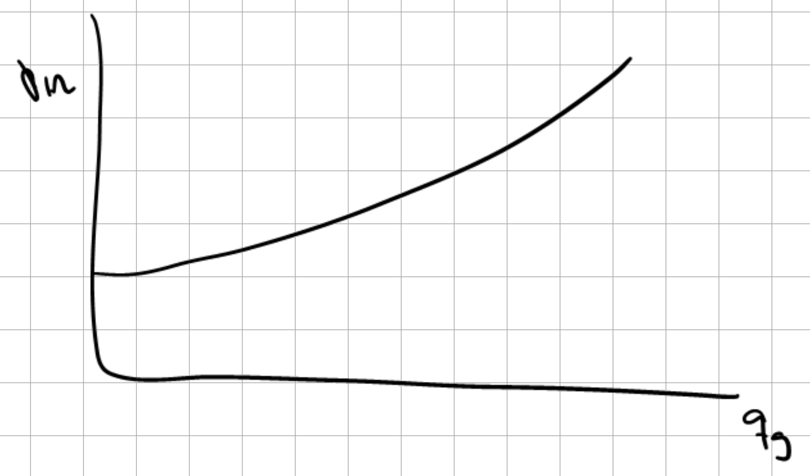
tubing_qg (P1, P2)

calc current → tubing_P1 (P2, qg)

tubing_P2 (P1, qg)







$$q_g = C_{FL} \cdot \left(\frac{p_1^2}{e^S} - p_2^2\right)^{0.5}$$

horizontal $S \to 0$

$$q_g = C_{FL}\left(p_1^2 - p_2^2\right)^{0.5}$$

required pressure

$$p_1 = \sqrt{p_2^2 + \left(\frac{q_g}{C_{FL}}\right)^2}$$

```
Function Linep1(Cfl, p2, qg)
Linep1 = (p2 ^ 2 + (qg / Cfl) ^ 2) ^ 0.5
End Function
```

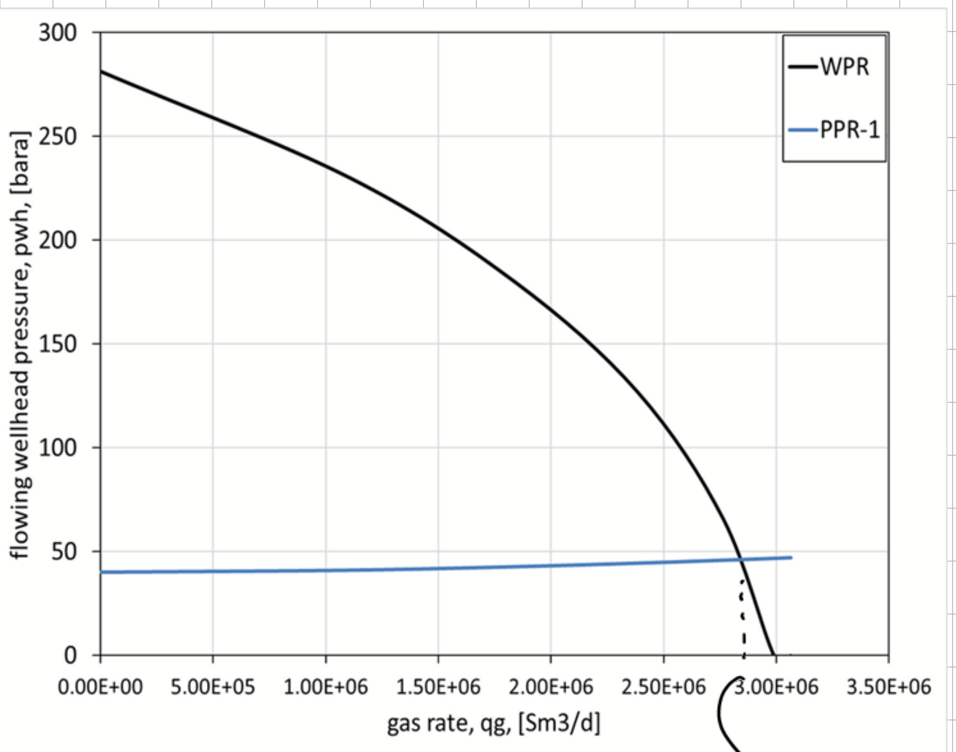
pwf [bara]	qg-IPR [Sm3/d]	pwf-TPR [Sm3/d]	WPR [Sm3/d]
304	0.00E+00	43	281
250	1.11E+06	52	230
200	1.84E+06	64	180
150	2.38E+06	74	127
100	2.76E+06	82	66
50	2.99E+06	87	#VALUE!
0	3.06E+06	89	#VALUE!

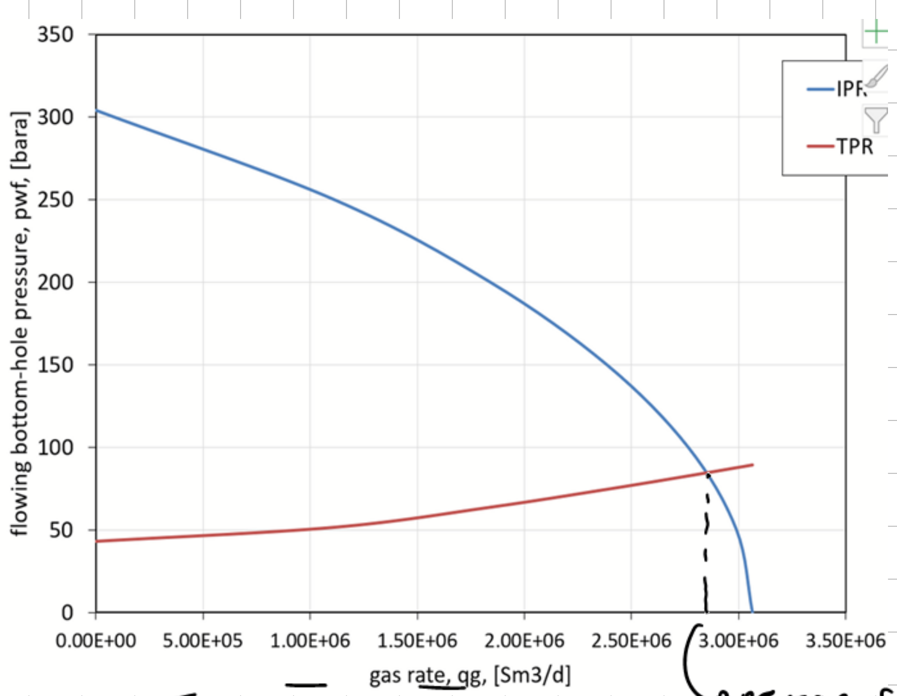
$$q_g = C_r\left(\frac{p_{wf}^2}{e^S} - p_{wh}^2\right)^{0.5}$$

$$p_{wh} = \sqrt{\frac{(p_{wf})^2}{e^S} - \left(\frac{q_g}{C_r}\right)^2}$$

(+)

(−)



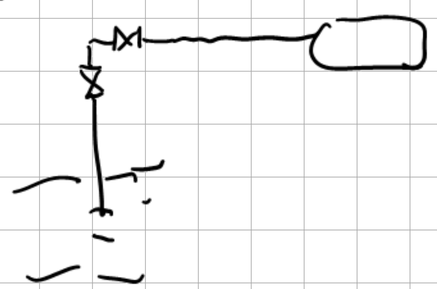


a bit lower

→ 2.85E06 Sm³/d

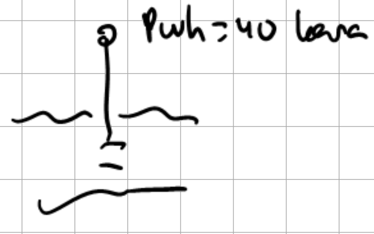
$P_{sep} = 40$ bara

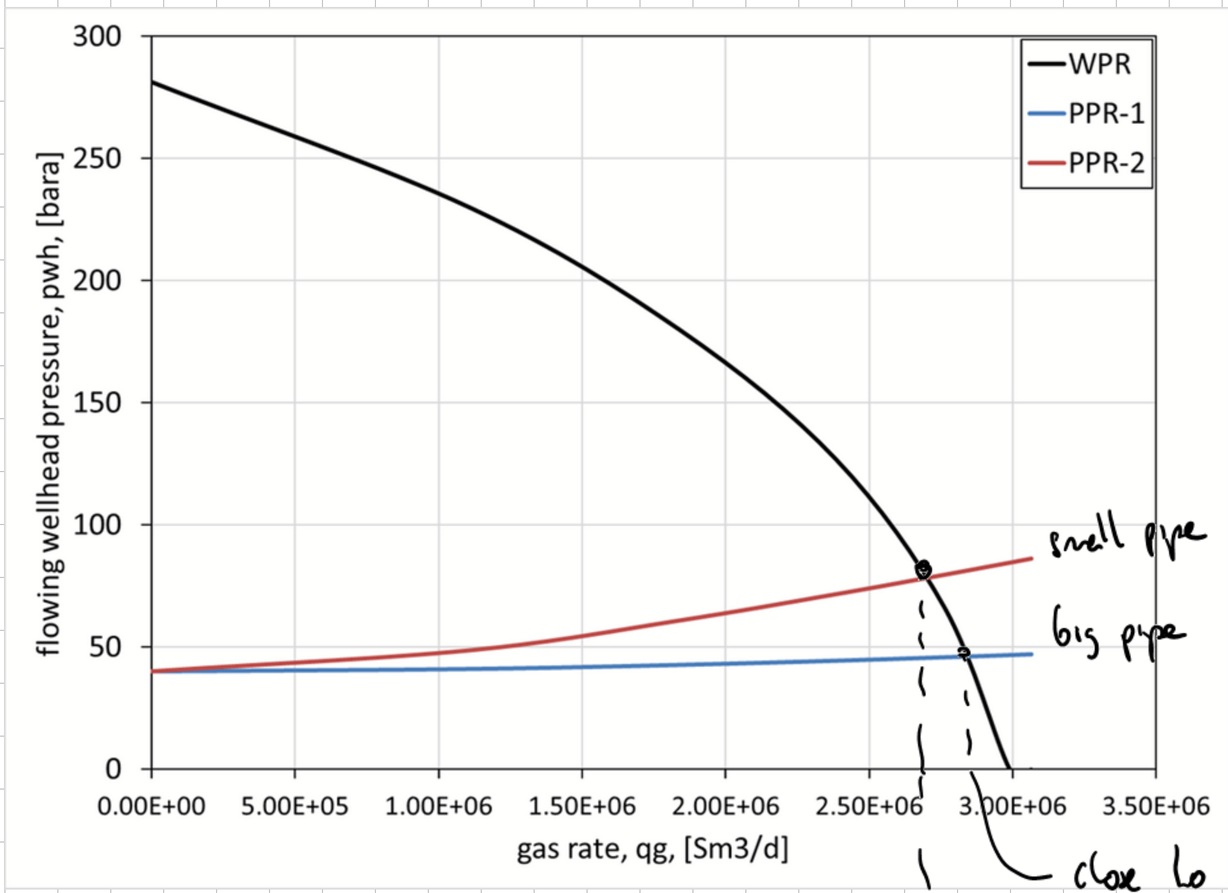
wellhead eq



bottom-hole eq

$P_{wh} = 40$ bara





small pipe

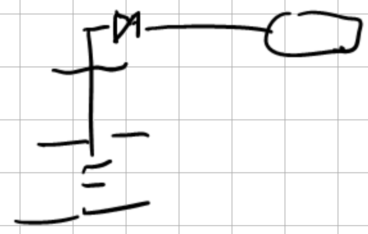
big pipe

close to 2.85 E06 Sm³/d

much lower than 2.85 E06 Sm³/d

wellhead eq

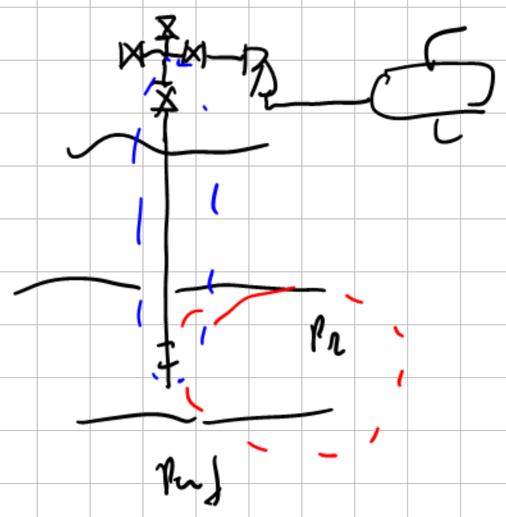
pipeline included



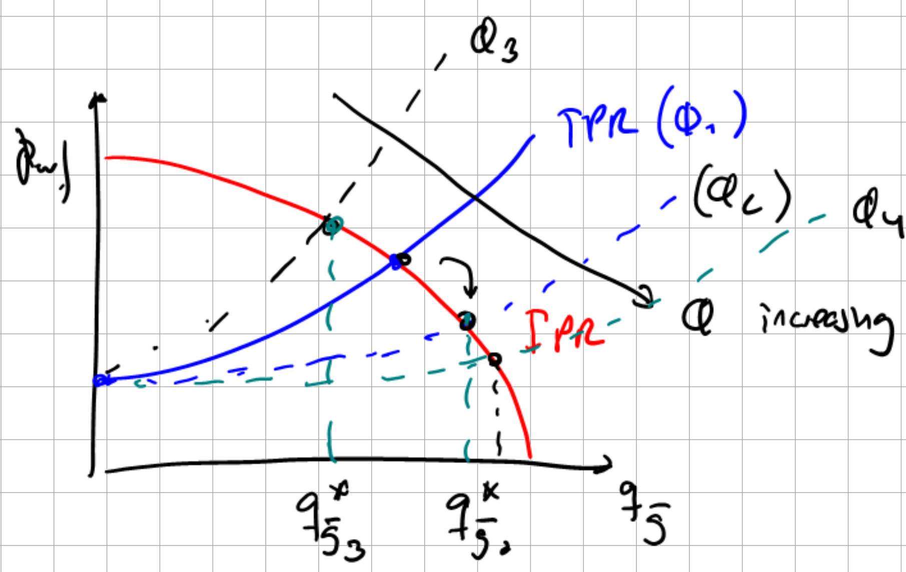
{ small pipeline
long pipeline

Tubing size considerations: (ϕ)

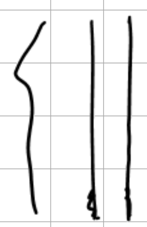
- Maximize q_o, q_g → increase equilibrium rate
  ↳ revenue



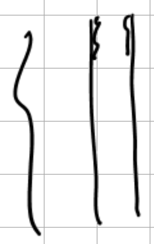
$P_{wh} = p_{sep}$



- minimize cost ↓ϕ, ↓ cost



tubing comes in discrete sizes $\phi \rightarrow \begin{cases} \phi_1 \\ \phi_2 \\ \phi_3 \end{cases}$



tubing according to API specs

used in pipe flow calculations

Tubing Size		Nominal Weight						Threaded Coupling					Joint Yield Strength		Capacity Table	
								Coupling Outside Dia.								
Nom. in.	OD in.	T & C Non-Upset lb/ft	T & C Upset lb/ft	Grade	Wall Thick-ness in.	Inside Dia. in.	Drift Dia. in.	Non-Upset in.	Upset Reg. in.	Upset Spec. in.	Col-lapse Resis-tance psi	Internal Yield Pres-sure psi	T & C Non-Upset lb	T & C Upset lb	Barrels per Linear ft	Linear ft per Barrel
3/4	1.05	1.14	1.20	H-40	0.113	0.824	0.730	1.313	1.660		7,200	7,530	6,360	13,300	0.0007	1516.13
				J-55							9,370	10,360	8,740	18,290		
				C-75							12,250	14,120	11,920	24,940		
				N-80							12,710	15,070	12,710	26,610		
1	1.315	1.700	1.800	H-40	0.113	1.049	0.955	1.660	1.900		6,820	7,080	10,960	19,760	0.0011	935.49
				J-55							8,860	9,730	15,060	27,160		
				C-75							11,590	13,270	20,540	37,040		
				N-80							12,270	14,160	21,910	39,510		
1 1/4	1.660	2.300	2.400	H-40	0.125	1.410	1.286	2.054	2.200		5,220	5,270			0.0019	517.79
				H-40	0.140	1.380					5,790	5,900	15,530	26,740	0.0018	540.55
				J-55	0.125	1.410					6,790	7,250			0.0019	517.79
				J-55	0.140	1.380					7,530	8,120	21,360	36,770	0.0018	540.55
				C-75	0.140	1.380					9,840	11,070	29,120	50,140	0.0018	540.55
				N-80	0.140	1.380					10,420	11,810	31,060	53,480	0.0018	540.55

Non-API tubing (tubulars)

VAM – Vallourec (French)

	VAM® 21	VAM® 21 HT	VAM TOP ®	VAM TOP ® HT	VAM TOP ® HC	VAM® HTTC	VAM® HP	VAM® HW ST	VAM® LOX	VAM® BOLT-II	VAM® HTF-NR	VAM® FJL	VAM® MUST	VAM® SG	VAM® EDGE SF	VAM® SLIJ-II	VAM® LIFT	DINO VAM®	BIG OMEGA ®	VAM TOP ® FE	VAM® TTR
2 3/8			✓									✓									
2 7/8			✓									✓									
3 1/2	✓		✓			✓						✓									
4			✓									✓									
4 1/2	✓	✓	✓	✓	✓	✓					✓	✓		✓		✓					
5	✓	✓	✓	✓	✓	✓		✓			✓	✓		✓	✓	✓					
5 1/2	✓	✓	✓	✓	✓	✓		✓			✓	✓	✓	✓	✓	✓					
5 3/4			✓																		
6	✓	✓	✓	✓	✓																
6 5/8	✓	✓	✓	✓	✓	✓		✓			✓	✓				✓				✓	
6 7/8																✓					
7	✓	✓	✓	✓	✓	✓	✓	✓			✓	✓				✓		✓		✓	
7 5/8	✓	✓	✓	✓	✓		✓	✓			✓	✓	✓			✓				✓	
7 3/4	✓		✓		✓						✓					✓				✓	
8																				✓	
8 1/8												✓									
8 5/8	✓		✓				✓					✓				✓				✓	✓
8 3/4	✓																				
9							✓														
9 3/8											✓	✓									
9 5/8	✓	✓	✓			✓		✓			✓	✓				✓		✓		✓	✓
9 3/4																✓		✓			
9 7/8	✓		✓								✓	✓				✓		✓		✓	
10	✓		✓													✓					
10 1/8	✓		✓				✓			✓						✓					
10 1/2							✓														
10 3/4	✓		✓				✓	✓				✓	✓			✓		✓		✓	✓

tub

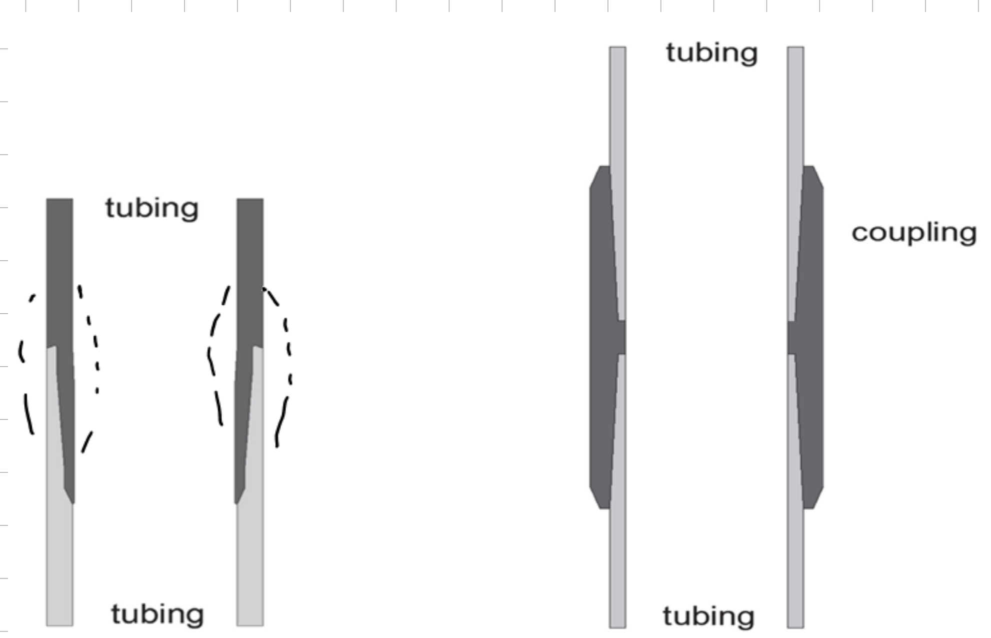
casing

riser
flowline

⋮
26

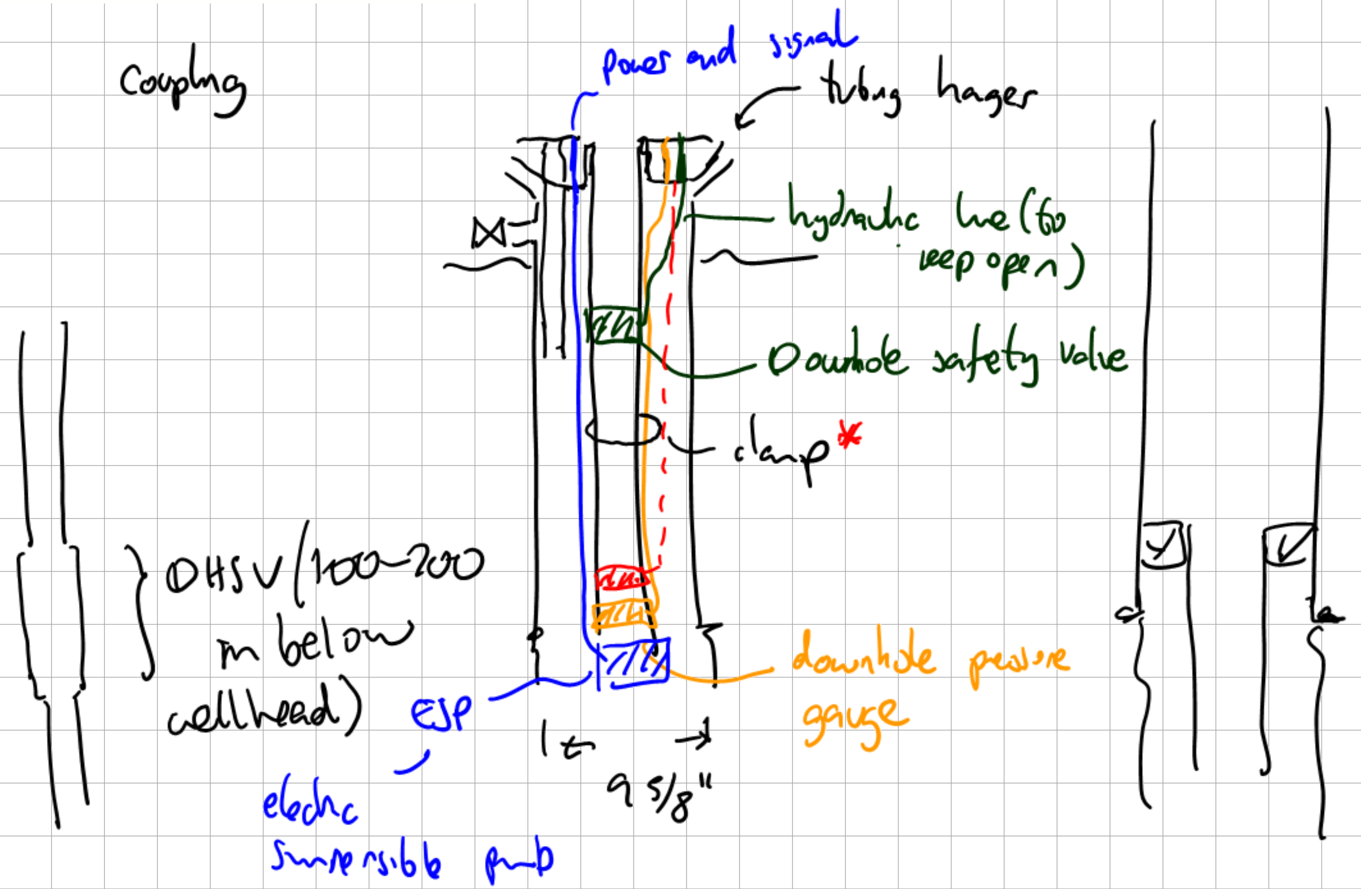
7'' - 32# - P-110 - BTC/LTC

- 7'': nominal size
- 32#: weight per length, pound per foot, ppf
- P-110: steel "type", grade, resistance
- BTC/LTC: type of thread



integrated joint Coupling

- tubing hanger considerations



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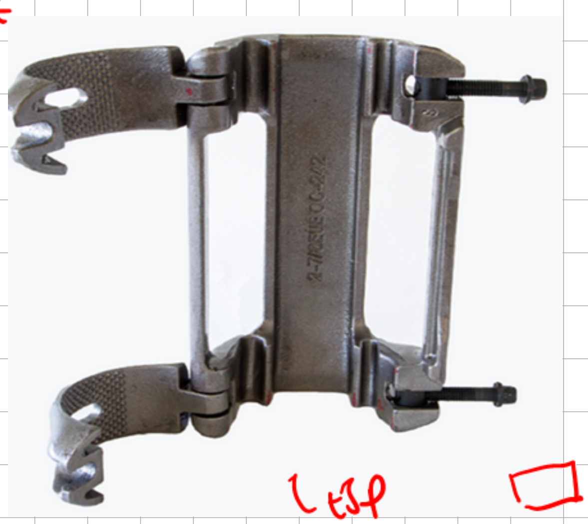
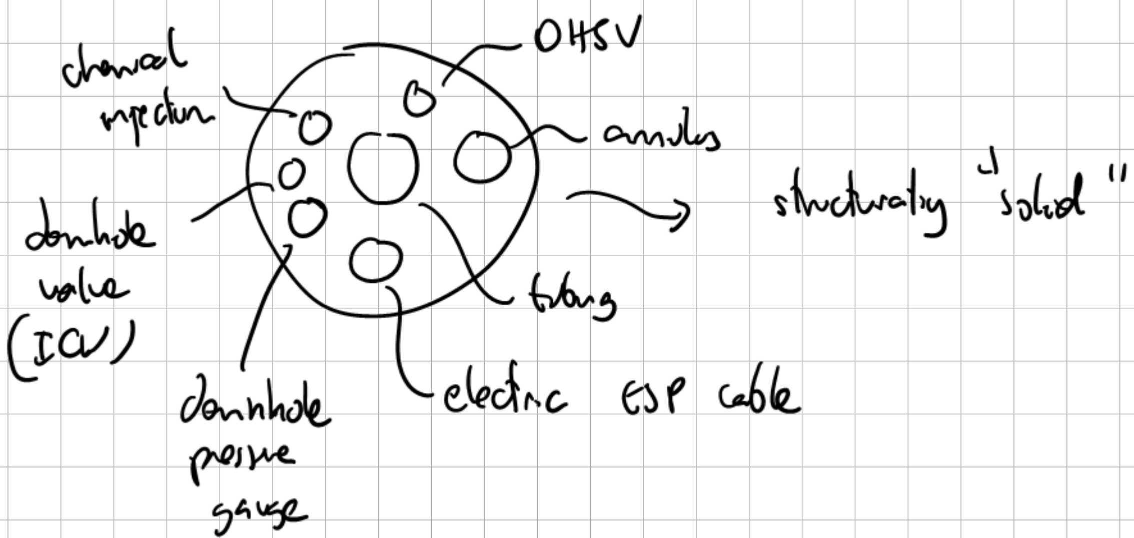
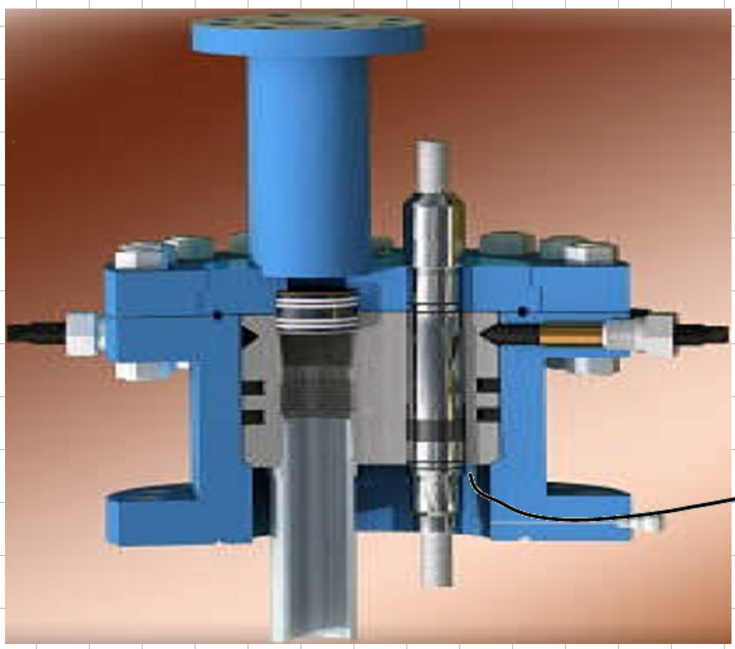






penetrator

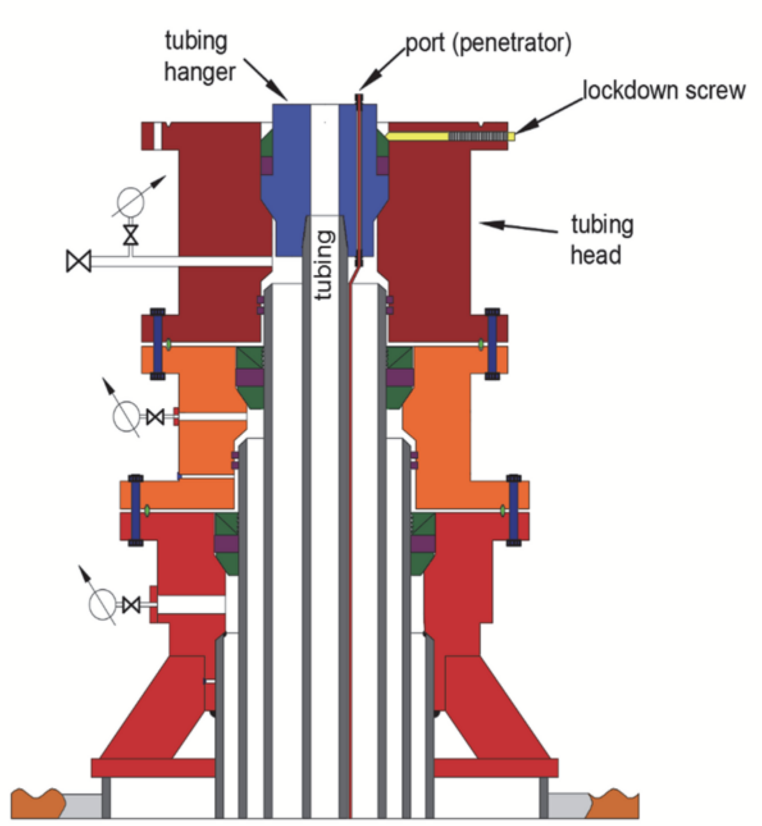
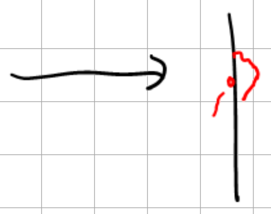


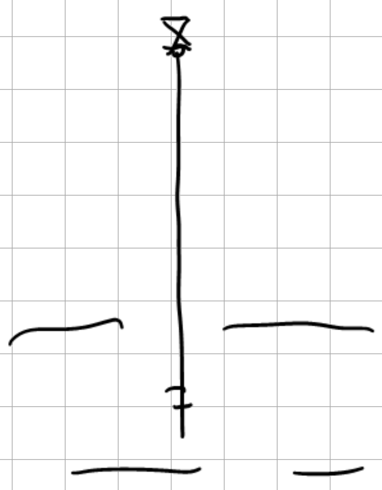
FIGURE 6-10. FINAL CONFIGURATION OF THE WELLHEAD

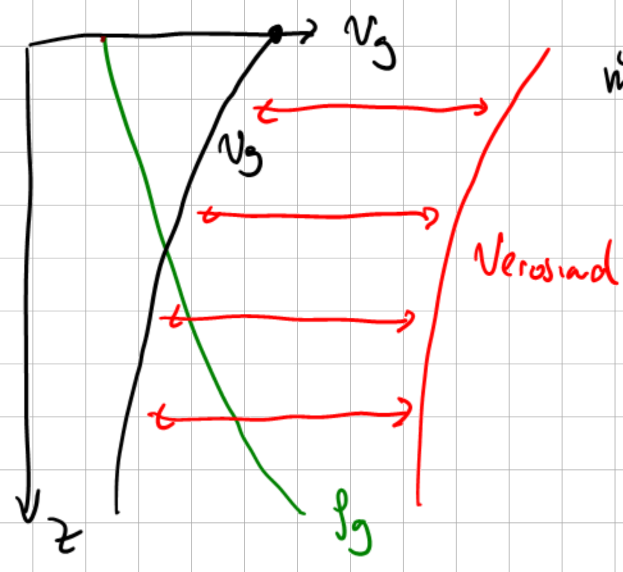
- erosional velocity

$$v_f < \underbrace{V_{erosional\ velocity}}$$



gas well $\quad v_f = v_g = \frac{q_g}{A}$





$$\dot{m}_g = \rho_{\bar{g}}\, q_{\bar{g}} = q_g \cdot \rho_g = v_g \cdot A \cdot \rho_g$$

$$v_g = \frac{\rho_{\bar{g}}\, q_{\bar{g}}}{A\, \rho_g}$$ constant with depth

API 14E $\sim$ $V_{erosional} = \frac{C}{\sqrt{\rho_m}} \rightarrow$ (100 – 125) "severe" $\left[ft/s \right]$

conservative

$\sim \left[lb/ft^3 \right]$

$$\rho_m = f(\rho_L, \rho_g)$$

e.g. $\quad \rho_m = \frac{q_o \rho_o + q_g \rho_g + q_w \rho_g}{q_o + q_g + q_w}$

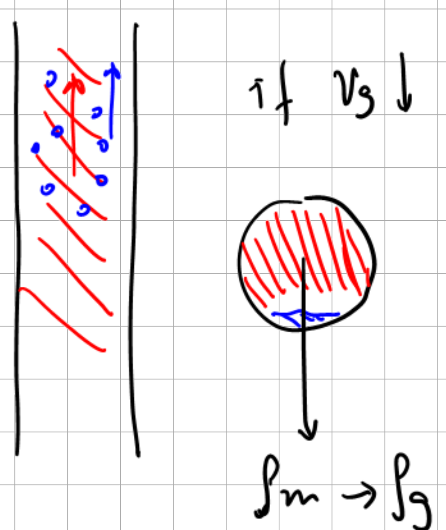
what happens if there is more than 1 fluid in the tubing?

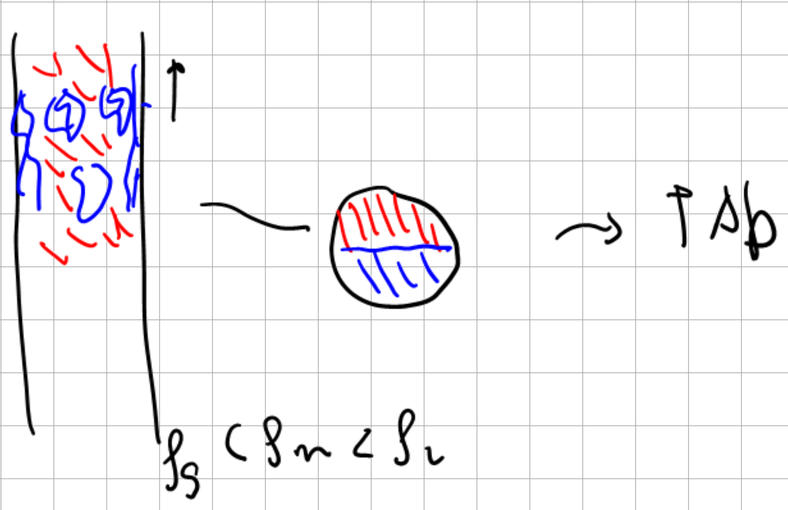
$$o + g + w$$

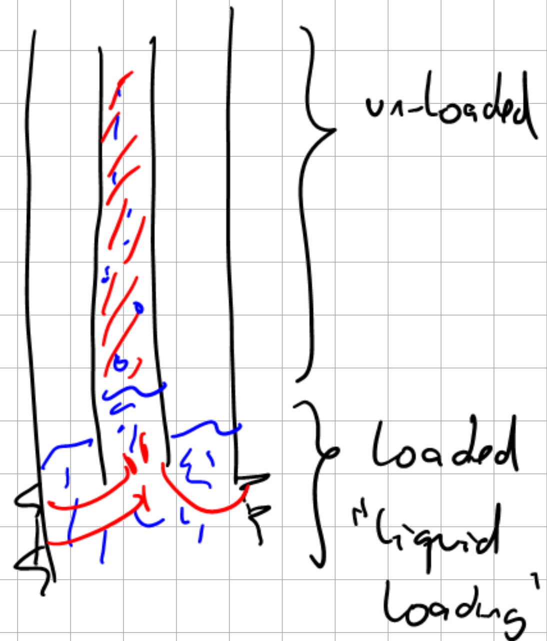
$$V_m = \frac{(q_o + q_g + q_w)}{A} = \underbrace{\frac{q_o}{A}}_{u_{so}} + \underbrace{\frac{q_g}{A}}_{u_{sg}} + \underbrace{\frac{q_w}{A}}_{u_{sw}}$$

↳ superficial velocity

- only in gas wells → avoid liquid loading



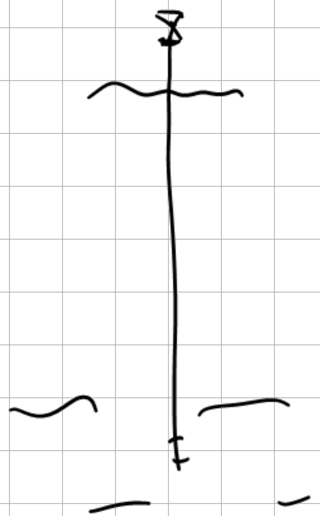


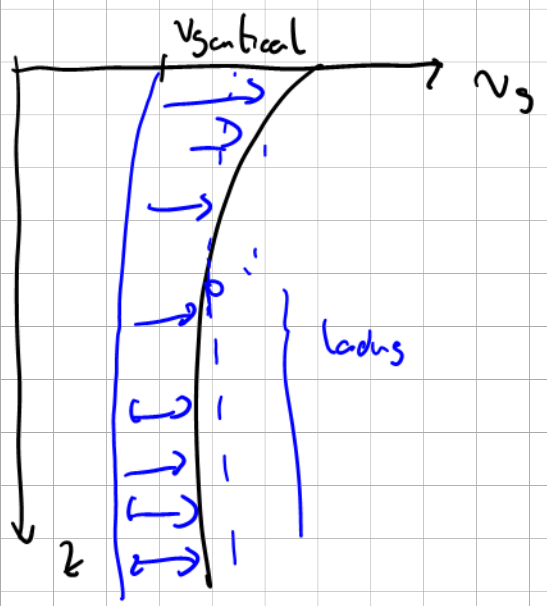


$$H_L = \frac{A_L}{A} \qquad \rho_m = (H_L)\rho_L + (1-H_L)\rho_g$$

in general $H_L \neq \frac{q_L}{q_L + q_g}$

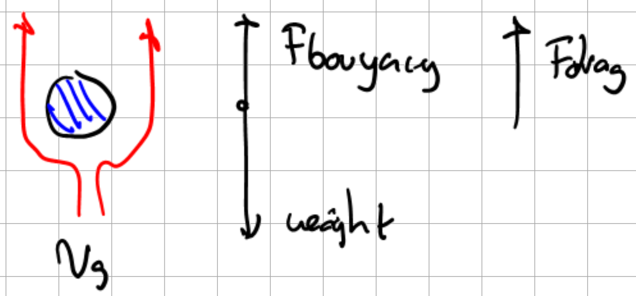
$v_g > v_{critical}$ for all tubing depths





How to calculate $V_{critical}$?

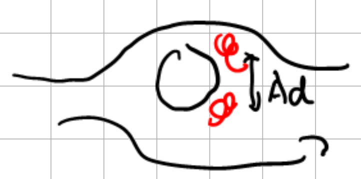
Turner criteria



$\underbrace{F_{drag}} + F_{buoyancy} = F_{weight}$

$$\frac{1}{2}\rho_g C_D v_g^2 A_d + V_d \rho_g g = V_d \cdot \rho_L \cdot g$$

$C_D \rightarrow 0.44$



$V_d = \frac{\pi \phi^3}{6}$ $\quad A_d = \frac{\pi \phi^2}{4}$

$$\frac{1}{2}\rho_g\, 0.44\, v_g^2 \frac{\cancel{\pi}\phi^{\cancel{2}}}{\cancel{4}} = (\rho_L - \rho_g)\, g\, \frac{\cancel{\pi}\phi^{\cancel{3}}}{\cancel{6}}$$

$$v_{crit} = \sqrt{\frac{4}{3}\frac{(\rho_L - \rho_g)}{\rho_g}\frac{\phi\, g}{0.44}}$$

Analysis and Prediction of Minimum Flow Rate for the Continuous Removal of Liquids from Gas Wells

R. G. Turner, SPE-AIME, Baker Oil Tools, Inc.
M. G. Hubbard, SPE-AIME, U. of Houston
A. E. Dukler, U. of Houston

1969

$N_{We} = \frac{\rho_g u_g^2 d}{\sigma_{gL}}$ — Drag

σ_{gL} — interfacial tension

Weber

Fundamentals of the Hydrodynamic Mechanism of Splitting in Dispersion Processes

J. O. Hinze, Royal Dutch Shell-Laboratory, Delft, Holland

1955

$N_{We} = 30 \rightarrow \phi_{max}$

$$30 = \frac{\rho_g u_g^2 \phi}{\sigma_{Lg}}$$

$$\phi = \frac{\sigma_{Lg}\, 30}{\rho_g u_g^2}$$

for every depth in tubing

$v_g \geq V_{crit}$

$$V_{crit} = 5.46\left[\frac{\sigma_{Lg}^{0.25}(\rho_L - \rho_g)^{0.25}}{\rho_g^{0.5}}\right] \quad [m/s]$$

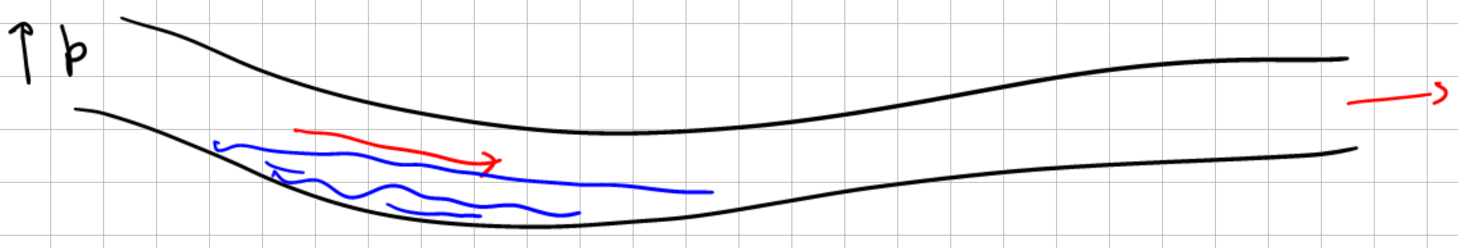
σ in $[N/m]$

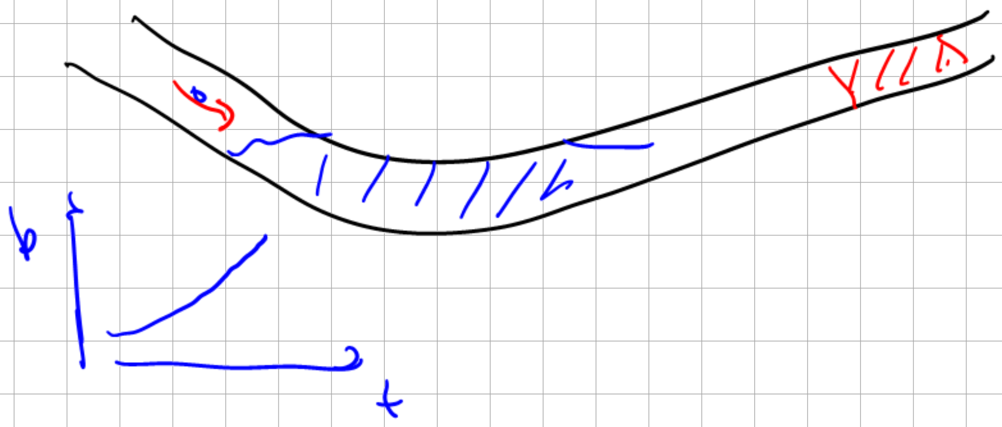
ρ in $[kg/m^3]$

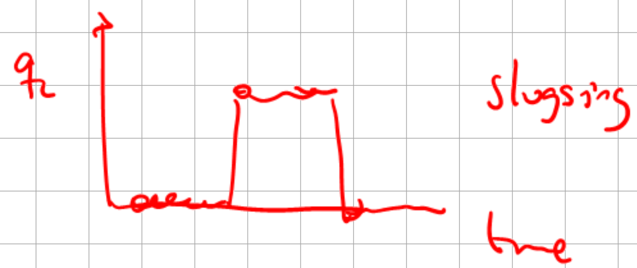
- pipeline sizing considerations

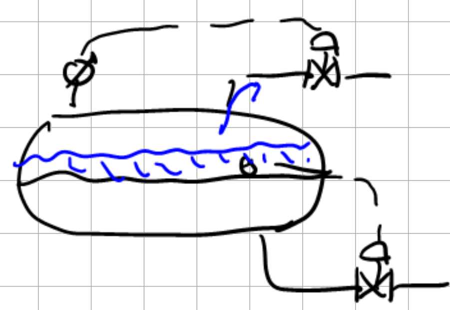
$\uparrow q_{g}$, $\uparrow q_{o}$ $\downarrow$ cost

- v_g high enough to avoid accumulation of liquid (gas condensate pipeline)









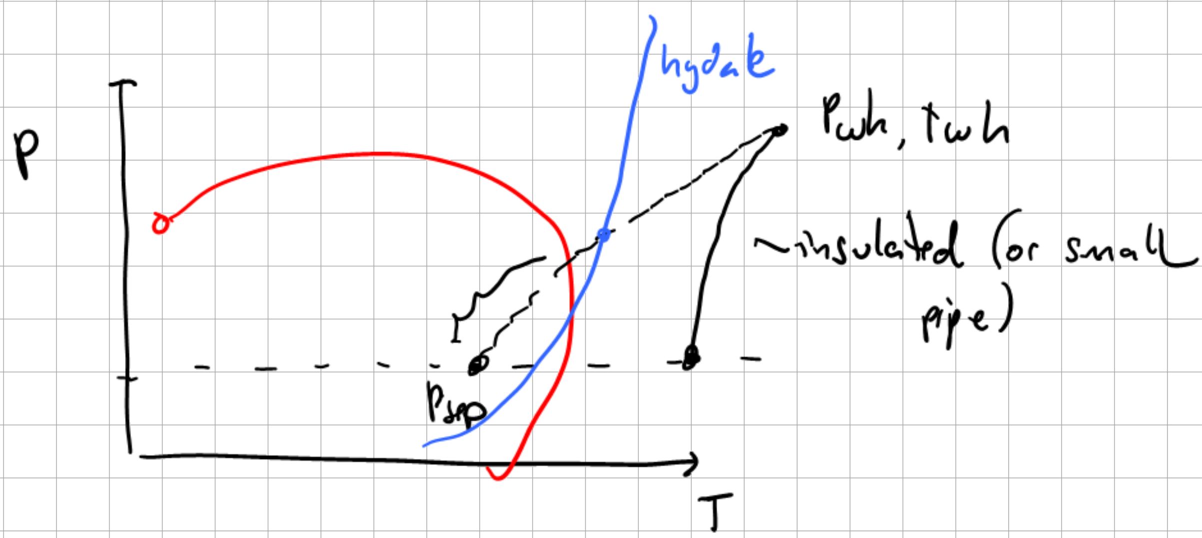
- heat transfer

$\dot{q} = A \cdot U (T_f - T_{amb})$

$\pi D \cdot L \, U (T_f - T_{amb})$

$D \uparrow$ $\dot{q} \uparrow$ $T_f \rightarrow \downarrow$

can also cause hydrate formation

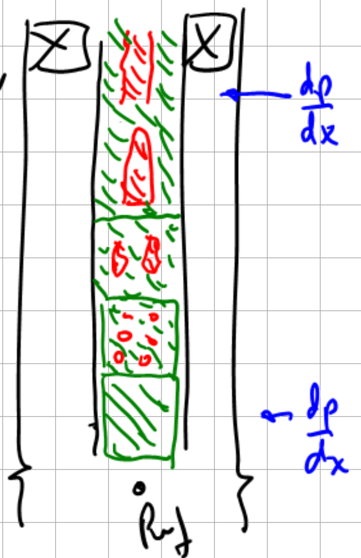


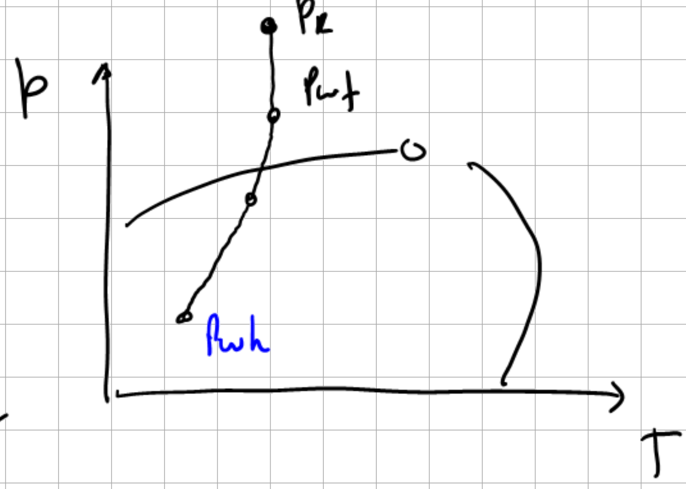
annular flow

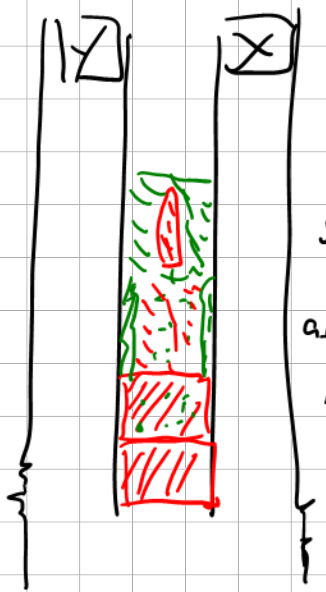
slug flow

churn flow

bubble flow







slug flow

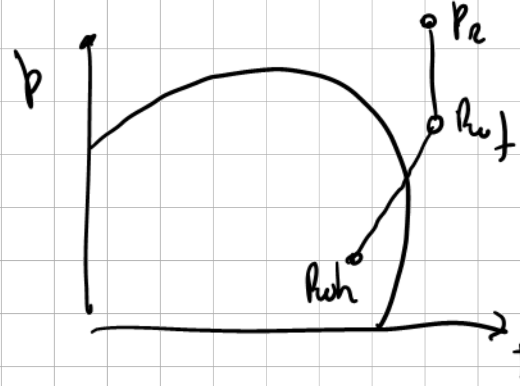
annular flow

mist flow

flow patterns

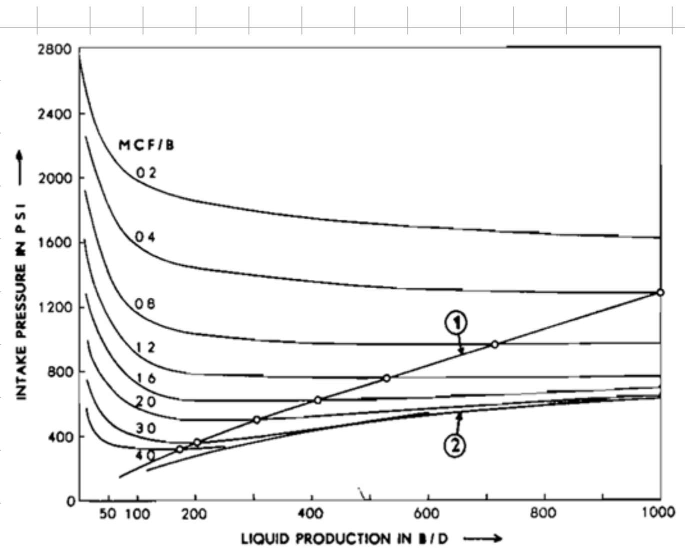
?

P_{wh} = constant



Flowing and Gas-lift Well Performance†

W. E. GILBERT*



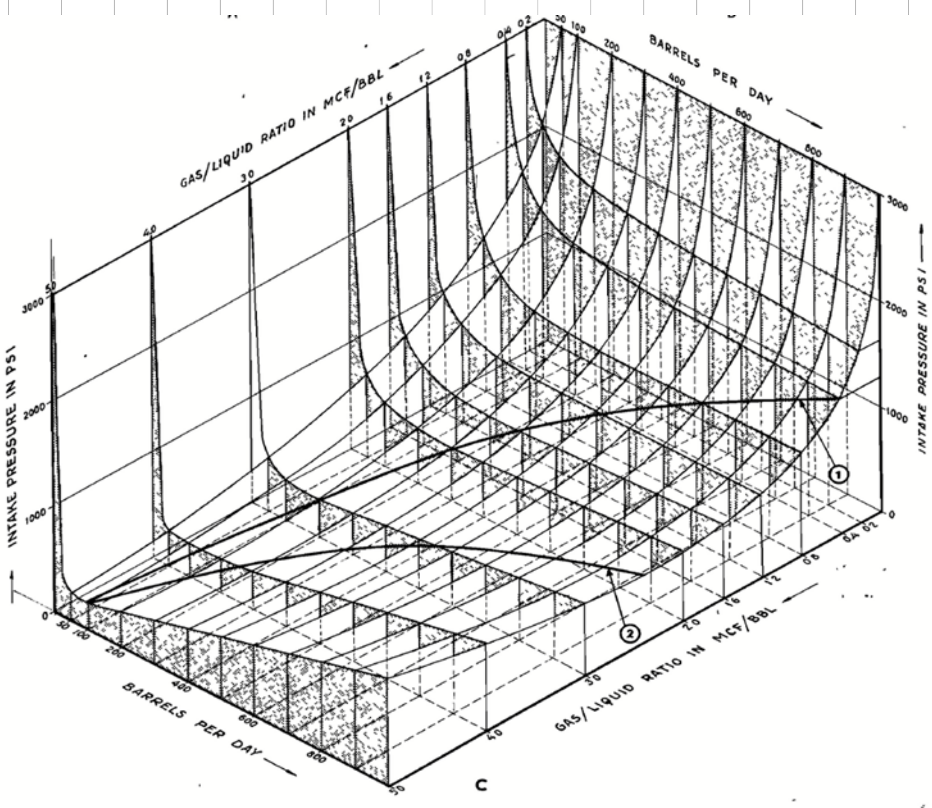
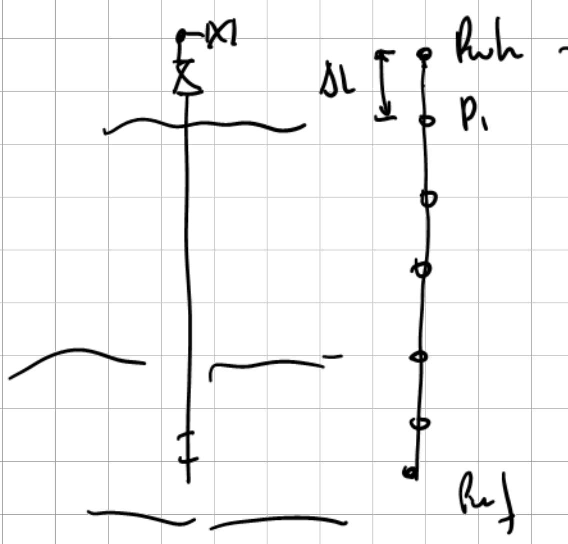


Fig. 6–The Two-phase Vertical-lift Function for 2.875-in. Tubing Set at 8,000 Ft
(Tubing Pressure = Zero Psi Gage)

$$\Delta p_{tubing} = \Delta p_{hydrostatic} + \Delta p_{friction}$$



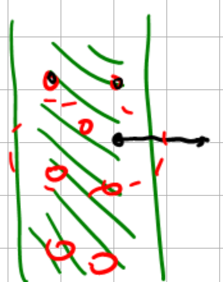
$$\left.\frac{dp}{dx}\right|_{p_{wh}} \rightarrow p_1$$

$$p_1 = p_{wh} + \left.\frac{dp}{dx}\right|_{p_{wh}} \cdot \Delta L \quad ?$$

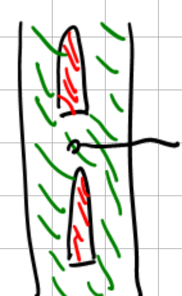
$$p_2 = p_1 + \left.\frac{dp}{dx}\right|_{p_1} \cdot \Delta L$$

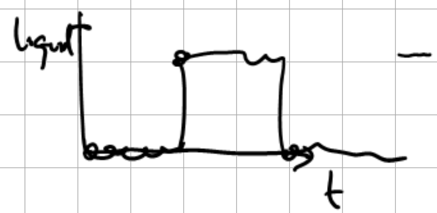
integration

marching algorithm



$$\frac{d}{dt} \neq 0$$





$x \quad \bar{x} \quad \int_t x \, dt \qquad \bar{x} = \int_A x \, dA$

$\frac{dp}{dx} \rightsquigarrow$ PVT properties $(\rho_L, \rho_g, \mu_L, \mu_g, \sigma_{Lg}, x_g)$ mass fraction

$\rightarrow$ velocity (q_g, q_L)

$\rightarrow A, \phi, \theta, e$ roughness

Steady-State Multiphase Flow—Past, Present, and Future, with a Perspective on Flow Assurance

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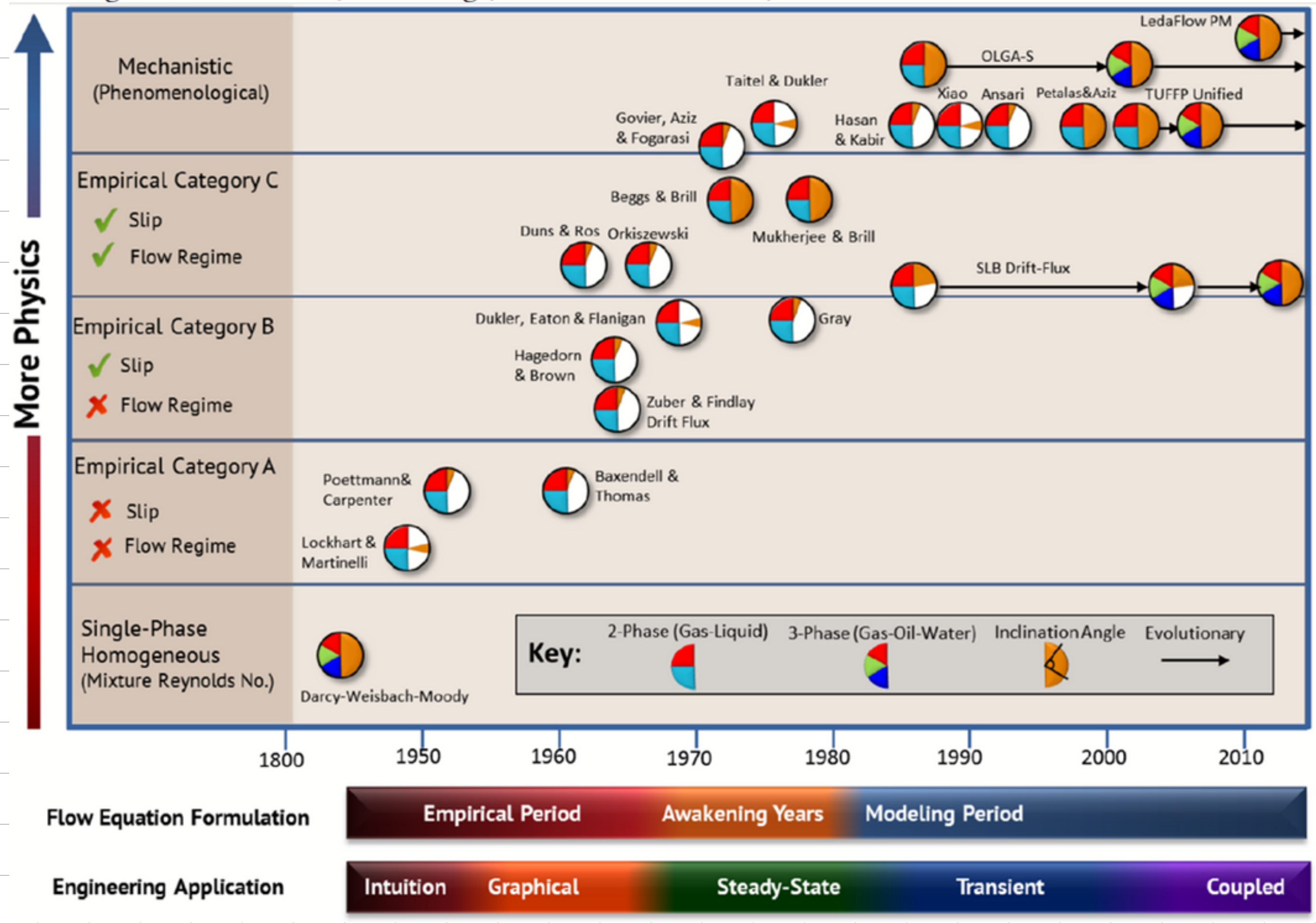
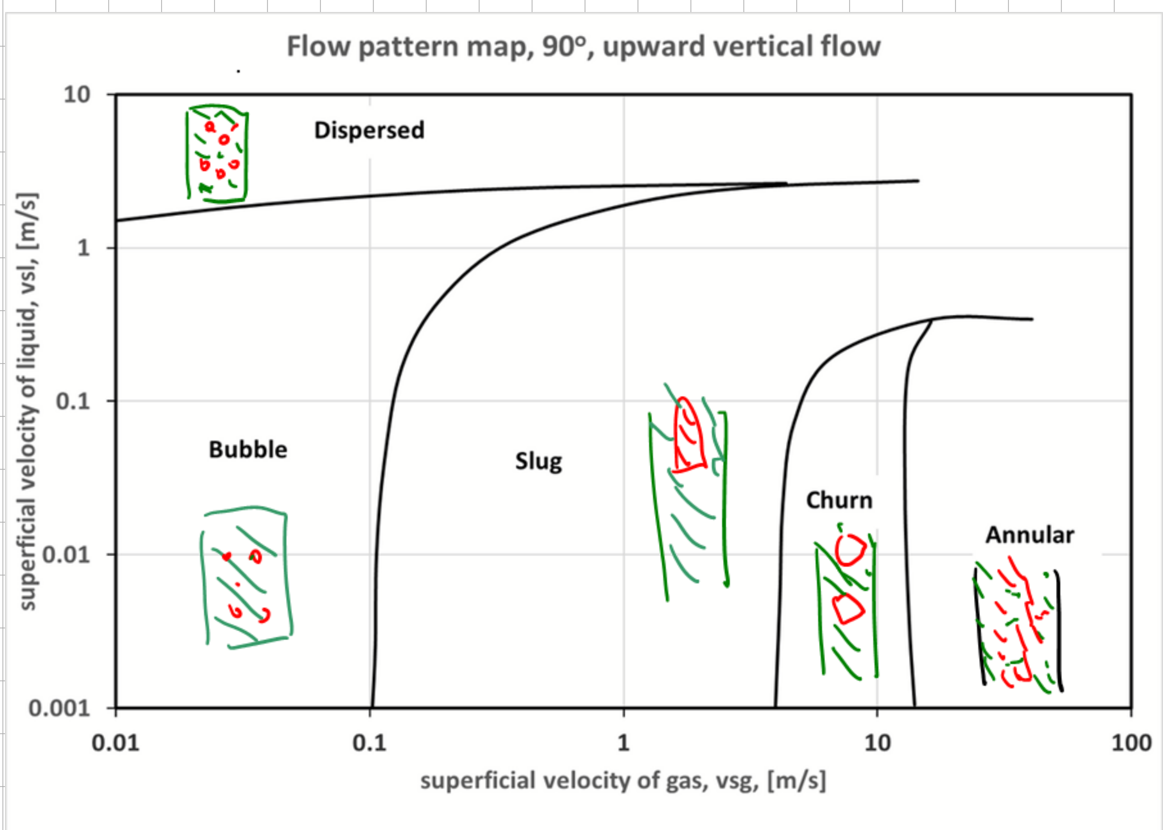
William J. Bailey

Schlumberger—Doll Research, Cambridge, Massachusetts 02139, United States



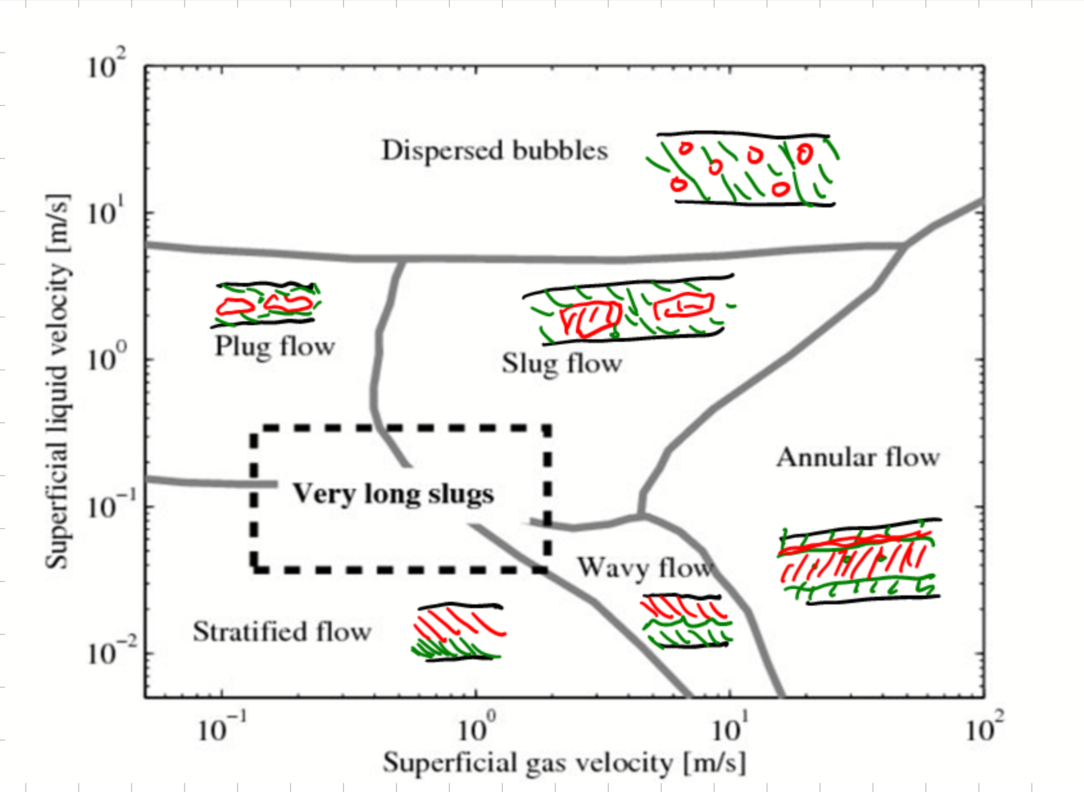
flow pattern map

← for a given fluid, pipe, inclination, etc

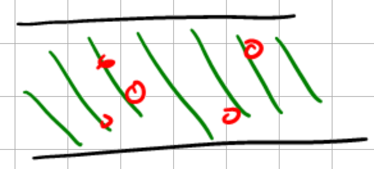


$u_{sL} = \frac{q_L}{A}$

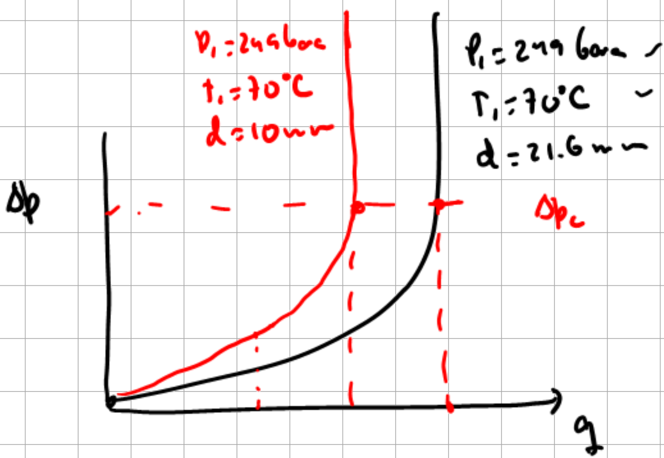
$u_{sg} = \frac{q_g}{A}$



Dispersed bubbles
Plug flow
Slug flow
Annular flow
Very long slugs
Wavy flow
Stratified flow
Superficial liquid velocity [m/s]
Superficial gas velocity [m/s]
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}



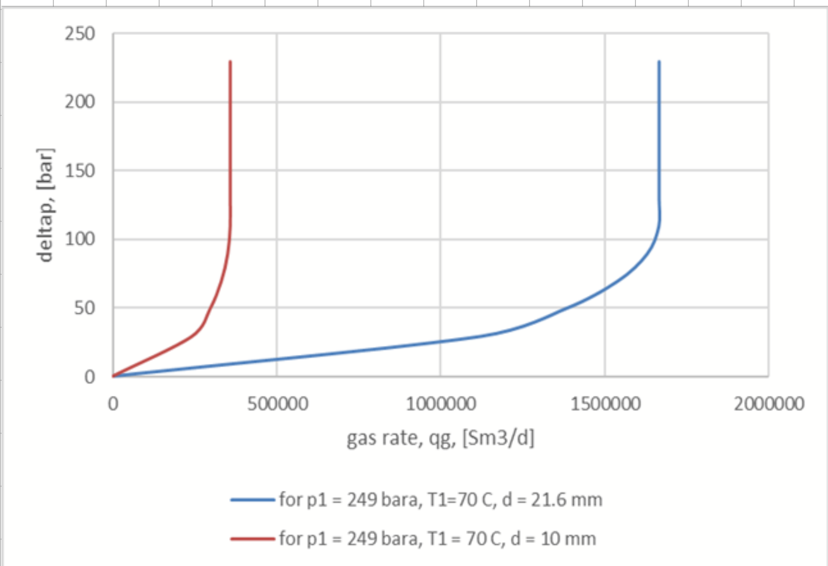
Δp $\qquad y = \frac{p_2}{p_1}$ $\qquad y_c = 0.546$

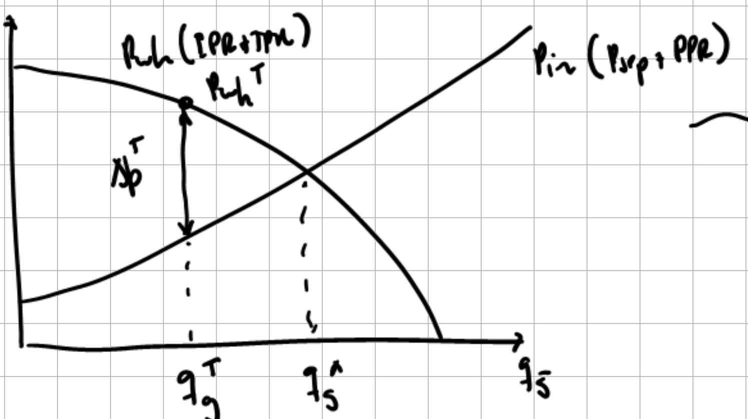


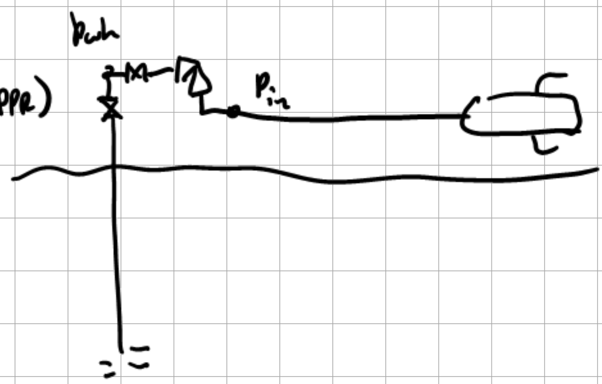
Δp_c ?

$$\Delta p = p_1 - p_2 = p_1 (1 - y)$$

$$\Delta p_c = p_1 (1 - y_c)$$





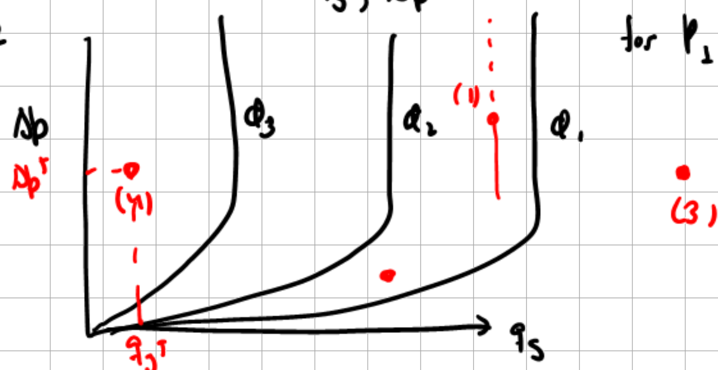


$q_s^T, \Delta p^T$

Choke performance

(1) critical
(2) subcritical

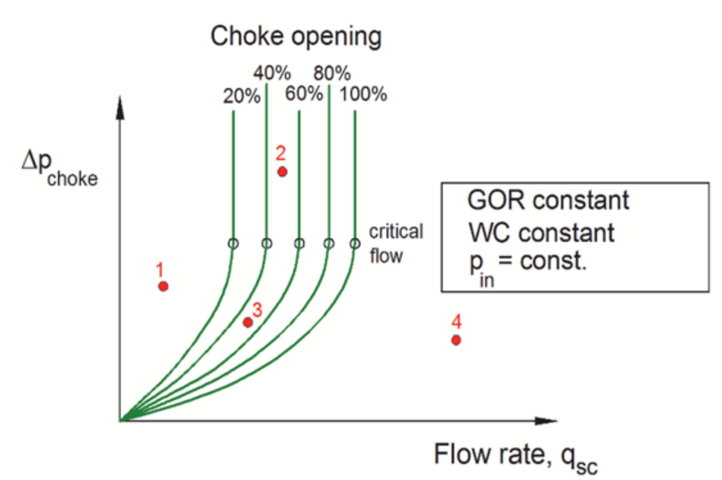
for $p_1 = p_{wh}^T$

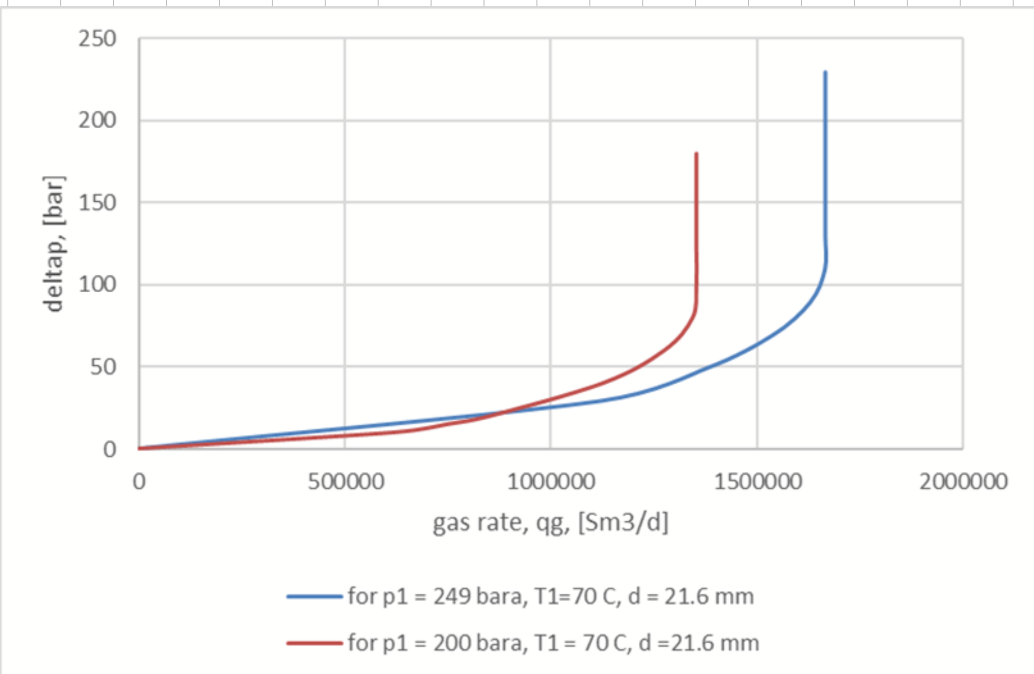


$Q_1 > Q_2 > Q_3$

(3) too small choke

(4) too large choke

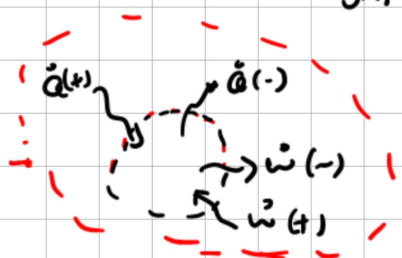




temperature change in choke

low GOR, high WC → expansion leads to an increase in temperature (modest)

high GOR → " " " a significant decrease in temperature



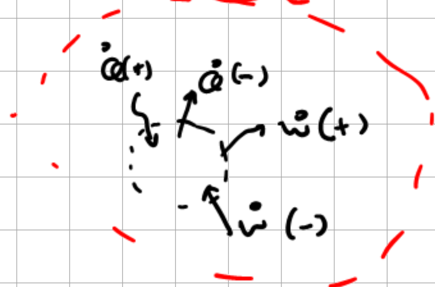
↳ ice formation → plug

↳ hydrate formation

$$\dot{Q} + \dot{W} + E_{in} - E_{out} = 0$$

mech eng

$$\dot{Q} - \dot{W} + E_{in} - E_{out} = 0$$



In the choke $\dot{Q} = 0$

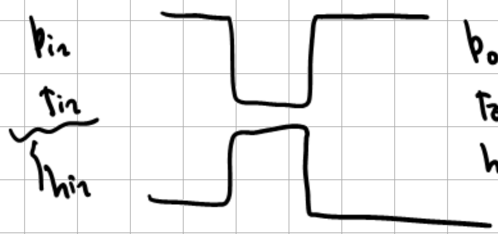
$\dot{W} = 0$

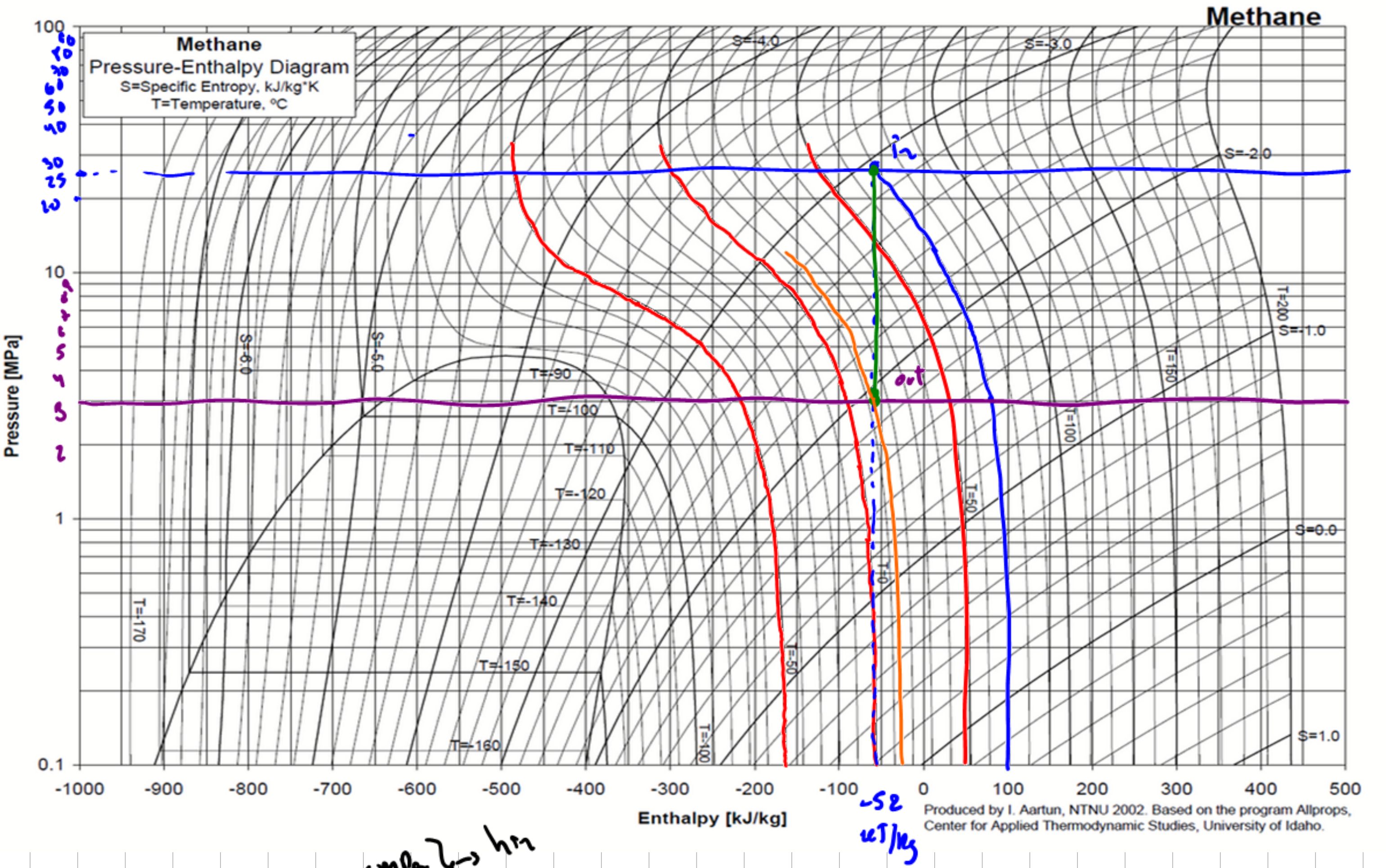
$V_{in} \simeq V_{out}$

$E_{in} = E_{out}$

$$\dot{m}\left(h_{in} + \cancel{\frac{V_{in}^2}{2}} + \dots\right) = \dot{m}\left(h_{out} + \cancel{\frac{V_{out}^2}{2}} + \dots\right)$$

$h_{in} = h_{out}$ isoenthalpic expansion





$p_{in} = 250$ bara $= 25$ MPa $\} \rightarrow h_{in}$ -52 kJ/kg

$T_{in} = 70°C$

$p_{out} = 30$ bara $= 3$ MPa

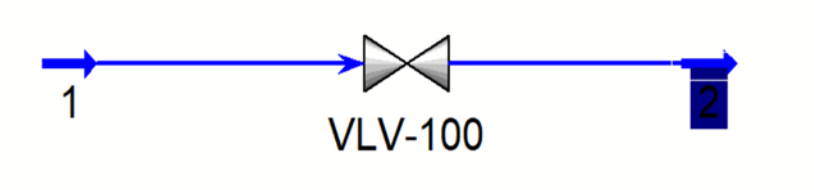
$T_{out} = ?$ $h_{out} = -52$ kJ/kg

$T_{out} = 12°C$

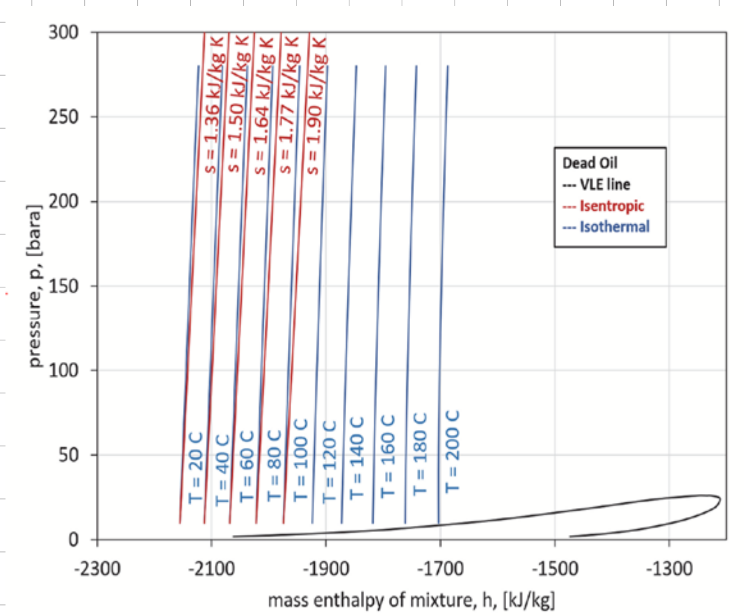
1 MPa $\rightarrow$ 10 bara

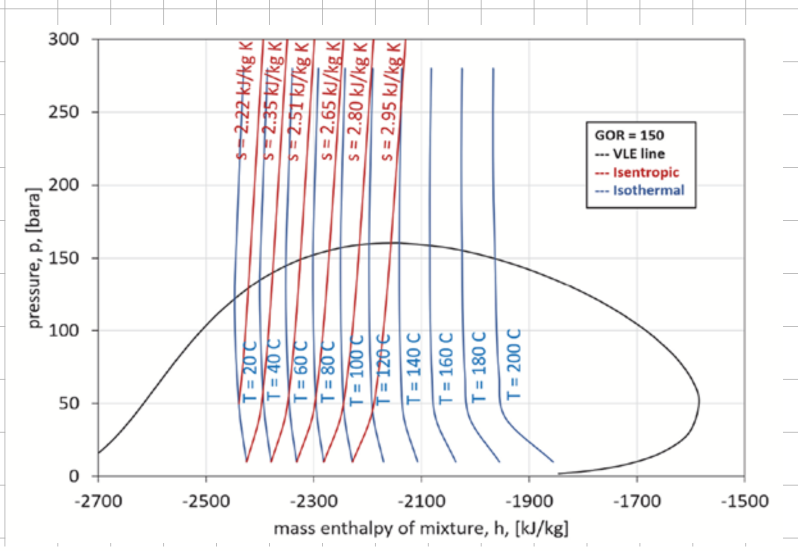
1 E6 Pa $\rightarrow$ 1E05 Pa

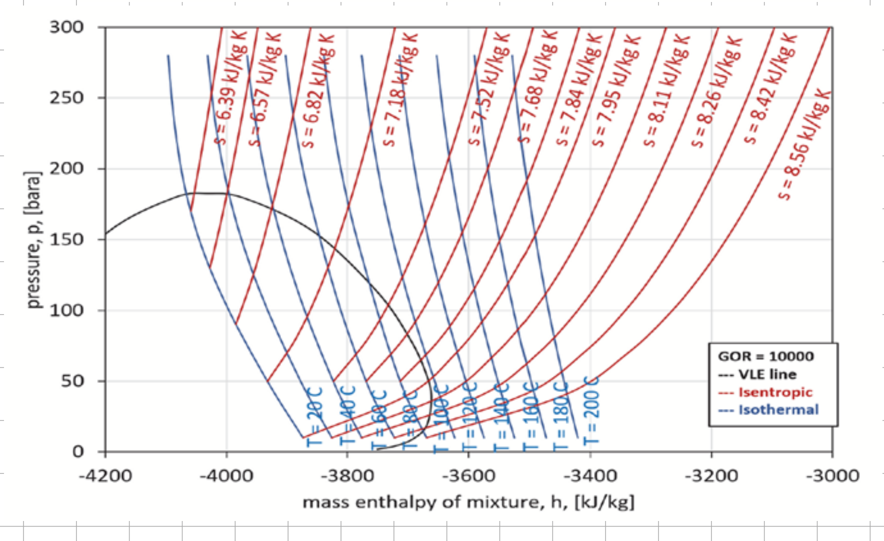
$T_{in} = 70°C \longrightarrow T_{out} = 12°C$



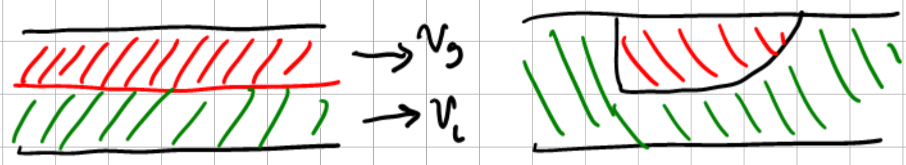
HYSYS Process simulation



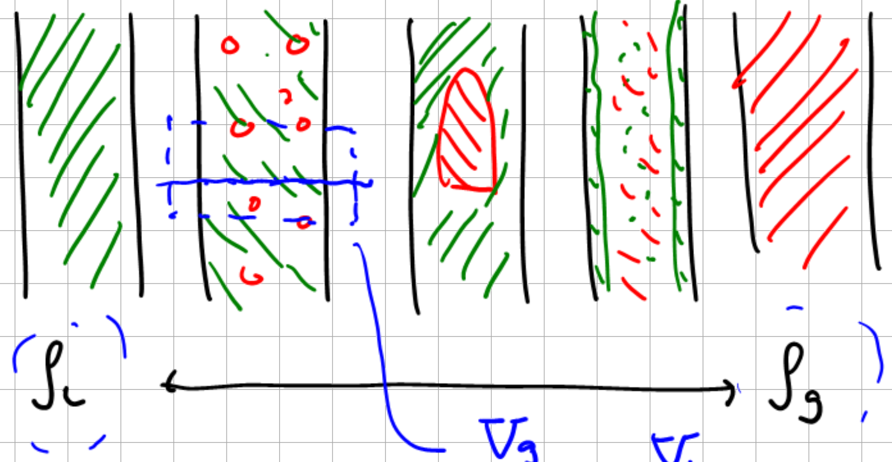


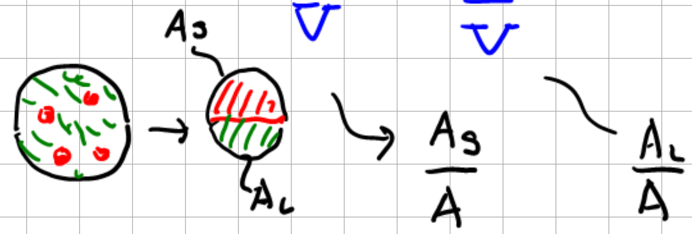


- flow patterns

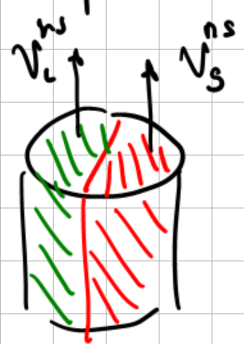


- phase velocity ⟶ friction Δp
- phase spatial distribution ⇝ hydrostatic Δp





Liquid and gas travel at the same velocity (no-slip)



$$v_L^{ns} = v_g^{ns} = v_m = \frac{q_g + q_L}{A} = \frac{q_g}{A} + \frac{q_L}{A} = u_{sg} + u_{sL}$$

$$\lambda_g = \frac{A_g^{ns}}{A} = 1 - \lambda_L = \frac{q_g}{q_g + q_L} \qquad q_g \uparrow\uparrow \gg q_L \longrightarrow \lambda_g \to 1$$

$$\lambda_L = \frac{A_L^{ns}}{A} = 1 - \lambda_g = \frac{q_L}{q_g + q_L} \qquad q_L \gg q_g \longrightarrow \lambda_L \to 1$$

gas and liquid move at different velocities $v_g \neq v_L$ (slip condition)

$$\overbrace{\varepsilon}^{\text{void fraction}} = \underbrace{H_g}_{\text{gas holdup}} = \frac{A_g}{A} = 1 - H_L$$

$$\underbrace{H_L}_{\text{liquid holdup}} = \frac{A_L}{A} = 1 - H_g$$

v_g ?

v_L ?

mass conservation between non-slip condition and slip condition

$$q_g = v_g^{ns} \cdot A_g^{ns} = v_g \cdot A_g \qquad \text{divide by } A$$

$$v_g^{ns} \lambda_g = v_g \cdot \varepsilon$$

$$\boxed{v_g = v_m \frac{\lambda_g}{\varepsilon}}$$

← $\lambda_g < \varepsilon$, $v_g < v_m$

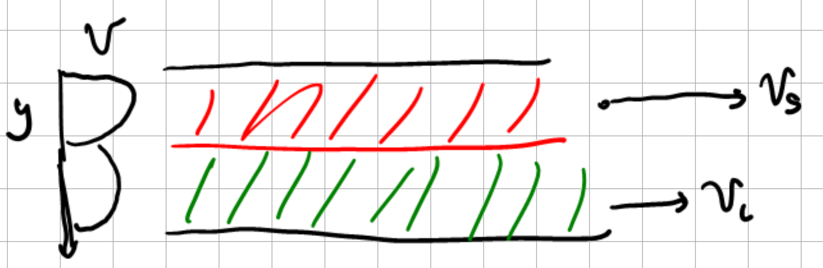
→ $\lambda_g = \varepsilon$, $v_g = v_m$

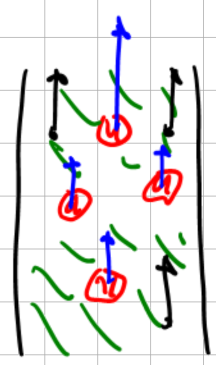
↓ $\lambda_g > \varepsilon$, $v_g > v_m$

$$\boxed{v_L = v_m \frac{\lambda_L}{H_L}}$$

$$v_g = \frac{v_m \lambda_g}{\varepsilon} = \frac{\cancel{q_g+q_L}}{A} \cdot \frac{q_g}{\cancel{q_g+q_L}} \cdot \frac{1}{\varepsilon} = \frac{u_{sg}}{\varepsilon}$$

$$\boxed{v_g = \frac{u_{sg}}{\varepsilon} \qquad v_L = \frac{u_{sL}}{H_L}}$$





ε, H_L, v_L, v_g is a result of solving our $\left\{ \begin{array}{l} \text{mass conservation} \\ \text{momentum conservation} \end{array} \right\}$

one example

$$v_g = C_0 v_m + \underbrace{u_0}_{\text{velocity of a bubble in stagnant liquid}}$$

TWO-PHASE FLOW IN VERTICAL TUBES

By D. J. NICKLIN, B.Sc. App.,* J. O. WILKES, M.A.*† and J. F. DAVIDSON, M.A., Ph.D., A.M.I.Mech.E.,*

SUMMARY

A study has been made of the properties of long bubbles in vertical tubes. It has been shown that these bubbles rise relative to the liquid ahead of them at a velocity exactly equal to the rising velocity of wakeless bubbles of the type studied by Dumitrescu and by Davies and Taylor. For 1 in. tubes, this velocity is closely predicted by Dumitrescu's theory and equals $0{\cdot}35\,(gD)^{\frac{1}{2}}$ where g is the acceleration of gravity and D the tube diameter. The motion of the bubbles in moving liquid streams has been studied, and the results applied to the problem of two-phase slug flow. An expression for the voidage in steady two-phase slug flow has been derived, and this predicted voidage agrees well with results reported here and elsewhere.

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TRANS. INSTN CHEM. ENGRS, Vol. 40, 1962

$$u_s = 1{\cdot}2\,\bar{u}_L + 0{\cdot}35\,(gD)^{\frac{1}{2}}$$

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J. A. FINDLAY
Knolls Atomic Power Laboratory. Mem. ASME
General Electric Co., Schenectady, N. Y.

Average Volumetric Concentration in Two-Phase Flow Systems

A general expression which can be used either for predicting the average volumetric concentration or for analyzing and interpreting experimental data is derived. The analysis takes into account both the effect of nonuniform flow and concentration profiles as well as the effect of the local relative velocity between the phases. The first effect is taken into account by a distribution parameter, whereas the latter is accounted for by the weighted average drift velocity. Both effects are analyzed and evaluated. The results predicted by the analysis are compared with experimental data obtained for various two-phase flow regimes, with various liquid-gas mixtures in adiabatic, vertical flow over a wide pressure range. Good agreement with experimental data is shown.

[2] Numbers in brackets designate References at end of paper.
Contributed by the Heat Transfer Division and presented at the Winter Annual Meeting, New York, N. Y., November 29–December 3, 1964, of The American Society of Mechanical Engineers. Manuscript received at ASME Headquarters, September 15, 1964.

"v_0"

THE FLOW OF LIQUID-GAS MIXTURES IN VERTICAL TUBES.

By

Hans Behringer

ZEITSCHRIFT FUR DIE GESAMTE KALTE-INDUSTRIE, 43, 55-58, 1936.

Z. angew. Math. Mech. Bd. 23 Nr. 3 Juni 1943 Dumitrescu, Strömung an einer Luftblase im senkrechten Rohr 139

Strömung an einer Luftblase im senkrechten Rohr.

Von *D. T. Dumitrescu* in Bukarest.

The mechanics of large bubbles rising through extended liquids and through liquids in tubes

By R. M. Davies and Sir Geoffrey Taylor, F.R.S.

(*Received* 13 *September* 1949)

$$\varepsilon = \frac{U_{SG}}{U_{SG}\left(1+\left(\frac{U_{SL}}{U_{SG}}\right)^{\left(\frac{\rho_G}{\rho_L}\right)^{0.1}}\right)+2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L-\rho_G)}{\rho_L^2}\right]^{0.25}(1.22+1.22\sin\theta)^{\frac{P_{atm}}{P_{system}}}}$$

Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

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Received 1 June 2006; received in revised form 13 September 2006

$$H_g = \frac{\lambda_g + 1 - \left[(\lambda_g+1)^2 - 4\cdot\lambda_g{}^2\right]^{0.5}}{2\cdot\lambda_g}$$

Pipe Fractional Flow Theory: Principles and Applications

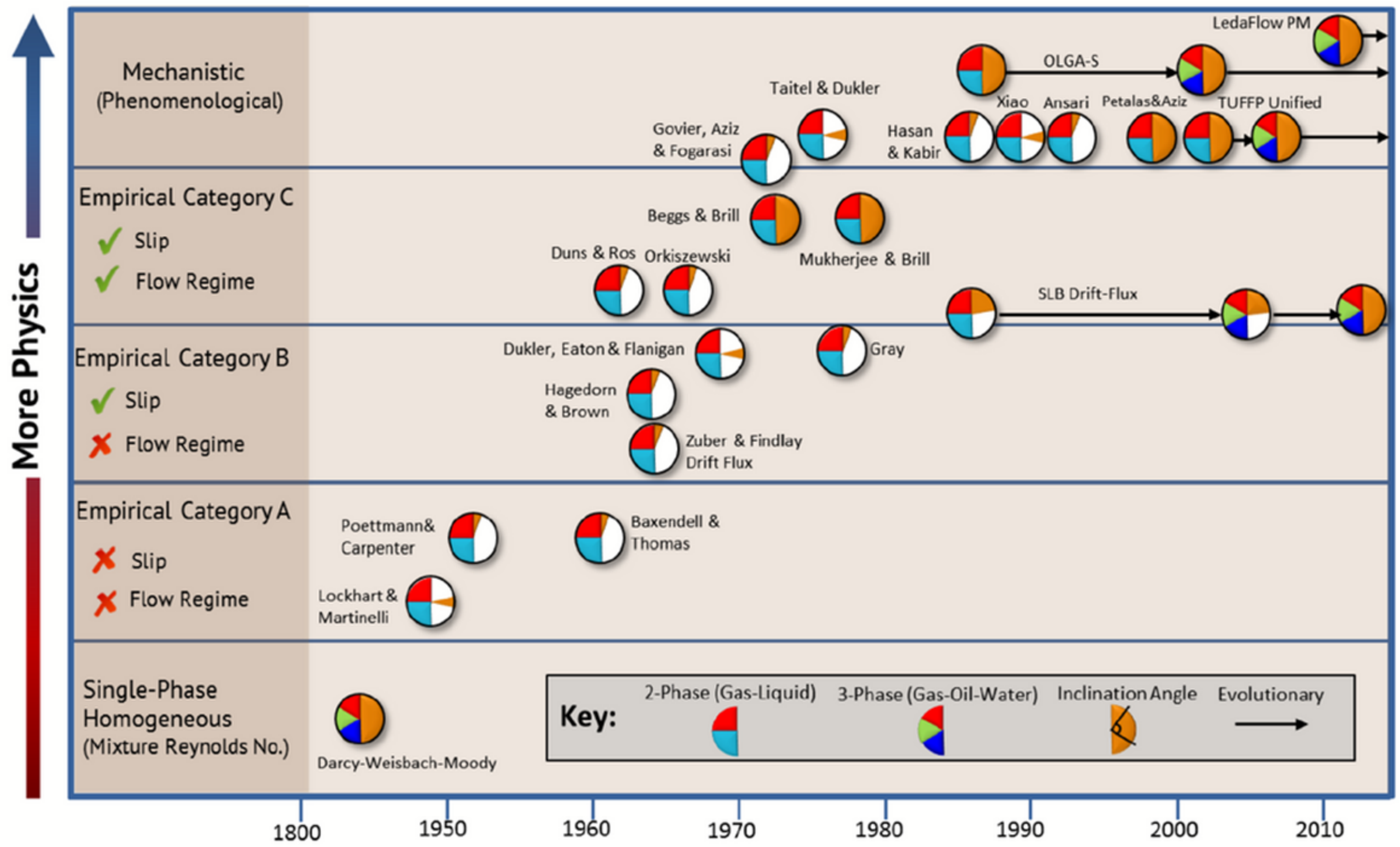
by

Anand Subhash Nagoo, B.Sc., M.S., M.S.

2013

- $U_S = V_g - V_L$ slip velocity
- $S = \frac{V_g}{V_L}$ slip ratio

Some examples of pressure drop models for multiphase flow



A Study of Two-Phase Flow in Inclined Pipes

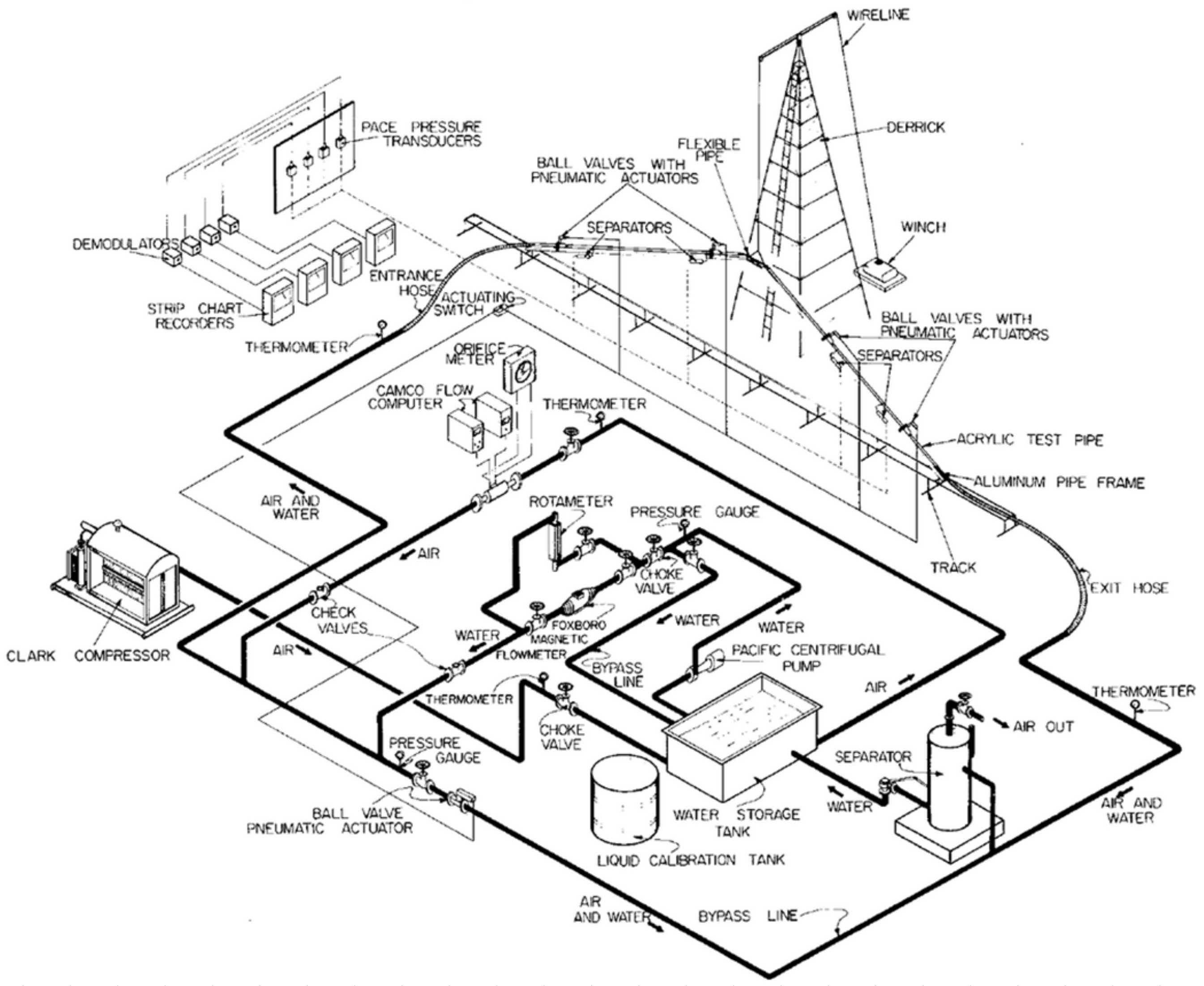
H. Dale Beggs,* SPE-AIME, U. of Tulsa
James P. Brill, SPE-AIME, U. of Tulsa

1973

$$-\frac{dp}{ds} = \left.\frac{dp}{ds}\right|_{grav.} + \left.\frac{dp}{ds}\right|_{fric.} + \left.\frac{dp}{ds}\right|_{accel.}$$

$$-\frac{dp}{dZ} = \frac{\dfrac{g}{g_c}\sin\theta\,[\rho_L H_L + \rho_g(1-H_L)] + \dfrac{f_{tp} G_m v_m}{2 g_c d}}{1 - \{[\rho_L H_L + \rho_g(1-H_L)]\, v_m v_{sg}\}/g_c p}$$

https://wiki.whitson.com/pipeflow/correlations/beggs_brill/



A UNIFIED MODEL FOR PREDICTING FLOW-PATTERN TRANSITIONS FOR THE WHOLE RANGE OF PIPE INCLINATIONS

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(*Received 2 February 1986; in revised form 9 June 1986*)

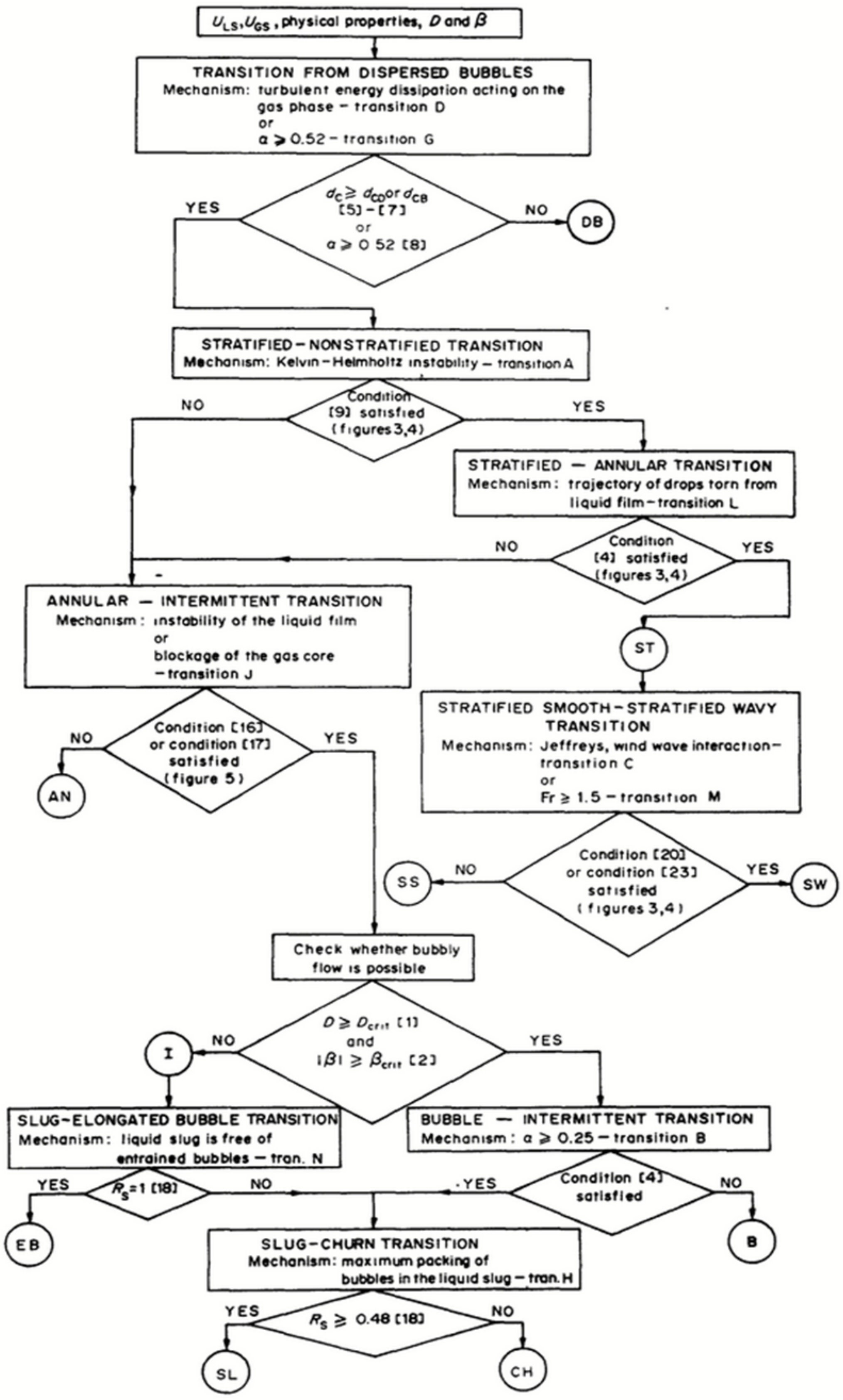
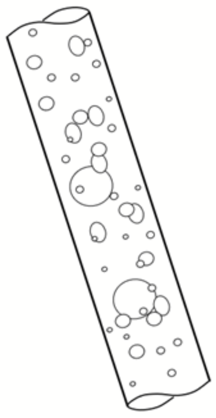


Figure 6. Logical pass for flow-pattern determination.

Bubble Flow-Pattern

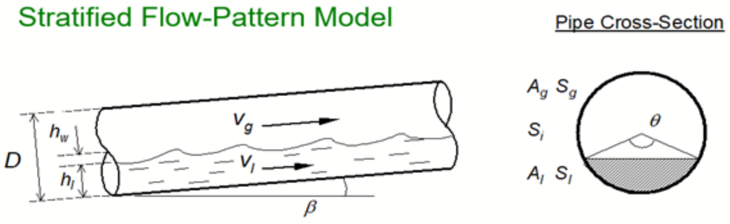
- Turbulent forces prevent bubble agglomeration and slip effect.
- Transition from bubble flow is given in the work of Barnea et al. (1987).
- The bubble flow-pattern is modeled as homogenous single fluid flow with averaged properties of liquid and gas.



Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = f_m \frac{2\rho_m v_m^2}{D} + \rho_m g \sin\beta$$

Stratified Flow-Pattern Model



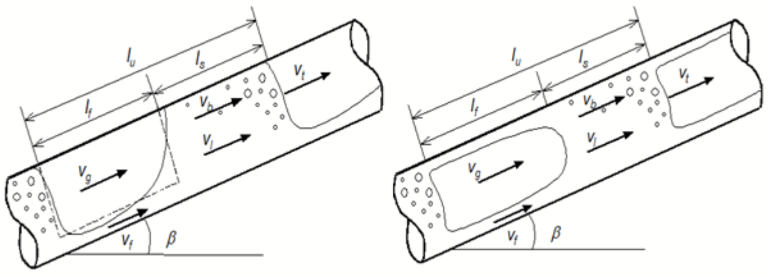
Combined momentum equation:

$$\frac{\tau_g S_g}{A_g} - \frac{\tau_l S_l}{A_l} + \tau_i S_i\left(\frac{1}{A_l}+\frac{1}{A_g}\right) - (\rho_l-\rho_g)\, g\sin\beta = 0$$

Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = \frac{\tau_l S_l + \tau_g S_g}{A} + \left(\frac{A_l}{A}\rho_l + \frac{A_g}{A}\rho_g\right) g\sin\beta$$

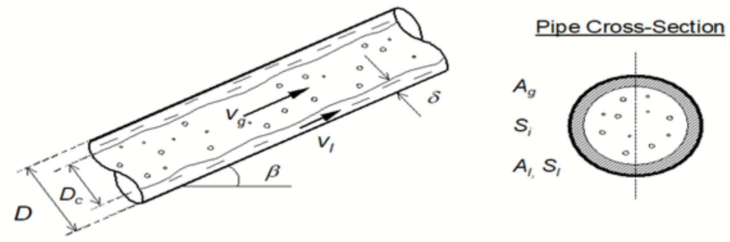
Intermittent Flow-Pattern Models



Pressure gradient equation:

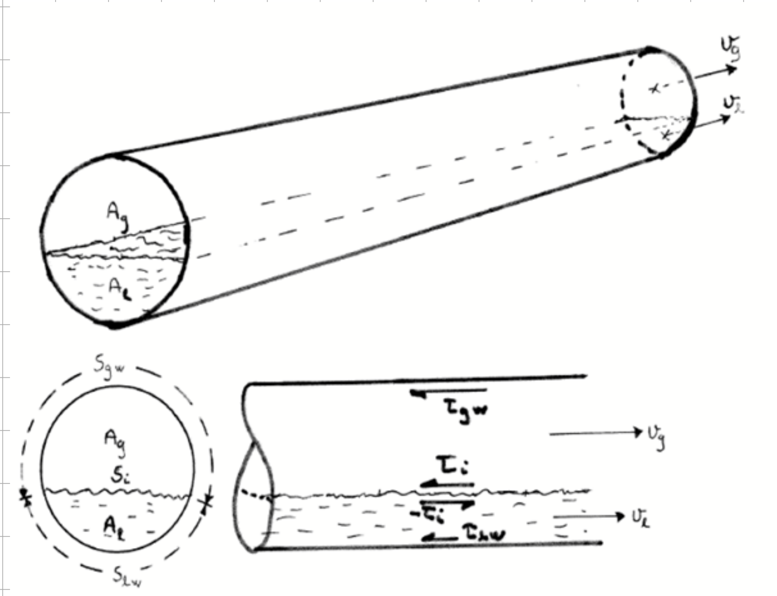
$$-\left(\frac{dP}{dx}\right) = \rho_u g\sin\beta + \frac{1}{l_u}\left[\left(\frac{\tau_s \pi D}{A} l_s\right) + \left(\frac{\tau_f S_f + \tau_g S_g}{A} l_f\right)\right]$$

Annular Flow-Pattern Model



Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = \frac{\tau_l S_l}{A} + \left(\frac{A_l}{A}\rho_l + \frac{A_g}{A}\rho_{gc}\right) g\sin\beta$$



Harald Asheim's drift flux model

$$A_g\,dp + A_g\,\rho_g\,g_x\,dx + A_g\,\rho_g\,v_g\,dv_g + \tau_{gw}\,S_{gw}\,dx + \tau_i\,S_i dx = 0$$

+

$$A_l\,dp + A_l\,\rho_l\,g_x\,dx + A_l\,\rho_l\,v_l\,dv_l + \tau_{lw}\,S_{lw}\,dx - \tau_i\,S_i dx = 0$$

$$dp + \left(\rho_g y_g + \rho_l y_l\right)g_x dx + \rho_g v_g y_g dv_g + \rho_l v_l y_l dv_l + \frac{\tau_g S_{gw} + \tau_{lw} S_{lw}}{A} dx = 0$$

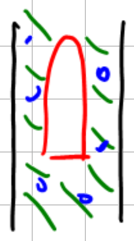
$$\tau_g = \frac{1}{8} f_g \rho_g v_g |v_g|$$

$$\tau_l = \frac{1}{8} f_l \rho_l v_l |v_l|$$

$$S_{gw} = \pi d y_g$$

$$S_{gw} = \pi d y_l$$

$$\frac{dp}{dx} + \rho_{TP} g_x + \rho_g v_{sg} \frac{dv_g}{dx} + \rho_l v_{sl} \frac{dv_l}{dx} + \frac{1}{2 \cdot d}\left(f_g \rho_g v_g |v_g| y_g + f_l \rho_l v_l |v_l| y_l\right) = 0$$



$p^*, T^* \rightarrow$

	o	g	w
ρ			
M			
	σ_{og}	σ_{wg}	σ_{ow}
h			

$\underline{\dot{m}_o \quad \dot{m}_g \quad \dot{m}_w} \rightarrow q_o \quad q_g \quad q_w \rightarrow u_{so} \quad u_{sg} \quad u_{sw} \rightarrow \dfrac{dP}{dx}, \dfrac{dT}{dx}$

2 approaches:

- Compositional $\quad z_i$ (mole frac): C_1, C_2, C_3, ⋮, i, pseudo components + EOS (Peng Robinson) + EOS parameters
  - mixing rules
  - Binary interaction parameters

→ mass fraction o, g, w

$$X_o = \frac{\dot{m}_o}{\dot{m}_o + \dot{m}_g} \qquad X_g = (1 - X_o)$$

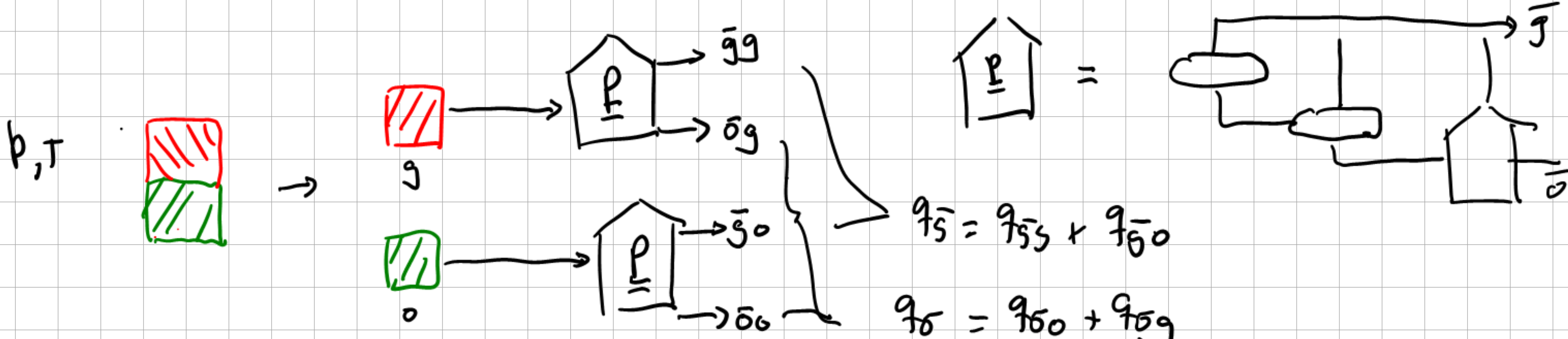
$$\dot{m}_o = \dot{m} X_o$$

$$\dot{m}_g = \dot{m} X_g$$

run in two ways

- live: flash calculation (equilibrium calculation) → property estimation
- precompute tables and interpolate on table

- Black oil tables $\quad$ generated by:
  - EOS
  - correlations
  - lab experiments



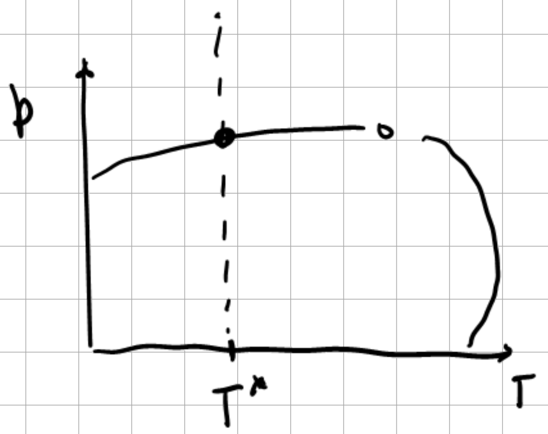
$$q_{\bar{g}} = q_{\bar{g}g} + q_{\bar{g}o}$$

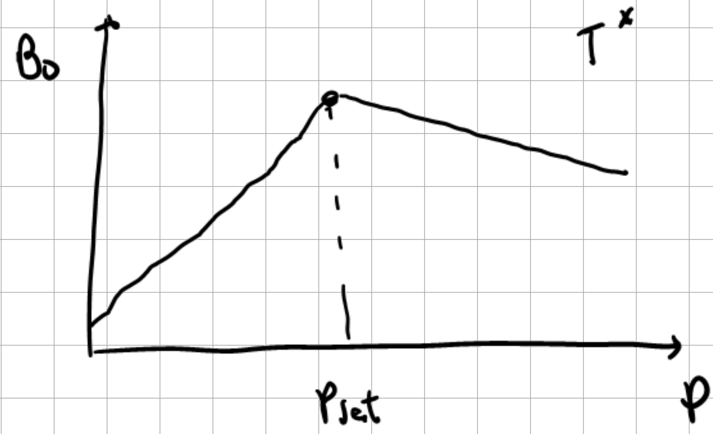
$$q_{\bar{o}} = q_{\bar{o}o} + q_{\bar{o}g}$$

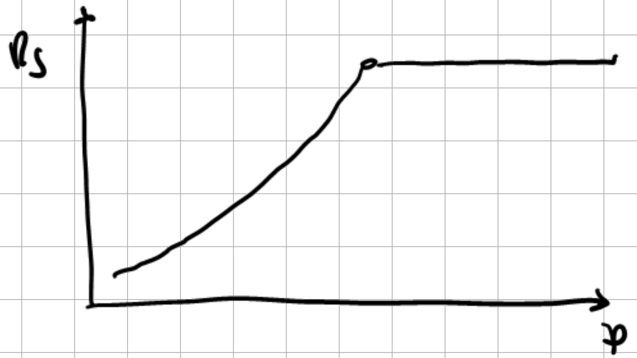
$$B_o = \frac{V_o}{V_{\bar{o}o}} \qquad B_g = \frac{V_g}{V_{\bar{g}g}} \qquad R_s = \frac{V_{\bar{g}o}}{V_{\bar{o}o}} \qquad r_s\,(r_v) = \frac{V_{\bar{o}g}}{V_{\bar{g}g}}$$

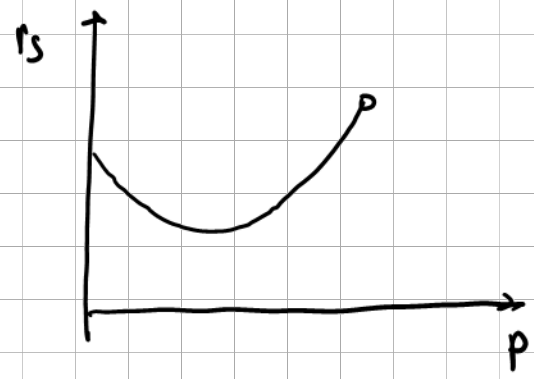
R_s: solution gas-oil ratio

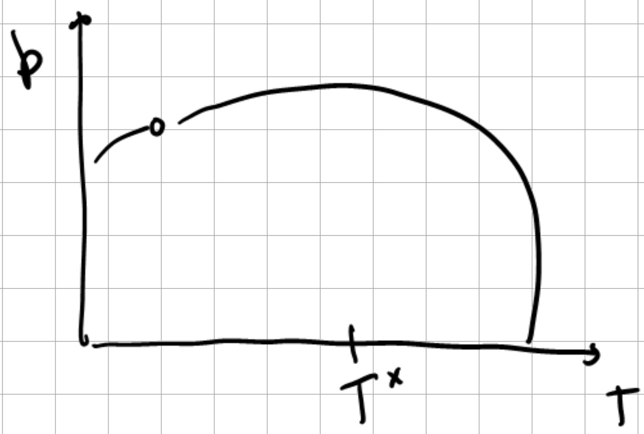
r_s: solution oil-gas ratio

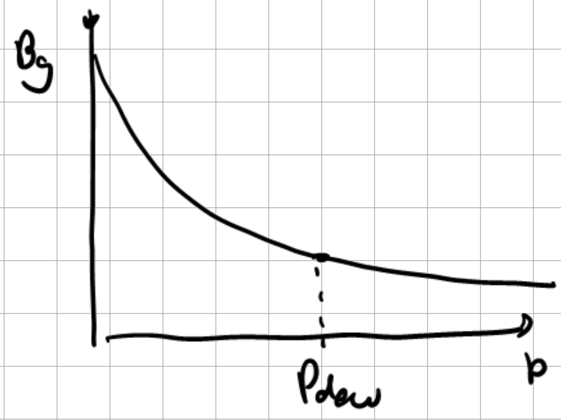


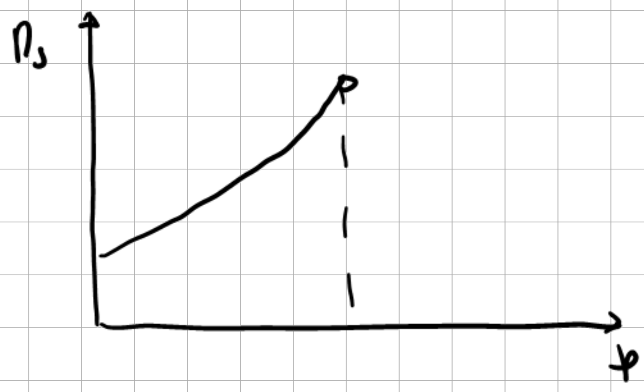


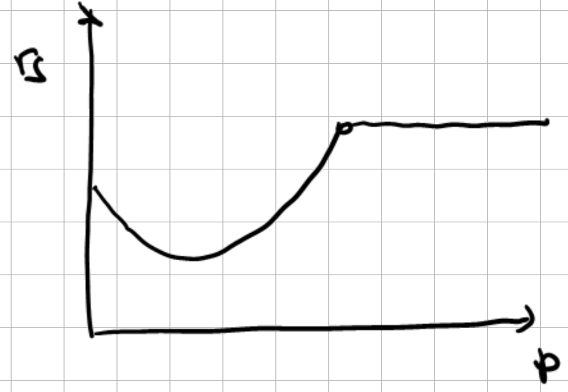












$\begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix} = \begin{bmatrix} \frac{1}{B_g} & \frac{R_s}{B_o} & 0 \\ \frac{r_s}{B_g} & \frac{1}{B_o} & 0 \\ 0 & 0 & \frac{1}{B_w} \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix}$	$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \frac{B_g}{1-\cancel{R_s \cdot r_s}\,0} & \frac{-R_s \cdot B_g}{1-R_s \cdot r_s} & 0 \\ \frac{-B_o . r_s}{1-R_s \cdot r_s} & \frac{B_o}{1-R_s \cdot r_s} & 0 \\ 0 & 0 & B_w \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix}$
Standard conditions calculated from local conditions	Local conditions calculated from standard conditions

$$q_{\bar{g}} = q_{\bar{g}g} + q_{\bar{g}o}$$

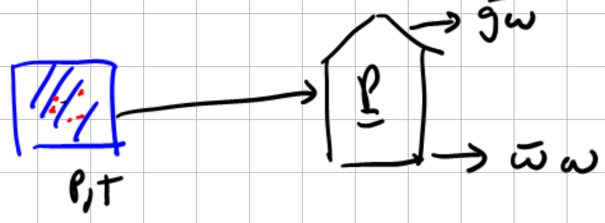
$$q_{\bar{o}} = q_{\bar{o}o} + q_{\bar{o}g}$$

for conventional / heavy oil $r_s = 0$

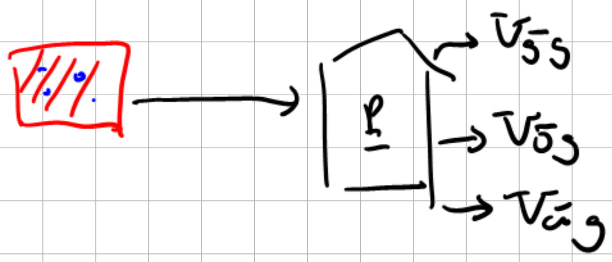
not adequate for volatile oils

gas condensate

water BO properties



$$B_w = \frac{V_w}{V_{\bar{w}w}}$$

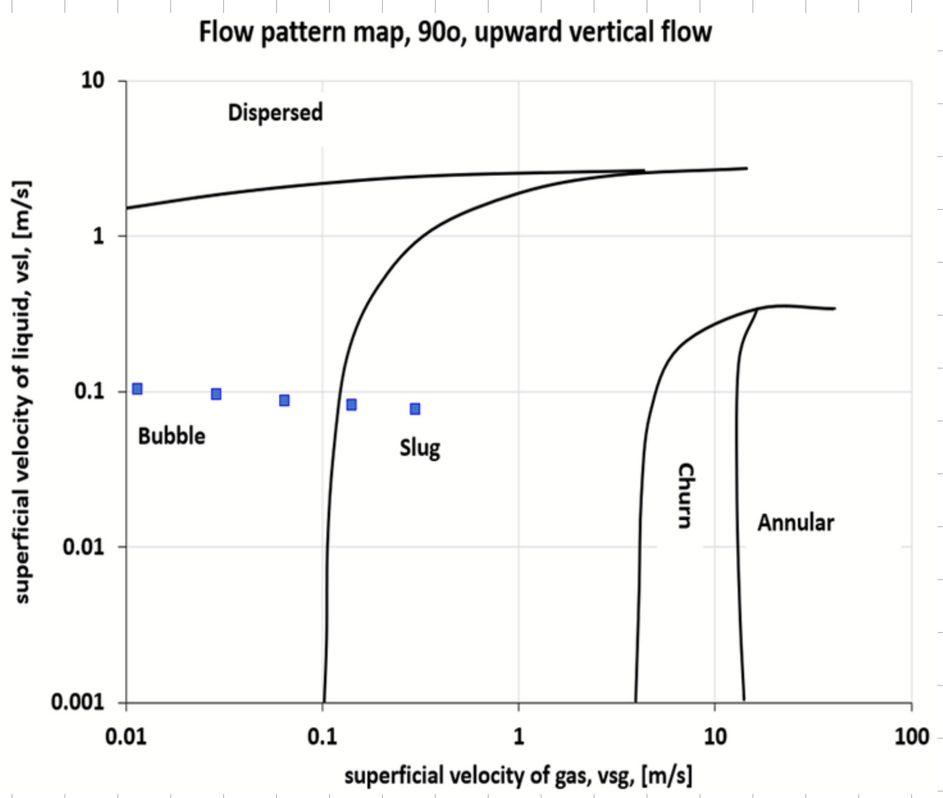


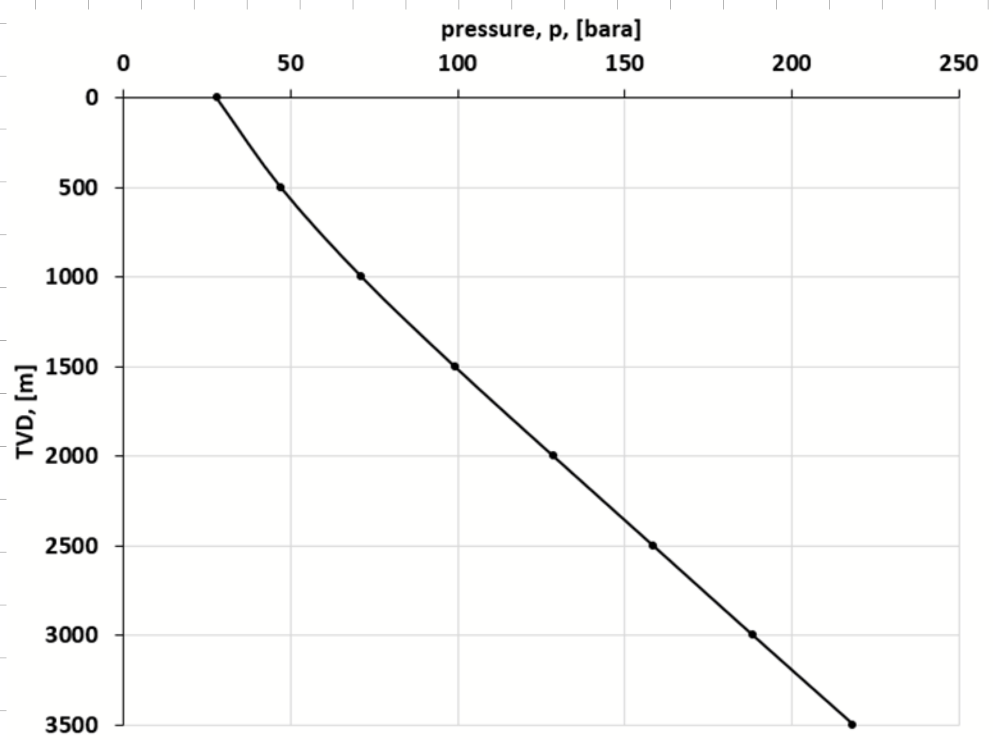
Some cases it could be important to include surface gas from local water and surface water from local gas.

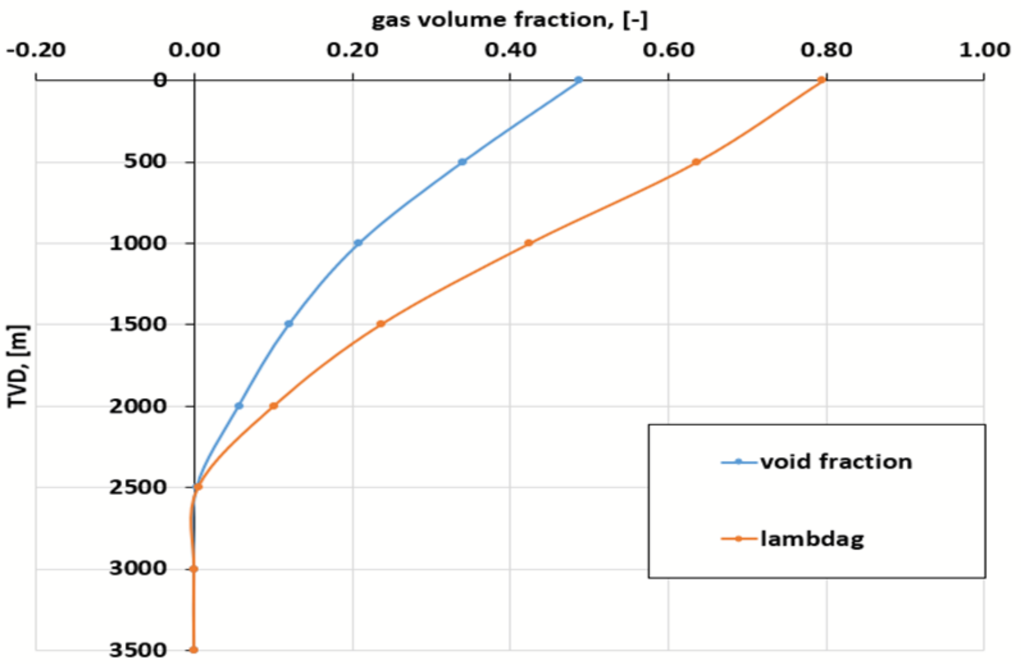
Exercise: Pressure drop calculation in saturated oil well, Prof. Milan Stanko (NTNU)

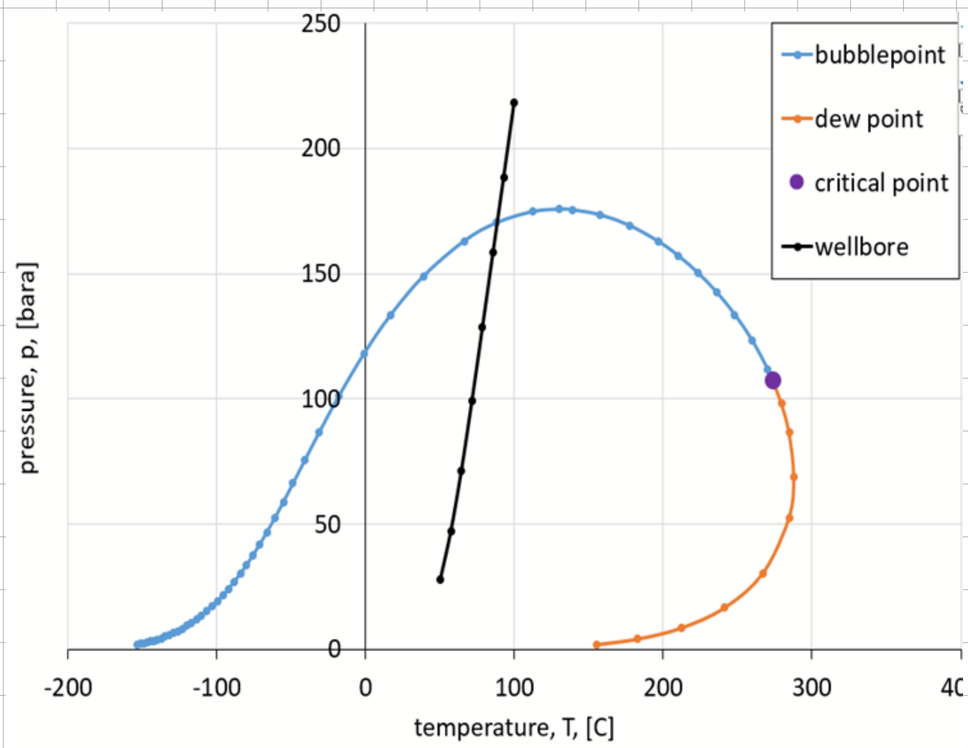
GOR	[Sm3/Sm3]	155.1
Pipe ID	[m]	0.15
Pipe cross section area	[m2]	0.0177
Pipe roughness	[m]	1.50E-05
Pipe inclination from hor	[deg]	90
qo	[Sm3/d]	100
qg	[Sm3/d]	1.55E+04

		BO table column	3	4	5	6	8	10	7	9	11						Woldesemayat and Ghajar	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	117.4	4.566E+02	0.077	0.299	0.80	0.49	0.0384
500	57.1	47.2	41.1	1.31E-05	1.2	1.90E-02	70.8	1.25E-02	708.8	1.2	8.37E-03	124.2	2.173E+02	0.081	0.142	0.64	0.34	0.0483
1000	64.3	71.4	65.3	1.43E-05	1.3	1.09E-02	119.4	1.49E-02	684.3	0.8	5.12E-03	133.2	9.832E+01	0.087	0.064	0.42	0.21	0.0556
1500	71.4	99.2	93.9	1.69E-05	1.4	7.29E-03	178.7	1.91E-02	657.3	0.6	2.64E-03	144.4	4.468E+01	0.095	0.029	0.24	0.12	0.0589
2000	78.6	128.6	124.4	2.13E-05	1.6	5.71E-03	228.2	2.38E-02	630.8	0.5	1.33E-03	156.8	1.761E+01	0.103	0.012	0.10	0.06	0.0597
2500	85.7	158.4	153.2	2.41E-05	1.7	4.49E-03	229.5	2.44E-02	607.9	0.4	6.71E-04	169.0	8.799E-01	0.111	0.001	0.01	0.00	0.0595
3000	92.9	188.2	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.3	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602
3500	100.0	218.3	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.4	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602



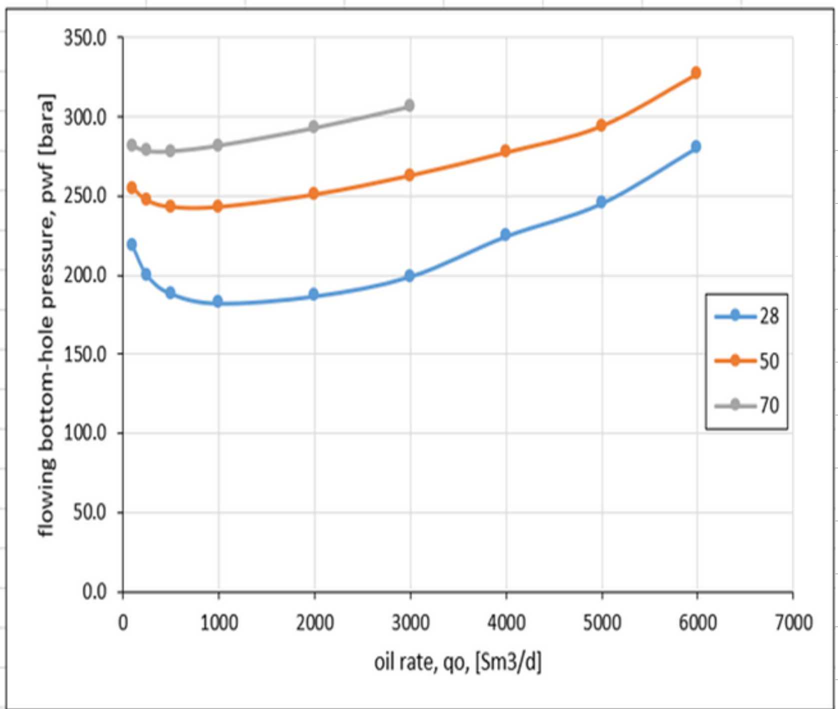






With Wolgha model

pwh[bara]	28	50	70
qo	pwf	pwf	pwf
[Sm3/d]	[bara]	[bara]	[bara]
100	218.3	254.9	281.4
250	200.0	247.1	278.3
500	188.5	243.0	278.2
1000	182.4	243.0	281.8
2000	186.9	250.8	292.9
3000	199.2	262.8	306.3
4000	224.5	277.3	
5000	245.2	294.0	
6000	280.5	327.0	



lculation in saturated oil well, Prof. Milan Stanko (NTNU)

[Sm3/Sm3]	155.1
[m]	0.15
[m2]	0.0177
[m]	1.50E-05
[deg]	90
[Sm3/d]	100
[Sm3/d]	1.55E+04

		BO table column	3	4	5	6	8	10	7	9	11						Woldesemayat and Ghajar	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	117.4	4.566E+02	0.077	0.299	0.80	0.49	0.0384
500	57.1	47.2	41.1	1.31E-05	1.2	1.90E-02	70.8	1.25E-02	708.8	1.2	8.37E-03	124.2	2.173E+02	0.081	0.142	0.64	0.34	0.0483
1000	64.3	71.4	65.3	1.43E-05	1.3	1.09E-02	119.4	1.49E-02	684.3	0.8	5.12E-03	133.2	9.832E+01	0.087	0.064	0.42	0.21	0.0556
1500	71.4	99.2	93.9	1.69E-05	1.4	7.29E-03	178.7	1.91E-02	657.3	0.6	2.64E-03	144.4	4.468E+01	0.095	0.029	0.24	0.12	0.0589
2000	78.6	128.6	124.4	2.13E-05	1.6	5.71E-03	228.2	2.38E-02	630.8	0.5	1.33E-03	156.8	1.761E+01	0.103	0.012	0.10	0.06	0.0597
2500	85.7	158.4	153.2	2.41E-05	1.7	4.49E-03	229.5	2.44E-02	607.9	0.4	6.71E-04	169.0	8.799E-01	0.111	0.001	0.01	0.00	0.0595
3000	92.9	188.2	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.3	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602
3500	100.0	218.3	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.4	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602

```
Function e_wolgha(usl, usg, denl, deng, sigma_lg, teta_deg, p, D)
    'p in bar
    'D in m
    'usl in m/s
    'usg in m/s
    'denl kg/m^3
    'deng kg/m^3
    'teta deg in deg
    'sigma_lg in N/m
    If usg = 0 Then
        e_wolgha = 0
    Else
        Pi = Atn(1) * 4
        teta = teta_deg * Pi / 180
        'void fraction correlation by Woldesemayat and Ghajar (2006)
        a = usg * (1 + ((usl / usg) ^ ((deng / denl) ^ 0.1)))
        B = 2.9 * ((9.81 * sigma_lg * D * (1 + Cos(teta)) * (denl - deng) / (denl ^ 2)) ^ 0.25)
        C = (1.22 + 1.22 * Sin(teta)) ^ (1.01325 / p)
        e_wolgha = usg / (a + (B * C))
    End If
End Function
```

```
Function dpdx_mpf(roughness, viscl, viscg, denl, deng, usl, usg, D, angle, voidfraction)
    'dpdx_mpf pressure gradient, in bar/m, for multiphase flow
    'denl, liquid density, [kg/m3]
    'deng, gas density, [kg/m3]
    'usl superficial liquid velocity, [m/s]
    'usg superficial gas velocity, [m/s]
    'angle, inclination angle of pipe with respect to horizontal [deg]
    'D hydraulic diameter of pipe [m]
    'roughness pipe roughness, [m]
    'viscl, liquid viscosity [cP]
    'viscg, gas viscosity, [cP]
    'voidfraction [-]
    Pi = Atn(1) * 4
    denm = voidfraction * deng + (1 - voidfraction) * denl
    If voidfraction = 0 Or usg = 0 Then
        ug = 0
        ul = usl
        fg = 0
        fl = ffactor(denl, viscl, D, roughness, ul)
    ElseIf voidfraction = 1 Or usl = 0 Then
        ug = usg
        ul = 0
        fl = 0
        fg = ffactor(deng, viscg, D, roughness, ug)
    Else
        ug = usg / voidfraction
        ul = usl / (1 - voidfraction)
        fg = ffactor(deng, viscg / 1000#, D, roughness, ug)
        fl = ffactor(denl, viscl / 1000, D, roughness, ul)
    End If
    dpdx_f = (fg * deng * (ug * Abs(usg)) * 0.5 / D) + (fl * denl * (ul * Abs(usl)) * 0.5 / D)
    dpdx_h = denm * 9.81 * Sin(angle * Pi / 180)
    dpdx_mpf = dpdx_f + dpdx_h
    dpdx_mpf = dpdx_mpf / 100000#
End Function
```

lculation in saturated oil well, Prof. Milan Stanko (NTNU)

[Sm3/Sm3]	155.1
[m]	0.15
[m2]	0.0177
[m]	1.50E-05
[deg]	90
[Sm3/d]	1000
[Sm3/d]	1.55E+05

		BO table column	3	4	5	6	8	10	7	9	11							Nagoo
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	1174.3	4.566E+03	0.769	2.991	0.80	0.61	0.0313
500	57.1	43.7	37.3	1.29E-05	1.2	2.08E-02	63.4	1.22E-02	711.9	1.2	8.90E-03	1229.4	2.451E+03	0.805	1.605	0.67	0.50	0.0387
1000	64.3	63.0	56.3	1.36E-05	1.3	1.34E-02	100.1	1.40E-02	691.6	0.9	6.20E-03	1301.4	1.320E+03	0.852	0.865	0.50	0.38	0.0461
1500	71.4	86.1	79.2	1.53E-05	1.4	8.91E-03	146.3	1.68E-02	668.7	0.6	3.75E-03	1390.6	6.776E+02	0.911	0.444	0.33	0.26	0.0525
2000	78.6	112.3	105.7	1.84E-05	1.5	6.67E-03	195.8	2.08E-02	644.3	0.5	2.05E-03	1497.1	3.306E+02	0.981	0.217	0.18	0.16	0.0567
2500	85.7	140.7	134.2	2.32E-05	1.6	5.56E-03	234.6	2.48E-02	620.2	0.4	1.12E-03	1615.5	1.167E+02	1.058	0.076	0.07	0.06	0.0588
3000	92.9	170.1	154.6	1.91E-06	1.7	3.87E-04	17.9	1.91E-03	604.6	0.4	6.30E-05	1701.8	2.006E-01	1.115	0.000	0.00	0.00	0.0597
3500	100.0	200.0	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	605.0	0.4	0.00E+00	1702.0	0.000E+00	1.115	0.000	0.00	0.00	0.0649

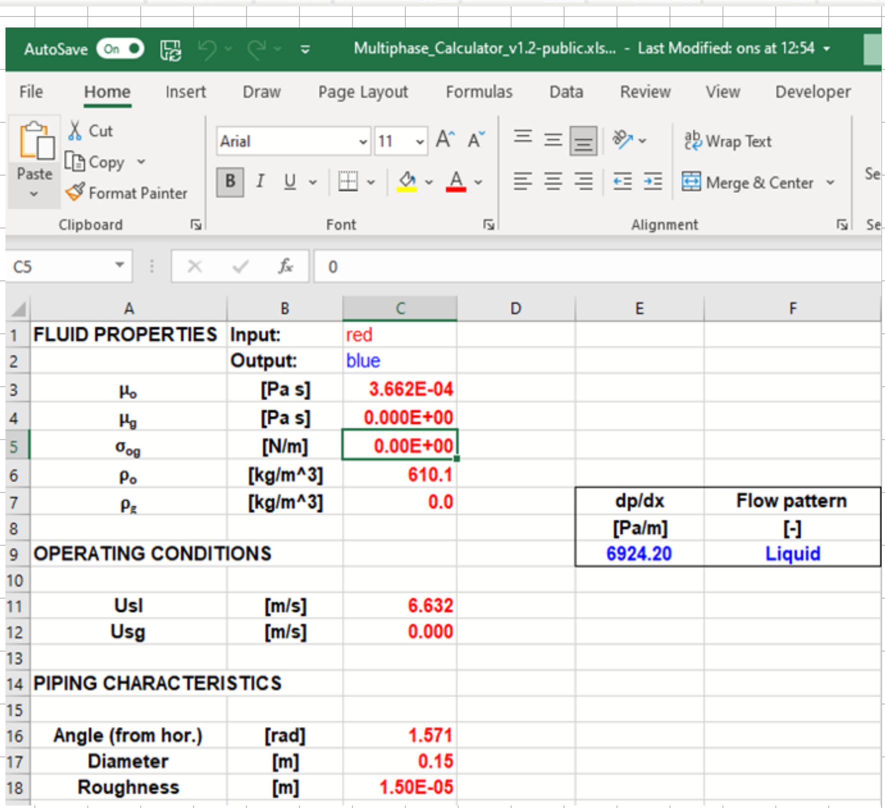
```
Function e_Nagoo(lambdag)
    ' e_Nagoo, the void fraction of gas, in fraction, using the ANSLIP equation by Nagoo, 2013
    'lambdag is non slip volume fraction of gas, in fraction
    If lambdag = 0 Then
        e_Nagoo = 0
    Else
        e_Nagoo = (lambdag + 1 - ((lambdag + 1) ^ 2 - 4 * (lambdag ^ 2)) ^ 0.5) / (2 * lambdag)
    End If
End Function
```


```
Function dpdx_mpf(roughness, viscl, viscg, denl, deng, usl, usg, D, angle, voidfraction)
    'dpdx_mpf pressure gradient, in bar/m, for multiphase flow
    'denl, liquid density, [kg/m3]
    'deng, gas density, [kg/m3]
    'usl superficial liquid velocity, [m/s]
    'usg superficial gas velocity, [m/s]
    'angle, inclination angle of pipe with respect to horizontal [deg]
    'D hydraulic diameter of pipe [m]
    'roughness pipe roughness, [m]
    'viscl, liquid viscosity [cP]
    'viscg, gas viscosity, [cP]
    'voidfraction [-]
    Pi = Atn(1) * 4
    denm = voidfraction * deng + (1 - voidfraction) * denl
    If voidfraction = 0 Or usg = 0 Then
        ug = 0
        ul = usl
        fg = 0
        fl = ffactor(denl, viscl, D, roughness, ul)
    ElseIf voidfraction = 1 Or usl = 0 Then
        ug = usg
        ul = 0
        fl = 0
        fg = ffactor(deng, viscg, D, roughness, ug)
    Else
        ug = usg / voidfraction
        ul = usl / (1 - voidfraction)
        fg = ffactor(deng, viscg / 1000#, D, roughness, ug)
        fl = ffactor(denl, viscl / 1000, D, roughness, ul)
    End If
    dpdx_f = (fg * deng * (ug * Abs(usg)) * 0.5 / D) + (fl * denl * (ul * Abs(usl)) * 0.5 / D)
    dpdx_h = denm * 9.81 * Sin(angle * Pi / 180)
    dpdx_mpf = dpdx_f + dpdx_h
    dpdx_mpf = dpdx_mpf / 100000#
End Function
```

Exercise: Pressure drop calculation in saturated oil well, Prof. Milan Stanko (NTNU)

GOR	[Sm3/Sm3]	155.1
Pipe ID	[m]	0.15
Pipe cross section area	[m2]	0.0177
Pipe roughness	[m]	1.50E-05
Pipe inclination from hor	[deg]	90
qo	[Sm3/d]	6000
qg	[Sm3/d]	9.31E+05

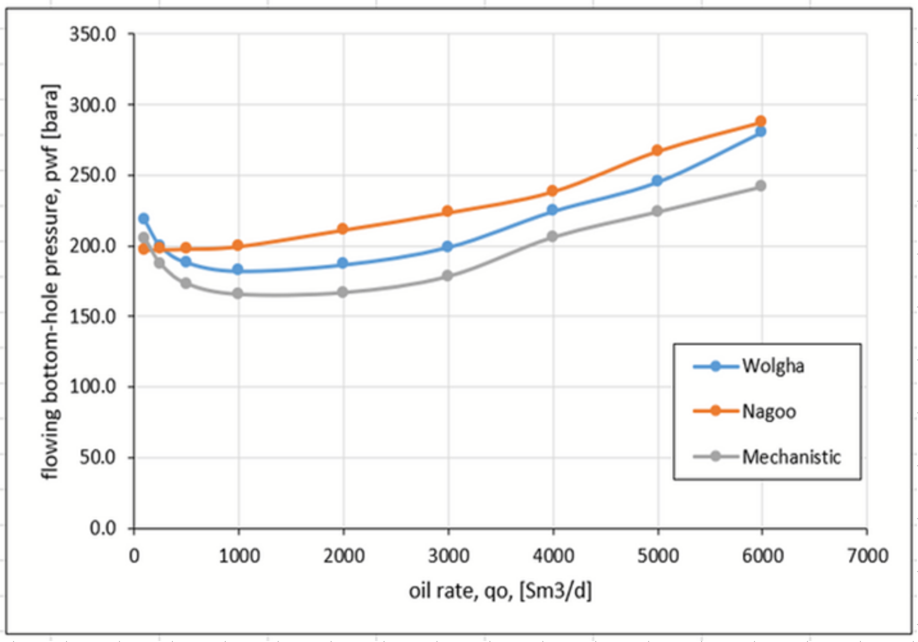
		BO table column	3	4	5	6	8	10	7	9	11						Mechanistic model	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	flowpattern	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	7046.0	2.740E+04	4.615	17.944	0.80	Slug	0.0492
500	57.1	52.6	46.8	1.34E-05	1.3	1.64E-02	82.1	1.29E-02	704.1	1.1	7.56E-03	7564.5	1.065E+04	4.954	6.976	0.58	Slug	0.0417
1000	64.3	73.5	67.6	1.45E-05	1.3	1.05E-02	125.0	1.53E-02	682.4	0.8	4.88E-03	8044.2	5.540E+03	5.269	3.628	0.41	Bubble	0.0605
1500	71.4	103.7	99.2	1.76E-05	1.5	6.90E-03	189.5	2.00E-02	653.3	0.6	2.35E-03	8781.2	2.322E+03	5.751	1.521	0.21	Bubble	0.0675
2000	78.6	137.5	134.8	2.33E-05	1.6	5.37E-03	243.1	2.55E-02	623.6	0.4	1.07E-03	9650.4	6.558E+02	6.321	0.430	0.06	Bubble	0.0708
2500	85.7	172.9	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	609.0	0.4	0.00E+00	10144.6	0.000E+00	6.644	0.000	0.00	Liquid	0.0693
3000	92.9	207.5	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	610.1	0.4	0.00E+00	10127.0	0.000E+00	6.633	0.000	0.00	Liquid	0.0693
3500	100.0	242.1	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	610.1	0.4	0.00E+00	10125.7	0.000E+00	6.632	0.000	0.00	Liquid	0.0692



	A	B	C	D	E	F
1	FLUID PROPERTIES	Input:	red			
2		Output:	blue			
3	μ_o	[Pa s]	3.662E-04			
4	μ_g	[Pa s]	0.000E+00			
5	σ_{og}	[N/m]	0.00E+00			
6	ρ_o	[kg/m^3]	610.1			
7	ρ_g	[kg/m^3]	0.0		dp/dx	Flow pattern
8					[Pa/m]	[-]
9	OPERATING CONDITIONS				6924.20	Liquid
10						
11	Usl	[m/s]	6.632			
12	Usg	[m/s]	0.000			
13						
14	PIPING CHARACTERISTICS					
15						
16	Angle (from hor.)	[rad]	1.571			
17	Diameter	[m]	0.15			
18	Roughness	[m]	1.50E-05			

pwh=28 bara

Models	Wolgha	Nagoo	Mechanistic
qo	pwf	pwf	pwf
[Sm3/d]	[bara]	[bara]	[bara]
100	218.3	197.5	205.5
250	200.0	197.6	187.5
500	188.5	198.2	173.2
1000	182.4	200.0	165.6
2000	186.9	211.5	166.8
3000	199.2	223.7	178.4
4000	224.5	238.5	206.2
5000	245.2	266.9	224.2
6000	280.5	287.5	242.1



- g
- l $\left\{ \begin{array}{l} \text{oil} \\ \text{water} \end{array} \right\} \longrightarrow$

$\frac{q_w}{q_o+q_w}$

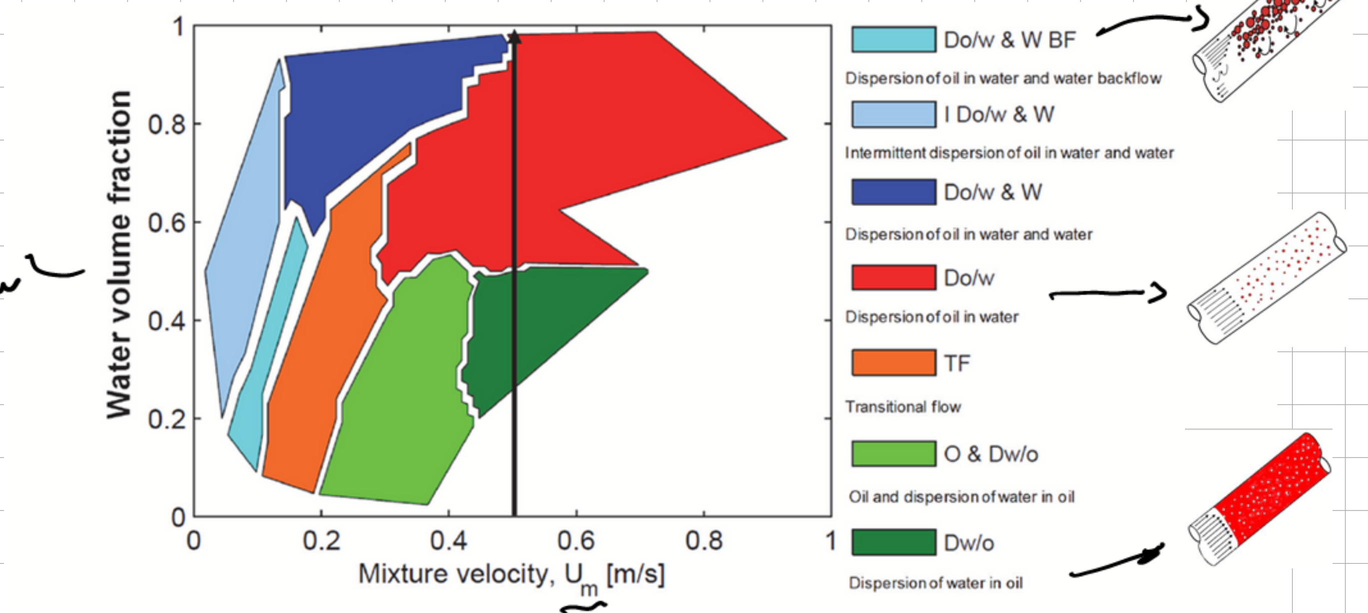


FIGURE 7-24. OIL-WATER FLOW PATTERN MAP OF WATER VOLUME FRACTION VERSUS MIXTURE VELOCITY FOR AN UPWARD PIPE INCLINATION OF 45°. FIGURE ADAPTED FROM RIVERA[7-5] [7-1].

$u_{so} + u_{sw}$

g	u_{sg}	λ_g	H_g	v_g
o	u_{so}	λ_o	H_o	v_o
w	u_{sw}	λ_w	H_w	v_w

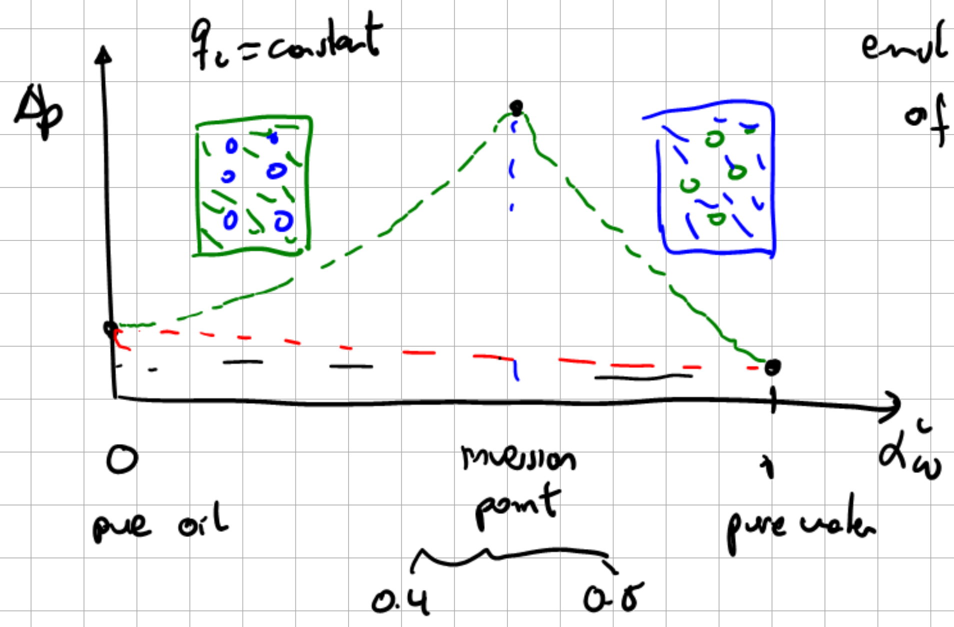
$v_o \approx v_w$ little slip

assume both travel at the same speed v_L

$$\rho_L = \rho_o \alpha_o^L + \rho_w \alpha_w^L \qquad \mu_L = \mu_o \alpha_o^L + \mu_w \cdot \alpha_w^L$$

$$\alpha_o^L = \frac{q_o}{q_o+q_w} \qquad \alpha_w^L = \frac{q_w}{q_o+q_w} = 1-\alpha_o^L$$

$\frac{q_L}{q_o+q_w} \rightarrow$



emulsion behavior of oil + water mixture

$$\mu_L = \mu_o \cdot e^{c_1(\alpha_w^L)} \qquad \mu_L = \mu_w \cdot e^{c_2(1-\alpha_w^L)}$$

$$wc = \frac{q_{\bar{w}}}{q_{\bar{o}}+q_{\bar{w}}} \qquad \alpha_w^L = \frac{q_w}{q_o+q_w}$$

sometimes in the literature, if $q_{\bar{o}} = q_o$

$wc \approx \alpha_w^L$

Pressure integration method

p_3

p_2

$\Big\} \Delta L$

p_1

$$p_2 = p_1 + \frac{dp}{dx}\Big|_{@p_1} \cdot \Delta L \quad \Big\} \text{ not so accurate for large } \Delta L$$

$$p_2 = p_1 + \frac{dp}{dx}\Big|_{@p_{av}} \Delta L$$

$$p_{av} = \frac{p_1 + p_2}{2}$$

implicit calculation

- assume a value of p_2^*
- compute $\frac{dp}{dx}\Big|_{p_{av}}$
- compute $p_2^{calc} = p_1 + \frac{dp}{dx}\Big|_{@p_{av}} \Delta L$
- check $p_2^{calc} = p_2^*$

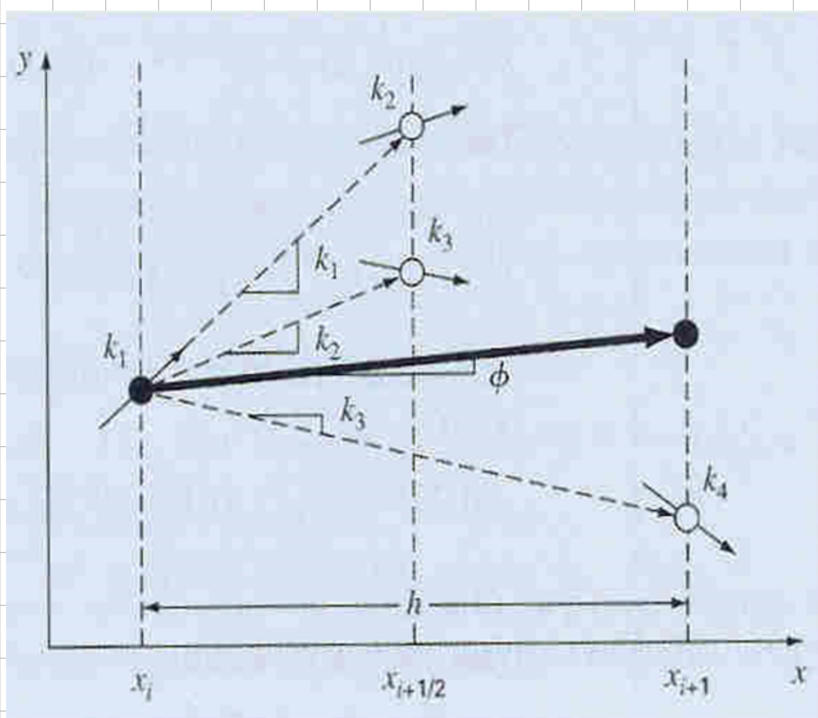
not → assume a value of p_2^*

yes ↓

proceed to next step

explicit approach (higher order)

Runge-Kutta 4th



p_1 $\quad$ p^* $\quad$ p_2

$$y_{n+1} = y_n + \frac{1}{6} \cdot h \cdot (k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4)$$

$$p_2 = p_1 + \frac{1}{6} \Delta L (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = \frac{dp}{dx}\Big|_{@p_1}$$

$$p^* = p_1 + \frac{dp}{dx}\Big|_{@p_1} \cdot \frac{\Delta L}{2}$$

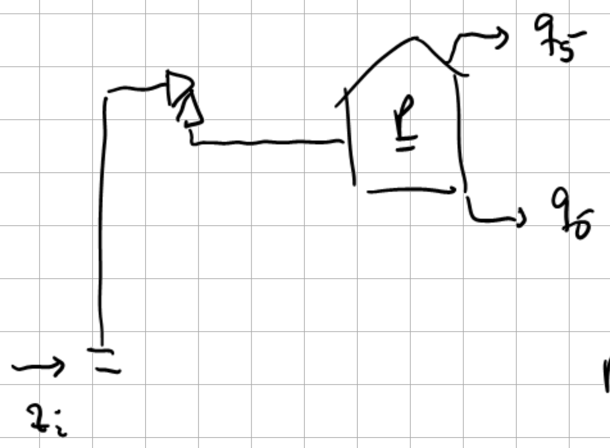
$$k_2 = \frac{dp}{dx}\Big|_{@p^*}$$

$$p^{**} = p_1 + k_2 \frac{\Delta L}{2}$$

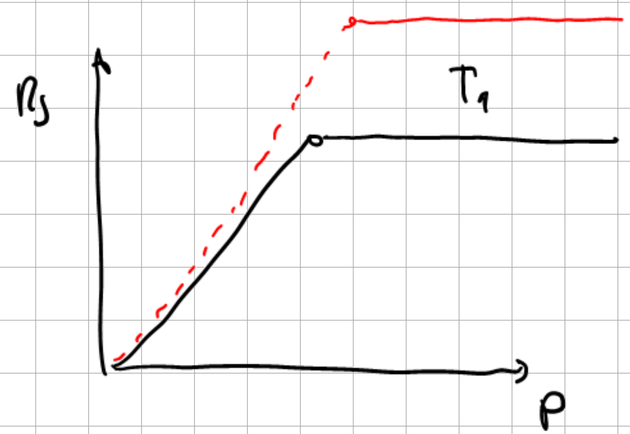
$$k_3 = \frac{dp}{dx}\Big|_{@p^{**}}$$

$$p^{***} = p_1 + k_3 \Delta L$$

$$k_4 = \frac{dp}{dx}\Big|_{@p^{***}}$$



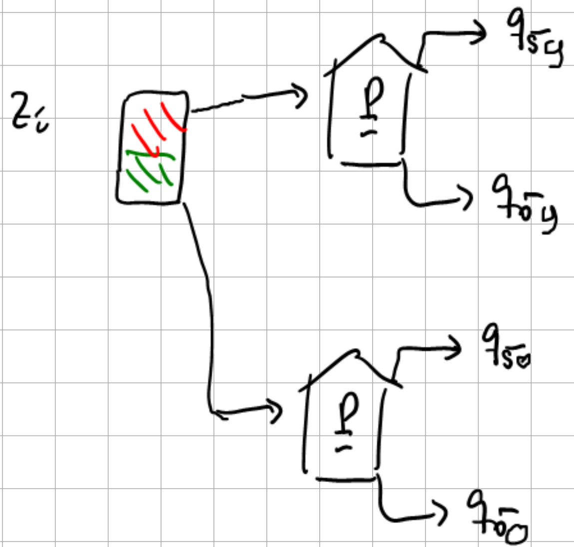
$R_p = \frac{q_{\bar{g}}}{q_{\bar{o}}}$ If z_i changes then R_p should also change

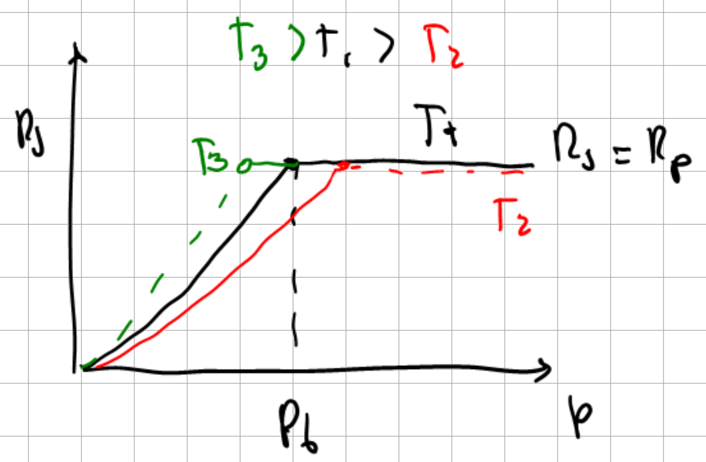


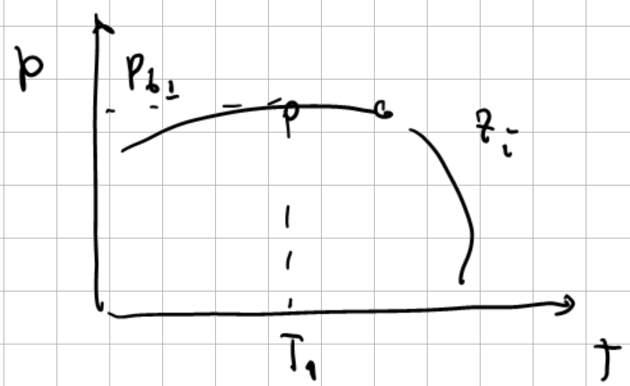
If z_i changes

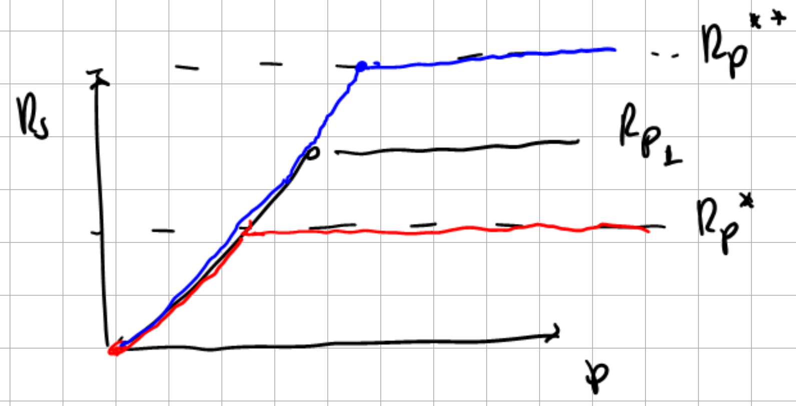
z_i^*

If z_i changes (or R_p) it is usually necessary to generate a new black oil table

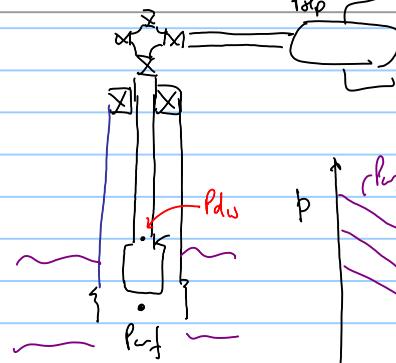


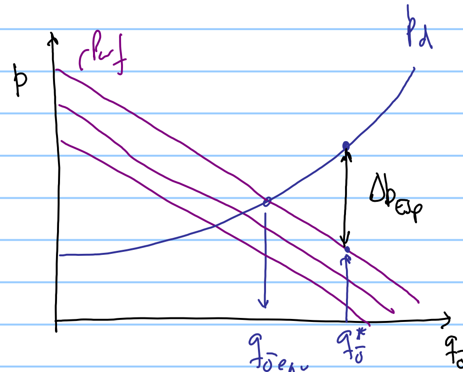


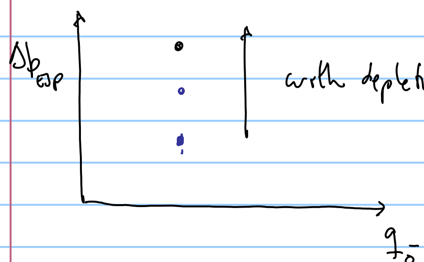




ESP electric submersible pump





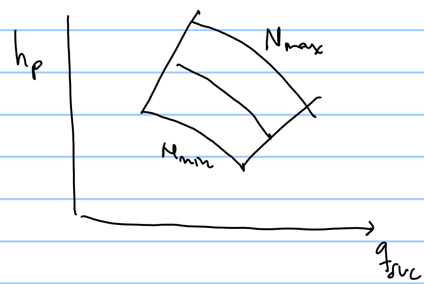


with depletion Δp_{esp} is increasing

the pump also has operational constraints

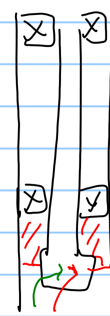
- limited available (maximum power) capacity
- operational map (envelope)
- $p_{suc} \geq p_b(T_R)$
  ↳ buble point pressure (no gas is allowed in the pump)

for compressor:



onshore

offshore

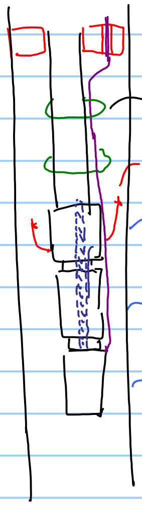


typical gas tolerance in esp is GVF = 10%
gas volume fraction

$$GVF = \frac{q_g}{q_L + q_g} \times 100\%$$

$q_L = q_o + q_w$

Armais Arutunoff



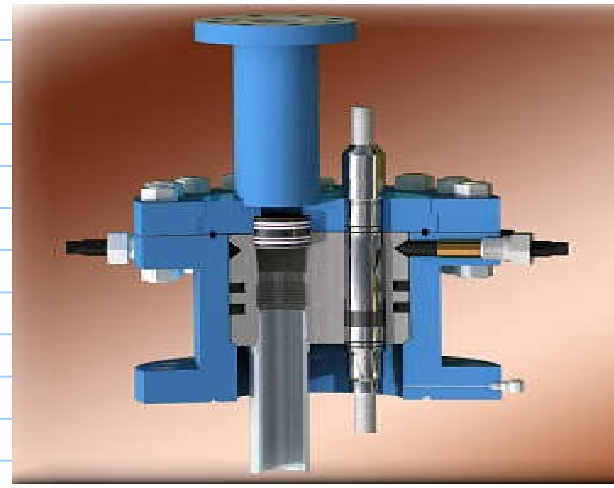
clamps

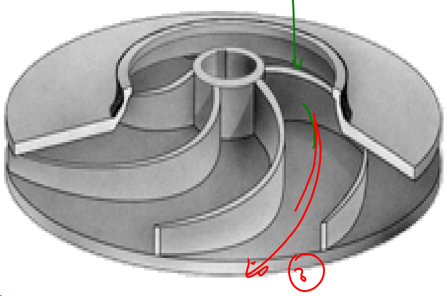
gas

pump might have a gas separator

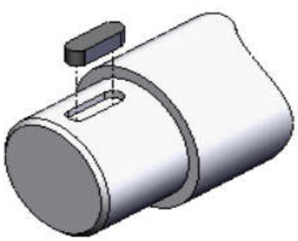
protector / seal

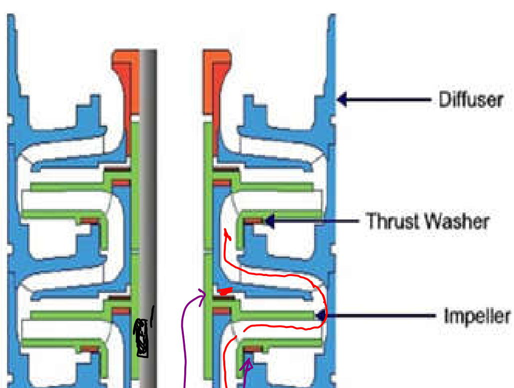
motor





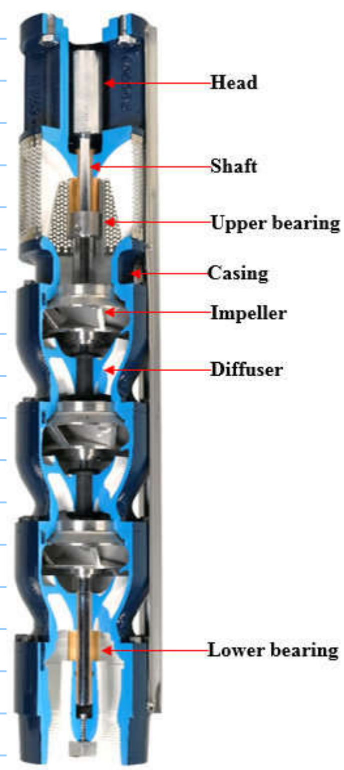
stage : impeller (rotor) + diffuser

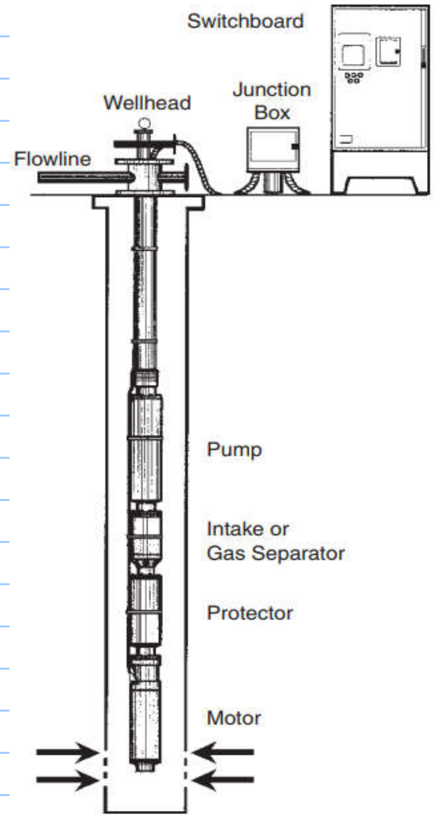


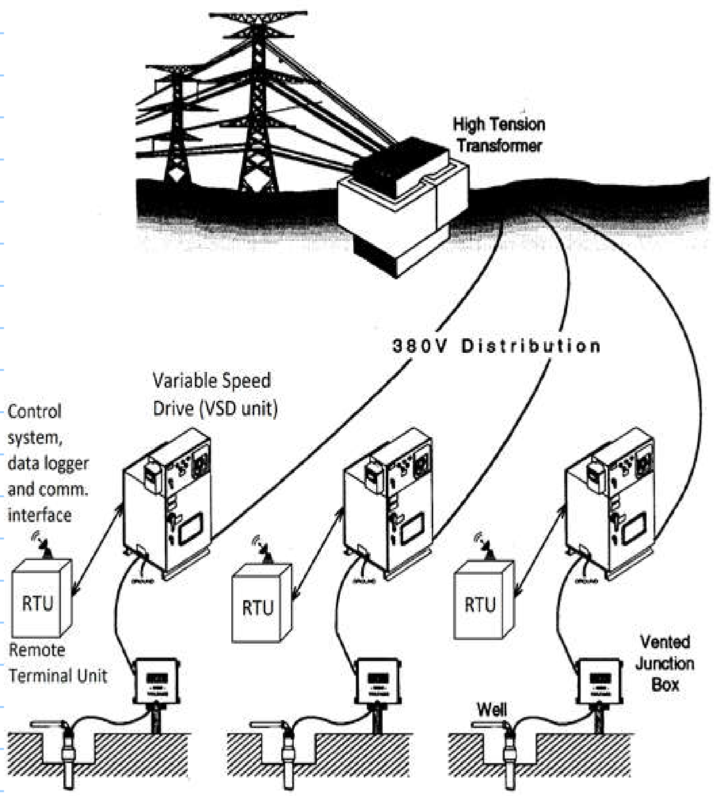


upthrust

downthrust



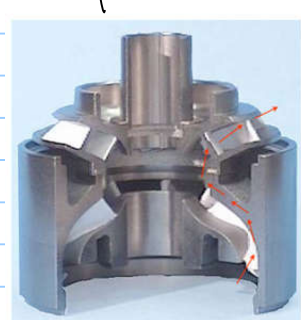




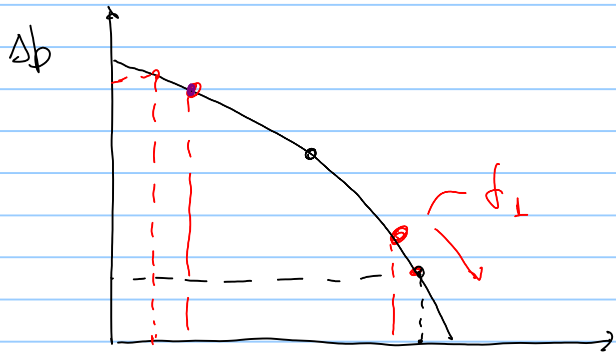
pump frequency

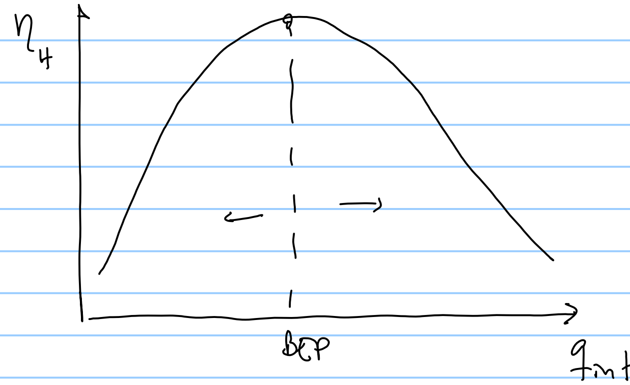
$f = 30 - 70\ hz$

gas impeller and diffuser more tolerant to gas



operational map of ESP





q_{inlet} $(q_o + q_w)$

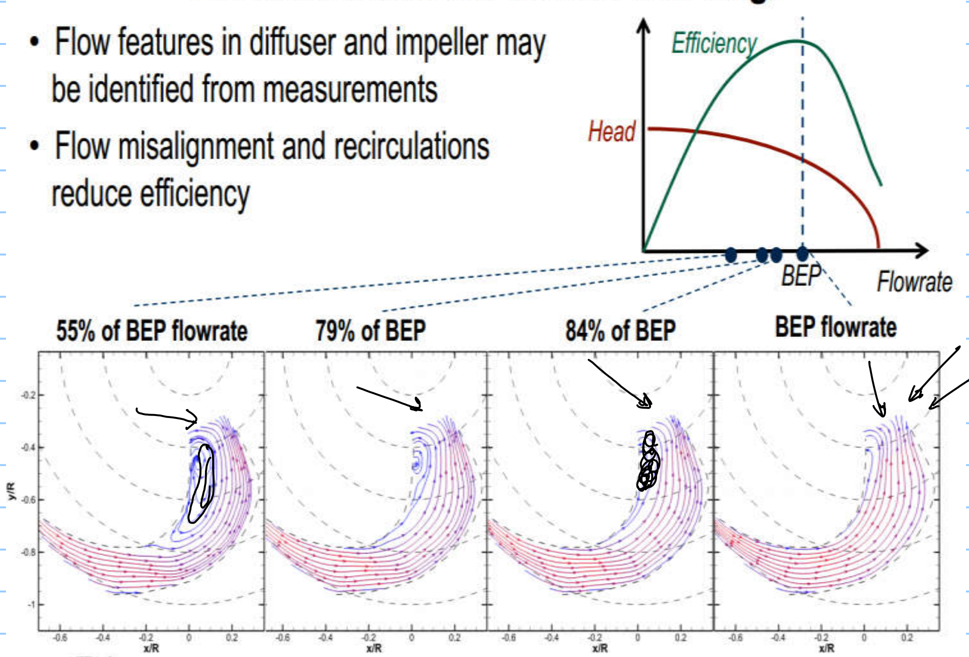
(downthrust

if $q > q_{max}$ then upthrusts but also low efficiency

pump hydraulic power $P_H = \dfrac{\Delta p \cdot q_{inlet}}{\eta_H}$

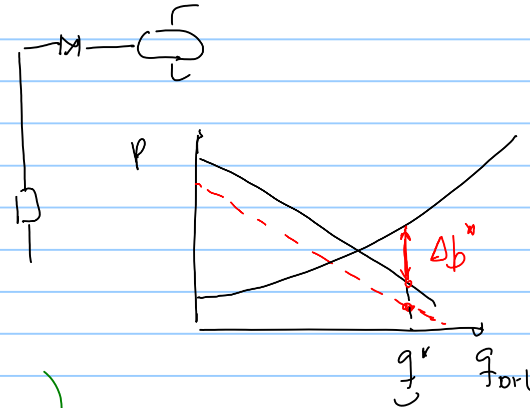
PIV measurement in a radial flow stage

- Flow features in diffuser and impeller may be identified from measurements
- Flow misalignment and recirculations reduce efficiency

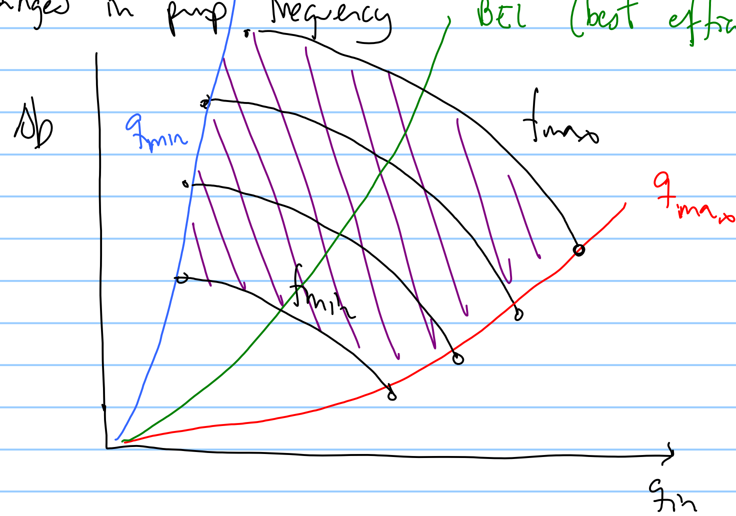


Example of stall region in diffuser passage (measured)

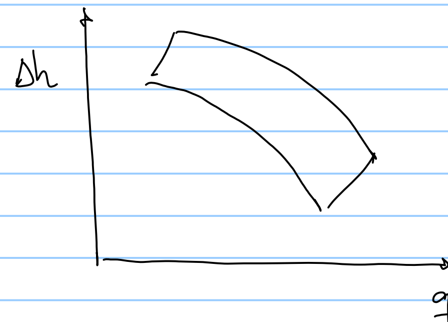
ARTIFICIAL LIFT SPE MIDDLE EAST CONFERENCE & EXHIBITION
SPE-14MEAL-14017-PP-MS • Measurement and Unsteady Simulation of Internal Flows within Stages • J Dusting

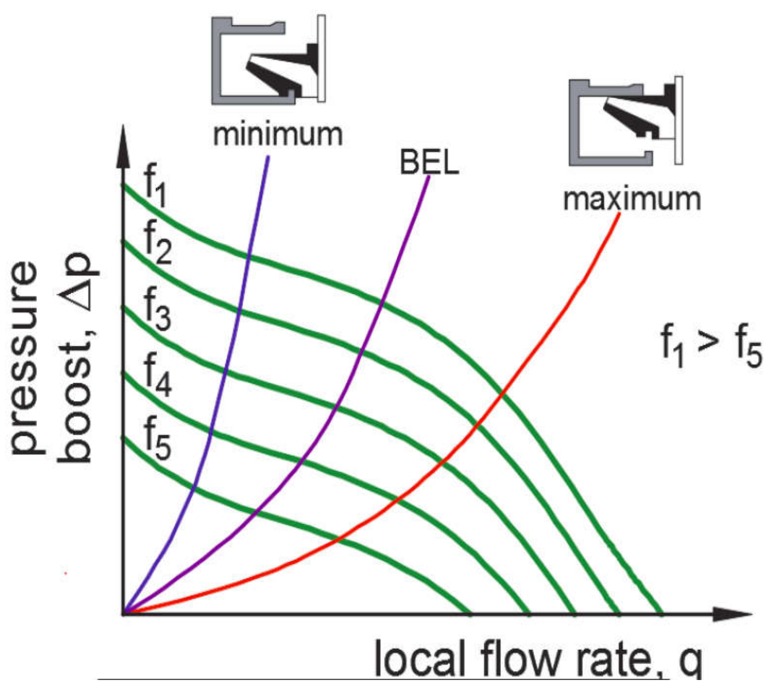


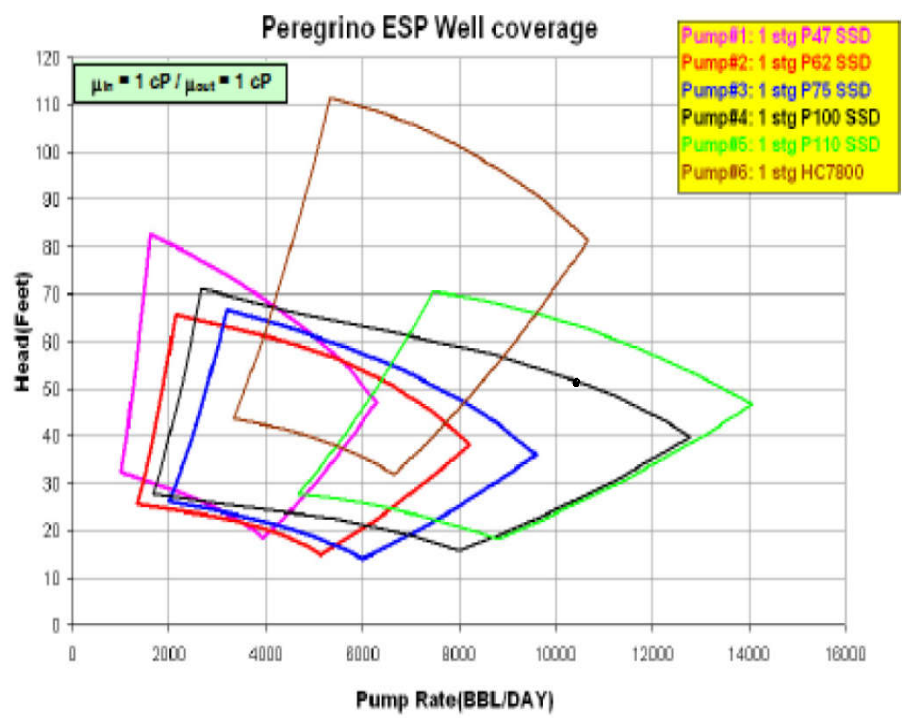
changes in pump frequency BEL (best efficiency line)



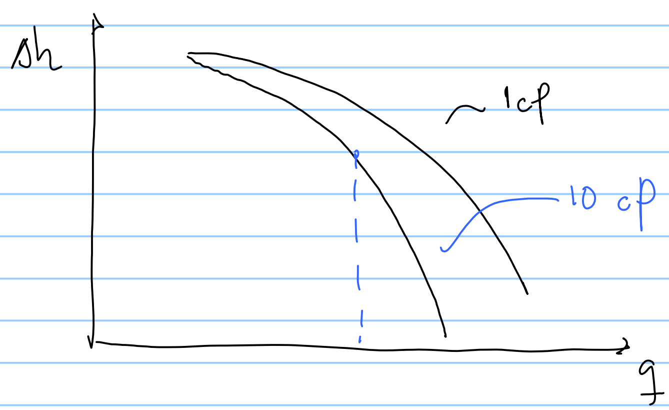
$$\Delta h = \text{head} = \frac{\Delta p}{\rho_{mix}}$$

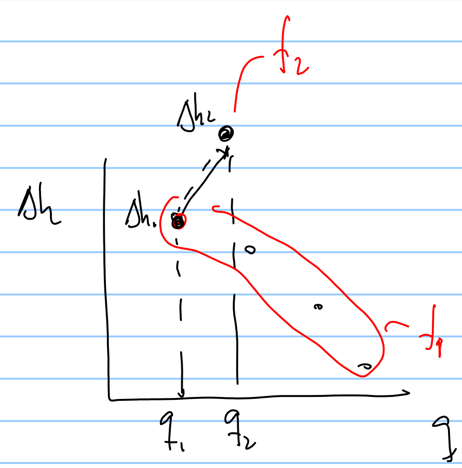






the viscosity of fluid also affects the performance of pump





for a given frequency

$$\Delta h = a q^4 + b q^3 + c q^2 + d q + e$$

$$\frac{\Delta h_{@f_1}}{\Delta h_{@f_2}} = \left(\frac{f_1}{f_2}\right)^2 \qquad \frac{q_1}{q_2} = \frac{f_1}{f_2} \qquad \text{similarity law}$$

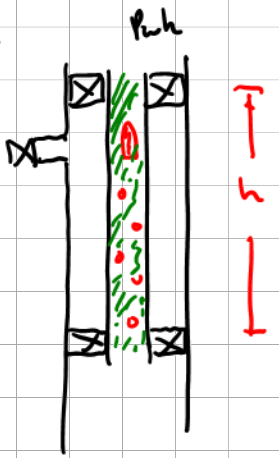
$$\frac{q_{ref}}{q} = \frac{f_{ref}}{f}$$

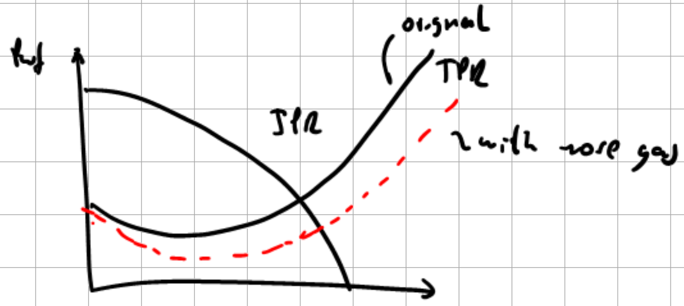
$$\Delta h(f) = \frac{f^2}{f_{ref}^2}\left[a\left(\frac{f_{ref}}{f}q\right)^4 + b\left(\frac{f_{ref}}{f}q\right)^3 + c\left(\frac{f_{ref}}{f}q\right)^2 + d\,\frac{f_{ref}}{f}q + e\right]$$

gas lift

oil wells

no gas lift

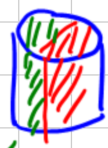




$$\Delta p = P_{wf} - P_{wh} = \underbrace{\Delta p_{hydros}}_{\rho_{mix} \cdot g \cdot h} + \underbrace{\Delta p_{frict}}_{f \frac{v^2}{2}} + \Delta p_{acceleration}$$

$v \propto (q_g + q_o)$

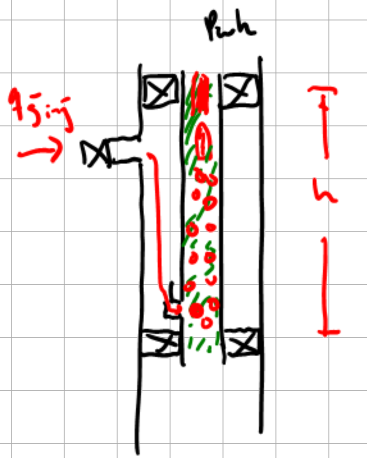
ρ_g — ρ_L



to reduce $\Delta p_{hydrostatic}$ try to $\rho_{mix} \rightarrow \rho_g$

more gas $\rightarrow \rho_{mix} \approx \rho_g \rightarrow \Delta p_{hydros} \downarrow$

$p_{wf} \downarrow \rightarrow q_{o\,equilibrium} \uparrow$

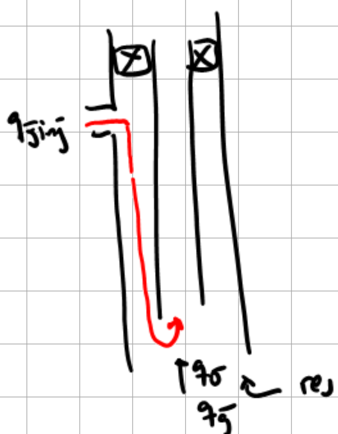


$\leftarrow q_{or}$

$\leftarrow q_{gr}$

$\leftarrow q_{wr}$

An approximation

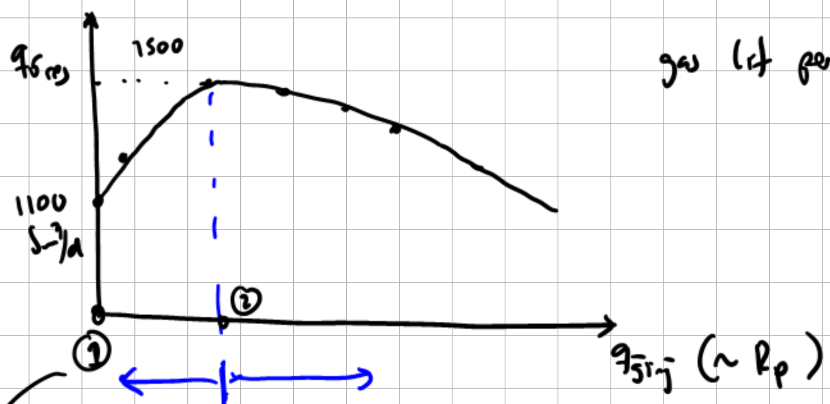


if R_p (GOR) $\uparrow$

$$R_p^{orig} = \frac{q_{g\,res}}{q_{o\,res}} = R_p^{res}$$

$$R_p^{gaslift} = \frac{q_g^{res} + q_{g\,inj}}{q_{o\,res}} = R_p^{gas\,lift}$$

gas lift performance curve



$\rho_{mix} \rightarrow \rho_g$

$\Delta p_{hyd} \downarrow$

$\rho_{mix} \rightarrow \rho_g$?

① $R_p = 150\ Sm^3/Sm^3$ $\quad q_o = 1100\ Sm^3/d \rightarrow q_g^{res} = 165\ E03\ Sm^3/d$

$q_{g\,inj} = 0\ Sm^3/d$

② $R_p = 350\ Sm^3/Sm^3$ $\quad q_o \approx 1500\ Sm^3/d \rightarrow q_g^{res} =$

$q_g^{inj} =$

$q_g^T = 350 \times 1500 = 525\ E03\ Sm^3/d$

$q_g^R = 150 \times 1500 = 225\ E03\ Sm^3/d$

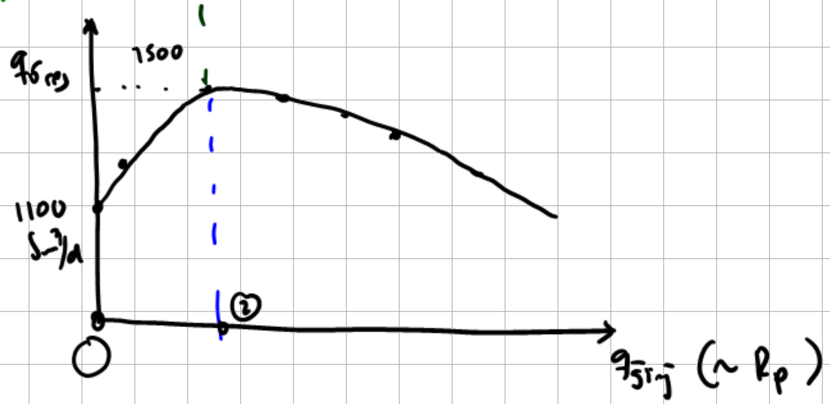
$q_g^{inj} = 300\ E03\ Sm^3/d$

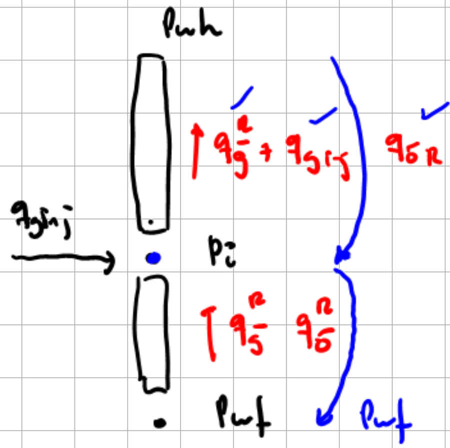
if $q_g \uparrow \rightarrow v \uparrow \rightarrow \Delta p_{friction} \uparrow \rightarrow p_{wf} \uparrow$ EFFECT ①

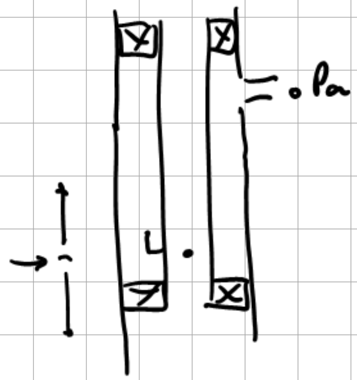
if $q_g \uparrow \rightarrow \rho_m \rightarrow \rho_g \rightarrow \Delta p_{hydr} \downarrow \rightarrow p_{wf} \downarrow$ EFFECT ②

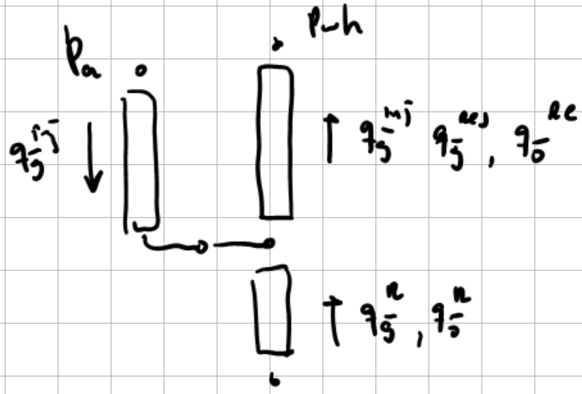
EFFECT ② is BIGGER THAN EFFECT ①

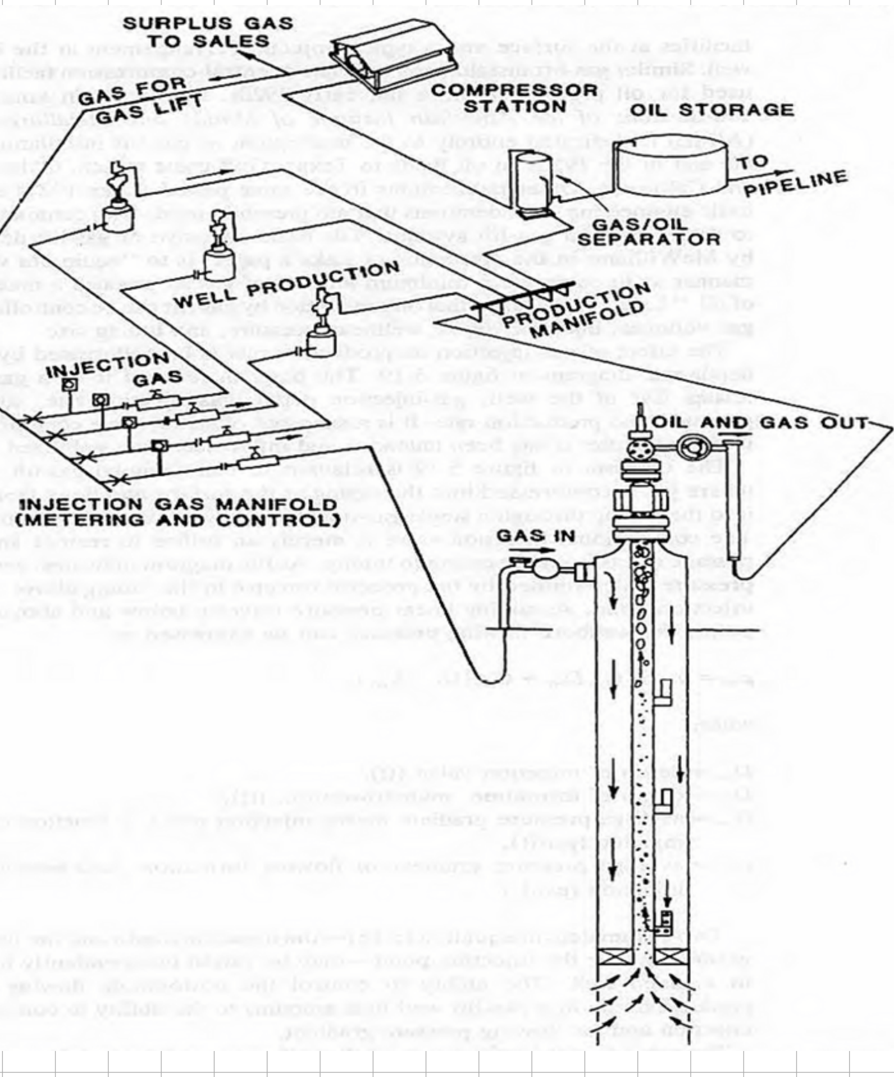
effect ② is smaller than effect ①

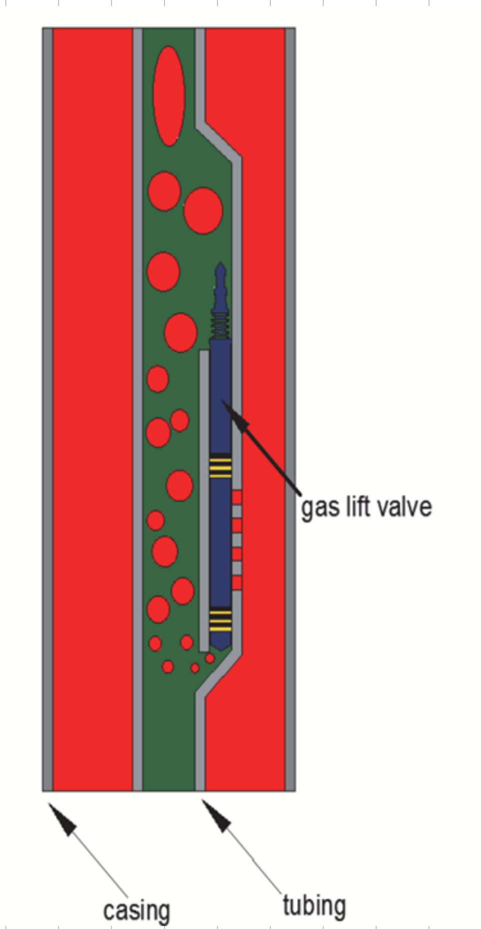


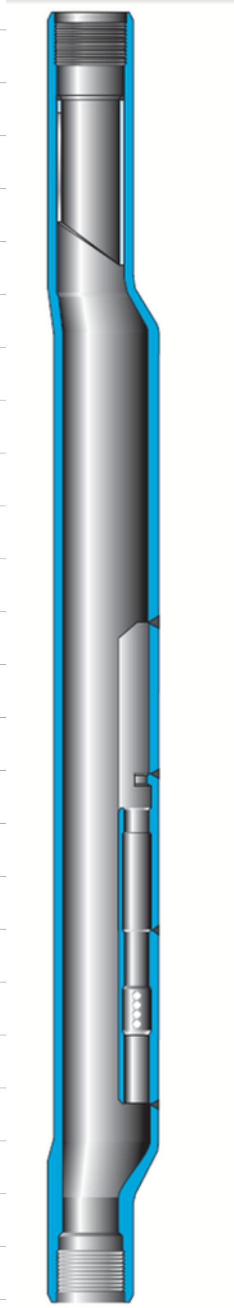

Pwh
qginj
Pi
Pwf
Pwf


Pa

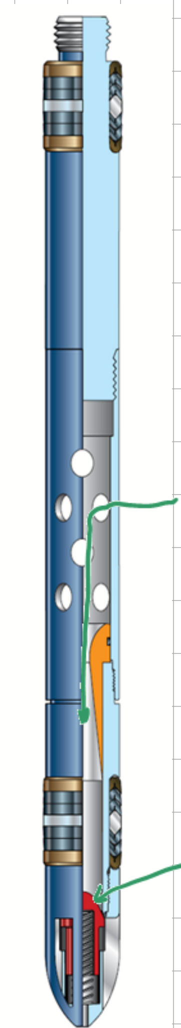

Pa
Pwh


SURPLUS GAS
TO SALES
GAS FOR
GAS LIFT
COMPRESSOR
STATION
OIL STORAGE
TO
PIPELINE
GAS/OIL
SEPARATOR
WELL PRODUCTION
PRODUCTION
MANIFOLD
INJECTION
GAS
OIL AND GAS OUT
INJECTION GAS MANIFOLD
(METERING AND CONTROL)
GAS IN



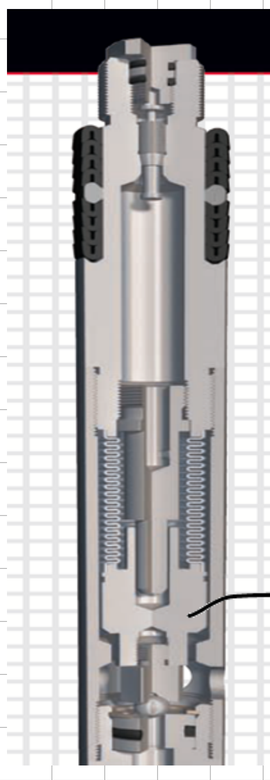


MMM Series
side pocket mandrel.



check valve, allows flow ↓
but not ↑

NOVA orifice venturi



pressure control port

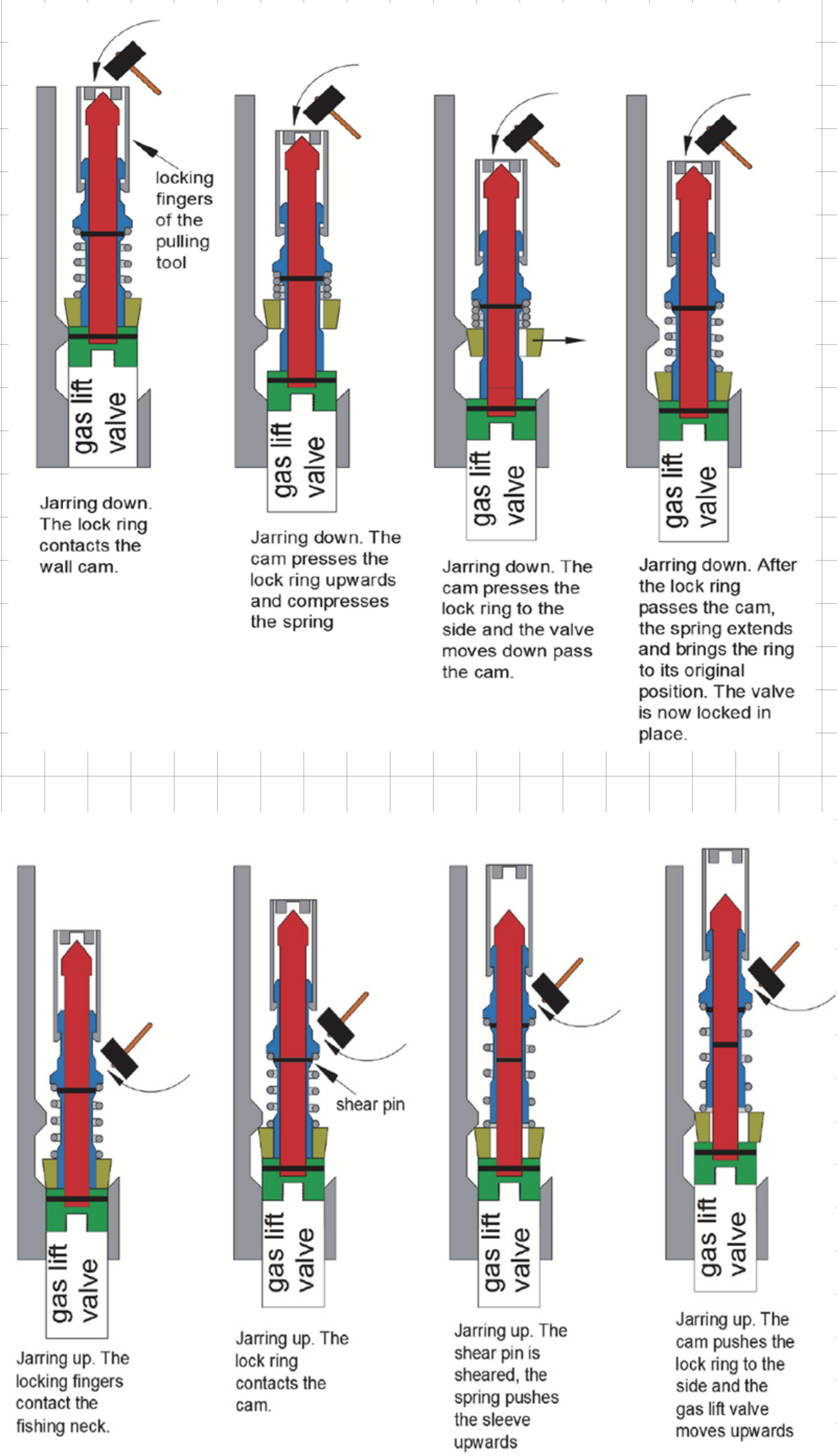
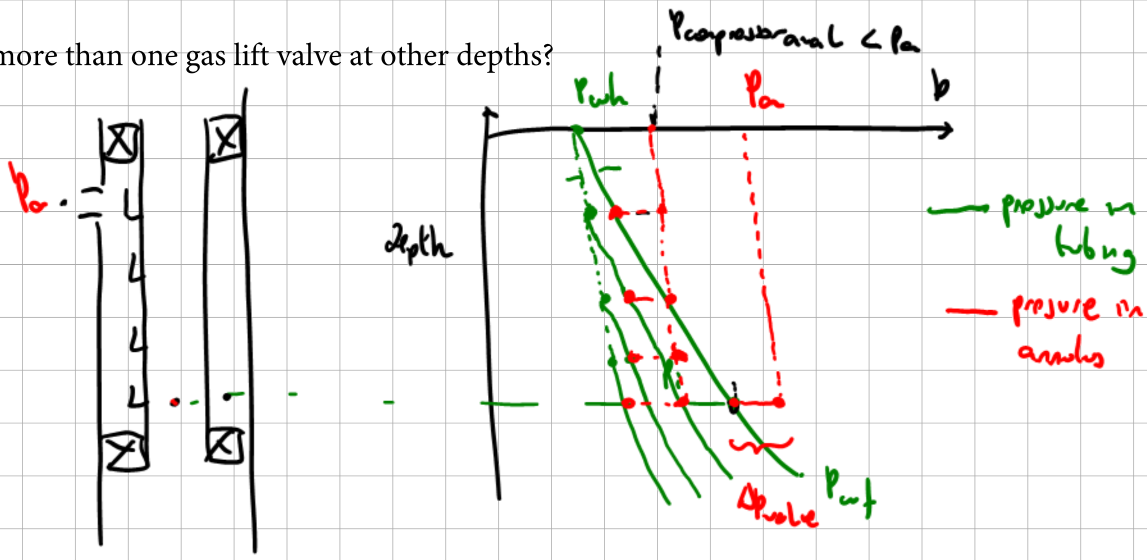


FIGURE 3-8. SEQUENCE TO RETRIEVE A GAS-LIFT VALVE FROM THE MANDREL POCKET

Why do we have more than one gas lift valve at other depths?



P_i is big, P_e is too big