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Optimization of Smart Well Production Through Nonlinear Model Predictive Control

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Abstract

In this paper, we present an algorithm for optimizing reservoir production using smart well technology. The term smart well is used to indicate an unconventional well equipped with down hole inflow control valves (ICVs) and instrumentation. This additional instrumentation extends the degree of freedom in the field production planning, since production can be efficiently distributed on the different well segments available. By proper utilization of the ICVs through optimal production planning, an increased oil recovery for the reservoir can be expected.

We propose a method for optimal closed-loop production known from control theory as model predictive control (MPC). A commercial reservoir simulator, `ECLIPSE`, is used for modeling and predictions. MPC is chosen for its ability to provide an optimal solution for the constrained multivariable control problem. To compute the optimal ICV settings, we propose using a nonlinear MPC (NMPC) application, which can handle the severe nonlinearities found in reservoir models. The NMPC uses a single shooting multi-step quasi-Newton (SSMQN) method to solve the optimization problem. As the term multistep suggests, this is an iterative method which solves a sequence of quadratic problems (QPs) in each time step.

We apply our method to a benchmark reservoir model with multiple geostatistical realizations. This model has already proven potential for increased oil recovery by using optimization techniques. We show an even additional increase over the former approach in production totals, using the SSMQN method, with as much as 68% increase in one case, and 30% on average compared to a reference case.

Introduction

Reservoir management has traditionally been performed on the basis of long and short term plans made by production engineers in a manual, ad hoc fashion. The overall goal is obviously to maximize the total hydrocarbon production and recovery factor while minimizing total cost and staying within operational constraints. But reservoir models have generally been viewed as too large and computer resources too scarce to apply full scale production optimization. Meanwhile, on the downstream end of the production line and in process industry in general, advanced control techniques have been gradually developing and implemented with prosperous results.

Recent technological advances have opened for new possibilities within reservoir production. New reservoir mapping techniques offer more accurate reservoir models and the computational cost of simulating the models has decreased significantly. Well completions are more sophisticated than ever and supply new dimensions of flexibility to the day to day field operation. This new well generation is better known as smart wells. A smart well is a unconventional well equipped down hole with ICVs. Smart wells offer control of the total flow through individual segments and branches, as well as temperature and pressure measurements. The potential benefits from proper use of ICVs in a real-time control application are substantial. This is because continuous redistribution of the production from the available branches can delay or avoid break through of gas and/or water for as long as possible.

These advances combined with a growing motivation from the oil companies' side to increase the recovery factor of each production field, have spawned a high level of attention to the reservoir optimization problem. Reservoir optimization is today an active field of research, and has already been investigated by a number of authors since the turn of the millennium. Although, some early attempts were made by Asheim [1] and Virnovsky [2], most acknowledge Sudaryanto and Yortsos [3] to be the first to systematically address the flooding problem. They used optimal control theory to maximize the sweep efficiency for a multiple source (injector), single sink (producer) system. By optimally allocating the injection rate for each injector they showed a "bang-bang" strategy to maximize the displacement efficiency, as this caused a simultaneous breakthrough from both sources at the producing end. Brouwer et al. [4] made a study from a less theoretical point of view, focusing on production potential available through smart well control. Using a heuristic algorithm for static optimization, they developed a production strategy for simple reservoir models. This work was extended by Dolle et al. [5] who developed an algorithm for dynamic optimization, using gradient-based optimization. In addition to improving the results from the static approach, they also addressed reservoirs with heterogeneous permeability fields.

The above mentioned work was made with an assumption of constant production rates. Brouwer and Jansen [6] recognized that this was hardly common in practice, and therefore investigated the problem further, as they compared the constant production rate case with the constant bottom hole pressure (BHP) case. These cases were argued to illustrate the two extremes of well-operating conditions, as practical production planning need to take them both into consideration. Yeten et al. [7] combined optimization, using a conjugate gradient method, with features available in a commercial reservoir simulator. The simulator is used for numerical gradients, as well as for efficient modeling of ICVs. Though costly in terms of computation time, their algorithm produced promising results. Sarma et al. [8] used an approximate feasible direction algorithm, in combination with a general purpose simulator. Exchanging exact gradient information directly with the simulator, and from the efficiency of the approximate feasible direction algorithm, they proved to match results by Yeten [9] using only a fraction of the CPU time.

In this paper we will extend this line of work by using a well known technique from the field of advanced process control known as NMPC to optimize reservoir production through closed-loop control. First we will describe the general principles of NMPC before outlining an NMPC algorithm specially designed to interface a black-box simulator which is to perform the reservoir modeling. We then apply our developed algorithm to a set of reservoir models, previously used in Yeten et al. [7] and Yeten [9]. The models are multiple geostatistical realizations of a fluvial reservoir with a horizontal multilateral well. All models are applied to the optimization routine, and results are compared both to base case numbers, and the results from previously published optimizations. We will show that our NMPC algorithm, by dynamic optimization and reducing the sample time, further increases production potentials over the previously used methods.

Model Predictive Control

MPC is one of today's most commonly used techniques within advanced process control (APC). It is the largest sub group of a general class of methods known as *predictive control,* which is claimed to be the only class of APC to have significant impact in industrial control engineering (Maciejowski, 2002) [10]. The main reason for the widespread acceptance is because MPC combines the principle of optimality with the robustness of closed-loop control, while efficiently handling constraints on system inputs and outputs at the same time. Mayne et al. [11] define MPC as "a form of control in which the current control action is obtained by solving *on-line*, at each sampling instant, a finite horizon open-loop optimal control problem". The controller's closed-loop property, which is an important feature on the issue of controller stability, is inherited from the repeated solving for optimality of the finite horizon open-loop problem for every subsequent time step starting from the observed process state. The control law is calculated for a given control horizon, T_c, and the dynamic behavior of the system is calculated over the prediction horizon, T_p, where $T_c \leq T_p$. The basic idea is illustrated in **Fig. 1**. A system is sought to be controlled to a set point, $r(t)$. The controller calculates an optimal input sequence, parameterized as a piecewise constant function of time, for the control horizon. At the next sampling interval all horizons are moved one step forward.

Consider a general class of continuous time systems described by the differential equation

$$x(t) = f(x(t), u(t)),\ x(t_0) = x_0$$

which is subject to input and state constraints of the form:

$$u(t) \in U, \forall t \geq t_0,$$
$$x(t) \in X, \forall t \geq t_0.$$

The inputs are given in the vector $u(t) \in \mathbf{R}^m$ and $x(t) \in \mathbf{R}^n$ denote the state vector. U is a set of input constraints assumed to be compact and X is a set of state constraints assumed to be connected [12]. These are the *model equations*. They play an important role in the controller, as it uses the model for system predictions to calculate the optimal inputs. The optimal open-loop control is given by solving at every time instant:

$$\min_{\bar{u}(t)} J(x(t), \bar{u}(t)) = \int_t^{t+T_p} F(\bar{x}(\tau), \bar{u}(\tau)) d\tau$$

subject to

$$\dot{\bar{x}}(\tau) = f(\bar{x}(\tau), \bar{u}(\tau)), \bar{x}(t) = x(t)$$
$$\bar{u}(\tau) \in U, \forall \tau \in [t, t + T_c]$$
$$\bar{u}(\tau) = \bar{u}(t + T_c), \forall \tau \in [t + T_c, t + T_p]$$
$$\bar{x}(\tau) \in X, \forall \tau \in [t, t + T_p]$$

where T_p and T_c refer to the control horizons for predictions and control, respectively. The $\bar{u}$ denotes the internal controller variables and $\bar{x}$ refers to the system response to the input vector $\bar{u}$, i.e. the solution to the defined problem. Controller variables are also called *manipulated variables* (MVs) and the *controlled variables* (CVs) are the significant process measurements available. The cost functional J is the sum of the performance costs, F, at each time step. F is often found in a quadratic form displaying some economical consideration for the underlying system:

$$F(x,u) = (x - x_{op})Q(x - x_{op}) + (u - u_{op})R(u - u_{op}).$$

In this case the cost is given as a result of deviations from an operational set point, specified by positive definite weighting matrices Q and R.

The complexity of an MPC application depends heavily on the complexity of the model. For a simple linear case with a quadratic cost function, linear model and linear constraints, the control problem is reduced to a standard quadratic program (QP), solvable by standard QP solver. This is a common class of controllers called *linear MPC*. Note that this does not imply a linear controller, as the linear MPC will show nonlinear closed loop dynamics in the presence of constraints [12]. Considering the severe nonlinearities in a reservoir model, a linear MPC will not at all be able to provide an optimal solution to the reservoir management problem. Such a problem calls for a nonlinear MPC scheme. Nonlinear MPC must solve a nonlinear program (NLP) at each time step. Solving an NLP is much more an area of research than solving a QP. In the next section we will present a scheme for solving the NLP using a single shooting multi-step quasi-Newton method (SSMQN), specially designed to solve this class of constrained nonlinear control problems.

A multistep quasi-Newton method

Reservoir models are fairly complex and highly nonlinear. This makes analytical model representations from first principles costly to develop. This paper will bypass this modeling exercise, and use the commercial simulator, `ECLIPSE` for predictions. As the simulator is a dedicated reservoir model it captures the first principle properties, while we are able to treat it as if it was a black-box. Also, large resources have been spent already to develop accurate `ECLIPSE` models of many reservoirs currently in production. This further motivates the choice of the simulator interface in the control application, as it opens up possibilities to optimize the production of mid-life reservoirs.

The system model enters the optimal control problem as a set of nonlinear constraints that must be satisfied in every time step. In addition, there are also state-path constraints, from operational considerations, which are possibly nonlinear in terms of the manipulated variables (MVs) [8]. In this paper we solve the corresponding NLP through a sequence of quadratic sub problems (QSPs), with the SSMQN method. An analogous method was described by Li and Biegler [13], extending a single-step method by Li et al. [14]. An algorithm very similar to the one used here can be found in Oliveira and Biegler [15], where more general objective functions are addressed.

The method uses an input-output linearization of a black-box model, i.e. the reservoir simulator, around a nominal input sequence, the input trajectory. The nominal trajectory can be seen as the solution sequence transferred from the previous sample. A quasi-Newton search direction, p_k, is found solving a QSP for the linearized model. For algorithm convergence, a line search is performed along the search direction p_k, ensuring descent in the objective function value. If no algorithm convergence is obtained, the procedure is repeated, starting from the input-output linearization. We give the algorithm in five steps:

1. Set the QSP counter to zero.
2. Compute input sensitivities for the nominal input trajectory (linearization).
3. Solve the QSP from the linearization for a search direction, p_k.
4. Employ a line search to determine a suitable step size along the search direction.
5. Set solution as nominal trajectory. Check for convergence. If satisfactory, input the first element of the solution to the system, and start over for the next sample. If no convergence is found, increment the QSP counter. Stop if maximum QSP iterations are reached, else return to step 2.

Some of the steps listed in the algorithm deserve a more detailed description. In Step 2 we find the linearized model calculating a sensitivity matrix, S, from every input to every output:

$$S_{j+1,i} = \frac{\partial x_{j+1}}{\partial u_i}, j \geq i,$$
$$S_{j+1,i} = 0, else,$$
$$j \in \left[k, k + T_p - 1\right], i \in \left[k, k + T_c\right]$$

where k is the current sample, T_p and T_c are the number of prediction samples, x_k is the state/output at sample k and u_k is the input at sample k. An illustration is given in **Fig. 2**. The elements in S are found from numerical gradients given by the ECLIPSE model, by perturbating the input trajectory by small, finite values.

Step 3 calls for solving a QSP. This is a second order approximation of the NLP, with linearized constraints, similar to the QP solved in a conventional SQP algorithm. As our objective function is quadratic in terms of the states, we can guarantee positive definiteness of the QSP, since the Hessian becomes an equivalent to the "Gauss-Newton" choice of Hessian. The QSP provides the Newton-direction in the QSP, making it a quasi-Newton step for the NLP. We solve our QSP with a dual method solver developed by Goldfarb and Idnani [16]. The dual method is preferred because of its lack of demand for a feasible starting point.

The Newton step has a natural step length of one. To ensure objective function value descent, a line search is needed to decide on the fraction of the step size p_k to choose, as described in Step 4. The line search uses a backtracking approach, found in Nocedal and Wright [17]. By decreasing the step length, α_k, for every line search iteration with a factor ξ, the objective function value descent is secured by terminating the algorithm when fractional step length satisfies a decrease condition.

In step 5 the algorithm checks for solution convergence using the norm of the input change between the nominal input trajectory and the new line searched QSP solution trajectory as proposed in [13]:

$$\sum_k \left\| \overline{u}_k - \overline{u}_{k-1} \right\|^2 \leq \varepsilon \; .$$

The value of ε should be chosen small for problems of such a degree of nonlinearity as in this paper to ensure convergence sufficiently close to the optimum.

The algorithm is implemented in SEPTIC (Statoil Estimation and Prediction Tool for Identification and Control) which is StatoilHydro's in-house tool for MPC. SEPTIC is today used in more than 70 installations. SEPTIC is designed to interface different models for prediction as black-box models. The link and interface to ECLIPSE were implemented in C++ and communication between the two software instances are executed through file I/O.

Applications

Model description

Our testing of the NMPC algorithm on a reservoir model is now presented. Although smaller than a typical reservoir model, the model is considered to show sufficient complexity while still being suitable for research and development in terms of simulation execution time. The model is of a strongly channelized reservoir involving three phase flows, previously used in Yeten [7] where it was proven to have a large potential for production optimization.

Rendering the reservoir description and model data given in [7], this model represents a virtual North Sea type fluvial reservoir made from the fluvsim software. **Fig. 3** shows a cross sectional area of the 3-D model from top view. The colors show the grid permeability according to the specified scale bar on the right, and clearly indicate the reservoir's channel structure. The model dimensions are 5000×5000×100 ft^3, split up into 50×50×6 grid blocks. A detailed list of other parameters is given in **Table 1**. A gas cap is present at the top of field, while water is represented by an aquifer at the bottom, giving pressure support.

The reservoir contains a multilateral horizontal smart well, which is also shown in **Fig. 3**. The red line show the imperforated main bore, while the white lines connected to this are four fully perforated laterals. As can be seen, the laterals intersect with both permeable and impermeable zones. The length of the laterals are approximately 2150 ft long, and the well is placed 15 ft above the water-oil contact, giving rapid water-breakthrough after production start up. Each lateral is controllable through an ICV located at the pipe segment connection between the main bore and the lateral, also shown in **Fig. 3** by the yellow circles. There is also a fifth valve at the start of the main bore, giving control over the total flow rate in the well (white circle).

Reservoir models used in practice are made from geological and seismic data. To reflect the uncertainty in such data this model is available in five independent realizations from a geostatistical distribution. In **Fig. 4** a histogram with the global statistics of the permeabilitiy distribution for all five models is shown, along with other basic statistical properties in **Table 2**. Well properties, like location, architecture and instrumentation are the same in all five models, so all that differs is the surrounding permeability fields. The five actual realizations are shown in **Fig. 5**. We will use these to investigate production increase potentials for equally probable models by applying our NMPC application on all models.

Base Case Definition

We now define a set of operational conditions which will function as the base case production philosophy. The conditions are chosen to be exactly the same as proposed in [7] in their base case definition, to maintain a foundation for comparison of results. The liquid production rate (LRAT) is specified to 10.0 MSTB/DAY, with a constraint on gas production not to exceed a gas/oil ratio (GOR) over 5.0 MSCF/STB. If the constraint on GOR is violated, the LRAT will cut back with 10% every time the constraint is reached. Hence, the base case is not an uncontrolled case, as it includes some simple constraint handling offered by `ECLIPSE` to avoid problems such as model unstability. There is however no control on the lateral valves, as they are all fully opened during the simulations. A water-cut (WCT) constraint on the LRAT is specified at 80%, causing the well to shut in. The minimum bottom hole pressure (BHP) is set to 1500 psi to ensure sufficient lift conditions, although BHP is never an issue in this model due to the presence of the strong aquifer and the gas cap.

SPE 79031 case definition

We also define an alternative case for comparison, which is the method presented in [7]. We will refer to this case as the SPE 79031 case. As previously mentioned [7] uses a conjugate gradient method combined with `ECLIPSE` for numerical gradients and ICV modeling. They optimize the field production over a fixed reservoir lifetime of 900 days. By dividing these 900 days into 180 day time steps, their algorithm performs a static optimization of the ICV settings for the whole 900 days, and then a re-optimization at each subsequent time step for the remaining lifetime. The MVs are the settings on the four ICVs and the target objective is to maximize the cumulative oil production. SPE 79031 offers no explicit constraint handling in the optimization algorithm itself, and so relies on `ECLIPSE` to provide this feature regarding the maximum GOR allowed.

Simulations

Implementation

The five model realizations were each simulated with `ECLIPSE` interfaced to the NMPC application for a total of 900 days. All simulations were performed with no modeling errors, i.e. the same model was used as both model and process plant. The MVs were chosen as the opening of each of the four ICVs in connection to the laterals, similar to SPE 79031. In addition, the LRAT was also chosen as an MV for control. It was recognized that this would be necessary to manage to meet the production constraints on GOR in some of the model realizations, as the simple control offered by `ECLIPSE` should preferably be overridden. The `ECLIPSE` ad hoc solution of cutting back 10% upon an active constraint is obviously suboptimal at best, and the better approach would be to produce at optimal LRATs instead. The CVs available are pressures and multi phase flow rates in the well and in each lateral respectively.

The prediction horizon and the control time horizon are fixed at 900 days after production start up. Though some of these models would possibly have a lifetime beyond 900 days, this was done to maintain a fair comparison to the SPE 79031 results. The application then becomes a *batch optimization* NMPC. The application is modified not to consider any production beyond 900 days. The sample time is chosen as 30 days, as an estimate of how often it is realistic to change ICV settings in a real reservoir considering valve impact and exhaustion. Notice how this is different from SPE 79031.

The control problem is formulated to maximize cumulative oil production, while satisfying the constraints on GOR and WCT. There is no explicit cost connected to the production of gas or water, other than reducing the oil production total.

Results

We compare our simulations to the base case performance. **Fig. 6** shows a plot typical of optimal valve settings over the time horizon for the models. Even though the different models all have different optimal valve settings, there are some common characteristics they all share. In all cases except for model 2 (SP2), the valve at the branch closest to the well heel (Valve1) is choked down the most. In three of the cases the second most chocked down valve is Valve2, at the second closest branch. This is explained by the fact that the pressure in the reservoir causes much more flow in the branches closest to the heel than the outer ones, when the valves are fully opened. A large difference in flow gives a more rapid breakthrough of water and gas for the high flowing branches, deteriorating the production performance. The solution is to distribute more equal rates on all the branches. **Fig. 7** shows that this is exactly the solution found by the SSMQN algorithm for SP1. Observe how the optimized case, the plot to the right, produces at more equal branch rates than the base case in the left plot, and therefore manages to be productive for a longer time. The other model simulations show the same tendencies.

Fig. 8 shows the optimal LRATs versus time for the models. Recalling the base case specified production rate at 10000 STB/DAY, the figure shows that for models model 1 (SP1), model 3 (SP3) and model 4 (SP4) this is close to the computed optimal LRATs. However, in models SP2 and model 5 (SP5) the controller uses this MV to a great extent. A natural question arises on how such severe decrease in liquid production is optimal to maximize the oil recovery. The answer is given in the well GOR plot of SP2 and SP5, **Fig. 9** left and right plots respectively. The decreased LRAT prevents the GOR from hitting the constraint at 5.0 MSCF/STB. The SSMQN case performs far better (i.e. has the lower GOR) than the base case 10% cut-back strategy. The three other cases also reduce the GOR to a substantial degree, using only the choke settings. This indicates that in cases where the presence of gas is not so extreme, valve control can be sufficient, but in the extreme case LRAT control will add an important contribution to increased production.

It is important to handle the gas production constraint efficiently, since minimizing the GOR will yield a higher OPR. Also, too high GOR can cause well instability. Equally important in this case is that the WCT constraint is satisfied to the longest extent possible, because of the specification of well shut-in when the constraint is reached. **Fig. 10** shows a typical time plot of the WCT behaviour for the different models. The optimized case outperforms the base case as the water breakthrough is either delayed or the time of WCT reaching the constraint of 0.8 is postponed, or in some cases – both. Also, we recognize how the WCT in **Fig. 10** meets the constraint at approximately 900 days. This is a direct result of the fixed prediction horizon, as the NMPC will not see any profit in holding the WCT back any longer beyond the horizon end. Some of the cases could therefore possibly gain from using a standard receding horizon NMPC instead.

The optimization problem was formulated to maximize the oil production total (OPT) for the models, see **Table 3**. This shows how the NMPC application manages to increase the production in all models, varying from substantial to minor increases. The table lists the production for every model realization on separate lines. The columns consist of the total oil production after 900 days for three cases, the base case, the optimized case and also the results found in SPE 79031. The gain columns show production increase found for the two optimization methods relative to the base case. The results from the SSMQN show large improvements over the base case production. The relative gains span from 5.4% to 67.6%, with an average of 29.9%.

A similar table was given in SPE 79031, without the SSMQN results, where [7] commented on the large variations in production gains for the five cases. Although able to show significant increase in some cases, SP2 only produced an extra 1.8% when using optimization and instrumentation. On this basis [7] suggested that with the geological uncertainty represented by the different models, optimizations did not give a consistent answer on whether or not instrumentation is economically justifiable. In some models, the gain found was fairly small; on the other hand, they stated that “significant resources might be lost by not deploying the control devices”.

Discussion

Comparing the results from the SSMQN to the results from SPE 79031, the former comes out on top for all the model realizations. On average, the SSMQN gives an additional 3% increase, relative to the base case, over the SPE 79031. The three main explanations behind this are:

- **Decreased sample time** – the NMPC used time steps of 30 days, whereas the SPE 79031 optimized with 180 day intervals.
- **Dynamic optimization** – in SPE 79031 the optimization problem parameters were kept to a minimum as each step found the optimal set of static inputs for the rest of the reservoir lifetime. The SSMQN opened for computation of dynamically changing inputs by increasing the number optimization parameters. Where SPE 79031 solved for one parameter per input, the SSMQN solved for 4, giving a total of 20.
- **Optimal LRAT** – the SSMQN included the LRAT as an optimization parameter. This way the constraint handling could be taken care of by the NMPC method, instead of by the heuristic method offered by `ECLIPSE`. Since the models are carrying information about the constrained rates, the GORs and WCTs, the natural thing is to include all the available MVs for controlling these in the optimization problem.

The listed reasons imply a higher computational cost for the SSMQN than for the SPE 79031. A larger number of optimization problems are solved, the problems are harder to solve and the application requires more `ECLIPSE` simulations. Still the SSMQN performs well within real-time demands, as the 900 days optimal control simulation took about 3 days each on a single desktop computer. Also, the results suggest that the extra computations are compensated for, as they give additional increase in all cases. It should be mentioned that very little effort was put in to decrease the simulation time. Implementing the sensitivity calculations on a parallel CPU architecture could easily reduce this time by a major fraction. For a full scale real life reservoir such solutions should be taken into consideration.

An interesting thing is to notice the SSMQN performance compared to the SPE79031 for SP2 and SP5. These where the two cases where the LRAT control was heavily used, recalling **Fig. 7**. **Table 3** shows that these are the two cases providing the most increase over SPE 79031, indicating the NMPC being most valuable for models with the toughest operational conditions. Significant improvements are also shown for SP2 and SP4, representing the most profitable cases in both this paper and in SPE 79031.

Summary

In this paper we have described an implemented NMPC application in StatoilHydro’s in-house tool for advanced process control, `SEPTIC`, using a single shooting multi-step quasi-Newton approach. The NMPC was interfaced to a reservoir simulator, used as a simulation and prediction model, to solve the optimal reservoir control problem. The application has been successfully tested against multiple realizations of a complex reservoir model, showing satisfactory control performance. The simulation results suggest that reservoir recovery can be increased by considerable amounts by using an NMPC controller compared to reference cases produced using simple constant rate strategies. For one case an increase with up to 68% was found.

The developed algorithm was applied to a benchmark reservoir model to validate the quality of the NMPC results. The application managed to show an increase in production over the previously published results on the model. Hence, the quality of the method was proven.

We have also compared our results to a previous publication using the same reservoir models. These confirm and further strengthen the results from [7] – valuable resources will potentially be lost if a reservoir like this is not equipped with smart wells. In addition to improving the most promising results in [7], our simulations improve the "worst case" scenario as well, by recovering 5.4% more oil in one of the models, compared with the 1.8% found in [7]. Proper decision making methods should still be considered before investing the extra costs of smart well optimization for a reservoir. However, these results can be view as an additional argument to the discussion on applying reservoir optimization for complicated reservoir structures.

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Nomenclature

f = general nonlinear function
F = time step performance cost
J = cost function
k = time, discrete
p = search direction
Q = state weighting matrix
R = control weighting matrix
S = sensitivity matrix
t = time, continous
T_c = control horizon
T_p = prediction horizon
u = control vector
U = set of control variable constraints
x = dynamic state vector
X = set of dynamic state constraints
α = backtracking fraction
ε = convergenve criteria
τ = time variable

Subscripts

i, j = matrix indices
k = time step
op = operational set point
0 = initial condition

Superscripts

$\bar{}$ = (bar sign) optimal solution

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Table 1: Model properties [7]

drainage area		5000×5000 ft^2
oil thickness		50 ft
gas cap thickness		50 ft
ϕ		0.20
gas cap PV		0.625 MMft^3
R_s		1.0 MSCF/STB
c	at p_{bub}	3.0×10^{-6} psi^{-1}
k_{ro}		0.8 at $S_{wc} = 0.20$; 0.8 at $S_{gr} = 0.05$
k_{rw}		0.4 at $S_{or} = 0.30$
k_{rg}		0.9 at $S_{wc} = 0.20$
γ	at 14.7 psi	
oil		0.85
water		1.0
gas		0.71
μ, cp	at p_{bub}	
oil		0.42
water		0.30
gas		0.02
B, V/V	at p_{bub}	
oil		1.55
water		1.02
gas		0.71

Table 2: Permeability Statistics [7]

Facies	Average (md)	Standard Deviation (md)	Coefficient of Variation
Channel Sand	1534	635	0.4
Mudstone	4.9	1.5	0.3

Table 3: Comparison of oil production totals

Model realization	Base case (MMSTB)	SPE 79031 (MMSTB)	Gain %	SSMQN (MMSTB)	Gain %
SP1	2.61	3.83	46.7	3.86	47.9
SP2	2.22	2.26	1.8	2.34	5.4
SP3	3.80	4.13	8.7	4.14	8.9
SP4	2.59	4.27	64.9	4.34	67.6
SP5	2.18	2.48	13.8	2.61	19.7
Average	2.68	3.40	27.2	3.46	29.9
Std. Dev.	0.66	0.95	27.2	0.92	26.9

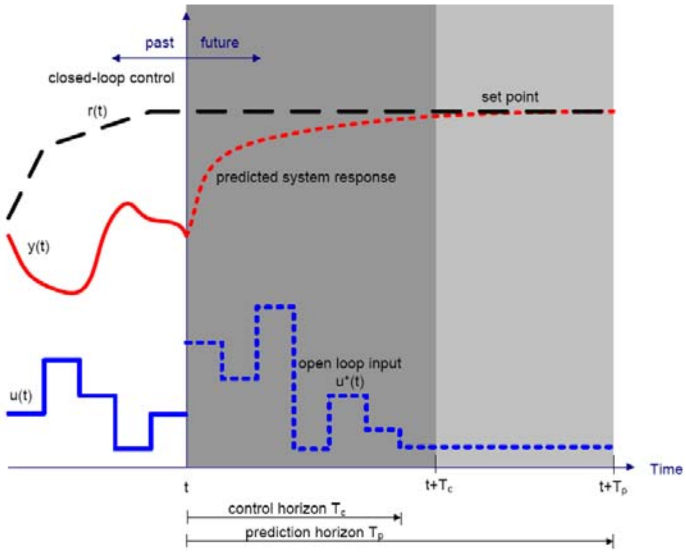

Fig. 1: Principle of MPC

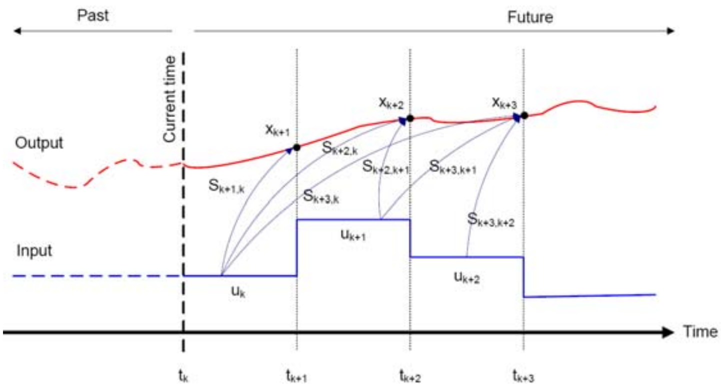

Fig. 2: Calculation of sensitivities

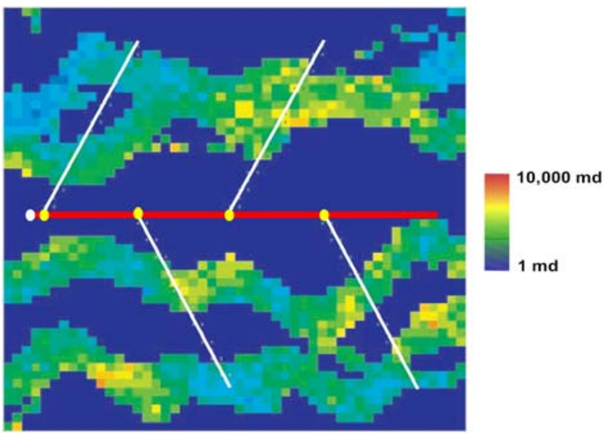

Fig. 3: Fluvial reservoir model [7]

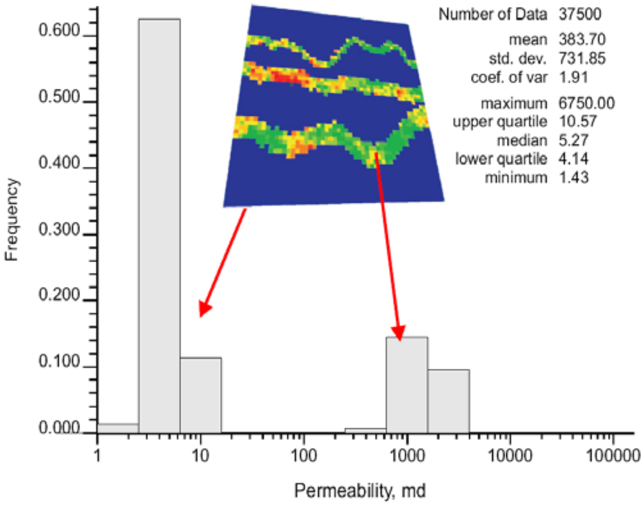

Fig. 4: Permeability distribution for the 5 model realizations [7]

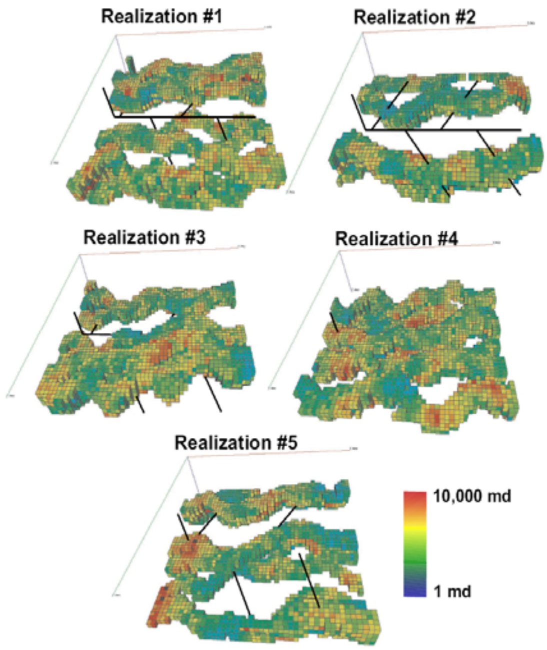

Fig. 5: The five geostatistical realizations [7]

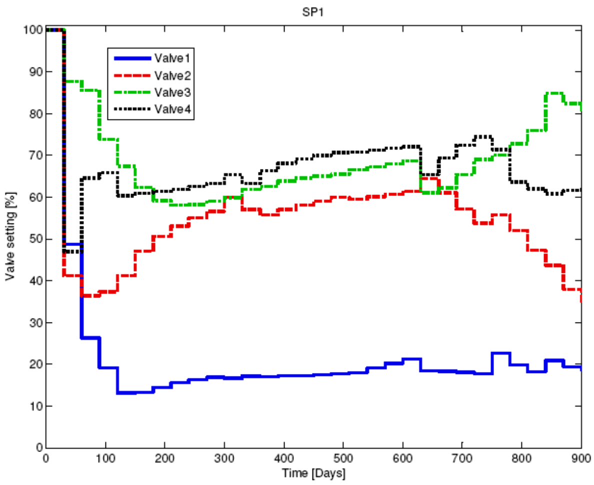

Fig. 6: Optimal valve settings typical

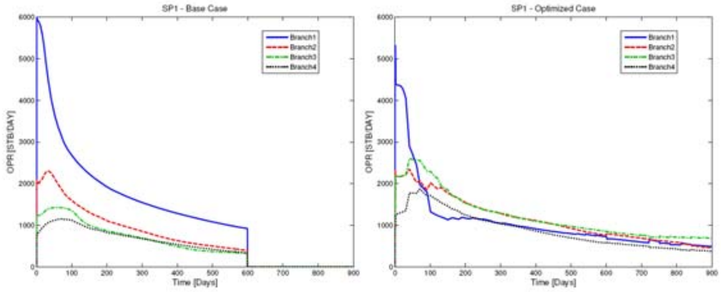

Fig. 7: Lateral oil production rates

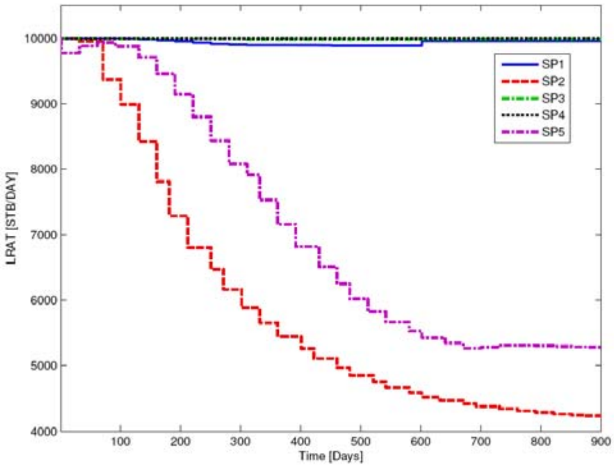

Fig. 8: Optimal liquid production rates

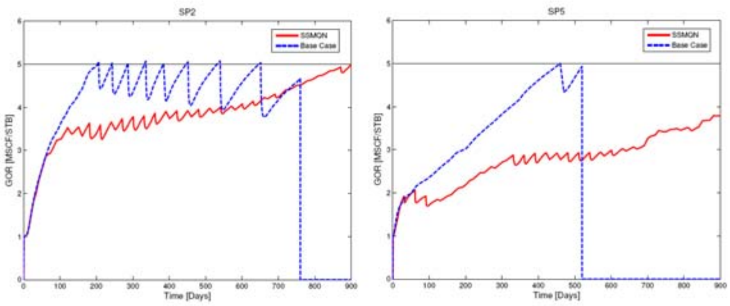

Fig. 9: Base case vs. optimal GOR

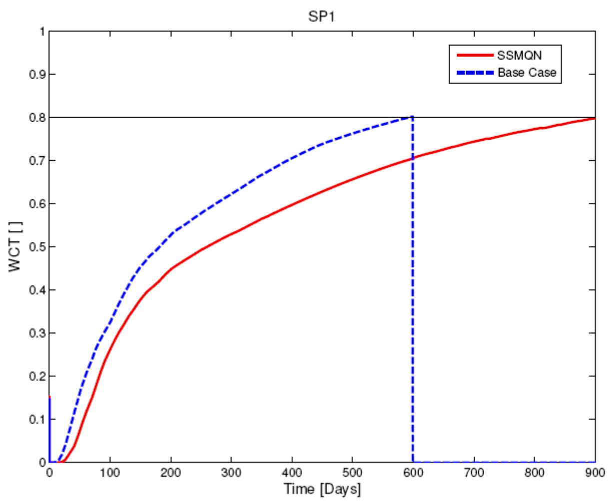

Fig. 10: Base case vs. optimal WCT