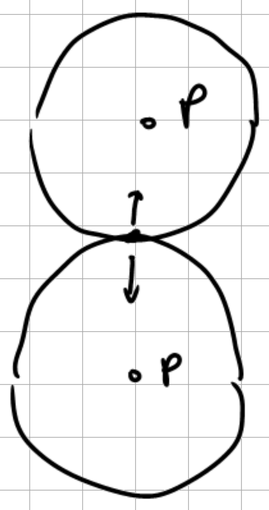
$$q_{\bar{o}} = \frac{kh}{(\mu_o B_o)_{av}\left(\underbrace{\ln\left(\frac{r_e}{r_w}\right)}_{8}-0.5\right)\cdot 18.68}(p_R - p_{wf})$$

pseudo steady-state pss @re → no-flow boundary



$$\int_{r_w}^{@p_e} \frac{dr}{r} = \frac{kh}{q_{\bar{o}}\cdot 18.68}\int_{p_{wf}}^{p_e}\frac{dp}{B_o\mu_o}$$

for pss r at which $p = p_R$ $r = 0.47\, r_e$

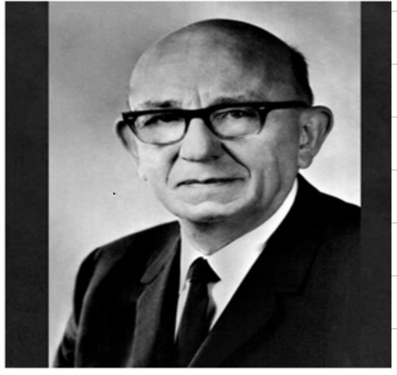
$$\ln\left(\frac{0.47\, r_e}{r_w}\right) = \ln\left(\frac{r_e}{r_w}\right) - 0.75$$

$$J = \frac{kh}{(\mu_o B_o)_{av}\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right] 18.68}$$

Skin factor

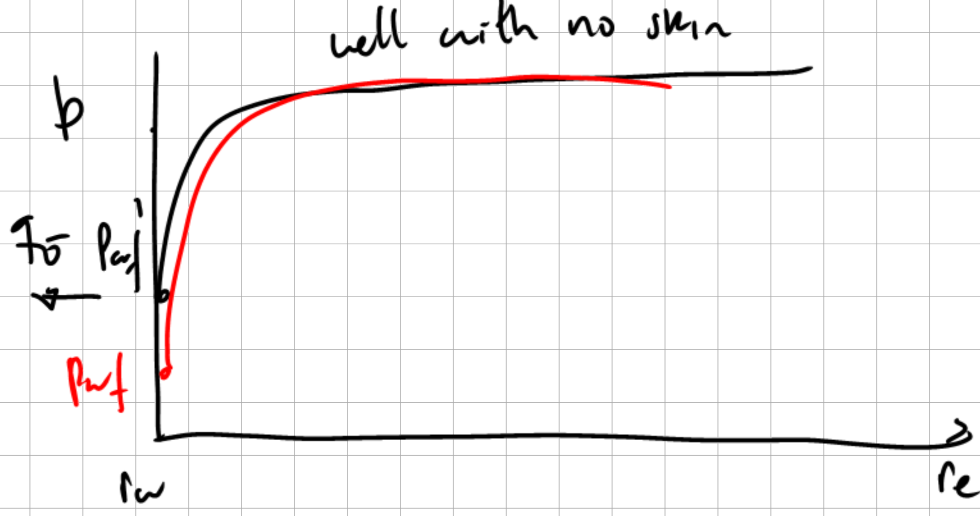
1953 → Hurst

↳ Antonius Van Everdingen (Dutch mining engineer) shell



THE SKIN EFFECT AND ITS INFLUENCE ON THE PRODUCTIVE CAPACITY OF A WELL

A. F. VAN EVERDINGEN, SHELL OIL CO., HOUSTON, TEXAS, MEMBER AIME



$$\left(\underbrace{p_{wf}'}_{\text{ideal}} - \underbrace{p_{wf}}_{\text{actual}}\right) = \Delta p_{skin} = S\underbrace{\left[\frac{18.68\,(\mu_o B_o)_{av}\cdot q_{\bar{o}}}{kh}\right]}_{\text{bar}}$$

ideal ↓ no skin

actual ~ with skin

$$p_R - \cancel{p_{wf}'} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{k.h}\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]$$

$$\cancel{p_{wf}'} - p_{wf} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{kh} S$$

$$p_R - p_{wf} = \frac{q_o \, 18.68 \, (\mu_o B_o)_{av}}{kh}\left[\ln \frac{r_e}{r_w} - 0.75 + S\right]$$

$$J = \frac{kh}{18.68 \, (\mu_o B_o)_{av}\left[\underbrace{\ln \frac{r_e}{r_w}}_{\text{convergence}} \underbrace{- 0.75}_{\text{boundary}} \underbrace{+ S}_{\text{skin (near wellbore non-idealities)}}\right]}$$

8

other definitions "equivalent" to skin

- flow efficiency $$E_F = \frac{p_R - p_{wf}'}{p_R - p_{wf}} = \frac{\cancel{q_o}/J'}{\cancel{q_o}/J} = \frac{\ln\left(\frac{r_e}{r_w}\right) - 0.75}{\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S} \approx \frac{7}{7 + S}$$

- apparent wellbore radius

$$\ln\left(\frac{r_e}{r_w}\right) + S = \ln\left(\frac{r_e}{r_{wa}}\right) \qquad r_{wa} = r_w e^{-S}$$

If $S > 0$ ⇝ damage
(less than ideal)
$(p_{wf}' - p_{wf}) > 0$

if $S = 0$ ⇝ ideal

if $S < 0$ ⇝ "stimulation" → productivity more than ideal

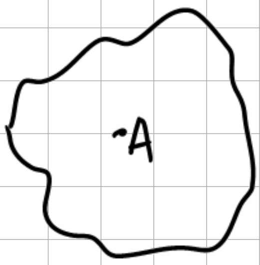
- Drainage area shape → 1965 Daniel Dietz

Determination of Average Reservoir Pressure From Build-Up Surveys

D. N. DIETZ
MEMBER AIME

KONINKLIJKE/SHELL LABORATORIUM
RIJSWIJK, THE NETHERLANDS

$$\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + \underbrace{S_A}_{\text{shape-area skin}}$$



$A = \pi \cdot r_e^2$ $r_e = \sqrt{\frac{A}{\pi}}$

← well performance (2nd edition)
Golan, Whitson

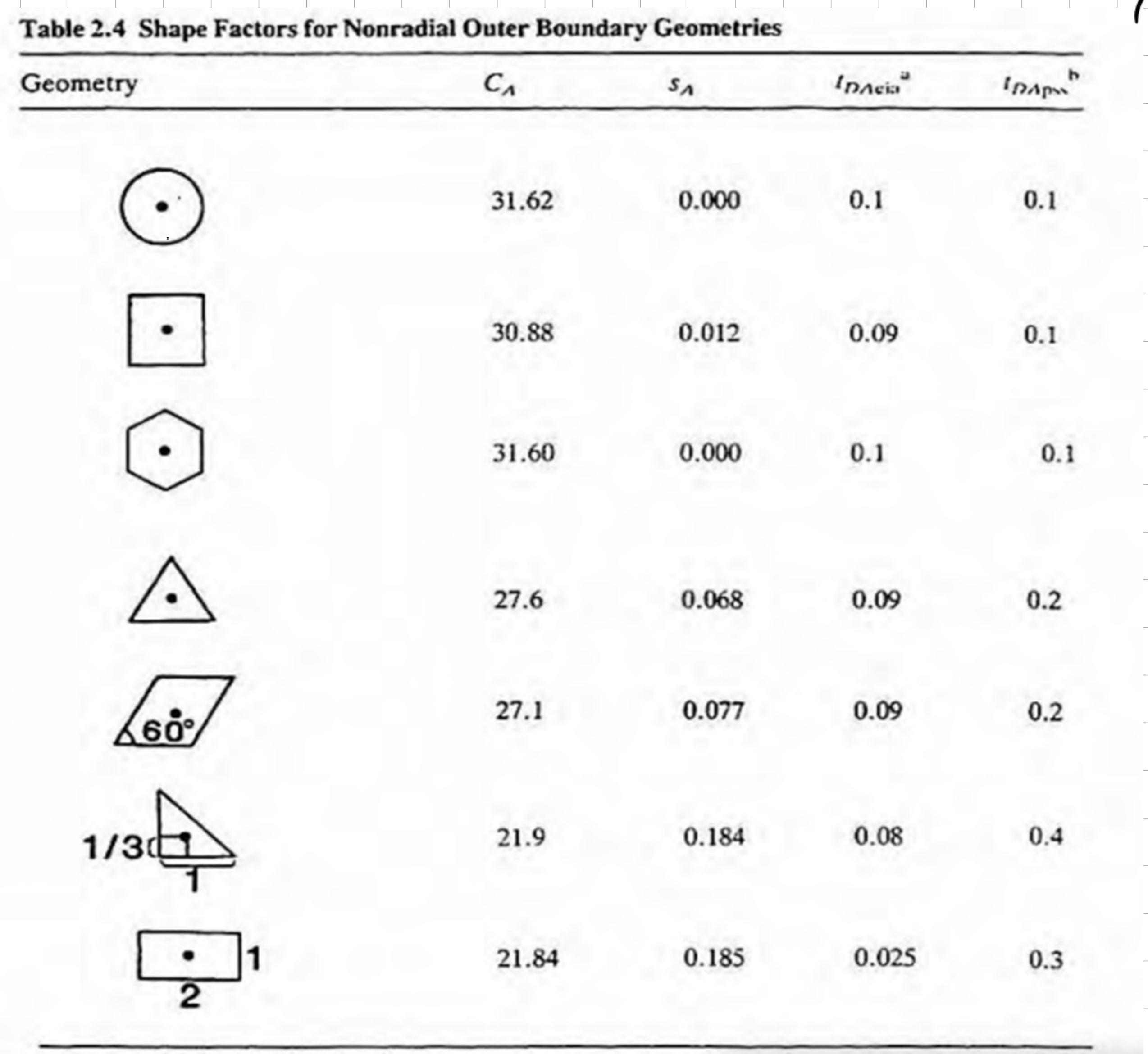
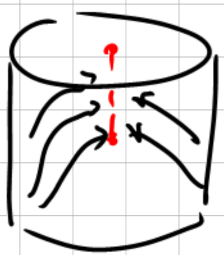


Table 2.4 Shape Factors for Nonradial Outer Boundary Geometries

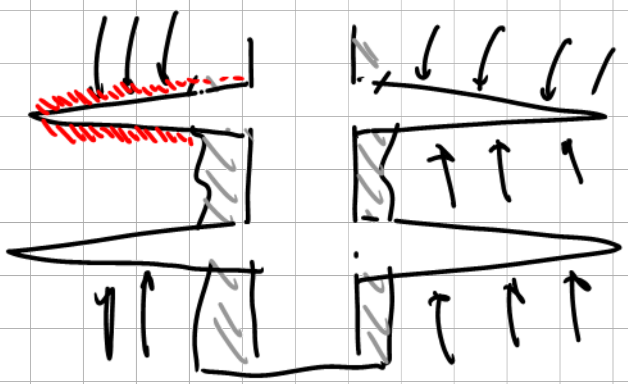
Geometry	C_A	s_A	t_{DAeia}[a]	t_{DApss}[b]
	31.62	0.000	0.1	0.1
	30.88	0.012	0.09	0.1
	31.60	0.000	0.1	0.1
	27.6	0.068	0.09	0.2
60°	27.1	0.077	0.09	0.2
1/3 1 1	21.9	0.184	0.08	0.4
1 2	21.84	0.185	0.025	0.3

- other cases than can be modeled (represented) with skin
- formation damage (due to drilling) $S > 0$
- partial penetration



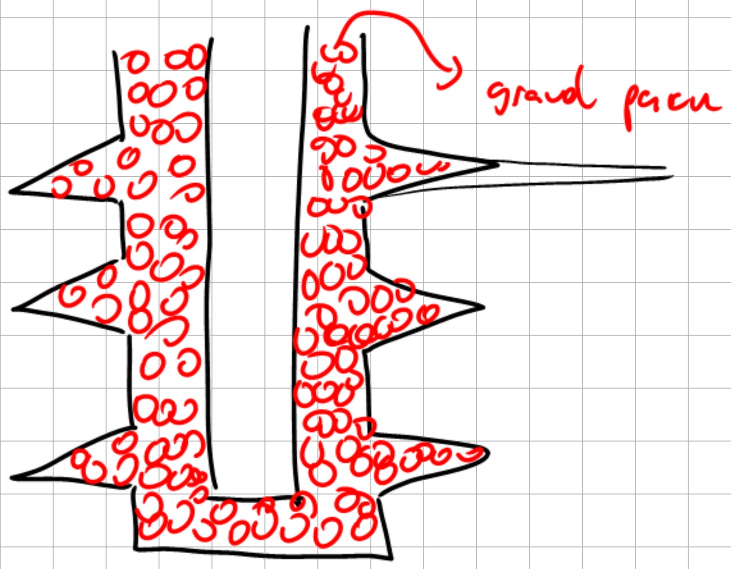
$k_v << k_H$ $S > 0$

- perforation



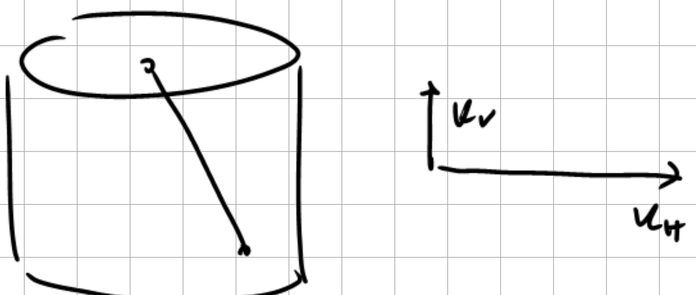
$S>0$

- Sand control (gravel pack)



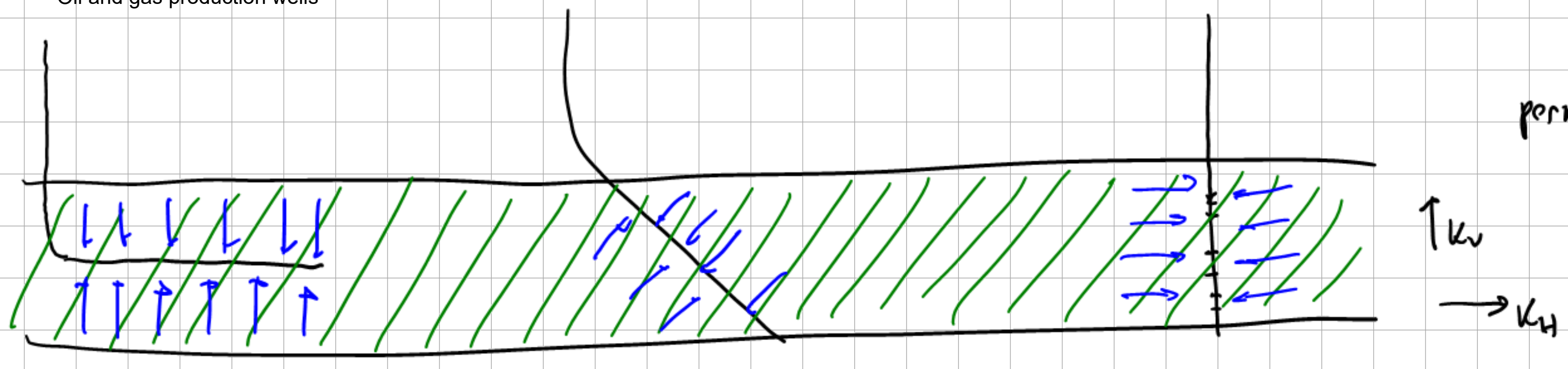
$S>0$

- wellbore inclination
  (permeability anisotropy)



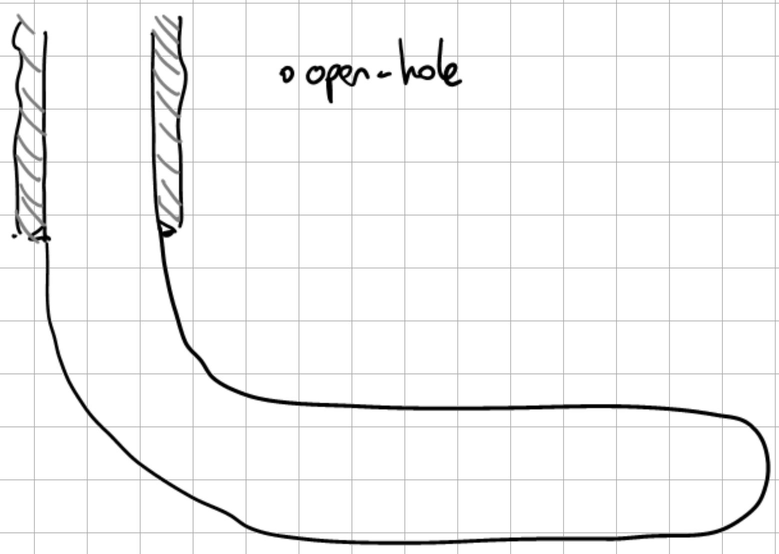
- acidizing (stimulation)
- fracking

$\}\ S<0$

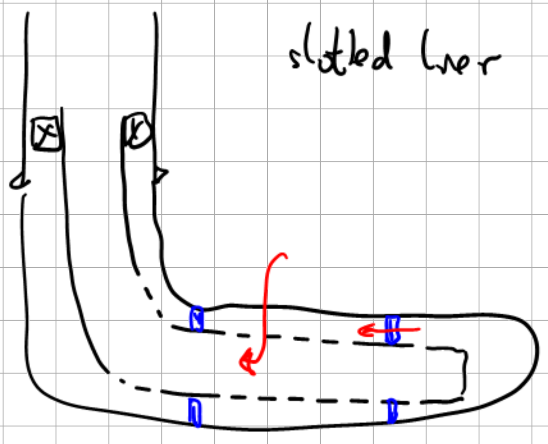


permeability anisotropy

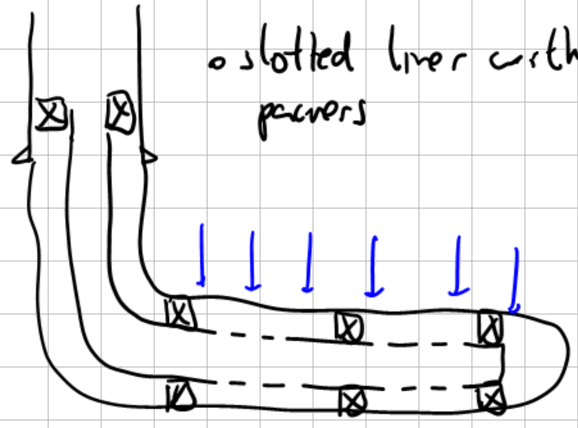
$\frac{k_H}{k_v} >> 1$ e.g. 10



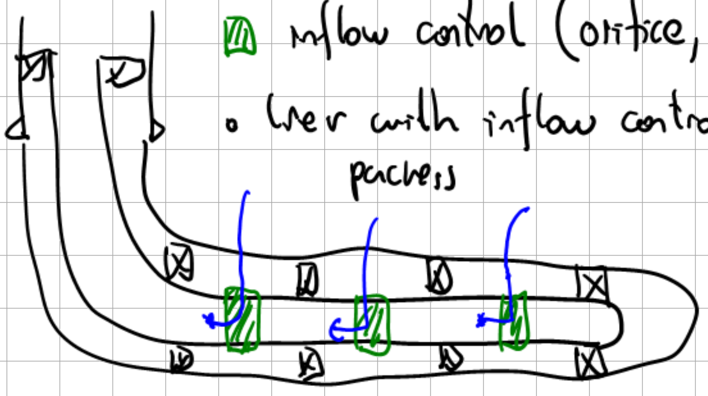
∘ open-hole



slotted liner



∘ slotted liner with packers



▨ inflow control (orifice, control valve)

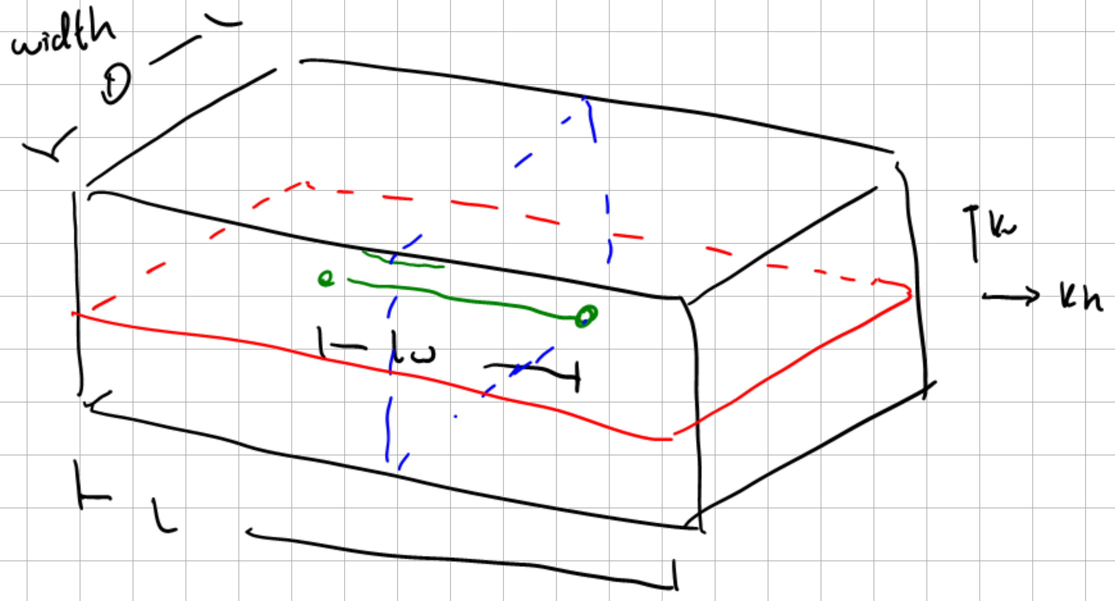
∘ liner with inflow control and packers

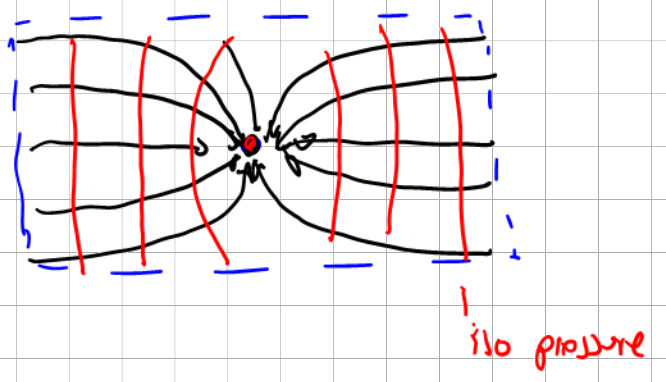
cemented and perforated liner

Morris

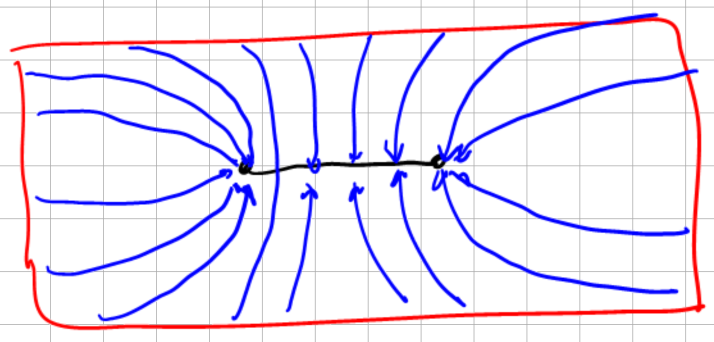
1937 - Muskat (1937) (Gulf oil)
1958 - Merkulov
1964 - Borisov
1974 - Gringarten (Imperial College), Ramey (Stanford)
1983 - Giger (IFP)
1988 - Joshi
1989 - Babuh, Odeh (Mobil)
1991 - Renard, Dupuy (IFP)
1994 - Buttler
Harald Asheim (NTNU)

http://www.ipt.ntnu.no/~asheim/prodbr.html





$P = f(x)$

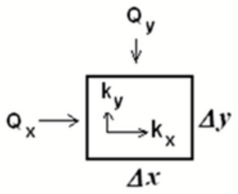


$\sim P \approx f(x)$

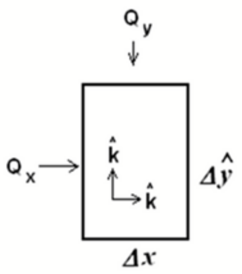
Superposition

Strategy to deal with permeability anisotropy

- Actual reservoir element



- Homogenous equivalent:



Permeability scaling: $\hat{k} = \sqrt{k_x k_y}$

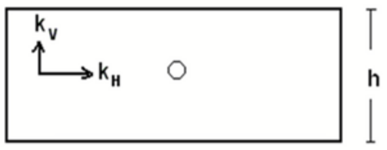
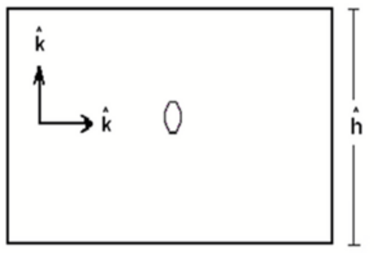


Figure 3.10 Anisotropic reservoir

$$k_H \frac{\partial^2 p}{\partial x^2} + k_V \frac{\partial^2 p}{\partial y^2} = \varphi \mu c \frac{\partial p}{\partial t}$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial \left(\sqrt{k_H / k_V}\, y\right)^2} = \frac{\varphi \mu c}{k_H} \frac{\partial p}{\partial t}$$



$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial \hat{y}^2} = 0$$

$$\hat{y} = \left(k_H / k_V\right)^{0.5} y$$

Height scaling: $\hat{h} = \left(k_H / k_V\right)^{0.5} h$

geometry related

$$q_{\dot{o}} = \frac{k_H \cdot h}{6.22 \cdot (\mu_o \cdot B_o)_{@p_{av}} \cdot \left[\frac{\pi \cdot D \cdot \hat{f}_a}{2 \cdot \bar{L}_w} + \frac{3 \cdot h \cdot \beta}{\hat{L}_w} \cdot \left(ln\left(\frac{h \cdot \beta}{2 \cdot \pi \cdot \hat{r}_w}\right) + \dot{s}_b\right)\right]} [p_R - p_{wf}]$$

function of PVT, fluid properties

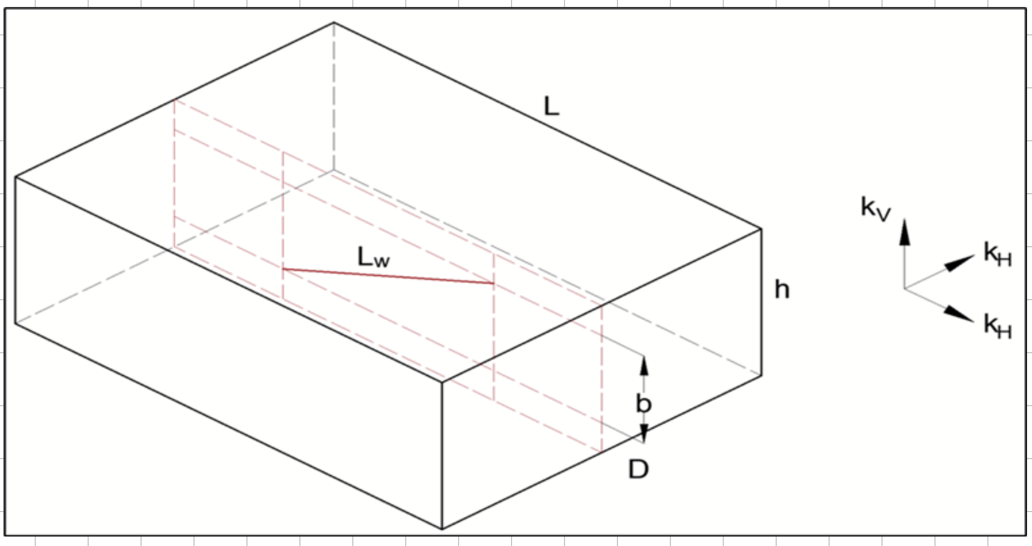
Harald Asheim's IPR equation

- undersaturated oil
- horizontal well
- pss
- scaling due to anisotropy

$$\beta = \sqrt{\frac{k_H}{k_V}}$$

$$\hat{L}_w = L_w \cdot \sqrt{1 - \left(\frac{b}{L_w}\right)^2 \cdot (\beta^2 - 1)}$$

$$\bar{L}_w = L_w \cdot \sqrt{1 - \left(\frac{b}{L_w}\right)^2}$$



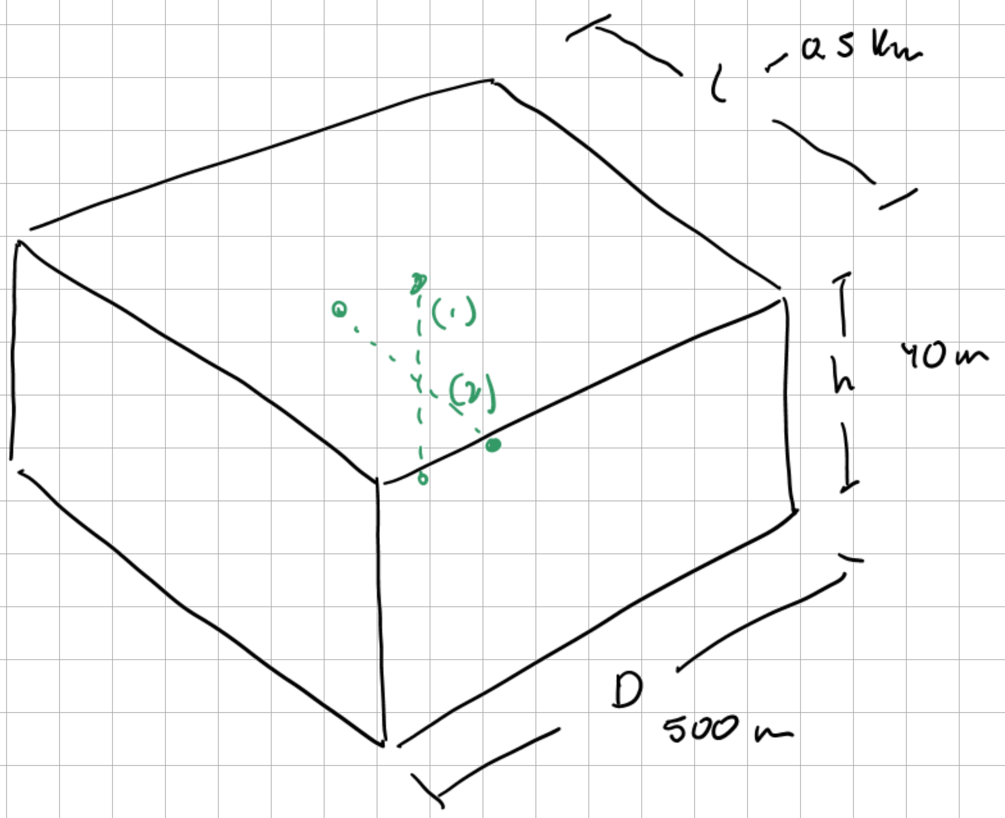
$$\hat{r}_w = r_w \cdot \frac{1 + \sqrt{1 + (\beta^2 - 1) \cdot \left(\frac{\bar{L}_w}{L_w}\right)^2}}{2}$$

well completed partially in the interval

$$\hat{f}_a = \frac{\bar{L}_w}{L} \cdot \left[1 + \left(0.53 \cdot \left(\frac{L}{D}\right)^2 + 1.15 \cdot \left(\frac{L}{D}\right) + 0.164\right) \cdot \left(\frac{1 - \left(\frac{\bar{L}_w}{L}\right)}{0.45 + \left(\frac{\bar{L}_w}{L}\right)}\right)\right]$$

$$\bar{s}_b = \left\{\begin{matrix} 0.69 \ if \ O(b) = O(h) \\ 0 \ if \ b = 0 \end{matrix}\right\}$$

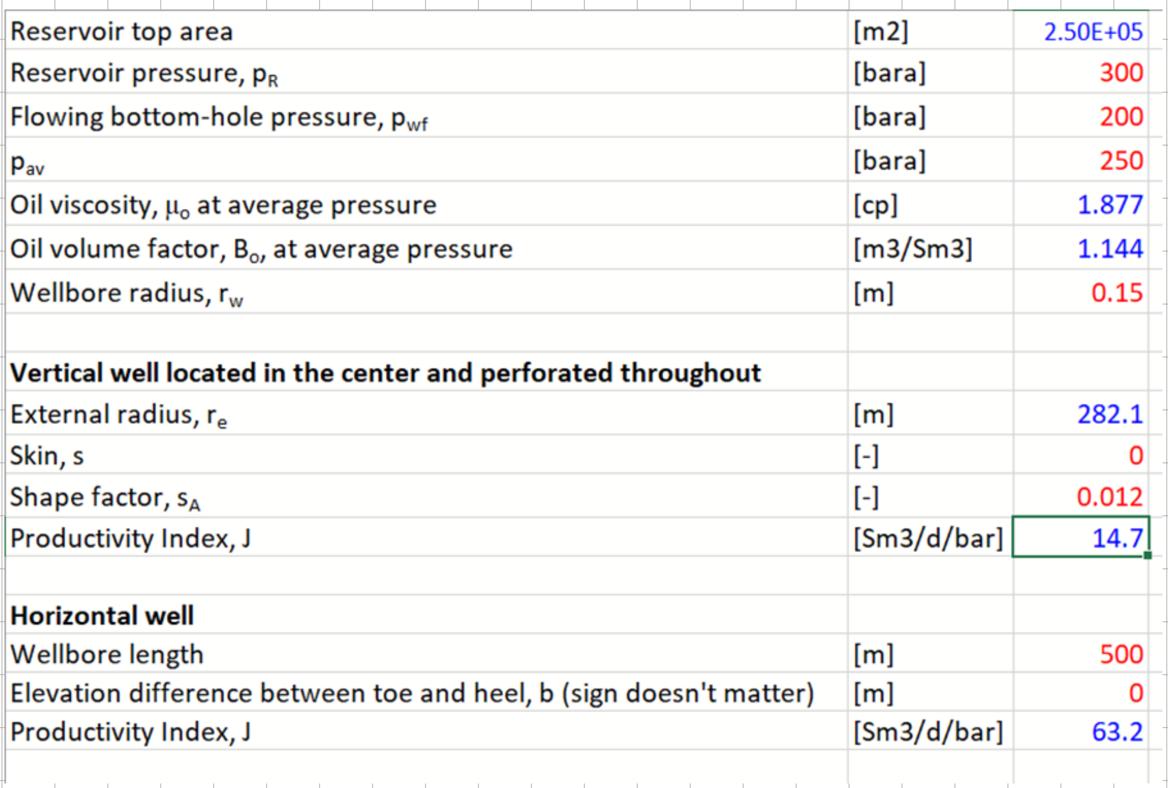
skin factor, "shielding" effect from top and bottom walls



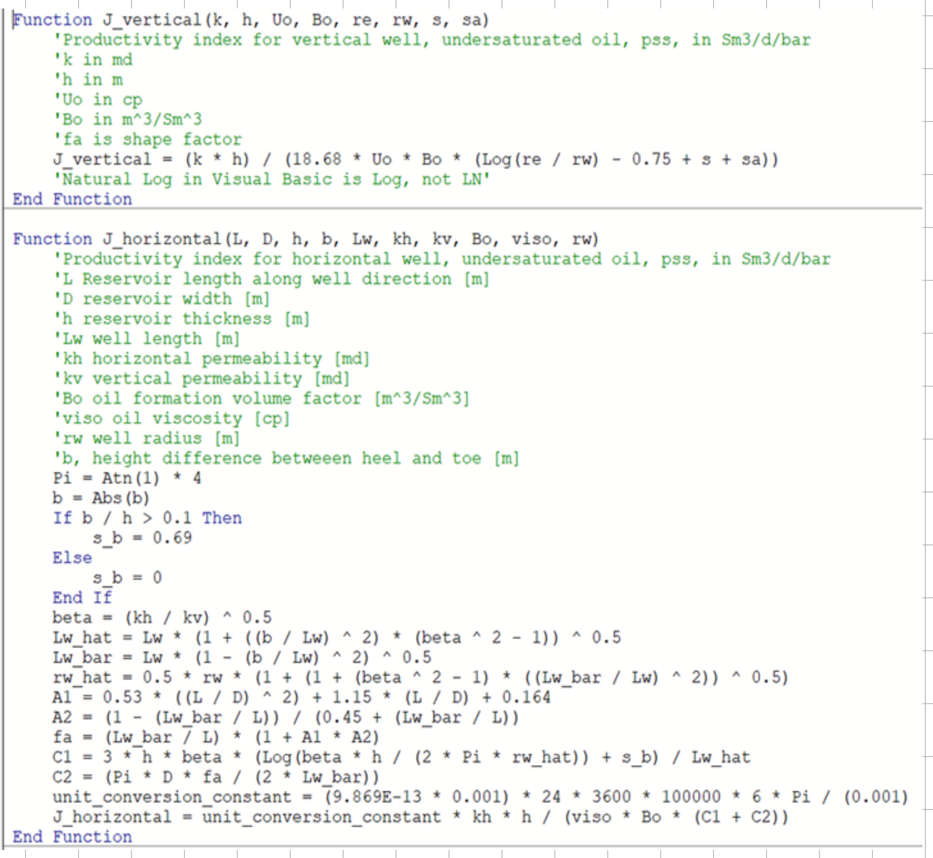
vertical well

$$\pi r_e^2 = A = 500.500$$

$$r_e = \sqrt{\frac{500.500}{\pi}}$$



Reservoir top area	[m2]	2.50E+05
Reservoir pressure, p_R	[bara]	300
Flowing bottom-hole pressure, p_{wf}	[bara]	200
p_{av}	[bara]	250
Oil viscosity, μ_o at average pressure	[cp]	1.877
Oil volume factor, B_o, at average pressure	[m3/Sm3]	1.144
Wellbore radius, r_w	[m]	0.15
Vertical well located in the center and perforated throughout		
External radius, r_e	[m]	282.1
Skin, s	[-]	0
Shape factor, s_A	[-]	0.012
Productivity Index, J	[Sm3/d/bar]	14.7
Horizontal well		
Wellbore length	[m]	500
Elevation difference between toe and heel, b (sign doesn't matter)	[m]	0
Productivity Index, J	[Sm3/d/bar]	63.2



```
Function J_vertical(k, h, Uo, Bo, re, rw, s, sa)
    'Productivity index for vertical well, undersaturated oil, pss, in Sm3/d/bar
    'k in md
    'h in m
    'Uo in cp
    'Bo in m^3/Sm^3
    'fa is shape factor
    J_vertical = (k * h) / (18.68 * Uo * Bo * (Log(re / rw) - 0.75 + s + sa))
    'Natural Log in Visual Basic is Log, not LN'
End Function
```

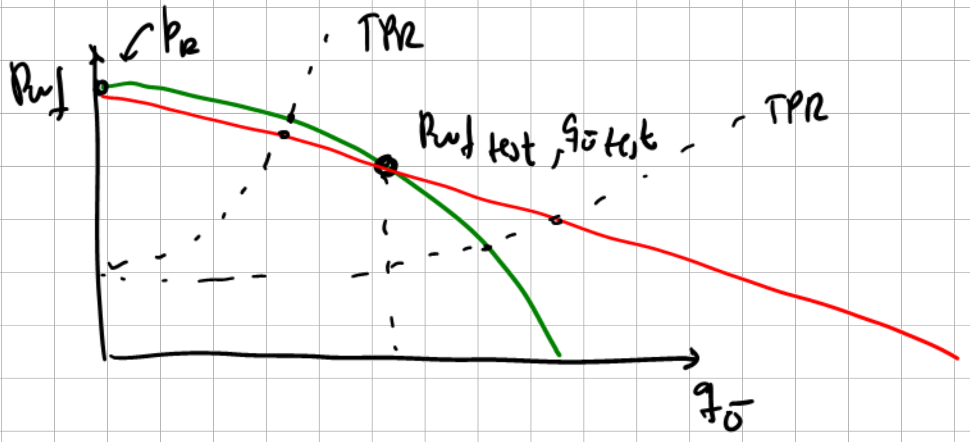
```
Function J_horizontal(L, D, h, b, Lw, kh, kv, Bo, viso, rw)
    'Productivity index for horizontal well, undersaturated oil, pss, in Sm3/d/bar
    'L Reservoir length along well direction [m]
    'D reservoir width [m]
    'h reservoir thickness [m]
    'Lw well length [m]
    'kh horizontal permeability [md]
    'kv vertical permeability [md]
    'Bo oil formation volume factor [m^3/Sm^3]
    'viso oil viscosity [cp]
    'rw well radius [m]
    'b, height difference betweeen heel and toe [m]
    Pi = Atn(1) * 4
    b = Abs(b)
    If b / h > 0.1 Then
        s_b = 0.69
    Else
        s_b = 0
    End If
    beta = (kh / kv) ^ 0.5
    Lw_hat = Lw * (1 + ((b / Lw) ^ 2) * (beta ^ 2 - 1)) ^ 0.5
    Lw_bar = Lw * (1 - (b / Lw) ^ 2) ^ 0.5
    rw_hat = 0.5 * rw * (1 + (1 + (beta ^ 2 - 1) * ((Lw_bar / Lw) ^ 2)) ^ 0.5)
    A1 = 0.53 * ((L / D) ^ 2) + 1.15 * (L / D) + 0.164
    A2 = (1 - (Lw_bar / L)) / (0.45 + (Lw_bar / L))
    fa = (Lw_bar / L) * (1 + A1 * A2)
    C1 = 3 * h * beta * (Log(beta * h / (2 * Pi * rw_hat)) + s_b) / Lw_hat
    C2 = (Pi * D * fa / (2 * Lw_bar))
    unit_conversion_constant = (9.869E-13 * 0.001) * 24 * 3600 * 100000 * 6 * Pi / (0.001)
    J_horizontal = unit_conversion_constant * kh * h / (viso * Bo * (C1 + C2))
End Function
```

$$q_o = \underset{\sim}{J}\,(p_R - p_{wf}) \longrightarrow \text{Undersaturated oil}$$

geometry →

fluid-properties → $(\mu_o B_o)_{@\,p_{av}}$

for gas, saturated oil using a linear IPR is not adequate !

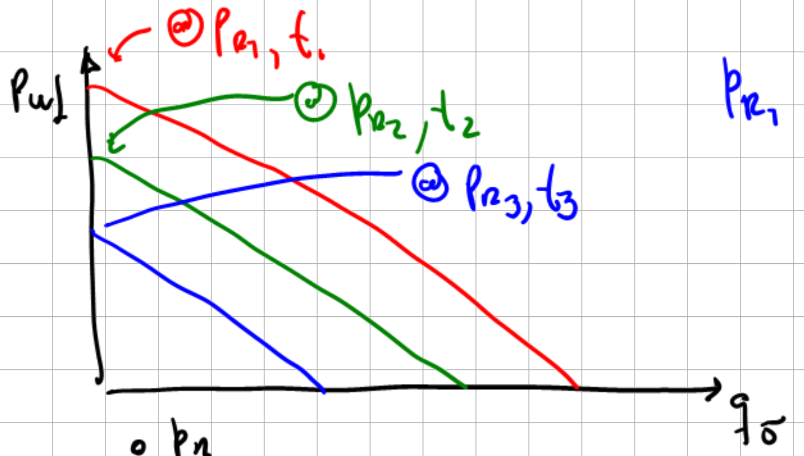


changes to IPR due to depletion

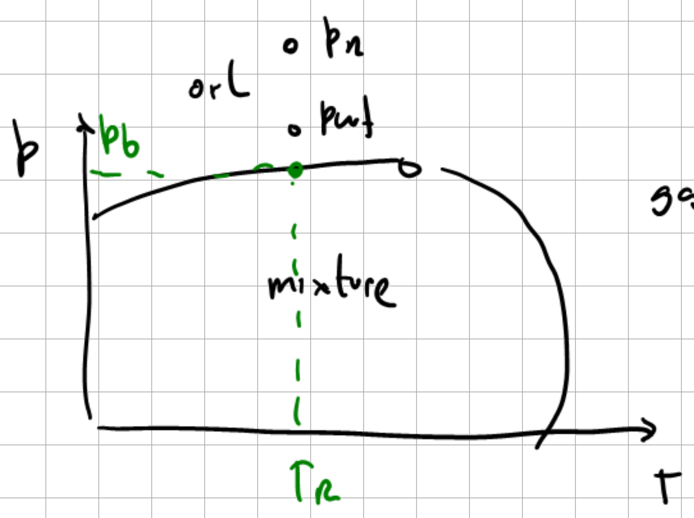
p_R → goes down

$J \sim (\mu_o B_o)_{@p_{av}}$, re, psi, ss, S, otherwise J remains fairly constant with time

$p_{av} = (p_R + p_{wf})\,0.5$



$p_{R1} > p_{R2} > p_{R3}$

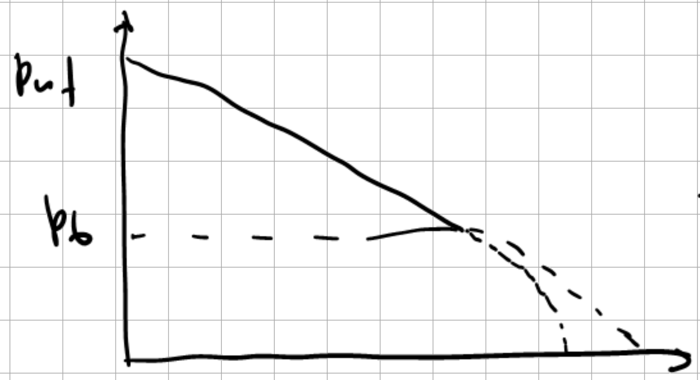


linear IPR valid if $p_{wf} > p_b$

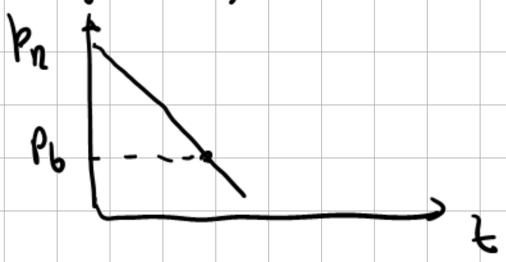
$p_e > p_b$

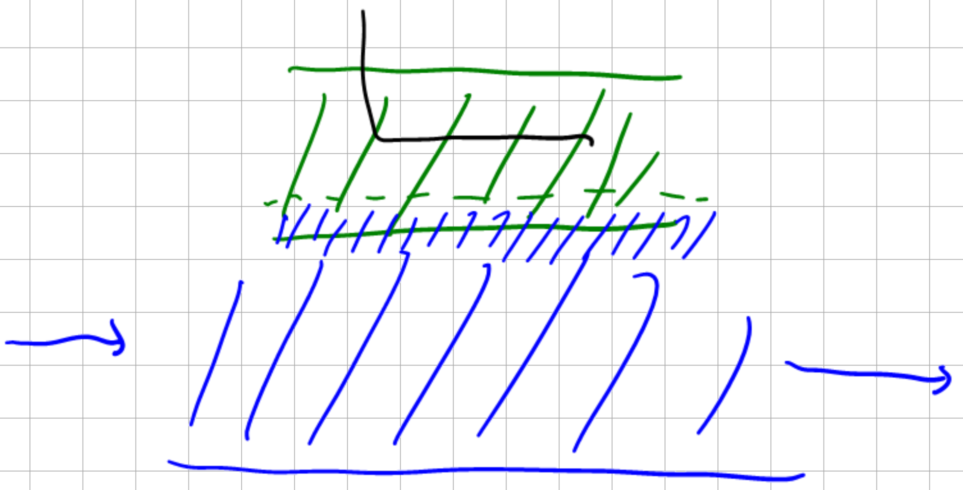
water injection

strong hydrodynamic aquifer



no aquifer, no injection, undersaturated oil reservoir





flow of water + oil (undersaturated) → Exhibits a linear IPR

liquid
incompressible

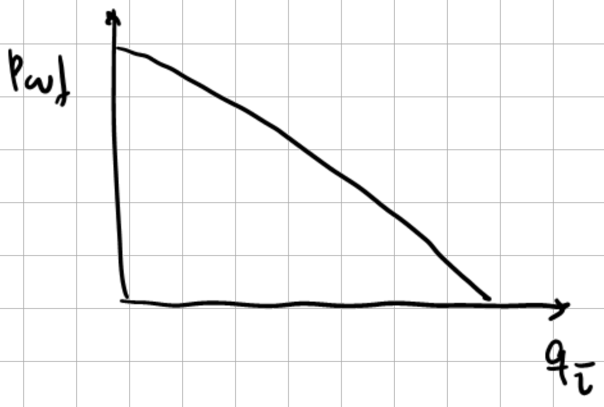
$$J_L = \frac{(q_o + q_w)}{(P_R - P_{wf})}$$

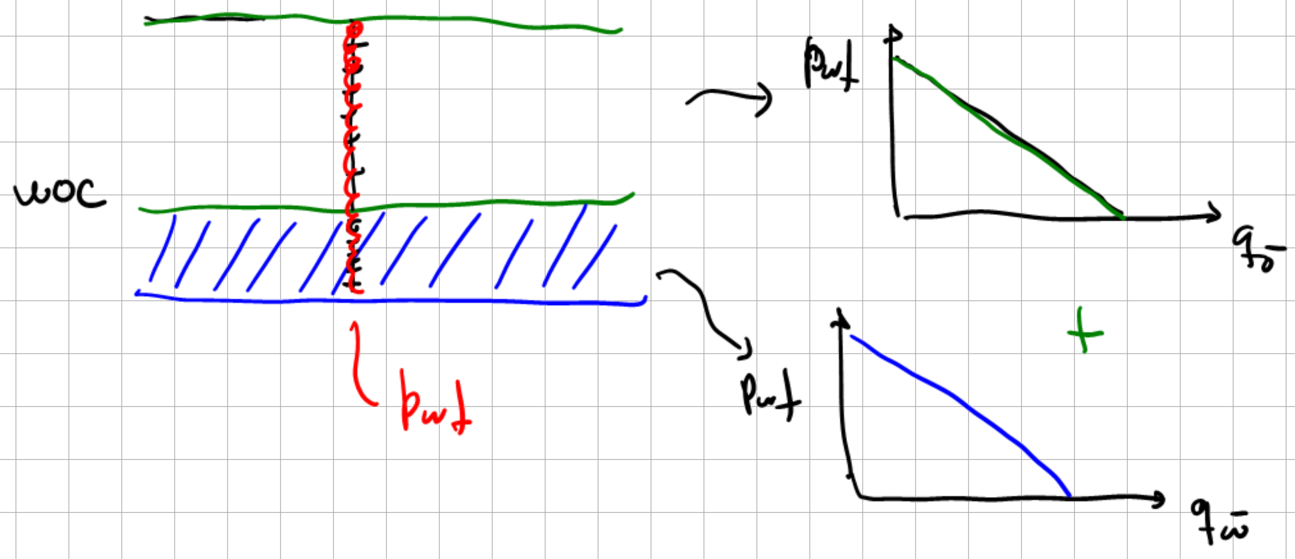
$$q_L = J_L (P_R - P_{wf})$$

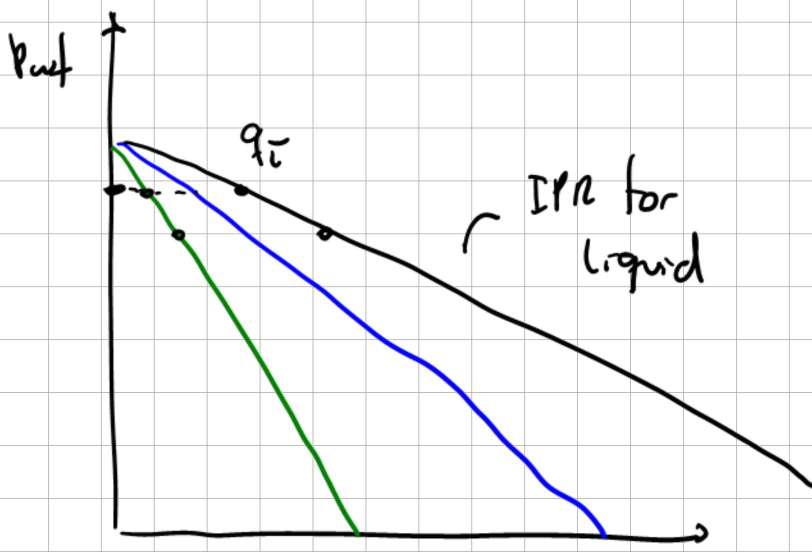
$$q_o = q_L (1 - w_c)$$

$$w_c = \frac{q_w}{q_o + q_w}$$

$$q_w = q_L (w_c)$$

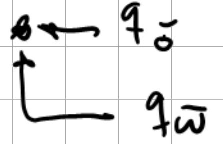


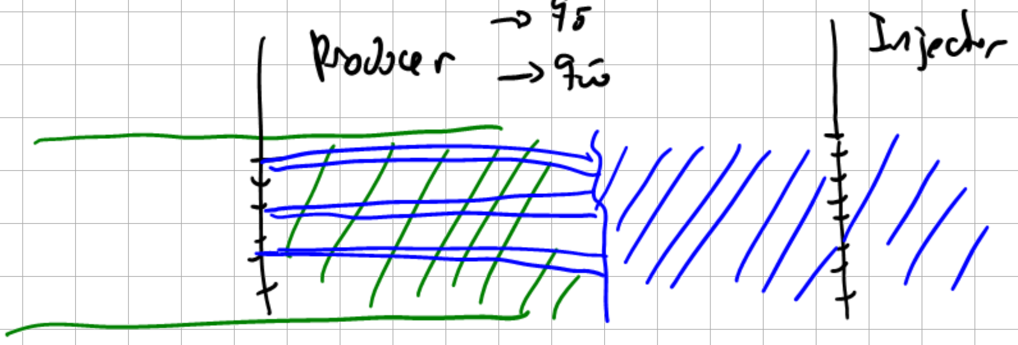


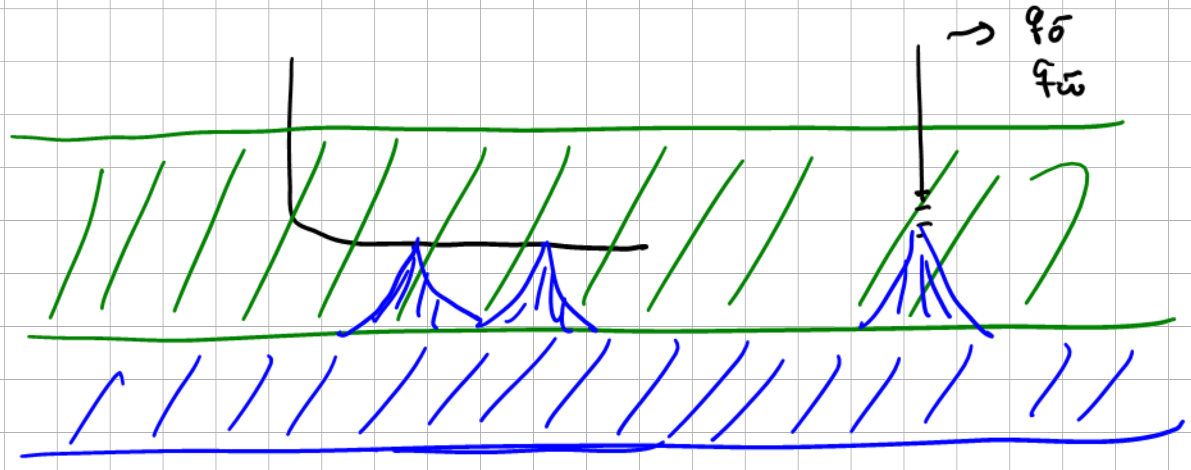


mobility $\lambda_o = \frac{k_{ro}}{\mu_o}$ $\lambda_w = \frac{k_{rw}}{\mu_w}$

$\lambda_o < \lambda_w$





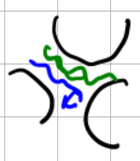


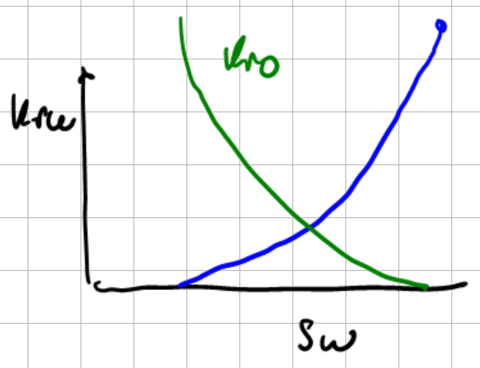
water coning - cusping

$J\ f(K)$ when two phases flowing

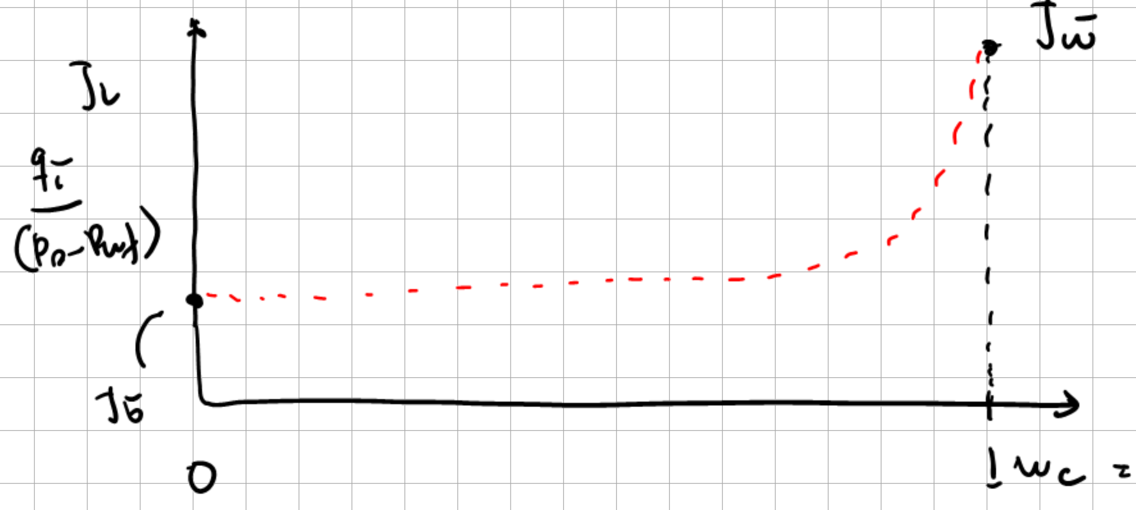
$K\ k_{ro}$

k_{rw}





$S_w = \dfrac{V_w}{V_T}$ in the pore

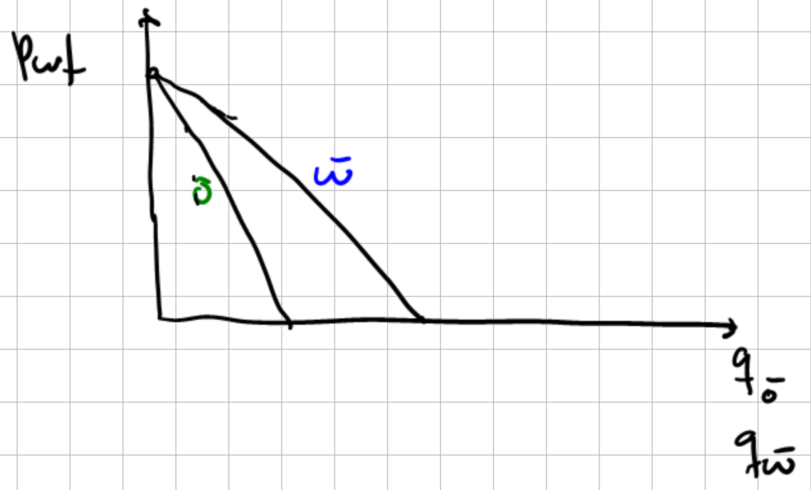


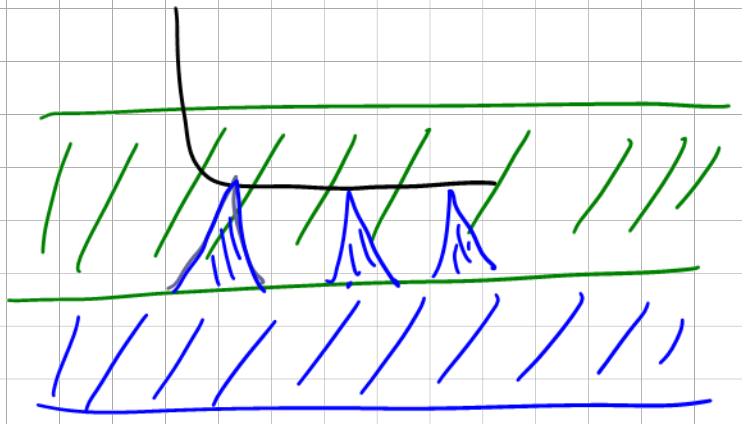
$wc = \dfrac{q_{\bar{w}}}{q_{\bar{o}}+q_{\bar{w}}}$

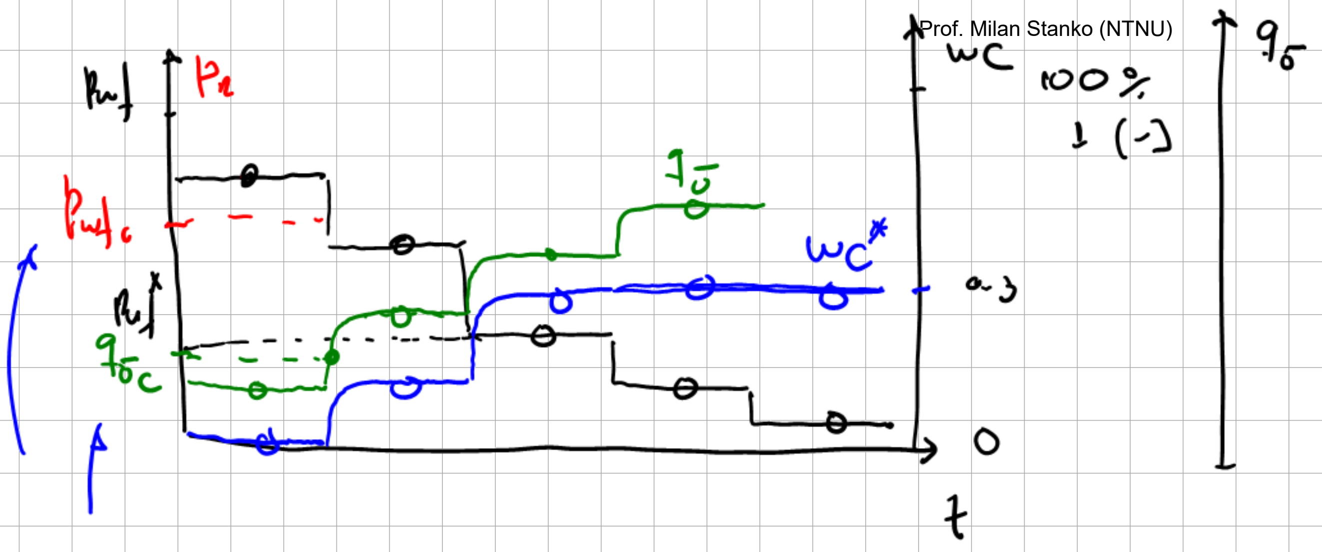
$q_{\bar{L}} = J_L\ (p_R - P_{wf})$

$q_{\bar{o}} = q_{\bar{L}} \cdot (1 - w_c)$

$q_{\bar{w}} = q_{\bar{L}}\ (wc)$

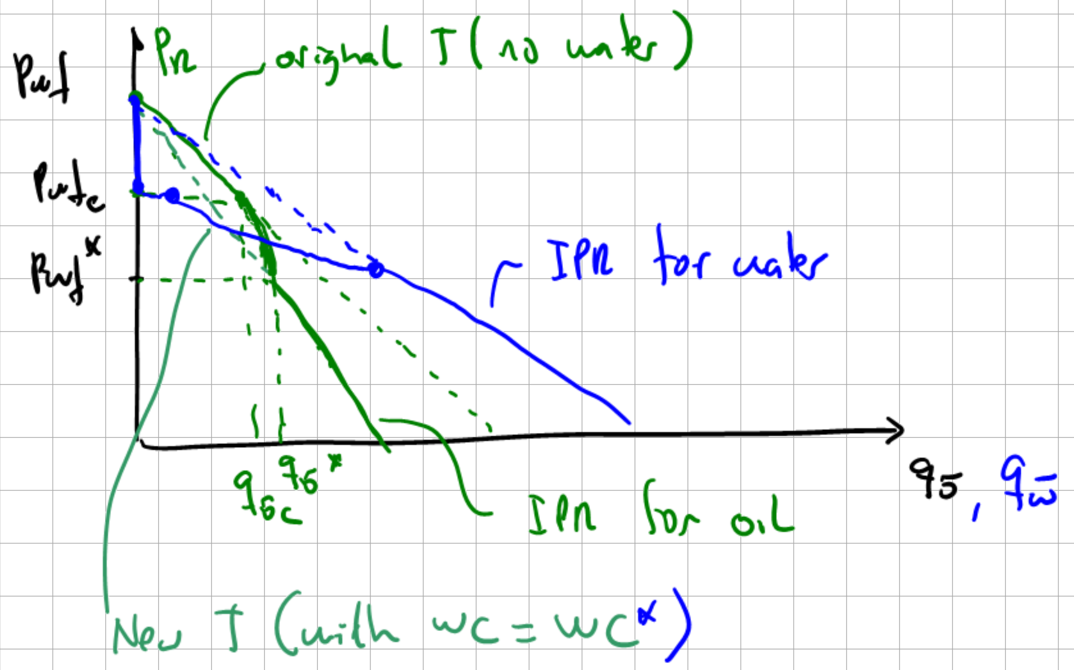






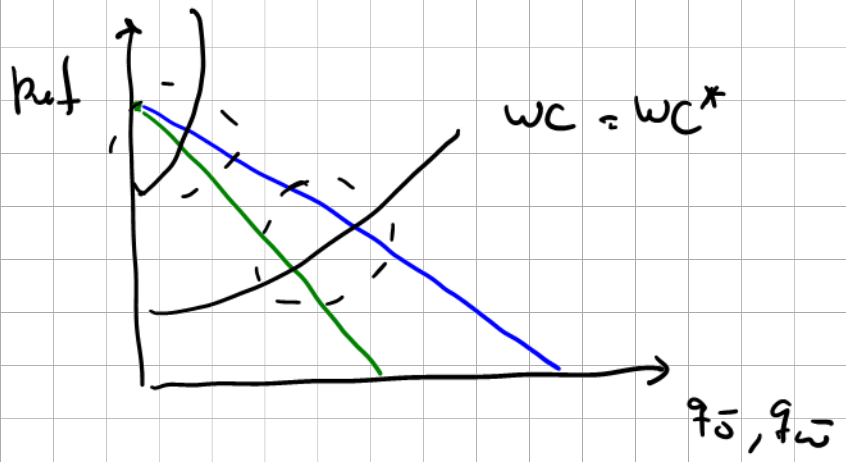
$q_{\bar{o}} < q_{\bar{o}c}$ $P_{wf_c} < P_{wf}$ ~> no coning

$q_{\bar{o}} \geq q_{\bar{o}c}$ $P_{wf_c} \geq P_{wf}$ ~> coning

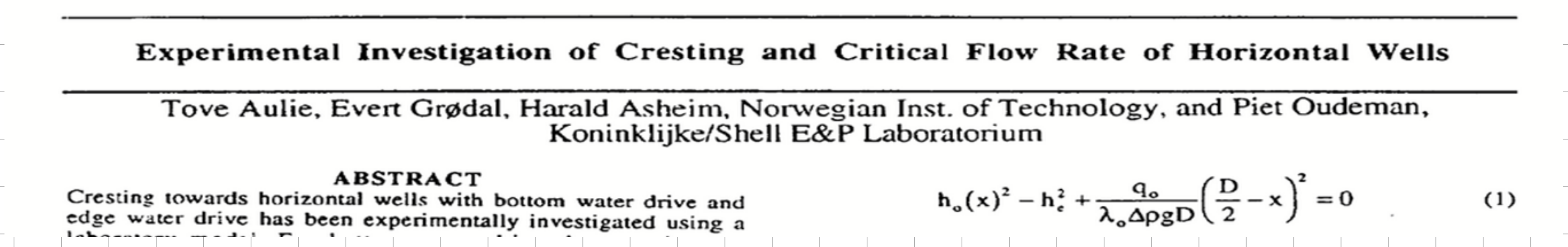


in many cases $q_{\bar{o}c}$ is very small

$q_{\bar{o}}^*$ is very small



Example: one model for water coning from the literature:



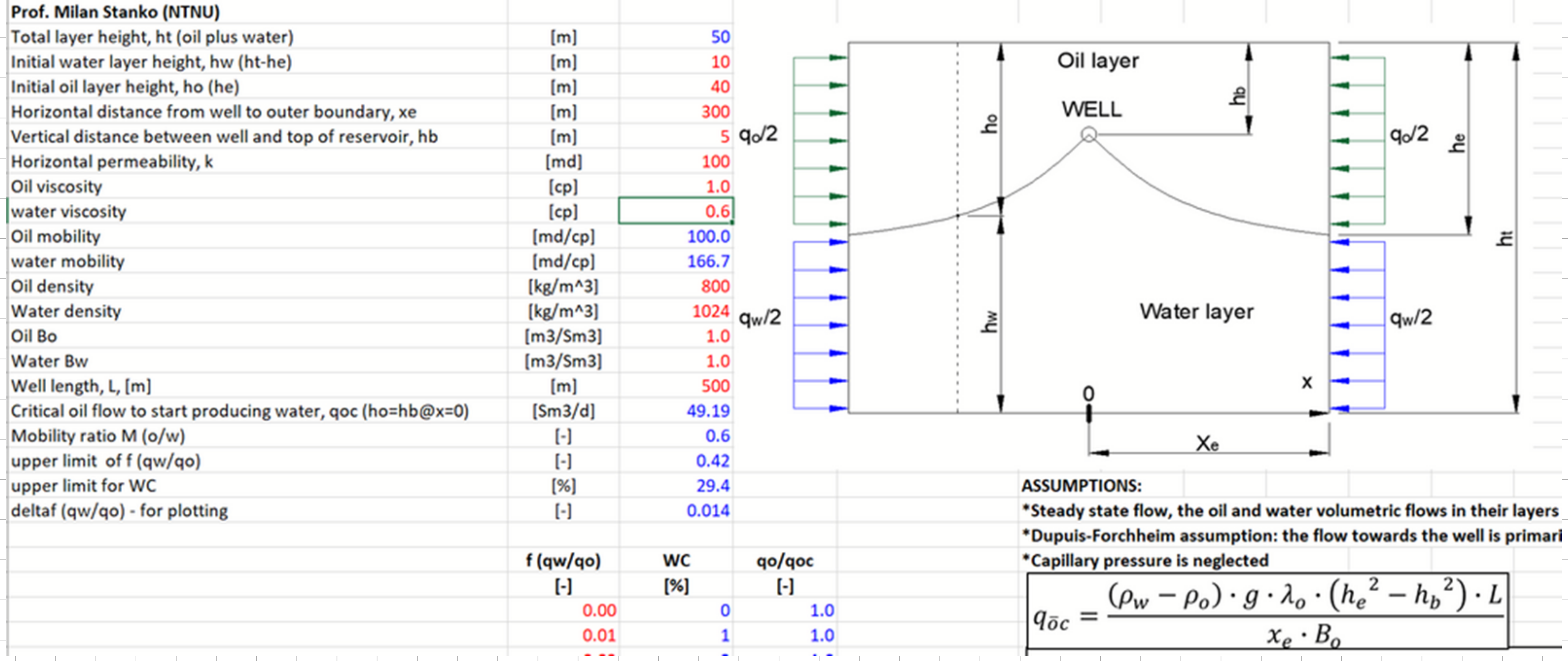
Experimental Investigation of Cresting and Critical Flow Rate of Horizontal Wells

Tove Aulie, Evert Grødal, Harald Asheim, Norwegian Inst. of Technology, and Piet Oudeman, Koninklijke/Shell E&P Laboratorium

ABSTRACT

Cresting towards horizontal wells with bottom water drive and edge water drive has been experimentally investigated using a

$$h_o(x)^2 - h_e^2 + \frac{q_o}{\lambda_o \Delta\rho g D}\left(\frac{D}{2} - x\right)^2 = 0 \qquad (1)$$



Prof. Milan Stanko (NTNU)		
Total layer height, ht (oil plus water)	[m]	50
Initial water layer height, hw (ht-he)	[m]	10
Initial oil layer height, ho (he)	[m]	40
Horizontal distance from well to outer boundary, xe	[m]	300
Vertical distance between well and top of reservoir, hb	[m]	5
Horizontal permeability, k	[md]	100
Oil viscosity	[cp]	1.0
water viscosity	[cp]	0.6
Oil mobility	[md/cp]	100.0
water mobility	[md/cp]	166.7
Oil density	[kg/m^3]	800
Water density	[kg/m^3]	1024
Oil Bo	[m3/Sm3]	1.0
Water Bw	[m3/Sm3]	1.0
Well length, L, [m]	[m]	500
Critical oil flow to start producing water, qoc (ho=hb@x=0)	[Sm3/d]	49.19
Mobility ratio M (o/w)	[-]	0.6
upper limit of f (qw/qo)	[-]	0.42
upper limit for WC	[%]	29.4
deltaf (qw/qo) - for plotting	[-]	0.014

f (qw/qo)	WC	qo/qoc
[-]	[%]	[-]
0.00	0	1.0
0.01	1	1.0

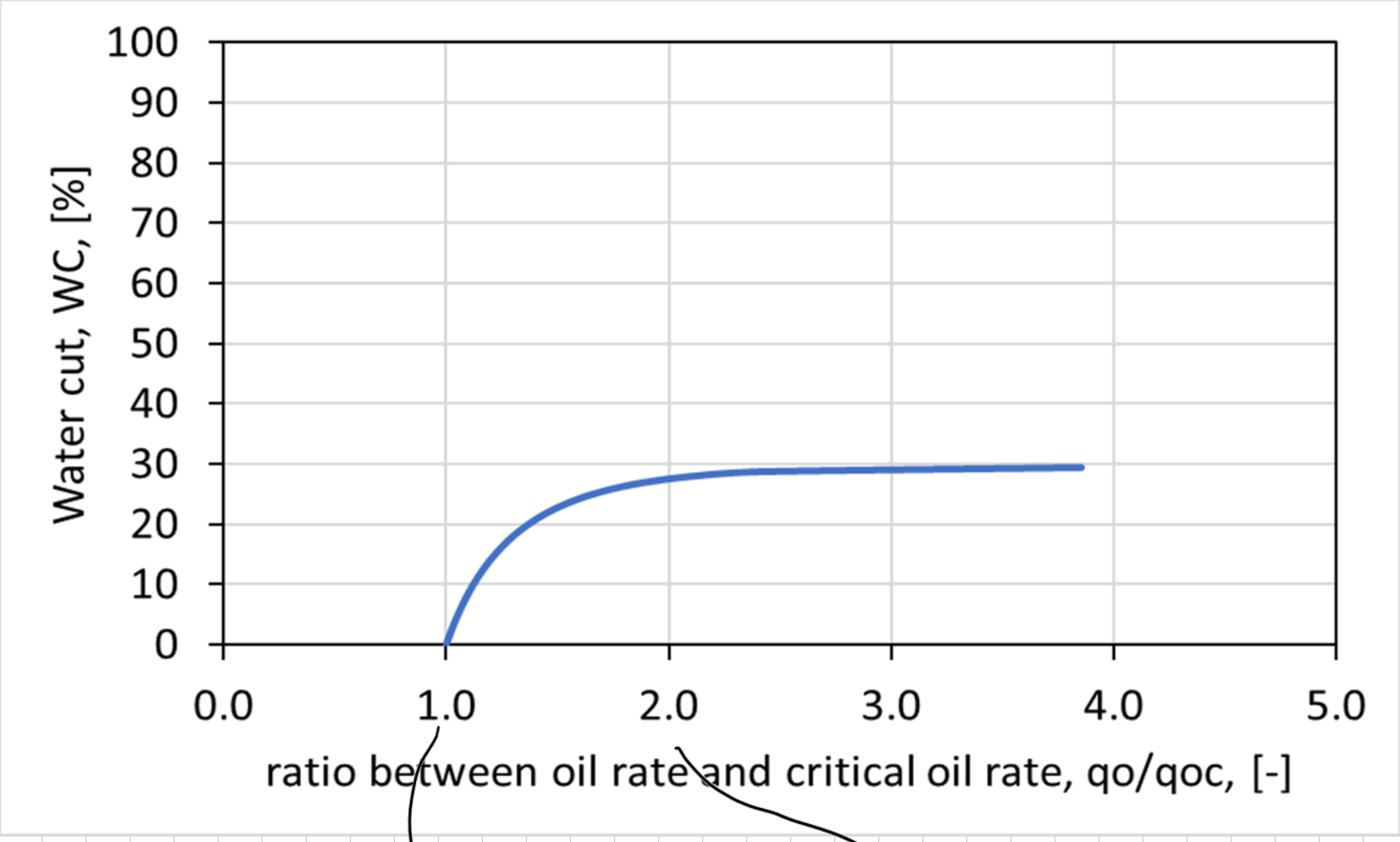
ASSUMPTIONS:

*Steady state flow, the oil and water volumetric flows in their layers

*Dupuis-Forchheim assumption: the flow towards the well is primari

*Capillary pressure is neglected

$$\boxed{q_{\bar{o}c} = \frac{(\rho_w - \rho_o)\cdot g \cdot \lambda_o \cdot \left({h_e}^2 - {h_b}^2\right)\cdot L}{x_e \cdot B_o}}$$



Ca 50 Sm^3/d

Ca. 100 Sm^3/d

Pwf