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Efficient Well Placement Optimization with Gradient-Based Algorithms and Adjoint Models

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Abstract

A key reservoir management decision taken throughout the life of a reservoir is the determination of optimal well locations that maximizes asset value (such as Net Present Value, NPV). Because this well placement optimization problem is a discrete-parameter problem (well locations are discrete parameters in the simulation model), gradients of the objective function (NPV) with respect to these parameters are not defined. Thus, gradient-based methods have not found much applicability to this problem, and most existing algorithms applied to this problem are stochastic in nature, such as genetic algorithms, simulated annealing, and stochastic perturbation methods. These methods are usually quite inefficient requiring hundreds of simulations and thus may have limited application to large-scale simulation models with many wells.

We propose a novel, continuous approximation to the original discrete-parameter well placement problem such that gradients can be calculated on the approximate problem, and gradient-based algorithms can then be employed for efficiently determining the optimal well locations. The basic idea is to first replace the discrete parameters (i, j well location indices) with their continuous counterparts in the spatial domain (x, y well locations) and then obtain a continuous functional relationship between the objective function and these continuous parameters. Such a functional relationship is obtained by replacing the discontinuous Dirac-delta functions (defining wells as point sources) in the underlying governing PDE with continuous functions (which in the limit tends to the Dirac-delta function, such as the bivariate Gaussian function). Numerical discretization of the modified PDE leads to well terms in the mass balance equations that are continuous functions of the continuous well location variables. As a result of this continuous functional relationship, adjoints and gradient-based optimizations algorithms can now be applied to obtain the optimal well locations. The efficiency and practical applicability of the approach is demonstrated on a few synthetic waterflood optimization problems.

Introduction

A key reservoir management decision necessary throughout the life of an oilfield is the determination of optimal well locations that maximize asset value. The current industry practice to do so is usually through manual approaches wherein the engineer essentially uses engineering judgment and numerical simulation to determine such locations. Although such an approach may be viable for small reservoirs with a small number of wells, it is unlikely that such approaches will be applicable when dealing with large reservoirs (with large-scale simulation models) and a large number of wells.

Recently however, there has been an increasing interest in solving this problem more efficiently with automatic optimization algorithms. This optimal well placement problem is usually formulated as a discrete parameter optimization problem, because the well location variables are discrete variables (i, j indices of grid blocks where wells are located). Because of the discrete nature of the problem, gradients of the objective function (NPV for example) with respect to these discrete variables do not exist. As a result, gradient-based optimization algorithms have not found much applicability to this problem, and most existing algorithms applied to this problem are stochastic gradient-free algorithms, such as genetic algorithms (Montes and Bartolome, 2001, Yeten, 2003), simulated annealing (Beckner and Song, 1995), and stochastic perturbation methods (Spall, 2003, Bangerth et al., 2006). Although these algorithms are easy to apply and are supposedly global in nature, they are usually quite inefficient requiring hundreds of simulations and thus may have limited application to large-scale simulation models with many wells. Furthermore, they do not guarantee a monotonic increase in the objective function with successive iterations, implying that increasing the computational effort may not necessarily provide a better optimum.

On the other hand, gradient-based algorithms (with associated adjoint models) are usually considered more efficient compared to stochastic algorithms requiring only few tens of simulations for convergence and guarantee a monotonically

increasing objective function with successive iterations. However, these algorithms have the potential drawback of being stuck in local minima. Considering the fact that most practical simulation models range from a few hundred thousand cells to a few million cells with hundreds of wells requiring many hours even for a single evaluation, efficiency of the optimization algorithm becomes imperative for practical applicability. Further, since any improvement of the objective function from the base case is always useful, it is desirable but not necessary to obtain the global optimum. As a result, gradient-based algorithms seem to be the algorithms of choice from a practical standpoint, provided that they can be applied to this problem. However, as mentioned above, since this is a discrete parameter problem, gradients of the objective function with respect to these discrete variables do not exist, and therefore a direct application is not possible. To the best of our knowledge, gradients and adjoints have been used indirectly for the well placement problem in two recent papers, one by Handels et al. (2007), and the other by Wang et al. (2007).

The basic idea of the Handles et al. (2007) paper is to surround each well to be optimized by eight "pseudo-wells" in the eight neighboring grid blocks in the 2D plane, with each pseudo-well producing at a very low rate to minimize their influence on the flow behavior of the reservoir. An adjoint model is then used to calculate the gradient of the objective function (NPV) over the life of the reservoir with respect to the rate at each pseudo-well. The largest positive gradient value among the eight gradients then determines the direction in which the original well should be moved to increase the objective function in the next iteration. That is, the improving direction is approximated as the direction of the pseudo-well location (from the original well location) with the largest positive gradient. This approach is an indirect application of gradients because gradients of the objective function with respect to rate of the pseudo-well is used instead of the gradient of the objective function with respect to actual optimization parameters (i, j well location indices). Further, a basic limitation of the approach is that only eight possible search directions can obtained at each iteration for each well (corresponding to the direction of the eight pseudo-wells), and this can be quite limiting as the optimal search direction resulting in the fastest increase in the objective function can be any arbitrary direction in the 2D plane. Also, as implemented in the paper, each well in moved only one grid block in each iteration, which will be very inefficient for practical problems, although this does not seem to be a limitation of the algorithm itself.

Wang et al. (2007) consider the placement of one or more injection wells in a 2D reservoir to maximize NPV. The basic idea behind their approach is to initialize the optimization problem with an injector well at each grid block that does not contain a producer well, and successively reduce the number of injectors at each iteration of the algorithm until the optimal number of injectors remain at the optimal locations. In order to do so, the objective function (NPV) is augmented with a drilling cost assigned to each well drilled. As a result, larger the number of injectors drilled, larger will be the total drilling cost and therefore more heavily will the modified objective function be penalized. The algorithm advances by calculating the gradient of the modified objective function with respect to the rate of each injector using an adjoint model. These gradients are used to calculate the next rate for each well using the steepest descent method, and as the rate of a well goes to zero, the well is eliminated. Ultimately, the optimum number of injectors at the optimal wells locations should remain. Again, this is an indirect application of gradients because the gradients of the objective function with respect to rate of the injectors is used instead of the gradient of the objective function with respect to actual optimization parameters (i, j well location indices). Further, because the algorithm starts by drilling an injector at each grid block, and only one injector can be eliminated at each iteration, it is clearly not a very practical or efficient approach for large-scale problems.

In this work, we propose a direct, efficient, and rigorous application of gradient-based algorithms and adjoints to solve the well placement problem. We define a continuous approximation to the original discrete-parameter well placement problem such that gradients can be calculated on the approximate problem, and gradient-based algorithms can then be employed for efficiently determining the optimal well locations. The basic idea is to first replace the discrete parameters (i, j well location indices) with their continuous counterparts in the spatial domain (x, y well locations) and then obtain a continuous functional relationship between the objective function and these continuous parameters. Such a functional relationship is obtained by replacing the discontinuous Dirac-delta functions (defining wells as point sources) in the underlying governing partial differential equations (PDE) with continuous functions (which in the limit tend to the Dirac-delta function, such as the bivariate Gaussian function). Numerical discretization of the modified PDE leads to well terms in the mass balance equations that are continuous functions of the continuous well location variables. An implication of the continuous approximation is that in the numerical model, the original wells are now surrounded by pseudo-wells whose geometric well indices are weighted by spatial integrals of the continuous (Gaussian) function, which itself is a function of the x and y locations of the original wells. As a result of this continuous functional relationship, adjoints and standard gradient-based optimizations algorithms can now be applied to obtain the optimal well locations. Although the creation of pseudo-wells is similar to that of the Handels et al. (2007) algorithm, the key advantages over the Handels algorithm are that the actual gradient with respect to the well location variables are obtained (optimization parameters), and any arbitrary search direction is possible at each iteration unlike the Handels algorithm, where the search direction is limited to 8 fixed directions. The efficiency and practical applicability of the approach is demonstrated on a few synthetic waterflood optimization problems.

Discrete Parameter Problem Definition

The well placement optimization problem discussed above requires finding the location of wells (*i* and *j* indices of grid blocks where wells are to be located, represented by control vector $\mathbf{u}$) to maximize (or minimize) a performance measure $J(\mathbf{u})$. The optimization can be described generally with the following mathematical formulation:

$$\max_{\mathbf{u}} \left[J = \sum_{n=0}^{N-1} L^n \left(\mathbf{x}^{n+1}, \mathbf{u} \right) \right]$$

subject to:

$$g^n(\mathbf{x}^{n+1}, \mathbf{x}^n, \mathbf{u}) = 0 \qquad \forall n \in (0,..,N-1) \tag{1}$$

$$\mathbf{x}^0 = \mathbf{x}_0 \qquad \text{(Initial Condition)}$$

$$\mathbf{u} \in \Omega$$

Here, $\mathbf{x}^n$ refers to the dynamic states such as pressures, saturations, compositions etc. of the simulation model g^n representing the reservoir at time step *n* (*N* is the total number of time steps). The simulation model g^n together with the initial conditions defines the dynamic system, which are basically the reservoir simulation equations for each grid block at each time step:

$$g^n \left(\mathbf{x}^{n+1}, \mathbf{x}^n, \mathbf{u} \right) = Accumulation - Flux - Well \tag{2}$$

The objective function J in this work is the Net Present Value (NPV). It is defined as a summation over all time steps of a function L^n known as the Lagrangian in control literature (Stengel, 1985). Since L^n usually consists of well parameters or quantities that are functions of well parameters, it is written here in a fully implicit form. The definition of L^n used in this work is as follows (see the Nomenclature for definition of symbols):

$$L^n \left(\mathbf{x}^{n+1}, \mathbf{u} \right) = \sum_{j=1}^{N_P} \left[\frac{P_{op}}{\rho_{o,SC}} W_{op,j}^n - \frac{C_{wp}}{\rho_{w,SC}} W_{wp,j}^n \right] \frac{\Delta t^n}{(1+\alpha)^{t^n}} - \sum_{j=1}^{N_I} \frac{C_{wi}}{\rho_{w,SC}} W_{wi,j}^n \frac{\Delta t^n}{(1+\alpha)^{t^n}} \tag{3}$$

The last constraint of Equation (1), where Ω represents the spatial domain encompassed by the reservoir (simulation model), simply states that the wells have to be located within the simulation model. For example, this constraint could be simple bound constraints for rectangular simulation models, or could be nonlinear functions of the spatial variables if the model boundaries are curvilinear. Note however that this constraint is usually not a function of the dynamic states $\mathbf{x}^n$, and are therefore easy to handle with standard nonlinear programming algorithms (NLP).

Since the control vector $\mathbf{u}$ consists of the *i* and *j* indices of the grid blocks where the wells to be optimized are located, and because these indices are discrete in nature (as the simulation model is defined on a discrete numerical grid), a gradient of the objective function J with respect to $\mathbf{u}$ does not exist, and therefore gradient-based optimization algorithms cannot be applied to solve this optimization problem directly. Thus, in order to apply gradient-based optimization algorithms on this problem, a continuous approximation of the original problem has to be formulated.

Continuous Approximation

Consider the simple schematic of a numerical grid with a well in it, as shown in Figure 1. In the figure, *i* represents the grid indices in the *x* spatial direction and *j* represents the grid indices in the *y* spatial direction. A well is present in grid block ω with grid block indices (i_ω, j_ω) and spatial location (x_ω, y_ω). The grid in uniform rectangular with dimension $2\Delta x$ and $2\Delta y$. Note that a uniform grid is shown for the ease of explanation and is not a limitation of the method. Now, one approach to obtain a continuous approximation to this discrete parameter problem is to replace the original discrete control parameters (the *ij* locations of wells) with their continuous counterparts of the underlying spatial domain, that is, the spatial locations of the wells in the continuous *xy* domain, which are continuous variables. However, this alone does not solve the problem because the spatial location variables of the wells (x_ω and y_ω) do not directly appear in the dynamic system (mass balance equations g^n) that provide the functional relationship between the location of the well (control parameter) and the objective function (NPV). The problem is therefore to obtain a continuous functional relationship between the continuous well location variables (x_ω and y_ω) and the objective function J. Once such a relationship is defined, we can then easily obtain the gradient of the objective function with respect to x_ω and y_ω, and thereby apply a gradient-based optimization algorithm to obtain the optimal well locations.

Since it is not possible to replace the discrete grid with any continuous approximation in terms of the underlying spatial variables, we have to look at other aspects of the problem in order to determine how we can introduce functional dependence of the mass balance equations g^n on the well locations variables x_ω and y_ω. The solution is to look at the original continuous governing equations (partial differential equations) of which the discrete simulation equations are an approximation. Note that the methodology described in this work is independent of the number of phases or components of the simulation model, and therefore for simplicity, we consider the single phase governing equations (Aziz and Durlofsky, 2001):

$$\nabla.\left(\frac{\rho}{\mu}k\nabla p\right)-\tilde{m}^w=\frac{\partial}{\partial t}(\phi\rho) \tag{4}$$

In the above equation, all terms have their usual meanings. The term of interest with respect to this work is the source/sink term $\tilde{m}^w$ representing the addition or removal of fluids from the dynamic system. For two dimensional systems which will be considered in this work, source/sink terms are usually point sources/sinks (Gunduz and Aral, 2005), and therefore, this source/sink term for a point source/sink at (x_ω, y_ω) can be defined as:

$$\tilde{m}^w = m^w\delta^2\left(x-x_\omega, y-y_\omega\right) \tag{5}$$

Here, m^w is the usual well term that appears in the simulation equations (strength of the source/sink, with units mass/time), and δ^2 is the two dimensional Dirac-delta function (Gunduz and Aral, 2005) defined as:

$$\delta^2\left(x-x_\omega, y-y_\omega\right)=\delta\left(x-x_\omega\right)\delta\left(y-y_\omega\right)=\begin{cases}\infty & x=x_\omega, y=y_\omega\\ 0 & otherwise\end{cases} \tag{6}$$

Numerical discretization of Equation (4) would result in integration of Equation (5) over the domain of the reservoir Ω, and the final well terms in the discretized mass balance equations (resulting from the presence of the point source/sink at (x_ω, y_ω)) for any arbitrary grid block υ is given as:

$$\iint_{\Omega_\upsilon} m^w\delta^2\left(x-x_\omega, y-y_\omega\right)=\begin{cases}m^w & \upsilon=\omega\\ 0 & \text{otherwise}\end{cases} \tag{7}$$

Here, Ω_υ is the part of the reservoir domain Ω in grid block υ. It is clear from Equation (7) that the well term m^w will only appear in the mass balance equations of grid block ω as required.

Instead of looking at a continuous approximation to the grid, we well look at a continuous approximation to this discontinuous Dirac-delta function. Essentially, we want a continuous approximation to the discontinuous Dirac-delta function, that is, a continuous function that in the limit tends to the Dirac-delta function. One such function is the bivariate Gaussian function (Wikipedia, 2007a), because:

$$\lim_{\sigma\to 0}\frac{1}{2\pi\sigma^2}\exp\left[-\frac{1}{2\sigma^2}\left\{\left(x-x_\omega\right)^2+\left(y-y_\omega\right)^2\right\}\right]=\delta^2\left(x-x_\omega, y-y_\omega\right) \tag{8}$$

Furthermore, both are probability densities, because (Wikipedia, 2007a, Loeve, 1977):

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{2\pi\sigma^2}\exp\left[-\frac{1}{2\sigma^2}\left\{\left(x-x_\omega\right)^2+\left(y-y_\omega\right)^2\right\}\right]dxdy=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\delta^2\left(x-x_\omega, y-y_\omega\right)dxdy=1 \tag{9}$$

This bivariate Gaussian function is shown in Figure 2. As σ (standard deviation) is reduced, the function becomes more and more steep and ultimately tends towards the Dirac-delta function.

Replacing the Dirac-delta function with the bivariate Gaussian function in the governing equations (PDE) and then discretizing them results in a modification to the well terms in the mass balance equations, and these terms are approximations to the original well terms given by Equation (7). The new well term for any arbitrary grid block υ associated with the well in grid block ω is given as:

$$\iint_{\Omega_\upsilon} m^w\frac{1}{2\pi\sigma^2}\exp\left[-\frac{1}{2\sigma^2}\left\{\left(x-x_\omega\right)^2+\left(y-y_\omega\right)^2\right\}\right]dxdy \tag{10}$$

Here Ω_υ is the part of the domain of the reservoir in grid block υ. The most important result of this approximation is that, as opposed to Equation (7) where the well term is non-zero only in grid block ω, the new well term as given by Equation (10) is non-zero in every grid block, and is a continuous function of the well location variables (x_ω, y_ω). Thus, the mass balance equations g^n are now a function these variables (x_ω, y_ω), and therefore, a functional relationship between these variables and the objective function J (NPV etc) is now obtained. In essence, m^w is now distributed over the entire reservoir, and the magnitude of the well term for an arbitrary grid block υ depends on the distance of grid block υ from grid block ω and its size and shape.

The fact that the application of the above approximation results in a well term that is non-zero in every grid block is clearly a problem, because this implies that a well has to be created in every grid block, which is not practically feasible. However, this problem can be eliminated by realizing that the larger the distance of a grid block from grid block ω, the smaller the magnitude of the well term. By making σ small enough (but not zero, as then we will get back the original discrete parameter problem), the well terms for all grid blocks except for the grid blocks nearest to grid block ω can be made very small and thus discarded. Further, a small σ implies that the Gaussian approximation is very close to the Dirac-delta function, and therefore the modified continuous problem is a close approximation to the original discrete problem.

As a result of the application of the above approach, the original well in grid block ω is now surrounded by a set of "pseudo-wells" in the neighboring grid blocks and are defined using Equation (10) (see Figure 3). From an implementation perspective, these pseudo-wells (and also the well in grid block ω) have the same geometric well indices as the original well but weighted with the integral of the bivariate Gaussian function as in Equation (10). As explained above, these weights for each pseudo-well (and also the original well) is dependent upon its distance from the original well and the size and shape of the grid in which it is located. That is, the well term for these wells for a general multi-phase, multi-component simulation model is now given as:

$$f(x_\omega, y_\omega, \sigma) WI^\omega \sum_p \lambda_p \rho_p X_{cp} \left(p_p - p^W \right) \tag{11}$$

Every term in the above equation is standard notation (see Nomenclature) except the function f which is the weight determined from the integral of the Gaussian function, given again for clarity:

$$f(x_\omega, y_\omega, \sigma) = \iint_{\Omega_\upsilon} \frac{1}{2\pi\sigma^2} \exp\left[-\frac{1}{2\sigma^2} \left\{ (x - x_\omega)^2 + (y - y_\omega)^2 \right\} \right] dxdy \tag{12}$$

To elaborate and clarify further, consider the uniform rectangular grid given in Figure 1. The weight function f for a grid block υ centered at (x_υ, y_υ) is then given as:

$$f(x_\omega, y_\omega, \sigma) = \int_{x_\upsilon - \Delta x}^{x_\upsilon + \Delta x} \int_{y_\upsilon - \Delta y}^{y_\upsilon + \Delta y} \frac{1}{2\pi\sigma^2} \exp\left[-\frac{1}{2\sigma^2} \left\{ (x - x_\omega)^2 + (y - y_\omega)^2 \right\} \right] dxdy \tag{13}$$

This double integral can be evaluated analytically, and is given as:

$$\begin{aligned} f(x_\omega, y_\omega, \sigma) = \frac{1}{4} & \left\{ erf\left(\frac{x_\upsilon + \Delta x - x_\omega}{\sqrt{2}\sigma} \right) - erf\left(\frac{x_\upsilon - \Delta x - x_\omega}{\sqrt{2}\sigma} \right) \right\} \\ & \left\{ erf\left(\frac{y_\upsilon + \Delta y - y_\omega}{\sqrt{2}\sigma} \right) - erf\left(\frac{y_\upsilon - \Delta y - y_\omega}{\sqrt{2}\sigma} \right) \right\} \end{aligned} \tag{14}$$

Here *erf* is the error function (Abramowitz and Stegun, 1972). However, for arbitrary shaped grid block (non-uniform grids), it may not be possible to obtain this integral analytically. In such a case, numerical integration may be applied. Note that this numerical integration is trivial computationally as it is done external to the simulator.

The final modification required to the original problem to complete the continuous approximation is to modify the original objective function to also include the pseudo-wells associated with each well to be optimized (see Equation (15)). In Equation (15), it is assumed that all the N_P producers and N_I injectors in the simulation model are to be optimized. Note that it is trivial to exclude certain wells from being optimized by not creating any pseudo-wells for such wells. Also note that the control vector $\mathbf{u}$ now refers to the continuous well location variables (x_ω, y_ω) and not the discrete grid block indices (i, j) as in the original problem.

$$L^n\left(\mathbf{x}^{n+1},\mathbf{u}\right)=\sum_{j=1}^{N_P}\left[\frac{P_{op}}{\rho_{o,SC}}\left\{W^n_{op,j}+\sum_{k=1}^{8}W^n_{op,jk}\right\}-\frac{C_{wp}}{\rho_{w,SC}}\left\{W^n_{wp,j}+\sum_{k=1}^{8}W^n_{wp,jk}\right\}\right]\frac{\Delta t^n}{\left(1+\alpha\right)^{t^n}}$$
$$-\sum_{j=1}^{N_I}\frac{C_{wi}}{\rho_{w,SC}}\left\{W^n_{wi,j}+\sum_{k=1}^{8}W^n_{wi,jk}\right\}\frac{\Delta t^n}{\left(1+\alpha\right)^{t^n}} \quad (15)$$

To summarize, the essence of Equation (12) and Equation (13) or Equation (14) is that now the well terms of all the pseudo-wells (and also the original well in grid block ω) in the mass balance equations g^n are continuous functions of the continuous well location variables $\left(x_\omega, y_\omega\right)$ of the original well, and thus a continuous functional relationship between these variables and the objective function J (NPV etc) is obtained. Therefore, the gradient of the objective function with respect to these variables can now be calculated either with numerical perturbation or with adjoints easily. These gradients will be an approximation to the gradients obtained from the underlying PDE on the continuous spatial domain of which the numerical model is an approximation. Note again that although the creation of pseudo-wells is similar to that of the Handels et al. (2007) algorithm, the key advantages over the Handels algorithm are that the actual gradient of objective function J with respect to the well location variables $\left(x_\omega, y_\omega\right)$ are obtained, and any arbitrary search direction is possible at each iteration unlike the Handels algorithm, where the search direction is limited to 8 fixed directions. Further, the step size is not limited to 1 grid block, but is determined using a line search procedure as in any standard gradient-based optimization algorithm.

Note again that by controlling σ, we can control the degree of approximation to the original problem. Ideally, σ should be such that the weights of the pseudo-wells are very small compared to the weight of the original well, so that the approximation is very small. Also note that although Figure 3 shows one ring of pseudo-wells around the original well, the approach is not limited to one ring. The possible benefit of having more than one ring could be that, although as the number of rings of pseudo-wells is increased (and σ increased), the model becomes more and more approximate and the gradient direction obtained would also be more approximate, having more rings implies that we are looking at a larger region around the well to calculate the descent direction, and therefore the search direction obtained should be a better direction in a global sense. Another point to note is that, although the bivariate Gaussian function is used in this work, in principle, any continuous function that satisfies Equation (8) and Equation (9) can be used. For example, it may be more advantageous and appropriate to use the exponential distribution (Wikipedia, 2007b) near the model boundaries. These issues will be investigated in future work.

Gradient Calculation with Adjoints

Adjoint models have been widely employed for calculating gradients for the optimal well control problem, that is, determining the optimal production and injection rates or bottom hole pressures of wells to maximize an objective such as NPV. Some papers in this category include those by Mehos and Ramirez (1989), Zakirov et al. (1996), Asheim (1988), Vironovsky (1991), Sudaryanto and Yortsos (2001), Brouwer and Jansen (2002), and Sarma et al. (2005). From an implementation perspective, the optimal well control adjoint can be used with minor modifications to calculate the gradient for the well placement problem. The adjoint equations essentially remain the same as in the optimal control problem (Sarma, 2006), and are given as:

$$\boldsymbol{\lambda}^{Tn}=-\left[\frac{\partial L^{n-1}}{\partial \mathbf{x}^n}+\boldsymbol{\lambda}^{T(n+1)}\frac{\partial g^n}{\partial \mathbf{x}^n}\right]\left[\frac{\partial g^{n-1}}{\partial \mathbf{x}^n}\right]^{-1}\quad \forall n=1,...,N-1$$
$$\boldsymbol{\lambda}^{TN}=-\left[\frac{\partial L^{N-1}}{\partial \mathbf{x}^N}\right]\left[\frac{\partial g^{N-1}}{\partial \mathbf{x}^N}\right]^{-1}\qquad \text{(Final Condition)} \quad (16)$$

Because the weighting function $f\left(x_\omega, y_\omega, \sigma\right)$ of Equation (12) is not a function of the state variables $\mathbf{x}^n$, only minor modifications to the $\partial L^{n-1}/\partial \mathbf{x}^n$ code is required to take the pseudo-wells into account (consistent with Equation (15)), and the rest of the terms in Equation (16) are virtually unchanged. Refer to Sarma et al. (2006) for details on the calculations of the various terms of the above adjoint equations.

Once the Lagrange multipliers $\boldsymbol{\lambda}^n$ are calculated using Equation (16), the final gradient of objective function J with respect to the well location variables $\left(x_\omega, y_\omega\right)$ is calculated as:

$$\frac{dJ}{d\mathbf{u}}=\sum_{n=0}^{N-1}\left[\frac{\partial L^n}{\partial \mathbf{u}}+\boldsymbol{\lambda}^{T(n+1)}\frac{\partial g^n}{\partial \mathbf{u}}\right] \quad (17)$$

Since both L^n and g^n are functionally dependent on $\mathbf{u}\left(x_\omega, y_\omega\right)$ only through the well terms (Equation (11)), which further depend on $\mathbf{u}$ only through $f\left(x_\omega, y_\omega, \sigma\right)$, the calculation of the partial derivatives in Equation (17) require $\partial f / \partial x_\omega$ and $\partial f / \partial y_\omega$, which can be calculated analytically for Equation (14) or calculated numerically for more complicated cases. An important point to note is that both the derivatives $\partial f / \partial x_\omega$ and $\partial f / \partial y_\omega$ are calculated externally to the simulator.

The essence of this discourse is that, if a control adjoint is available and is implemented as described in Sarma (2006), then all modifications necessary to implement the well placement adjoint are external to the forward simulator, and can thus be accomplished relatively easily.

Example 1

This is a very simple example to validate that the gradients calculated with the above approach are indeed the right gradient directions. The simulation model is that of a simple 2D 2-phase black oil model of a horizontal square reservoir with a quarter 5-spot pattern. The reservoir covers an area of 1476×1476 ft^2 and has a thickness of 33 ft and is modeled by a 45×45×1 horizontal 2D grid. The fluid system is an essentially incompressible two-phase unit mobility oil-water system, with zero connate water saturation and zero residual oil saturation. The porosity (0.3) and permeability (10 d) are completely homogeneous. The producers are placed at the four corners, and the objective is to find the location of the injector such that NPV is maximized. All the wells are set at constant BHP values (not changed during optimization) and the model is run for 950 days. Thus, the only unknowns in the optimization problem are the x and y locations of the injector.

The objective of this study is to determine if the gradient direction (of NPV with respect to. injector location x_ω and y_ω) obtained with the above-described approach is correct. In order to do so, first an exhaustive search is done over the entire search space, that is, the injector is placed at all the 45*45 = 2025 grids and NPV is calculated. This provides the objective function over the entire search space as is shown in the Figure 4. It is clear that because the porosity and permeability is homogeneous, and the model is completely symmetric, the objective function is a nice convex function with the maximum at the center, that is, the maximum NPV is obtained when the injector is located at the center as expected. Now, the gradient direction is calculated for the injector well placed at various different locations. These directions are plotted along with the contours of the objective function in Figure 5.

In Figure 5, the blue point is the optimum, the red point is the injector location where the gradient is calculated, and the black arrow is the negative of the gradient direction (steepest ascent direction) calculated with proposed algorithm. It is clear that for all cases the gradient direction obtained is the right gradient direction as this direction is orthogonal to the contour line at the injector location for all cases. Further, for this very simple, convex and almost bowl shaped objective function, the gradient direction directly points towards the optimum for all well locations shown above, which implies that for all the above starting points, the optimum will be obtained in one iteration with any gradient-based optimization algorithm.

Example 2

The second example is the same as above, except for the permeability field, which is now heterogeneous (multi-Gaussian) as seen in Figure 6 (log permeability), with permeabilities ranging from 100md to 10d. This is generated using the *'sgsim'* software (Deutsch and Journel, 1998). The problem is to determine the optimal location of the injector such that NPV is maximized over a period of 950 days. Again, to validate the approach, an exhaustive search is done to obtain the NPV over the entire search space, and the contours of NPV are plotted in Figure 7. We observe that there is a large global maximum, but there are quite a few local minimum is this problem.

Figure 8 to Figure 11 show the results of optimization using the projected gradient optimization algorithm (Kelley, 1999) with various initial guesses. In each figure, the left figures show the NPV contours with the current injector location (black dot), and the right figures show the final water saturation after 950 days given the current injector location. The top figure in each case is the initial guess, and the bottom figure is the converged solution.

It is clear that in the first 3 cases, we are able to get to the global optimum, with the first and third case taking 4 iterations, and the second case taking 8 iterations of the optimization algorithm. Each iteration requires about 4-5 simulations including adjoint simulations. Note that the line search algorithm currently implemented is quite simple, and thus there is scope of further reduction in the number of iterations. For the fourth case, where the initial guess is quite close to a local optimum, we get stuck at the local optimum, which is expected as gradient-based algorithms are local algorithms. However, given the nature of the objective function for this case, it is clear that the global optimum will be obtained starting from most locations except possibly from regions close to the local optima.

Example 3

In this third and final example, the simulation model is again that of a 2D 2-phase black oil model of a horizontal square reservoir but with 4 injectors and 9 producers. The reservoir covers an area of 3333×3333 ft^2 and has a thickness of 33 ft and is modeled by a 101×101×1 horizontal 2D grid. The fluid, rock, and rock-fluid properties are the same as above, and porosity is homogeneous (0.2). The permeability field (log perm) shown in Figure 12 is generated using a multi-point geostatistical algorithm called *'filtersim'* (Zhang, 2006), and is the main source of complexity in this optimization problem. The red colors represent high permeability channels and the blue colors represent a low permeability background. The objective of this study

is to find the optimal locations of all the 13 wells such that NPV is maximized. All the wells are set at constant BHP values (not changed during optimization) and the model is run for 1900 days.

The optimization is started with an arbitrary initial guess for the well locations as seen in Figure 13 (injectors as blue circles and producers as black circles). The figure shows final water saturation map after 1900 days for this well configuration. Figure 14 shows the final water saturation map and well locations after convergence of the optimization algorithm (projected gradient, Kelly, 1999), which took 5 iterations resulting in about 25 simulations. This is evidently a very reasonable and affordable number of simulations. It is also clear that the increase in sweep efficiency and therefore NPV is quite significant at about 40% (see Figure 15). Furthermore, the sweep and NPV are also improved by about 8% compared to a standard pattern drive, as seen in Figure 16.

Conclusions

In this work, a novel, direct and efficient well placement optimization algorithm using gradient-based optimization algorithms and adjoint models was proposed. A continuous approximation to the original discrete-parameter well placement problem is defined such that gradients can be calculated on the approximate problem, and gradient-based algorithms can then be employed for efficiently determining the optimal well locations. As a result of application of the continuous approximation, the original wells are surrounded by pseudo-wells whose geometric well indices are weighted by spatial integrals of a continuous (Gaussian) function, which is a function of the x and y locations of the original wells (continuous variables). Thus, adjoints and standard gradient-based optimizations algorithms can now be applied on the continuous approximate problem to obtain the optimal well locations. To our knowledge, this is the first direct application of gradients and adjoints to the well placement problem. Previous applications of gradients and adjoints to this problem were indirect in nature, and suffer from certain limitations as a result of which practical application is not feasible. Also, because the proposed approach is gradient-based, it is generally more efficient compared to existing stochastic algorithms such as genetic algorithms. The efficiency and practical applicability of the approach was demonstrated on a few synthetic waterflood optimization problems.

Nomenclature

$C_{w,p}$	*Water production costs per Bbl*
$C_{w,i}$	*Water injection costs per Bbl*
f	*Weight function of geometric well index*
g	*Dynamic system equations*
J	*Objective function*
k	*Permeability*
L	*Lagrangian*
$\tilde{m}^w$	*Source/sink term*
m^w	*Source/sink term strength or well term in mass balance equations*
N	*Number of control steps*
N_g	*Number of grid blocks*
N_P	*Number of producers*
N_I	*Number of injectors*
p	*Grid block pressure*
p^W	*Well bottom hole pressure*
$P_{o,p}$	*Oil price per Bbl*
t	*Time*
$\mathbf{u}$	*Control vector*
W	*Well terms of simulation equations*
WI	*Geometric well index*
x	*Spatial variable*
$\mathbf{x}$	*Dynamic states*
X	*Component mass fraction*
y	*Spatial variable*

Greek

α	*Discounting factor*
δ	*Dirac delta function*

ϕ *Porosity*
Φ *Nonlinear mapping to feature space*
λ *Mobility*
$\boldsymbol{\lambda}$ *Lagrange multipliers*
ρ *Fluid density*
σ *Standard deviation*
ω *Grid block where well is located*
υ *Arbitrary grid block*
Ω *Simulation grid spatial domain*

Subscripts

c *Component*
i *Summation index, grid block index*
j *Summation index, grid block index*
k *Summation index*
o *Oil*
p *Phase*
w *Water*
SC *Standard conditions*
ω *Grid block where well is located*

Superscripts

n *Time level*
W *Well*
ω *Grid block where well is located*

Abbreviations

BHP *Well bottom hole pressure*
NLP *Nonlinear programming*
NPV *Net present value*
PDE *Partial differential equations*

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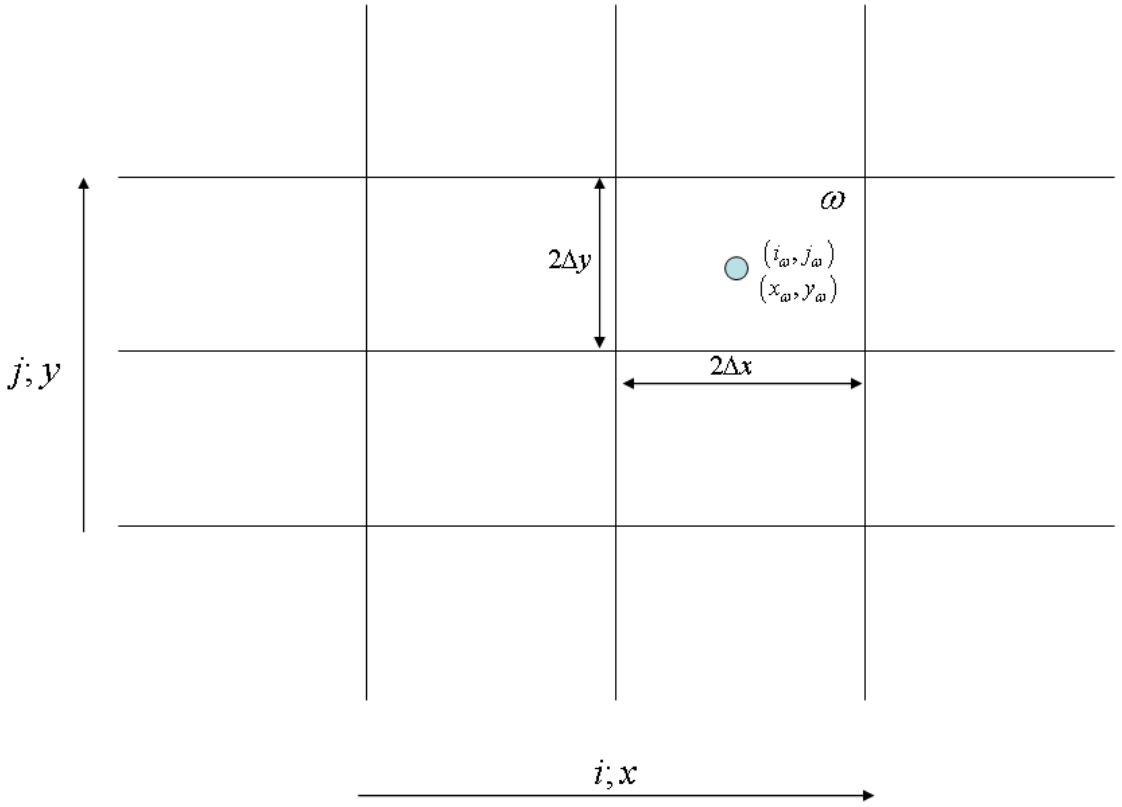

Figure 1 Schematic of a uniform rectangular grid with a well located in grid block ω

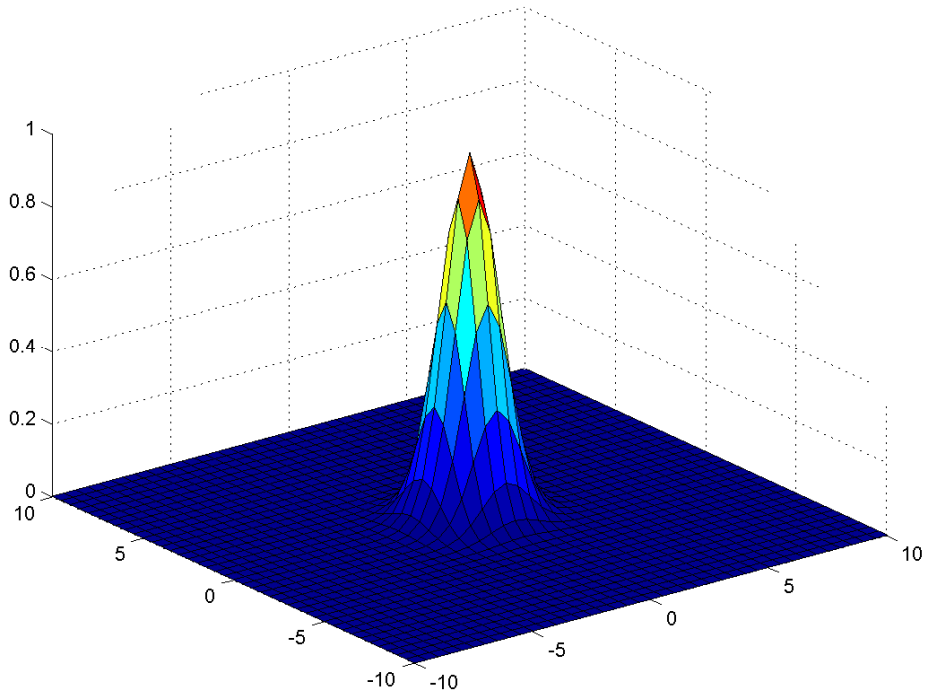

Figure 2 The bivariate Gaussian function

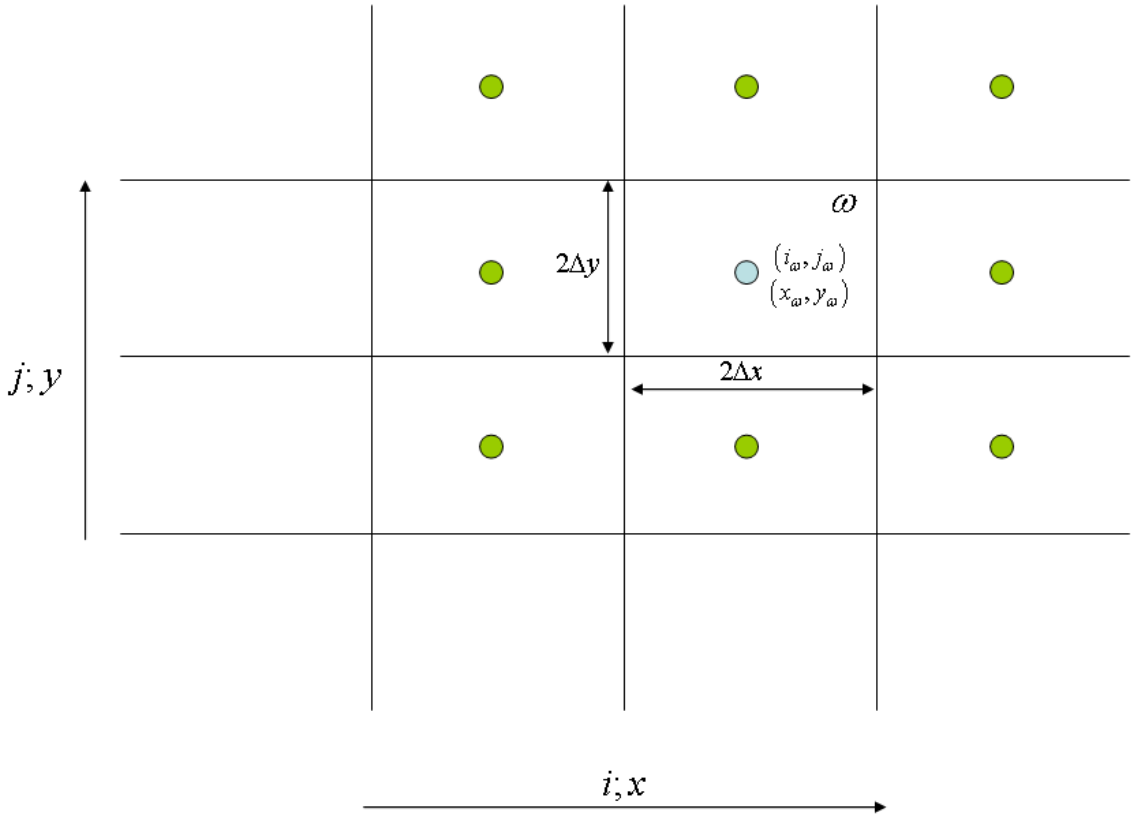

Figure 3 Original well with surrounding pseudo-wells

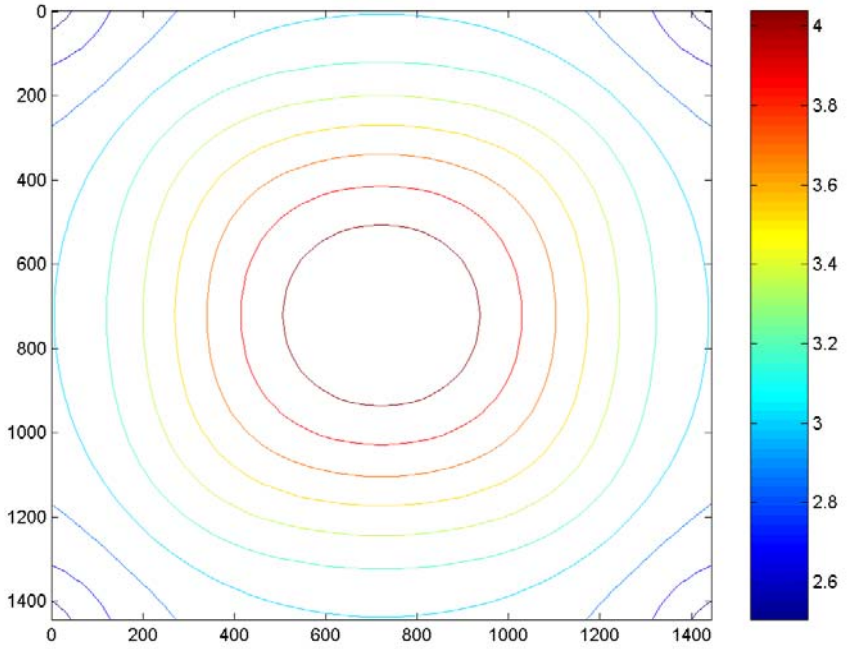

Figure 4 The contours of the objective function NPV with respect to injector location on xy domain

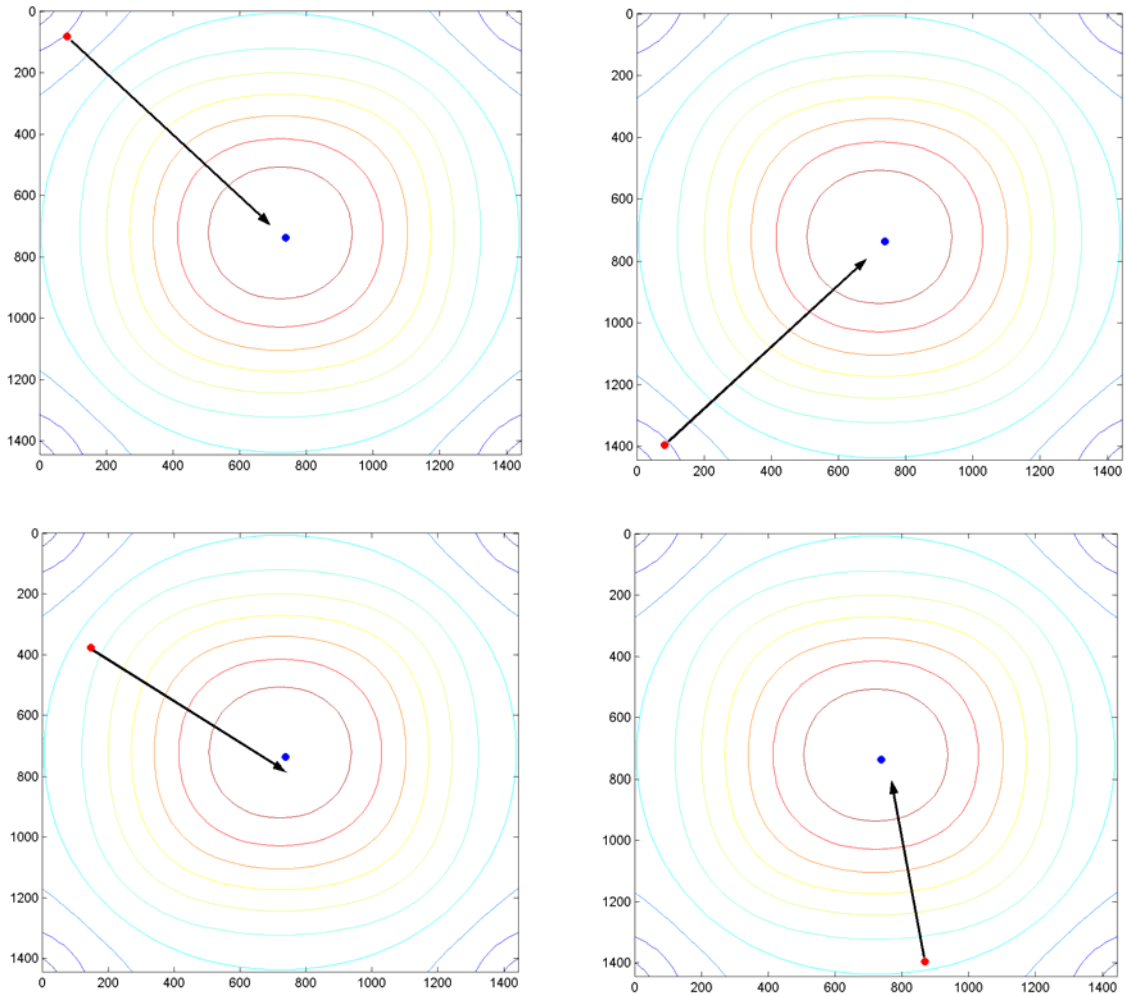

Figure 5 The gradient direction of NPV w.r.t. injector well location for various locations

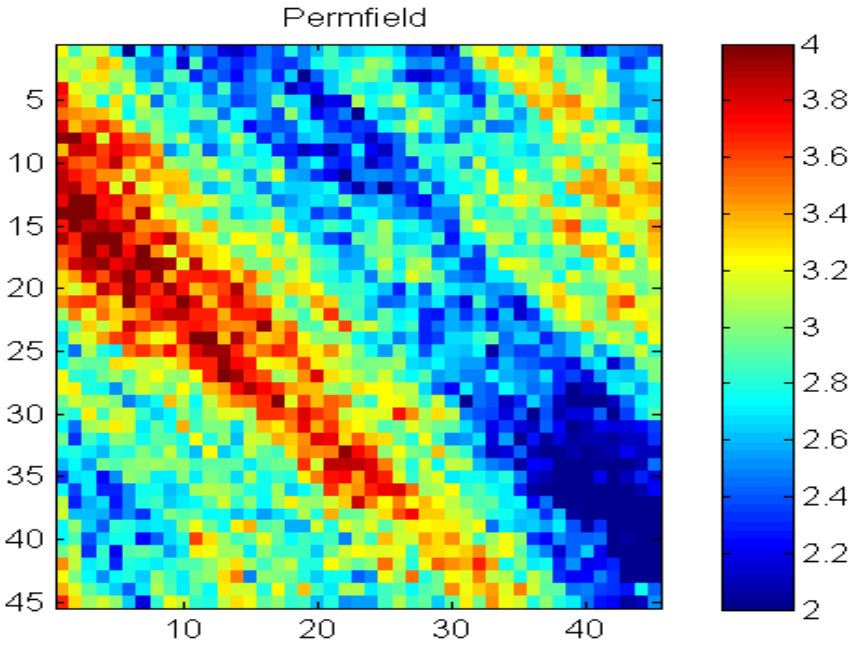

Figure 6 Permeability field (log perm) generated using *'sgsim'* (Deutsch and Journel, 1998)

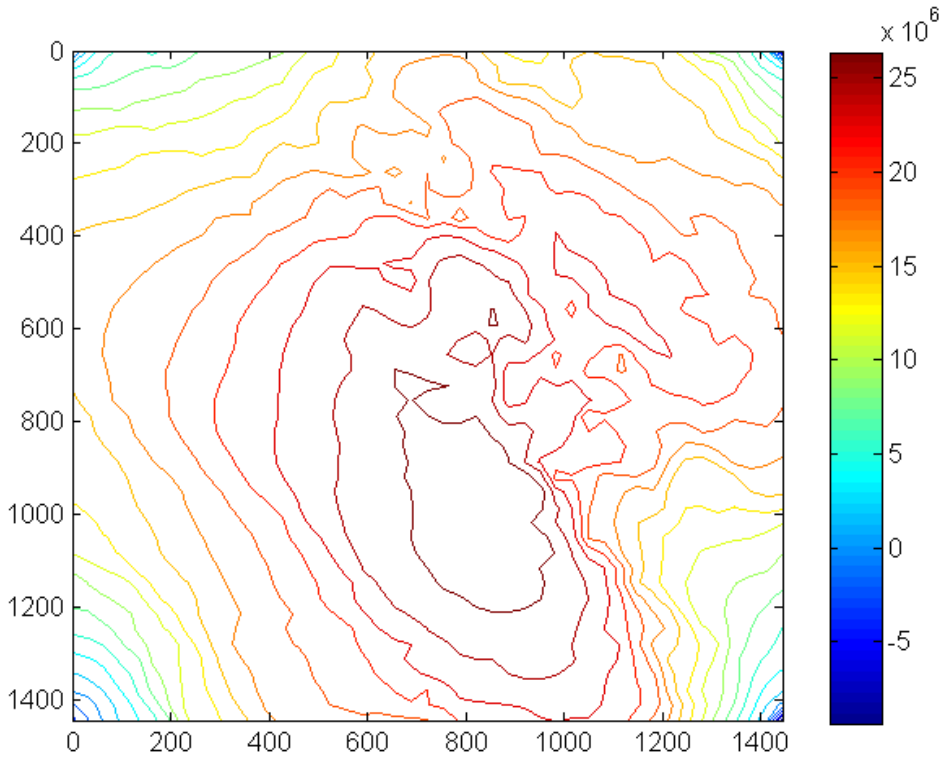

Figure 7 Contours of NPV w.r.t injector location obtained from exhaustive search

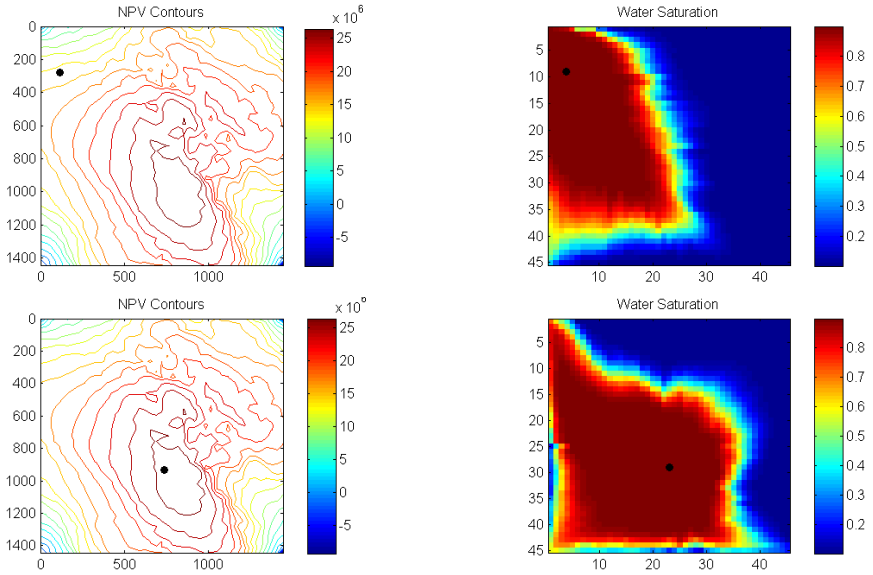

Figure 8 Initial and final well location plotted on the NPV contour and final water saturation maps

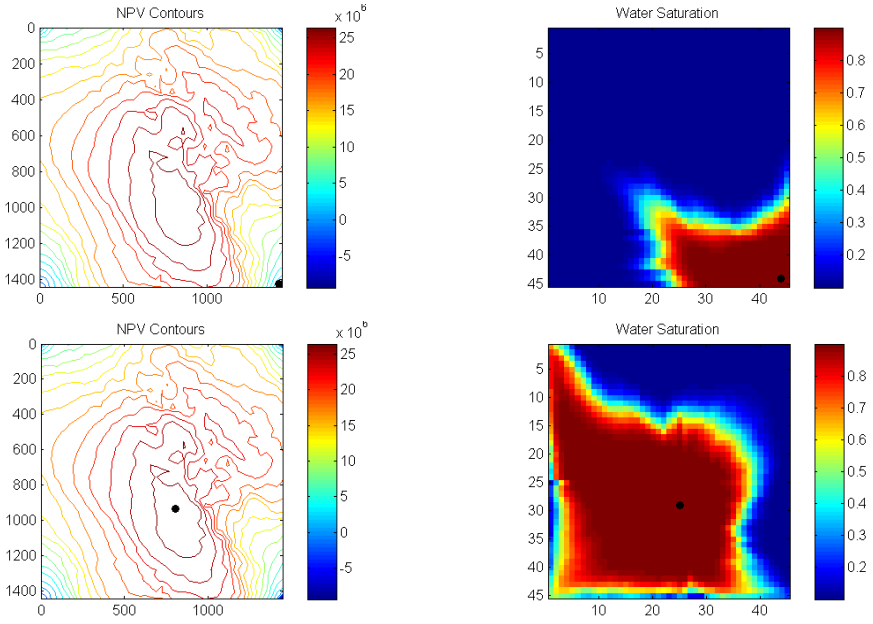

Figure 9 Initial and final well location plotted on the NPV contour and final water saturation maps

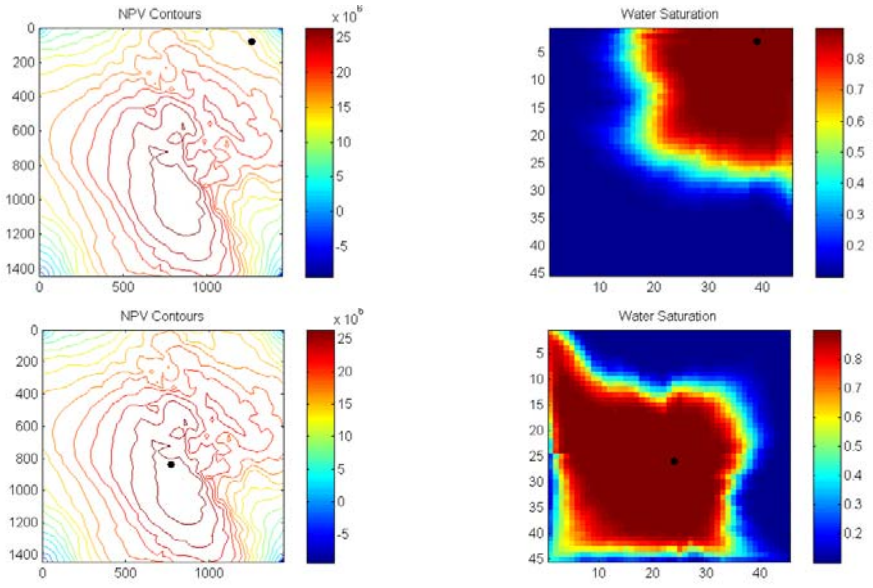

Figure 10 Initial and final well location plotted on the NPV contour and final water saturation maps

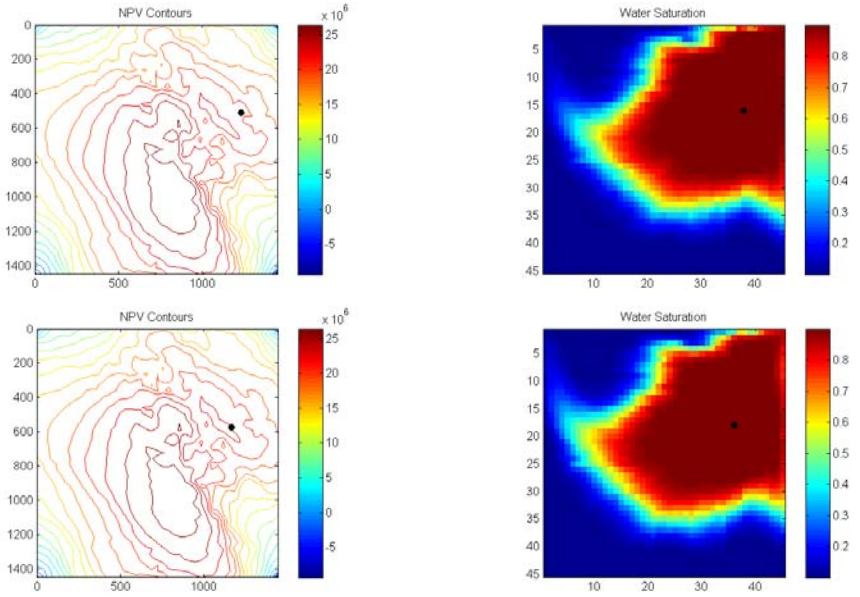

Figure 11 Initial and final well location plotted on the NPV contour and final water saturation maps

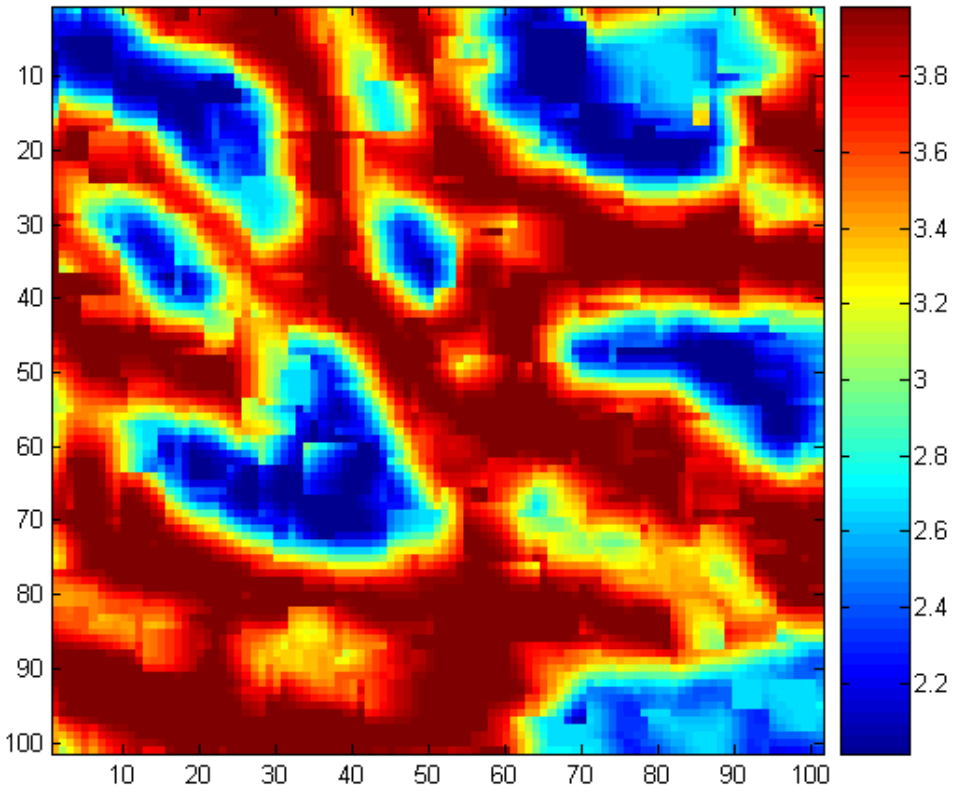

Figure 12 Channelized permeability field (log perm) obtained with *'filtersim'* (Zhang, 2006)

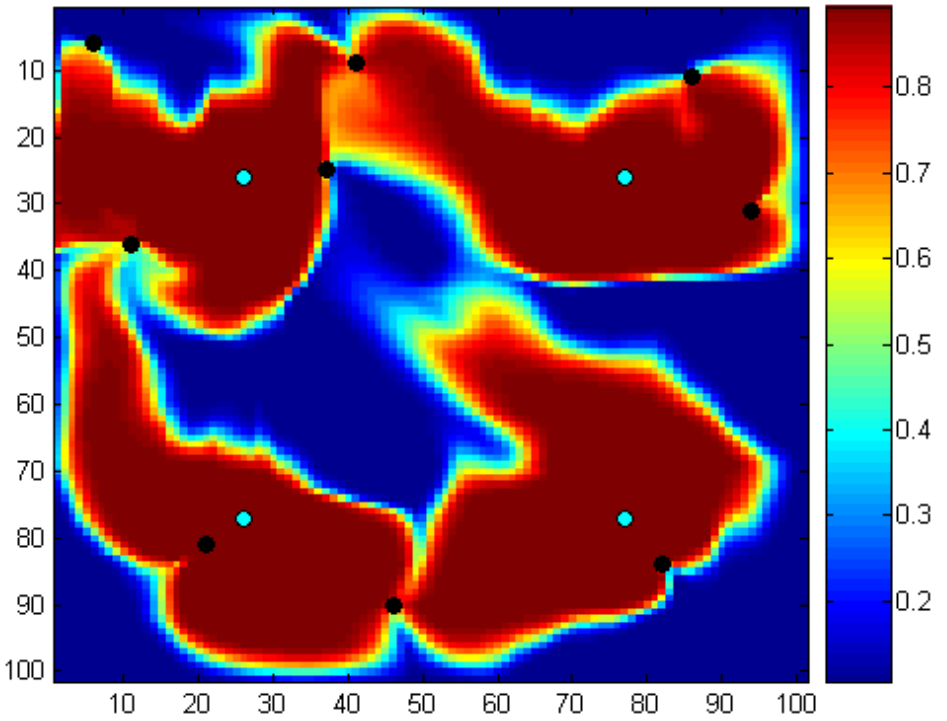

Figure 13 Final water saturation map for initial guess well configuration

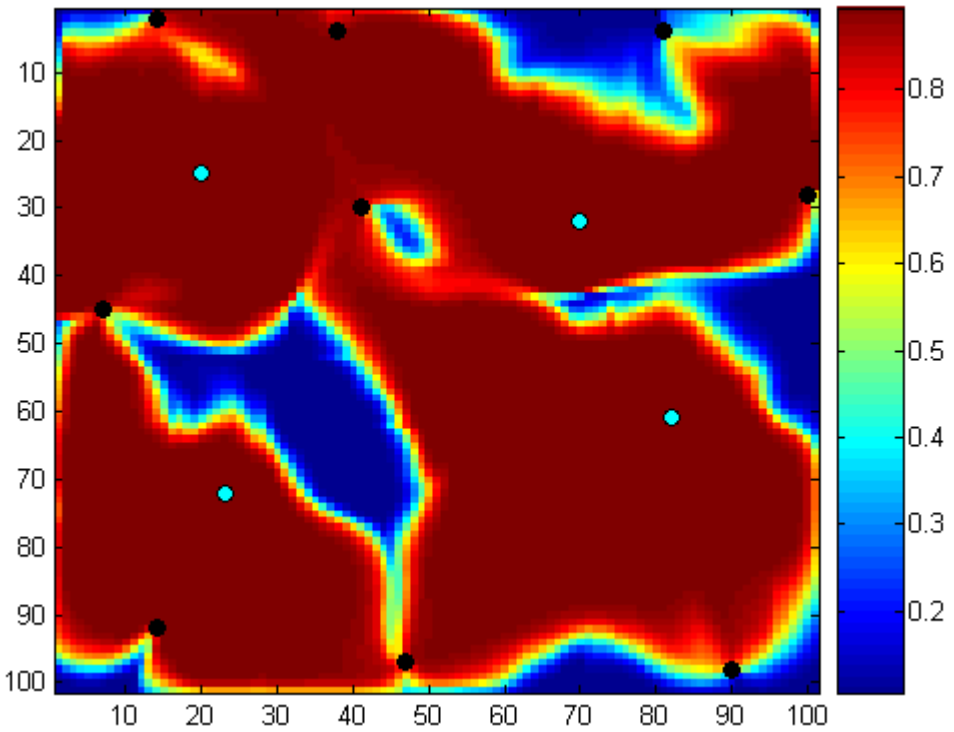

Figure 14 Final water saturation map for optimized well configuration

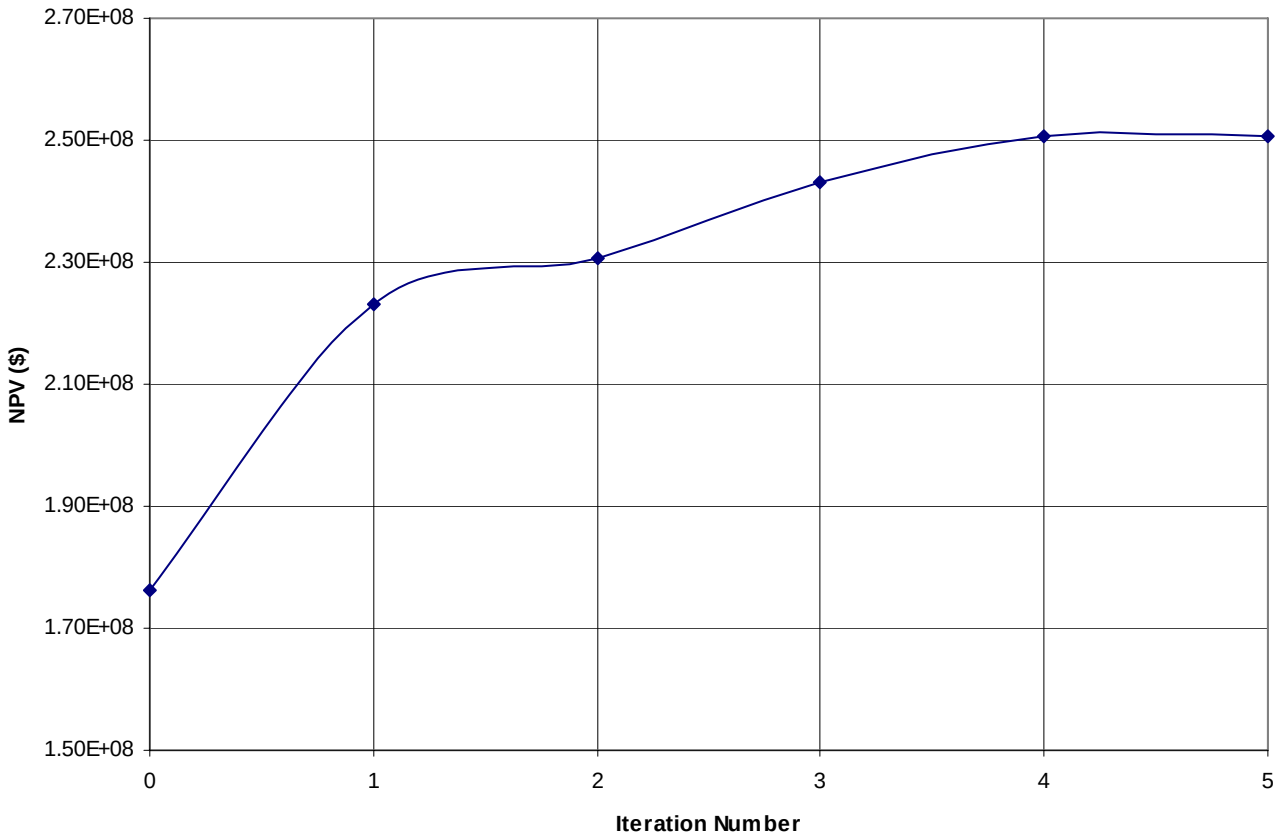

Figure 15 Increase in NPV with iteration

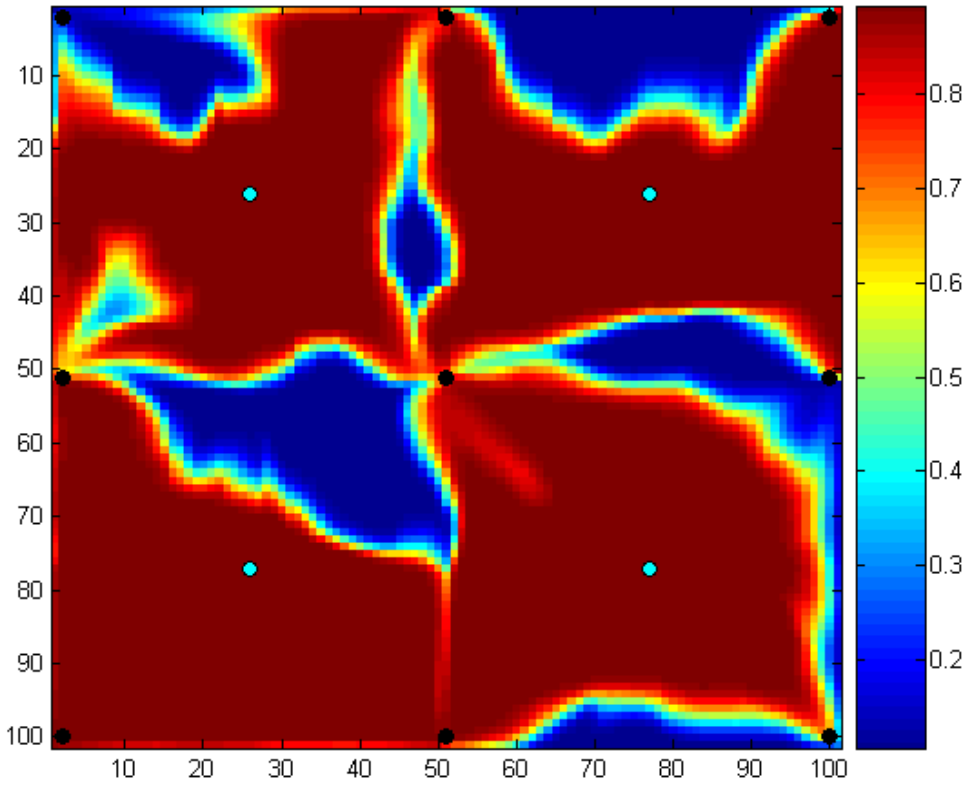

Figure 16 Final water saturation map for standard pattern drive