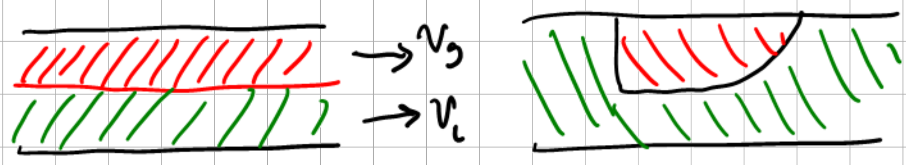
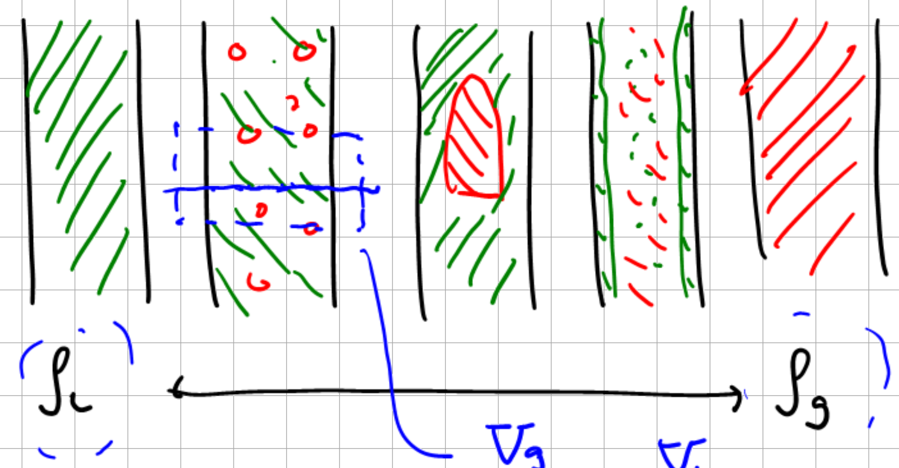
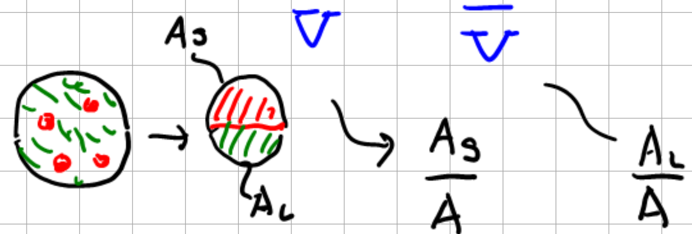
- flow patterns



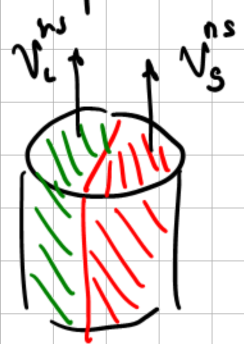
- phase velocity $\rightsquigarrow$ friction Δp

- phase spatial distribution $\rightsquigarrow$ hydrostatic Δp

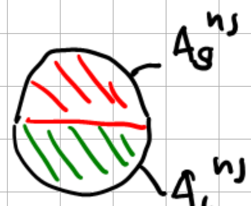




Liquid and gas travel at the same velocity (no-slip)



$$v_L^{ns} = v_g^{ns} = v_m = \frac{q_g + q_L}{A} = \frac{q_g}{A} + \frac{q_L}{A} = u_{sg} + u_{sL}$$



$$\lambda_g = \frac{A_g^{ns}}{A} = 1 - \lambda_L = \frac{q_g}{q_g + q_L} \qquad q_g \uparrow\uparrow >> q_L \longrightarrow \lambda_g \to 1$$

$$\lambda_L = \frac{A_L^{ns}}{A} = 1 - \lambda_g = \frac{q_L}{q_g + q_L} \qquad q_L >> q_g \longrightarrow \lambda_L \to 1$$

gas and liquid move at different velocities $v_g \neq v_L$ (slip condition)

void fraction

$$\varepsilon = \underbrace{H_g}_{\text{gas holdup}} = \frac{A_g}{A} = 1 - H_L$$

$$\underbrace{H_L}_{\text{liquid holdup}} = \frac{A_L}{A} = 1 - H_g$$

v_g ?
v_L ?

mass conservation between non-slip condition and slip condition

$$q_g = v_g^{ns} \cdot A_g^{ns} = v_g \cdot A_g \qquad \text{divide by } A$$

$$v_g^{ns} \lambda_g = v_g \cdot \varepsilon$$

$$\boxed{v_g = v_m \frac{\lambda_g}{\varepsilon}}$$

$\lambda_g < \varepsilon$

$v_g < v_m$

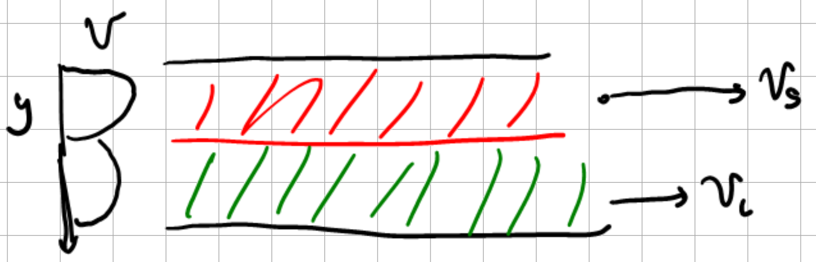
$\lambda_g = \varepsilon$
$v_g = v_m$

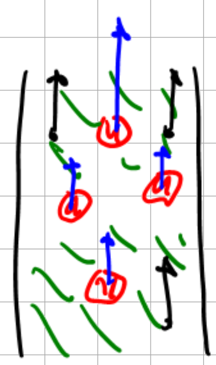
$\lambda_g > \varepsilon$
$v_g > v_m$

$$\boxed{v_L = v_m \frac{\lambda_L}{H_L}}$$

$$v_g = \frac{v_m \lambda_g}{\varepsilon} = \frac{\cancel{q_g + q_L}}{A} \cdot \frac{q_g}{\cancel{q_g + q_L}} \cdot \frac{1}{\varepsilon} = \frac{u_{sg}}{\varepsilon}$$

$$\boxed{v_g = \frac{u_{sg}}{\varepsilon} \qquad v_L = \frac{u_{sL}}{H_L}}$$





$\varepsilon, H_L, v_L, v_g$ is a result of solving our $\left\{ \begin{array}{l} \text{mass conservation} \\ \text{momentum conservation} \end{array} \right\}$

one example

$$v_g = C_0 v_m + \underbrace{u_0}_{\text{velocity of a bubble in stagnant liquid}}$$

TWO-PHASE FLOW IN VERTICAL TUBES

By D. J. NICKLIN, B.Sc. App.,* J. O. WILKES, M.A.*† and J. F. DAVIDSON, M.A., Ph.D., A.M.I.Mech.E.,*

SUMMARY

A study has been made of the properties of long bubbles in vertical tubes. It has been shown that these bubbles rise relative to the liquid ahead of them at a velocity exactly equal to the rising velocity of wakeless bubbles of the type studied by Dumitrescu and by Davies and Taylor. For 1 in. tubes, this velocity is closely predicted by Dumitrescu's theory and equals $0{\cdot}35\,(gD)^{\frac{1}{2}}$ where g is the acceleration of gravity and D the tube diameter. The motion of the bubbles in moving liquid streams has been studied, and the results applied to the problem of two-phase slug flow. An expression for the voidage in steady two-phase slug flow has been derived, and this predicted voidage agrees well with results reported here and elsewhere.

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$$u_s = 1{\cdot}2\,\bar{u}_L + 0{\cdot}35\,(gD)^{\frac{1}{2}}$$

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Average Volumetric Concentration in Two-Phase Flow Systems

A general expression which can be used either for predicting the average volumetric concentration or for analyzing and interpreting experimental data is derived. The analysis takes into account both the effect of nonuniform flow and concentration profiles as well as the effect of the local relative velocity between the phases. The first effect is taken into account by a distribution parameter, whereas the latter is accounted for by the weighted average drift velocity. Both effects are analyzed and evaluated. The results predicted by the analysis are compared with experimental data obtained for various two-phase flow regimes, with various liquid-gas mixtures in adiabatic, vertical flow over a wide pressure range. Good agreement with experimental data is shown.

[2] Numbers in brackets designate References at end of paper.
Contributed by the Heat Transfer Division and presented at the Winter Annual Meeting, New York, N. Y., November 29–December 3, 1964, of The American Society of Mechanical Engineers. Manuscript received at ASME Headquarters, September 15, 1964.

"v_0"

THE FLOW OF LIQUID-GAS MIXTURES IN VERTICAL TUBES.

By

Hans Behringer

ZEITSCHRIFT FUR DIE GESAMTE KALTE-INDUSTRIE, 43, 55-58, 1936.

Z. angew. Math. Mech. Bd. 23 Nr. 3 Juni 1943 Dumitrescu, Strömung an einer Luftblase im senkrechten Rohr 139

Strömung an einer Luftblase im senkrechten Rohr.

Von *D. T. Dumitrescu* in Bukarest.

The mechanics of large bubbles rising through extended liquids and through liquids in tubes

By R. M. Davies and Sir Geoffrey Taylor, F.R.S.

(*Received* 13 *September* 1949)

$$\varepsilon = \frac{U_{SG}}{U_{SG}\left(1 + \left(\frac{U_{SL}}{U_{SG}}\right)^{\left(\frac{\rho_G}{\rho_L}\right)^{0.1}}\right) + 2.9\left[\frac{gD\sigma(1+\cos\theta)(\rho_L - \rho_G)}{\rho_L^2}\right]^{0.25}(1.22 + 1.22\sin\theta)^{\frac{P_{atm}}{P_{system}}}}$$

Comparison of void fraction correlations for different flow patterns in horizontal and upward inclined pipes

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Received 1 June 2006; received in revised form 13 September 2006

$$H_g = \frac{\lambda_g + 1 - \left[(\lambda_g + 1)^2 - 4 \cdot {\lambda_g}^2\right]^{0.5}}{2 \cdot \lambda_g}$$

Pipe Fractional Flow Theory: Principles and Applications

by

Anand Subhash Nagoo, B.Sc., M.S., M.S.

2013

- $U_s = V_g - V_L$ slip velocity
- $S = \frac{V_g}{V_L}$ slip ratio