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Control of a Displacement Front in Potential Flow Using Flow-Rate Partition

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Abstract

We consider the problem of the control of a displacement front in a porous medium, via flow-rate partition in a well. We assume that the flow is potential and that the displacement is at a unit mobility ratio. These assumptions are for mathematical convenience only and can be relaxed. They allow, however, significant insight into the problem. The specific question we address is how to partition the flow rate within the injection well, so that the induced displacement front can be steered according to pre-determined dynamics.

When the reservoir is homogeneous and isotropic, we derive an integral equation in an analytical form, the solution of which determines the desired injection rate profile. We provide illustrative applications. A similar approach applies for an anisotropic or a heterogeneous system, except that the kernel in the integral equation must be determined numerically. This can be obtained by repeated calculations of the Green's function in a heterogeneous system or a modified two well system.

For the solution of the integral (Fredholm) equation, a regularization technique is necessary. However, it is found that numerical instabilities do develop, even with the use of regularization, for later times, when tranverse cross-flow is large. Conversely, the instabilities diminish with a more stratified structure.

The results find applications to the rapidly emerging field of smart wells and the optimization of displacement problems in oil reservoirs using flow rate control.

Introduction

Optimizing the recovery efficiency of displacement processes can be obtained in a most direct way by the optimal control of the rates of injection and production wells. Various measures of the recovery efficiency, or the objective function, can be defined that lead to corresponding optimal rate profiles. The literature in the subject has been sparse, but it is considerably expanding in recent years. Asheim[1] studied methods to increase the waterflooding efficiency by controlling rates in production/injection wells. Sudaryanto and Yortsos[2] provided a systematic approach for the dynamic optimization of displacement problems, in which the objective function was the displacement efficiency at breakthrough. Using potential flow in 2-D and point sources (vertical wells) for injection and production wells (Figure 1), they showed that the optimal injection policy is "bang-bang".

Smart wells and intelligent completion have gained significant attention in recent years in the the field of dynamic optimization of EOR processes. The ability to implement separate downhole control valves enables the application of optimal rate control in real time, through the appropriate valve settings in injection and production wells. In recent work, Brouwer and colleagues[3,4] published a series of papers investigating the optimization of waterflooding in heteregenous reservoirs having multiple segments along both injectors and producers. Two smart horizontal wells, in both injection and production, with multiple Inflow Conrol Valves (ICVs) were used. In the static approach, optimization of the waterflooding process was based on heuristic algorithms, resulting in fixed valve settings, which were kept fixed during the displacement. The approach was then extended to dynamic conditions, using optimal control theory for maximizing ultimate recovery by controlling the valve settings continuously. Yetan[5] applied an optimization technique based on conjugate gradients to optimize the operation of smart wells. He linked the optimization algorithm with a commercial simulator to model the inflow control devices. As in all previous studies, significant improvement in ultimate recovery was obtained, compared to the non-optimized case.

In this paper, we consider a fundamental question in rate-control flow problems, namely the control of a displacement front via flow-rate partition in a horizontal well. The specific question we address is how to partition the flow rate within the well, so that the displacement front can be steered according to pre-determined dynamics, or as is necessary. To our knowledge, this problem has not been addressed before. The ability to steer a front at will is of obvious significance. Given the reservoir geology, one can for example, steer the front away from flow obstacles, affect a piston-like displacement in heterogeneous formations and otherwise control the displacement process as needed. We address this problem in a simple rectangular geometry based on potential flow. These simplifications allow for the problem to be posed through an exact formalism, resulting in significant insight. Subsequent extensions to more complex geometries and processes will follows.

We show that, in principle, the injection profile that will result into specified front dynamics can be determined from the solution of an integral equation. The kernel of the equation is obtained analytically, in the case of a homogeneous and isotropic reservoir, or numerically in the general case. Because the integral equation is ill-posed, however, as is often the case in inverse problems, a regularization technique is necessary. We discuss the application of such techniques and the extent to which they can provide useful solutions.

Problem Formulation

In this section we present the general methodology and formulation. A simplified schematic of the problem considered is shown in Figure-2 with boundary conditions and a typical front location. Under the assumption of incompressible miscible fluids of equal mobility, and in the absence of gravity and dispersion effects, the governing equation is

$$\nabla \cdot \left(k(x,y)\nabla P\right) = 0 \qquad (1)$$

where P is a normalized flow potential. For the geometry of Figure 2, the appropriate boundary conditions in dimensionless notation are

$$-\left.\frac{\partial P}{\partial x}\right|_{x=0} = -q(y,t),$$

$$P\big|_{x=1} = 0$$

$$-\left.\frac{\partial P}{\partial y}\right|_{y=0,\,y=1} = 0.0$$

In the above the transverse and the flow direction have been scaled by H and L, respectively, and we assumed that the ratio $R_L = \frac{L}{H}\sqrt{\frac{k_V}{k_L}}$ (where the two permeability values in the y and x directions were defined) is equal to one. In the general case when this does not apply, equation (1) should be replaced by

$$R_L^{-2}\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = 0 \qquad (2)$$

(where we considered only the homogeneous problem). Yortsos[6] examined in detail the effect of parameter R_L on displacements, with the limit $R_L \to \infty$ corresponding to the so-called limit of Transverse Flow Equilibrium and the limit $R_L \to 0$ to the Dykstra-Parsons limit (see also Yang et al.[7]). Because the problem is linear, in the general case, the pressure field can be represented by a superposition of the rate *q(y,t)* with the Green's function

$$P(x,y) = \int_0^1 G(0,\eta;x,y)\,q(\eta,t)\,d\eta \qquad (3)$$

For example, when the problem is homogeneous and $R_L = 1$ the Green's function is

$$G(0,\eta;x,y) = (1-x) + \sum_{n=1}^{\infty}\left(\frac{2\sinh(\lambda_n(1-x)\cos\lambda_n y}{\lambda_n \cosh\lambda_n}\cos\lambda_n\eta\right) \qquad (4)$$

where $\lambda_n = n\pi$. Analytical results are always possible in the homogeneous case (e.g. equation (2)), but numerical solutions will be needed in the heterogenous case (see below). Assume now that the front is given by the general equation $F(x,y,t) = 0$, which can be also recast as

$$F(x,y,t) = x - f(y,t) \qquad (5)$$

where *f* is the front location. Because the front moves with the fluid, the material derivative is zero,

$$dF(x,y,t) = 0 = \frac{\partial F}{\partial t} - R_L^{-2}\frac{\partial P}{\partial x}\frac{\partial F}{\partial x} - \frac{\partial P}{\partial y}\frac{\partial F}{\partial y} \qquad (6)$$

where we rescaled time appropriately and used Darcy's law. Using (5) we further obtain the following kinematic equation

$$\frac{\partial f}{\partial t} = -R_L^{-2}\frac{\partial P}{\partial x} + \frac{\partial P}{\partial y}\frac{\partial f}{\partial y} \qquad (7)$$

Then, using equation (3) for the pressure we rearrange (7) in the following integral equation

$$\frac{\partial f(y,t)}{\partial t} = \int_0^1 K(0,\eta;f(y,t),y)\,q(\eta,t)\,d\eta \qquad (8)$$

where the kernel is a function of the front dynamics

$$K(0,\eta;f(y,t),y) = \frac{\partial f(y,t)}{\partial y}\frac{\partial G}{\partial y}(0,\eta;f(y,t),y) - R_L^{-2}\frac{\partial G}{\partial x}(0,\eta;f(y,t),y) \qquad (9)$$

Equation (8) is a Fredholm integral equation of the first kind, the solution of which will allow in principle for the control profile to be determined, hence the front to be steered according to the prescribed dynamics.

<u>Heterogeneous Porous Media</u>

A similar technique can also be applied for displacement problems in heteregenous media. Alternatively, one can apply a different superposition technique, as elaborated in Sudaryanto and Yortsos[2]. The idea is to express the displacement as the superposition of the response of individual well segments of point source strength, in the horizontal well. For example, assume that $R_L = 1$, in which case the kinematic equation can be written in terms of the velocities at the front as

$$\frac{\partial f}{\partial t} = v_x - v_y\frac{\partial f}{\partial y} \qquad (10)$$

Because of the linearity of the problem, the velocities can be represented as the superposition of an (infinite) number of velocity fields v_x^i and v_y^i corresponding to well doublets (namely one injection and one production point sources and sinks)

$$v_x = \sum_{i=1}^{Nwi} \alpha_i v_x{}^i \qquad (11)$$
$$v_y = \sum_{i=1}^{Nwi} \alpha_i v_y{}^i$$

Here, α_i is the correspondent injection rate for each well doublet (along each injection segment), and *Nwi* is the number of well segments along the injection well (theoretically infinite, but in practice finite). In the production well, segments are represented as one well by imposing the same Dritchlet boundary condition for all segments. By substituting (11) into (10) we find an integral equation similar to equation (8), the discretized form of which is

$$\left.\frac{\partial f_j}{\partial t}\right|_t = \sum_{i=1}^{Nwi} K_{j,i}\alpha_i(t) \qquad j,i = 1,2,...Nwi \qquad (12)$$

where

$$K_{j,i} = v_x{}^i - \frac{\partial f}{\partial y}\, v_y{}^i$$

The solution of (12) gives the rate partition along the injection well as a function of time.

Solution of the Inverse Problem

Fredholm integral equations of the first kind arise in many engineering applications involving inverse problems. The generic form of the equation is

$$\int_a^b K(s,t) f(t) dt = g(s) \qquad c \le s \le d \qquad (13)$$

where the kernel $K(s,t)$ and the right-hand side $g(s)$ are known, while $f(t)$ is the unknown function to be determined. These types of equations are inherently ill-posed and the solution is sensitive to the perturbation of the system, therefore the classical numerical techniques fail to compute a meaningful solution once the integral has been discretized. To compute a unique and meaningful solution one must impose stability by specifying additional information which helps to singles out a smooth solution close to the correct solution. One such technique is regularization. (Hansen [8])

Tikhonov Regularization Method

Regularization techniques, such as Tikhonov Regularization, have been used to get a meaningfull regularized solution. In theory regularization is applicable to the integral equation as proposed by Tikhonov in his original work. However, in practice it is simpler to implement it to the linear system produced by discretization of the integral equation. Then, Tikhonov regularization produces the solution x_λ from the following optimization problem

$$x_\lambda = Arg\min\left\{\|Ax-b\|_2^2 + \lambda^2\|x-x_o\|_2^2\right\} \qquad (14)$$

The regularization parameter λ controls the amount of weight given to the side constraint. The idea behind Tikhonov regularization is to introduce a way to control the trade-off between the residual norm (first term above) and the constraint (second term). The optimum value of λ is the one that balances the two. This is the concept behind the L-Curve technique which visually helps to choose the best λ .

Results and Discussion

Results corresponding only to the homogeneous problem are shown below. The heterogeneous case will be discussed in a future report. In all figures x and y axes are as depicted in Figure-2. Two simple front profiles were used.
The first front profile has the sinusoidal form

$$f(y,t) = t + 0.2t\cos(4\pi y) \qquad (15)$$

corresponding to a front translating at a y-dependent velocity. Figure 3a shows the desired evolution of the front at different times. Figure 3b shows the corresponding rate profile obtained after applying regularization and for the case $R_L = 1$. For the times shown, the front is well controlled and with good accuracy. Indeed, using the obtained rate solution into a forward solving algorithm with front tracking, produces the results shown in Fig. 3c. The comparison between desired and achieved is good although one notes that there is progressive deterioration as time increases. Regularization allowed us to obtain a stable solution for a substantially larger values of time than would have otherwise been possible. The deterioration of the solution as time increases is a reflection of the extensive cross-flow that develops. In early times (or at small values of R_L as also noted by Yang et al. [7]), cross-flow is limited and the problem behaves as in a non-communicating layered system (see further discussion below). In fact, the early-time solution for the rate (Figure 3b) is identical to that corresponding to a layered system, which for the front above has the simple form

$$q(y,0) = 1 + 0.2\cos(4\pi y) \qquad (16)$$

As time increases, the rate profile has the same variation but with a larger amplitude, as shown in the Figure. At the same time, at larger times the intensity of cross-flow increases and the sensitivity of the front to the rate diminishes, causing ill-posedness and instability. Extending the applicability to larger times is the subject of continuous study.
The second example corresponds to a front evolving according to

$$f(t) = 0.5t(\kappa-1)\tanh((y-0.5)/\lambda) + 0.5t(1+\kappa) \qquad (17)$$

Parameters κ and λ dictate the sharpness of the front. A characteristic example is shown in Figure 4a, corresponding to $\kappa = 0.5$ and $\lambda = 0.25$, respectivly. The injection rate profile and the corresponding front dynamics achieved are shown in Figures 4b and 4c, respectively. As with the first profile, good control is possible at early times, when the system is almost behaving as layered. Indeed, the initial rate, as shown in Figure 4b, is very close to the layered-system profile

$$q(y,0) = 0.5(\kappa-1)\tanh((y-0.5)/\lambda) + 0.5(1+\kappa) \qquad (18)$$

The control is progressively weakened at larger times, when substantial cross-flow is experienced. Again, regularization was found to be necessary for a stable solution to be obtained.
Using the first profile we computed solutions for the case when the TFE parameter is different than unity. Figures 5a and 5b show results for the case $R_L = 0.1$. As the system approaches conditions of a non-communicating layer, the flow rate profile becomes closer to (16) and control can be exerted at later times. This behavior is more pronounced in the more extreme case shown in Figures 6a and 6b, corresponding to $R_L = 0.01$. The rate profile becomes independent of time and it is identical to (16).

Conclusions

In this paper we considered the problem of the control of a displacement front in a porous medium, via flow-rate partition in a well. Under the assumptions of potential flow and that the displacement is at a unit mobility ratio we derived a formalism that allows controlling displacement fronts of predetermined dynamics using rate control. The formulation results into the solution of an integral equation, which is in an analytical form, for the case of a rectangular geometry and a homogeneous reservoir.
For the solution of the integral (Fredholm) equation, a regularization technique is necessary. However, it is found that numerical instabilities do develop, even with the use of regularization, for later times, when tranverse cross-flow is large. Conversely, the instabilities diminish with a more stratified structure. Work is currently under way to extend the applicability of the method to larger times and more general conditions.
The results find applications to the rapidly emerging field of smart wells and the optimization of displacement problems in oil reservoirs using flow rate control.

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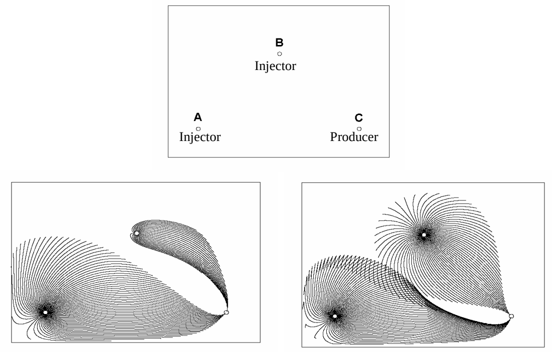

Figure.1 Performance comparison of conventional equal rate injection policy (left) vs. optimized Bang-Bang injection policy (right). (Sudaryanto and Y. Yortsos)

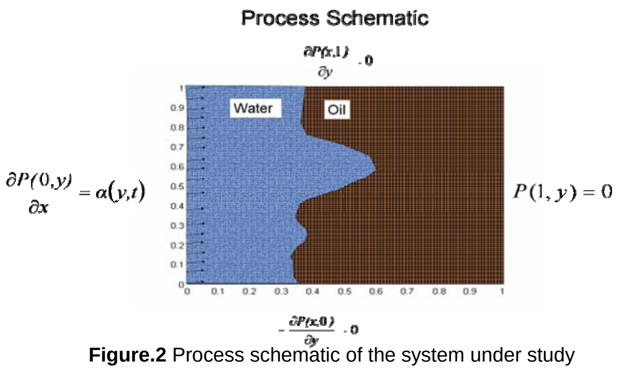

Figure.2 Process schematic of the system under study

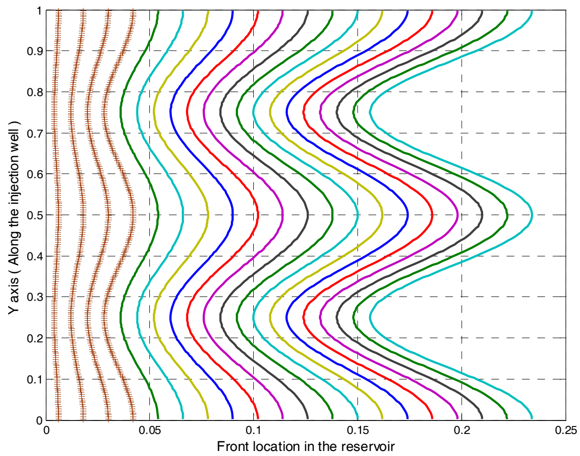

Figure.3a One example of desired front location as a function of time.

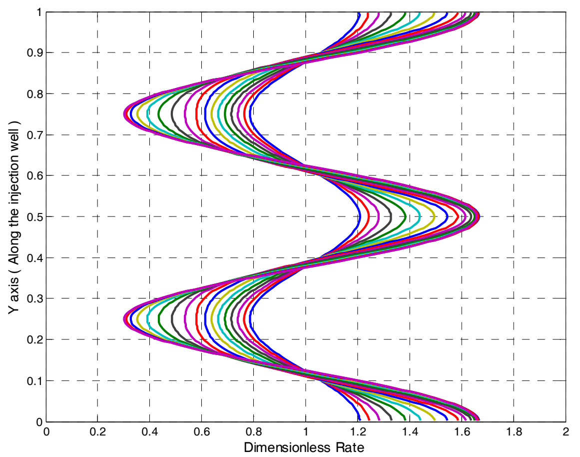

Figure.3b: Injection rate profile that produces the front location as a function of time specified in Fig. 3a. ($R_L = 1$)

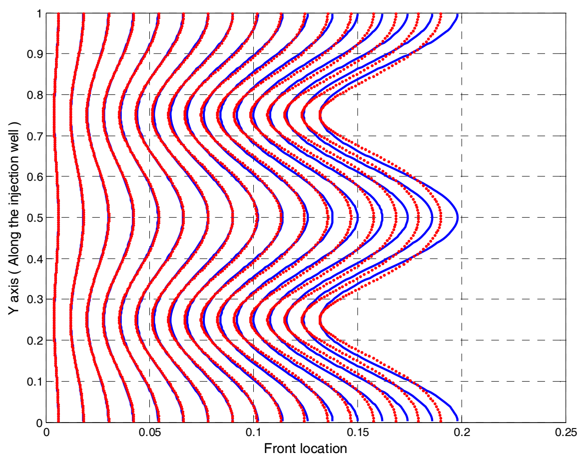

Figure.3c Comparison between desired (blue color) (Fig. 3a) and achieved front dynamics (red color) using the profile of Fig. 3b. ($R_L = 1$)

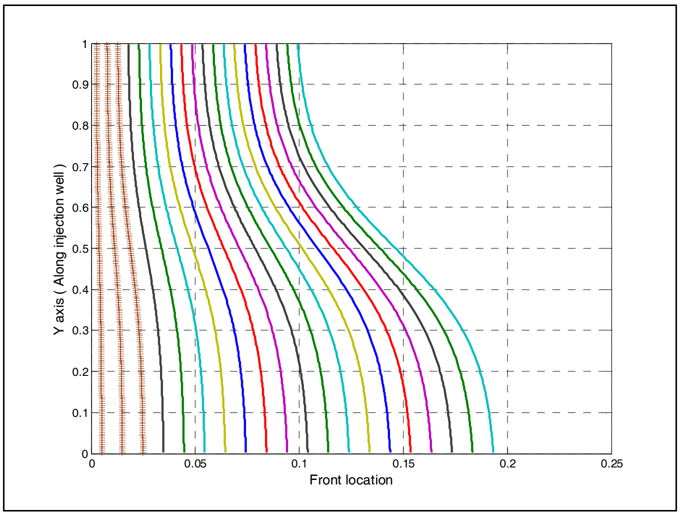

Figure.4a A different example of desired front location as a function of time.

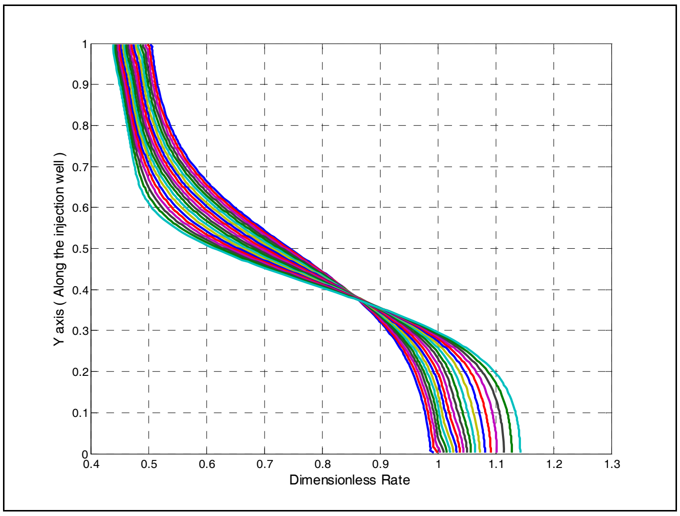

Figure.4b Injection rate profile that produces the front location as a function of time specified in Fig. 4a. **(** $R_L = 1$**)**

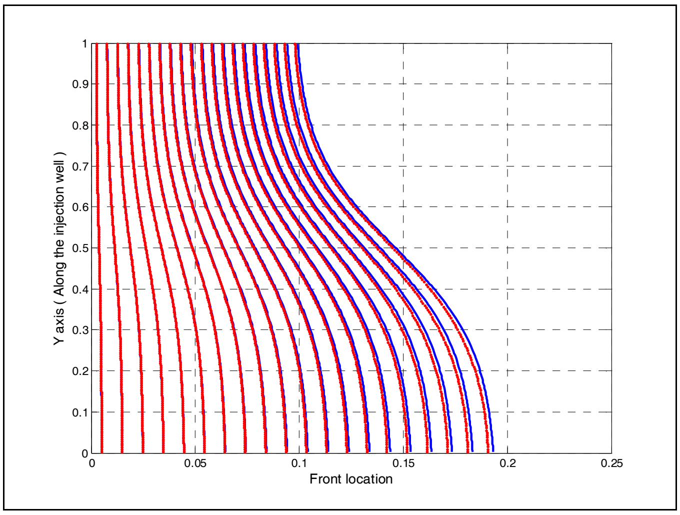

Figure.4c Comparison between desired (blue color) (Fig. 4a) and achieved front dynamics (red color) using the profile of Fig. 4b. **(** $R_L = 1$)

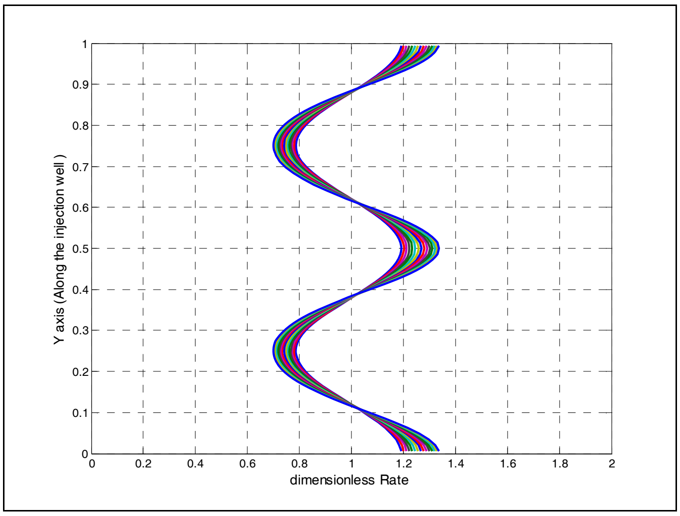

Figure.5a Injection rate profile that produces the front location as a function of time specified in Fig. 3a. $R_L = 0.1$

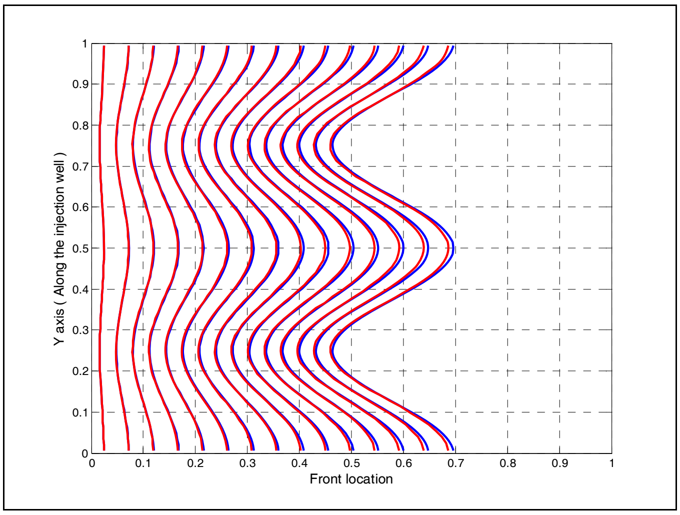

Figure 5b. Comparison between desired (blue color) (Fig. 3a) and achieved front dynamics (red color) using the profile of Fig. 5a. $R_L = 0.1$

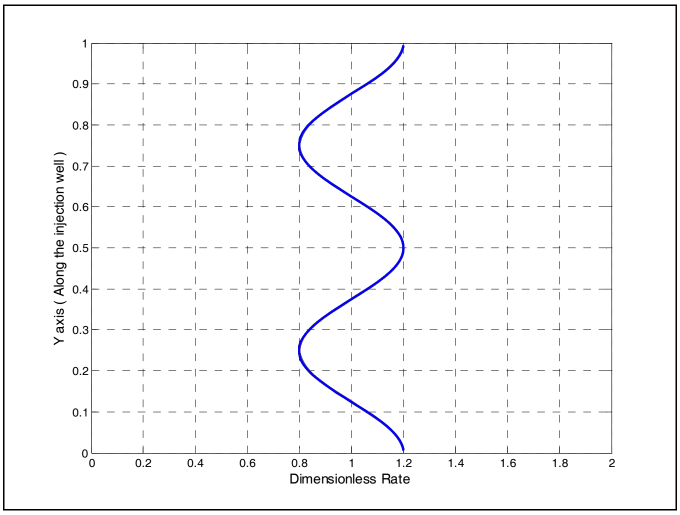

Figure.6a Injection rate profile that produces the front location as a function of time specified in Fig. 3a. $R_L = 0.01$

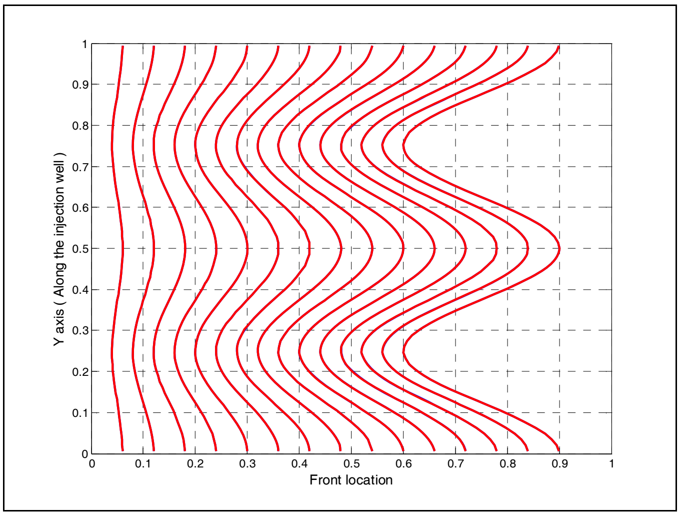

Figure.6b Achieved front dynamics using the profile of Figure.6a