U. S. DEPARTMENT OF THE INTERIOR
HAROLD L. ICKES, Secretary

BUREAU OF MINES
John W. Finch, Director

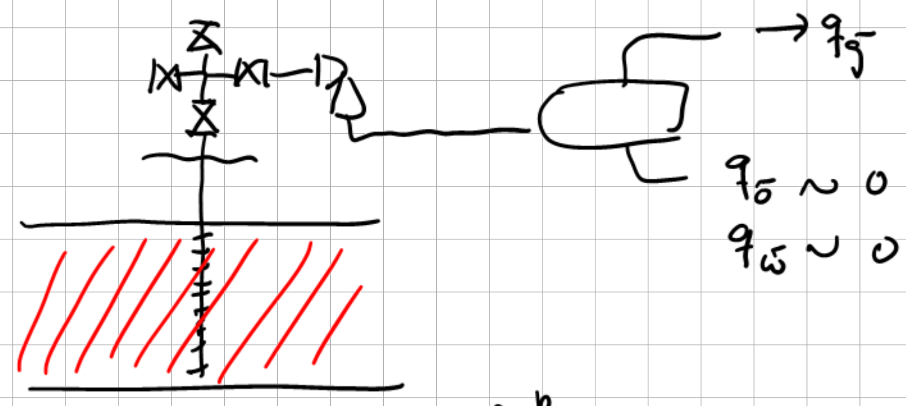
(1935)

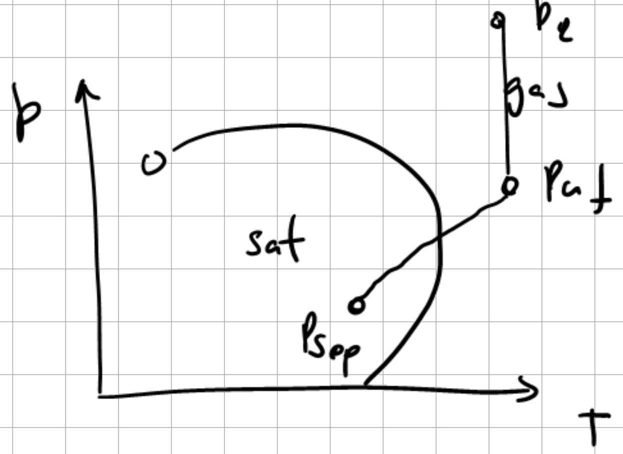
Monograph 7

Back-Pressure Data on Natural-Gas Wells and Their Application to Production Practices

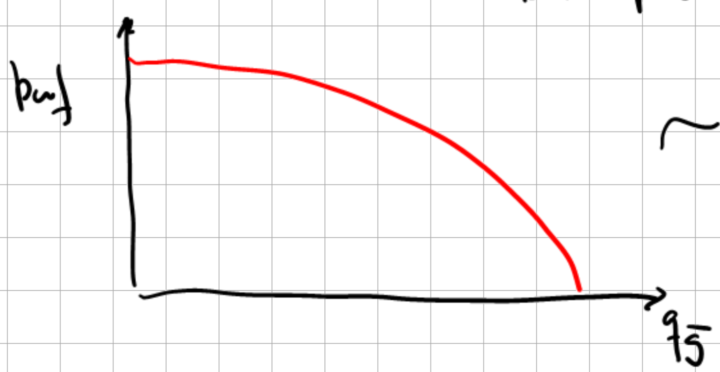
By

E. L. RAWLINS AND M. A. SCHELLHARDT





Back-pressure equation



empirically, measured in the field

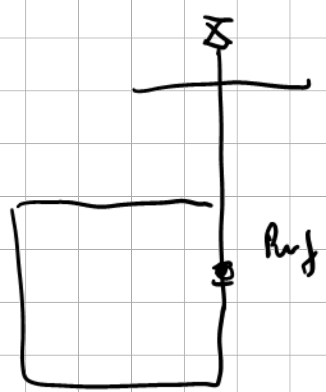
$$\sim \quad q_g = C\left(P_R^2 - P_{wf}^2\right)^n \rightarrow 0.5 \leq n \leq 1$$

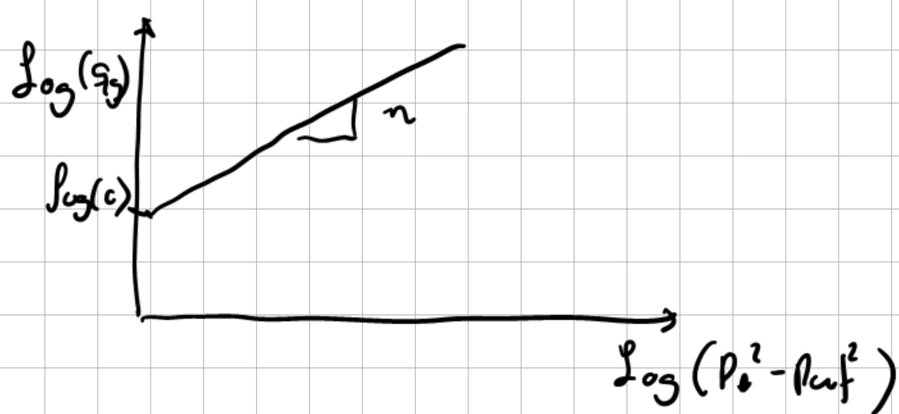
$$\frac{q_g}{C} = \left(P_R^2 - P_{wf}^2\right)^n \qquad \text{Log}$$

$$\text{Log}\left(\frac{q_g}{C}\right) = \text{Log}\left(\left(P_R^2 - P_{wf}^2\right)^n\right)$$

$$\text{Log}(q_g) - \text{Log}(C) = n \cdot \text{Log}\left(P_R^2 - P_{wf}^2\right)$$

$$\text{Log}(q_g) = \text{Log}(C) + n\ \text{Log}\left(P_R^2 - P_{wf}^2\right)$$





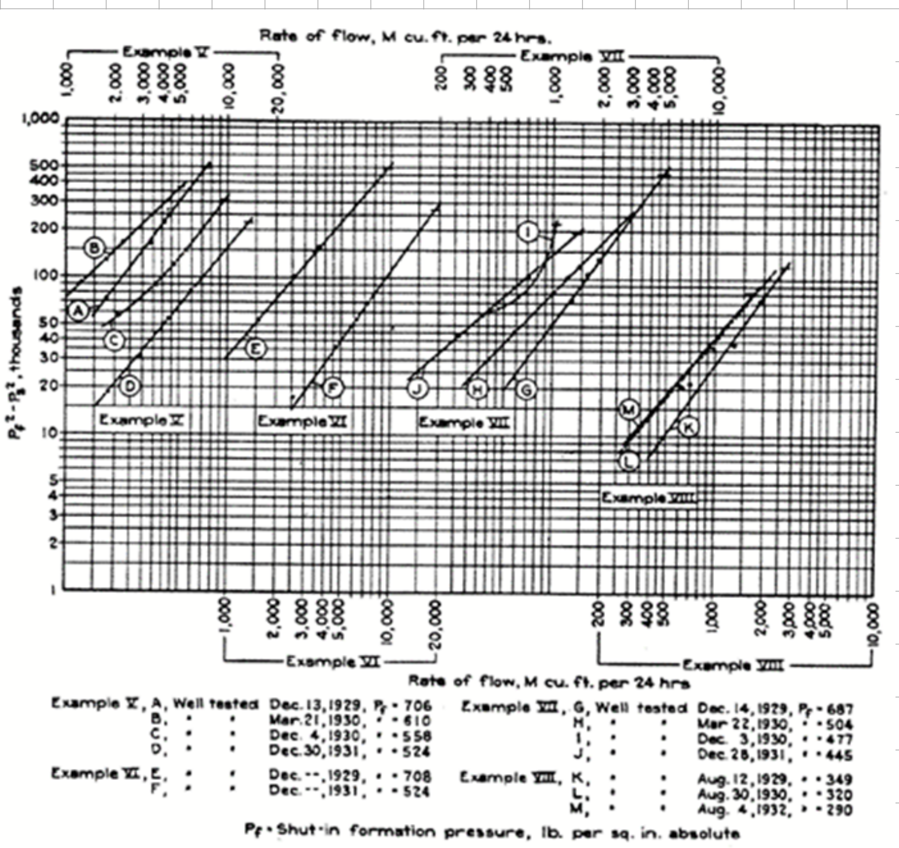
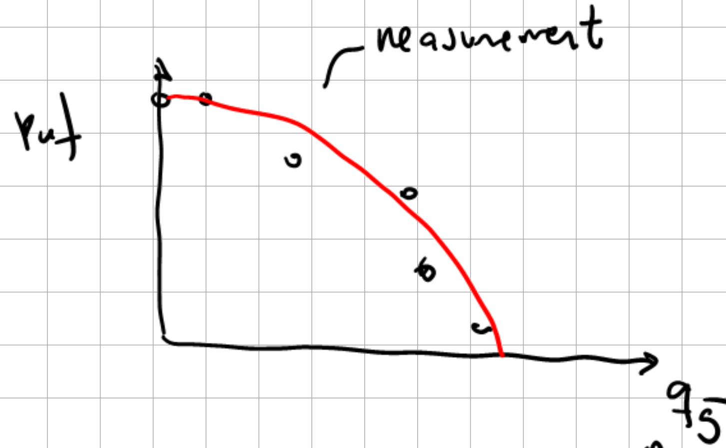


FIGURE 24.—Variation in delivery capacities of gas wells at different times in their productive lives, examples V, VI, VII, and VIII



$$q_g = C\left(P_R^2 - P_{wf}^2\right)^n$$

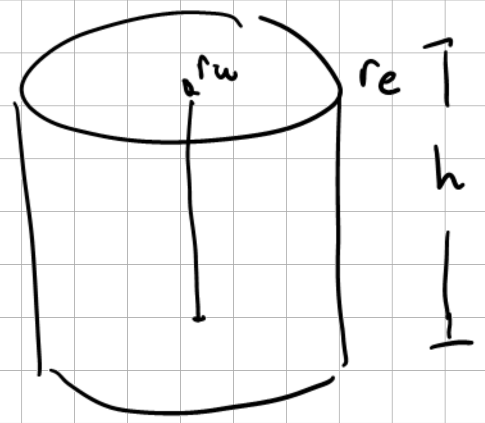
in Excel change C, n to match data

analytical derivation of Dry gas IPR

$q_{\bar{g}}$

$$v = \frac{k}{\mu_g}\frac{dp}{dr}$$

$$\frac{q_g}{2\pi r h} = \frac{k}{\mu_g}\frac{dp}{dr}$$

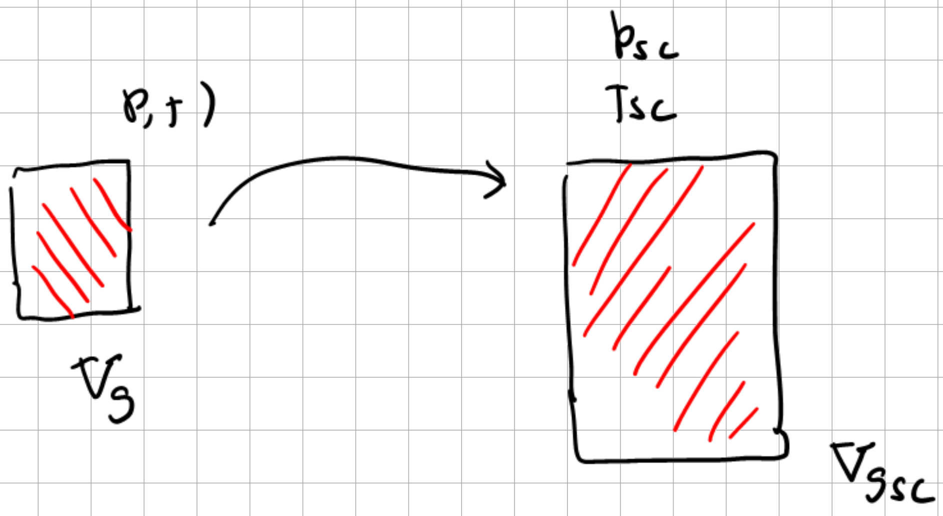


$$q_g = f(q_{\bar{g}})$$

$$B_g = \frac{q_g}{q_{\bar{g}}}$$

$$q_g = q_{\bar{g}} \cdot B_g$$

$$\int_{r_w}^{r_{@pR}} \frac{q_{\bar{g}}}{r} = \int_{p_{wf}}^{p_R} \frac{2\pi k h}{B_g \mu_g}\frac{dp}{dr}$$



$$B_g = \frac{V_g}{V_{\bar{g}}}$$

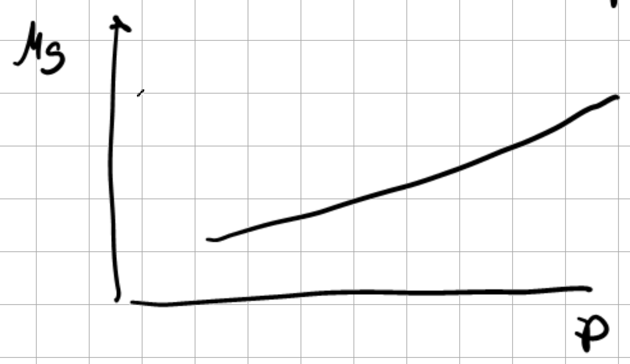
gas volume factor

$$B_g = f(p,T)$$

$$B_g \sim 1E-2 \rightarrow 1E-3$$

for PSS
$$q_{\bar{g}} = \frac{2\pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)}$$

$$\int_{p_{wf}}^{p_R} \frac{1}{B_g \mu_g} dp \rightarrow ?$$



mass conservation

$$q_{\bar{g}} \cdot \rho_{\bar{g}} = q_g \rho_g$$

$$B_g = \frac{q_g}{q_{\bar{g}}} = \frac{\rho_{\bar{g}}}{\rho_g}$$

Real gas equation

$$\frac{p}{\rho} = zRT \qquad R = \frac{R_u}{MW}$$

gas deviation factor

deviation from ideal gas

$$\rho = \frac{p}{zRT}$$

Boyle (English)

Hooke (English)

Charles (French)

Gay-Lussac (French)

Avogadro (Italian)

$z \sim$

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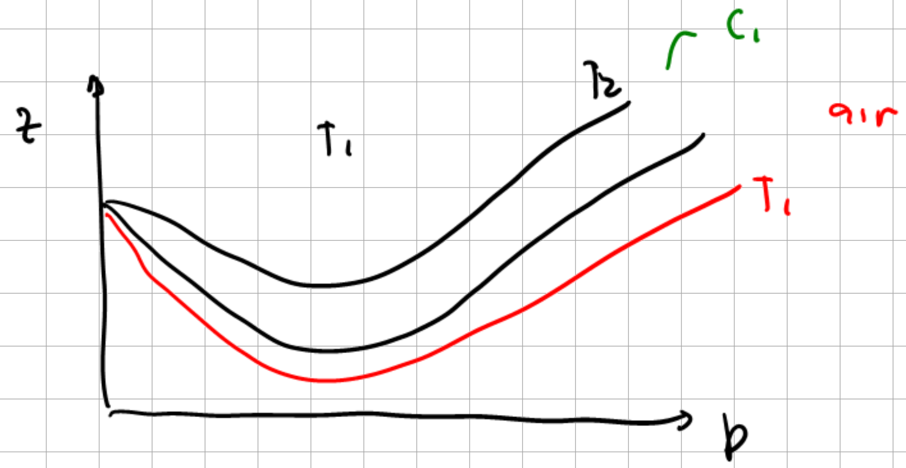
Density of Natural Gases

By Marshall B. Standing* and Donald L. Katz,* Member A.I.M.E.

(New York Meeting, February 1941)



$$T_r = \frac{T}{T_c} \qquad p_r = \frac{p}{p_c}$$

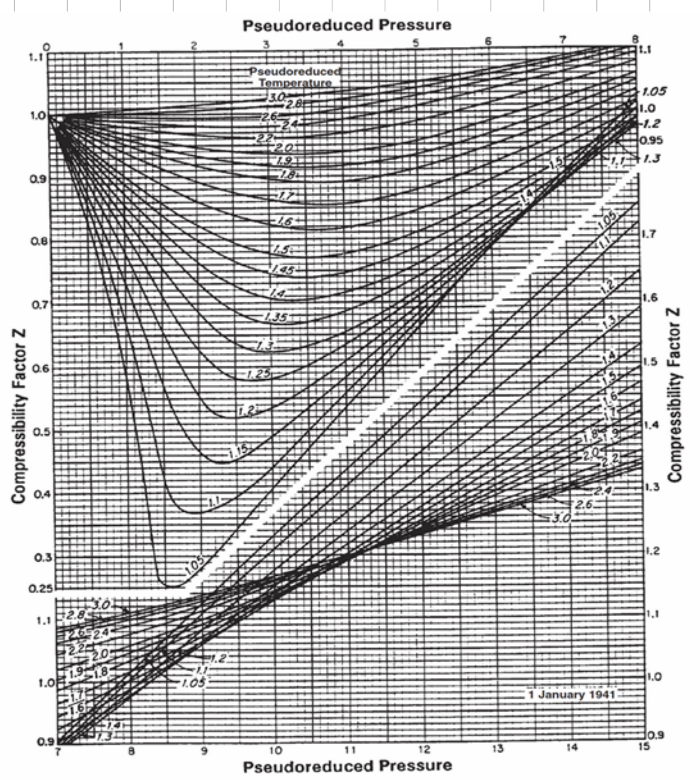
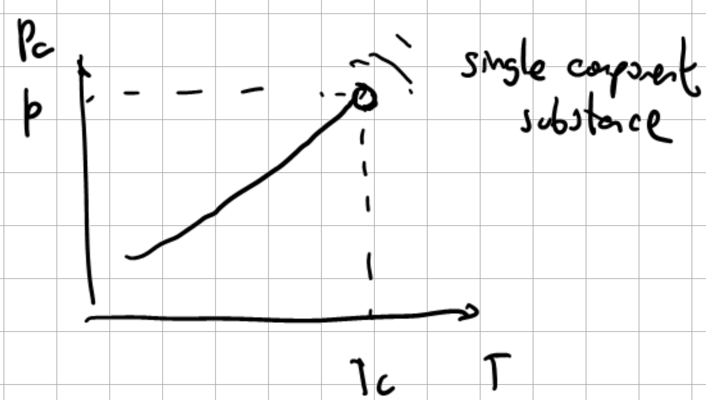


Fig. 3.6—Standing-Katz[4] Z-factor chart.

T_c, p_c

$T_c, p_c = f(MW)$



Gas composition

Component	mole fraction
C_1	z_{c_1}
C_2	z_{c_2}
C_3	z_{c_3}

$$\sum_{1}^{N} z_i = 1$$

$$MW_{mix} = \sum_{1}^{N} Mw_i\, z_i \qquad T_c, p_c = f(MW_{mix}) \rightsquigarrow \text{Sutton}$$

$$B_g = \frac{\rho_{\bar{g}}}{\rho_g} \qquad \rho_{\bar{g}} = \frac{p_{sc}}{\cancel{z_{sc}}\, R\, T_{sc}} \qquad \rho_g = \frac{p}{z\, R\, T}$$

1

$$B_g = \frac{p_{sc}}{T_{sc}} \cdot \frac{z\, T}{p}$$

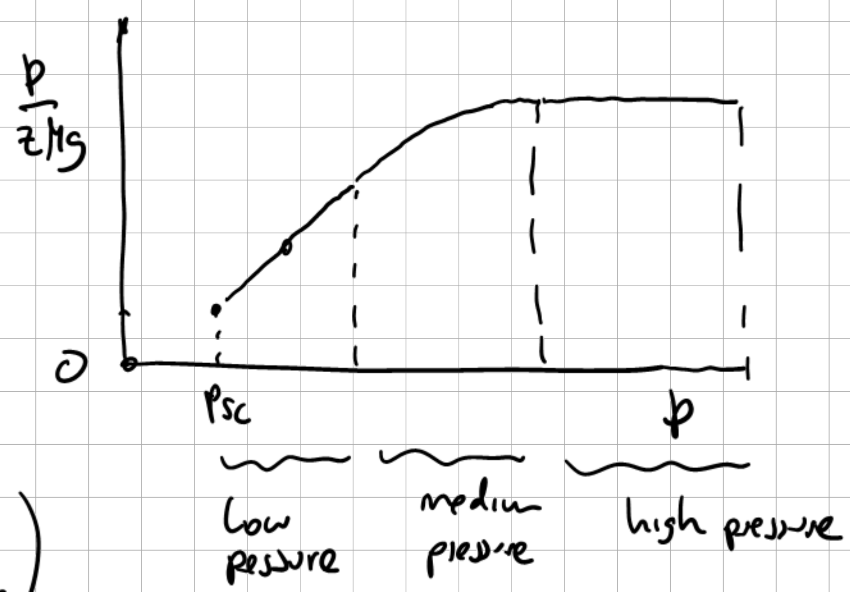
p_{sc} 1.01325 bara

T_{sc} 15.56 °C

$T = T_R$

$$q_{\bar{g}} = \frac{2\pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)} \int_{p_{wf}}^{p_R} \frac{1}{\mu_g B_g}\, dp$$

$$q_{\bar{g}} = \frac{2\pi k h}{\ln\left(\frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \underbrace{\int_{p_{wf}}^{p_R} \frac{p}{z\, \mu_g}\, dp}$$



low pressure (p_{wf}, p_R)

$$q_g = \frac{2\pi k h}{\left(\ln\frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R}\; 0.5 \left[p_R^2 - p_{wf}^2\right] \left(\frac{1}{z \mu_g}\right)_{@\, p_R}$$

in the low pressure region $\left(\frac{1}{\mu_g z}\right) = \text{const}$

$$q_{\bar{g}} = C\left(p_R^2 - p_{wf}^2\right)^n \quad \text{if } n = 1 \quad \text{it's a match!}$$

high pressure

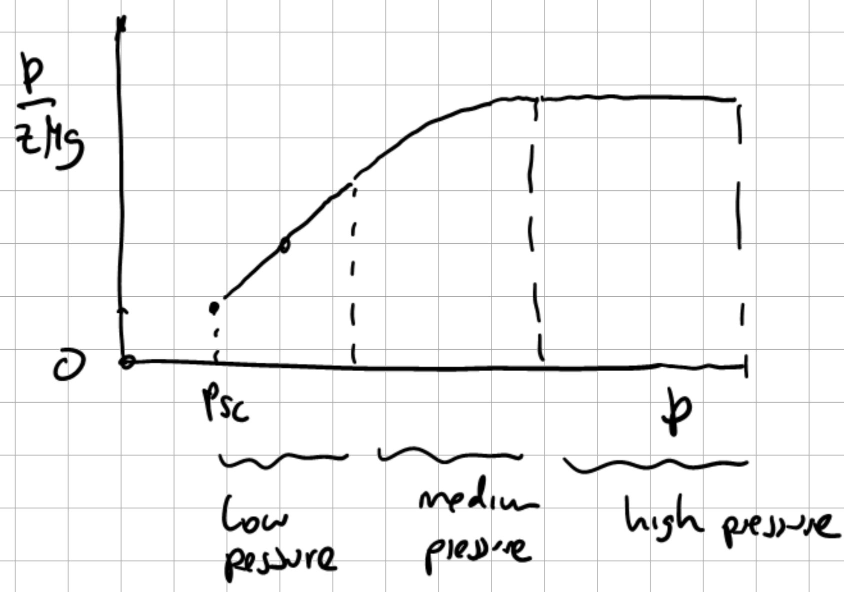
$$q_{\bar{g}} = \frac{2\pi k h}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \underbrace{\left(\frac{p}{\mu B_g}\right)_{@}}_{\text{avg pressure}} \int_{p_{wf}}^{p_R} dp = \underbrace{\frac{2\pi k h}{\left(\ln\frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \frac{p_R}{(\mu_g z)_{@\, p_R}}}_{\text{constant}} (p_R - p_{wf})$$

similar to undersaturated oil IPR

doesn't resemble $q_{\bar{g}} = C\left(p_R^2 - p_{wf}^2\right)^n$

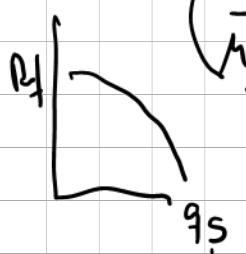
$$q_{\bar{o}} = J\,(p_R - p_{wf})$$

$$q_g = \frac{2\pi k h}{\ln\left(\frac{r_e}{r_w} - 0.75\right)} \frac{T_{sc}}{p_{sc} T_R} \underbrace{\int_{p_{wf}}^{p_R} \frac{p}{z \mu_g} dp}$$



LP

$$\left(\frac{1}{\mu_g z}\right)_{@p_R} \frac{(p_R^2 - p_{wf}^2)}{2}$$



?

HP

$$\frac{p_R}{(\mu_g z)_{@p_R}} (p_R - p_{wf})$$

p [bara] $\quad q_g = [Sm^3/d] \quad p_{sc} = 1.01325$ bara

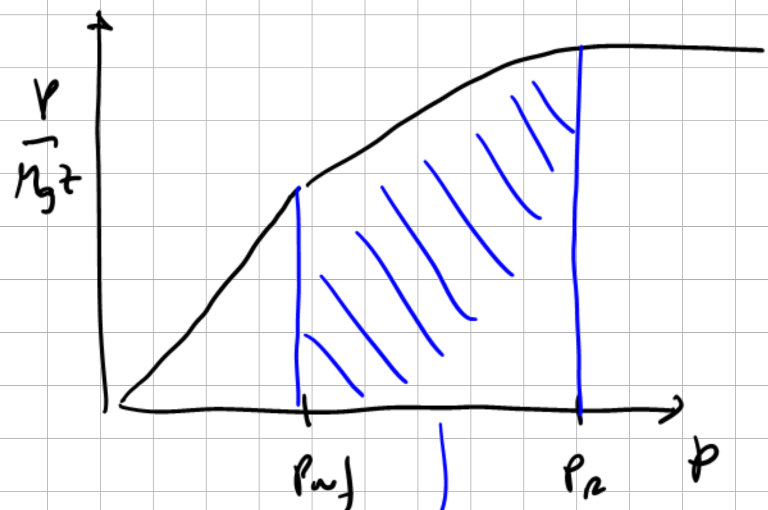
T [K] $\quad \mu_g = [cp] \quad T_{sc} = 15.56$ °C

k [md]

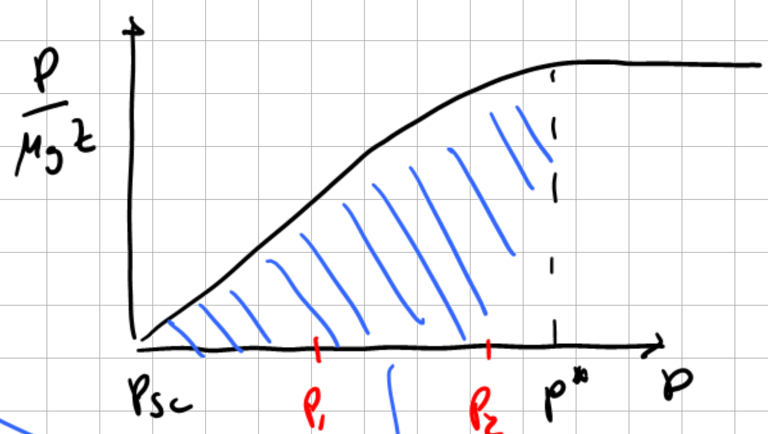
excluding the 2 !

$$q_g = \frac{k h \; 7.63}{\left(\ln \frac{r_e}{r_w} - 0.75 + S\right) T_R} \; 2 \underbrace{\int_{p_{wf}}^{p_R} \frac{p}{\mu_g z} dp}$$

$$\Big(\underbrace{\bigcirc}_{p_R} - \underbrace{\bigcirc}_{p_{wf}} \Big)$$

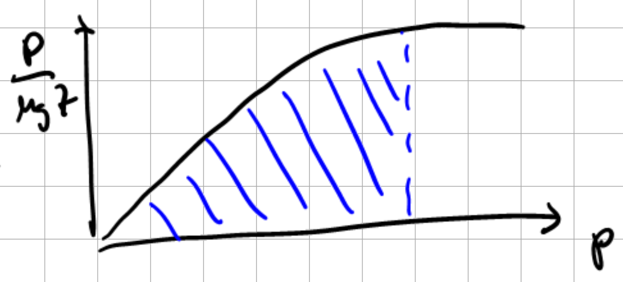


$$m(p^*) = 2 \int_{0 \to p_{sc}}^{p^*} \frac{p}{\mu_g z} dp$$



$$2 \int_{p_1}^{p_2} \frac{p}{\mu_g z} dp = 2 \int_{p_{sc}}^{p_2} \frac{p}{\mu_g z} dp - 2 \int_{p_{sc}}^{p_1} \frac{p}{\mu_g z} dp = m(p_2) - m(p_1)$$

=



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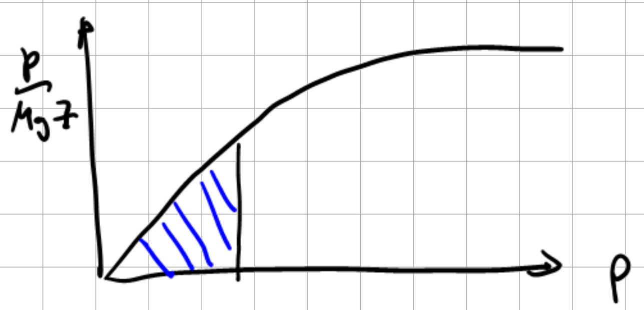
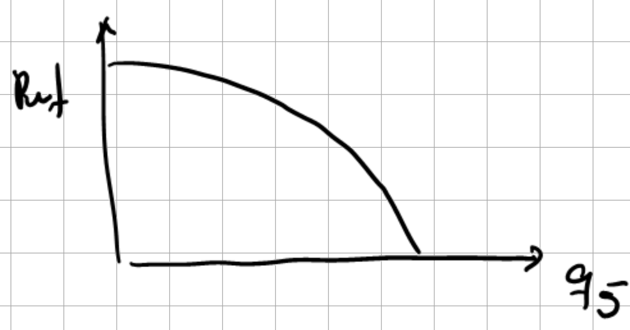


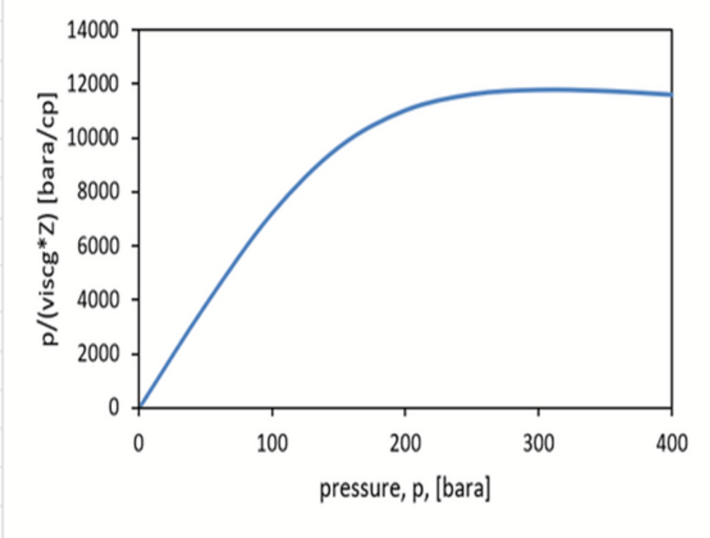
IPR og

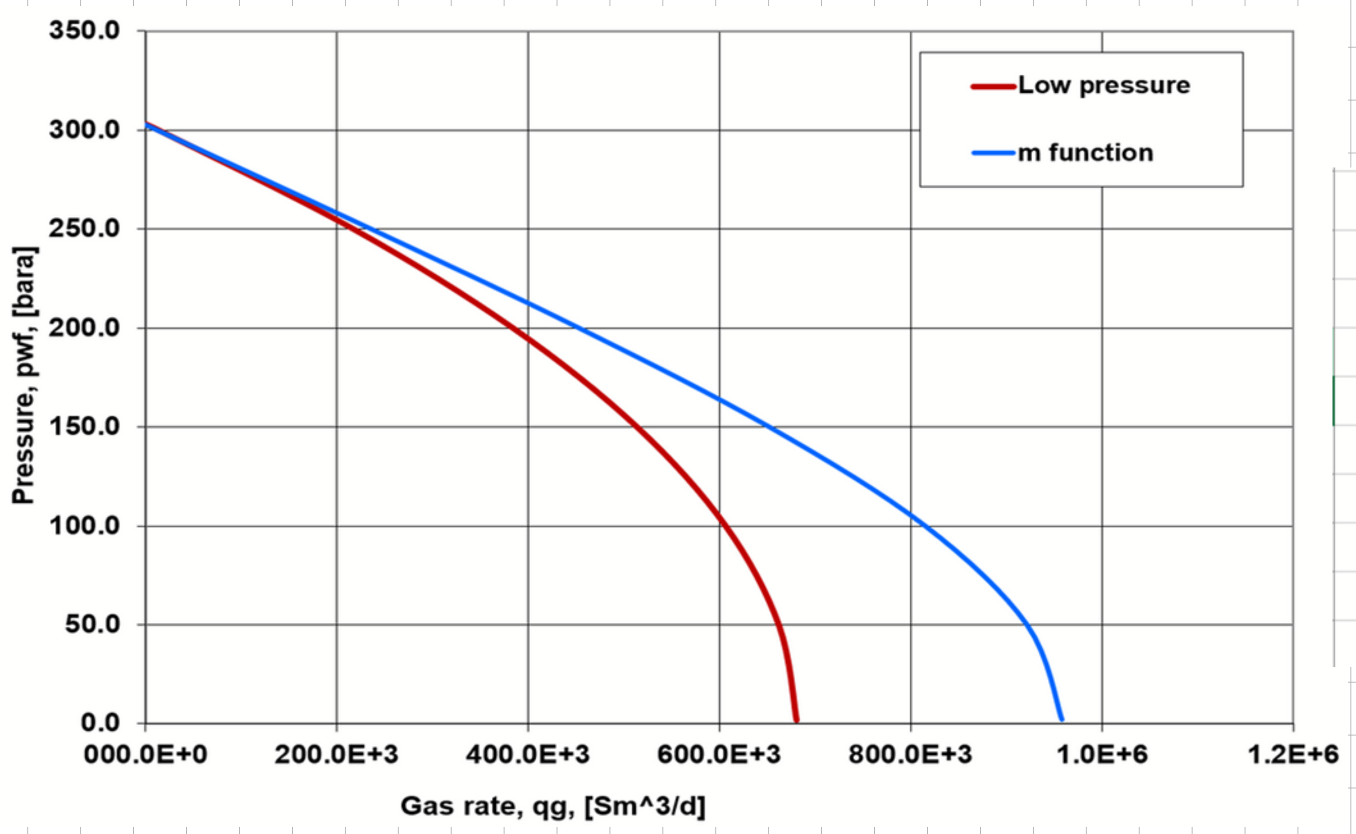
$$q_g = \frac{2.63\, kh}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right)} \frac{1}{T_R} \left[m(p_R) - m(p_{wf})\right] \quad ?$$



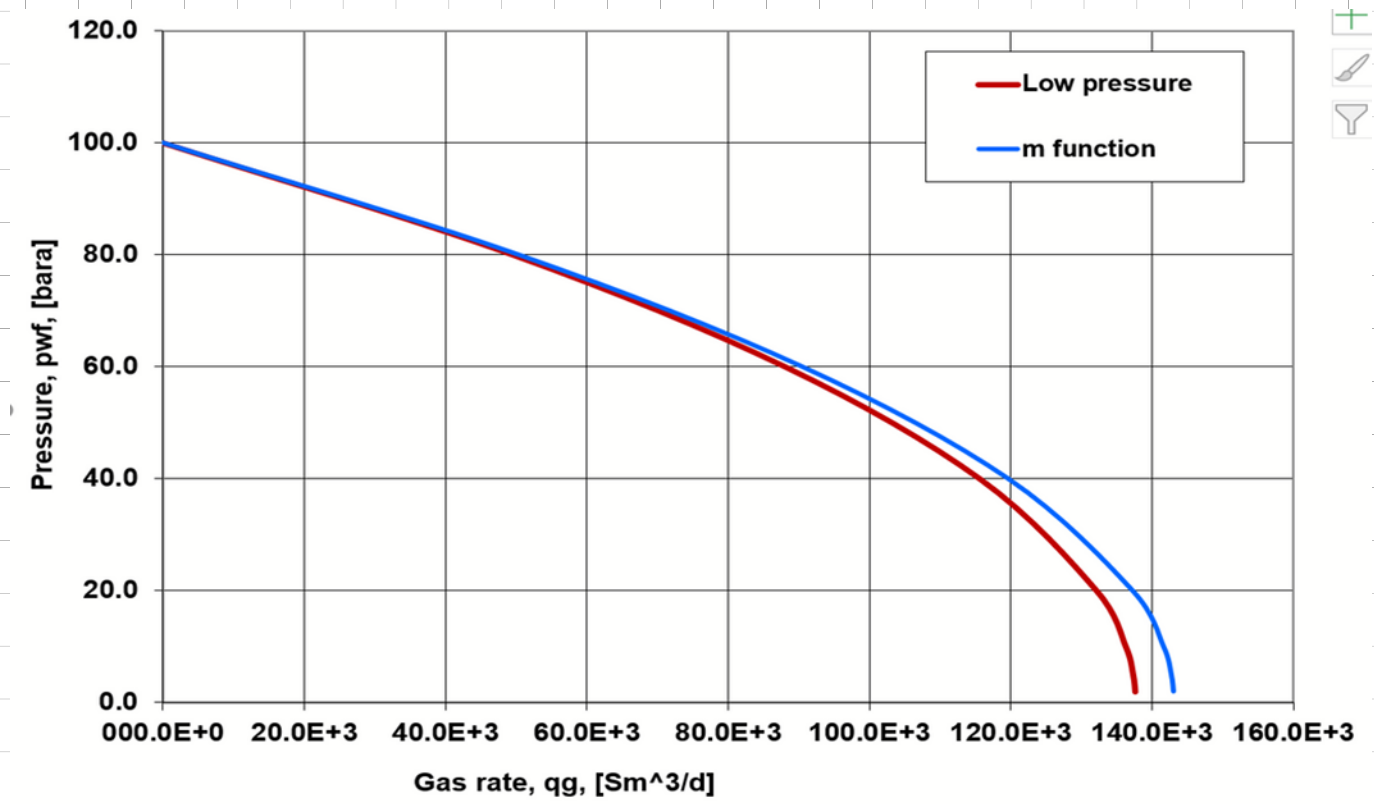
Well Data		
pR	[bara]	400
TR	[C]	105
TR	[K]	378
z_R	[-]	1.069
$deng_R$	[kg/m3]	276
$Viscosity_R$	[cp]	0.032

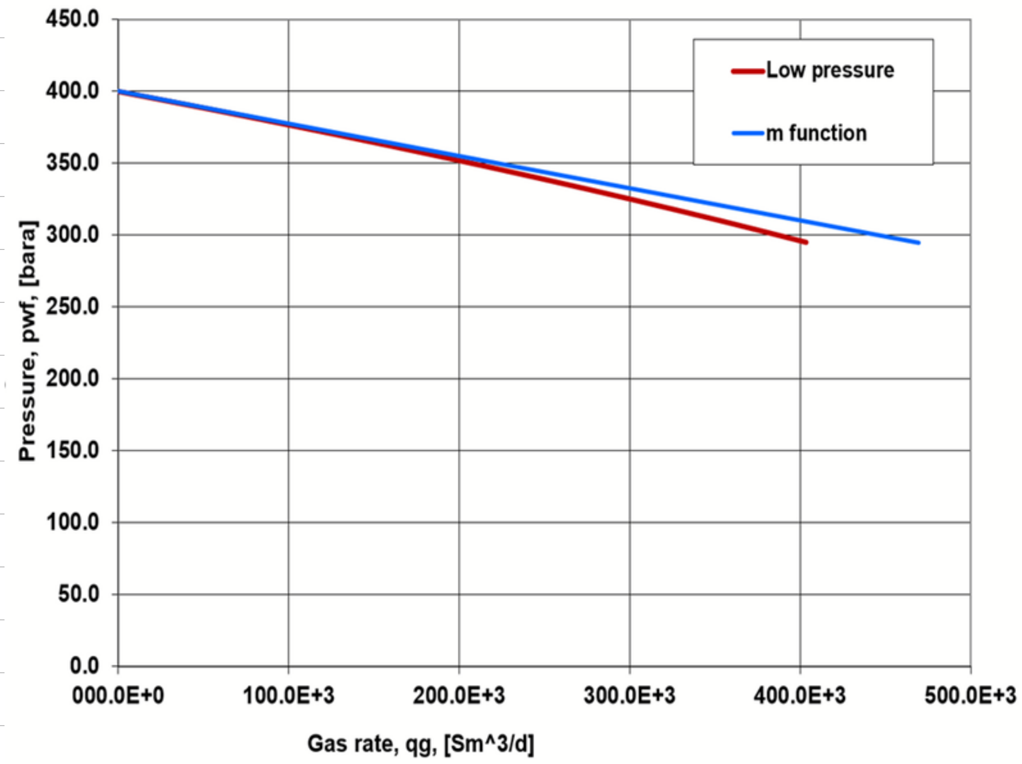
p [bara]	Z [-]	deng [kg/m3]	viscg [cp]	p/viscg*Z [bara/cp]
2	0.997	1.5	0.013	152
50	0.932	39.6	0.014	3825
100	0.882	83.6	0.016	7224
150	0.859	128.8	0.018	9662
200	0.867	170.2	0.021	11029
250	0.899	205.1	0.024	11622
300	0.948	233.6	0.027	11788
350	1.006	256.8	0.030	11745
400	1.069	276.1	0.032	11607





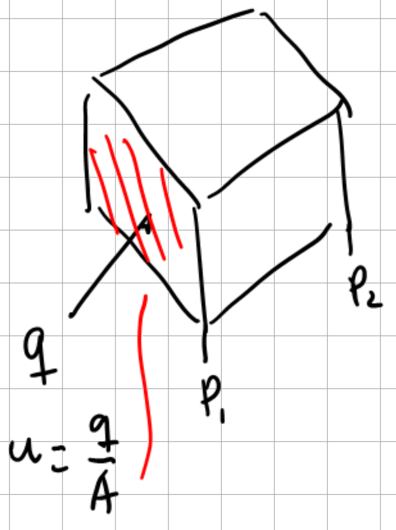
pwf bara	LP - qg [Sm3/d]	m(p) [bara2/cp]	qg [Sm3/d]
303.0	000.0E+0	5026555	000.0E+0
300.0	13.4E+3	4955852	13.5E+3
250.0	217.4E+3	3783006	237.1E+3
200.0	384.3E+3	2645766	453.9E+3
150.0	514.1E+3	1603483	652.6E+3
100.0	606.8E+3	750299.5	815.2E+3
50.0	662.5E+3	191885.9	921.7E+3
2.0	681.0E+3	226.2864	958.2E+3

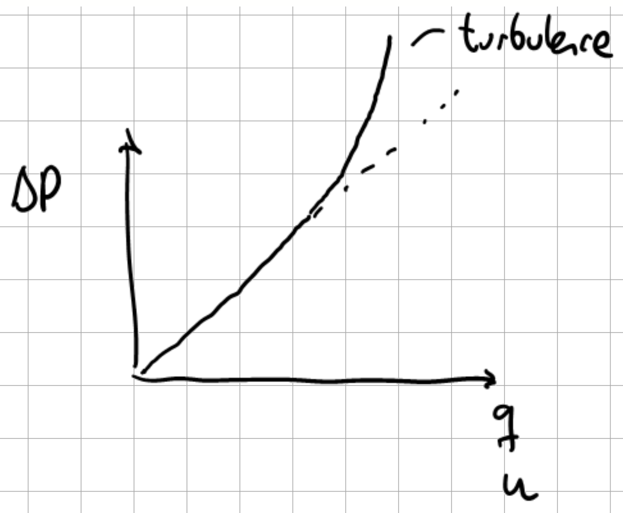


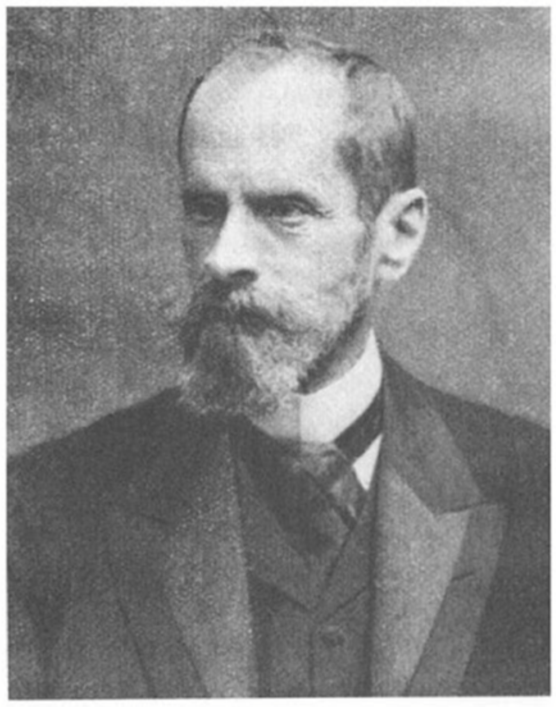


450.0
400.0
350.0
300.0
250.0
200.0
150.0
100.0
50.0
0.0
Pressure, pwf, [bara]
000.0E+0
100.0E+3
200.0E+3
300.0E+3
400.0E+3
500.0E+3
Gas rate, qg, [Sm^3/d]
Low pressure
m function

[19] P. Forchheimer. *Wasserbewegung durch Boden.* Zeitschrift des Vereines Deutscher Ingenieuer, 45 edition, 1901.

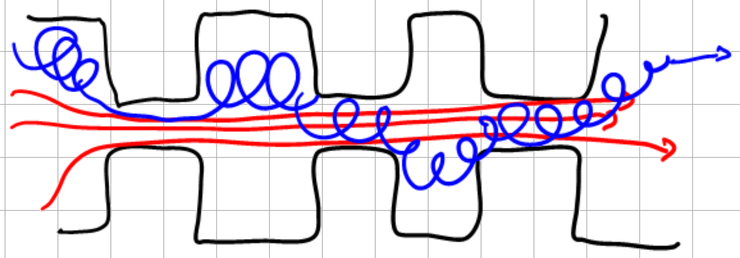






Professor Philipp Forchheimer.

not valid for medium-high rate gas wells
high rate sat oil



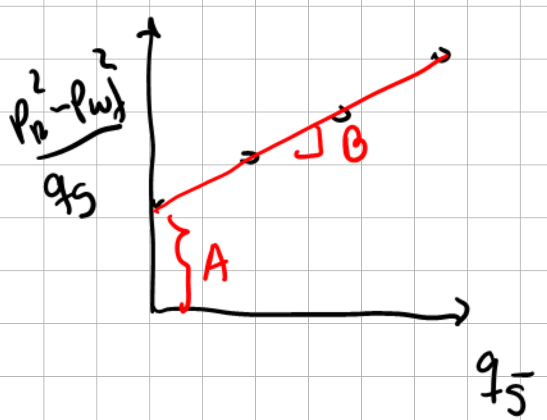
$$\Delta P = A \cdot u + B u^2$$

$$\Delta P = A u \left(1 + \frac{B}{A} u\right)$$

Use of Short Term Multiple Rate Flow Tests To Predict Performance of Wells Having Turbulence (1976)

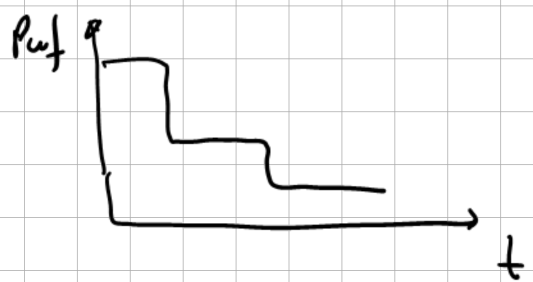
Lloyd G. Jones and E. M. Blount, Mobil Research and Development Corp., and O. H. Glaze, Mobil Oil Corp., Members SPE-AIME

LP dry gas wells



$$\left(p_R^2 - p_{wf}^2\right) = A\, q_{\bar{g}} + B\, q_{\bar{g}}^2$$

$$\frac{\left(p_R^2 - p_{wf}^2\right)}{q_{\bar{g}}} = A + B\, q_{\bar{g}}$$



quadratic equation

$$a x^2 + b x + c = 0$$

$$q_{\bar{g}} = \frac{7.63\, k h}{T_R M_g z} \frac{\left(p_R^2 - p_{wf}^2\right)}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right)}$$

$$\left(p_R^2 - p_{wf}^2\right) = \frac{T_R M_g z}{7.63\, k h} \left[\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right) q_{\bar{g}}\right] + B\, q_{\bar{g}}^2$$

rate dependent skin

$$B = D \frac{z M_g T_R}{7.63 \cdot k h}$$

$$\left(p_R^2 - p_{wf}^2\right) \frac{7.63\, k h}{T_R M_g z} = \left(\ln \frac{r_e}{r_w} - 0.75 + S\right) q_{\bar{g}} + D\, q_{\bar{g}}^2$$

$$\frac{(p_R^2 - p_{wf}^2)}{\left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + \underbrace{D\, q_g}_{\text{rate-dependent skin}}\right)} \frac{7.63\, kh}{T_R\, \mu_g\, z} = q_g$$

$$D = \left[\frac{d}{Sm^3}\right]$$

$$\hookrightarrow D = f(?)$$

alternative approach

USBM (1935) $\quad q_g = C(p_R^2 - p_{wf}^2)^n$

n ? $\nearrow$ 1 laminar (Darcy flow)

$\searrow$ 0.5 turbulent (HVF)

$$q_g = C\, \underline{(p_R^2 - p_{wf}^2)^{n}}$$ — accounts for turbulent flow

$$C = \frac{(7.63\; kh)^n}{(T\mu_g Z)^n D^{1-n}[\ln(r_e/r_w) - 0.75 + s]^{2n-1}}.$$

extends to other IPRs

$$q_g = C\left[m(p_R) - m(p_{wf})\right]^n$$

$$C = \frac{(7.63\quad kh)^n}{T^n D^{1-n}[\ln(r_e/r_w) - 0.75 + s]^{2n-1}}.$$

$$q_{\bar g} = \frac{2\pi k h}{\ln\left(\frac{r_e}{r_w}-0.75\right)} \frac{7.63}{T_R} \underbrace{\int_{p_{wf}}^{p_e} \frac{p}{z\,\mu_g}\,dp}$$

LP: $$\left(\frac{1}{\mu_g z}\right)_{@p_R} \frac{(p_R^2-p_{wf}^2)}{2}$$

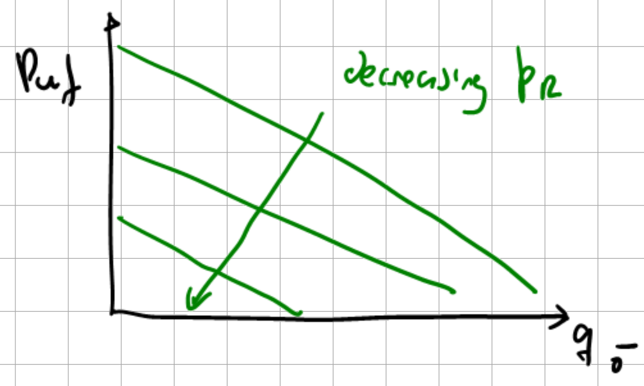
$$\frac{\left(m(p_e)-m(p_{wf})\right)}{2}$$

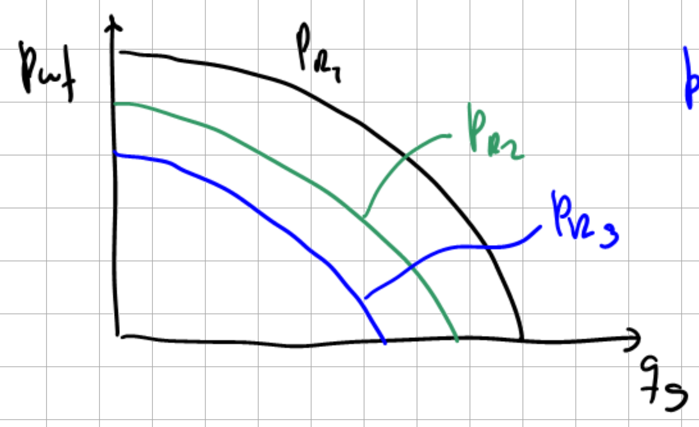
HP: $$\frac{p_R}{(\mu_g z)_{@p_R}}(p_R-p_{wf})$$

$$q_{\bar o} = \underset{\sim}{J}\,(p_R - p_{wf})$$

geometry →

fluid-properties → $(\mu_o B_o)_{@p_{av}}$



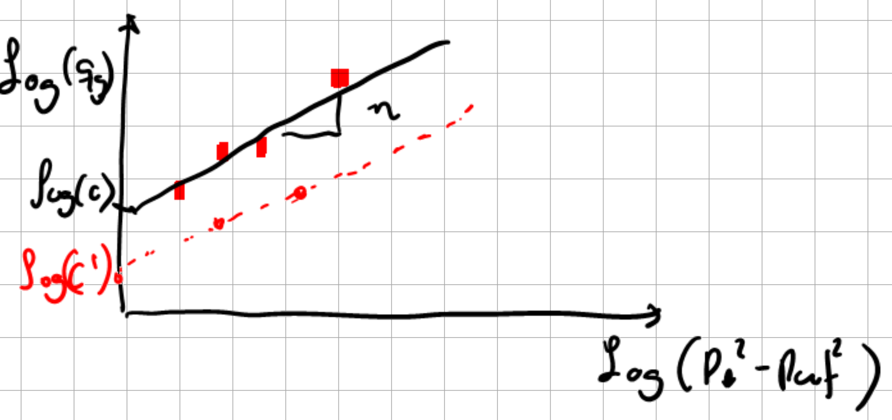


$p_{R_1} > p_{R_2} > p_{R_3}$

Diagnostic of well productivity issues

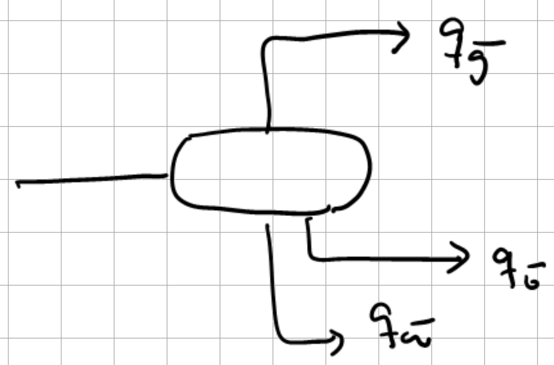
$$q_{\bar g} = C\,(p_R^2 - p_{wf}^2)^n$$

$$\log(q_{\bar g}) = \log(C) + n \log(p_R^2 - p_{wf}^2)$$

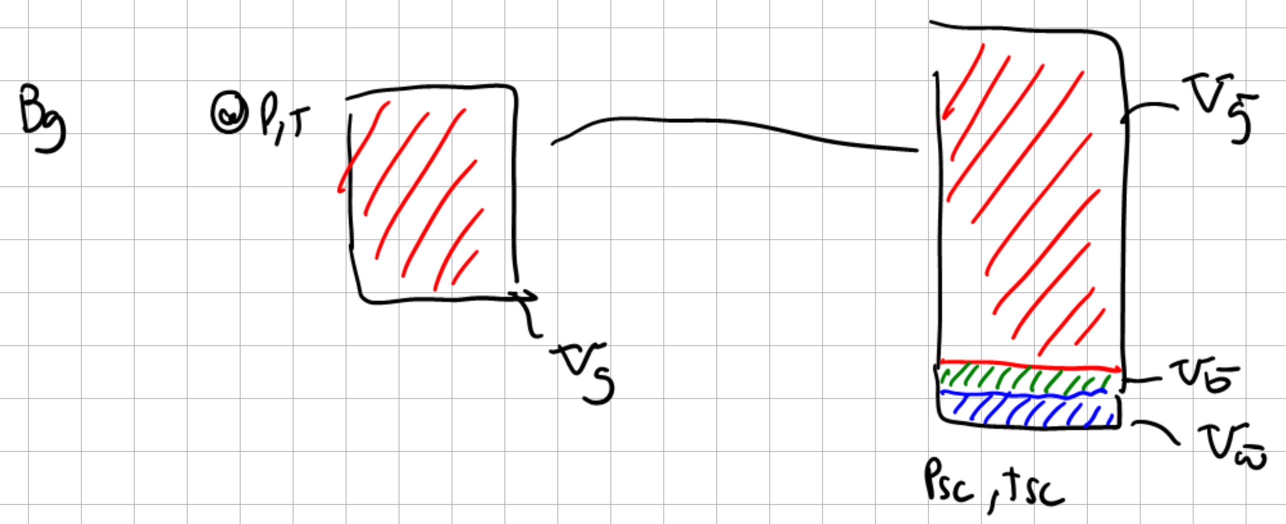


- Gas wells produce condensate $q_{\bar o}$, water $q_{\bar w}$

$$CGR = \frac{q_{\bar o}}{q_{\bar g}} \qquad WGR = \frac{q_{\bar w}}{q_{\bar g}}$$



- the same IPR equations can be used as long as flow $p_R \rightarrow p_{wf}$ is single phase



$$B_{gd} = \frac{V_g}{V_{\bar{g}}}$$

$$B_{gw} = \frac{V_g}{V_{\bar{g}} + V_{\bar{o}} + V_{\bar{w}}}$$

$\frac{p}{\rho} = z R T$ → assumes single-phase gas

$\int_{p_{wf}}^{p_R} \frac{p}{z \mu_g} dp$ → neglects the dropout of condensate / water

If there is dropout of condensate and water → generate a table of B_{gd} { e.g. EOS }

$\int_{p_{wf}}^{p_R} \frac{1}{\mu_g B_{gd}} dp$ ← integrate B_{gd} directly

---

Expressions are available for horizontal wells:

SPE 99712

Generalized Horizontal Well Inflow Relationships for Liquid, Gas, or Two-Phase Flow

R. Kamkom and D. Zhu, Texas A&M U.

base pressure. The IPR equation for horizontal gas wells in term of the real gas pseudo-pressure is

$$q_g = \frac{kL\left(m(\overline{p}) - m(p_{wf})\right)}{1424T\left(\ln\left[\frac{hI_{ani}}{r_w(I_{ani}+1)}\right] + \frac{\pi y_b}{hI_{ani}} - 1.224 + s\right)} \quad (13)$$

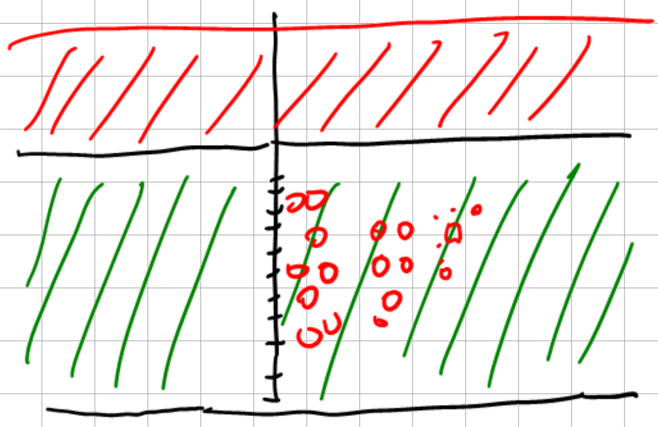
$$I_{ani} = \sqrt{\frac{k_H}{k_V}}$$

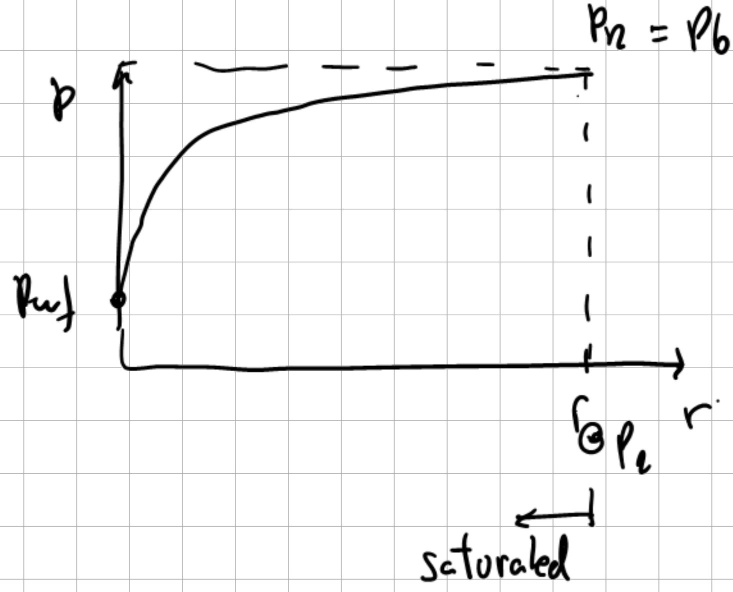
IPR:

$$q_g = \underbrace{C}_{\text{geometry}} \underbrace{\int_{p_{wf}}^{p_R} F(p)\, dp}_{\text{function of fluid properties}}$$

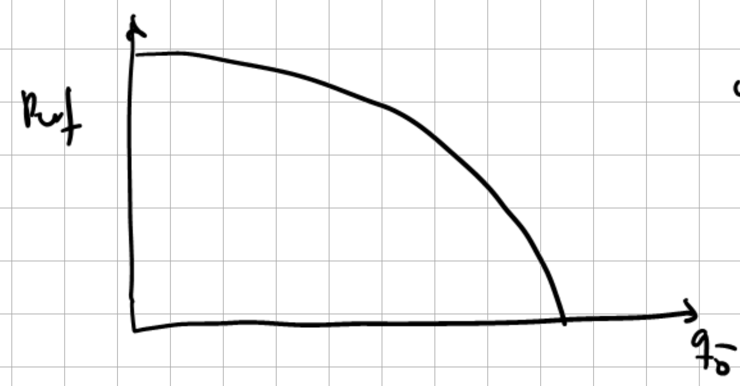
Saturated oil

$p_R = p_b$ @ T_R





solution-gas drive



what equation? $\sim q_o = C\left(p_R^2 - p_{wf}^2\right)$

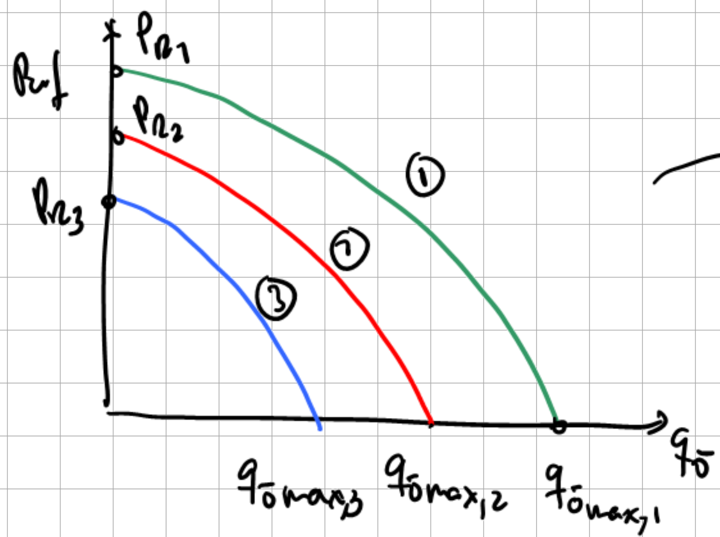
$q_o = J\left(p_R - p_{wf}\right)$

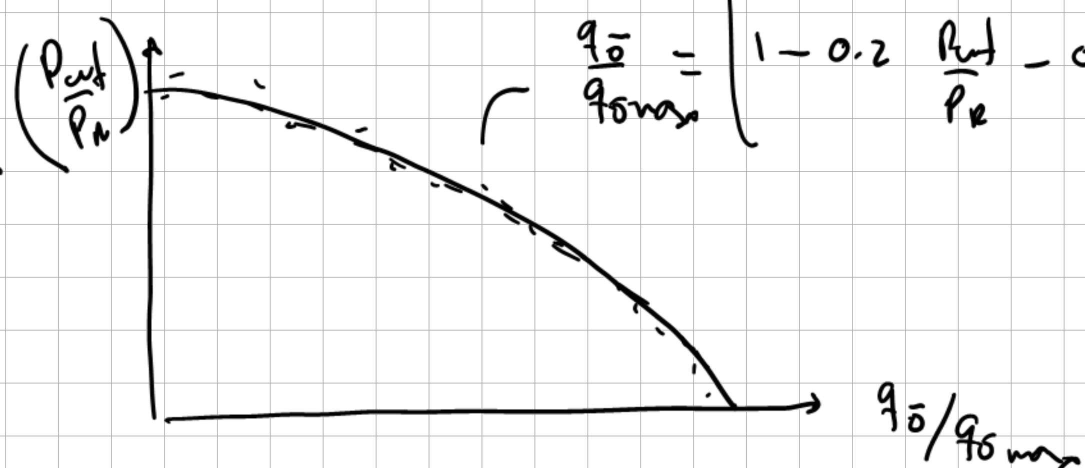
?

Reservoir simulation → 1968 Vogel

Inflow Performance Relationships for Solution-Gas Drive Wells

J. V. VOGEL
MEMBER AIME | SHELL OIL CO.
BAKERSFIELD, CALIF.





$$\frac{q_o}{q_{o max}} = \left[1 - 0.2\,\frac{p_{wf}}{p_R} - 0.8\left(\frac{p_{wf}}{p_R}\right)^2\right]$$

q_o	p_{wf}

→

$\frac{q_o}{q_{o max}}$	$\frac{p_{wf}}{p_R}$

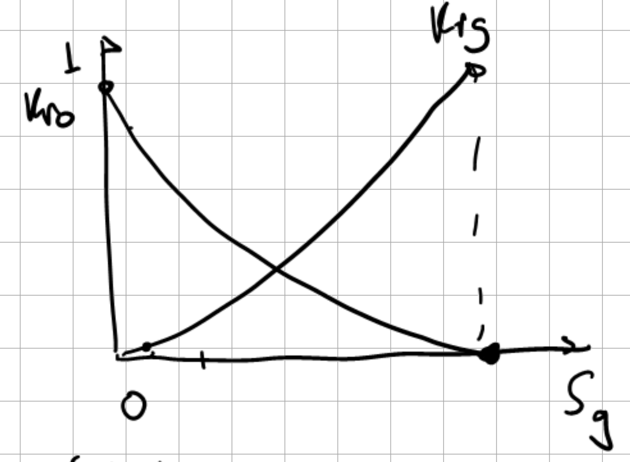
Analytical derivation

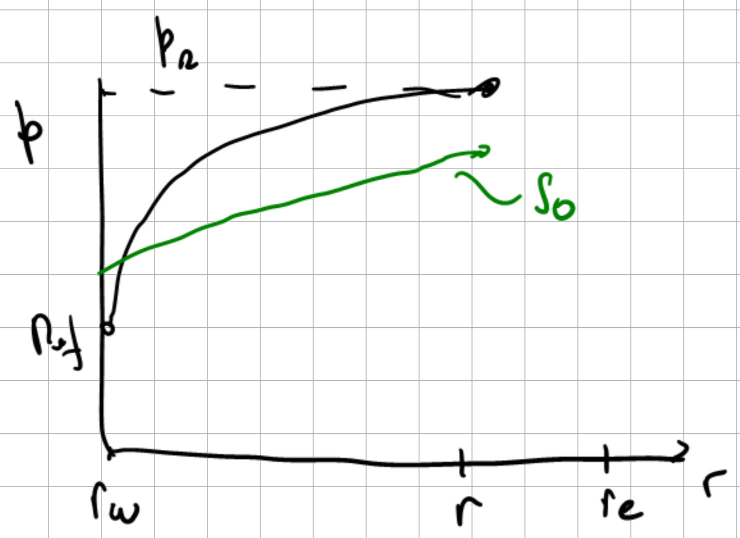
$$q_o = \frac{h\ k}{1868\left(\ln\frac{r_e}{r_w} - 0.75 + S\right)}$$

$$\int_{p_{wf}}^{p_R} \frac{k_{ro}}{\mu_o B_o}\, dp$$

effective permeability $(k \cdot k_{ro})$

$k_{ro} = f(S_o)$

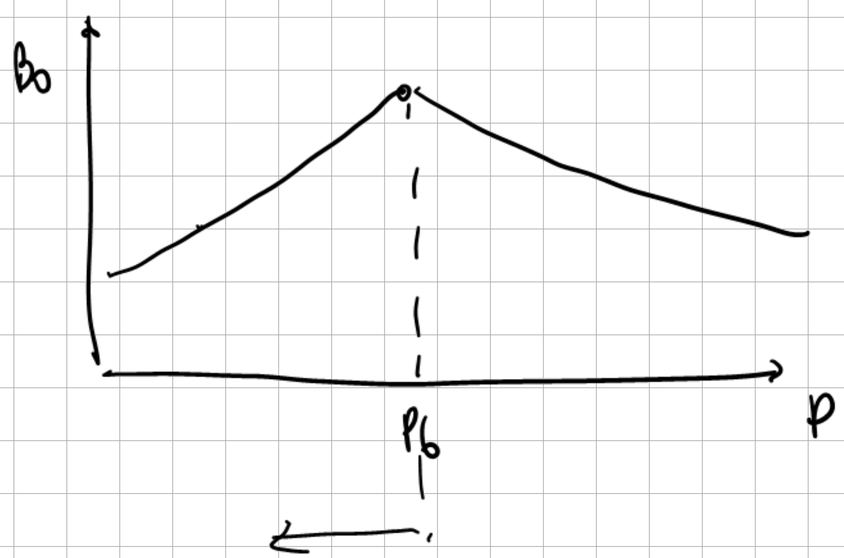


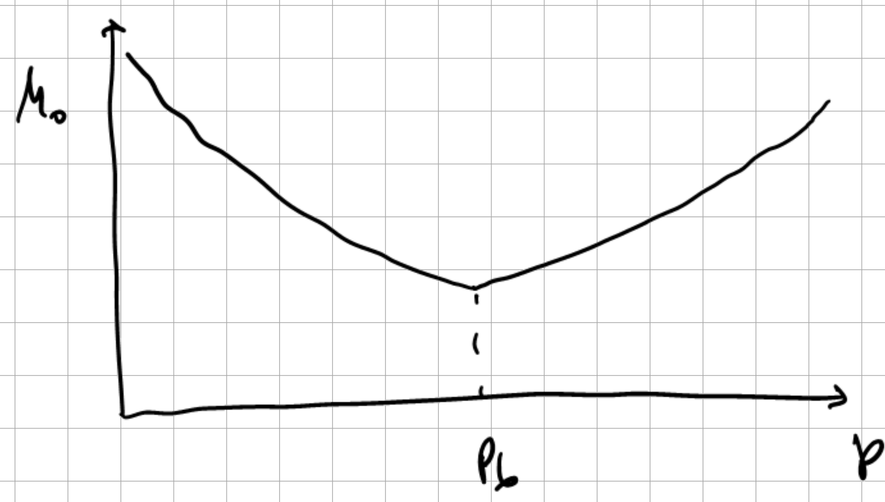


$S_o = f(p) \rightarrow k_{ro} = f(p)$

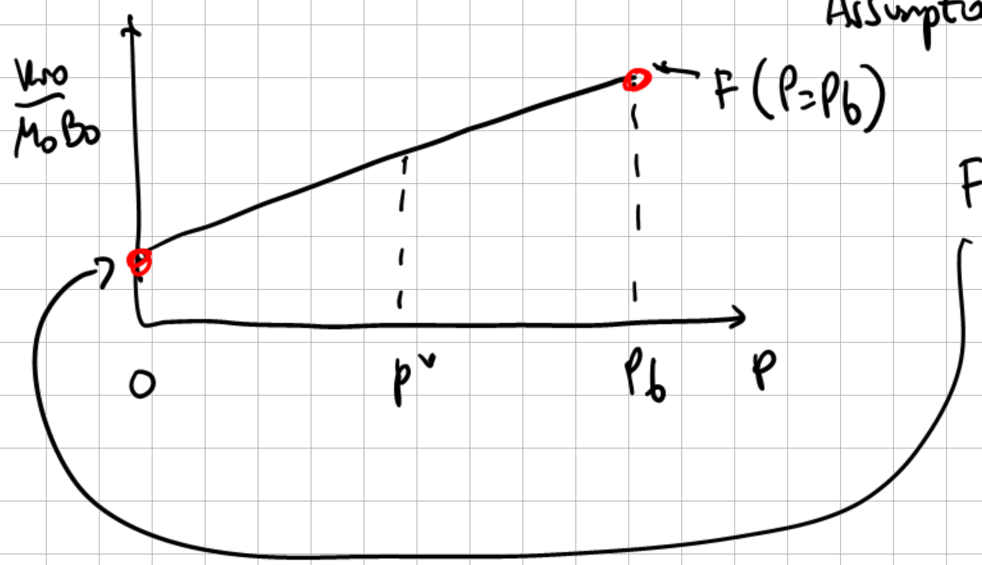
$$\int_{p_{wf}}^{p_R} \frac{k_{ro}}{\mu_o B_o}\, dp$$

$k_{ro} = f(S_o) = f(p)$





Assumption



$F(P=0)$

? $P = P_b$ single phase oil

$k_{ro} = 1$

$$\int_{p_{wf}}^{p_R} F(p)\, dp$$

$$F(p) = F(p=0) + \frac{F(p_R) - F(p=0)}{(p_R - 0)}(p^* - 0)$$

$$\int_{p_{wf}}^{p_R} (a + b\,x)\, dx$$

$$F(p) = F(p=0) + [F(p_R) - F(p=0)] \cdot \frac{p}{p_R} \qquad \text{EQ. 2-16}$$

Therefore, the solution of the pressure function integral will have a linear term in addition to the quadratic term:

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0) \cdot (p_R - p_{wf}) + [F(p_R) - F(p=0)] \cdot \frac{1}{p_R \cdot 2}(p_R{}^2 - p_{wf}{}^2) \qquad \text{EQ. 2-17}$$

Expanding terms:

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0) \cdot p_R - F(p=0) \cdot p_{wf} + [F(p_R) - F(p=0)] \cdot \frac{1}{p_R \cdot 2}(p_R{}^2 - p_{wf}{}^2) \qquad \text{EQ. 2-18}$$

$$\int_{p_{wf}}^{p_R} F(p)dp = F(p=0) \cdot p_R - F(p=0) \cdot p_{wf} + F(p_R) \cdot \frac{p_R}{2} - F(p_R) \cdot \frac{p_{wf}{}^2}{p_R \cdot 2} - F(p=0) \cdot \frac{p_R}{2} + F(p=0) \cdot \frac{p_{wf}{}^2}{p_R \cdot 2} \qquad \text{EQ. 2-19}$$

Grouping terms by pressure:

$$\int_{p_{wf}}^{p_R} F(p)dp = [F(p=0) + F(p_R)] \cdot \frac{p_R}{2} - F(p=0) \cdot p_{wf} - \frac{[F(p_R) - F(p=0)]}{2} \cdot \frac{p_{wf}{}^2}{p_R} \qquad \text{EQ. 2-20}$$

Dividing by $[F(p=0) + F(p_R)] \cdot \frac{p_R}{2}$

$$\frac{2}{[F(p=0) + F(p_R)] \cdot p_R} \cdot \int_{p_{wf}}^{p_R} F(p)dp = 1 - \frac{F(p=0) \cdot 2}{[F(p=0) + F(p_R)]} \cdot \frac{p_{wf}}{p_R} - \frac{[F(p_R) - F(p=0)]}{[F(p=0) + F(p_R)]} \cdot \left(\frac{p_{wf}}{p_R}\right)^2 \qquad \text{EQ. 2-21}$$

Defining a variable "V"

$$V = \frac{F(p=0) \cdot 2}{[F(p=0) + F(p_R)]} \qquad \text{EQ. 2-22}$$

Therefore:

$$1 - V = \frac{F(p_R) - F(p=0)}{[F(p=0) + F(p_R)]} \qquad \text{EQ. 2-23}$$

Substituting back in the integral of the pressure function:

$$\frac{2}{[F(p=0) + F(p_R)] \cdot p_R} \cdot \int_{p_{wf}}^{p_R} F(p)dp = 1 - V \cdot \frac{p_{wf}}{p_R} - (1 - V) \cdot \left(\frac{p_{wf}}{p_R}\right)^2 \qquad \text{EQ. 2-24}$$

Substituting Eq. 2-24 back in the IPR equation:

$$q_{\ddot{o}} = \frac{k \cdot h \cdot [F(p=0) + F(p_R)] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} \left[1 - V \cdot \frac{p_{wf}}{p_R} - (1 - V) \cdot \left(\frac{p_{wf}}{p_R}\right)^2\right] \qquad \text{EQ. 2-25}$$

Making $q_{\ddot{o},max}$:

$$q_{\ddot{o},max} = \frac{k \cdot h \cdot [F(p=0) + F(p_R)] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} \qquad \text{EQ. 2-26}$$

The following expression is obtained:

$$q_o = q_{o,max}\left[1 - V \cdot \frac{p_{wf}}{p_R} - (1 - V) \cdot \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

Vogel says $V = 0.2 \rightarrow$

Vogel found this same equation using data points generated with reservoir simulator, with V = 0.2.

Using Eq. 2-22, and assuming V = 0.2, $F(p=0)$ is then:

$$F(p=0) = \frac{F(p_R)}{9} \qquad \text{EQ. 2-28}$$

Eq. 2-26 can then be further simplified:

$$q_{o,max} = \frac{k \cdot h \cdot \left[\frac{10}{9} \cdot F(p_R)\right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o}\right)_{@p_R}\right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 1.8} = \frac{J}{1.8} \cdot pR \qquad \text{EQ. 2-29}$$

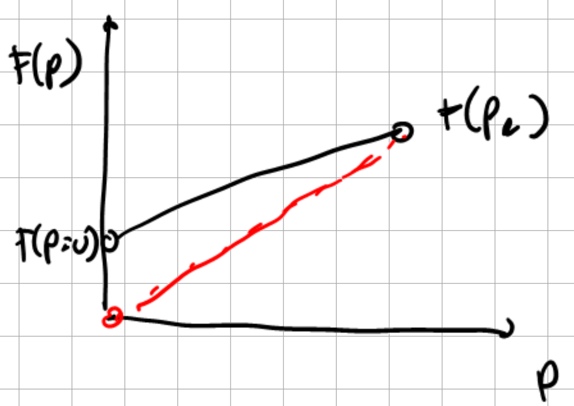
The Isochronal Testing of Oil Wells

By

M. J. Fetkovich, Member AIME, Phillips Petroleum Co.

1973

Fetkovich proposed



$V = 0$

$$\frac{q_o}{q_{o\,max}} = \left[1 - \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

$$q_{o,max} = \frac{k \cdot h \cdot [F(p_R)] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{k \cdot h \cdot \left[\left(\frac{k_{ro}}{\mu_o \cdot B_o}\right)_{@p_R}\right] \cdot p_R}{18.68 \cdot \left(ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right) \cdot 2} = \frac{J}{2} \cdot pR$$

$$q_o = \frac{J}{2} p_R \left(1 - \left(\frac{p_{wf}}{p_R}\right)^2\right]$$

$$q_o = \frac{J}{2} \left[p_R - \frac{(p_{wf})^2}{p_R}\right]$$

$$q_o = \frac{J}{2 p_R} \left[p_R^2 - p_{wf}^2\right] \rightsquigarrow q_g = C\left(p_R^2 - p_{wf}^2\right)^n$$