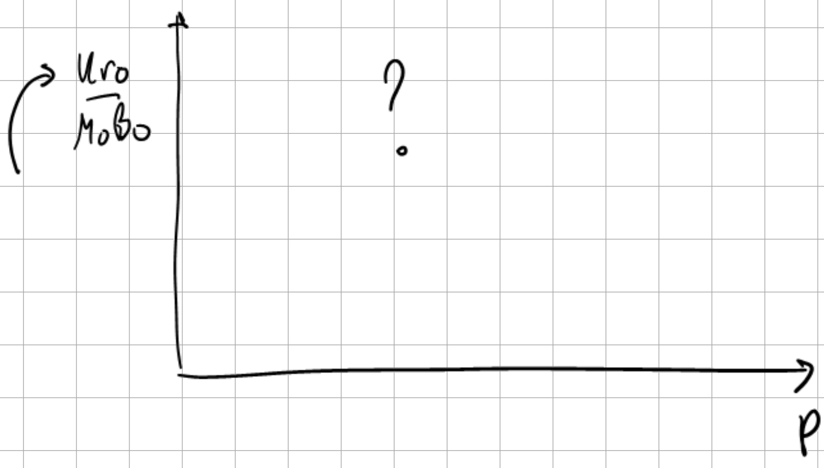


(1941) Evinger and Muskat

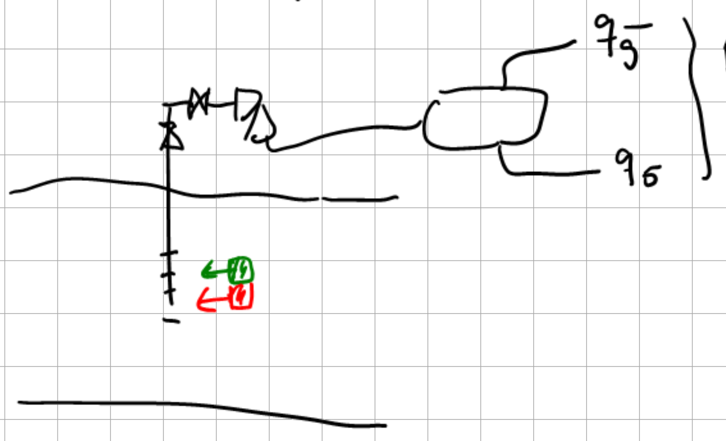
Calculation of Theoretical Productivity Factor

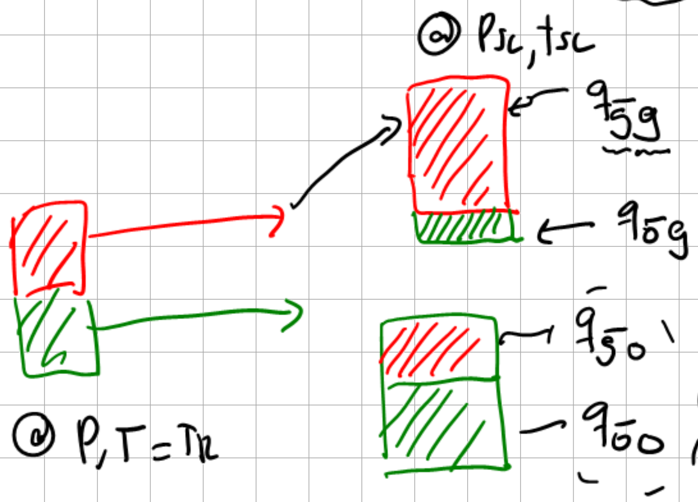
By H. H. Evinger* and M. Muskat*

(New York Meeting, February 1941)

$k_{ro} = f(p)$

$\frac{q_{\bar{g}}}{q_{\bar{o}}}$ $GOR = R_p = \frac{q_{\bar{g}}}{q_{\bar{o}}} = \frac{q_{\bar{g}g} + q_{\bar{g}o}}{q_{\bar{o}o}} = \frac{q_{\bar{g}g}}{q_{\bar{o}o}} + \frac{q_{\bar{g}o}}{q_{\bar{o}o}} = \frac{q_{\bar{g}g}}{q_{\bar{o}o}} + R_s(p)$





$q_{\bar{g}} = q_{\bar{g}g} + q_{\bar{g}o}$

$q_{\bar{o}} = q_{\bar{o}g} + q_{\bar{o}o}$

very small

$q_{\bar{o}g} << q_{\bar{o}o}$

$q_{\bar{o}} = q_{\bar{o}o}$

$$\frac{q_{\bar{g}g}}{q_{\bar{o}o}} = \frac{q_g}{B_g}\frac{B_o}{q_o} = \frac{k_{rg}}{k_{ro}}\frac{\mu_o B_o}{\mu_g B_g}$$

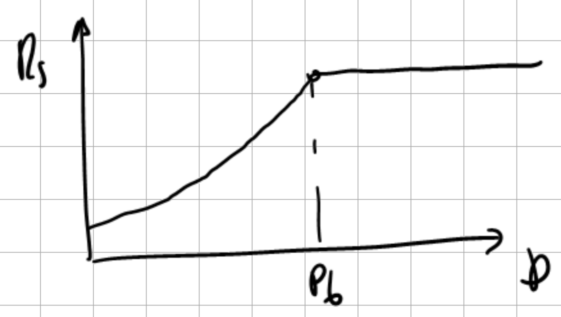
$v_o = \frac{q_o}{A}$ $v_g = \frac{q_g}{A}$

$R_s = \frac{q_{\bar{g}o}}{q_{\bar{o}o}}$ $R_s(p)$ solution gas oil ratio

$$\frac{q_g}{q_o} = \frac{k_{rg}}{\mu_g}\frac{\mu_o}{k_{ro}}$$

$v_g = \frac{k_{rg} k}{\mu_g}\frac{dP}{dx}$

$v_o = \frac{k_{ro} k}{\mu_o}\frac{dP}{dx}$



$\frac{V_o}{V_{\bar{o}}} = B_o = \frac{q_o}{q_{\bar{o}o}}$

$\frac{V_g}{V_{\bar{g}}} = B_g = \frac{q_g}{q_{\bar{g}g}}$

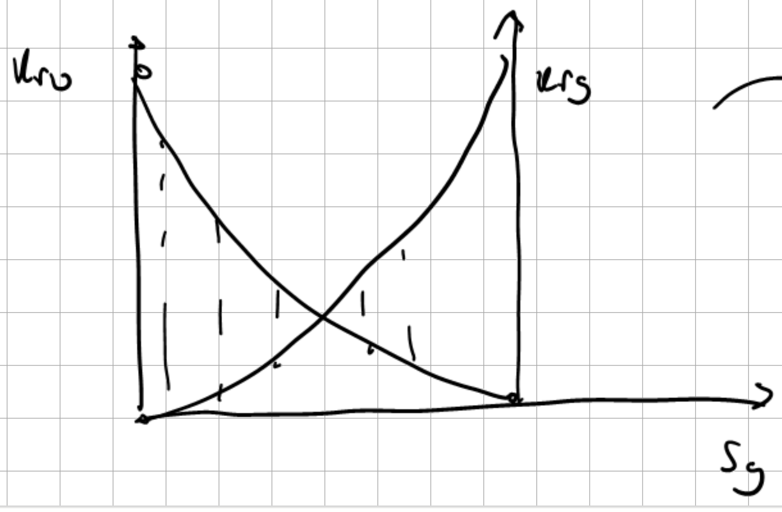
$$\underset{\text{measured}}{R_p} = \frac{k_{rg}}{k_{ro}}\frac{\mu_o B_o}{\mu_g B_g} + R_s(p)$$

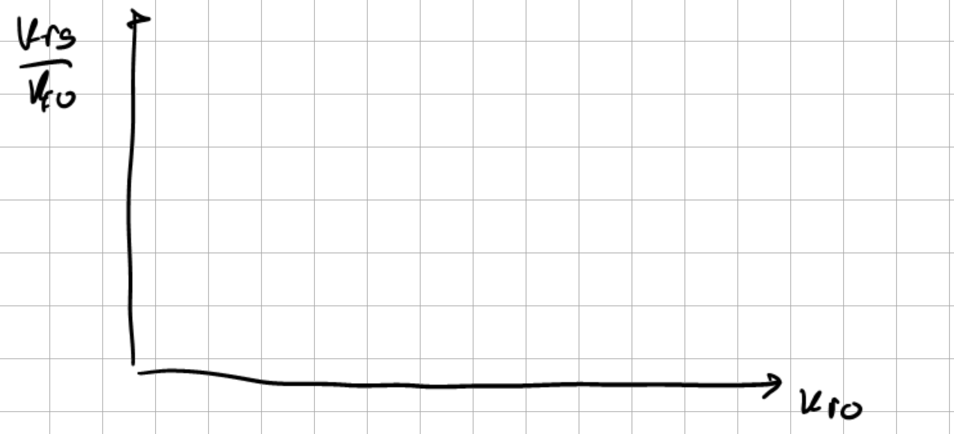
f(p)

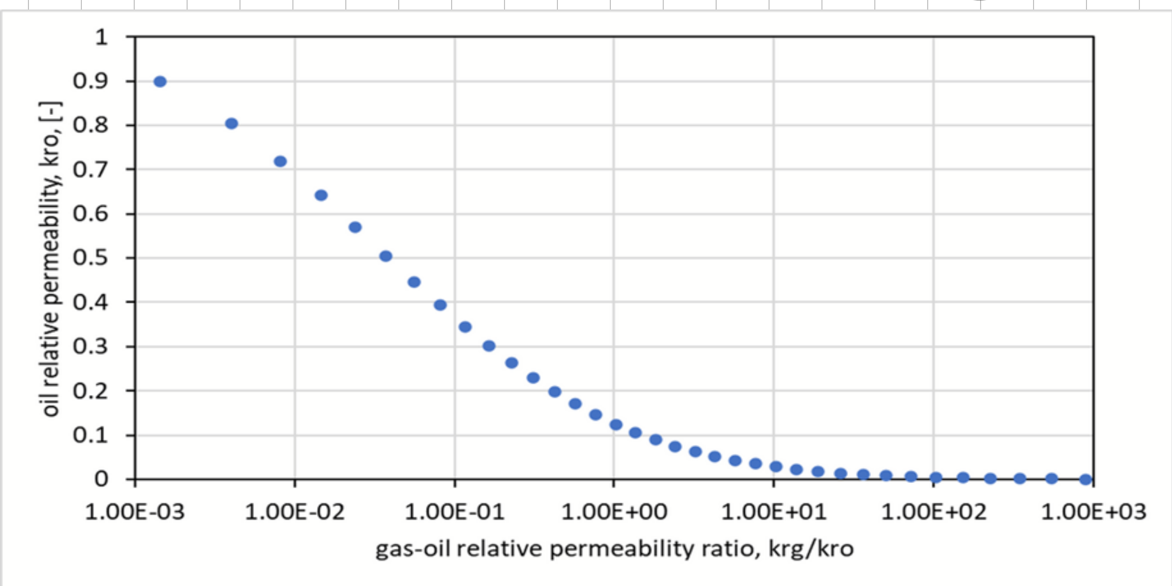
f(p) ?

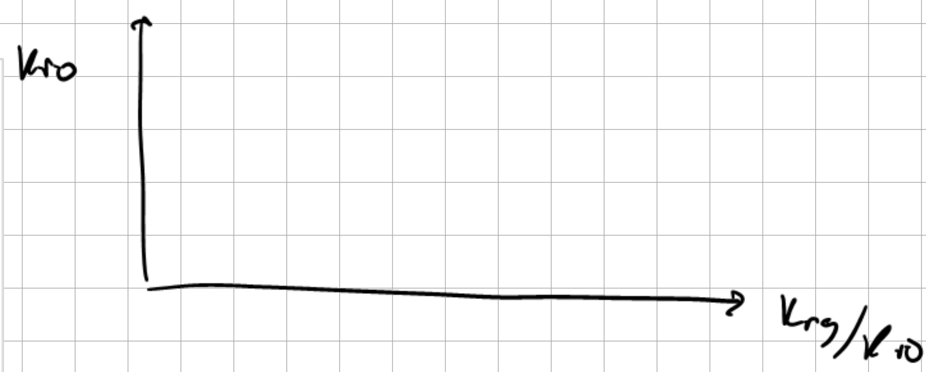
$$\frac{k_{rg}}{k_{ro}} = \frac{\mu_g B_g}{\mu_o B_o}(R_p - R_s) \quad (1)$$

But we need k_{ro}, NOT $\frac{k_{rg}}{k_{ro}}$?







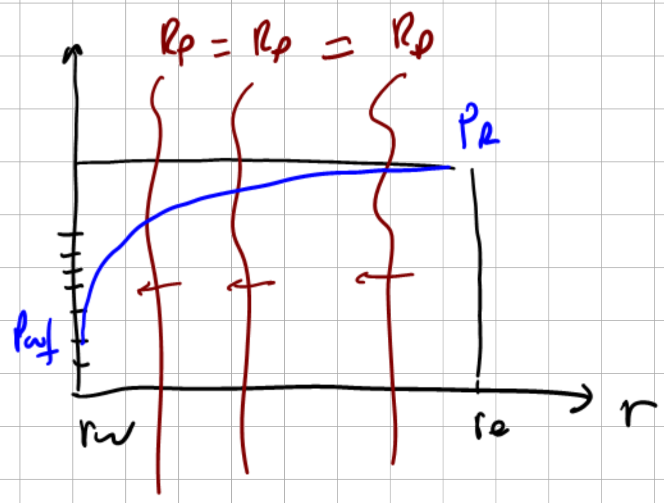


(2)

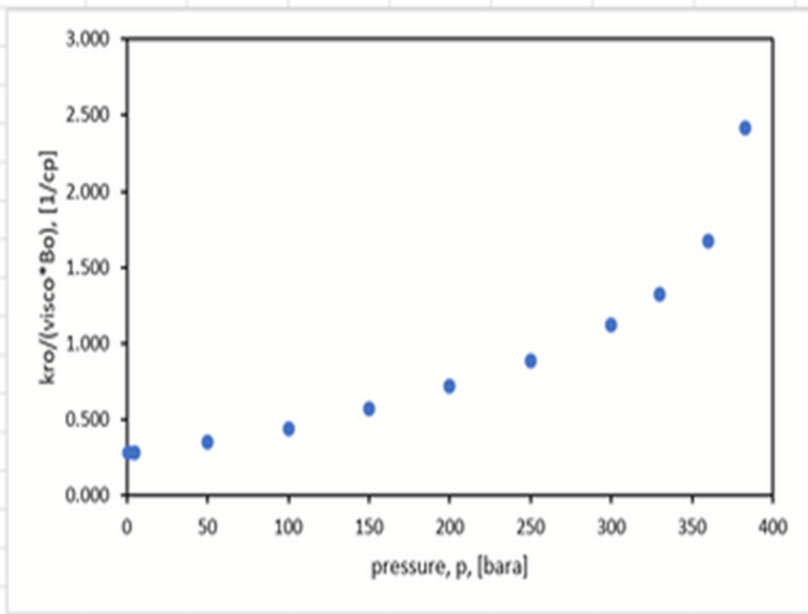
1) from (1) at p ⤳ $\frac{k_{rg}}{k_{ro}}$

2) from (2) with $\frac{k_{rg}}{k_{ro}}$ ⤳ k_{ro}

3) k_{ro} f(p)



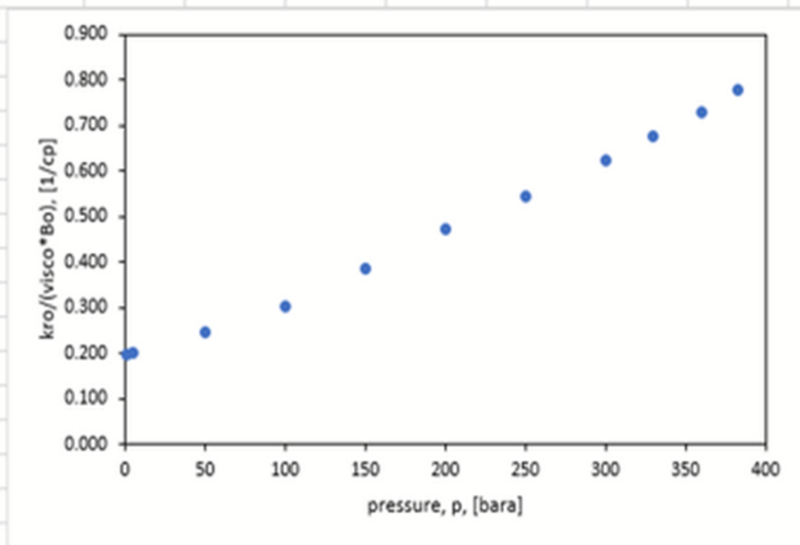
Rp	[Sm3/Sm3]	300								
COLUMN	4	6	3	5	2					
p	Rs	viscg	Bg	Visco	Bo	krg/kro	kro	kro/(visco*Bo)	So	
[bara]	[Sm3/Sm3]	[cp]	[m3/Sm3]	[cp]	[m3/Sm3]	[-]	[-]	[1/cp]	[-]	
1	0.5	1.12E-02	2.66E-02	1.4	1.2	5.54E-02	4.49E-01	0.277	0.60	
5	2.6	1.14E-02	2.59E-02	1.4	1.2	5.50E-02	4.50E-01	0.282	0.60	
50	25.5	1.32E-02	1.81E-02	1.1	1.2	4.96E-02	4.67E-01	0.352	0.60	
100	48.8	1.58E-02	1.22E-02	0.9	1.2	4.46E-02	4.82E-01	0.443	0.60	
150	76.4	1.82E-02	8.20E-03	0.7	1.3	3.81E-02	5.03E-01	0.573	0.61	
200	107.2	2.14E-02	6.25E-03	0.5	1.4	3.65E-02	5.09E-01	0.722	0.61	
250	143.9	2.51E-02	5.29E-03	0.4	1.5	3.59E-02	5.12E-01	0.888	0.61	
300	187.9	2.90E-02	4.69E-03	0.3	1.6	3.23E-02	5.29E-01	1.121	0.62	
330	220.4	3.15E-02	4.45E-03	0.3	1.7	2.65E-02	5.57E-01	1.322	0.62	
360	258.3	3.41E-02	4.26E-03	0.2	1.8	1.61E-02	6.29E-01	1.672	0.64	
383	292.6	3.60E-02	4.14E-03	0.2	1.9	3.19E-03	8.34E-01	2.414	0.67	



NOT LINEAR!!!

For higher GOR

Rp	[Sm3/Sm3]	800							
COLUMN	4	6	3	5	2				
p	Rs	viscg	Bg	Visco	Bo	krg/kro	kro	kro/(visco*Bo)	So
[bara]	[Sm3/Sm3]	[cp]	[m3/Sm3]	[cp]	[m3/Sm3]	[-]	[-]	[1/cp]	[-]
1	0.5	1.12E-02	2.66E-02	1.4	1.2	1.48E-01	3.17E-01	0.196	0.56
5	2.6	1.14E-02	2.59E-02	1.4	1.2	1.47E-01	3.17E-01	0.199	0.56
50	25.5	1.32E-02	1.81E-02	1.1	1.2	1.40E-01	3.24E-01	0.244	0.56
100	48.8	1.58E-02	1.22E-02	0.9	1.2	1.33E-01	3.30E-01	0.304	0.56
150	76.4	1.82E-02	8.20E-03	0.7	1.3	1.23E-01	3.40E-01	0.387	0.56
200	107.2	2.14E-02	6.25E-03	0.5	1.4	1.31E-01	3.32E-01	0.471	0.56
250	143.9	2.51E-02	5.29E-03	0.4	1.5	1.51E-01	3.14E-01	0.545	0.56
300	187.9	2.90E-02	4.69E-03	0.3	1.6	1.77E-01	2.95E-01	0.624	0.55
330	220.4	3.15E-02	4.45E-03	0.3	1.7	1.93E-01	2.85E-01	0.675	0.54
360	258.3	3.41E-02	4.26E-03	0.2	1.8	2.09E-01	2.75E-01	0.730	0.54
383	292.6	3.60E-02	4.14E-03	0.2	1.9	2.19E-01	2.69E-01	0.778	0.54



LINEAR??

$$q_{\bar{o}} = \frac{k \cdot h}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s \right)} \underbrace{\int_{p_{wf}}^{p_R} \frac{k_{ro}}{\mu_o \cdot B_o} dp}_{\left[m(p_R) - m(p_{wf}) \right]} =$$

$$\int_0^{p_{wf}} \frac{k_{ro}}{\mu_o B_o} dp = m(p_{wf})$$

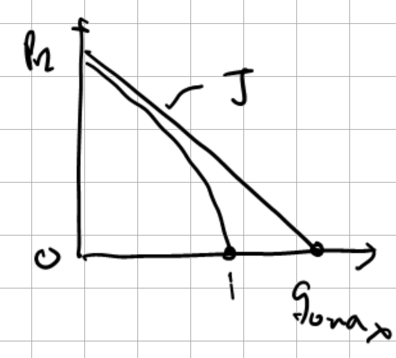
Not the same gas m function !

$$\int \frac{p \, dp}{Z \mu_g}$$

Vogel (V=0.2)

$$q_{\bar{o}} = q_{\bar{o},max} \left[1 - V \cdot \frac{p_{wf}}{p_R} - (1 - V) \cdot \left(\frac{p_{wf}}{p_R} \right)^2 \right]$$

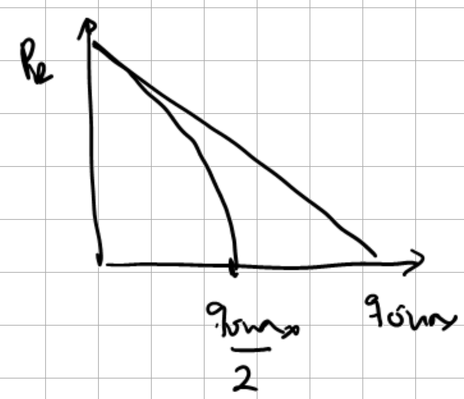
$$\underbrace{\frac{\overbrace{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o} \right)_{@pR} \right]}^{J} \cdot (p_R - 0)}{18.68 \cdot \left(ln\left(\frac{r_e}{r}\right) - 0.75 + s \right) \cdot 1.8}}_{q_{\bar{o}max}} = \frac{J}{1.8} \cdot pR$$



Fetkovich

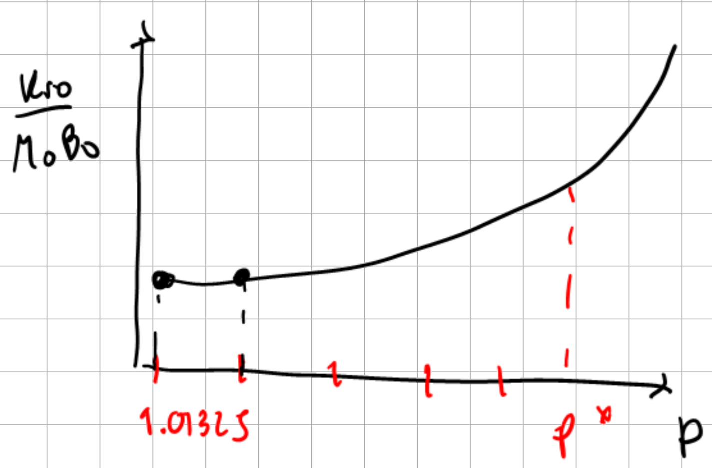
$$q_{\bar{o}} = q_{\bar{o},max} \left[1 - \left(\frac{p_{wf}}{p_R} \right)^2 \right]$$

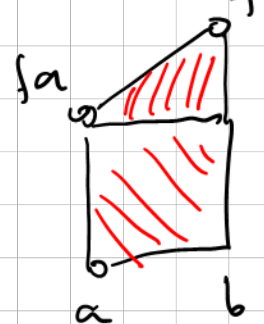
$$\frac{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o} \right)_{@pR} \right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s \right) \cdot 2} = \frac{J}{2} \cdot pR$$



oil m function in VBA:

```
Function m_function_oil(p, col, Matrix As Range)
    'p in bara
    'n number of intervals to use in the integration
    p1 = 1.01325
    p2 = p
    If p2 - p1 > 40 Then
        n = Round((p2 - p1) / 10, 0)
    Else
        n = 10
    End If
    DP = (p2 - p1) / n
    pj = p1
    Sum = 0
    fa = tabinterpol(pj, col, Matrix)
    For J = 1 To n
        pj = pj + DP
        fb = tabinterpol(pj, col, Matrix)
        Sum = Sum + (DP * (fa + fb)) * 0.5
        fa = fb
    Next
    m_function_oil = Sum
End Function
```

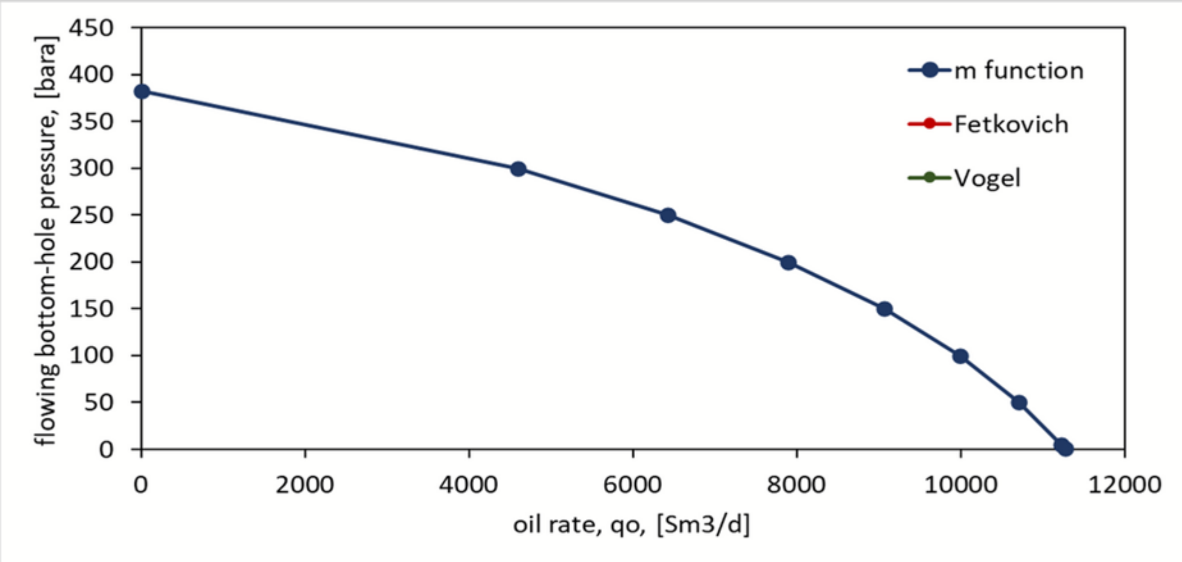




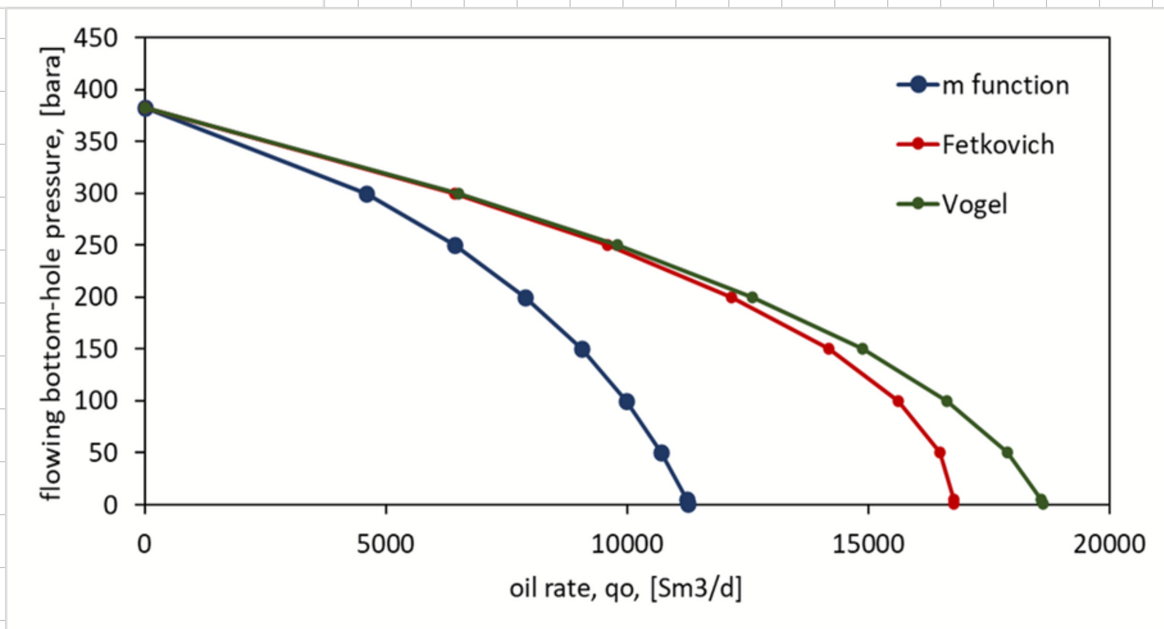
fb (from interpolation on table)

$$Area = (fa + fb) \cdot \frac{(b-a)}{2}$$

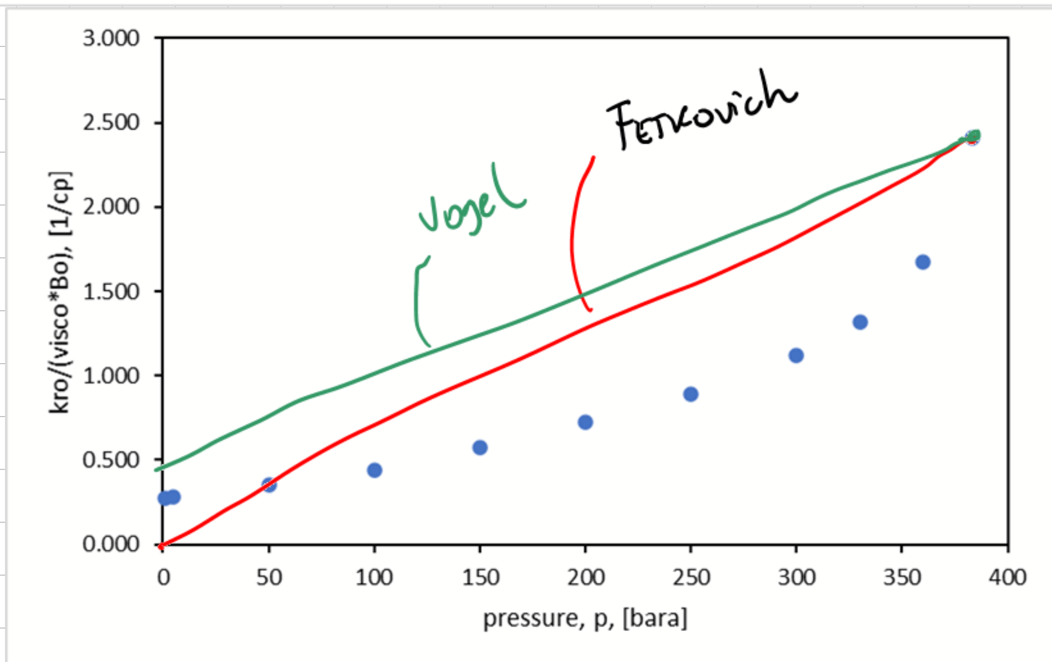
```
Function ipr_sat_oil_qo_m_function(k, h, re, rw, s, mpR, mPwf)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'mpR, m function at reservoir pressure [bara/cp]
    'mpwf, m functino at flowing bottom-hole pressure [bara/cp]
    ipr_sat_oil_qo_m_function = k * h * (mpR - mPwf) / ((Log(re / rw) - 0.75 + s) * 18.68)
End Function
```



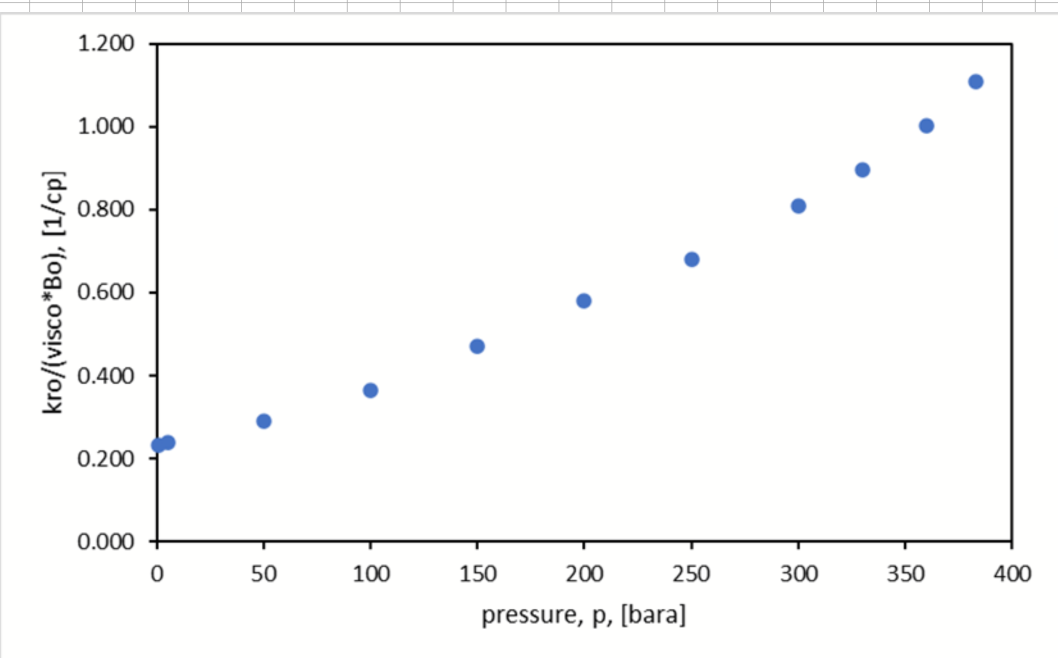

```
Function ipr_sat_oil_qo_Fetkovich(k, h, re, rw, s, pR, Pwf, Bo_pR, visco_pR, kro_pR)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'pR, reservoir pressure [bara]
    'pwf, flowing bottom-hole pressure [bara]
    'kro_pR oil relative permeability at reservoir pressure, [-]
    'visco_pR oil viscosity at reservoir pressure [cp]
    'Bo_pR oil formation volume factor at reservoir pressure, [m3/Sm3]
    J = k * h * (kro_pR / (visco_pR * Bo_pR)) / ((Log(re / rw) - 0.75 + s) * 18.68)
    qomax = J * (pR - 0) / 2
    ipr_sat_oil_qo_Fetkovich = qomax * (1 - (Pwf / pR) ^ 2)
End Function
```




```
Function ipr_sat_oil_qo_Vogel(k, h, re, rw, s, pR, Pwf, Bo_pR, visco_pR, kro_pR)
    'ipr_sat_oil, oil rate in Sm3/d, calculated with the m function
    'k, permeability, [md]
    'h, layer height, [m]
    're external radius of reservoir [m]
    'rw, wellbore radius [m]
    's, skin factor [-]
    'pR, reservoir pressure [bara]
    'pwf, flowing bottom-hole pressure [bara]
    'kro_pR oil relative permeability at reservoir pressure, [-]
    'visco_pR oil viscosity at reservoir pressure [cp]
    'Bo_pR oil formation volume factor at reservoir pressure, [m3/Sm3]
    J = k * h * (kro_pR / (visco_pR * Bo_pR)) / ((Log(re / rw) - 0.75 + s) * 18.68)
    qomax = J * (pR - 0) / 1.8
    ipr_sat_oil_qo_Vogel = qomax * (1 - 0.2 * (Pwf / pR) - 0.8 * (Pwf / pR) ^ 2)
End Function
```



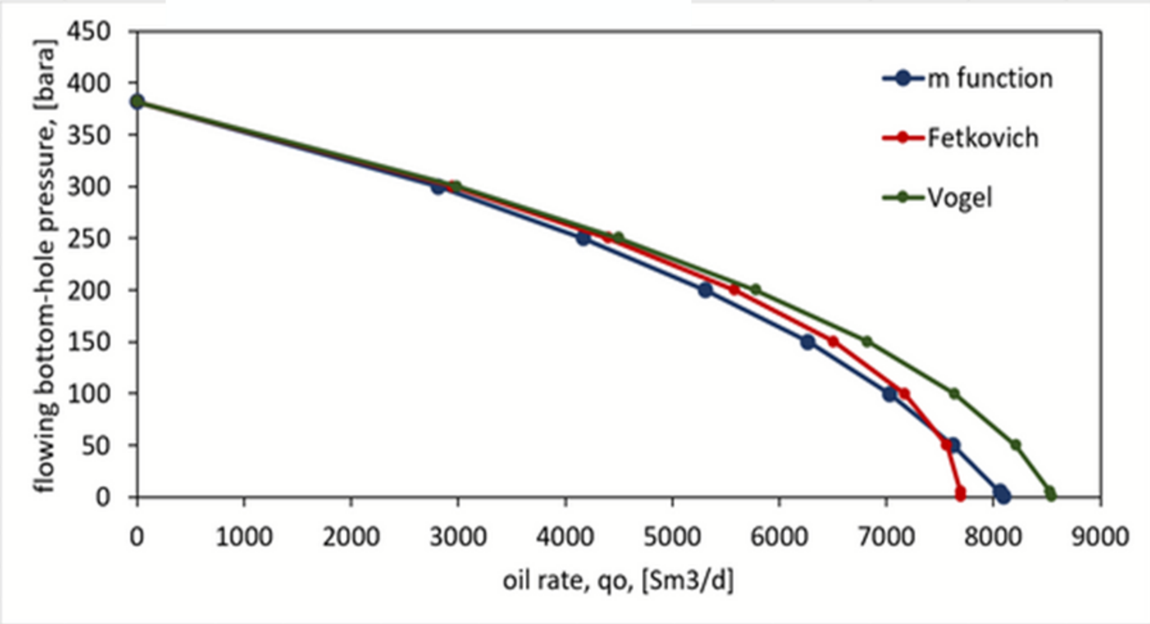
Changing the GOR (Rp) manually to 500 Sm3/Sm3

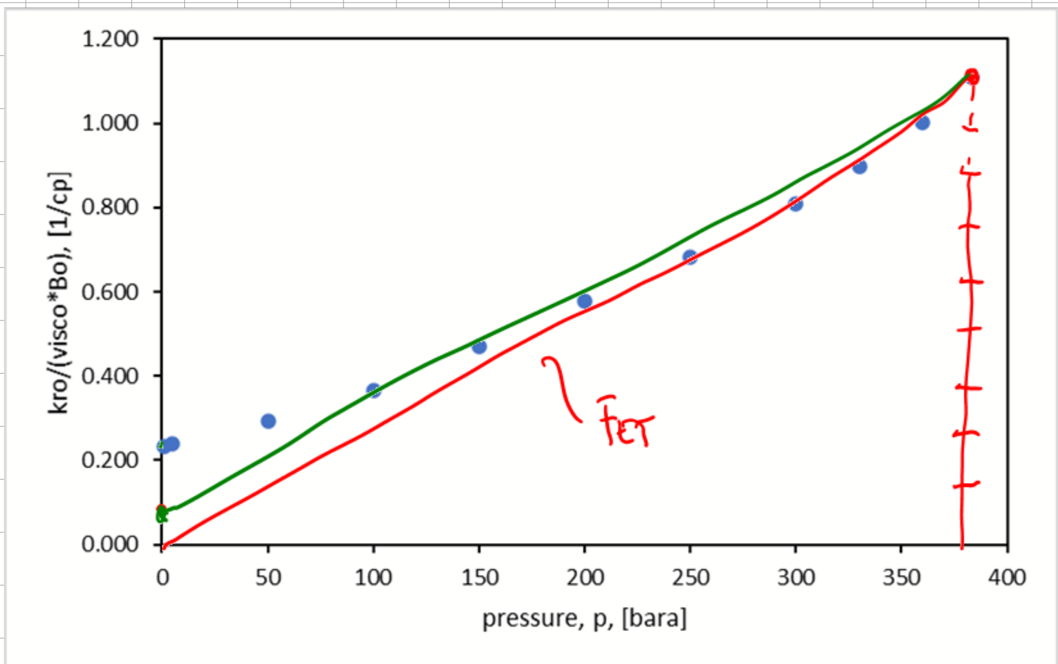


MORE LINEAR....

THE PREDICTION IS BETTER

pwf [bara]	m(p) [bara/cp]	qo [Sm3/d]	qo - Fet [Sm3/d]	qo -Vogel [Sm3/d]	qo - Fet-v2 [Sm3/d]	qo -Vogel-v2 [Sm3/d]
382	222.6	0	0	0		
300	145.3	2810	2950	2990		
250	108.0	4166	4401	4503		
200	76.5	5312	5588	5782		
150	50.2	6268	6511	6826		
100	29.3	7028	7170	7637		
50	12.9	7625	7566	8212		
5	0.9	8059	7697	8530		
1	0.0	8094	7698	8549		





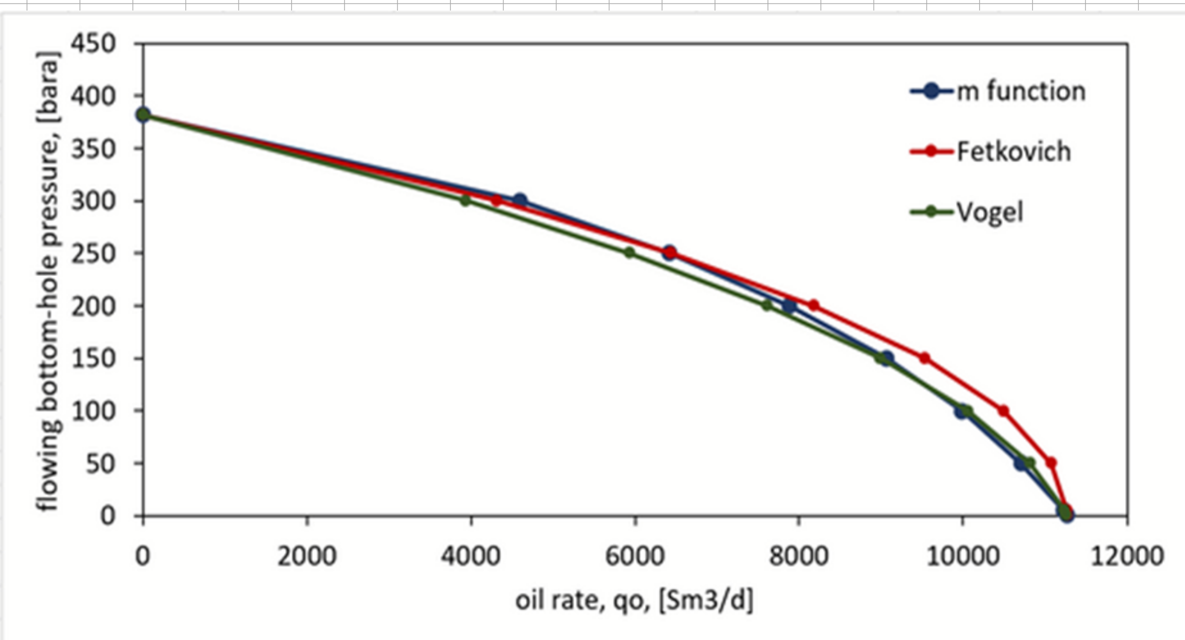
WE CHANGE THE RP BACK TO 300...

But, what if we use the q_{omax} from the m function equation? (11268 Sm³/d)

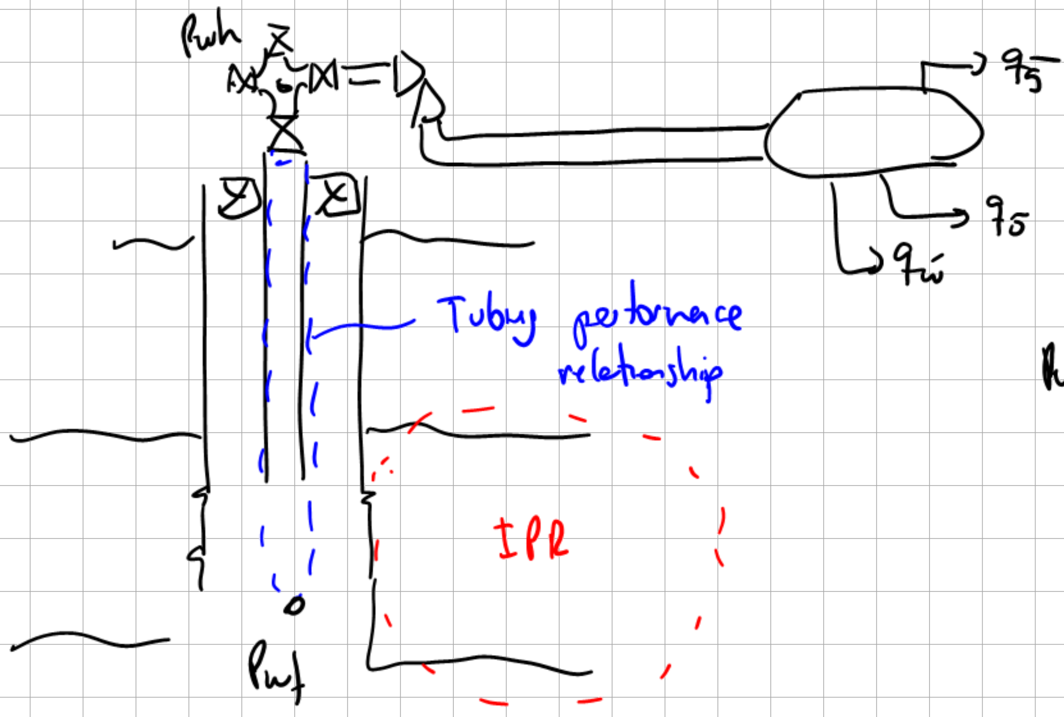
$$q_o = q_{omax}\left(1 - \left(\frac{P_{wf}}{P_R}\right)^2\right)$$

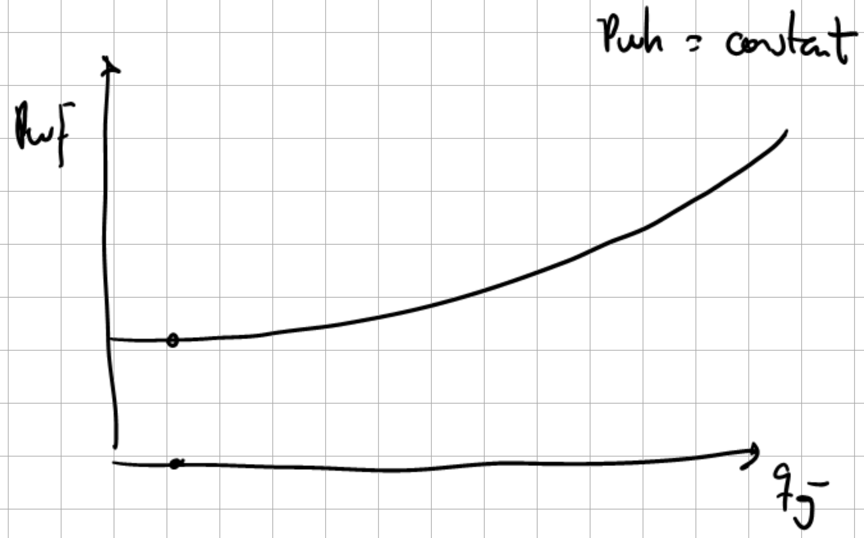
$$q_o = q_{omax}\left(1 - 0.2\left(\frac{P_{wf}}{P_R}\right) - 0.8\left(\frac{P_{wf}}{P_R}\right)^2\right)$$

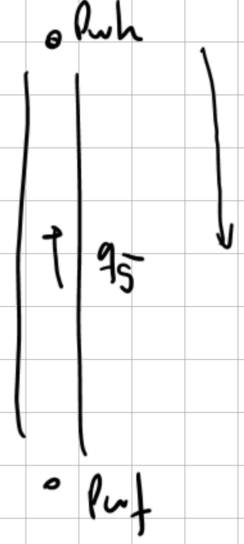
pwf [bara]	m(p) [bara/cp]	qo [Sm3/d]	qo - Fet [Sm3/d]	qo -Vogel [Sm3/d]	qo - Fet-v2 [Sm3/d]	qo -Vogel-v2 [Sm3/d]
382	309.9	0	0	0	0	0
300	183.5	4596	6425	6511	4318	3938
250	133.3	6421	9584	9807	6442	5932
200	93.1	7884	12169	12592	8179	7617
150	60.7	9061	14180	14867	9530	8993
100	35.3	9985	15616	16631	10495	10060
50	15.4	10708	16477	17884	11075	10818
5	1.1	11227	16762	18576	11266	11236
1	0.0	11268	16765	18617	11267	11261



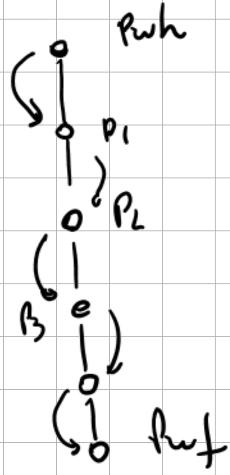
the prediction is improved considerably!

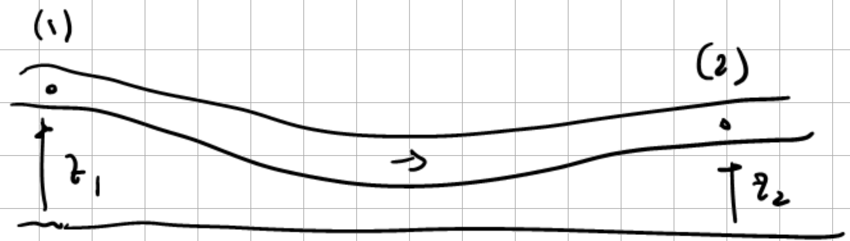






Carlos current to calculate P_{wf} (pressure integration)





head $\rightarrow$ $h_1 = h_2$ (no energy losses) $\qquad h = z + \frac{p}{\rho g} + \frac{v^2}{2g}$ incompressible

$h_1 = h_2 + \Delta h_{losses} \qquad \Delta h_{losses} = \underbrace{\cancel{\Delta h_{accessories}}}_{\text{valves restrictions}} + \Delta h_{friction} \qquad \Delta h_{friction} = f \frac{L}{\Phi} \frac{v^2}{2g}$

moody friction factor

$$h_1 = z_1 + \frac{p_1}{\rho g} + \cancel{\frac{v_1^2}{2g}} = z_2 + \frac{p_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + f \frac{L}{\Phi} \frac{v^2}{2g}$$

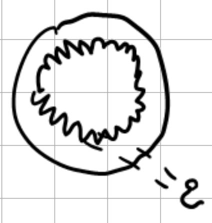
its incompressible, so $v_1 = v_2 = v$
Q = constant

$$\Delta p = (p_1 - p_2) = \rho g \cdot \left((z_2 - z_1) + f \frac{L}{\Phi} \frac{\rho g}{2g} v^2 \right)$$

$$(p_1 - p_2) = \underbrace{\Delta z \cdot \rho g}_{\text{hydrostatic}} + \underbrace{f \frac{L}{\Phi} \rho \frac{v^2}{2}}_{\text{friction}}$$

for gas $\rho = f(p, T)$

e/D dimensionless roughness

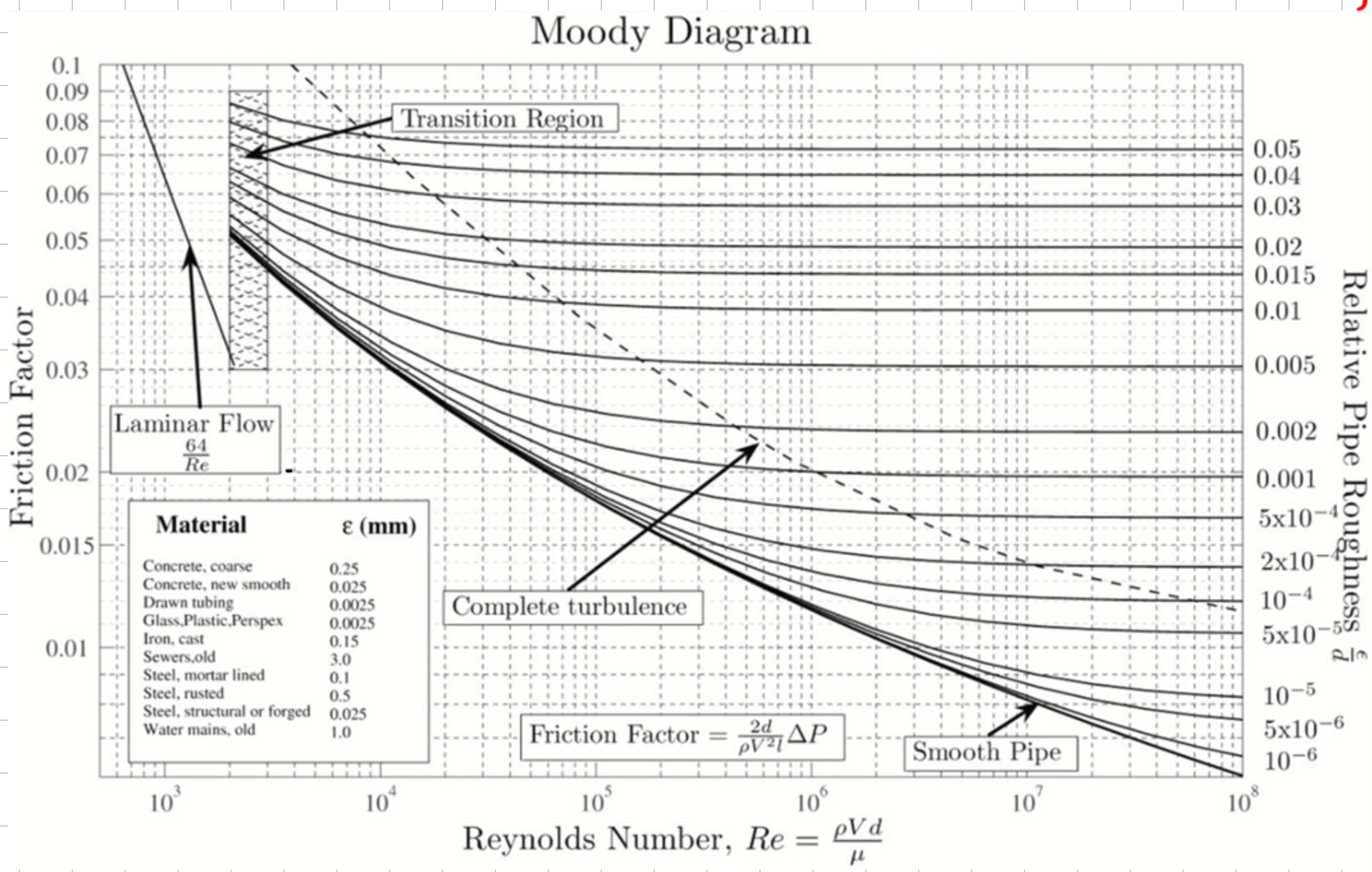


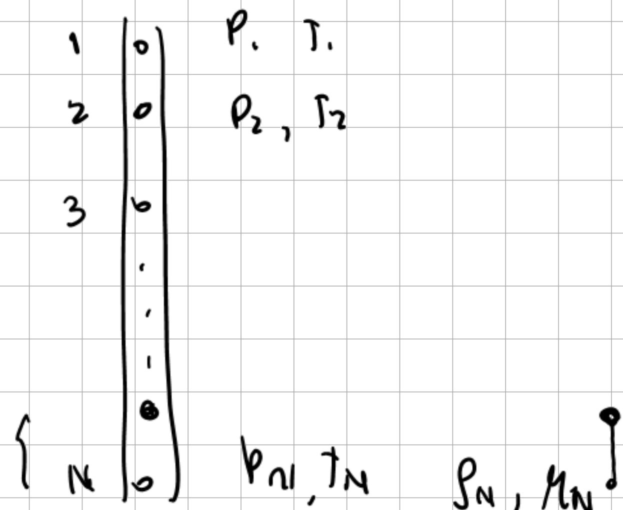
laminar region

$Re < 2500$

$$Re = \frac{\rho v D}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

$f(p,T)$





$N \to N-1$, gas properties don't change much

valid if ΔL is small

ΔT, Δp is small

$$V_g = \frac{q_g}{A} = \frac{q_{\bar{g}} B_g(p,T)}{A}$$

$$B_g = \frac{q_g}{q_{\bar{g}}}$$

$$q_{\bar{g}} \cdot \rho_{\bar{g}} = q_g \cdot \rho_g$$

$$B_g = \frac{q_g}{q_{\bar{g}}} = \frac{\rho_{\bar{g}}}{\rho_g} = \frac{zT}{p}\left(\frac{p_{sc}}{T_{sc}}\right)$$

$$\rho_{\bar{g}} = \frac{p_{sc}}{R z_{sc} T_{sc}}$$

$$\rho_g = \frac{p}{zRT}$$

m^3/d $\quad \frac{1\,d}{24\,h}$ $\quad \frac{1\,h}{3600\,s}$

qg [Sm3/d]	2.85E+06
Gas gravity	0.7
Tubing ID [m]	0.157
Tubing cross section area [m2]	0.019
Tubing roughness [m]	1.50E-06

TVD [m]	p [bara]	T [C]	Z [-]	deng [kg/m3]	Bg [m3/Sm3]	viscg [cp]	qg [m3/d]	vg [m/s]	p-calc [bara]
0	40	87	0.948	28.6	3.00E-02	1.36E-02	8.54E+04	51.03	
284	46	89	0.942	32.9	2.60E-02	1.38E-02	7.41E+04	44.28	
567	51	90	0.938	36.8	2.32E-02	1.39E-02	6.63E+04	39.61	
851	56	92	0.934	40.4	2.12E-02	1.41E-02	6.04E+04	36.14	
1135	61	94	0.930	43.6	1.96E-02	1.43E-02	5.59E+04	33.43	
1418	66	96	0.928	46.7	1.83E-02	1.45E-02	5.23E+04	31.25	
1702	70	98	0.926	49.5	1.73E-02	1.46E-02	4.92E+04	29.44	
1986	74	99	0.924	52.2	1.64E-02	1.48E-02	4.67E+04	27.92	
2269	78	101	0.922	54.8	1.56E-02	1.49E-02	4.45E+04	26.61	
2553	81	103	0.921	57.3	1.49E-02	1.51E-02	4.26E+04	25.47	
2837	85	105	0.920	59.6	1.44E-02	1.52E-02	4.09E+04	24.46	

2837 - 2553

∘ p_{N-1} ∘ p_{wf} ?

$$p_{N-1} = p_{wf} - \underbrace{\rho g \Delta z - f \frac{L}{D} \rho \frac{v^2}{2}}_{\text{using conditions at } p_{wf} \text{ !}}$$

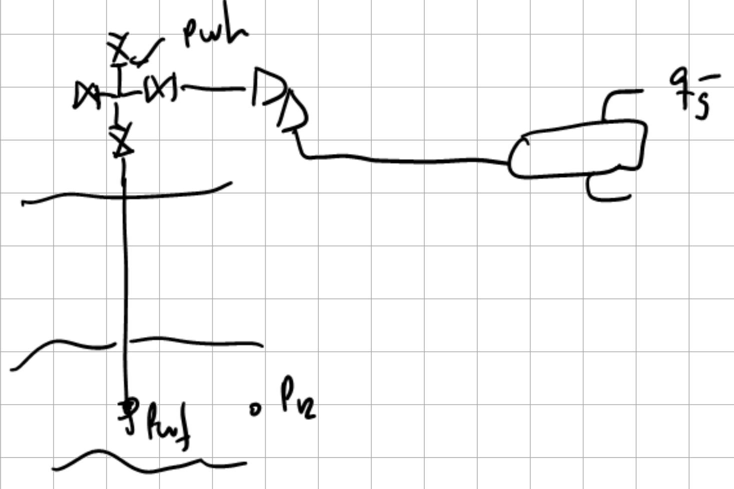
```
Function Pout(qtl, ID, den, visc, Length, teta, pin, roughness)
    'Function that give pressure available at the oulet of a pipe with a flow qt and inlet pressure pressure pin
    'Calculation made for liquid single phase flow
    'Takes in data in SI
    'qt flow [m^3/d]
    'ID inner diameter of pipe [m]
    'den density of fluid, [kg/m^3]
    'visc viscosity of fluid, [Pa s]
    'Length, pipe length, [m]
    'teta inclination angle of pipe with respect to horizontal [°]
    'pin, discharge pressure required, [bara]
    'roughness of pipe [m]

    'Gravitational acceleration g, [m/s(2]
    g = 9.81
    'Pi number
    Pi = 4 * Atn(1)
    qt = qtl / (3600# * 24#)  '[m^3/s]

    'Calculating area and velocity
    Area = Pi * (ID ^ 2) / 4
    v = qt / Area

    Presscalcl = pin - (Length * Sin(teta * Pi / 180) * den * g / 100000#) - (ffactor(den, visc, ID, roughness, v) * Length * (v ^ 2) * den / (ID * 200000#))
    Pout = Presscalcl

End Function
```

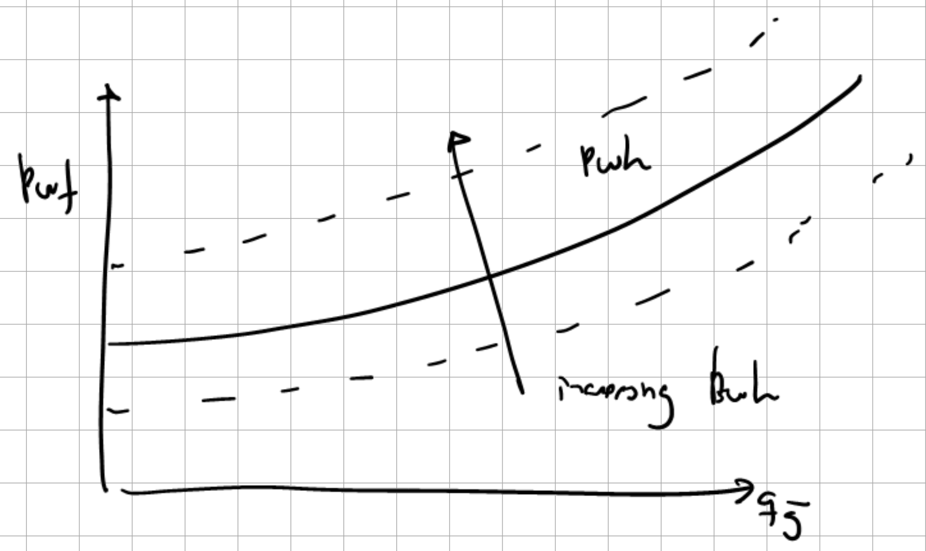
change for m^3/d to m^3/s

dividing by 1E5 to take from Pa to bara

TVD [m]	p [bara]	T [C]	Z [-]	deng [kg/m3]	Bg [m3/Sm3]	viscg [cp]	qg [m3/d]	vg [m/s]	p-calc [bara]
0	40	87	0.948	28.6	3.00E-02	1.36E-02	8.54E+04	51.03	35.55
284	46	89	0.942	32.9	2.60E-02	1.38E-02	7.41E+04	44.28	41.50
567	51	90	0.938	36.8	2.32E-02	1.39E-02	6.63E+04	39.61	47.03
851	56	92	0.934	40.4	2.12E-02	1.41E-02	6.04E+04	36.14	52.27
1135	61	94	0.930	43.6	1.96E-02	1.43E-02	5.59E+04	33.43	57.29
1418	66	96	0.928	46.7	1.83E-02	1.45E-02	5.23E+04	31.25	62.15
1702	70	98	0.926	49.5	1.73E-02	1.46E-02	4.92E+04	29.44	66.88
1986	74	99	0.924	52.2	1.64E-02	1.48E-02	4.67E+04	27.92	71.52
2269	78	101	0.922	54.8	1.56E-02	1.49E-02	4.45E+04	26.61	76.08
2553	81	103	0.921	57.3	1.49E-02	1.51E-02	4.26E+04	25.47	80.59
2837	85	105	0.920	59.6	1.44E-02	1.52E-02	4.09E+04	24.46	85.04

surprisingly close !





$$q_{\bar{g}} = C_T \cdot \left(\frac{P_{wf}^2}{e^S} - P_{wh}^2\right)^{0.5}$$

tubing coefficient, tubing elevation coefficient

for same P_{wf}, P_{wh}:

- C_T ↑ q_g ↑
- S ↑ q_g ↓

$$u_g = \frac{q_g}{A} = \frac{\dot{m}_g}{\rho_g \cdot A} \qquad \rho_g = \frac{p}{Z\, R\, T}$$

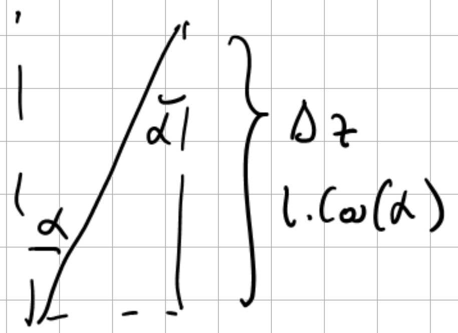
R_0 / MW

$$u_g^2 = \left(\frac{\dot{m}_g}{\rho_g A}\right)^2$$

$f(v, \rho, \mu)$

$$-\frac{dp}{dl} = \frac{p \cdot M_g}{\underbrace{Z}_{f(p)} \cdot R \cdot T} \cdot g \cdot \cos(\alpha) + \frac{8 \cdot f_M \cdot \dot{m}^2}{\pi^2 \cdot D^5} \cdot \frac{Z \cdot R \cdot T}{p \cdot M_g}$$

"R_0"



$$S = 2 \cdot L \cdot C_a = 2 \cdot \frac{M_g}{Z_{av} \cdot R \cdot T_{av}} \cdot L \cdot g \cdot \cos(\alpha)$$

$$q_{sc} = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left[\left(p_{wf}^2 - p_t^2 \cdot e^S\right) \cdot \left(\frac{S}{e^S - 1}\right)\right]^{0.5}$$

By defining the tubing constant C_T:

$$C_T = \left(\frac{\pi}{4}\right) \cdot \left(\frac{R}{M_{air}}\right)^{0.5} \cdot \left(\frac{T_{sc}}{p_{sc}}\right) \cdot \left(\frac{D^5}{f_M \cdot L \cdot \gamma_g \cdot Z_{av} \cdot T_{av}}\right)^{0.5} \cdot \left(\frac{S \cdot e^S}{e^S - 1}\right)^{0.5} \qquad \text{EQ. A-24}$$

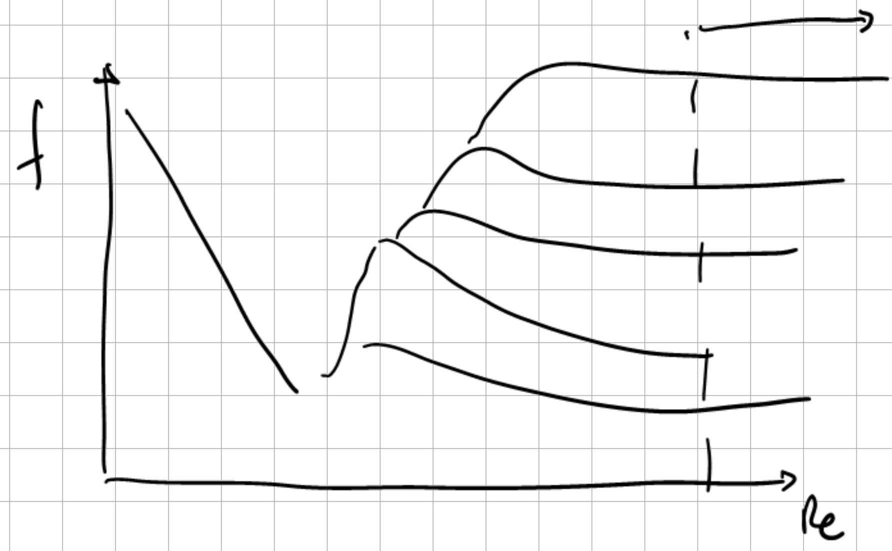
This yields:

$$q_{sc} = C_T \cdot \left[\left(\frac{p_{wf}^2}{e^S} - p_t^2 \cdot\right)\right]^{0.5} \qquad \text{EQ. A-25}$$

to calculate C_T, S, a seed P_{wf}, P_{wh}, T_{wf}, T_{wh} are required!

Z_{av}, T_{av}

Luckily the effect of seed is modest



fully turbulent regime $\rightarrow f = f(e/D)$

$$Re = \frac{v D \rho}{\mu}$$

ρ is low

v is high

μ is high

Re is very high

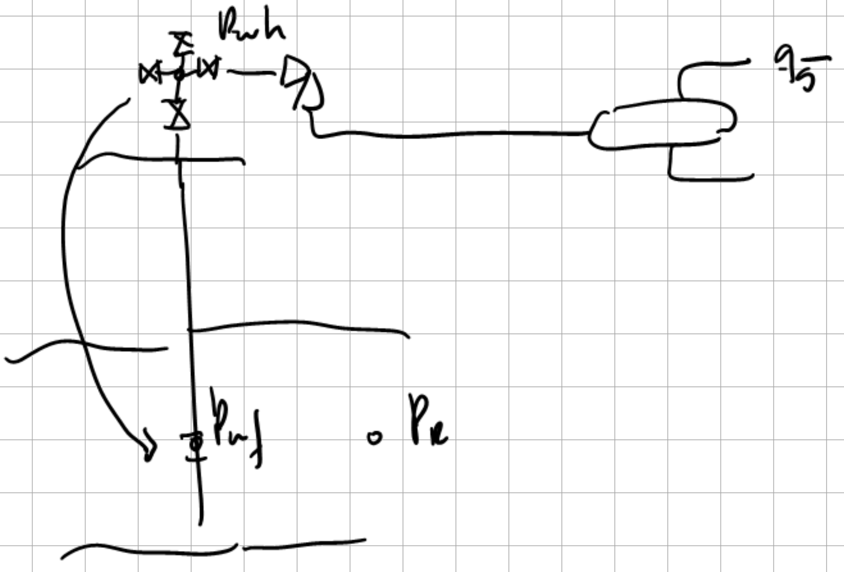
$e = f(D)$ Smith (1950)

$\hookrightarrow f = f(D)$

DETERMINING FRICTION FACTORS FOR MEASURING PRODUCTIVITY OF GAS WELLS

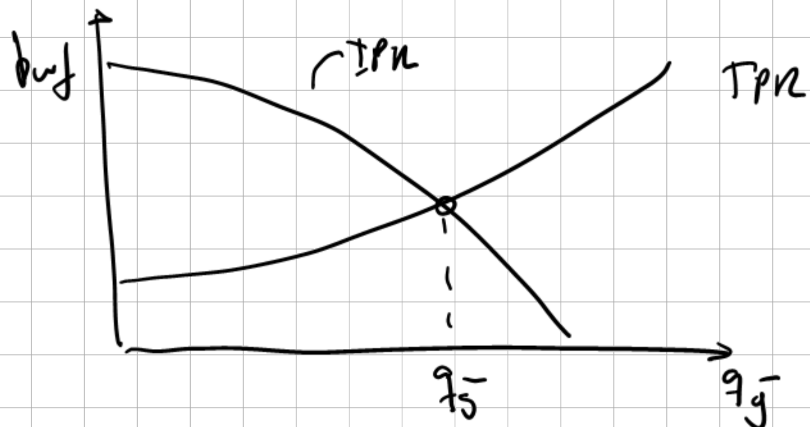
R. V. SMITH, U. S. BUREAU OF MINES, BARTLESVILLE, OKLAHOMA

$$f_M = \frac{0.01748}{D^{0.224} \cdot \left(|1\,m| \cdot \left|\frac{39.37\,in}{1\,m}\right|\right)^{0.224}} = \frac{0.0077}{D^{0.224}}$$



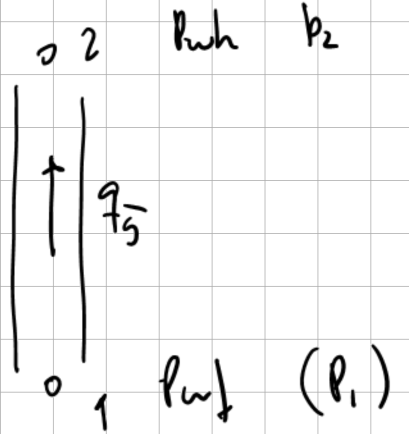
1) neglecting flowline $\rightsquigarrow$ $p_{wh} = p_{sep}$
fully open choke

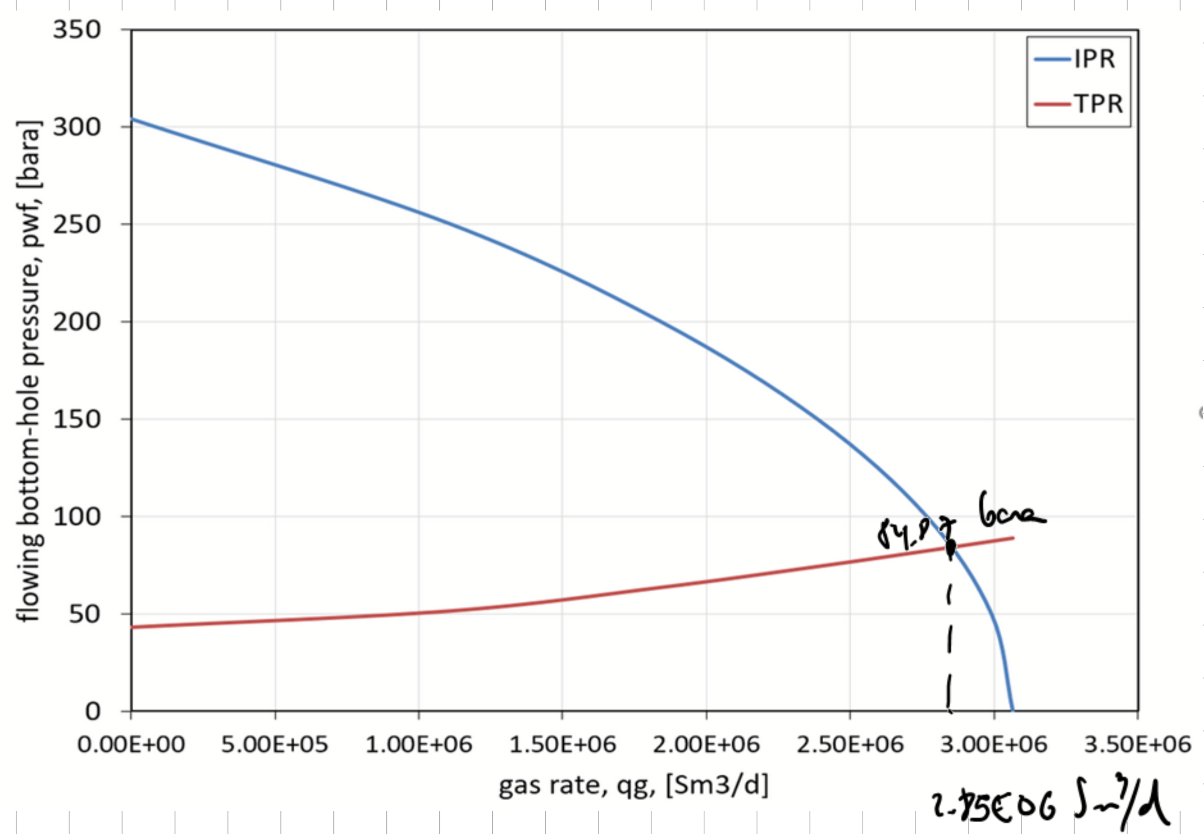
• select an equilibrium point $\rightarrow$ bottom-hole

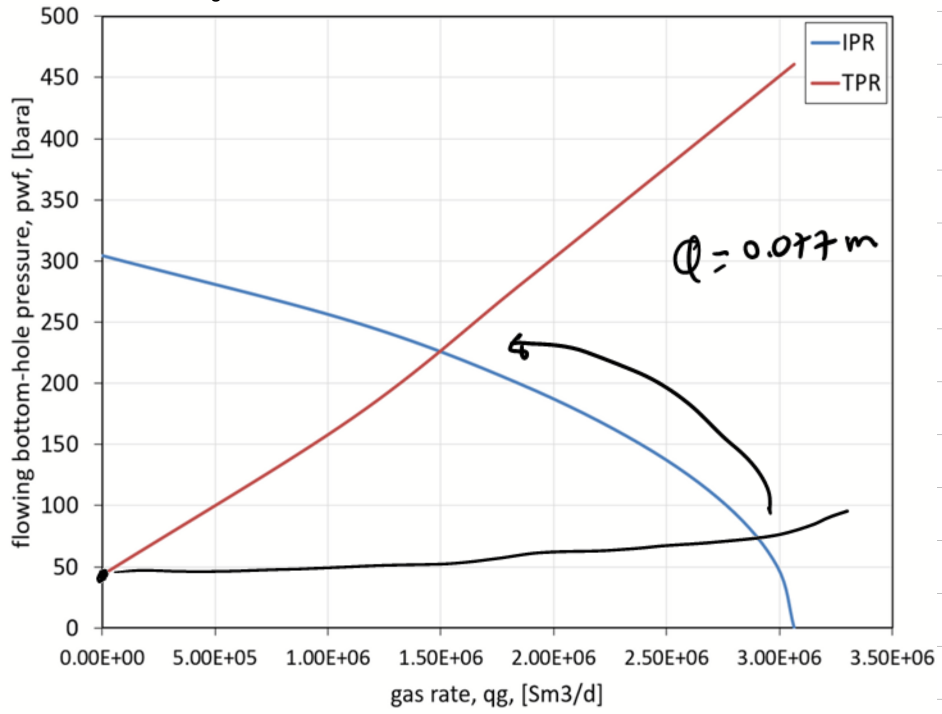


$$q_g = C_R \left(p_R^2 - p_{wf}^2\right)^{n}$$

$$q_g = C_T \left(\frac{p_{wf}^2}{e^S} - p_{wh}^2\right)^{0.5} \rightarrow p_{wf} = \sqrt{\left(p_{wh}^2 + \left(\frac{q_g}{C_T}\right)^2\right) e^S}$$







effect of tubing size

$D \downarrow \rightarrow G \downarrow \rightarrow q_g^* \downarrow$