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## Innovative Approach to Assist History Matching Using Artificial Intelligence

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### Abstract

The study objective is to investigate the use of Artificial Intelligence (AI) methods to accelerate the history matching process.

A new criterion for measuring the deviation of the simulation model from measured and /or observed parameters has been introduced. Instead of comparing parameter deviations in wells to input changes on regional basis, it is proposed to calculate a regional RMS (Root Mean Square)-error, so that the impact of input changes can be directly evaluated. Instead of grouping grid blocks based on geology, it is proposed here, to generate regions of similar trends based on all available information. Artificial intelligence (AI) is used via Self Organizing Maps (SOM) to cluster grid blocks of similar behavior. SOMs can process any kind of information; in this case these types of parameters have been particularly used:

- geological description: lithofacies type
- hydraulic flow units (HFU): permeabilities, porosities
- initialization: water saturations (initial and critical), initial pressure
- discretization: spatial discretization (e.g. DZ), grid block pore volumes
- secondary phase movement: relative permeability endpoints

A three fold approach for improving and/or assisting the history matching (AHM) work-flow using Artificial Intelligence has been tested:

1. Use production plots, Neural Networks and “Material Balance with Interference (MBI) method for quality control and consistency check of time dependent and static data.
2. Use the multi-dimensional cross-plot and SOM to evaluate reservoir and well performance.

3. Use SOMs and the region RMS error to evaluate the performance of history matching runs.

This new approach is simple and leads to a clear improvement of the match quality and significantly reduces the number of runs needed to achieve the match. Different field models have been used to develop this new AHM workflow. Finally in this paper, two of them are selected to demonstrate the improvement of model pressure and watercut matches using this new method.

### Introduction

History matching a numerical simulation model is an inverse problem, which cannot be solved directly. An iterative procedure has to be applied to reduce the deviation of the model calculations and measured values. Assisted history matching allows the automation of low level processes, without taking over the key decisions from the reservoir engineer.

The workflow can be divided into two categories. The first one uses gradient based optimization methods, requiring additional programming in the numerical simulation program code itself (Ref.1-7). The second category consists of algorithms and workflows, which do not require calculations inside the simulator (Ref. 8-12). The approach presented in this paper does not need the calculation of gradients. However gradients and even automated history matching tools can be used in combination with this new procedure. The creation of clusters (subsequently referred to as regions) represents a major step forward for the use of any assisted or automated history match process.

The second part of the paper (weighted RMS factor) directly relates to the problem described in Ref. 17. By using the weighted RMS-error, the objective function (it quantifies the misfit between simulated and observed data) will be defined much better, discarding a lot of the otherwise possible solutions to the inverse problem.

### A New Approach to Assist History Matching (AHM)

History matching is defined by finding a set of model parameters that minimize the difference between calculated and observed measurement values like pressure and fluid production rates.

Investigating a process to leverage Artificial Intelligence to improve and speed-up history match simulation models by incorporating all reservoir and field data is the main objective of this study. A user friendly process where the simulation engineer can interact and control the parameter modifications on their own (see Figure 1) has been developed.

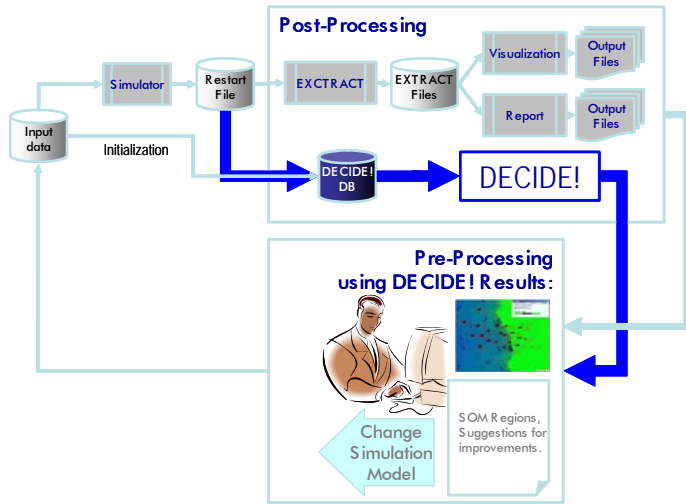


Figure 1: History Matching Cycle

A sound understanding of all data involved is essential. The quality control of production data as well as the understanding of the influence of the geology on the development of flow units in the reservoir is crucial for the engineer to understand the impact of his modifications on pressure/saturation changes.

Visualization of all simulation input data and identifying trends and dependencies between parameters enable engineers to learn and understand the behavior of reservoir regions and individual wells. Furthermore, Artificial Intelligence can assist in this effort and perform time-consuming tasks. For example, Self Organizing Maps can be used to group wells or simulation grid blocks of similar behavior. During the study, different simulation models with different heterogeneities scales have been used to develop and test the new workflow.

### Description of the Workflow

The workflow is divided into three different phases. The first phase includes data quality control, and the second phase investigates reservoir and well performance. The third phase covers the actual history matching process and is considered to be the main subject of this study.

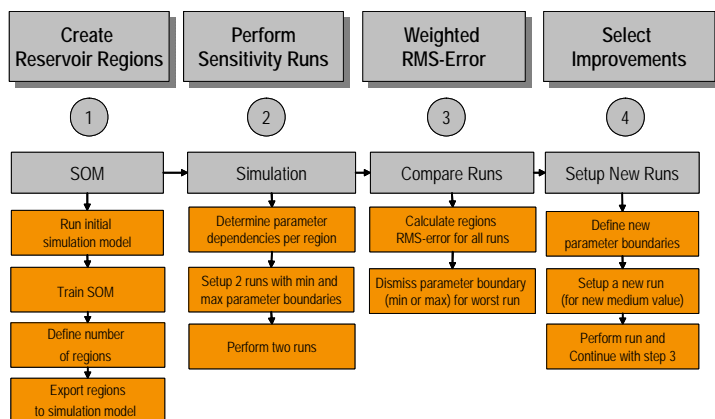


Figure 2: Description of Workflow

This paper describes only the third phase of the workflow. Quality control and performance analysis are too extensive to be properly illustrated here. The actual history matching workflow (Figure 2) is described in details below:

- Step 1: use SOMs to create the regions. Depending on the input parameters of the regions, the clustering process can include geological information (e.g. lithofacies type), the definition of hydraulic flow units, the special discretization, initial equilibrium and dynamic properties.
- Step 2: the identification of parameter dependencies is a very important task during history matching. This is the only way to ensure, that the simulation engineer is aware of his modifications and the impact on other parameters. Without taking these dependencies into account, violations of these fundamental relationships can easily happen. It is proposed to setup two sensitivity runs (plus the original run) using the lower and upper limits for the modification of the input parameter for each region. The third run has already been performed with the initial settings.
- Step 3: to be able to correlate the impact of input parameter changes, the correct RMS-error has to be calculated. This approach suggests using regions to adjust the simulation input parameter and therefore suggest calculating the RMS-error for regions. The procedure is described in the chapter 'Weighted RMS Factor'. The three runs are evaluated in a way that the RMS-error for each region of all three runs is compared. The modification factor of a region, which resulted in the highest RMS-error of this region, is dismissed. The values of the remaining two modifiers are set as new lower and upper bounds.
- Step 4: from the two modification limits left, a third modifier is generated, which has a value in between the new lower and upper bounds. The input parameters have to be changed for each region and according to each region modifier. After finishing the new run, again the RMS-errors per region are compared and the region modifier leading to the highest RMS-error in each region is dismissed. This leads again to new modifier boundaries (See Figure 3).
- Steps 5&6: now the engineer can go back to step #3 and repeat the process as many times as he wants. The other possibility is, to switch to an automated history matching tool using the new bounds of the modifiers. The previous five steps should have lead to a drastically reduced search space for the history matching tool, so that it should converge to a solution pretty fast. Unfortunately, many of these tools are limited in the number of variables to change. Therefore the regions with the highest impact on the global match should be used. The determination of these regions is easy; they just have to be ranked according to their RMS-error.

As shown in Figure 3, the sensitivity analysis can also be run using several steps for the parameter multiplier. In this example, the x-axis shows the value of the multiplier for the water relative permeability ( $k_{rw}$ ) endpoint, the y-axis shows the RMS-error value for a specific region. It is clear to see, that a range between 0.6 and 0.8 will lead to the best history

match of this region. This represents already a major reduction in the uncertainty of this match parameter in this region.

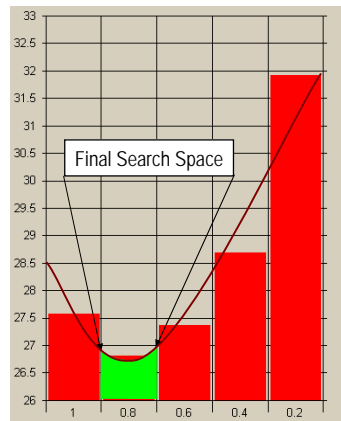


Figure 3: Reduction of Search space

The steps and the resource consumption are shown in the next table. Provided, that the simulation model is setup and the quality control workflow has been performed, the amount of time to apply this workflow is reduced to a minimum.

Depending on the model and the parameters, which are adjusted during the match, the time consumption may grow. This can happen, if the necessary changes cannot be included in the simulation model automatically, e.g. if some formatting work is needed.

The table shows also the number of simulation runs needed for the complete workflow. To decrease the search space for an automatic history matching tool, only four runs are needed. Of course, the loop of steps 3 & 4 can be performed several times, which would also increase the number of runs. The estimation of 20 runs for the automated HM tool is based on a gradient technique for optimization. Other methods e.g. Experimental Design (ED) may need a much higher number of runs. Nevertheless, the workflow can be used independently of any simulator type and it is not bound to any commercial software. Below is a table showing the sequence of the steps needed to take.

	Step	User days	Computer Sim-Runs
1.	Identify regions using SOM	1	
1.1	Determine PORO-Perm Relationship	1	
2.	Perform two sensitivity runs		2
3.	Calculate weighed RMS factors	0.5	
3.1	Evaluate the three runs	0.25	
4.	Set new boundaries	0.25	
4.1	Perform one run		1
4.2	Evaluate the new run		
5.	Rank the regions by sensitivity	-	-
6.	Run automated HM tool	1	20
	Total Time	4	23

Table 1: Steps of HM workflow

Clustering (SOM)

After getting familiar with all parameters, using various options of cross plots, a selection of these parameters might be used to create reservoir regions of similar behavior.

A common way to history match numerical simulation models is, to multiply the value of a parameter (e.g. permeability) with a certain value. It is clear, that the multiplier cannot be the same number for each grid block. Therefore, the engineer defines regions, where he thinks the same multiplier would be suitable to improve the match. The selection of these regions might be done in alliance with the geological model. The geological model was definitely built by distinguishing different lithofacies types, which have been generated in the geological life of the reservoir. Therefore grouping grid blocks with a similar geological description sounds reasonable.

One problem associated with this approach is, that the geological model is very often based on static geological information, but not directly related to hydraulic parameters (e.g. permeability is derived from a correlation with porosity after the creation of the geological model).

Instead of grouping grid blocks based on geology, it is proposed to generate regions of similar behavior based on all available information. This includes also model parameters from the initialization, like initial pressure and saturation. If pressures are recorded over time and reliable pressure maps can be created, also this information can flow into the clustering algorithm and influence the creation of regions.

The advantage of this approach is its simplicity. As the self organizing map (SOM) is a special kind of neural network with a self-learning approach; no expert knowledge is required to use this technology. The only decision, which the user has to make, is how many regions he wants to create.

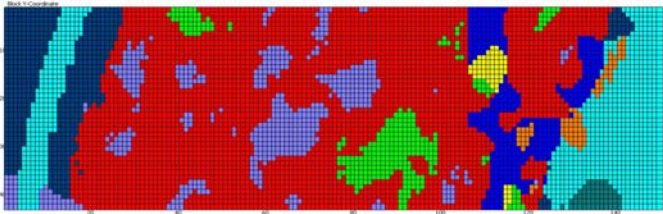


Figure 4: Example of Automatic Creation of Regions

Figure 4 is an example for clustering; each color represents a region. The total number of regions is ten. This number might seem to be too low, but it is better to start with a course and simple creation of regions, than with the most complicated to think of. Later during the history match – at any point of time - the engineer can come back and increase the number of regions. Of course, he has also to modify all steps of the workflow, which are successors of this step.

If the clustering is based on geologic parameters, the regions strongly correlate with the geological description. Including permeabilities, water saturations (initial and critical) as well as initial pressure a good description of hydraulic flow units (HFU) can be achieved. In addition, spatial discretization can be taken into account (e.g. DZ). Depending on the importance of the parameter, its influence can be controlled using a weight factor. This factor is normalized between 0 and 1. The parameter gets the highest weight, if the factor is one.

He has no influence on the clustering, if the weight factor is set to 0.

### Create Regions

After the creation of the SOM and the definition of the number of regions to use, the SOM is using *Ward's Hierarchical Clustering* method to define which grid block belongs to which region. This information is exported in a format, which the simulator can read. The region information can be used as descriptor for FIPNUM, SATNUM or any other region parameter in the simulator.

The values of a certain grid block parameter can be modified using a multiplier, which is constant for each region. In this way, grid block properties can be changed on a regional base respecting the natural occurrence and distribution of this parameter. It is also possible to write out the modified parameter values per grid block and include them in the simulation run.

### Modify Permeability

For the pressure history match in the sector model, the XY permeability has been used as modification parameter. The SOM has clustered all available grid blocks according to the available grid block properties. As the input parameters for the SOM also include porosity, a quick quality control of the porosity - permeability parameter dependency can be done by cross-plotting the parameter versus each other.

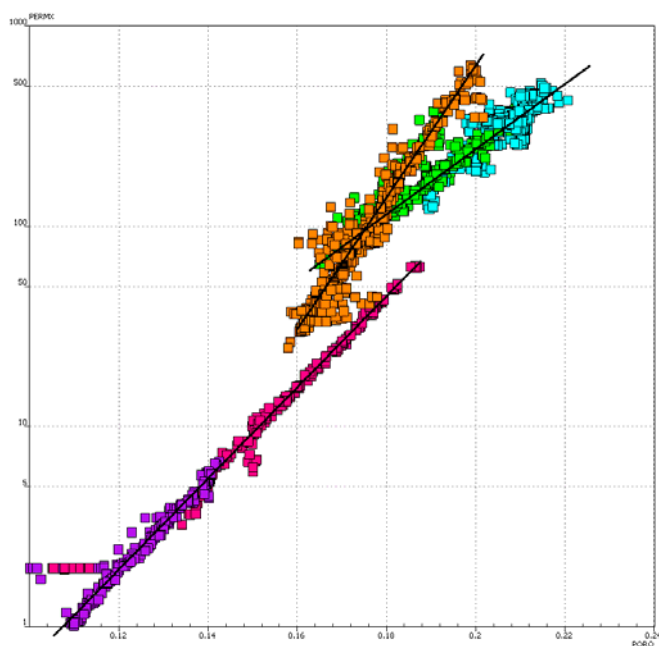


Figure 5: Cross-Plot of Porosity versus Permeability

Figure 5 shows an example of the cross-plot. Each data point represents a grid block. The color of the points is equal to its region affiliation. It is clear to see, that different regions follow different trends. Some of them have a clear linear semi-log character (e.g. magenta region). Some have a more fuzzy description (e.g. orange region).

When the permeability of one region is changed by a multiplier, all grid blocks belonging to this region are shifted vertically on the cross-plot. This is possible only in a limited data range; otherwise, the correlation with the porosity is disturbed too much.

To enlarge the possible data range, the porosity could be tied to the permeability through a linear semi-log function. In this case, the natural dependencies of porosity and permeabilities are preserved during the history match. Any changes in the pore volume of these regions should of course result also in a change in the pore-volume of the other regions, so that in total the original pore volume is preserved.

### Weighted RMS Factor

#### Problem Statement

When the setup of the simulation model is finished and first model has been executed, the deviation of the model behavior from reality has to be established. Usually engineers compare the calculated output to parameters measured during the field production history.

One way to describe the deviation in form of an equation is the calculation of the root mean square (RMS) error. The Root Mean Square (RMS) error represents the difference between the measured points and the calculated points of a parameter.

The RMS-error (RMSE) is given by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (x_i)^2}{n-1}}$$

Where  $x$  is the difference of the calculated to the measured value of a point,  $n$  is the number of points in the sample.

Many post-processing simulation tools use the RMS error to quantify the quality of the history match. In most cases, the total RMS error is used to do that. It is also possible, that the RMS error per well is displayed. Some tools allow picking so called 'match points' and calculate the RMS error of these points. Nevertheless, all of the methods described above show a value, which cannot be directly related to the changes made to adjust the simulation model.

Usually, the modifications, which are performed by the engineer to increase the match quality, are based on geological rock properties like permeability etc. These modifications should be done in a way, so that the geological description is not destroyed. Therefore the possibility of using rock-regions has been introduced and become a standard way to describe rock properties in simulation models. The quality of the match is then measured by the impact of changes in rock-regions on the dynamic behavior of the wells (e.g. pressure, GOR or water cut). As each well is very likely influenced by more than one rock-region, the analyzed well performance does not relate to a single change of the input parameters. It represents the mixture of all changes, which are performed at the same step, i.e. before launching a new simulation run. This leads to the conclusion, that only one parameter of one rock-region should be changed at a time. Obeying this rule, it would take a high number of simulation runs to be able to match a simulation model.

### Assumptions

This study lead to the development of a new approach to overcome the limitations described above. If it is possible to directly quantify the impact of an input change to the output result, many model modifications can be performed simultaneously. The need to run a new simulation for each modification vanishes.



The solution proposed in this study is simple to apply and therefore is bound to some basic assumptions, which are explained in the following. This approach assumes,

- that the geology of the initial model is correct
- the description of rock parameters is consistent within each lithofacies
- the geological model and the population of the rock parameters is not destroyed during the up scaling process

Many efforts to automate history matching allow the change of permeability and porosity in a way, so that the whole geological description of the model is changed (e.g. in a fluvial environment the paths of the channels). This approach relies on the work of geologists. It is assumed that the reservoir description is correct, within a limited range of uncertainty.

A difficult task is the population of the reservoir model with parameters which cannot be directly measured. One example is the permeability distribution. Usually, permeability is established from core laboratory measurements and then correlated to porosity. Naturally, each lithofacies type has a different dependency of porosity and permeability. This approach assumes that this dependency has been established in the best possible manner and it should not be changed during the history matching process.

Furthermore, it is assumed, that the upscaling process has not changed the above described dependency of porosity and permeability – or more generally spoken – any logical dependency of parameters, which are needed to describe the static and dynamic behavior of the reservoir.

### Region RMS

The solution proposed here is to calculate a RMS error based on regions. It means that the direct impact of a parameter change of a region can be compared to the quality of the match in that region.

To be able to do that, it is needed to split up the RMS error per well into the fractions, which are contributed by each individual region. Each region, in which a well is perforated, contributes in a different way to the well behavior. As the well behavior is mainly driven by its production, it is also clear, that the importance of a region in the well is depending on the product of permeability and thickness (kh). The higher the kh of a region in the perforated part of a well is, the higher its contribution to production will be.

This principle is used to split up the well RMS error into an error for each region in which the well is perforated. Therefore the fraction of the region kh compared to the total kh of the perforated section of a well is determined. This fraction is called 'region weight' and calculated for each well. The region weight is then multiplied with the well RMS-error to obtain the RMS-error contribution of a region to the well. Now the region RMS-errors of all wells can be summed up to one value per region. In this way, the direct impact of a change in the region input parameter can be quantified directly.

This is a very simple principle, which leads to a clear improvement of the match quality and the number of runs needed to achieve the match.

Unfortunately, some facts complicate the application of this approach. The region weight is not constant over time. Completion changes are also changing the importance of regions during the live of a well. E.g. it is common practice to shut-in deeper zones after water-breakthrough. These zones might have a high kh and therefore a high influence on the well behavior and the RMS-error. When such a zone is shut-in, the contribution to the calculation of the weights drops to zero. This example demonstrates that it is necessary to calculate time-dependent weights and apply them during the calculation of the RMS error.

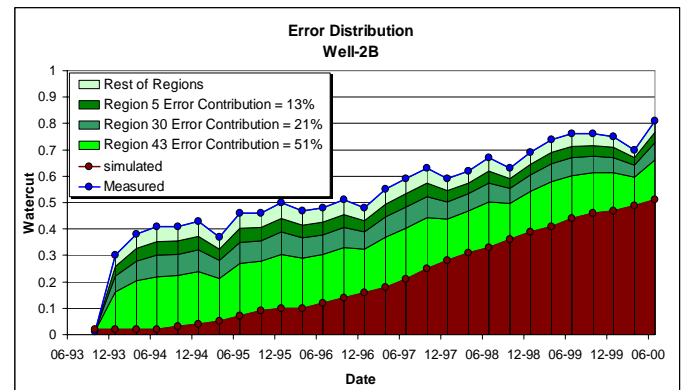


Figure 6: Example of Error Contribution of Well-2B

Figure 6 shows an example of the error contribution of all penetrated regions. The difference, between the measured water cut (blue curve) and the simulated watercut (brown curve) can be split and allocated to each region, which is penetrated by the well. This allows ranking the regions according to their impact on the match quality. In this example, the highest error contribution comes from region 43; i.e. that this regions is responsible for more than 50% of the error. It is clear, that the engineer will first concentrate on region 43 to improve the history match. Region 5 and region 30 are together responsible for 34% of the error. If the engineer manages to estimate the correct values for the modified input parameter for only these three regions, the history match of this well can be improved by 85%.

Of course, other wells might increase their RMS error, when these three regions are manipulated. Therefore, the RMS error of each region is also calculated and used as objective function; i.e. the overall quality of the history match shall be improved, not just one well. The match quality of Well-2B will be less than the theoretical improvement of 85%, but still show a satisfactory improvement.

## Results

### Pressure Match Case

The first example model contains approximately 180 wells. The difficulty of the history lies in the presence of a tar zone at one of the flanks of the reservoir. The tar zone has been modeled as a low permeability area. To achieve a pressure match, the horizontal permeability of the reservoir is modified using multipliers.

The parameters permeability (PERMXY, PERMZ), initial pressure and saturation (Pini, Sw), porosity (PORO), and the block thickness (DZ) of the initial are used to create 14 regions. The Self Organizing Map (SOM) has clustered all

simulation grid blocks with similar parameter values into regions.

These regions have been used to modify the permeabilities in horizontal direction. For each region, the minimum and maximum permeability multipliers have been defined. In the first level, the minimum has been set to 0.5 and the maximum to 2. Two additional runs have been calculated and the RMS error of each region has been calculated. Based on these results, new multipliers have been defined. Some of the region permeability multipliers did not change; some have been increased to a maximum of 5.

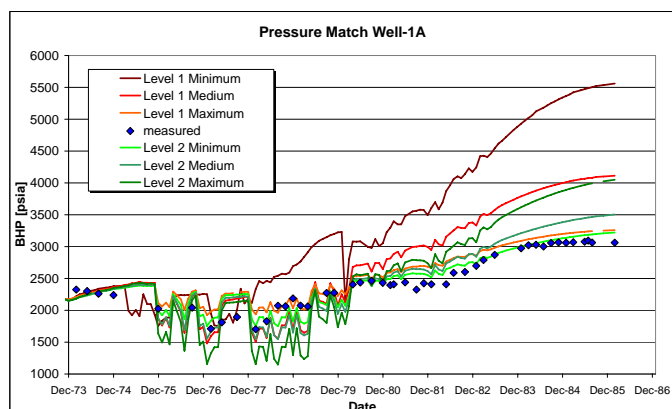


Figure 7: Pressure Match of Well-1A

Figure 7 shows the sensitivity of all six runs, which have been performed in this model, for one well. It is clear to see the improvement. In this case, the well pressure is already matched. If not, this quick sensitivity analysis can reduce the search space (i.e. the starting constraints) for the automated history matching tool, which could perform the fine tuning.

From the sensitivity analysis, the importance of the individual regions on the match performance can be derived. The difference of the RMS-Errors of the individual runs can be used to rank the regions according to their sensitivity. The result of the ranking can be seen in Figure 8.

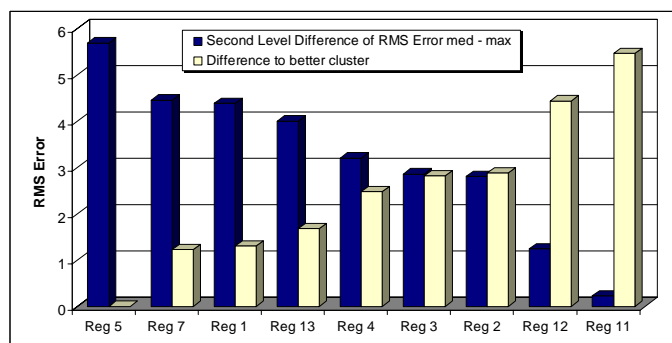


Figure 8: Pressure Match of Well-1A

One of the strong advantages of this approach is that it does not destroy the geological structure of the model. This can be seen in Figure 9. In the left picture, the initial permeability distribution in a small part of the model is shown. After the pressure matching workflow, leading to the results presented here, the permeability distribution did not change in shape, but in the magnitude in some of the areas. Low permeability areas did not change much, but in some of the

higher permeable areas a clear increase of permeability can be observed.

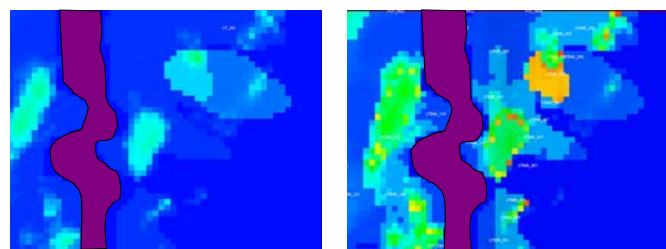


Figure 9: Comparison of Initial Model (left) and Final Model (right)

### Watercut Match Case

A second model is chosen to demonstrate the new approach for improving the water cut match quality. The model shows an excellent pressure match, but needs some refinement in modeling the second phase fluid flow.

The same procedure as in the previous study was applied. The initial run parameters were used to train a SOM. The simulation grid blocks have been clustered into 48.

As already illustrated in the description of the workflow, the first level modification is performed using the minimum and maximum parameter multipliers for all regions simultaneously. After the second iteration of the workflow, it is easy for the engineer to pick the results and compose a new run. Figure 10 shows the difference of the solutions on the well 3C. The grey area marks the sensitivity of the water cut when all regions are multiplied with the minimum and the maximum multiplier (in this case for the  $k_{rw}$  endpoint). The green points describe the solution which is obtained from a composition of the best multipliers of all regions. It is clear to see how superior this run is compared to the others.

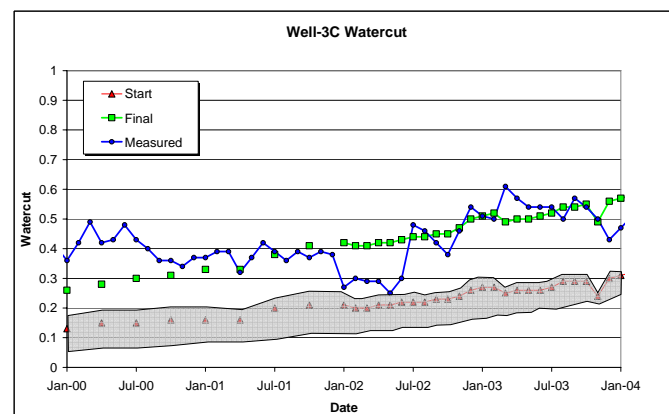


Figure 10: Result of Sensitivity Analysis for Well-3C

### Conclusions

This paper introduces a new approach for assisted history matching. The engineer is able to define the regions and the magnitude of necessary modifications using only a small amount of simulation runs. Thus the search space is reduced tremendously, so that any automated history tool will converge fast to an accurate solution.

The SOM is able to cluster grid blocks of similar behavior using geological, as well as hydraulic (static and dynamic) information. The regions, which are built by the SOM are a critical and missing step in many history matching workflows.

By calculating a region RMS error, the changes in the input can be directly correlated to the resulting increase or decrease of the match quality in this region. Contradictory effects when using multiple modifications in the same simulation run vanish.

The overall match error can now be regarded in two ways. A RMS error per region is created, which allows increasing the match quality on regional bases. The information about the error contribution of each region on each well gives the possibility to better understand the influence factors which lead to an improvement of the match performance on well level.

The error function can be described more precisely. Therefore the match converges closer towards an unique solution. By maintaining the geological definition through the whole history matching process, many scenarios, which would have been created by geo-statistics can be disregarded in this way.

The approach is not limited in size. It has been applied to multi-million block models with more than 400 wells.

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### References

1. Anterion, F. and Eymard, R.: "Use of Parameter Gradients for Reservoir History Matching," , SPE 18433, SPE Symposium on Reservoir Simulation, Houston, 6–8 February 1989.
2. Arenas, E.; van Kruijsdijk C.; Oldenziel; T. "Semi-Automatic History Matching Using the Pilot Point Method Including Time-Lapse Seismic Data", SPE 71634, SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, 30 September–3 October 2001.
3. Bissell, R.C. *et al.*: "Combining Geostatistical Modelling With Gradient Information for History Matching: The Pilot Point Method," ,SPE 38730, SPE Annual Technical Conference and Exhibition, San Antonio, Texas, 5–8 October 1997.
4. Brun, B.; Gosselin, O and Barker, J. W. "Use of Prior Information in Gradient-Based History-Matching" , SPE 66353, SPE Reservoir Simulation Symposium, Houston, Texas, 11-14 February 2001.
5. Gomez, S.; Gosselin, O. and Barker, J. W. "Gradient-Based History Matching With a Global Optimization Method" SPE 71307, SPE Journal, pp. 200-208, June 2001.
6. Le'pine O.J. *et al.*: "Uncertainty Analysis in Predictive Reservoir Simulation Using Gradient Information," *SPEJ* (September 1999) p251.
7. Wu, Z. "A Newton-Raphson Iterative Scheme for Integrating Multiphase Production Data Into Reservoir Models" SPE 74143, SPE Journal, pp. 343-351, September, 2001.
8. Maschio, C., Schiozer, D.J.: "Development and Application of Methodology for Assisted History Matching", SPE 94882, SPE Latin American and Caribbean Petroleum Conference, Rio de Janeiro, Brazil, 20-23 June 2005.
9. Rodrigues, J. R. P. "Calculating Derivatives for History Matching in Reservoir Simulators" , SPE 93445, SPE Reservoir Simulation Symposium, Houston, Texas, 31 January – 2 February 2005.
10. Romero, C. E.; Carter, L. N., Zimmerman, R. W. and Gringarten, A. C. "Improved Reservoir Characterization through Evolutionary Computation", SPE 62942, SPE Annual Technical Conference and Exhibition, Dallas, Texas, U.S.A., 1 – 4 October 2000.
11. Schulze-Riegert, R. W.; Axmann, J. K.; Haase, O. *et al.* "Evolutionary Algorithms Applied to History Matching of Complex Reservoirs", SPE 77301, SPE Reservoir Evaluation and Engineering, pp. 163-173, April, 2002.
12. Uldrich, D., Matar, S.: "Using Statistics To Evaluate A History Match", SPE 75223, SPE/DOE Improved Oil Recovery Symposium, Tulsa, Oklahoma, 13–17 April 2002.
13. Dogru, A.H. and Seinfeld, J.H.: "Comparison of Sensitivity Coefficient Calculation Methods in Automatic History Matching," SPEJ (December 1981) p551.
14. Hirasaki, G.J.: "Sensitivity Coefficients for History Matching Oil Displacement Processes," SPEJ (March 1973) p39.
15. Watson, A.T.: "Sensitivity Analysis of Two-Phase Reservoir History Matching," SPERE (August 1989) p319.
16. Shah, S., Gavalas, G.R., and Seinfeld, J.H.: "Error Analysis in History Matching: The Optimum Level of Parameterization," SPEJ (June 1978) p219.
17. Tavassoli, Z., Carter, J.N., and King, P.R.: "Errors in History Matching", SPE 86883, SPEJ (Sept. 2004), p 352.
18. Yamada, T.: "Nonuniqueness of History Matching," , SPE 59434, SPE Asia Pacific Conference on Integrated Modeling for Asset Management, Yokohama, Japan, 25–26 April 2000.