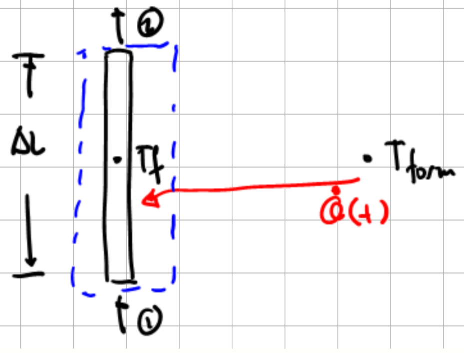
Lecture 41: Temperature calculations in wellbore



8. HEAT TRANSFER FOR FLOW IN CONDUITS

The equation for conservation of energy for a section of a conduit is

$$\dot{Q} + \dot{W} = \dot{m} \cdot (e_{out} - e_{in}) \quad \text{EQ. 8-1}$$

The specific energy that the stream has is usually split in internal energy (u), potential energy (z·g) and kinetic energy ($V^2/2$).

A conduit doesn't exchange work with the surroundings, but the fluid must perform work to enter and leave the system. This specific work is: $(p_{in} \cdot v_{in} - p_{out} \cdot v_{out})$ (Here v is specific volume).

By combining the inlet and outlet specific internal energy "u" with the specific work to enter and leave the system to obtain specific enthalpy, the energy conservation equation is written as:

$$\dot{Q} = \dot{m} \cdot \left(h_{out} + z_{out} \cdot g + \frac{(V_{out})^2}{2} - h_{in} - z_{in} \cdot g - \frac{(V_{in})^2}{2} \right) \quad \text{EQ. 8-2}$$

Or, alternatively

$$\dot{Q} = \dot{m} \cdot \left(\Delta h + \Delta z \cdot g + \frac{(V_{out})^2}{2} - \frac{(V_{in})^2}{2} \right) \quad \text{EQ. 8-3}$$

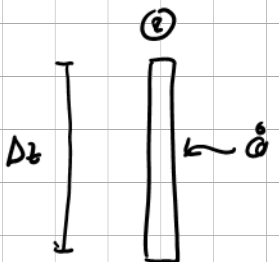
Here Δ represents outlet minus inlet.

In differential form (for an infinitesimally small length of pipe) the equation can be expressed as follows:

$$\frac{d\dot{Q}}{dL} = \dot{m} \cdot \left(\frac{dh}{dL} + g \cdot \frac{dz}{dL} + v \cdot \frac{dv}{dL} \right) \quad \text{EQ. 8-4}$$

≈ 0

Heat leaving the system is negative (the temperature of the outlet fluid is lower than the temperature at the inlet and the term Δh is usually negative). Heat entering the system is positive.



$\dot{Q} = \dot{m}(h_2 - h_1 + \Delta z \cdot g)$

p distribution is known $p(z)$

① $P_{wf}, T_{wf} (T_R), h_1 \{x_{g1}, x_{o1}, x_{w1}, h_{g1}, h_{o1}, h_{w1}\}$

$\dot{Q} = f(T_f, T_{form})$ → T_2

$h_2 = \frac{\dot{Q}}{\dot{m}} + h_1 - \Delta z \cdot g$

with P_2, assume T_2 → h_2^{calc}

specific enthalpy (h):

-Another BO property, just like viscosity, density, etc
-calculated at p-T with e.g. EOS

Mixture specific enthalpy:

$$h_{mix} = x_g \cdot h_g + x_o h_o + x_w h_w$$

$$x_g = \frac{\dot{m}_g}{\dot{m}_T} \quad \text{mass fraction}$$

p [bara]	T [C]	ho [kJ/kg]	hg [kJ/kg]
300.0	148.0	-2108.26	-3529.99
300.0	137.1	-2136.36	-3602.31
300.0	126.2	-2164.67	-3672.1
300.0	115.3	-2193.21	-3739.34
300.0	104.4	-2221.97	-3803.95
300.0	93.6	-2250.97	-3865.83
300.0	82.7	-2280.24	-3924.82
300.0	71.8	-2305.53	0
300.0	60.9	-2329	0
300.0	50.0	-2351.97	0
285.7	148.0	-2094.13	-3547.64
285.7	137.1	-2122.44	-3619.91
285.7	126.2	-2150.93	-3689.79
285.7	115.3	-2179.62	-3757.24

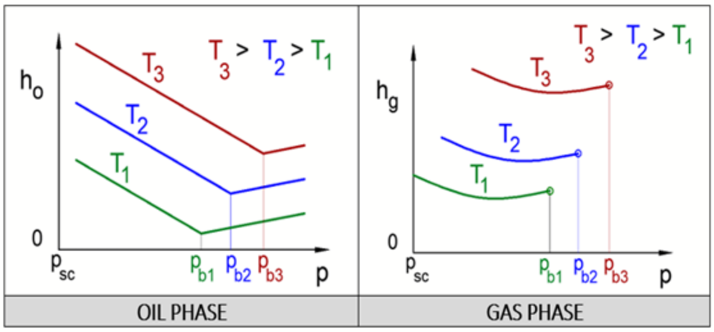


FIGURE 8-6. BEHAVIOR OF SPECIFIC ENTHALPY OF GAS AND OIL VS. PRESSURE FOR THREE TEMPERATURES

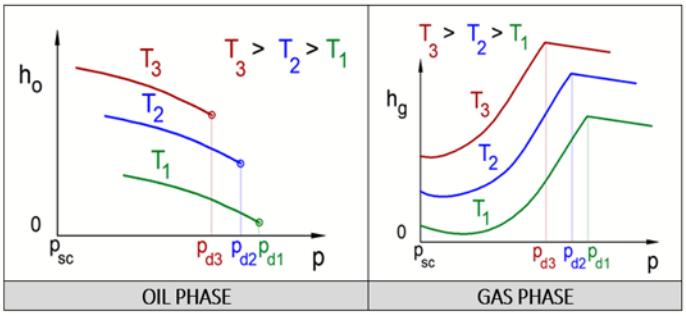
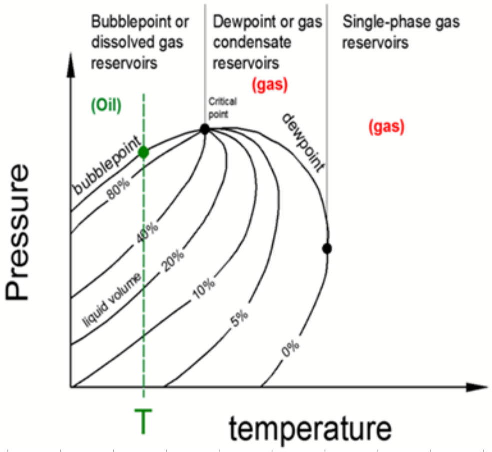
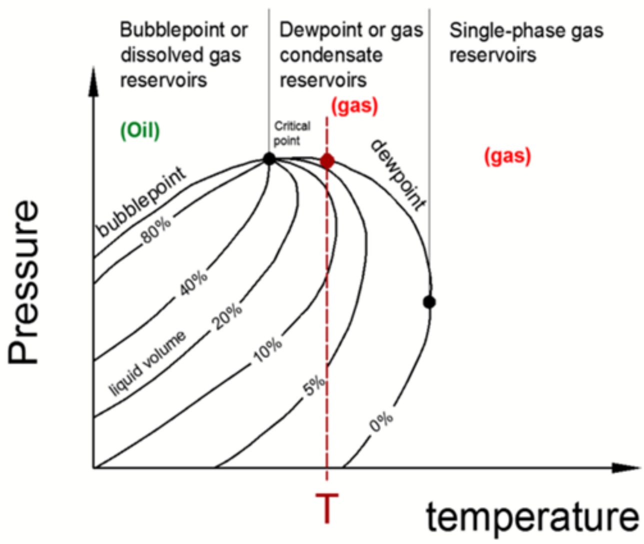


FIGURE 8-8. BEHAVIOR OF SPECIFIC ENTHALPY OF GAS AND OIL VS. PRESSURE FOR THREE TEMPERATURES





$\Delta h = C_p \cdot \Delta T$ for liquids

for gases $C_p = f(p,T)$ and requires frequent update

Heat term ($\dot{Q}$)

sign

$$\dot{Q} = -2 \cdot \pi \cdot L \cdot r \cdot U \cdot (T_f - T_\infty) \qquad \text{Eq. 8-20}$$

Where:

r	reference radius [m]
U	Overall heat transfer coefficient, expressed in terms of the reference radius r [W/m^2.K]
T_∞	Mean ambient temperature [K or °C]
T_f	Mean fluid temperature in the section [K or °C]

In this section we will work with the heat by unit of conduit length $\frac{\dot{Q}}{L} = \frac{d\dot{Q}}{dL}$.

Heat transfer mechanisms:

-Forced convection
-Free convection
-Conduction

In pipes:

$$\mathrm{Nu}_L = \frac{\text{Convective heat transfer}}{\text{Conductive heat transfer}} = \frac{h}{k/L} = \frac{hL}{k}$$

Forced and free convection:

$$\frac{d\dot{Q}}{dL} = -2 \cdot \pi \cdot r_i \cdot h_i \cdot (T_f - T_i)$$

forced: $\mathrm{Nu} = f(\mathrm{Re}, \mathrm{Pr})$

free: Nu= f(Gr,Pr)

IMPLICIT! T is required to find h!!

Conduction:

steady state

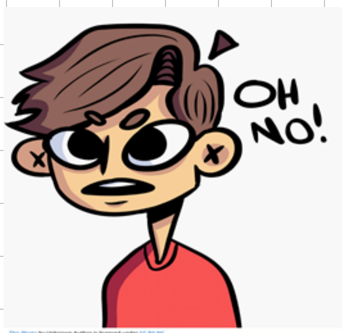
$$\frac{d\dot{Q}}{dL} = -2 \cdot \pi \cdot k_p \cdot \frac{(T_i - T_o)}{\ln\left(\frac{r_o}{r_i}\right)}$$

Transient?

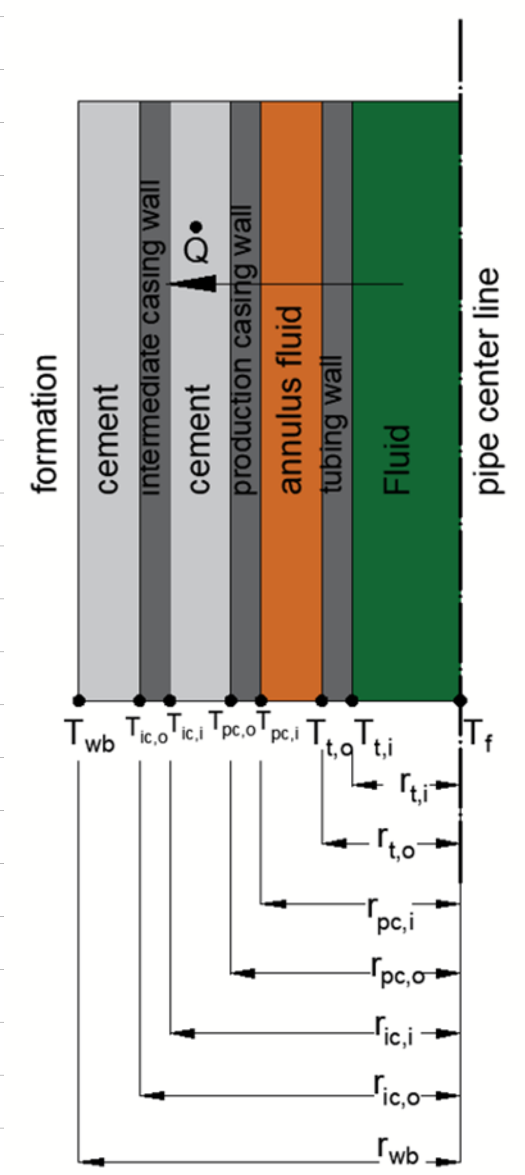
ATTENTION!!!

-h can be pump head (deltap/(den*g))
-h can be specific enthalpy (u+p*v)
-h can be forced/free convection coefficient

BE CAREFUL!!!



Example:



Overall heat transfer coefficient:

$$(T_f - T_{t,i}) + (T_{t,i} - T_{t,o}) + (T_{t,o} - T_{pc,i}) + (T_{pc,i} - T_{pc,o}) + (T_{pc,o} - T_{ic,i}) + (T_{ic,i} - T_{ic,o}) + (T_{ic,o} - T_{wb}) = \frac{-\frac{d\dot{Q}}{dL}}{2 \cdot \pi \cdot r_{t,i} \cdot h_i} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2 \cdot \pi \cdot k_t}{\ln\left(\frac{r_{t,o}}{r_{t,i}}\right)}} + \frac{-\frac{d\dot{Q}}{dL}}{2 \cdot \pi \cdot r_{t,o} \cdot h_{ann}} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2 \cdot \pi \cdot k_{pc}}{\ln\left(\frac{r_{pc,o}}{r_{pc,i}}\right)}} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2 \cdot \pi \cdot k_c}{\ln\left(\frac{r_{ic,i}}{r_{pc,o}}\right)}} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2 \cdot \pi \cdot k_{ic}}{\ln\left(\frac{r_{ic,o}}{r_{ic,i}}\right)}} + \frac{-\frac{d\dot{Q}}{dL}}{\frac{2 \cdot \pi \cdot k_c}{\ln\left(\frac{r_{wb}}{r_{ic,o}}\right)}}$$

Clearing the temperature difference between fluid and wellbore wall:

$$(T_f - T_{wb}) = -\frac{d\dot{Q}}{dL} \cdot \left[\frac{1}{2\cdot\pi\cdot r_{t,i}\cdot h_i} + \frac{1}{\frac{2\cdot\pi\cdot k_t}{\ln\left(\frac{r_{t,o}}{r_{t,i}}\right)}} + \frac{1}{2\cdot\pi\cdot r_{t,o}\cdot h_{ann}} + \frac{1}{\frac{2\cdot\pi\cdot k_{pc}}{\ln\left(\frac{r_{pc,o}}{r_{pc,i}}\right)}} + \frac{1}{\frac{2\cdot\pi\cdot k_c}{\ln\left(\frac{r_{ic,i}}{r_{pc,o}}\right)}} + \frac{1}{\frac{2\cdot\pi\cdot k_{ic}}{\ln\left(\frac{r_{ic,o}}{r_{ic,i}}\right)}} + \frac{1}{\frac{2\cdot\pi\cdot k_c}{\ln\left(\frac{r_{wb}}{r_{ic,o}}\right)}}\right]$$

If the inner tubing radius will be used as reference radius, we then we divide by the inner perimeter of the inner tubing:

$$(T_f - T_{wb}) = -\frac{d\dot{Q}}{dL}\cdot\frac{1}{2\cdot\pi\cdot r_{t,i}} \cdot \left[\frac{1}{h_i} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{t,o}}{r_{t,i}}\right)}{k_t} + \frac{r_{t,i}}{r_{t,o}\cdot h_{ann}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{pc,o}}{r_{pc,i}}\right)}{k_{pc}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{ic,i}}{r_{pc,o}}\right)}{k_c} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{ic,o}}{r_{ic,i}}\right)}{k_{ic}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{wb}}{r_{ic,o}}\right)}{k_c}\right]$$

Then:

$$U = \left(\frac{1}{h_i} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{t,o}}{r_{t,i}}\right)}{k_t} + \frac{r_{t,i}}{r_{t,o}\cdot h_{ann}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{pc,o}}{r_{pc,i}}\right)}{k_{pc}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{ic,i}}{r_{pc,o}}\right)}{k_c} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{ic,o}}{r_{ic,i}}\right)}{k_{ic}} + \frac{r_{t,i}\cdot\ln\left(\frac{r_{wb}}{r_{ic,o}}\right)}{k_c}\right)^{-1}$$

- Inner forced convection: The inner forced-convection coefficient (h_i) is usually in the range 100-50 000 W/m^2 K.[23] It is lower for low velocities and for gas flow. This gives a term in the range O(1E-5) to O(1E-2).
- Conduction in metal: Inner radii of well tubulars and pipelines are usually in the range 0.01-0.25 m. The ratio between inner and outer radius is usually between 1.05-1.3 (thickest pipe walls are usually for the small pipe diameters), thus the natural log of it is between 0.04-0.24. Lastly, the conductivity of the steel is around 45 W/m^2 K. This gives a term O(1E-4).

Free convection in the annulus (Term 3): The free convection coefficient in the annulus usually has values around 100 W/m^2 K. The ratio between outer and inner tubing diameter can range from 1.05 to 1.3. Therefore, this term is usually O(1E-2).

Conduction in cement (terms 5 and 7): The thermal conductivity of cement (k_c) is usually in the range between 0.3 to 2 W/m K. The ratio between the outer and inner diameter of the annular space is usually around 1.2. The inner tubing diameter is usually 0.02-0.2. Therefore, this term is usually O(1E-2).

Heat transfer in formation or soil

$$\frac{\partial^2 T_e}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T_e}{\partial r} = \frac{c_e \cdot \rho_e}{k_e} \cdot \frac{\partial T_e}{\partial t}$$

k_e Thermal conductivity, soil [W/m.K]

C_e Specific heat capacity, soil [J/K.kg]

t Time [s]

$$T_e(r, t = 0) = T_{ei}$$

$$\frac{\partial T_e}{\partial r}(r \to \infty, t) = 0$$

$$\frac{d\dot{Q}}{dz} = -2 \cdot \pi \cdot k_e \cdot r_{wb} \cdot \left.\frac{\partial T_e}{\partial r}\right|_{r=r_{wb}}$$

input

Transient!!!!

An approximate, analytical solution:

Wellbore Heat Transmission

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Heat Transfer During Two-Phase Flow in Wellbores: Part I—Formation Temperature

A.R. Hasan, U. of North Dakota, and C.S. Kabir, Chevron Oil Field Research Co.
SPE Members

Using that solution, and doing some math:

$$U_{eff}(t) = \left(\frac{U \cdot k_e}{k_e + T_D \cdot r_{t,i} \cdot U}\right)$$

$$T_D = 1.1281 \cdot \sqrt{t_D} \cdot (1 - 0.3 \cdot \sqrt{t_D}), \quad for \quad t_D \leq 1.5$$

$$T_D = (0.4063 + 0.5 \cdot \ln(t_D))\left(1 + \frac{0.6}{t_D}\right) \quad for \quad t_D > 1.5$$

Dimensionless time, $t_D = \frac{\alpha_e \cdot t}{{r_{wb}}^2}$

And α_e is the thermal diffusivity of the soil, [m2/s], equal to $\frac{k_e}{\rho_e \cdot C_e}$