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A New Approach for Dynamic Optimization of Waterflooding Problems

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Abstract

Technological advances have resulted in use of smart wells, which are typically equipped with remotely operated downhole chokes. We present an approach for controlling these chokes so that the water flooding is optimized. The optimization problem is done by maximizing either total cumulative oil production or net present value.

The new methodology presented here avoids the limitations related to using optimal control as no adjoint equations are needed and the model equations are treated as a "black box".

In the new approach the ensemble Kalman filter is used as an optimization routine, and the methodology is compared to the Partial Enumeration Method.

We demonstrate the methodologies on a simple synthetic reservoir with five layers of different permeabilities. The conclusions from this work are that the ensemble Kalman filter approach is working robustly, and the results are in agreement with, or superior to, the results obtained with the Partial Enumeration Method and a reference solution.

Introduction

Controlling downhole choke settings in smart wells for optimal water flooding represents a great challenge. Traditionally, the solution of this problem has been to apply optimal control. Optimal control falls under the category of gradient-based optimization, and does require the construction and solution of an adjoint set of equations. This approach was pursued in the work by Brouwer and Jansen[1] and Virnovsky[2]. These papers also contain references to other works within this area.

A disadvantage with the adjoint approach is that explicit knowledge of the model equations is necessary. In addition, extensive programming is needed to implement the equations. A remedy for the latter drawback was suggested by Sarma et. al.[3]. Here an approach was introduced which simplified the calculation of the adjoint equations. However, the approach requires specific forms of the cost function and a fully implicit forward model.

We introduce a new approach for solving the optimization problem which is completely independent of the model equations used. That is, the model is treated as a "black box". The approach is not gradient-based, so no implementation of adjoint equations is necessary. Here, the methodology is used to optimize either net present value (NPV), or total cumulative oil production. In addition, for further validation, the methodology is compared to the Partial Enumeration Method (see Wang[4]), which is a discrete non-gradient based method.

The new approach introduced here is based on utilizing the ensemble Kalman filter (see Evensen[5]). The ensemble Kalman filter was originally developed for estimation of state variables, but is in this work used as an optimization routine. The ensemble Kalman filter updates a state vector based on a set of measurements. In this new application, the measurements are replaced by values representing an upper limit for the possible cumulative oil production or

NPV. The filter will then return choke settings which results in cumulative oil production, or NPV, as close as possible to the predefined value.

Optimization of water flooding problems

In this section we give a brief introduction to the ensemble Kalman filter. This is followed by a thorough description of how the filter is used for controlling the downhole choke settings in order to maximize the total cumulative oil production or NPV. Thereafter we introduce the Partial Enumeration Method. Note that this method is only used to maximize total cumulative oil production, and not NPV. In both approaches, the reservoir simulator can be treated as a "black box", where no explicit knowledge of the model equations is necessary.

The ensemble Kalman filter The ensemble Kalman filter has been applied in a variety of physical problems, including both pure state estimation and combined state and parameter estimation. A comprehensive descriptions of the ensemble Kalman filter is found in the work by Evensen[5]. Here we present an abridgment of the ensemble Kalman filter theory necessary for our application. The ensemble Kalman filter is a Monte Carlo approach where an ensemble of model states is used to approximate the necessary covariance matrices. This implies that no linearization of the model function is necessary.

One iteration of the ensemble Kalman filter consists of two steps, a forecast step (giving $\mathbf{U}_f$) and an analysis step (giving $\mathbf{U}_a$). Here $\mathbf{U}$ represents the state vector. The forecast step is calculated by using the model function to propagate the state vectors (ensemble) from timestep $n-1$ to timestep n

$$\mathbf{U}_f^{n,i} = \mathbf{f}(\mathbf{U}_a^{n-1,i}),$$

where i runs from 1 to the number of ensemble members (N).

In the analysis step, the forecast state vectors $\mathbf{U}_f^n$ are updated by taking into account the mismatch between measurements and the corresponding predictions from the ensemble members. The state vectors are related to the measured variables through the following equation

$$\mathbf{D} = \mathbf{H}\mathbf{U}, \tag{1}$$

where $\mathbf{H}$ is a matrix that selects measured variables from the state vector. Note here that it may be necessary to include the measured variables explicitly in the state vector, in order to get a linear relationship between $\mathbf{U}$ and $\mathbf{D}$. Further we assume that the true observation vector at time n is given by $\mathbf{D}_o^n$. The analyzed states are now computed as

$$\mathbf{U}_a^{n,i} = \mathbf{U}_f^{n,i} + \mathbf{K}^n(\mathbf{D}_o^{n,i} - \mathbf{H}\mathbf{U}_f^{n,i}),$$

where $\mathbf{K}^n$ is called the Kalman gain matrix and is given by

$$\mathbf{K}^n = \mathbf{P}_f^n\mathbf{H}^T(\mathbf{H}\mathbf{P}_f^n\mathbf{H}^T)^{-1}.$$

The matrix $\mathbf{P}_f^n$ is an approximation to the model error covariance matrix, and can be written as

$$\mathbf{P}_f^n = \mathbf{L}_f^n(\mathbf{L}_f^n)^T,$$

where $\mathbf{L}_f^n$ is given by

$$\mathbf{L}_f^n = \frac{1}{\sqrt{N-1}}\left[(\mathbf{U}_f^{n,1} - \widehat{\mathbf{U}}_f^n)\ ...\ (\mathbf{U}_f^{n,N} - \widehat{\mathbf{U}}_f^n)\right].$$

Here $\widehat{\cdot}$ represents the ensemble mean.

Choke regulation using the ensemble Kalman filter The ensemble Kalman filter corrects a state vector as measurements become available. If the measurement error is small, the result will honour the data correspondingly. The idea behind the approach described here is to replace the measurements by an upper limit for the total cumulative oil production or NPV. This scalar value is then stored in $\mathbf{D}_o$. The motivation for constructing this value, is to force the ensemble Kalman filter to produce a solution that fits, or lies as close as possible to the point stored in $\mathbf{D}_o$. With this objective, it is clear that the measurement error should be chosen small or zero.

In standard applications of the ensemble Kalman filter, the state vector usually contains space distributed data at a specific time. That approach is not pursued here, but instead the state vector is constructed from different choke settings ($\mathbf{c}$) over a time interval, in addition to a calculated value for total cumulative oil production or NPV

$$\mathbf{U} = \left[\begin{array}{cc} m & \mathbf{c} \end{array}\right]^T, \qquad \mathbf{c} = [\mathbf{c}_1\ \mathbf{c}_2\ ...\ \mathbf{c}_M].$$

Here m represents the total cumulative oil production or NPV, and corresponds to calculated measured values ($\mathbf{D}$ in Eq. 1).

Further, the production interval is divided into a set of M regulation intervals. Choke settings are constant within each regulation interval. The choke settings within each regulation interval are here vectors where each entry represents one choke. Note that in our case, these choke settings are represented by discrete numbers.

An initial ensemble of choke settings, $\mathbf{c}^0$, is constructed by choosing random integers with equal probability of getting every allowed choke setting, and $\mathbf{m}^0$ is calculated by running forward simulations using $\mathbf{c}^0$.

The updating procedure is as follows: First, data ($\mathbf{m}^{j-1}$) and choke settings ($\mathbf{c}^{j-1}$) are collected. Then the ensemble Kalman filter is run for each ensemble member (i) to produce $\tilde{\mathbf{m}}^j$ and $\tilde{\mathbf{c}}^j$. I.e.

$$\tilde{\mathbf{U}}^{j,i} = \mathbf{U}^{j-1,i} + \mathbf{K}^{j-1}(\mathbf{D}_o^{j-1} - \mathbf{H}\mathbf{U}^{j-1,i}).$$

This results in continuous choke settings, $\tilde{\mathbf{c}}^j$. These are rounded to the closest allowed discrete setting to produce $\mathbf{c}^j$. Finally, forward simulations using $\mathbf{c}^j$ are run to produce $\mathbf{m}^j$. This ensures that there is consistency between

$\mathbf{c}^j$ and $\mathbf{m}^j$. In addition, to accelerate the convergence, the new ensemble ($\mathbf{U}^j$) is modified by selecting the N best ensemble members from $\mathbf{U}^j$ and $\mathbf{U}^{j-1}$. We also replace duplicated ensemble members with random numbers drawn in the same manner as for the initial ensemble.

The value stored in $\mathbf{D}_o^{j-1}$ is calculated by using the known ensemble of total cumulative oil production or NPV, $\mathbf{m}^{j-1}$, according to the formula

$$\mathbf{D}_o^{j-1} = \max(\mathbf{m}^{j-1}) + \mathrm{std}(\mathbf{m}^{j-1}),$$

where 'std' denotes the standard deviation.

Note the change in notation when compared to the previous section. The vector $\mathbf{U}_a^{n,i}$ is replaced by $\tilde{\mathbf{U}}^{j,i}$ and $\mathbf{U}_f^{n,i}$ is replaced by $\mathbf{U}^{j-1,i}$

In order to find the optimal choke settings ($\mathbf{c}_{max}^j$) at a given iteration (j), the ensemble of state vectors are searched to find the maximum total cumulative oil production or NPV, and $\mathbf{c}_{max}^j$ is defined to be the corresponding choke values. Note that the superscript j here represents iterations, and not timestep or ensemble member.

Formula for net present value As mentioned, the ensemble Kalman filter is used to optimize either total cumulative oil production or NPV. Here we introduce the notation and formula for the NPV.

Let cumulative oil production be given by $\mathbf{p}^{op} = [p_1^{op}\ p_2^{op}\ ...\ p_M^{op}]$, where the time interval between the values is Δt (measured in days). The value Δt is for simplicity assumed constant. Similarly, let cumulative water production be given by $\mathbf{p}^{wp} = [p_1^{wp}\ p_2^{wp}\ ...\ p_M^{wp}]$ and cumulative water injection by $\mathbf{p}^{wi} = [p_1^{wi}\ p_2^{wi}\ ...\ p_M^{wi}]$. The objective function for the NPV is then given by

$$J = \sum_{k=1}^{M} \frac{r_o \Delta p_k^{op} - r_{wp}\Delta p_k^{wp} - r_{wi}\Delta p_k^{wi}}{(1+b/100)^{k\frac{\Delta t}{360}}}, \tag{2}$$

where $\Delta p_k^{op} = p_k^{op} - p_{k-1}^{op}$, and $p_0^{op} \equiv 0$. The quantities Δp_k^{wp} and Δp_k^{wi} are defined similarly. Further, r_o is the benefit factor for oil production, and r_{wp} and r_{wi} are the cost factors for water production and injection, respectively. The interest rate (in percent) is given by b.

Choke regulation using Partial Enumeration Method The Partial Enumeration Method (see Wang[4]) consists of sequentially perturbations of choke settings. It is quickly summarized by the following steps:

1. Iteration index k = 0.
2. Select choke j.
3. For choke j, do:
   (a) Switch to one of the allowed settings.
   (b) Run simulator for a given period of time.
   (c) Repeat a-b for all allowed settings and choose the setting that results in highest oil production.
4. Repeat 2-3 for all chokes.
5. Increase k by 1.
6. Repeat 2-5 until convergence.

The steps are run to maximize the cumulative production for the current time period, in contrast to the ensemble Kalman filter which performs a global optimization. This means that the choke settings for the first interval $\mathbf{c}_1$ are computed by maximizing m_1, then the choke settings for the second interval are maximized, an so on. The algorithm above is used to compute each of the choke settings $\mathbf{c}_i,\ i = 1...M$. Note that m_i here represents the cumulative oil production corresponding to interval i.

The ensemble Kalman filter approach is compared to the Partial Enumeration Method in order to validate the performance. Note that there is a difference between how the ensemble Kalman filter and the Partial Enumeration Method are set up. The ensemble Kalman filter estimates choke settings throughout the whole period in one iteration. The Partial Enumeration Method estimates the choke settings sequentially. I.e. choke settings for a given interval are estimated and then fixed. The method then proceeds to the next interval. This difference makes the comparison not completely fair, as the objective function is slightly different for the two methods.

Example

We consider an example with a simple synthetic reservoir. The reservoir dimensions are 1020 m x 510 m horizontally and 50 m vertically. The reservoir is divided into 30 x 3 x 20 grid blocks, with five horizontal layers with thickness 10 m. The layers have permeability (mD) 100, 1000, 50, 750 and 50 from top to bottom. The vertical permeability between layers is 1% of the horizontal. There are two wells penetrating the reservoir, one producer and one injector. Both are located in the middle of the reservoir's extension in the y direction. In the x-direction, the producer is located 153m away from the left end of the reservoir whereas the injector is 867m away (i.e. both are 153m away from a reservoir boundary). The producer has four inflow zones and the injector has five injection zones (which gives a total of nine chokes). The production chokes have three positions: open, half open, and closed. The injection chokes have two positions: open and closed. In the forward simulator this is represented by the discrete values 1 (closed), 2 (half-open) and 3 (open) for the producer and 1 (closed) and 2 (open) for the injector. Maximum allowed oil production is 2500 scm/day. Further, minimum allowed bottom hole pressure for producer is 215 bar, and maximum allowed bottom hole pressure for injector is 285 bar. The water injection is by voidage replacement (controlled by reservoir fluid volumerate). Figure 1 shows the schematic reservoir.

When NPV is optimized, the following economic parameters are used: $r_o = 50$ \$/bbl, $r_{wp} = 10$ \$/bbl and $r_{wi} = 0$ \$/bbl. The interest rate b is set to 10%.

In this example we have used an ensemble size of 100, and the number of iterations is 31 ($j = 1, 2, ..., 31$). Regulation interval is 180 days, and number of regulation intervals is 10 ($M = 10$, which gives a total production duration of 5 years).

Figure 2 shows the calculated maximum total cumulative oil production versus iterations (left figure). The green line represents the reference solution. The reference solution is calculated by keeping all chokes open over the entire production time. The figure also shows the calculated maximum NPV versus iterations (right figure). The reference value is not included here as this value is very low compared to the optimized NPV. The reason for this is high cost due to water production, which again is a consequence of keeping the chokes open all the time. We see that the optimized values are increasing as a function of iterations, for both approaches. The gain is 12 % and 15 % respectively.

Figure 3 shows the choke settings for the ensemble Kalman filter approaches and the Partial Enumeration approach. Chokes 1-4 represent the four chokes in the producer, whilst the next five chokes represent the chokes in the injector. The "Demo" choke (in the lower left corner of each figure) shows the color codes. The control of choke 4 is standing out as very different for the NPV optimization compared to the other two approaches. In the NPV optimization this choke, which is connected to a high permeability zone, is kept closed all the time. The consequence of this is reduced water production and reduced oil production. This is also seen on subsequent figures.

Figure 4 shows the cumulative oil production and the cumulative oil gain. The cumulative gain is the difference between the cumulative oil production using the ensemble Kalman filter approaches and the reference solution (solid lines), and the difference between the cumulative oil production using the Partial Enumeration Method and the reference solution (dashed line). Optimization of cumulative oil production using the ensemble Kalman filter approach shows the most profitable development as high values are obtained between 600 and 1800 days. It approaches the same value as the Partial Enumeration Method. Optimization of NPV is not giving high values for the oil production, as cost related to water production is making the oil production unprofitable at an early stage.

Figure 5 shows the cumulative water production and water cut. Here we see that optimization of NPV is giving the decided lowest water production. Optimization of the cumulative oil production using the ensemble Kalman filter approach is also giving lower water production compared to the Partial Enumeration Method.

Figure 6 shows the water and oil production rates. We see that the water production rate when NPV is optimized is some places close to zero. This is also seen for the oil production rate.

Figure 7 shows the difference between the oil production rate using the ensemble Kalman filter approaches and the reference solution (solid lines), and the difference between the oil production rate using Partial Enumeration Method and the reference solution (dashed line).

Figure 8 shows the cumulative water injection and the water injection rate. Here we see the same trends as for the cumulative water production. Lowest water injection is obtained when NPV is optimized.

Figure 9 shows the water saturation after 1800 days (5 years). For each of the three figures we show the three layers in the y-direction. We see that almost identical results are obtained when cumulative oil production is optimized using the ensemble Kalman filter approach and the Partial Enumeration Method, with the exception of a small pocket of oil which is not recovered with the Partial Enumeration Method. This pocket is seen at $x = 15$ and $z = 9$. Not surprisingly, less oil is recovered when NPV is optimized. We also see that far from all the oil is recovered from layer one and layer five.

Table 1 shows some numerical results. We see that the decided highest NPV value is obtained when the NPV is optimized. The ensemble Kalman filter approach when cumulative oil production is optimized is superior to the Partial Enumeration Method when is comes to NPV. All approaches are better than the reference solution. The other numbers is a quantification of the results shown on the figures.

Comparison of the approaches For this particular example the Partial Enumeration Method used 440 Eclipse simulations with duration 6 months. The Ensemble Kalman filter method used $31 \cdot 100 = 3100$ Eclipse simulations with duration 5 years. This means that the ensemble Kalman filter is more time consuming, but is producing better solutions. On the other hand, the Partial Enumeration Method is using restart files, which makes the simulations somewhat slower. The number of forward simulations will for the Partial Enumeration Method increase rapidly when the number of chokes and allowed choke settings increase. An advantage with the ensemble Kalman filter approach is that it can easily be extended to handel a variety of objective functions (cumulative oil production and NPV in this example), and it can be extended to handle continous choke settings.

Convergence criteria for the ensemble Kalman filter approach In this example we have simply used a fixed number of iterations to obtain an optimized value for the total cumulative oil production or NPV. As we can see from Figure 2, the development of the optimized values are at some places constant for a limited number of iterations, before the values again are increasing. This implies that it is difficult extract a convergence criteria based on these values. It is also difficult to use the spread (standard deviation) of the optimized values as convergence criteria, as inspection of these values reveals that they are not necessarily decreasing towards a small value. We have therefore, at this time, chosen to use the current approach with a fixed number of iterations, but this is a topic suited for further research.

Conclusions and further work

We have demonstrated a new approach for controlling downhole chokes so that water flooding is optimized. The methodology is based on utilizing the ensemble Kalman filter. This new application is verified by comparing the results to the Partial Enumeration Method. The ensemble Kalman filter is used to optimize either total cumulative oil production or NPV. The results reveals that the approach is producing better results compared to the Partial Enumeration method. This is seen as more optimal oil production and more optimal water production and injection. The NPV values for the ensemble Kalman filter cases are therefore higher than the NPV value for the Partial Enumeration approach. At the current stage, the ensemble Kalman filter approach is however somewhat more time consuming.

There are several topics suited for further research. It remains some work to develop a good convergence criteria for the ensemble Kalman filter approach. It is also possible that a more thorough investigation of the approach opens for faster development towards a high value for the optimized quantity. We have in this example not utilized the reference solution in the optimization. This reference solution could be included in the initial ensemble. We also wish to investigate the effect of the ensemble size for the optimized values. In addition, the approach is in this work only applied to a simple synthetic example. The next step is to apply the methodology to a large scale field example.

Acknowledgments

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Nomenclature

$\mathbf{U}$	=	state vector
$\mathbf{D}$	=	measurements
$\mathbf{f}$	=	model function (reservoir simulator)
$\mathbf{H}$	=	measurement matrix
$\mathbf{K}$	=	Kalman gain matrix
$\mathbf{P}$	=	covariance matrix for model uncertainty
$\mathbf{L}$	=	left factor of covariance matrix
N	=	ensemble size
m	=	optimized value
$\mathbf{c}$	=	choke settings
$\mathbf{p}^{op}$	=	cumulative oil production
$\mathbf{p}^{wp}$	=	cumulative water production
$\mathbf{p}^{wi}$	=	cumulative water injection
b	=	interest rate

Subscripts

f	=	forecast (a priori)
a	=	analyzed (a posteriori)
o	=	observation
M	=	number of regulation intervals
i	=	regulation interval
k	=	summation index

Superscripts

n	=	timestep index
i	=	ensemble member index
T	=	matrix transpose
j	=	iteration

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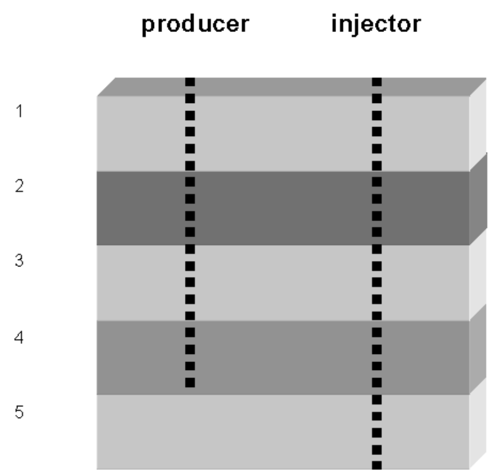
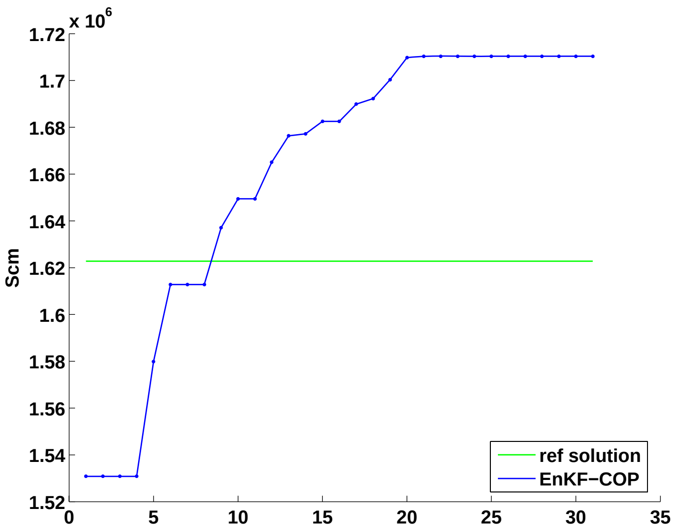
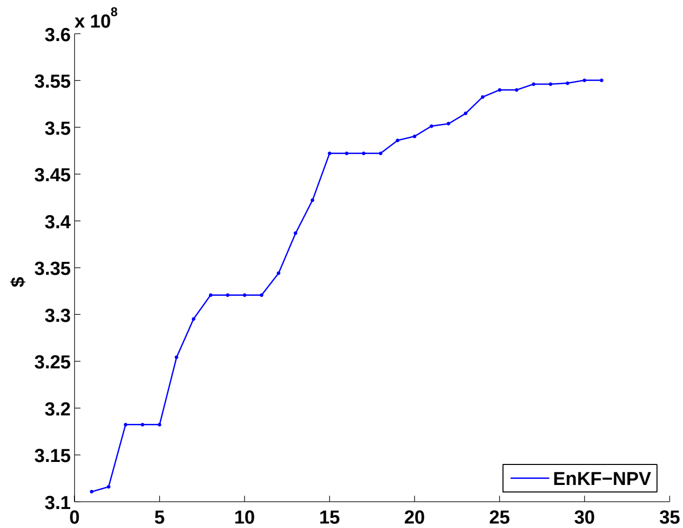


Figure 1: Schematic reservoir.



(a) Total cumulative oil production vs. iterations.

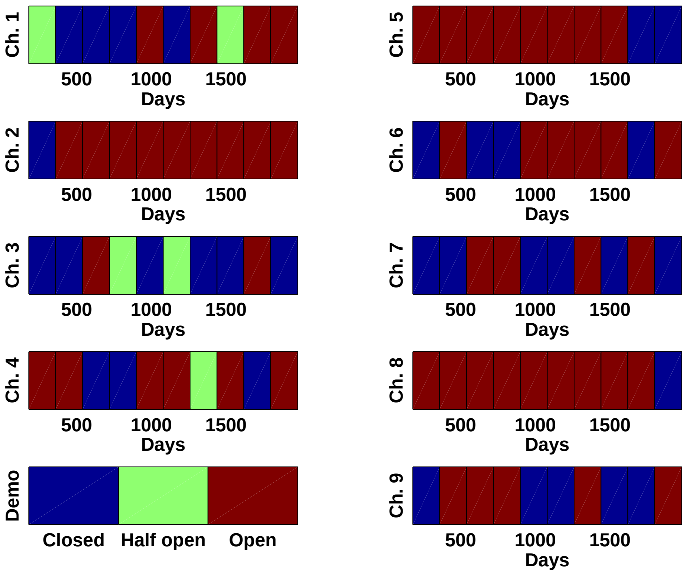


(b) Net present value vs. iterations.

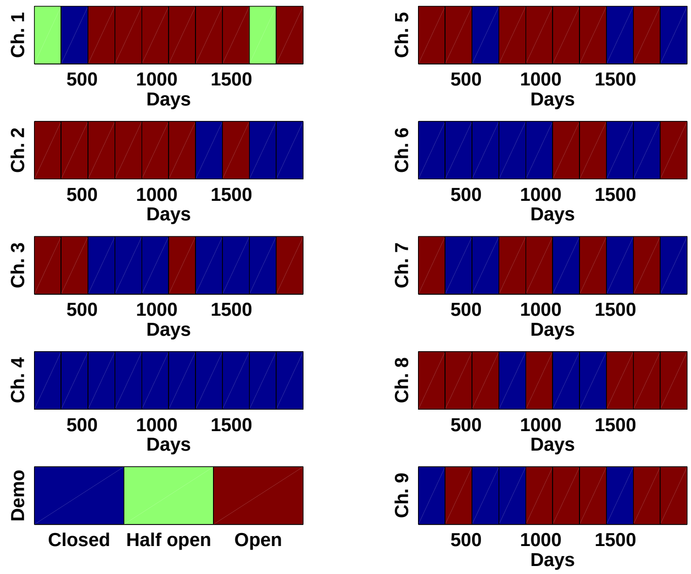
Figure 2: Development of optimized value.

	NPV $\times 10^6$ (\$)	$p_M^{op} \times 10^4$ (scm)	$p_M^{wp} \times 10^4$ (scm)	$p_M^{wi} \times 10^4$ (scm)
Ref	118.4	162.3	672.5	881.0
EnKF-COP	240.1	171.3	476.6	696.2
EnKF-NPV	366.7	142.4	76.6	258.0
PEM	193.1	170.8	561.4	780.7

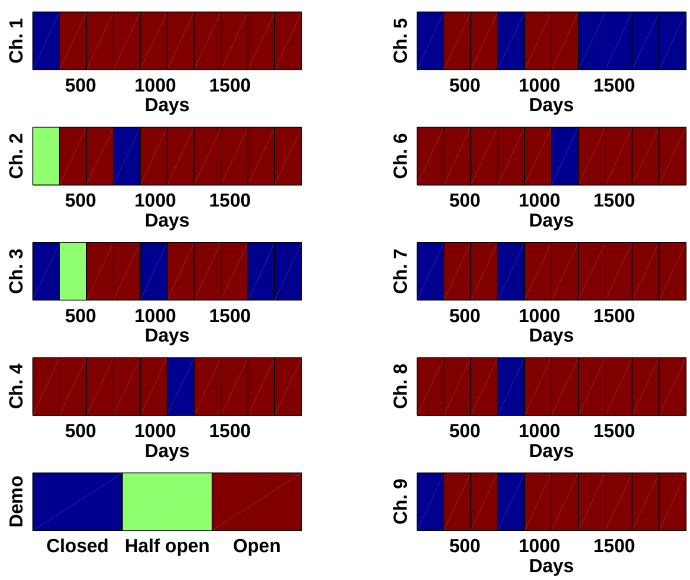
Table 1: The NPV, total cumulative oil production p_M^{op}, total cumulative water production p_M^{wp} and total cumulative water injection p_M^{wi} for the different runs.



(a) Optimal choke settings for EnKF-COP.

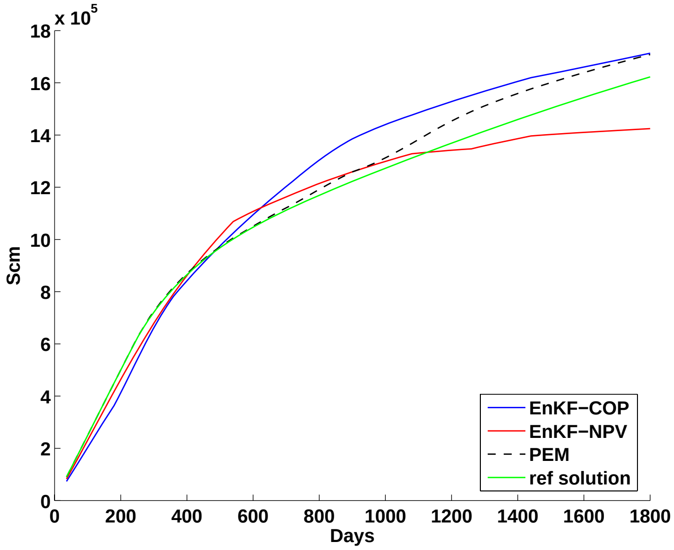


(b) Optimal choke settings for EnKF-NPV.

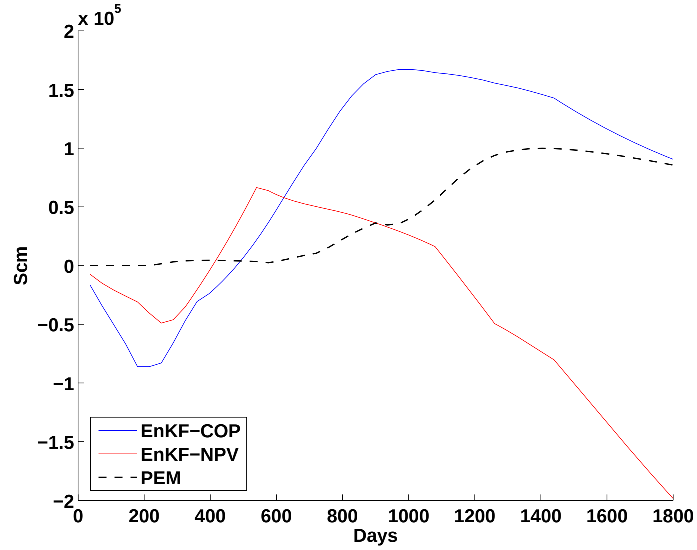


(c) Optimal choke settings for PEM.

Figure 3: Choke settings for the different runs. Chokes 1-4 represent the four chokes in the producer, whilst the next five chokes represent the chokes in the injector. The "Demo" choke shows the color codes.

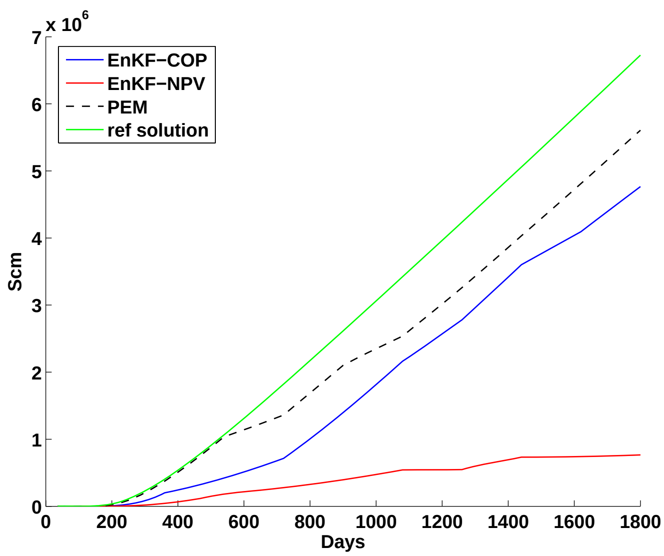


(a) Cumulative oil production.

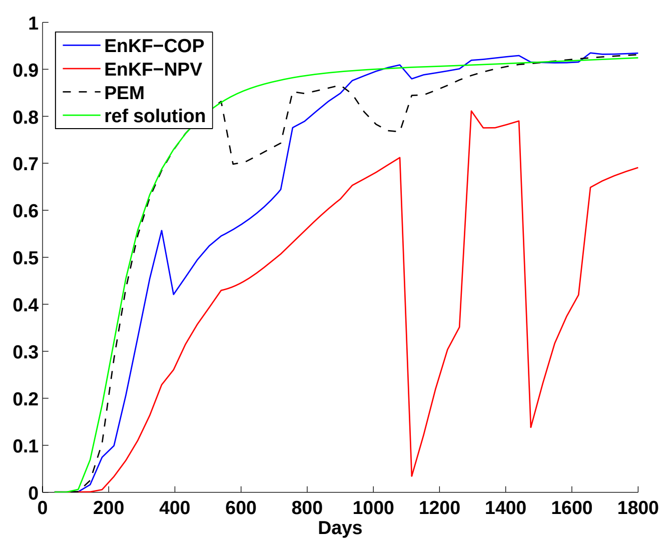


(b) Cumulative oil gain.

Figure 4: Cumulative oil production and cumulative gain. The cumulative gain is the difference between the cumulative oil production using the ensemble Kalman filter approaches and the reference solution (solid lines), and the difference between the cumulative oil production using the Partial Enumeration Method and the reference solution (dashed line).

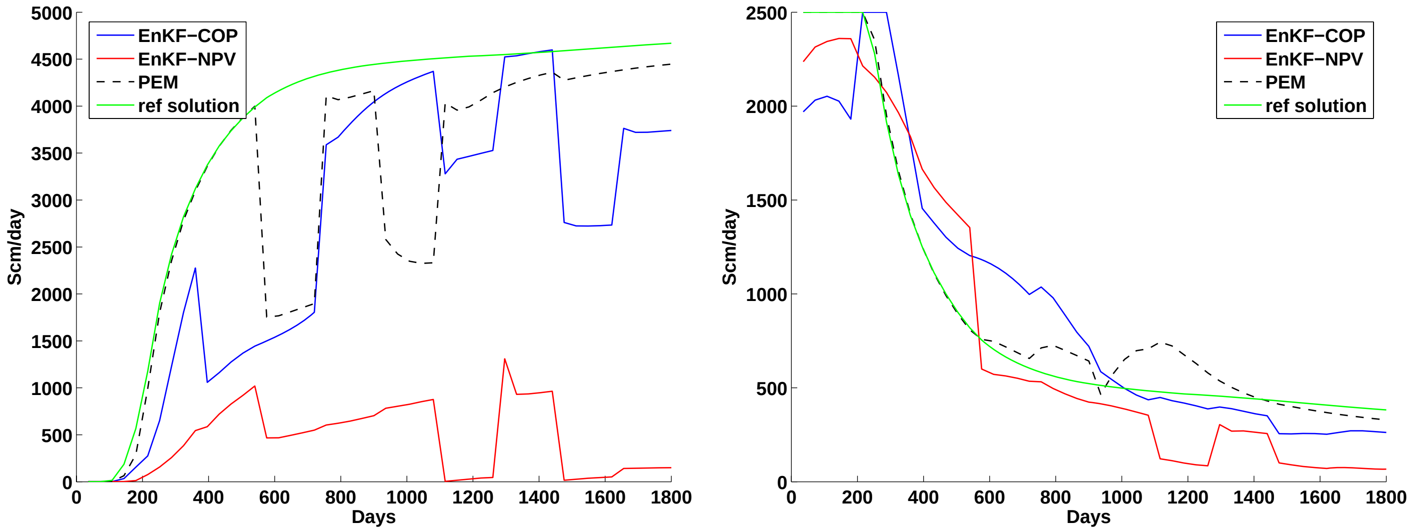


(a) Cumulative water production.



(b) Water cut.

Figure 5: Cumulative water production and water cut.



(a) Water production rate.

(b) Oil production rate.

Figure 6: Water and oil production rates.

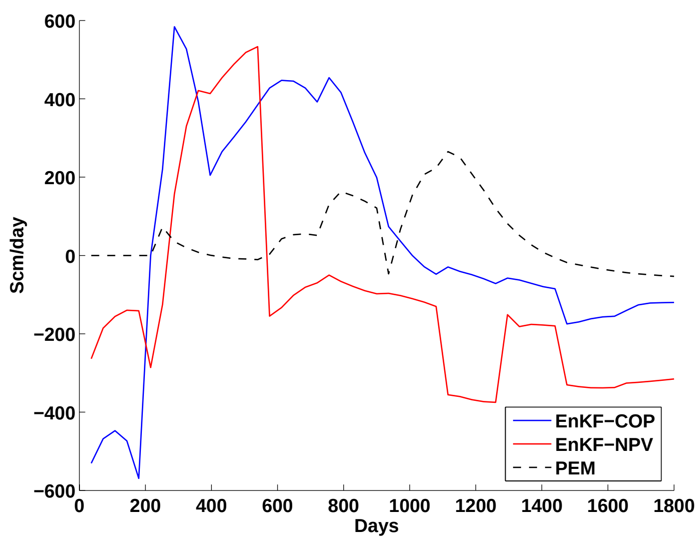
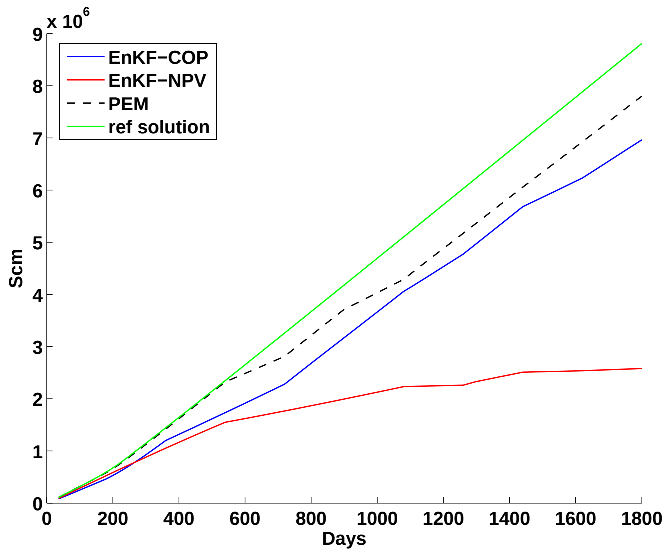
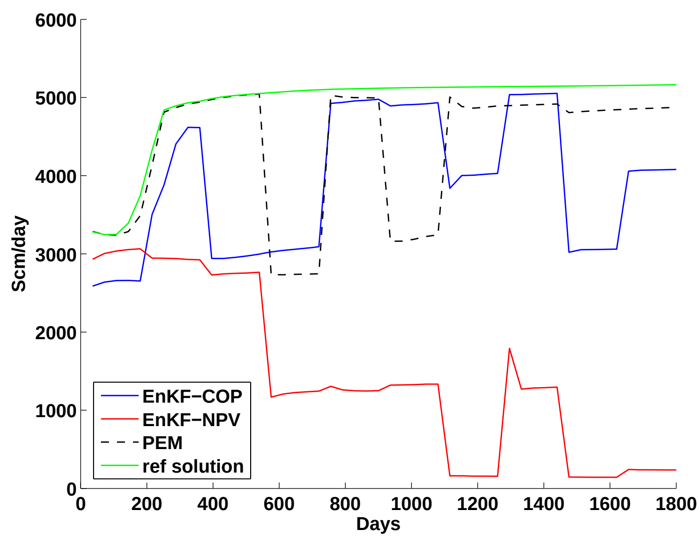


Figure 7: Extra oil. This is the difference between the oil production rate using the ensemble Kalman filter approaches and the reference solution (solid lines), and the difference between the oil production rate using Partial Enumeration Method and the reference solution (dashed line).

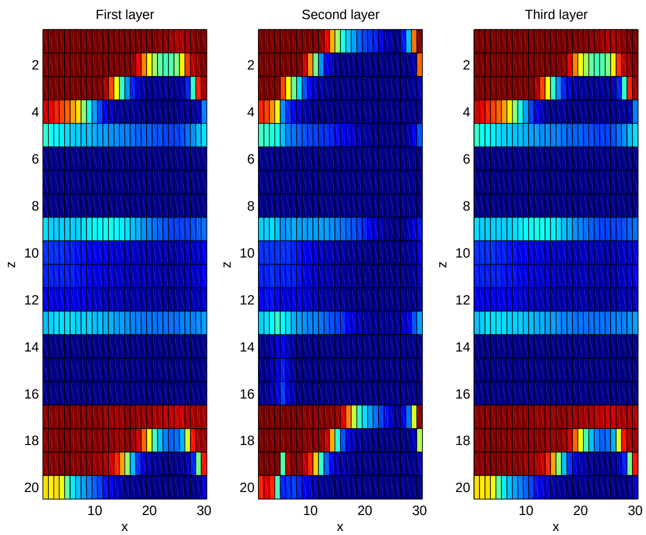


(a) Cumulative water injection.

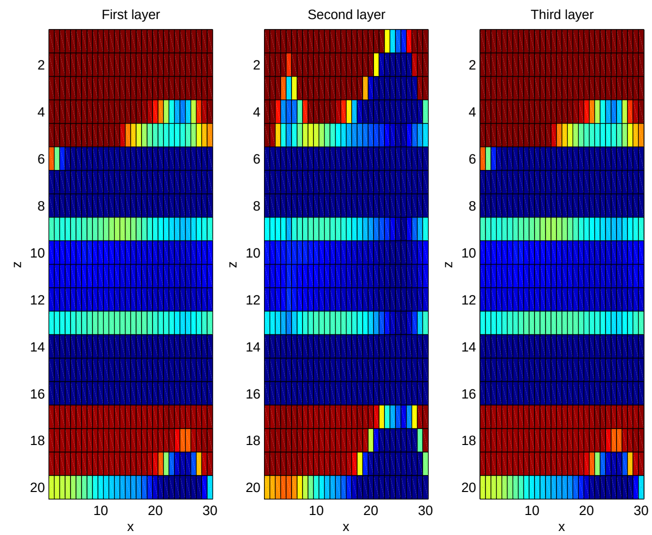


(b) Water injection rate.

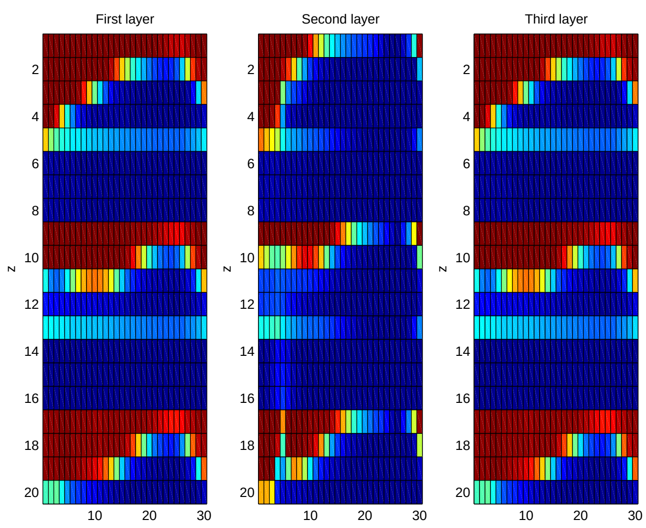
Figure 8: Cumulative water injection and water injection rate.



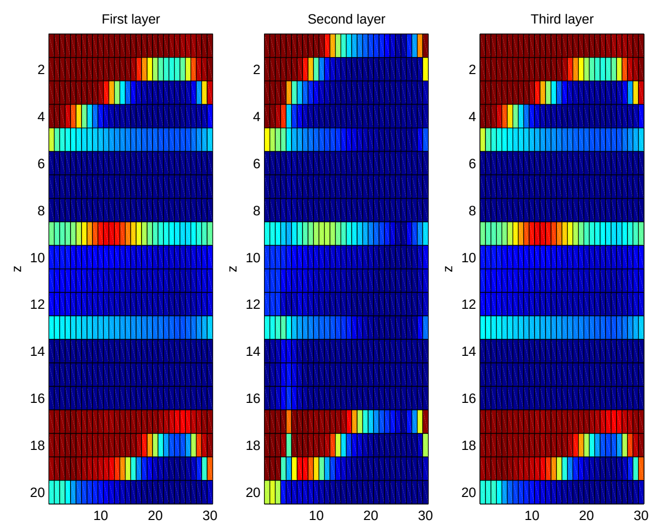
(a) Water saturation for EnKF-COP after 1800 days.



(b) Water saturation for EnKF-NPV after 1800 days.



(c) Water saturation for reference solution after 1800 days.



(d) Water saturation for PEM after 1800 days.

Figure 9: Water saturation for the different runs. Minimum water saturation is 0.2 (dark red) and maximum water saturation is 0.8 (dark blue).