SPE 99451

# A Consistent Approach Toward Reservoir Simulation at Different Time Scales

M. Nikolaou, SPE, U. of Houston; A.S. Cullick, L. Saputelli, G. Mijares, and S. Sankaran, SPE, Halliburton Digital and Consulting Solutions; and L. Reis, Petrobras



This paper was prepared for presentation at the 2006 SPE Intelligent Energy Conference and Exhibition held in Amsterdam, The Netherlands, 11–13 April 2006.



## Abstract

Reservoir simulation has become the *de facto* design and analysis tool to plan, develop, and manage oil and gas assets. With increasing complexity of flow networks and advanced recovery mechanisms in the fields, the model description and features of the reservoir simulator have also been progressively advancing.

The goal of a single, evolving, life-cycle model for oil and gas assets has many benefits for effective and efficient field development and exploitation. However, the size and complexity of the reservoir models often require characterization at several resolutions, thus ranging from full field strategic models to short range operational models. Full field strategic models can be used to evaluate various production scenarios and development strategies and to estimate future drilling and facilities requirements. Short range operational models concentrate on issues such as rate requirements, production decline analysis, etc. However, the approach to integrate and maintain these separate reservoir models while describing the same field is often *ad hoc* and many times, inconsistent.

This paper describes a new methodology for enhanced and effective use of reservoir simulation. Specifically, the application of a new method is presented to consistently integrate the full field strategic models and the short range operational models using a parametric system identification approach. The measurements from the field are used to continuously update the short range operational models over a moving time horizon, while simultaneously preparing the data for a history match of the full-field, strategic model. This hierarchical model structure at different scales avoids frequent and costly history-match runs of the larger strategic models without compromising on short term accuracy, for example, those required by production optimization. In addition, the hierarchical model structure improves effectiveness and efficiency in carrying out the simulation objectives. A case study of a full-field performance is presented to highlight the benefits of the method.

## Introduction

The increasing availability of real-time measurements and remotely activated valves in an oilfield has made oilfield-wide optimization of operations a distinct possibility[1]. While the term real-time optimization (RTO) is certainly not new and RTO is practiced in elements of drilling or production operations[2-4], the extent to which RTO is now feasible has increased dramatically. At the same time, the increased scope of RTO of oilfield operations entails significant complexity and creates challenges.

RTO technologies have been advanced, either within the oil and gas industry or in related industries, such as oil refining. While it would certainly be beneficial to further develop technologies for field-wide RTO, it is also useful to identify existing technologies suitable for the task, streamline such technologies for use in the oilfield, and ensure that such technologies are used prudently and ultimately add value Because elements of field-wide RTO can be manifested in many activities related to production optimization, one may be overwhelmed by the multitude of approaches and breadth of scope of field-wide RTO. Putting field-wide RTO in a concrete framework, as discussed in the next section, offers clear development and implementation benefits, in that it can catalyze progress by suggesting the path to long-term benefits which might not be immediately obvious from incremental improvements stemming from individual projects.

The essence of the RTO strategy considered in this paper is a multi-level (multi-scale) approach[1, 5], as depicted in Fig. 1 [adapted from Saputelli et al., 2003[1]]. The nested feedback loops in Fig. 1 involve decision making at each level (scale). Available measurements from a lower level are used, in "real time", to make a decision, which will be fed back to the lower measurement-providing level. In such decision making, it is crucial that some sort of a model be available, such that future outcomes of potential decisions can be predicted and assessed before optimal decisions are selected. When the corresponding level in Fig. 1 concerns production policies, a reservoir model is needed. Such a model is usually based on first principles, and is used in numerical simulation that guides the design of optimal production policies. In a companion paper[6], we present a moving-horizon optimization approach that focuses on using a reservoir simulator to make decisions within a few weeks to months. As useful as reservoir simulation is, it may be cumbersome and time consuming, particularly when it has to be performed repeatedly for the purpose of assessing an objective function to be optimized. An obvious alternative to a reservoir simulator would be a simplified (proxy) reservoir model that could generate predictions suitable for real-time decision making. In this paper, we discuss the possibility of using such a model as a computationally efficient alternative to the flow simulator for high-frequency decisions, i.e., on the order of hours to days. Such a model can be built from available data, on a real-time basis, while its basic structure should be consistent with first principles describing reservoir behavior.

In the rest of this paper, we first give a brief overview of the multi-scale RTO approach, followed by a discussion of modeling issues related to real-time decision making in the time-scale of days. A reservoir modeling approach that uses real-time field measurements to build a short-term proxy model is presented thereafter. The proposed approach is illustrated through a field-scale example.

## Proposed Approach

**Model Hierarchy.** Integration of asset-wide operations at different time scales and decision levels through a single model superstructure has always remained elusive. Some of the challenges of this problem may be attributed to its large computational needs and inability to capture multi-scale physical phenomena in a unified modeling framework. However, as a remedy, hierarchical decomposition of the problem at different time scales for multi-level decision-making can be employed.

Localized, high-frequency decision processes are often performed at the lowest levels, which are guided by governing targets calculated at the higher levels, which themselves focus on increasingly asset-wide, low frequency decision processes. Though this approach is well established in the downstream processes in the areas of simulation, control and optimization, it is still at its infancy in the upstream industry. By extrapolation, the multi-level model hierarchy for (oil and gas) field operations can be categorized as shown in Fig. 1.

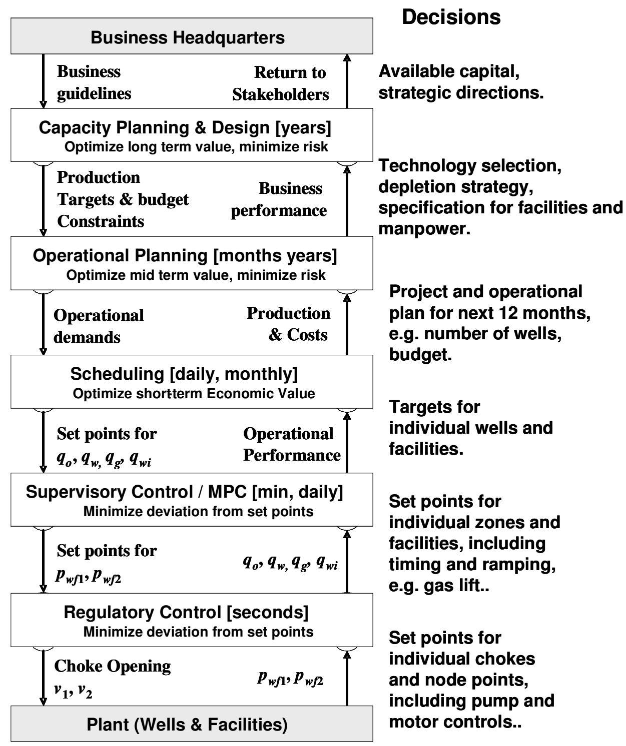


**Fig. 1 – Oil and gas field operations model hierarchy.**

The phenomena and principles governing the processes at the macro level have seldom the same requirements or drivers as at the micro level. As a result, the modeling framework and the underlying technology are often chosen to meet the specific needs at each hierarchical level. For example, a first-principle based, (pseudo) steady state model may be used for unit wide optimization, whereas an empirical, dynamic model may better suit the regulatory control operation such as adjusting the choke position.

The flow of information in Fig. 1 is bi-directional at each level, where the real time information from the field is passed to the level(s) above, while the decisions made are passed to the level(s) below and subsequently implemented in the field. The implicit assumption in the above decomposition is that the aggregate of the individual optimum decisions at each level will be close to the overall optimal decision at each point in time. This assumption is justified by the fact that decisions made at a certain level pass corresponding targets downwards to underlying level, which in turn attain such target almost instantly, with respect to the time scale of the decision-making level. Even though the multi-level decomposition cannot rigorously guarantee achievement of the global optimum, it nevertheless makes an otherwise unsolvable problem feasible.

**Framework.** Fig. 2 provides an overall framework for making production decisions within different time scales. A key component of the framework is a predictive reservoir model for the asset. Reservoir simulation with a finite-difference model is often preferred because of its rigor[7]. However, its role is often limited to the long term decisions because (1) the history match updating of the model to fit new production data can take many months; and (2) the model takes many hours to compute. The simulation model is often disconnected from the actual field production by many months. Often in practice, proxy models are used for short-term decisions. These proxy models can be spreadsheets with simple decline curve analysis, single-well material balance models, etc. The consequences of these factors are that the simulator, with its rigor, is not used for short term production optimization decisions, and the production engineers and reservoir engineers may work at cross-purposes.

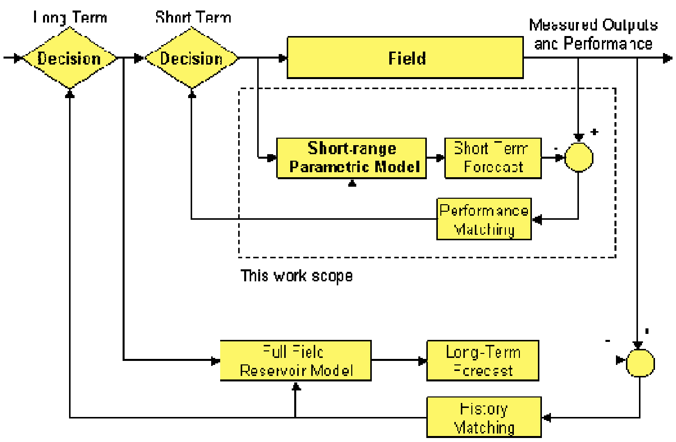


**Fig. 2 – A consistent framework towards production decision-making at different time scales.**

The opportunity presented in this paper is to utilize a rigorous proxy model of the reservoir.[8] The proxy is developed to provide predictions over a short period and is updated as new measurements become available. In contrast to a simple model, e.g., a decline curve, the proxy model is a closer representation of the short-term behavior of the reservoir. Thus, model-based optimization can be effected in a short-term decision framework, e.g., in the order of days.

**Full Field Reservoir Model.** In practice, reservoir simulation is the *de facto* industry standard for reservoir management. With technological advances, it is now possible to measure and acquire increasingly more data from an oil field. This explosion of available data further enables the development of advanced reservoir simulation technology to accurately characterize, model and simulate reservoirs in a multi-disciplinary manner. The development of advanced reservoir simulation technology leads to larger and more complex reservoir models, which have become the major source of forecasting for decision making.

While larger and complex models result in better long term forecasts and overall field management, it is often achieved at the cost of high computational time. Even with advanced schemes such as local grid refinement, grid computing, and parallel processing, there is a time gap which mandates the requirement of simplified reservoir models developed with fit-for-purpose requirements. The recent rise of the smart field initiatives warrants prediction tools capable of responding immediately based on real-time field information.

Also, the full field reservoir models need to be constantly updated through history matching (adjustment of model parameters to match production history). History matching is often a laborious, lengthy, and unwieldy task. In fact, history matching may sometimes take a year or so to complete, by which time additional discrepancies arise between the data used to update the model and actual production. As a result, full field reservoir simulators are not suitable for the accurate short term predictions that are necessary for optimization of daily production.

Consequently, there is significant value and demand to develop a modeling approach which can make more accurate short-term predictions based on real-time field data. In the past few years, several data-driven approaches for real time decision making have appeared, such as neural networks[9], Kalman filtering[10], wavelets[11], optimal control[12,13], system identification, and principal component analysis[14]. In this context, we examine below a parametric modeling framework to facilitate the development and use of tools for asset-wide optimal decision making, based on mature and proven techniques.

**Short-range Parametric Reservoir Model.** In this paper, we take the approach that a full field reservoir model is built with the vision of long term-field development (months to years), while the short range parametric reservoir model considers short-term field responses (days). Therefore, the primary objective of the short-range parametric reservoir model is to simulate the phenomena at this time-scale (days) that relate the field production and injection rates to static reservoir pressure and hydrocarbon saturation. The methodology of developing a parametric model for the hydrocarbon flow processes in the reservoir involves identifying the input and output variables and establishing a statistical relationship between them based on observed results.

Consider an injector-producer pair (as shown in Fig. 3) for a single-layer reservoir. The well production rates for oil, water and gas ($q_o$, $q_w$, $q_g$) and water injection rates ($q_{inj}$) that can be manipulated by adjusting their respective chokes are taken as inputs of the parametric model. The outputs of the model include static reservoir pressure ($P_b$); water saturation ($S_w$) and gas saturation ($S_g$) associated with each well.

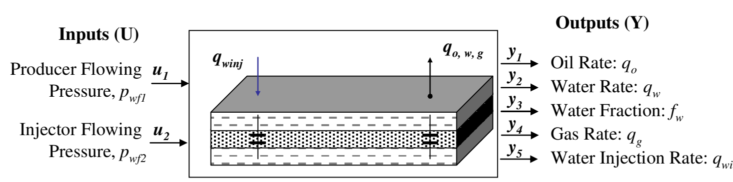


**Fig. 3 – Injector-producer pair example.**

From first principles (conservation of mass) and constitutive equations (Darcy's law, compressibility equations, and capillary pressure equations) — after discretization of derivatives with respect to the spatial co-ordinates — one can get a reservoir model in vector-matrix form as follows (equation (A.57) in Brouwer's Ph.D. Thesis[15], based on a summary of Ref. [16]):

$$\hat{\mathbf{B}}\frac{d\hat{\mathbf{p}}}{dt} = \hat{\mathbf{T}}\hat{\mathbf{p}}(t) - \mathbf{T}_4\mathbf{h} + \hat{\mathbf{q}}(t) \tag{1}$$

where the vector $\hat{\mathbf{p}}$, defined as

$$\hat{\mathbf{p}}_{i,j,k} = \begin{bmatrix} P_o \\ S_w \\ S_g \end{bmatrix}_{i,j,k} \tag{2}$$

contains values of variables that are sufficient to characterize the distribution of fluids in the reservoir at all discretization points (grid blocks), indexed by $\{i,j,k\}$; the vector $\hat{\mathbf{q}}$, defined as

$$\hat{\mathbf{q}}_{i,j,k} \triangleq \begin{bmatrix} q_o \\ q_w \\ q_g \end{bmatrix}_{i,j,k} \tag{3}$$

refers to all external fluid flows, obviously being non-zero only at injection or production points; and the matrices $\hat{\mathbf{B}}$, $\hat{\mathbf{T}}$ and $\mathbf{T}_4$ are associated with formation volume factors, mobilities, and gravitational forces, respectively, and vary with time.

**State-space System Representation.** The time-dependence of the matrices $\mathbf{B}$, $\hat{\mathbf{T}}$ and $\mathbf{T}_4$ in equation (1) is relatively weak. Therefore, for "short" periods of time, these matrices can be considered to be approximately constant. Applying this simplifying assumption to equation (1), one can formulate a simplified input-output model of the reservoir in the standard state-space form[17-19] as follows:

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) \approx \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}(t)\mathbf{x}(t) \approx \mathbf{C}\mathbf{x}(t) \end{aligned} \tag{4}$$

where the vector $\mathbf{x}$ comprises the states of the system, namely the values of $P_o$, $S_w$, and $S_g$ at all discretization points in the reservoir (indexed by $\{i,j,k\}$ in equation (2)); the vector $\mathbf{u}$ captures the effect of external manipulated inputs, i.e., flow rates or bottomhole pressures at all injection or production points (3); the matrix $\mathbf{A}$ captures the internal dynamics of the reservoir; the matrix $\mathbf{B}$ captures the effect of manipulated inputs; and the matrix $\mathbf{C}$ generates measurable outputs from system states $\mathbf{x}$.

Equation (4) can be obtained by streamlining of equation (1) as follows:

- As already mentioned, for "short-time" predictions, i.e., on the order of days, the matrices **A**, **B**, **C** can be considered approximately constant. However, they will require a continuous evaluation scheme to maintain the accuracy of the estimated model for short-term prediction purposes.
- External flow rates at injection or production points are related linearly to well flowing pressures via equations of the form

$$\hat{\mathbf{q}} = \hat{\mathbf{W}}(\hat{\mathbf{p}}_{wf} - \hat{\mathbf{p}}) + \hat{\mathbf{w}}_c \tag{5}$$

  (equation (B.20) in Ref. [15], based on a summary of Ref. [16]) where $\hat{\mathbf{p}}_{wf}$ is the well flowing pressure; and $\hat{\mathbf{w}}_c$ captures capillary pressure effects. Consequently, substitution of $\hat{\mathbf{q}}$ from equation (5) into equation (1) results in a manipulated input vector $\mathbf{u}$ to the entire system which comprises the desired values of either bottomhole pressures or total flow rates, corresponding to valve openings of producers or injectors.
- While equation (4) describe the time evolution of $P_o$, $S_w$, and $S_g$ at all grid blocks in the reservoir, only values of $P_o$, $S_w$, and $S_g$ at grid blocks associated with producers or injectors can be measured. Consequently, the output vector $\mathbf{y}$ contains values of $\hat{\mathbf{p}}$ at grid blocks associated with injectors or producers.
- The state vector $\hat{\mathbf{p}}$ of the system in equation (1) has physical significance, as indicated by equation (2). Therefore, the natural order of the system dynamics is relatively high, corresponding to the number of grid blocks considered in the discretization of the reservoir. However, the input-output behavior of the overall system, i.e., the effect of injection and production flow rates or bottomhole pressures on pressures and saturations at injector and producer grid blocks, is expected to be represented by a reduced-order model. This model captures the dominant modes of the system dynamics, e.g., the modes corresponding to the dominant eigenvalues of the matrix $\hat{\mathbf{B}}^{-1}\hat{\mathbf{T}}$ in equation (1). Note that if the well flowing pressures are used as inputs instead of flow rates, then the system dynamics will change according to equation (5). Consequently, the state vector $\mathbf{x}$ in equation (4) can be of considerably smaller dimension than that of $\hat{\mathbf{p}}$. Of course, $\mathbf{x}$ does not need to (and typically does not) have physical significance in the same fashion as $\hat{\mathbf{p}}$ but simply serves to capture the dynamics of the input-output behavior of the reservoir.

Given a model connecting manipulated inputs and measured outputs, one expects to be able to optimize production by using measured outputs to select values of manipulated inputs in a moving-horizon framework, as discussed briefly next. A more detailed discussion and related references are offered in a companion paper.[6]

**Moving Horizon Approach.** Production optimization ideally would have to account for the entire future effect of decisions made at a certain time. Because predictions of outcomes in the "distant" future are uncertain (where the term "distant" is commensurate with the time-scale at the corresponding level of the hierarchy in Fig. 1), the decisions up to a bounded future point at the end of a moving horizon are considered, as shown in Fig. 4. Optimal decisions are identified and acted upon until new measurements arrive at the next decision making point where new optimal decisions are made. To make decisions at each point in time, a model is used to assess future outcomes. This model is updated using available data with higher emphasis placed on recent data. Similarly to the prediction horizon, a moving horizon can be used for model refinement, such that data prior to the beginning of the horizon are not used to refine the model.

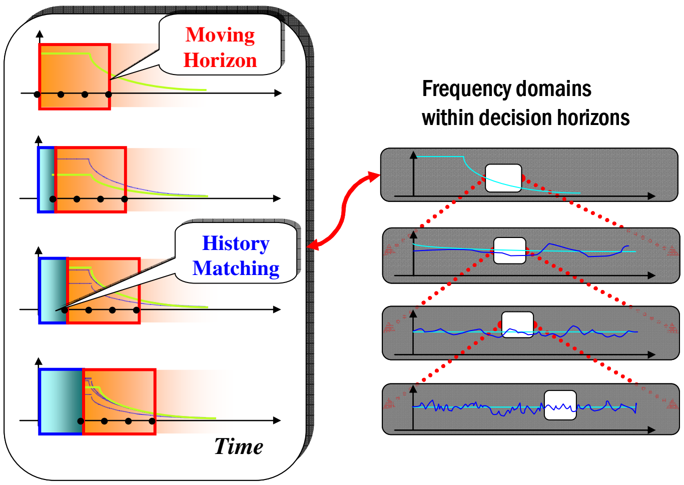


**Fig. 4 – Moving horizon for real-time reservoir management[6] in the time-scale of months to years (Fig. 1). A similar strategy can be used at all hierarchy levels. At each level, model-based predictions become increasingly uncertain as one moves within the finite (prediction) horizon into the future. Similarly, old data used for model refinement become increasingly less relevant as one moves within the finite (modeling) horizon into the past.**

Short-term decisions, namely on the order of a few days, include evaluating pressure and flow rate profiles against valve set points. Small adjustments to valve settings can have important consequences on production of oil, gas, and water. Models that can provide such predictions can be built in real–time using data over a period of several days or months. A multitude of well known techniques exist for this task[20, 21], one of which we discuss next.

**Parametric system identification.** At each point in time, the matrices **A, B,** and **C** must be identified from available measurements over a period of time extending to the past in a moving horizon (Fig. 4). Because there are multiple inputs and multiple outputs in any realistic reservoir, a system identification method suitable for multivariable models must be used. A class of methods known as Subspace Identification (SI) methods[22-24] has been fairly popular in recent years, because of its relative simplicity, generality, and numerical robustness. The general idea of SI methods is that they use input and output data (u and y in equation (4)) while avoiding the difficulty of not having direct measurements of the state vector x. A model identified using SI methods can be used directly, or it can serve as an initial guess for identification of a multivariable model via other methods, such as the multivariable prediction-error method (PEM).[20, 21]

With the availability of continuous field data, it is straightforward to implement the parameter updating procedure on a daily basis. Parameter updating ensures that the parametric model used for short-term forecasting is accurate and reflects the actual field conditions.

## Case Study

**Caratinga Field.** To illustrate the approach presented, a case study was undertaken using data from the Caratinga field, a deepwater oil field in offshore Brazil. The field is produced through a floating, production, storage, offtake (FPSO) vessel. Water injection began early in the reservoir life. There are, in total, ten producers and eight injectors. We used a history-matched simulated model as the source of information to generate daily data.

Assumptions and constraints of the simulated data include:

- Average daily well bottomhole pressures are known.
- Average daily total and well flowrates are known for each phase.
- Block pressures and saturations would not be used for the field for building the model, except for grid blocks associated with injectors or producers.

**Parametric Reservoir Model.**

***Case Study I.*** In the early days of the reservoir, only two producers are active. For these producers, the two total production flow rates are the manipulated input variables; and the two pressures of the grid blocks corresponding to the producers are the measured outputs. Thus, the corresponding multivariable model is $2\times2$. Using a moving identification horizon of 600 days, a new linear model of the form in equation (4) was identified daily for a period of 360 days, using SI to provide an initial guess to prediction error method (PEM). Each of the daily identified linear models was used with corresponding input values (production flow rates) shown in Fig. 5 to make 5-day-ahead (Fig. 6) and 15-day-ahead (Fig. 7) predictions of the resulting pressures (output variables) for each of the two producer grid blocks.

The model order (dimension of the state vector x in equation (4)) is automatically selected by the SI method, as shown in Fig. 8.

Fig. 9 indicates that the identified model is essentially an integrator (it has a maximum eigenvalue at 1) as expected.[15]

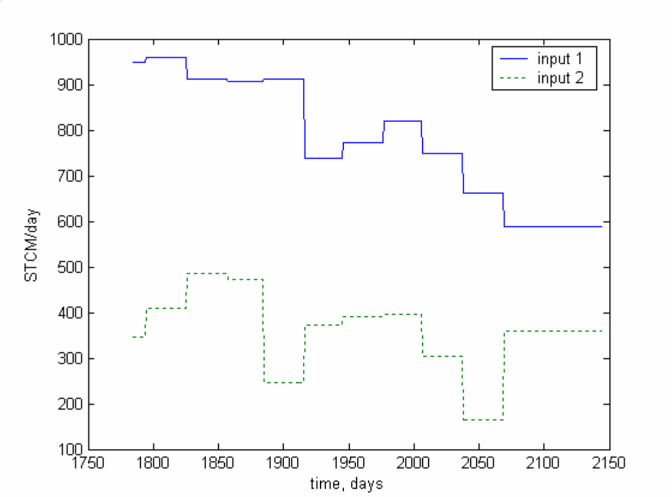


**Fig. 5 – Total flow rates (input variables) for the two producers considered in Case I of the Caratinga Case Study.**

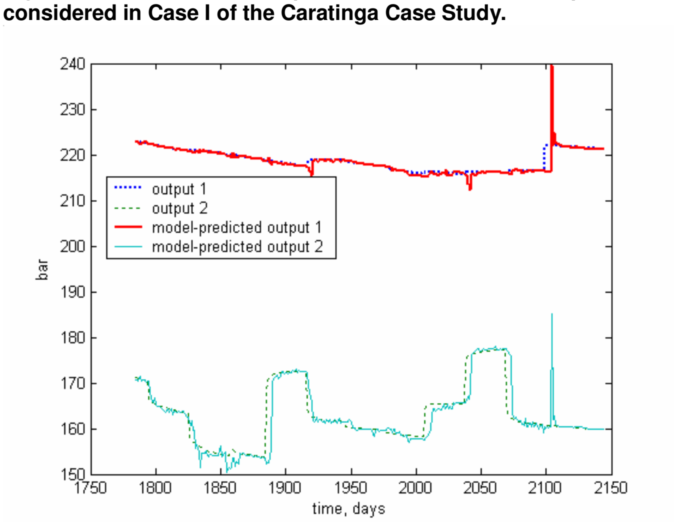


**Fig. 6 – The 5-day-ahead predictions of block pressure using the daily updated model of the two producers considered in Case I of the Caratinga Case Study.**

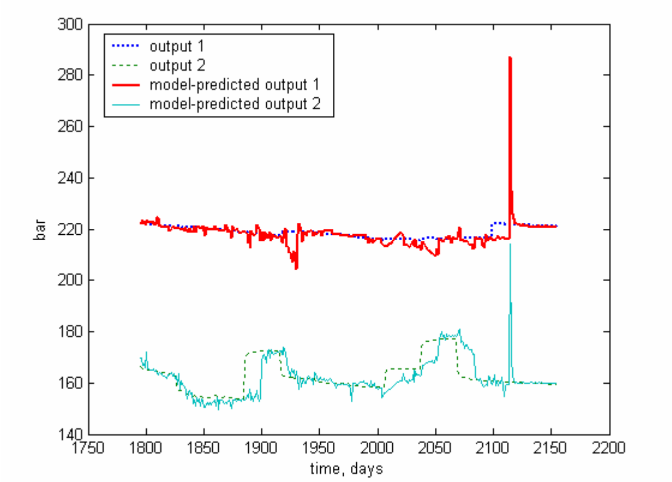


**Fig. 7 – Fifteen-day-ahead predictions of block pressure for the daily updated model of the two producers considered in Case I of the Caratinga Case Study.**

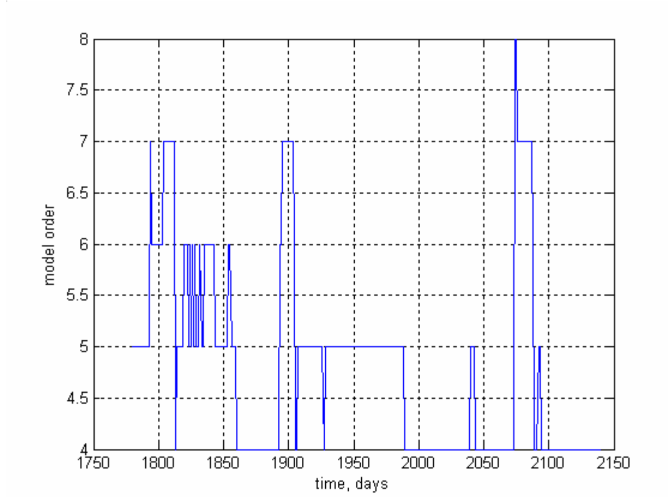


**Fig. 8 – Dimension of state space (model order) for the daily updated model of the two producers considered in Case I of the Caratinga Case Study.**

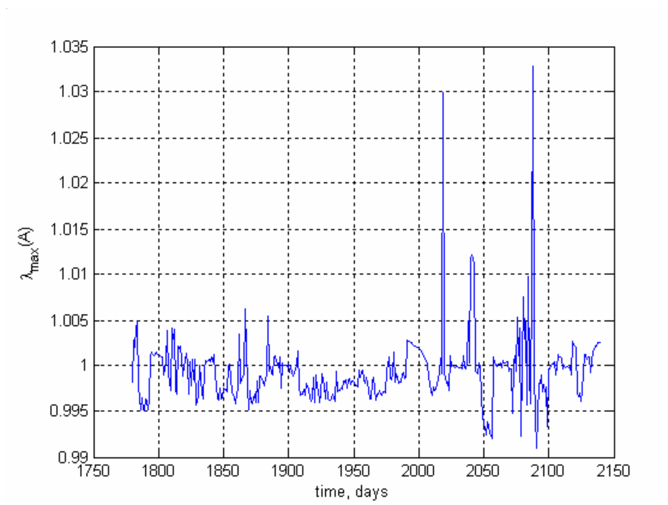


**Fig. 9 – Maximum eigenvalue of the matrix $\mathbf{A}$ (equation (4)) of the daily updated model for the two producers considered in Case I of the Caratinga Case Study.**

Note the discrepancy between model predictions and data around 2100 days. The reason for this discrepancy is not clear. It could manifest severe nonlinearity or simply a non-convergence problem with the numerical simulation that produced the data. The discrepancy warrants further investigation.

***Case Study II.*** After approximately 4500 days, additional producers as well as injectors have become operational, creating a total of 14 (seven production and seven injection) manipulated flow rates (see Fig. 10). Five wells have already experienced water breakthrough. Therefore, a model was built to capture the effect of total flow rate at each injector and producer on water cut or water production flow rates. Similarly to Case I, data from a past moving horizon of 600 days (see Fig. 10) are used to identify a model each day, such that future predictions of water production flow rates can be made. The model order produced by SI was 28, indicating interesting dynamics for this reservoir.

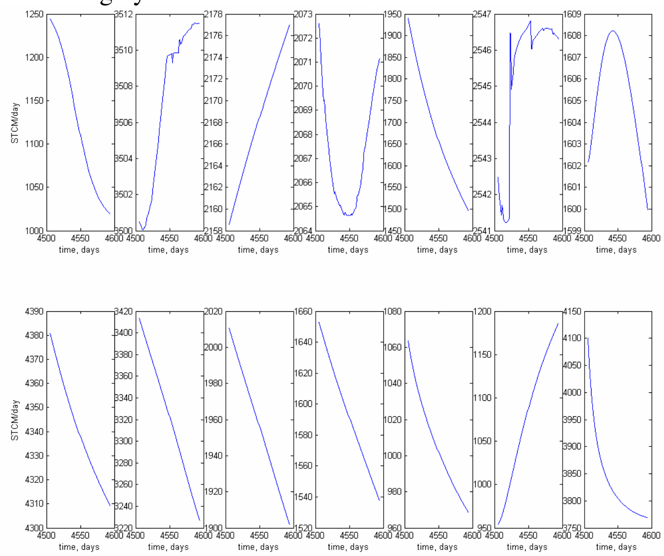


**Fig. 10 – Total flow rates (input variables) for the seven producers and seven injectors considered in Case II of the Caratinga Case Study.**

Note that the discrepancies between predictions and data that appear in Fig. 11 are due, in part, to the fact that the identified model is mildly unstable as indicated by the maximum eigenvalue of the matrix **A** in equation (4) (see Fig. 12). Because of this instability, long-term predictions are clearly infeasible. However, there also appear to be discrepancies between model predictions and data around 4530 days. They cannot be attributed to this instability and warrant further investigation.

## Conclusions and Discussion

We have illustrated how well-established methods of model identification for multivariable dynamic systems can be used to continuously develop proxy reservoir models. These models can effectively provide short-term predictions for the purpose of optimizing production operations within a multi-scale framework. However, a number of questions should be further investigated to establish the value of the proposed approach, including the following:

- How significant is the effect of nonlinearities on the accuracy of the predictions provided by the approximate linear model of equation (4)?
- Over what time horizon length can reasonable predictions be provided by the proxy model of equation (4)?
- Would transport delays be useful for the proxy model of equation (4)?
- Can the effect of injector flow rates on producer water cut be captured by a proxy model, particularly before water breakthrough?
- Can the behaviors of pressure and saturation be separated, thus simplifying the proxy model structure[15]?

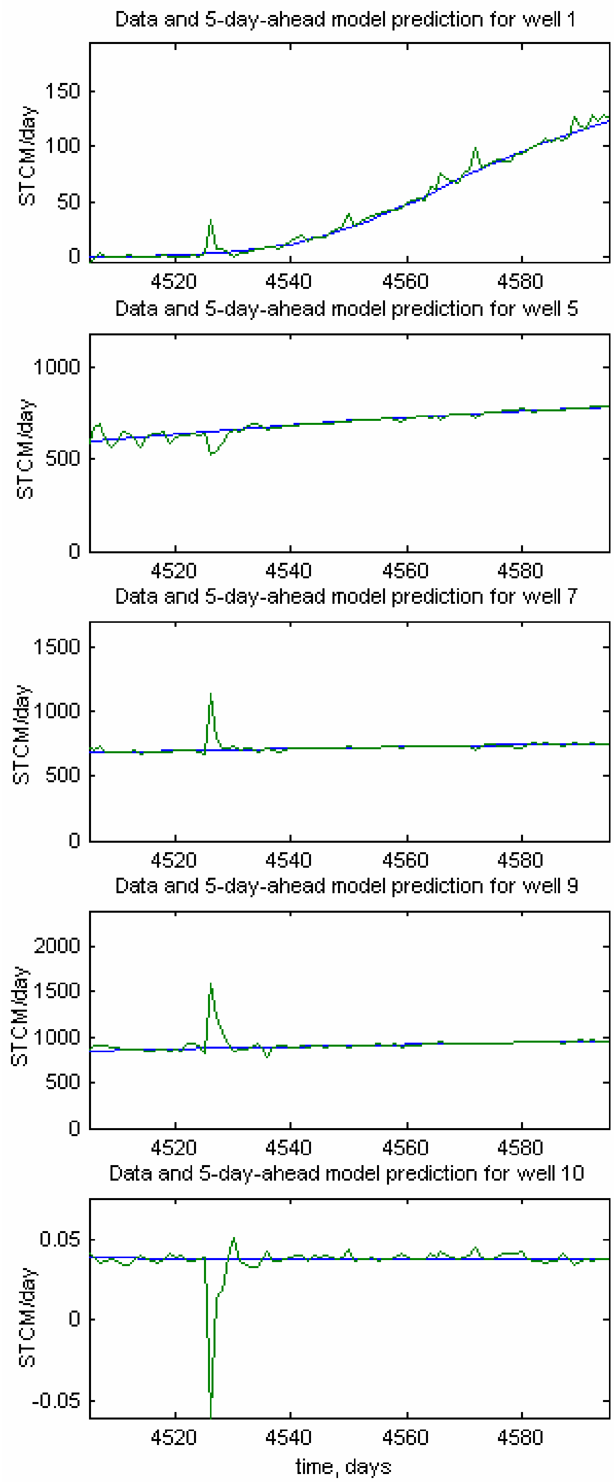


**Fig. 11 – The 5-day-ahead predictions of water production flow rates using the daily updated model of the seven injectors and seven producers considered in Case II of the Caratinga Case Study.**

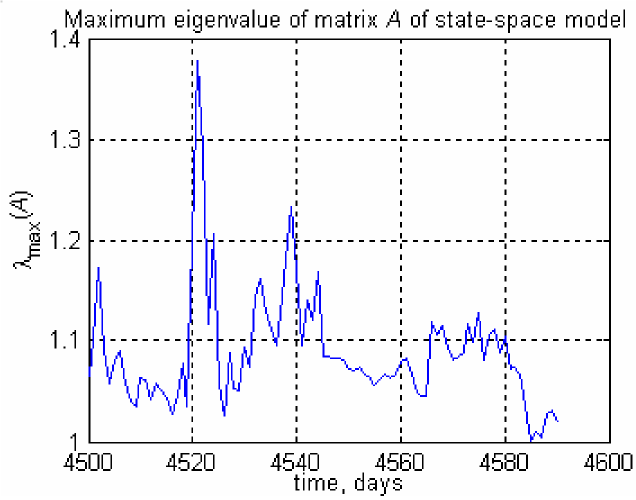


**Figure 12 – Maximum eigenvalue of the matrix A (equation (4)) of the daily updated model of the seven injectors and seven producers considered in Case II of the Caratinga Case Study.**

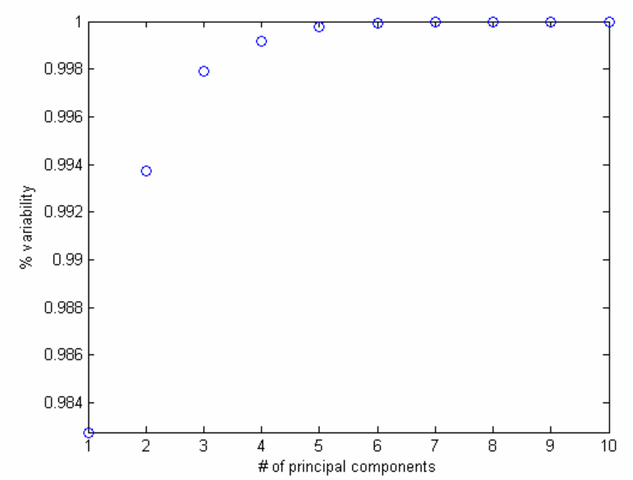


**Fig. 13 – Principal component analysis of the water flow rates for all 10 producers of the Caratinga Case Study. Significant interaction – with the potential for model reduction – is evident.**

- Can other techniques for dimensionality reduction (e.g., principal component analysis[25] (PCA)) be used to simplify the proxy model? For example, PCA reveals that there is strong interaction among the water flow rates for all ten producers of the Case Study (Fig. 13).
- What features of a proxy model are crucial for production optimization rather than for overall model quality (in the least squares sense)?
- If the linear proxy model structure of equation (4) is not adequate, what alternatives can be used (e.g. neural networks[3])?
- How effectively can a proxy model be used for production optimization?

We intend to address such issues in forthcoming publications.


## Acknowledgements

The authors acknowledge Petrobras for permission to use the data from Caratinga field and publish this work. The authors acknowledge Halliburton for permission to publish this work. The authors acknowledge the following software contributions: Landmark Graphics for Desktop VIP and Matlab System Identification Toolbox.


## Nomenclature

Boldface uppercase: Matrix
Boldface lowercase: Vector
q – flow rate
S – Saturation
**u** – input vector
**y** – output vector
**x** – state vector
**A** – matrix determining system dynamics
**B** – matrix determining input effects
**C** – matrix determining system outputs

*Subscripts*
w – water
*g* – gas
*o* – oil

## References


1. Saputelli, L.A., M. Nikolaou, and M.J. Economides. *Self-Learning Reservoir Management*. in *paper SPE 84064, Annual Technical Conference and Exhibition*. 2003. Denver, CO.
2. Mochizuki, S., L. Saputelli, C.S. Kabir, R. Cramer, M.J. Lochmann, R.D. Reese, L.K. Harms, C.D. Sisk, J.R. Hite, and A. Escorcia. *Real Time Optimization: Classification and Assessment*. in *SPE 90213*. 2004.
3. Saputelli, L., H. Malki, J. Canelon, and M. Nikolaou, *A critical overview of artificial neural network applications in the context of continuous oil field optimization.* SPE-77703, 2002.
4. Saputelli, L., S. Mochizuki, L. Hutchins, R. Cramer, M.B. Anderson, J.B. Mueller, A. Escorcia, A.L. Harms, C.D. Sisk, S. Pennebaker, J.T. Han, A. Brown, C.S. Kabir, R.D. Reese, G.J. Nunez, K.M. Landgren, C.J. McKie, and C. Airlie, *Promoting Real-Time Optimization of Hydrocarbon Producing Systems.* SPE 83978, 2003.
5. Saputelli, L., M. Nikolaou, and M.J. Economides, *Real-Time Reservoir Management: A Multi-Scale Adaptive Optimization and Control Approach.* Computational Geosciences, Special Issue on Modeling, 2005: p. to appear.
6. Nikolaou, M., A.S. Cullick, and L. Saputelli. *Production Optimization – A Moving-Horizon Approach*. in *SPE 99358, Intelligent Energy 2006*. 2006. Amsterdam.
7. Brouwer, D.R., G. Nævdal, J.D. Jansen, E.H. Vefring, and C.P.J.W. van Kruijsdijk. *Improved Reservoir Management Through Optimal Control and Continuous Model Updating*. in *SPE 90149*. 2004.
8. Heijn, T., R. Markovinović, and J.D. Jansen, *Generation of Low-Order Reservoir Models Using System-Theoretical Concepts.* SPE Journal, paper 88361, 2004. **9**(2): p. 202-218.
9. Ternyik, J., H. Bilgesu, S. Mohaghegh, and D. Rose. *Virtual Measurement in Pipes, Part 1: Flowing Bottom Hole Pressure*

*under Multi-phase flow and inclined wellbore conditions*. in *SPE Paper 30975*. 1995.

10. Zhu, X., Y.C. Soh, and L. Xie, *Design and analysis of discrete-time robust Kalman Filters.* Automatica, 2002. **38**: p. 1069-1077.
11. Jansen, F.E. and M.G. Kelkar. *Application of Wavelets to production data in describing inter-well relationships*. in *SPE 38876*. 1997.
12. Ramirez, W.F., *Application of Optimal Control Theory to Enhanced Oil Recovery*. 1987: Elsevier.
13. Brouwer, D.R. and J.D. Jansen, *Dynamic Optimization of Waterflooding With Smart Wells Using Optimal Control Theory.* SPE Journal, paper 78278, 2004. **9**(4): p. 391-402.
14. Renard, G., D. Dembele, J. Lessi, and J.L. Mari. *System Identification Approach Applied to Watercut Analysis in Waterflooded Layered Reservoirs*. in *SPE 39606*. 1998.
15. Brouwer, D.R., *Dynamic water flood optimization with smart wells using optimal control theory*. 2004: Ph.D. Thesis, Technische Universiteit Delft.
16. Aziz, K. and A. Settari, *Petroleum Reservoir Simulation*. 1979: Elsevier.
17. Brogan, W.L., *Modern control theory*. 3rd ed. 1991: Prentice Hall.
18. Chen, C.-T., *Linear system theory and design*. 3rd ed. 1999: Oxford University Press.
19. Kailath, T., *Linear systems*. 1980: Prentice-Hall.
20. Ljung, L., *System Identification: Theory for the User*. 2nd ed. 1999: Prentice Hall.
21. Söderstrom, T. and P. Stoica, *System Identification*. Series in Systems and Control Engineering. 1989: Prentice Hall International.
22. van Overschee, P. and B. DeMoor, *Subspace Identification of Linear Systems: Theory, Implementation, Applications*. 1996: Kluwer.
23. Verhaegen, M., *Identification of the deterministic part of MIMO state space models.* Automatica, 1994. **30**: p. 61-74.
24. Larimore, W.E. *Canonical variate analysis in identification, filtering and adaptive control*. in *29th IEEE Conference on Decision and Control*. 1990. Honolulu, HI.
25. Jolliffe, I.T., *Principal Component Analysis*. 1986: Springer-Verlag.