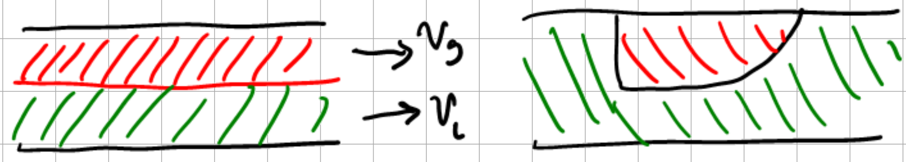
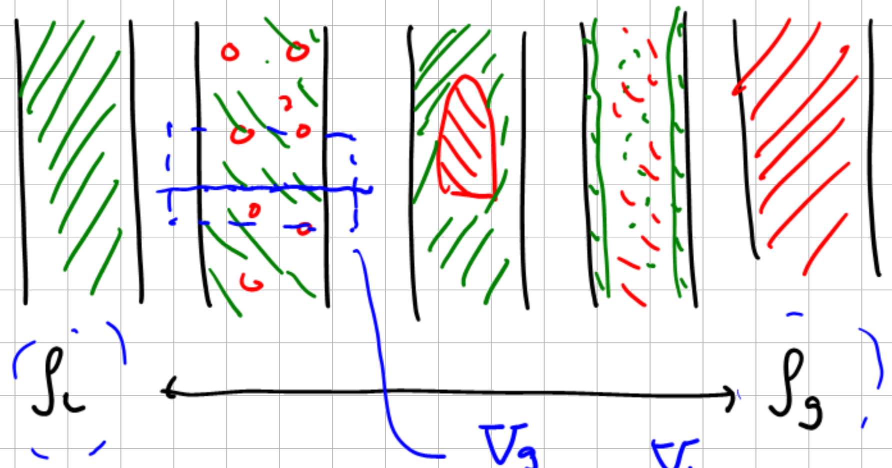
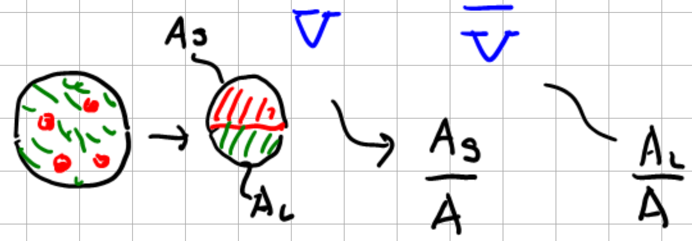
- flow patterns



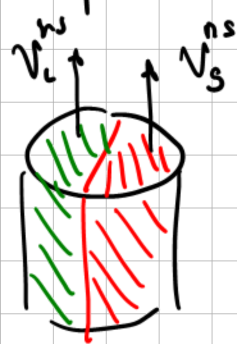
- phase velocity $\rightarrow$ friction Δp

- phase spatial distribution $\rightarrow$ hydrostatic Δp

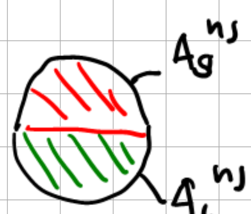




Liquid and gas travel at the same velocity (no-slip)



$$v_L^{ns} = v_g^{ns} = v_m = \frac{q_g + q_L}{A} = \frac{q_g}{A} + \frac{q_L}{A} = u_{sg} + u_{sL}$$



$$\lambda_g = \frac{A_g^{ns}}{A} = 1 - \lambda_L = \frac{q_g}{q_g + q_L} \qquad q_g \uparrow\uparrow \gg q_L \longrightarrow \lambda_g \to 1$$

$$\lambda_L = \frac{A_L^{ns}}{A} = 1 - \lambda_g = \frac{q_L}{q_g + q_L} \qquad q_L \gg q_g \longrightarrow \lambda_L \to 1$$

gas and liquid move at different velocities $v_g \neq v_L$ (slip condition)

void fraction

$$\varepsilon = H_g = \frac{A_g}{A} = 1 - H_L$$

(H_g: gas holdup)

$$H_L = \frac{A_L}{A} = 1 - H_g$$

(H_L: liquid holdup)

v_g ?
v_L ?

mass conservation between non-slip condition and slip condition

$$q_g = v_g^{ns} \cdot A_g^{ns} = v_g \cdot A_g \qquad \text{divide by } A$$

$$v_g^{ns} \lambda_g = v_g \cdot \varepsilon$$

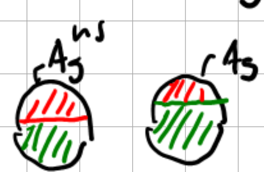
$$\boxed{v_g = v_m \frac{\lambda_g}{\varepsilon}}$$

$\lambda_g < \varepsilon$

$v_g < v_m$

$\lambda_g = \varepsilon$
$v_g = v_m$

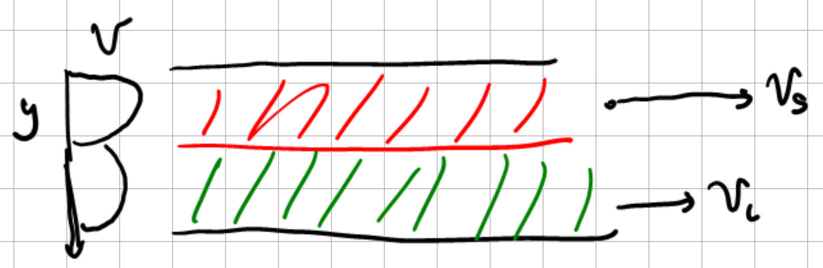
$\lambda_g > \varepsilon$
$v_g > v_m$

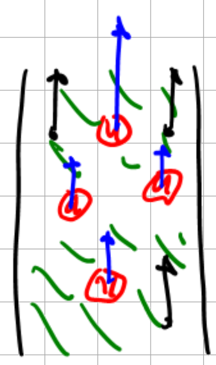


$$\boxed{v_L = v_m \frac{\lambda_L}{H_L}}$$

$$v_g = \frac{v_m \lambda_g}{\varepsilon} = \frac{q_g + q_L}{A} \cdot \frac{q_g}{q_g + q_L} \cdot \frac{1}{\varepsilon} = \frac{u_{sg}}{\varepsilon}$$

$$\boxed{v_g = \frac{u_{sg}}{\varepsilon} \qquad v_L = \frac{u_{sL}}{H_L}}$$





ε, H_L, v_L, v_g is a result of solving our {mass conservation, momentum conservation}

one example

$$v_g = C_0 v_m + u_0$$

u_0: velocity of a bubble in stagnant liquid

TWO-PHASE FLOW IN VERTICAL TUBES

By D. J. NICKLIN, B.Sc. App.,* J. O. WILKES, M.A.*† and J. F. DAVIDSON, M.A., Ph.D., A.M.I.Mech.E.,*

SUMMARY

A study has been made of the properties of long bubbles in vertical tubes. It has been shown that these bubbles rise relative to the liquid ahead of them at a velocity exactly equal to the rising velocity of wakeless bubbles of the type studied by Dumitrescu and by Davies and Taylor. For 1 in. tubes, this velocity is closely predicted by Dumitrescu's theory and equals $0{\cdot}35\,(gD)^{1/2}$ where g is the acceleration of gravity and D the tube diameter. The motion of the bubbles in moving liquid streams has been studied, and the results applied to the problem of two-phase slug flow. An expression for the voidage in steady two-phase slug flow has been derived, and this predicted voidage agrees well with results reported here and elsewhere.

* University of Cambridge, Department of Chemical Engineering, Pembroke Street, Cambridge.
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TRANS. INSTN CHEM. ENGRS, Vol. 40, 1962

$$u_s = 1{\cdot}2\,\bar{u}_L + 0{\cdot}35\,(gD)^{1/2}$$

N. ZUBER
Advanced Technology Laboratories. Mem. ASME

J. A. FINDLAY
Knolls Atomic Power Laboratory. Mem. ASME
General Electric Co., Schenectady, N. Y.

Average Volumetric Concentration in Two-Phase Flow Systems

A general expression which can be used either for predicting the average volumetric concentration or for analyzing and interpreting experimental data is derived. The analysis takes into account both the effect of nonuniform flow and concentration profiles as well as the effect of the local relative velocity between the phases. The first effect is taken into account by a distribution parameter, whereas the latter is accounted for by the weighted average drift velocity. Both effects are analyzed and evaluated. The results predicted by the analysis are compared with experimental data obtained for various two-phase flow regimes, with various liquid-gas mixtures in adiabatic, vertical flow over a wide pressure range. Good agreement with experimental data is shown.

[2] Numbers in brackets designate References at end of paper.
Contributed by the Heat Transfer Division and presented at the Winter Annual Meeting, New York, N. Y., November 29–December 3, 1964, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received at ASME Headquarters, September 15, 1964.

"v_0"

THE FLOW OF LIQUID-GAS MIXTURES IN VERTICAL TUBES.

By

Hans Behringer

ZEITSCHRIFT FUR DIE GESAMTE KALTE-INDUSTRIE, 43, 55-58, 1936.

Z. angew. Math. Mech. Bd. 23 Nr. 3 Juni 1943 — Dumitrescu, Strömung an einer Luftblase im senkrechten Rohr — 139

Strömung an einer Luftblase im senkrechten Rohr.

Von *D. T. Dumitrescu* in Bukarest.

The mechanics of large bubbles rising through extended liquids and through liquids in tubes

BY R. M. DAVIES AND SIR GEOFFREY TAYLOR, F.R.S.

(*Received* 13 *September* 1949)

$$\varepsilon = \frac{U_{SG}}{U_{SG}\left(1+\left(\frac{U_{SL}}{U_{SG}}\right)^{\left(\frac{\rho_G}{\rho_L}\right)^{0.1}}\right)+2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L-\rho_G)}{\rho_L^2}\right]^{0.25}(1.22+1.22\sin\theta)^{\frac{P_{atm}}{P_{system}}}}$$

Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

Melkamu A. Woldesemayat, Afshin J. Ghajar *

School of Mechanical and Aerospace Engineering, Oklahoma State University, Stillwater, OK 74078, USA

Received 1 June 2006; received in revised form 13 September 2006

$$H_g = \frac{\lambda_g + 1 - \left[\left(\lambda_g+1\right)^2 - 4\cdot{\lambda_g}^2\right]^{0.5}}{2\cdot\lambda_g}$$

Pipe Fractional Flow Theory: Principles and Applications

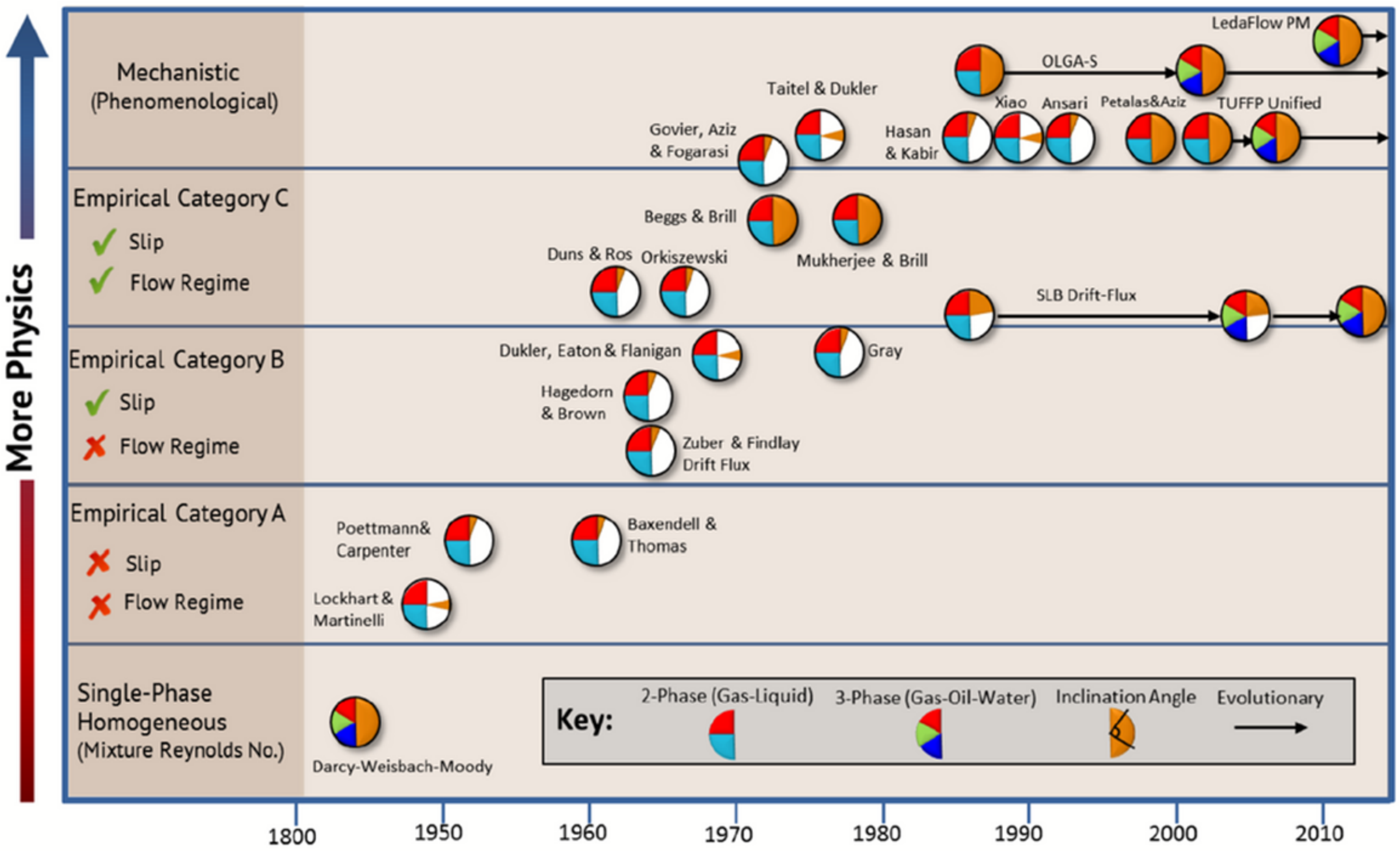
by

Anand Subhash Nagoo, B.Sc., M.S., M.S.

2013

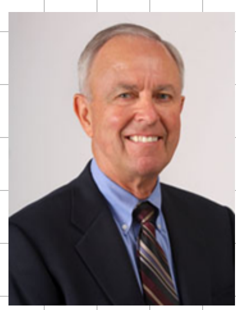
- $U_s = V_g - V_L$ slip velocity
- $S = \frac{V_g}{V_L}$ slip ratio

Some examples of pressure drop models for multiphase flow



A Study of Two-Phase Flow in Inclined Pipes

H. Dale Beggs,* SPE-AIME, U. of Tulsa
James P. Brill, SPE-AIME, U. of Tulsa

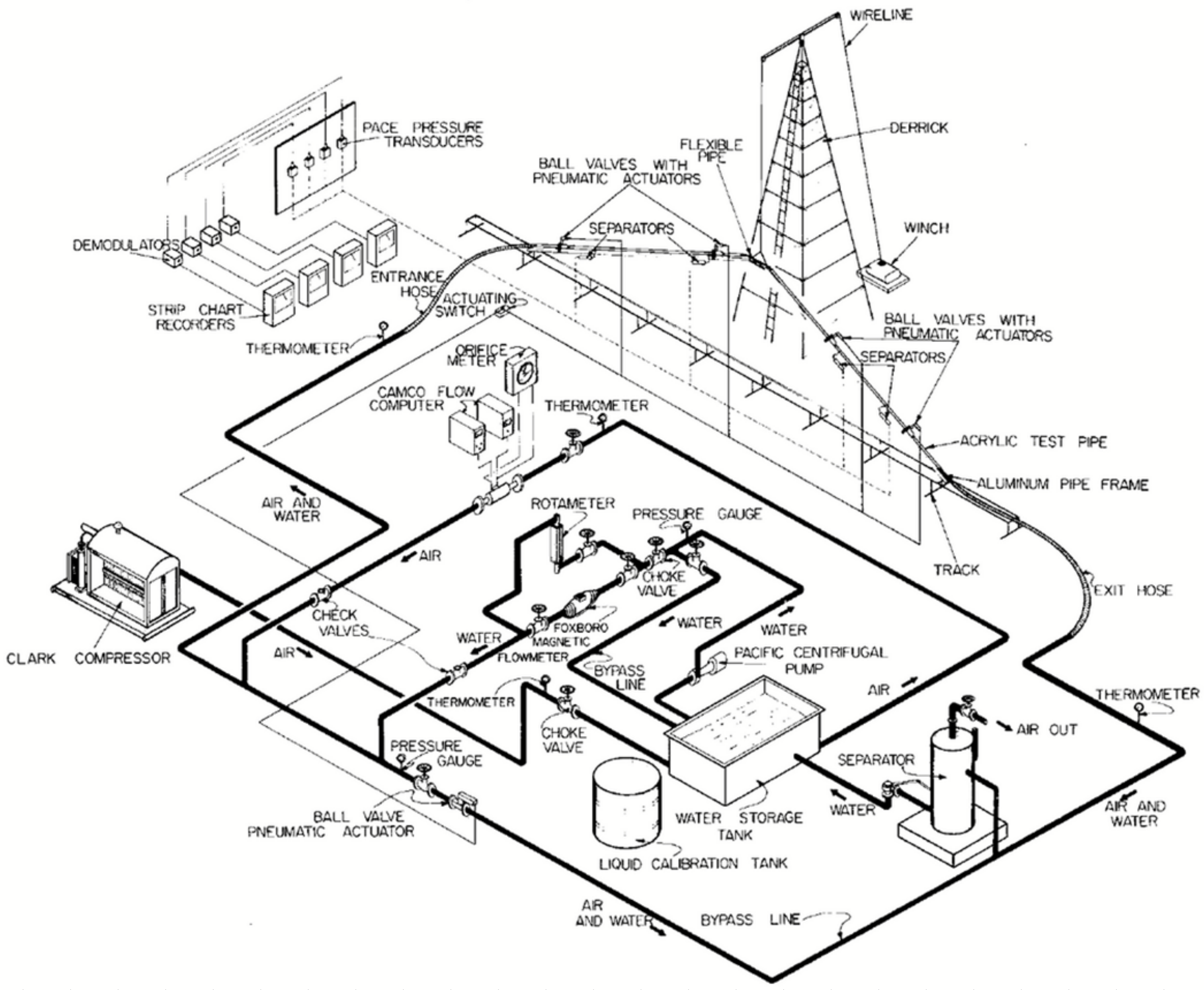


1973

$$-\frac{dp}{ds} = \left.\frac{dp}{ds}\right|_{grav.} + \left.\frac{dp}{ds}\right|_{fric.} + \left.\frac{dp}{ds}\right|_{accel.}$$

$$-\frac{dp}{dZ} = \frac{\frac{g}{g_c}\sin\theta\,[\rho_L H_L + \rho_g(1-H_L)] + \frac{f_{tp} G_m v_m}{2 g_c d}}{1 - \{[\rho_L H_L + \rho_g(1-H_L)]\, v_m v_{sg}\}/g_c p}$$

https://wiki.whitson.com/pipeflow/correlations/beggs_brill/



A UNIFIED MODEL FOR PREDICTING FLOW-PATTERN TRANSITIONS FOR THE WHOLE RANGE OF PIPE INCLINATIONS

D. BARNEA
Faculty of Engineering, Department of Fluid Mechanics and Heat Transfer, Tel-Aviv University, Ramat-Aviv 69978, Israel

(*Received 2 February 1986; in revised form 9 June 1986*)

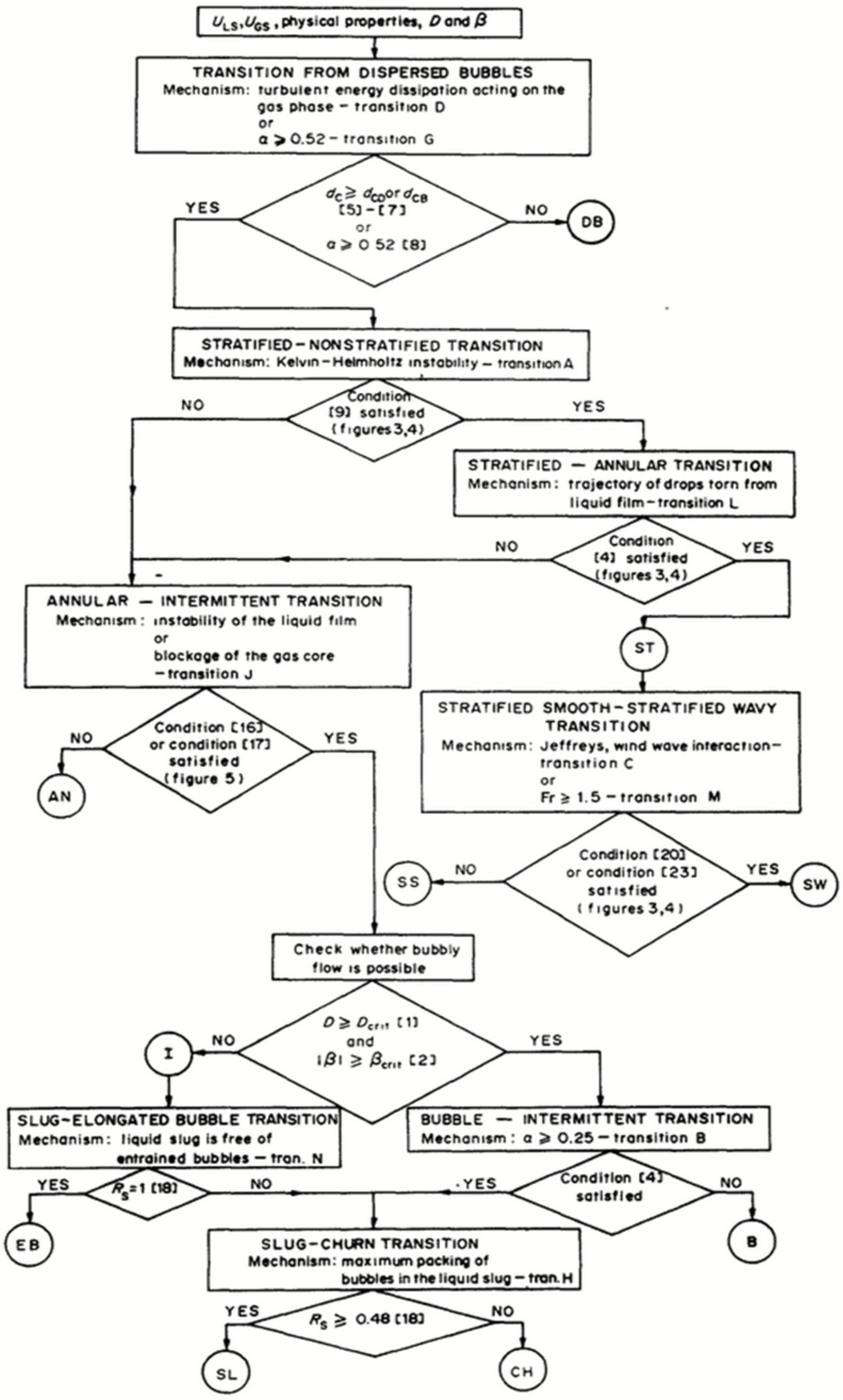
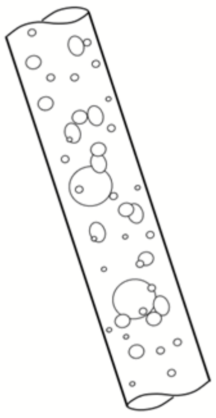


Figure 6. Logical pass for flow-pattern determination.

Bubble Flow-Pattern

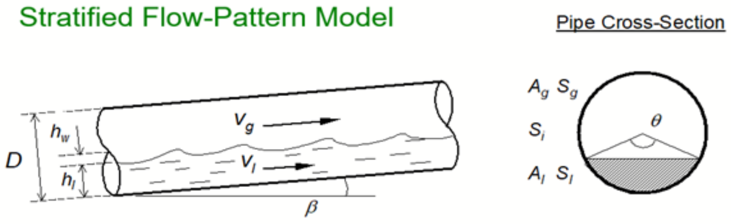


- Turbulent forces prevent bubble agglomeration and slip effect.
- Transition from bubble flow is given in the work of Barnea et al. (1987).
- The bubble flow-pattern is modeled as homogenous single fluid flow with averaged properties of liquid and gas.

Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = f_m \frac{2\rho_m v_m^2}{D} + \rho_m g \sin\beta$$

Stratified Flow-Pattern Model



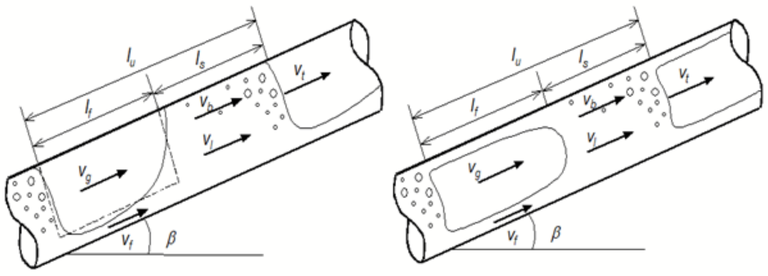
Combined momentum equation:

$$\frac{\tau_g S_g}{A_g} - \frac{\tau_l S_l}{A_l} + \tau_i S_i\left(\frac{1}{A_l} + \frac{1}{A_g}\right) - (\rho_l - \rho_g) g \sin\beta = 0$$

Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = \frac{\tau_l S_l + \tau_g S_g}{A} + \left(\frac{A_l}{A}\rho_l + \frac{A_g}{A}\rho_g\right) g \sin\beta$$

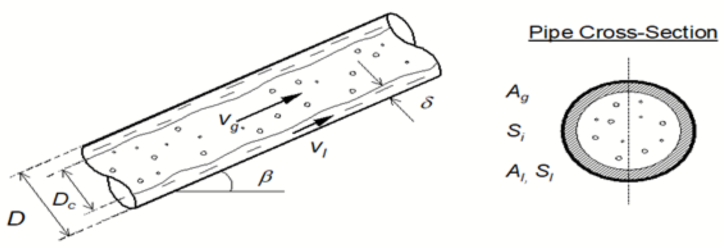
Intermittent Flow-Pattern Models



Pressure gradient equation:

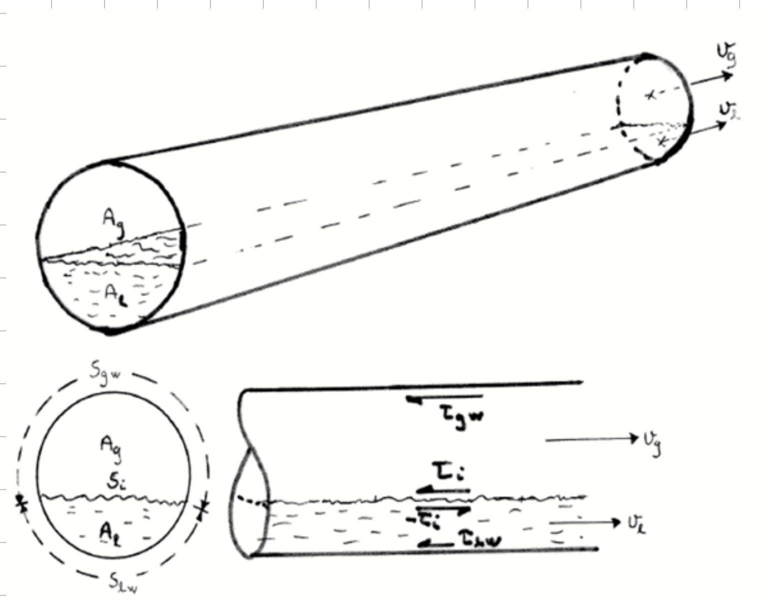
$$-\left(\frac{dP}{dx}\right) = \rho_u g \sin\beta + \frac{1}{l_u}\left[\left(\frac{\tau_s \pi D}{A} l_s\right) + \left(\frac{\tau_f S_f + \tau_g S_g}{A} l_f\right)\right]$$

Annular Flow-Pattern Model



Pressure gradient equation:

$$-\left(\frac{dP}{dx}\right) = \frac{\tau_l S_l}{A} + \left(\frac{A_l}{A}\rho_l + \frac{A_g}{A}\rho_{gc}\right) g \sin\beta$$



Harald Asheim's drift flux model

$$A_g\,dp + A_g\,\rho_g\,g_x\,dx + A_g\,\rho_g\,v_g\,dv_g + \tau_{gw}\,S_{gw}\,dx + \tau_i\,S_i dx = 0$$

+

$$A_l\,dp + A_l\,\rho_l\,g_x\,dx + A_l\,\rho_l\,v_l\,dv_l + \tau_{lw}\,S_{lw}\,dx - \tau_i\,S_i dx = 0$$

↓

$$dp + \left(\rho_g y_g + \rho_l y_l\right) g_x dx + \rho_g v_g y_g dv_g + \rho_l v_l y_l dv_l + \frac{\tau_g S_{gw} + \tau_{lw} S_{lw}}{A} dx = 0$$

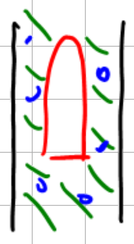
$$\tau_g = \frac{1}{8} f_g \rho_g v_g |v_g|$$

$$\tau_l = \frac{1}{8} f_l \rho_l v_l |v_l|$$

$$S_{gw} = \pi d y_g$$

$$S_{gw} = \pi d y_l$$

$$\frac{dp}{dx} + \rho_{TP} g_x + \rho_g v_{sg} \frac{dv_g}{dx} + \rho_l v_{sl} \frac{dv_l}{dx} + \frac{1}{2 \cdot d}\left(f_g \rho_g v_g |v_g| y_g + f_l \rho_l v_l |v_l| y_l\right) = 0$$



$p^*, T^* \rightarrow$

	o	g	w
ρ			
M			
	σ_{og}	σ_{wg}	σ_{ow}
h			

$\dot{m}_o \; \dot{m}_g \; \dot{m}_w \rightarrow q_o \; q_g \; q_w \rightarrow u_{so} \; u_{sg} \; u_{sw} \rightarrow \frac{dp}{dx}, \frac{dT}{dx}$

2 approaches:

- Compositional: z_i (mole frac) for $C_1, C_2, C_3, \ldots, i$, pseudo components + EOS (Peng Robinson) + EOS parameters
  - mixing rules
  - Binary interaction parameters

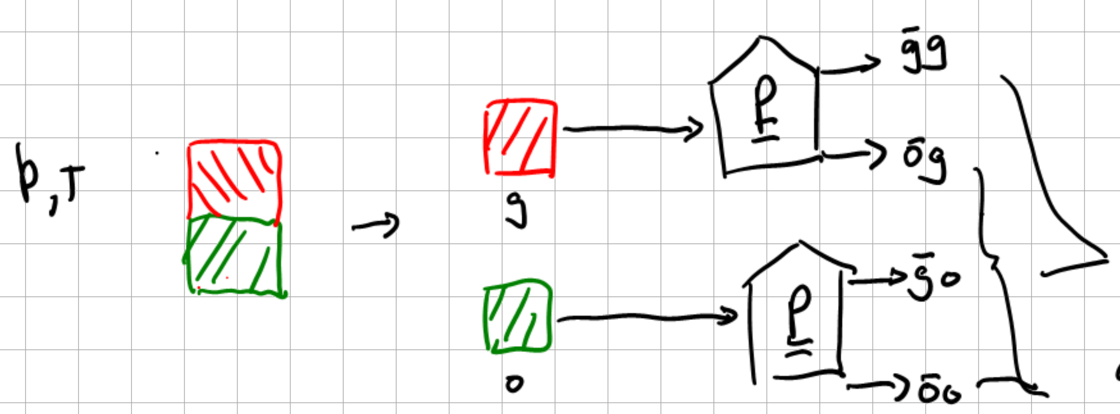
  → mass fraction o, g, w: $x_o = \frac{\dot{m}_o}{\dot{m}_o + \dot{m}_g}$, $x_g = (1 - x_o)$

  $\dot{m}_o = \dot{m}\, x_o$

  $\dot{m}_g = \dot{m}\, x_g$

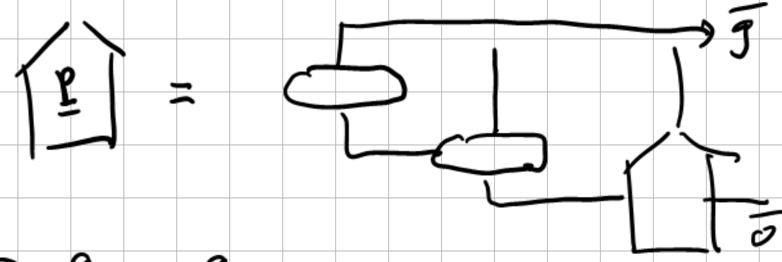
  run in two ways
  - live: flash calculation (equilibrium calculation) → property estimation
  - pre compute tables and interpolate on table

- Black oil tables generated by:
  - EOS
  - correlations
  - lab experiments



$q_{\bar{g}} = q_{\bar{g}g} + q_{\bar{g}o}$

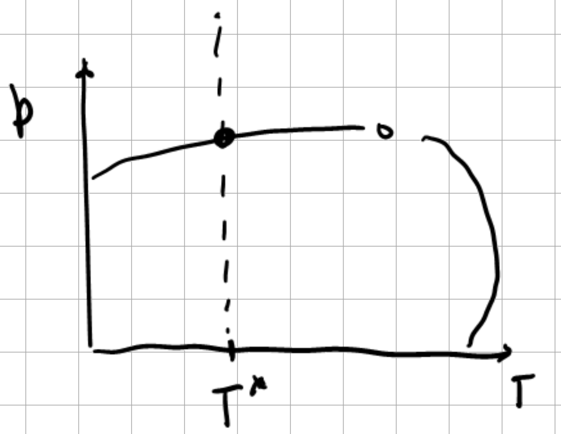
$q_{\bar{o}} = q_{\bar{o}o} + q_{\bar{o}g}$

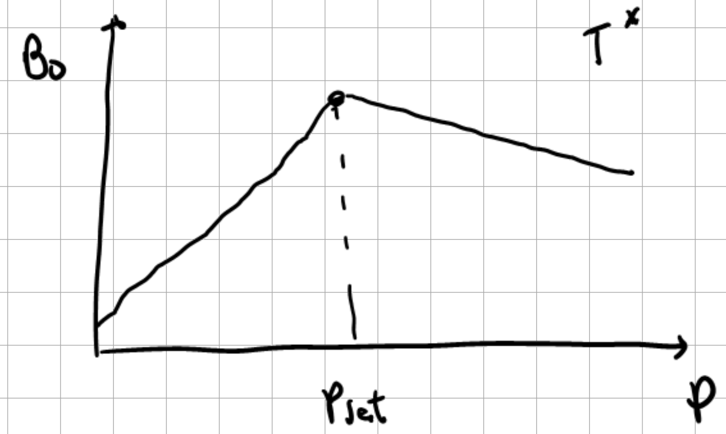


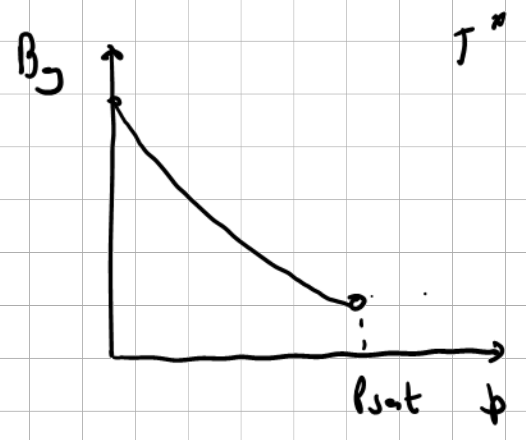
$B_o = \frac{V_o}{V_{\bar{o}o}}$ $\qquad B_g = \frac{V_g}{V_{\bar{g}g}}$ $\qquad R_s = \frac{V_{\bar{g}o}}{V_{\bar{o}o}}$ $\qquad r_s (r_v) = \frac{V_{\bar{o}g}}{V_{\bar{g}g}}$

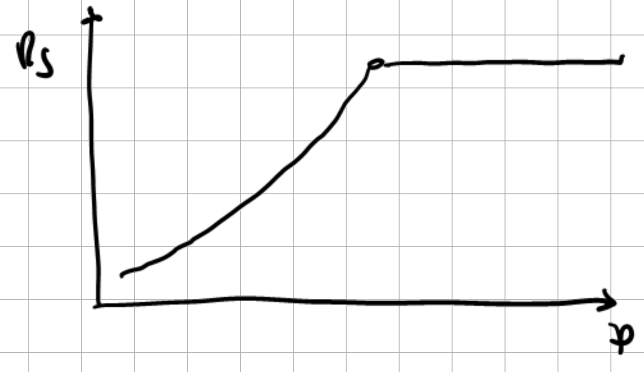
R_s: solution gas-oil ratio

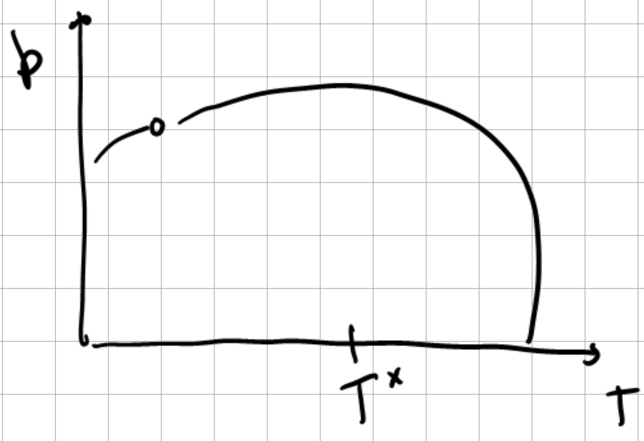
r_s: solution oil-gas ratio

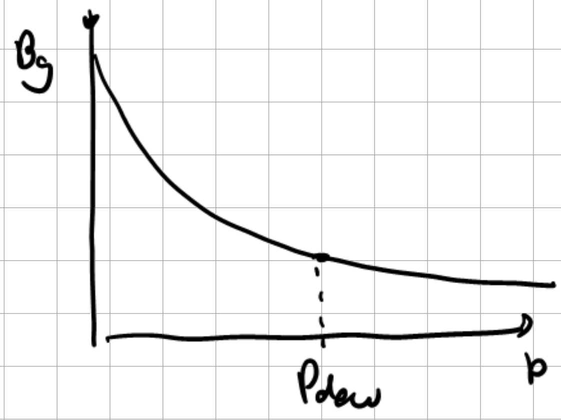


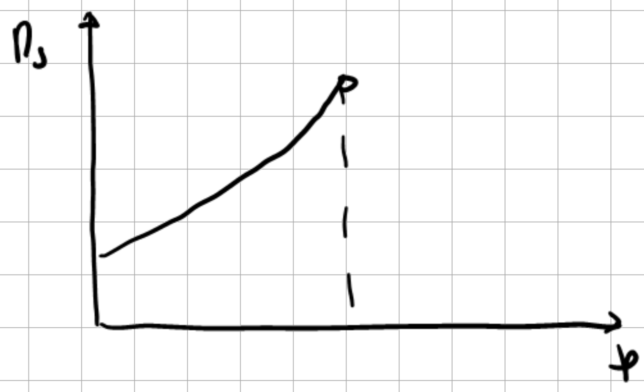












$\begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix} = \begin{bmatrix} \frac{1}{B_g} & \frac{R_s}{B_o} & 0 \\ \frac{r_s}{B_g} & \frac{1}{B_o} & 0 \\ 0 & 0 & \frac{1}{B_w} \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix}$	$\begin{bmatrix} q_g \\ q_o \\ q_w \end{bmatrix} = \begin{bmatrix} \frac{B_g}{1-R_s \cdot r_s} & \frac{-R_s \cdot B_g}{1-R_s \cdot r_s} & 0 \\ \frac{-B_o . r_s}{1-R_s \cdot r_s} & \frac{B_o}{1-R_s \cdot r_s} & 0 \\ 0 & 0 & B_w \end{bmatrix}_{(p,T)} \cdot \begin{bmatrix} q_{\bar{g}} \\ q_{\bar{o}} \\ q_{\bar{w}} \end{bmatrix}$
Standard conditions calculated from local conditions	Local conditions calculated from standard conditions

$q_{\bar{g}} = q_{\bar{g}g} + q_{\bar{g}o}$

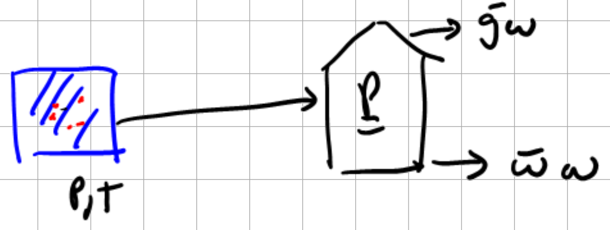
$q_{\bar{o}} = q_{\bar{o}o} + q_{\bar{o}g}$

for conventional / heavy oil $r_s = 0$

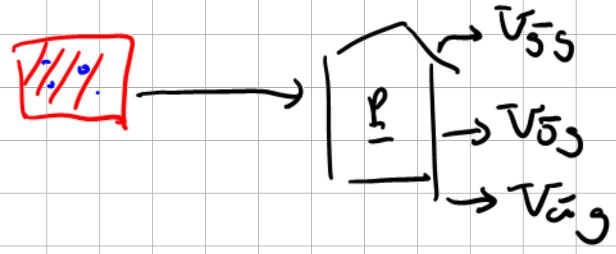
not adequate for volatile oils

gas condensate

water BO properties



$$B_w = \frac{V_w}{V_{\bar{w}w}}$$

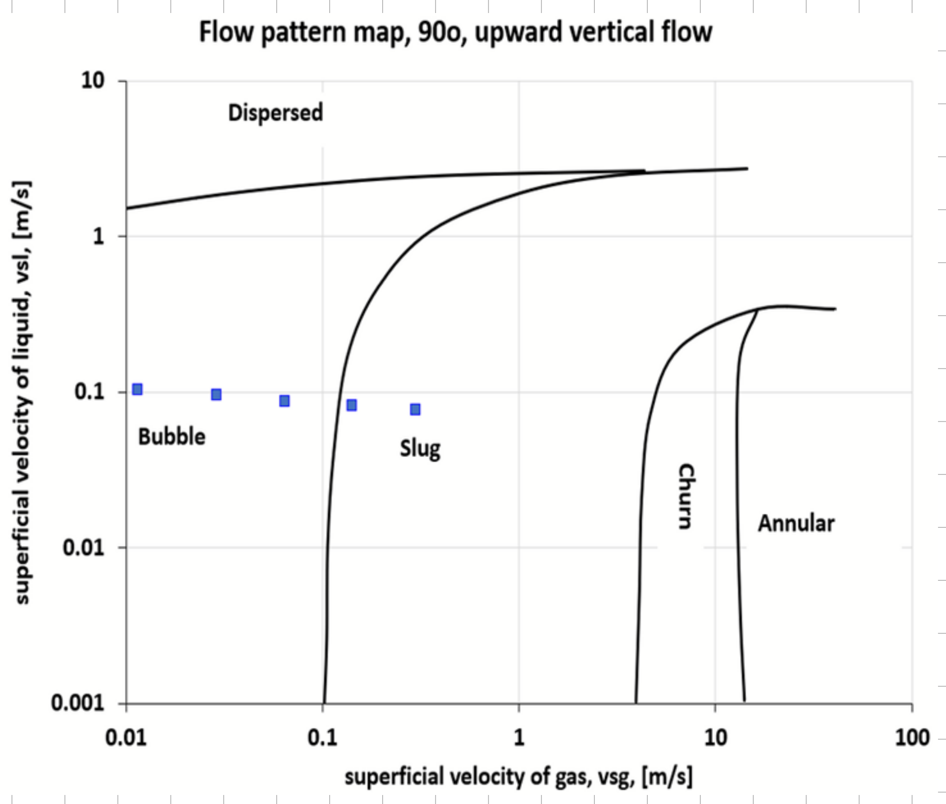


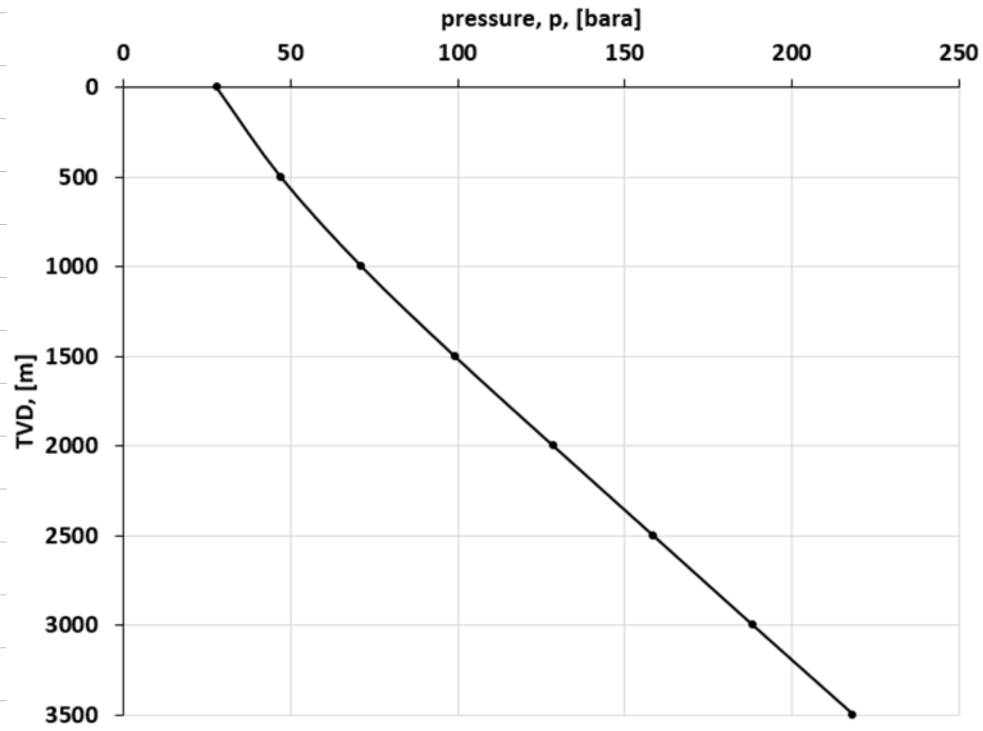
Some cases it could be important to include surface gas from local water and surface water from local gas.

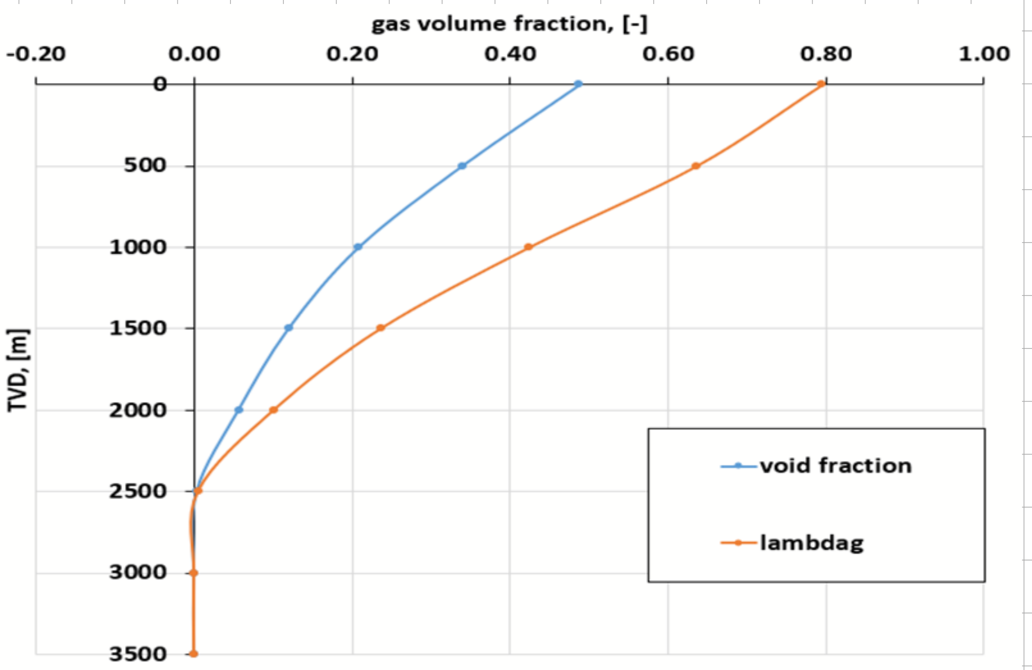
Exercise: Pressure drop calculation in saturated oil well, Prof. Milan Stanko (NTNU)

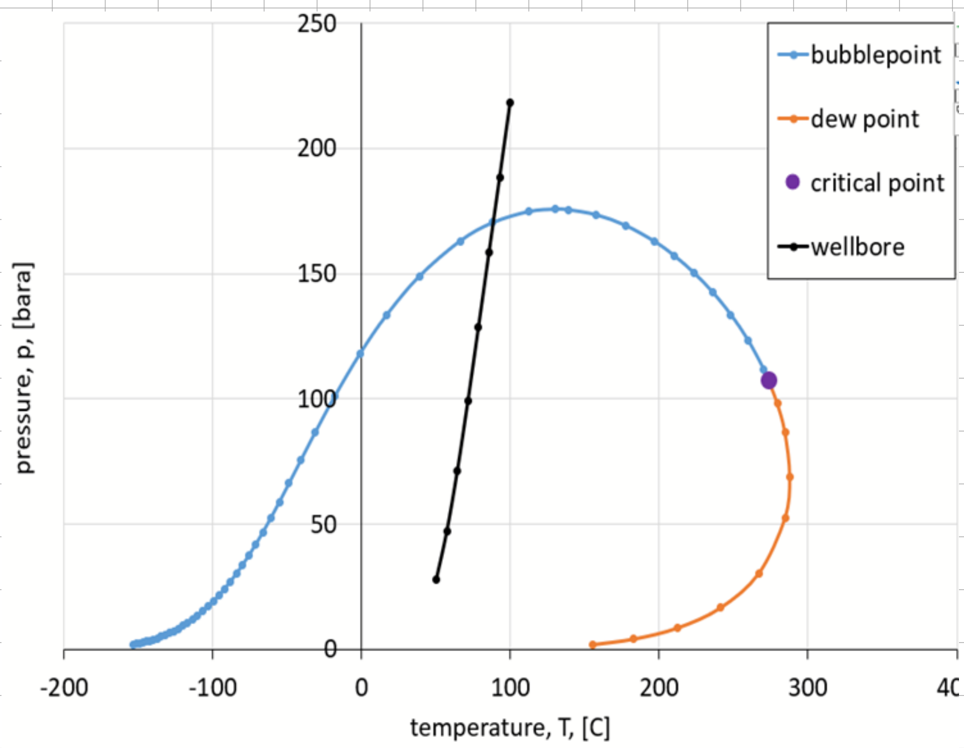
GOR	[Sm3/Sm3]	155.1
Pipe ID	[m]	0.15
Pipe cross section area	[m2]	0.0177
Pipe roughness	[m]	1.50E-05
Pipe inclination from hor	[deg]	90
qo	[Sm3/d]	100
qg	[Sm3/d]	1.55E+04

		BO table column	3	4	5	6	8	10	7	9	11						Woldesemayat and Ghajar	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	117.4	4.566E+02	0.077	0.299	0.80	0.49	0.0384
500	57.1	47.2	41.1	1.31E-05	1.2	1.90E-02	70.8	1.25E-02	708.8	1.2	8.37E-03	124.2	2.173E+02	0.081	0.142	0.64	0.34	0.0483
1000	64.3	71.4	65.3	1.43E-05	1.3	1.09E-02	119.4	1.49E-02	684.3	0.8	5.12E-03	133.2	9.832E+01	0.087	0.064	0.42	0.21	0.0556
1500	71.4	99.2	93.9	1.69E-05	1.4	7.29E-03	178.7	1.91E-02	657.3	0.6	2.64E-03	144.4	4.468E+01	0.095	0.029	0.24	0.12	0.0589
2000	78.6	128.6	124.4	2.13E-05	1.6	5.71E-03	228.2	2.38E-02	630.8	0.5	1.33E-03	156.8	1.761E+01	0.103	0.012	0.10	0.06	0.0597
2500	85.7	158.4	153.2	2.41E-05	1.7	4.49E-03	229.5	2.44E-02	607.9	0.4	6.71E-04	169.0	8.799E-01	0.111	0.001	0.01	0.00	0.0595
3000	92.9	188.2	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.3	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602
3500	100.0	218.3	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.4	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602



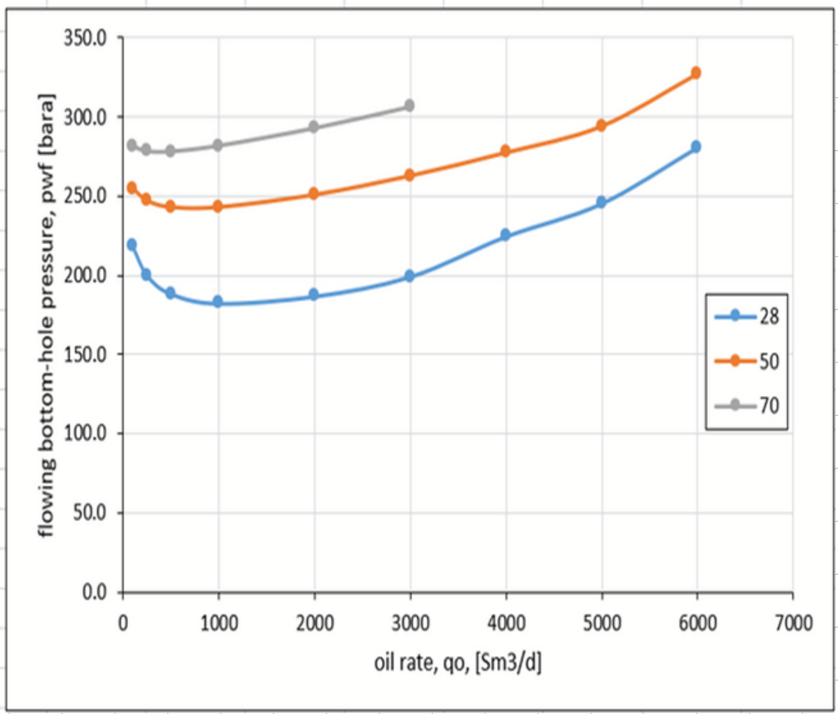






With Wolgha model

pwh[bara]	28	50	70
qo	pwf	pwf	pwf
[Sm3/d]	[bara]	[bara]	[bara]
100	218.3	254.9	281.4
250	200.0	247.1	278.3
500	188.5	243.0	278.2
1000	182.4	243.0	281.8
2000	186.9	250.8	292.9
3000	199.2	262.8	306.3
4000	224.5	277.3	
5000	245.2	294.0	
6000	280.5	327.0	



lculation in saturated oil well, Prof. Milan Stanko (NTNU)

[Sm3/Sm3]	155.1
[m]	0.15
[m2]	0.0177
[m]	1.50E-05
[deg]	90
[Sm3/d]	100
[Sm3/d]	1.55E+04

		BO table column	3	4	5	6	8	10	7	9	11						Woldesemayat and Ghajar	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	117.4	4.566E+02	0.077	0.299	0.80	0.49	0.0384
500	57.1	47.2	41.1	1.31E-05	1.2	1.90E-02	70.8	1.25E-02	708.8	1.2	8.37E-03	124.2	2.173E+02	0.081	0.142	0.64	0.34	0.0483
1000	64.3	71.4	65.3	1.43E-05	1.3	1.09E-02	119.4	1.49E-02	684.3	0.8	5.12E-03	133.2	9.832E+01	0.087	0.064	0.42	0.21	0.0556
1500	71.4	99.2	93.9	1.69E-05	1.4	7.29E-03	178.7	1.91E-02	657.3	0.6	2.64E-03	144.4	4.468E+01	0.095	0.029	0.24	0.12	0.0589
2000	78.6	128.6	124.4	2.13E-05	1.6	5.71E-03	228.2	2.38E-02	630.8	0.5	1.33E-03	156.8	1.761E+01	0.103	0.012	0.10	0.06	0.0597
2500	85.7	158.4	153.2	2.41E-05	1.7	4.49E-03	229.5	2.44E-02	607.9	0.4	6.71E-04	169.0	8.799E-01	0.111	0.001	0.01	0.00	0.0595
3000	92.9	188.2	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.3	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602
3500	100.0	218.3	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	607.4	0.4	0.00E+00	169.5	0.000E+00	0.111	0.000	0.00	0.00	0.0602

```
Function e_wolgha(usl, usg, denl, deng, sigma_lg, teta_deg, p, D)
    'p in bar
    'D in m
    'usl in m/s
    'usg in m/s
    'denl kg/m^3
    'deng kg/m^3
    'teta deg in deg
    'sigma_lg in N/m
    If usg = 0 Then
        e_wolgha = 0
    Else
        Pi = Atn(1) * 4
        teta = teta_deg * Pi / 180
        'void fraction correlation by Woldesemayat and Ghajar (2006)
        a = usg * (1 + ((usl / usg) ^ ((deng / denl) ^ 0.1)))
        B = 2.9 * ((9.81 * sigma_lg * D * (1 + Cos(teta)) * (denl - deng) / (denl ^ 2)) ^ 0.25)
        C = (1.22 + 1.22 * Sin(teta)) ^ (1.01325 / p)
        e_wolgha = usg / (a + (B * C))
    End If
End Function
```

```
Function dpdx_mpf(roughness, viscl, viscg, denl, deng, usl, usg, D, angle, voidfraction)
    'dpdx_mpf pressure gradient, in bar/m, for multiphase flow
    'denl, liquid density, [kg/m3]
    'deng, gas density, [kg/m3]
    'usl superficial liquid velocity, [m/s]
    'usg superficial gas velocity, [m/s]
    'angle, inclination angle of pipe with respect to horizontal [deg]
    'D hydraulic diameter of pipe [m]
    'roughness pipe roughness, [m]
    'viscl, liquid viscosity [cP]
    'viscg, gas viscosity, [cP]
    'voidfraction [-]
    Pi = Atn(1) * 4
    denm = voidfraction * deng + (1 - voidfraction) * denl
    If voidfraction = 0 Or usg = 0 Then
        ug = 0
        ul = usl
        fg = 0
        fl = ffactor(denl, viscl, D, roughness, ul)
    ElseIf voidfraction = 1 Or usl = 0 Then
        ug = usg
        ul = 0
        fl = 0
        fg = ffactor(deng, viscg, D, roughness, ug)
    Else
        ug = usg / voidfraction
        ul = usl / (1 - voidfraction)
        fg = ffactor(deng, viscg / 1000#, D, roughness, ug)
        fl = ffactor(denl, viscl / 1000, D, roughness, ul)
    End If
    dpdx_f = (fg * deng * (ug * Abs(usg)) * 0.5 / D) + (fl * denl * (ul * Abs(usl)) * 0.5 / D)
    dpdx_h = denm * 9.81 * Sin(angle * Pi / 180)
    dpdx_mpf = dpdx_f + dpdx_h
    dpdx_mpf = dpdx_mpf / 100000#
End Function
```

lculation in saturated oil well, Prof. Milan Stanko (NTNU)

[Sm3/Sm3]	155.1
[m]	0.15
[m2]	0.0177
[m]	1.50E-05
[deg]	90
[Sm3/d]	1000
[Sm3/d]	1.55E+05

		BO table column	3	4	5	6	8	10	7	9	11							Nagoo
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	e[-]	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	1174.3	4.566E+03	0.769	2.991	0.80	0.61	0.0313
500	57.1	43.7	37.3	1.29E-05	1.2	2.08E-02	63.4	1.22E-02	711.9	1.2	8.90E-03	1229.4	2.451E+03	0.805	1.605	0.67	0.50	0.0387
1000	64.3	63.0	56.3	1.36E-05	1.3	1.34E-02	100.1	1.40E-02	691.6	0.9	6.20E-03	1301.4	1.320E+03	0.852	0.865	0.50	0.38	0.0461
1500	71.4	86.1	79.2	1.53E-05	1.4	8.91E-03	146.3	1.68E-02	668.7	0.6	3.75E-03	1390.6	6.776E+02	0.911	0.444	0.33	0.26	0.0525
2000	78.6	112.3	105.7	1.84E-05	1.5	6.67E-03	195.8	2.08E-02	644.3	0.5	2.05E-03	1497.1	3.306E+02	0.981	0.217	0.18	0.16	0.0567
2500	85.7	140.7	134.2	2.32E-05	1.6	5.56E-03	234.6	2.48E-02	620.2	0.4	1.12E-03	1615.5	1.167E+02	1.058	0.076	0.07	0.06	0.0588
3000	92.9	170.1	154.6	1.91E-06	1.7	3.87E-04	17.9	1.91E-03	604.6	0.4	6.30E-05	1701.8	2.006E-01	1.115	0.000	0.00	0.00	0.0597
3500	100.0	200.0	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	605.0	0.4	0.00E+00	1702.0	0.000E+00	1.115	0.000	0.00	0.00	0.0649

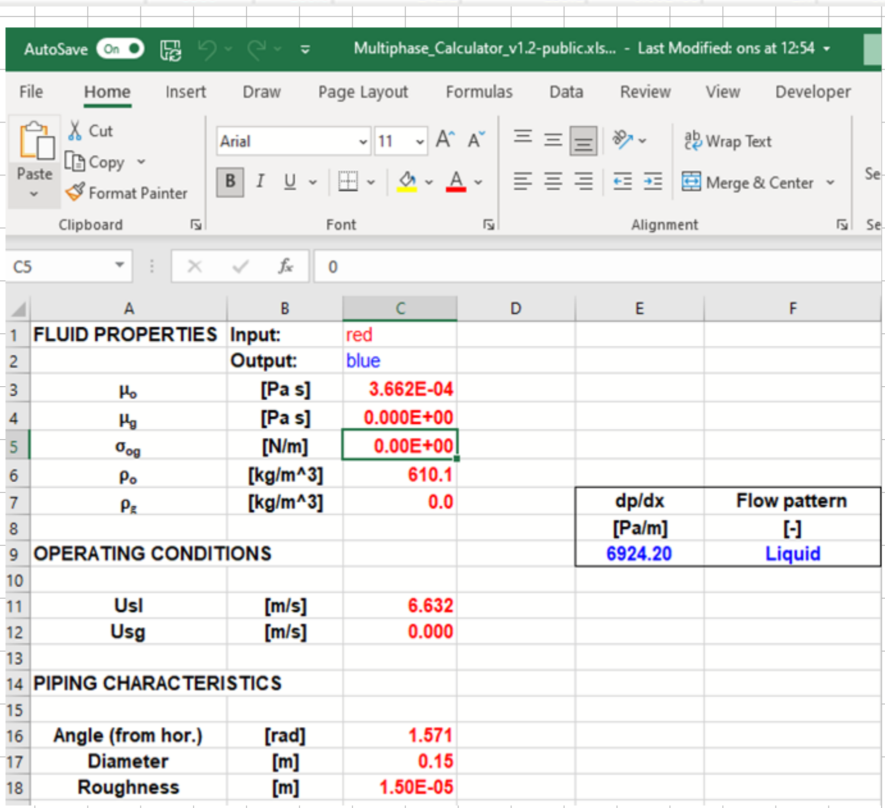
```
Function e_Nagoo(lambdag)
    ' e_Nagoo, the void fraction of gas, in fraction, using the ANSLIP equation by Nagoo, 2013
    'lambdag is non slip volume fraction of gas, in fraction
    If lambdag = 0 Then
        e_Nagoo = 0
    Else
        e_Nagoo = (lambdag + 1 - ((lambdag + 1) ^ 2 - 4 * (lambdag ^ 2)) ^ 0.5) / (2 * lambdag)
    End If
End Function
```


```
Function dpdx_mpf(roughness, viscl, viscg, denl, deng, usl, usg, D, angle, voidfraction)
    'dpdx_mpf pressure gradient, in bar/m, for multiphase flow
    'denl, liquid density, [kg/m3]
    'deng, gas density, [kg/m3]
    'usl superficial liquid velocity, [m/s]
    'usg superficial gas velocity, [m/s]
    'angle, inclination angle of pipe with respect to horizontal [deg]
    'D hydraulic diameter of pipe [m]
    'roughness pipe roughness, [m]
    'viscl, liquid viscosity [cP]
    'viscg, gas viscosity, [cP]
    'voidfraction [-]
    Pi = Atn(1) * 4
    denm = voidfraction * deng + (1 - voidfraction) * denl
    If voidfraction = 0 Or usg = 0 Then
        ug = 0
        ul = usl
        fg = 0
        fl = ffactor(denl, viscl, D, roughness, ul)
    ElseIf voidfraction = 1 Or usl = 0 Then
        ug = usg
        ul = 0
        fl = 0
        fg = ffactor(deng, viscg, D, roughness, ug)
    Else
        ug = usg / voidfraction
        ul = usl / (1 - voidfraction)
        fg = ffactor(deng, viscg / 1000#, D, roughness, ug)
        fl = ffactor(denl, viscl / 1000, D, roughness, ul)
    End If
    dpdx_f = (fg * deng * (ug * Abs(usg)) * 0.5 / D) + (fl * denl * (ul * Abs(usl)) * 0.5 / D)
    dpdx_h = denm * 9.81 * Sin(angle * Pi / 180)
    dpdx_mpf = dpdx_f + dpdx_h
    dpdx_mpf = dpdx_mpf / 100000#
End Function
```

Exercise: Pressure drop calculation in saturated oil well, Prof. Milan Stanko (NTNU)

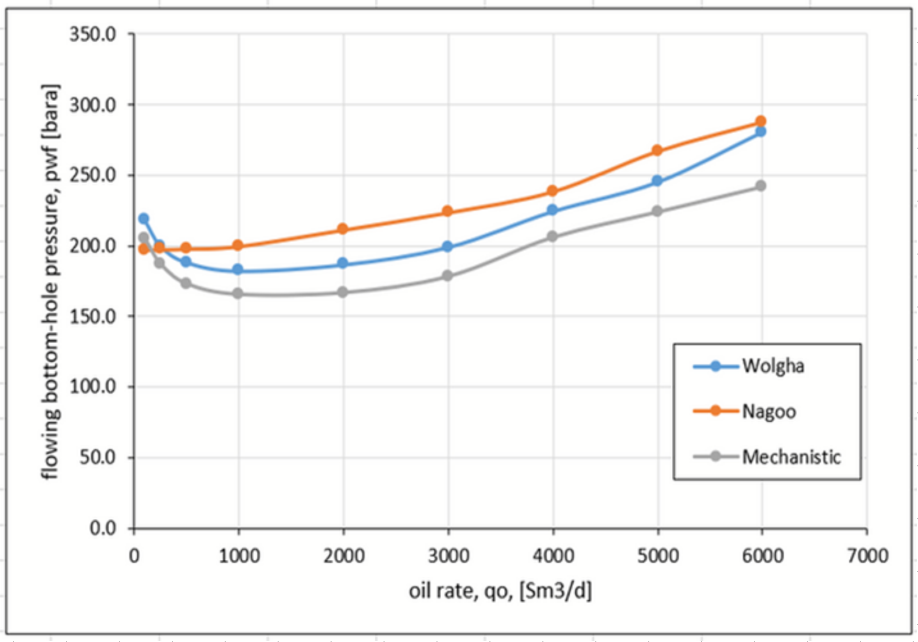
GOR	[Sm3/Sm3]	155.1
Pipe ID	[m]	0.15
Pipe cross section area	[m2]	0.0177
Pipe roughness	[m]	1.50E-05
Pipe inclination from hor	[deg]	90
qo	[Sm3/d]	6000
qg	[Sm3/d]	9.31E+05

		BO table column	3	4	5	6	8	10	7	9	11						Mechanistic model	
TVD [m]	T [C]	p[bara]	Rs [Sm3/Sm3]	rs [Sm3/Sm3]	Bo [m3/Sm3]	Bg [m3/Sm3]	deng [kg/m3]	viscg [cp]	deno [kg/m3]	viso [cp]	sigma_o_g [N/m]	qo [m3/d]	qg[m3/d]	uso [m/s]	usg [m/s]	lambdag[-]	flowpattern	dp/dx [bara/m]
0	50.0	28	22.6	1.28E-05	1.2	3.44E-02	37.8	1.10E-02	728.8	1.8	1.15E-02	7046.0	2.740E+04	4.615	17.944	0.80	Slug	0.0492
500	57.1	52.6	46.8	1.34E-05	1.3	1.64E-02	82.1	1.29E-02	704.1	1.1	7.56E-03	7564.5	1.065E+04	4.954	6.976	0.58	Slug	0.0417
1000	64.3	73.5	67.6	1.45E-05	1.3	1.05E-02	125.0	1.53E-02	682.4	0.8	4.88E-03	8044.2	5.540E+03	5.269	3.628	0.41	Bubble	0.0605
1500	71.4	103.7	99.2	1.76E-05	1.5	6.90E-03	189.5	2.00E-02	653.3	0.6	2.35E-03	8781.2	2.322E+03	5.751	1.521	0.21	Bubble	0.0675
2000	78.6	137.5	134.8	2.33E-05	1.6	5.37E-03	243.1	2.55E-02	623.6	0.4	1.07E-03	9650.4	6.558E+02	6.321	0.430	0.06	Bubble	0.0708
2500	85.7	172.9	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	609.0	0.4	0.00E+00	10144.6	0.000E+00	6.644	0.000	0.00	Liquid	0.0693
3000	92.9	207.5	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	610.1	0.4	0.00E+00	10127.0	0.000E+00	6.633	0.000	0.00	Liquid	0.0693
3500	100.0	242.1	155.1	0.00E+00	1.7	0.00E+00	0.0	0.00E+00	610.1	0.4	0.00E+00	10125.7	0.000E+00	6.632	0.000	0.00	Liquid	0.0692



pwh=28 bara

Models	Wolgha	Nagoo	Mechanistic
qo [Sm3/d]	pwf [bara]	pwf [bara]	pwf [bara]
100	218.3	197.5	205.5
250	200.0	197.6	187.5
500	188.5	198.2	173.2
1000	182.4	200.0	165.6
2000	186.9	211.5	166.8
3000	199.2	223.7	178.4
4000	224.5	238.5	206.2
5000	245.2	266.9	224.2
6000	280.5	287.5	242.1



- g
- l $\begin{cases} \text{oil} \\ \text{water} \end{cases}$ →

$$\frac{q_w}{q_o+q_w}$$

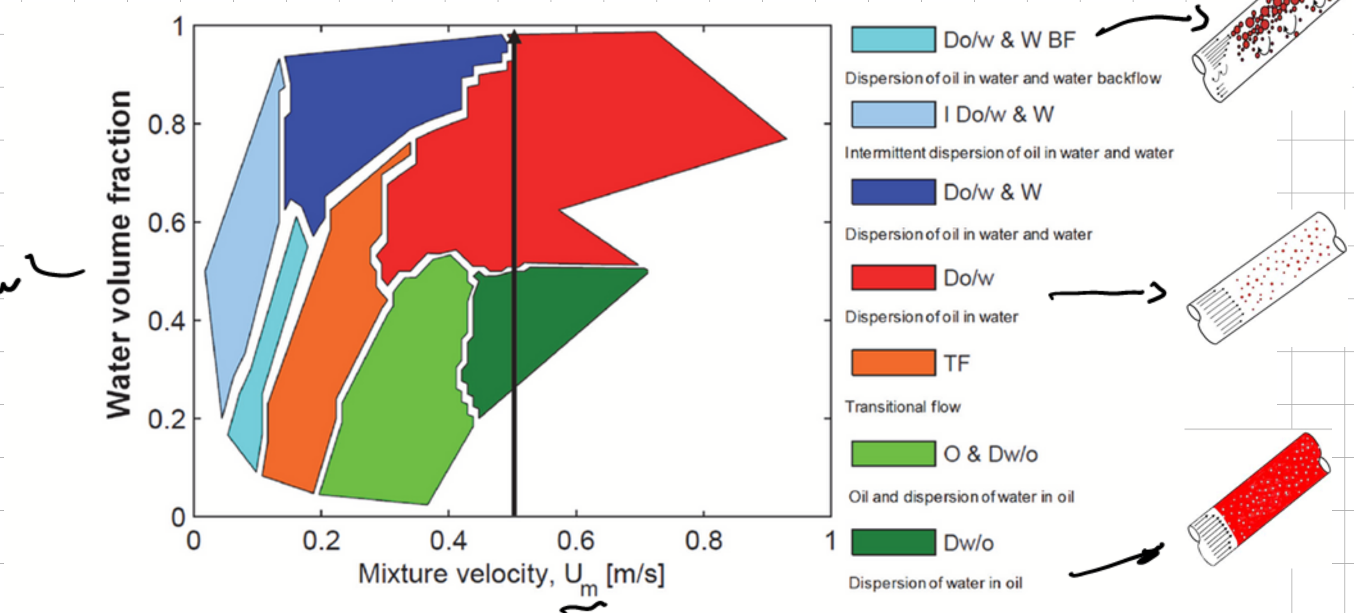


FIGURE 7-24. OIL-WATER FLOW PATTERN MAP OF WATER VOLUME FRACTION VERSUS MIXTURE VELOCITY FOR AN UPWARD PIPE INCLINATION OF 45°. FIGURE ADAPTED FROM RIVERA[7-5] [7-1].

U_m: $u_{so} + u_{sw}$

g	u_{sg}	λ_g	H_g	v_g
o	u_{so}	λ_o	H_o	v_o
w	u_{sw}	λ_w	H_w	v_w

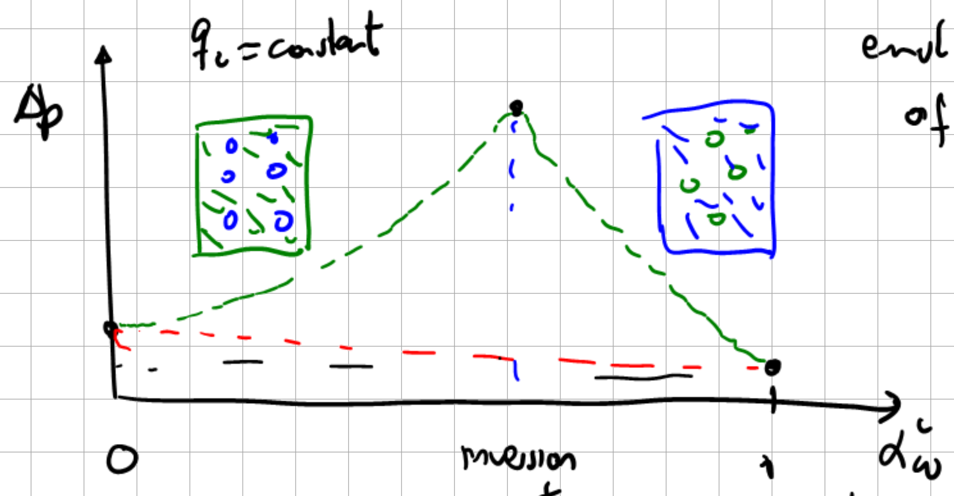
$v_o \approx v_w$ — little slip

assume both travel at the same speed v_L

$$\rho_L = \rho_o \alpha_o^L + \rho_w \alpha_w^L \qquad \mu_L = \mu_o \alpha_o^L + \mu_w \cdot \alpha_w^L$$

$$\alpha_o^L = \frac{q_o}{q_o+q_w} \qquad \alpha_w^L = \frac{q_w}{q_o+q_w} = 1-\alpha_o^L$$

q_L (q_o+q_w) →



emulsion behavior of oil + water mixture

$$\mu_L = \mu_o \cdot e^{c_1(\alpha_w^L)} \qquad \mu_L = \mu_w \cdot e^{c_2(1-\alpha_w^L)}$$

$$wc = \frac{\bar{q}_w}{\bar{q}_o+\bar{q}_w} \qquad \alpha_w^L = \frac{q_w}{q_o+q_w}$$

sometimes in the literature, if $\bar{q}_o = q_o$

$wc = \alpha_w^L$

Pressure integration method

p_3 o
⋮
p_2 o
} ΔL
p_1 o

$$p_2 = p_1 + \left.\frac{dp}{dx}\right|_{@ p_1} \cdot \Delta L$$

} not so accurate for large ΔL

$$p_2 = p_1 + \left.\frac{dp}{dx}\right|_{@ p_{av}} \Delta L$$

$$p_{av} = \frac{p_1 + p_2}{2}$$

implicit calculation

- assume a value of p_2^*
- compute $\left.\frac{dp}{dx}\right|_{p_{av}}$
- compute $p_2^{calc} = p_1 + \left.\frac{dp}{dx}\right|_{@ p_{av}} \Delta L$
- check $p_2^{calc} = p_2^*$

not → (back to assume)

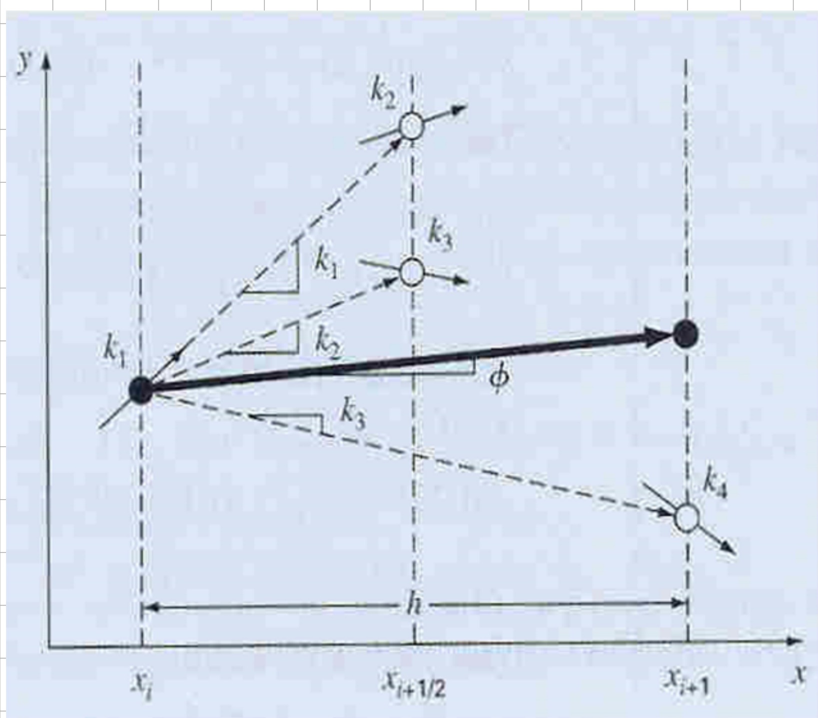
↓ yes

proceed to next step

explicit approach (higher order)

Runge-Kutta 4th

$$y_{n+1} = y_n + \frac{1}{6} \cdot h \cdot (k_1 + 2 \cdot k_2 + 2 \cdot k_3 + k_4)$$



p_1 p^* p_2

$$p_2 = p_1 + \frac{1}{6} \Delta L (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = \left.\frac{dp}{dx}\right|_{@ p_1}$$

$$p^* = p_1 + \left.\frac{dp}{dx}\right|_{@ p_1} \cdot \frac{\Delta L}{2}$$

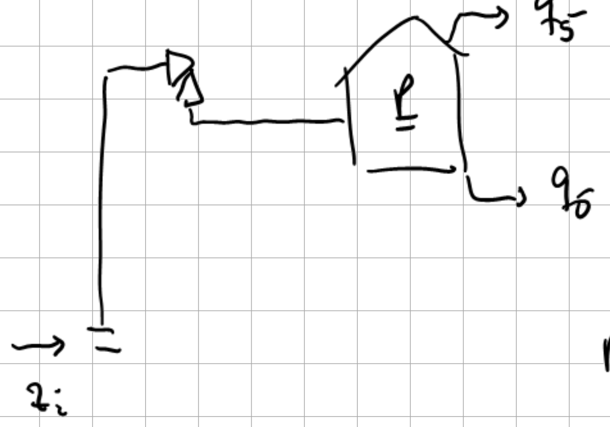
$$k_2 = \left.\frac{dp}{dx}\right|_{@ p^*}$$

$$p^{**} = p_1 + k_2 \frac{\Delta L}{2}$$

$$k_3 = \left.\frac{dp}{dx}\right|_{@ p^{**}}$$

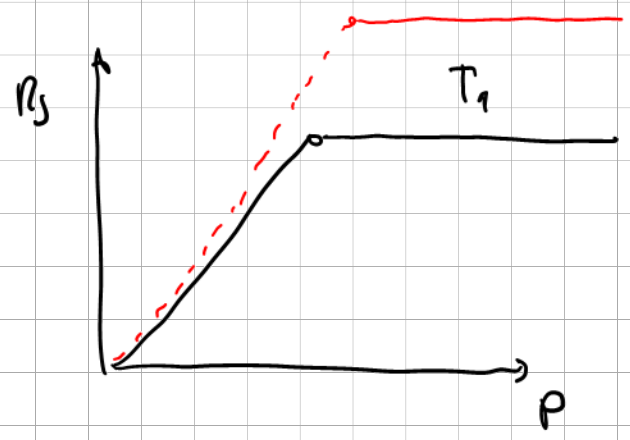
$$p^{***} = p_1 + k_3 \Delta L$$

$$k_4 = \left.\frac{dp}{dx}\right|_{@ p^{***}}$$



$$R_p = \frac{q_{\bar{g}}}{q_{\bar{o}}}$$

If z_i changes then R_p should also change



If z_i changes

z_i^*

If z_i changes (or R_p) it is usually necessary to generate a new black oil table