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Optimization of the WAG Process Under Uncertainty in a Smart Wells Environment: Utility Theory Approach

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Abstract

Smart well technology has progressed significantly over the last few years. Earlier research[1,2] has concentrated on the application of the technology to secondary recovery. More recent studies[3,4,16] have aimed to advance the technical application to tertiary recovery concentrating on WAG processes. A utility theory approach to valuing information and risk attitude is used in this study.

Incorporating the economics into the decision making process and taking into account risk attitude complicates the decision making process. Earlier the goal was optimization of the global sweep efficiency under economic constraints through the control of the injection size of each slug, the controlled injection rate of each well, the injection location along the wellbore, and the production rates and locations. The control parameters stay the same but the goal is a risk based optimization of the project economics. Traditional real options based approach requires a normal distribution of outcomes which was not found to be true in this study. Therefore a utility theory approach is used to incorporate risk attitude.

The WAG process is sensitive to reservoir, fluid, and economic parameters which justify the need to quantify the uncertainty in production economics and associated risk. Gradients are determined from the proxy model. The gradients provide optimal control settings for the injection and production settings. This study demonstrates the feasibility of creating a response surface proxy model, using experimental design and analysis, to facilitate Monte Carlo simulation, uncertainty analysis and optimization of the expected value utility. The proxy model is orders of magnitude faster allowing a statistical analysis of the uncertainty, value of information, value of flexibility and associated risk. Results on this model show significant improvements over an uncontrolled WAG production and the ability to incorporate risk attitude into the optimization process.

Introduction

Smart well technology involves the measurement and control of well bore and reservoir flow. This paper concentrates on the classic quarter 5-spot pattern common in pilot study design and pattern flooding. Smart well applications in the current technical environment are viable tools to control or minimize production problems.

Experimental Design applications are used to determine the optimal initial settings of the wells and the initial WAG slug sizes. A detailed sensitivity analysis[3] was performed to determine the sensitivity of production to several reservoir parameters. An optimization without reservoir uncertainty was also performed on the model to quantify the scope for improvement[3]. Two reservoir parameters and five control parameters are chosen to test the viability of this approach in a smart well WAG environment.

A d-optimal design is used at the start of the simulation to determine the set of runs to perturb the variables and determine the uncertainty and gradients for optimization. A proxy model is built and optimized with respect to the opening and closing of completions and the slug sizes of water and gas.

Results on this model show significant scope for the application of smart well technology to the WAG process. In making a decision as to use smart wells or invest further capital, both an expected monetary value (EMV) and utility approach provide insight into the decision making process.

The optimization technique applied requires a simulation model that is fast enough for multiple runs and accurate enough to produce valid results. This was achieved by performing the study with a commercial simulator on a quarter 5-spot pattern.

Water Alternating Gas (WAG) Technique

Water-alternating-gas (WAG) injection is a tertiary oil recovery process that has been growing in popularity since it was first introduced in the 1950's. Christensen, Stenby and Skauge[10] provided a review of 60 fields where WAG has been applied. The study identifies the use of WAG in several formation types with differing injectant gases and drive mechanisms. A commonality is that several of the projects reported channeling problems and / or reduced injectivity. Smart well flow allocation has the potential to mitigate these two primary problems and increase the recovery from a WAG project.

The first WAG process reported in literature was in Canada 1957. As the process is approaching half a century old, much of the fundamentals require more understanding through research. The majority of published literature discussing field cases does not provide details of the simulation model used or the decision analysis by management. Therefore the process of WAG is not well understood yet. In addition, there always exists uncertainty in the reservoir model even though technology has advanced significantly. The uncertainty in the reservoir model is attributed to the ambiguity in the reservoir and geological parameters.

The tertiary recovery process known as WAG is a combination of the two secondary recovery processes of water flooding and gas injection. The WAG process was proposed originally to aim for the ideal system of oil recovery: improvements in macroscopic and microscopic sweep efficiency at the same time. The water is used to control the mobility of the gas as can be seen in equations 1 and 2. The cyclic nature of the WAG process causes an increase in water saturation during the water injection half cycle and a decrease of water saturation during the gas injection half cycle. This process of inducing cycles of imbibition and drainage causes the residual oil saturation to WAG to be typically lower than that of water flooding and similar to those of gas flooding.

$$f_w = \frac{k_w/\mu_w}{k_w/\mu_w + k_o/\mu_o + k_g/\mu_g} \quad (1)$$

$$f_g = \frac{k_g/\mu_g}{k_w/\mu_w + k_o/\mu_o + k_g/\mu_g} \quad (2)$$

The oil recovery factor can be described by two factors that are the macroscopic sweep efficiency and the microscopic sweep efficiency. Further more the macroscopic sweep efficiency is defined by the horizontal and the vertical sweep efficiencies. This can be formulated as:

$$R_f = E_v.E_h.E_m \quad (3)$$

The horizontal sweep efficiency is related to the mobility ratio (Eq.4) and the vertical sweep efficiency depends on viscous to gravity forces ratio (Eq.5):

$$M = \frac{k_{rg}/\mu_g}{k_{ro}/\mu_o} \quad (4)$$

$$R_{v/g} = \left(\frac{v\mu_o}{kg\Delta\rho}\right)\left(\frac{L}{h}\right) \quad (5)$$

Gas injection alone decreases the residual oil saturation in the reservoir significantly. Gas has lower density and higher mobility thereby it could easily sweep the oil trapped in the attic and cellar parts of the reservoir. However, gas injection is expensive and there are some major problems associated with that like early breakthrough and fingering. Hence using only gas injection leads to low macroscopic sweep efficiency. On the other hand, water flooding generally leaves a large portion of oil unswept in the reservoir. Hence it delivers low microscopic sweep efficiency. However, in water injection, flooding front is more stable than that of gas injection, hence; it leads to better macroscopic sweep efficiency. Moreover, water injection process is relatively cheap. By injecting water and gas alternatively, it is possible to reduce the residual oil saturation significantly. Furthermore, water cycles attempt to stabilize the flooding front. In fact in the WAG technique, both water and gas cycles contribute to production increment. Gas cycle drains the oil that exists in the attic part of the reservoir to the place where the subsequent water cycle displaces it towards the production wells.

Literature on the WAG process[10,11,14,15] typically discusses two major management parameters that affect the economics of a WAG project. These operational aspects are the half-cycle slug sizes and the WAG ratio. The two major problems faced are early breakthrough and injectivity losses. It is therefore proposed that the third parameter to be studied is the operation of the smart wells.

The two most common distinctions in the classification of the WAG process are miscible WAG injection and immiscible WAG injection. Miscible WAG injection occurs when the reservoir is above the minimum miscibility pressure (MMP) and is immiscible when below the MMP. In this study the initial reservoir pressure is just above the MMP and therefore often moves in and out of miscibility in part or all of the reservoir.

Reservoir Model

The reservoir model is 1,320 by 1,320 by 144 feet, modeled using 19 by 19 grid blocks aerially and 4 in the vertical. A standard quarter 5-spot pattern with an injector and producer is used with all sides bounded by no flow boundaries. The reservoir model is implemented in a commercial reservoir simulator to model the WAG process. A detailed fluid description with 7 pseudo components describes the oil and gas.

An initial WAG ratio of 1:1 is used with 3 months per injection phase. For testing the influence of the WAG process two additional choices were implemented in the model. A ratio of 1:3 with 1 months and 3 months, and 3:1 with 3 months water and 1 month gas are implemented along with the original WAG setting.

The reservoir model was extracted from the statistics of a giant reservoir model with the size of 20x30 km aerially and roughly 40 m vertically. The average grid block size for the base reservoir was too large (300x300 m) to allow a detailed pattern flood analysis. Additionally the run time for the full field model was in the order of a week. Therefore, based on geo-statistical information derived from the full field model, an area in the flank of the reservoir was considered and a sector model was constructed.

Described in the WAG introduction, two primary problems faced in WAG processes are early breakthrough and loss of injectivity. The economic constraints placed on the wells are a maximum water cut of 0.5 stb/stb and a maximum GOR of 5 Mscf/stb at which point the well is shut-in. The wells are also tested every 100 days and connections can be reopened if the test shows the well can operate below the GOR and water cut constraints. This initial model with a 7% HCPV slug size and conventional wells experienced major breakthrough problems forcing the wells to be shut in, resulting in lost production.

The wells perforate all 4 grid blocks. The smart well, similar to that is **Figure 1** has completion zones that may be controlled. Both the injector and the producer are set up as smart wells.

Methods Theory

Design of Experiment[5,6,12]

Many applications of Experimental design have been reported in literature in many areas of petroleum engineering including sensitivity analysis, upscaling, performance prediction, uncertainty modeling and optimization. For example, Narayanan[23] used the design of experiment and the response surface method in order to study a model with water flooding. White[24] employed experimental design for estimating parameters and assessing uncertainty. Friedmann[25] applied experimental design methodology to quantify uncertainty in production forecasts for a population of deep-water channelized sandstone reservoirs. They obtained a simplified proxy to the simulation model.

A large number of numerical simulations are required to investigate the sensitivity of production with respect to many geologic, fluid and engineering parameters in a reservoir model. Technically, implementing all of these simulations in a reservoir simulator is time-consuming and expensive. Therefore, the need for a method, which can reduce this high amount of simulation runs to a reasonable number with adequate accuracy, is crucial especially, where sensitivity of many parameters should be studied.

Experimental design (DOE) and related response surface model (RSM) deliver tools to select efficiently a reasonable number of runs, which give maximum information from the design space. Design of experiment is defined as a structured and organized method, based on statistical principles that can be used to identify the impact of different parameters affecting a process. The objective of using DOE is to achieve the most reliable results with optimal use of time and money. Experimental design, in fact, changes different parameters systematically and simultaneously within a limited number of experiments to give an overall view of the process.

To evaluate a full 3-level factorial design, 3K (K: number of factors) experiments are needed. An experiment is defined as the combination of these factors. Obviously, as the number of factors increases, the number of experiments becomes more unmanageable and impossible to implement in a reservoir simulator as seen in **Table 1**.

The first step to construct a design is to identify those factors that are expected to have a large influence on the response (cumulative oil production). Afterwards, the factor ranges are usually scaled to lie between “-1” and “1” to represent factor’s maximum (1), minimum (-1) and mean (0) values. Factor ranges should be chosen carefully to avoid dominance of experimental error on response (small ranges) and to decrease the possibility of construction of a complicated response model (large range). Then a design depending on time and computer power can be constructed. The combination of factors derived from DOE is used to feed into a simulator or to implement experiments.

Table 2 shows a sample screening design. This 2 level design allows for a fast assessment of the ranking of the parameters. For the detailed study performed a 3 level optimal design as seen in **Table 3** is used in creation of the proxy model. Each table only shows a few of the runs required.

The proxy model to simulation is denoted as:

$$y = X\beta + \varepsilon \qquad (6)$$

where X is the design matrix with the row dimension equal to the number of experiments and column dimension equal to the number of terms in the model (regressors). The design matrix depends on both regression model (linear, quadratic, cubic, etc) and the design of experiment type (among classical or optimal). ‘y’ is the vector of simulation or experimental result. ‘ε’ denotes a random vector with distribution of N (0, σ^2), which represents the error. ‘ $\hat{\beta}$ ’, given by:

$$\hat{\beta} = (X'X)^{-1}X'y \qquad (7)$$

is the least square estimate of β which delivers the best set of coefficients by minimizing the error (ε). It has the covariance matrix of $(X'X)^{-1}\sigma^2$ where X’ is the transpose of the design matrix (X).

Plackett-Burman screening design

Initially a large number of factors must be screened at each time step. These are the status of each valve for each well at the time step being optimized. To screen a large number of parameters the interaction terms are confounded with the new main effects. These new saturated designs would then have 2^k runs. Plackett and Burman[12] provided a new way to fractionalize the full factorial yielding designs where the numbers of runs are 4*k rather than 2^k.

D-Optimal Design

A D-optimal design attempts to minimize the average size of the variance matrix by minimizing the average eigen-value. A maximization of D-efficiency results in finding a design where the factor effects are maximally independent.

$$D\text{-}efficiency = 100 * (|X'X|^{1/p}/n) \qquad (8)$$

Where X is the regression matrix, p is the number of parameters in the model, and n is the number of observation point or simulations.

Optimal designs offer significant advantages over classic experimental designs. A D-optimal design can be modified to allow the number of simulation the experimenter wants. Classic designs typically are not as flexible in allowing the experimenter to control the parameters. The design can also be augmented to add additional runs if the experimenter feels they are necessary. This is done while maintaining the optimality given the set of runs already performed therefore maximizing the additional information gained. Additionally the design is optimized for the response surface model the data will be fit to.

Response Surface Model (RSM)

The response surface model (RSM) is finally used to fit the simulation or experimental results to a model. Response surfaces are 3D visualizations of the response surface model. These surfaces allow a visualization of two of the parameters effects on the observation. Usually the model being fit is a polynomial function, which considers the linear, second order and interaction terms. Equation (9) shows a general form of this quadratic polynomial model.

$$\eta = \beta_o + \sum_{j=1}^{k} \beta_j x_j + \sum_{j=2}^{k} \beta_{jj} x_j^2 + \sum_{i<j}\sum \beta_{ij} x_i x_j \qquad (9)$$

The correlation coefficient and the R^2-adjust (Eq.10) is a measure of how well the model fits data.

$$R^2_{adj} = 1 - \frac{(1 - R^2)(n - 1)}{(n - p)} \qquad (10)$$

n: number of observations
p: number of terms
R^2: correlation coefficient squared
R^2 or R^2_{Adj} of much less than 1 represents poor fit.

Significance testing of the models is performed in order to validate the results. Standard R^2 regression analysis is performed on the resulting model. This provides the fit of the model to the input data to the training set. Additionally, an adjusted R^2 taking into account the number of terms in the model is calculated.

Oil Prices

Oil price forecasting is far from an exact science. Companies and countries take different approaches to integrating oil prices into their decision making process. Most tend to use a single price, usually on the conservative side due to the natural risk averse nature of corporations and governments, and not a range of possibilities or a stochastic price model.

The oil price has a significant on the Net Present Value (NPV) of the project as defined by:

$$NPV = \Delta x \Delta y h \left[\sum_{k=1}^{N_{prod}} \frac{-r_w \left(q_w\right)_k^n + r_o \left(q_o\right)_k^n}{\left(1 + b/100\right)^{t^n}} \right] \Delta t^n \qquad (11)$$

This allowing for the assignment of positive and negative revenue and to give a time value to the cash flow. This cash flow is all discounted to time zero using the discount rate *b* for comparison. The capital cost is subtracted from this to get the project NPV as discussed later

In December 2003 a survey of 27 oil analysts by Bloomberg showed an average estimate of a 13% decline in oil prices for 2004. In actuality oil prices were about 45% higher in 2004 than in 2003 and oil companies had a very profitable year due to the high oil prices. The analysts were not "wrong" inasmuch as world wide economic factors did not react as anticipated.

Historical price data[8] shows that this is true on several different scales due to different reasons. **Figure 2** shows inflation adjusted average annual oil prices from 1865 to 1998. Several things occurred during that time but a few can be highlighted to see how unforeseen events affect the oil prices. 1945 is primarily due to post war reconstruction, 1974 the OPEC oil embargo, 1979/80 the Iranian revolution, 1990 the Gulf War and the 1998 Asian economic crisis.

The past century has shown times of relative stability like the 50's and 60's to extremely volatile times like the 80's. Developing a 5 or 10 year production plan based on a single estimated oil price could cause some potential problems.

For the purposes of this paper oil prices are set at 50$/bbl in NPV calculation unless otherwise stated.

Decision & Risk Analysis

Risk and reward are ever present in the decision making process. This paper will qualitatively describe some of the additional benefits of smart wells. The fast proxy model allows the study of the distribution of NPV due to the distribution of the input parameters. A comparison of an EMV approach to a utility theory approach will be presented in brief.

The history of the incorporation of risk and uncertainty in economics has a relatively brief history. Utility can be loosely defined as a measure of happiness and therefore a utility function has a higher value for preferred choices. The translation of an economic value such as NPV into utility can therefore take into account the risk attitude of the decision maker.

The concept of marginal utility was first introduced by Daniel Bernoulli[20] in his 1738 solution to the St. Petersburg Paradox[22] posed by his cousin Nicholas Bernoulli in 1713. The two concepts arising from this was that people's utility from wealth is not linearly related to wealth and that the associated value of a risky venture is not the EMV but the expected utility.

The formal incorporation of utility theory[17] came in 1944. This formally incorporated choice based on preferences of distributions of outcomes.

By Bernoulli's logic, in order to value a risky venture the expected utility is:

$$E(u \mid p, X) = \sum_{x \in X} p(x)u(x) \qquad (12)$$

where X is the set of possible outcomes, p(x) is the probability of a particular outcome $x \in X$ and $u: X \to R$ is a utility function over outcomes. This implies if u(x) > u(y) then x is preferred to y.

Although there are several criticisms of the theory I wish to point out the one major difficulty. This difficulty is in creating the utility function as it can take several additional forms as seen in **Figure 3** this study will only use the logarithmic approach such that:

$$U(x) = a\log(x) \qquad (13)$$

Where x is the NPV.

The approach used to quantify the uncertainty in the NPV calculations is Monte Carlo simulation. NPV can then be directly converted into utility to facilitate the decision making process. The speed and accuracy of the proxy model allows this to be done.

WAG

The problem being addressed is the optimal recovery of oil under the economic constraints imposed on the wells. The problem of uncertainty in reservoir and fluid parameters is added to the mix to further compound the decision making process. In one case the oil price is varied to show the consequences of taking into account economics and not just recovery.

The first step taken in addressing the issue looks at sensitivity issues of the WAG process. This phase of the study used the initial WAG parameters and well settings without optimization to study the uncertainty of several reservoir properties. This work was performed on the same model used in this study. Gas breakthrough problems occurred in all the simulation runs and loss of water injectivity occurred in a significant number of the runs. Very few of these scenarios suffered major water production problems due to the injectivity abnormalities. Several reservoir parameters were studied to examine the sensitivity of the oil recovery[3].

The first step in the optimization was reducing the scope of the study for the purposes of testing this methodology of optimizing under uncertainty. The problem was reduced to two uncertainty parameters by nondimentionalizing the relative permeability and viscosity parameters into a single mobility term and studying the effect of uncertainty in the absolute permeability. The ranges used in the study are provided in **Table 4** along with the control setting.

A primary reason for choosing the absolute permeability is so that the range covers different flank sectors of the reservoir that the model was derived from. This has the added benefit of applying the technique to different sectors using different probability distribution functions (PDF) in the Monte Carlo analysis of the results.

The next step is deciding the control parameters available for optimization of the oil recovery. To maintain a reasonable number of simulations to test the process and validate the results the number of control parameters was reduced to five. An initial WAG ratio of 1:1 is used with 3 months per injection phase and two additional options with a ratio of 1:3 with 1 month water and 3 months gas, and 3:1 with 3 months water and 1 month gas are implemented along with the original WAG setting. Additionally the producer and injector have two completion intervals than can each be closed or open to flow.

The initial goal is determining the optimum WAG parameters at the start of the project with only a reactive control scheme. This is done by setting up a design with three parameters that include the permeability, mobility, WAG slug size, and well type. For ease of analysis two separate proxy models were generated, one for each well type. A single model with well type as a forth parameter was tested and it worked, but this would not be convenient for the next stage of the optimization. All production wells are placed on a reactive control mode at this time. The optimum WAG parameters determined at this stage remain constant throughout the simulation. To calculate NPV and asses the value of the smart well, a cost of $200,000 is taken for the conventional wells and $500,000 for the smart wells.

The parameters are coded and scaled such that the low value is coded to -1, the median to 0 and the maximum value to +1. The ranges for the WAG parameters and reservoir uncertainty parameters and their corresponding coded values can be seen in **Table 4**. There are then some assumptions made that all runs can be performed at any combination of these settings. If this is not true the design is modified with constraints. In this case closing all connections can be simulated but provides no useful results for the optimization process. These runs would also populate the data set with results that would pollute the proxy model fit.

Oil prices are incorporated in the post processing of the simulation data. A decision is made on the valve setting at the current time step and then the simulation moves forward. This process is repeated till the end of the 10 year simulation.

Results and Discussion

Monte Carlo Analysis

The average simulation time was between 10 and 20 minutes while the proxy model to determine the oil recovery takes approximately one second. The multiple order of magnitude increase in speed has facilitated the performing of Monte Carlo simulations to provide a reasonable PDF for the base and optimized oil recovery, the NPV, and the utility of the project under reservoir uncertainty. **Figure 4** shows the assessment of recovery distribution for the conventional well and optimized smart well. **Figure 5** shows the assessment of the NPV distribution for the conventional and optimized smart well. **Figure 6** shows the assessment of the utility distribution for the conventional and optimized smart well.

Ultimately a properly optimized flood is robust enough to increase the ultimate recovery significantly and reduce the uncertainty in the recovery.

WAG Flood

An experimental design and response surface proxy model based approach integrates the optimization with the model sensitivities. This approach treats the well operations as a parameter in the design space rather than a secondary control parameter. Therefore the well status becomes a term in the proxy model. Secondly this integrated approach optimizes simultaneously over the entire design space to provide a robust control algorithm.

The reservoir simulation model was run for 10 years with the recovery and NPV at 10 years modeled by proxy models. The optimization routine initially was aimed at maximizing the ultimate recovery of the pattern flood. The same approach using proxy models for NPV was applied to optimize the NPV.

In order to visualize the process 2 contour maps are shown in **Figure 7** and **Figure 8**. **Figure 7** comes from the beginning of the conventional well run prior to any optimization. As this is not a smart well there is only one control parameter, the WAG ratio. This figure clearly shows that a WAG ratio of "1" is best and referring to **Table 4** this corresponds to a 3:1 WAG.

Figure 8 comes from a late time in the simulation of the smart well. It clearly indicated the valve should be opened at this time but may not be true. This figure is plotted with the remaining parameters set at a mid level. The goal of this exercise is to optimize all 5 control parameters under the reservoir uncertainty. All these decisions are made from the proxy model but are somewhat more difficult. All 5 control parameters must be simultaneously optimized.

The results of the analysis at $50/stb are summarized in **Table 5**. The base model provided an average recovery of about 15% after 10 years. The results of the complete optimization of recovery provided much better results. Taking advantage of the derivatives provided by the proxy model optimizing injection and production as well as the WAG, results in an increase in the mean recovery to 39%. This provides an increase in the expected recovery and an increased value in the optimization.

The oil recovery uncertainties for the conventional and optimized smart well are shown in **Figure 4**. To represent the recovery over the entire design space the Monte Carlo Simulation drew samples of the reservoir properties distributed evenly over the entire design space.

In the period between World War I and the Gulf War inflation adjusted average annual oil prices in year 2000 dollars ranged from under $9.00 in the Early 1930's to over $60.00 during the height of the Iranian revolution of the early 1980's. Even the 1980's showed a range of under $20.00 to over $60.00. The base price of $50.00/stb was used in all previous calculations. To assess the risk of spending extra capital on the project under uncertain economic conditions two different oil price scenarios were used.

> "I don't think it's going to go to 100 us$ but if it does the crash is going to be even more spectacular…It will make the hi-tech bubble look like a picnic … this thing is not going to last." Steve Forbes

An optimistic price of $100 and a pessimistic price of $10 are used to compare risk attitudes.

An interesting observation can be made comparing the $10 and $50 results for the 2 simulations. The NPV after subtracting capital costs was positive for all 4 cases with mean NPV values of 1.23 MM$ and 7.78 MM$ for the conventional well and 13.51 MM$ and 21.35 mm$ for the smart well.

Initially these all seem very profitable and good returns on investment of $400,000 and $1,000,000 respectively. The utility approach tells a very different story

In a risk averse world a 50% chance of winning or losing $1 may have an expected monetary value of $0 but in truth has a negative utility. This means no one would choose this venture unless of course they are in Las Vegas.

Cost overruns were not a part of the data presented above. To illustrate the point above for one example, the low oil price conventional well, additional costs were added after the fact. The project with $1,000,000 in cost overruns had a large negative utility even though the NPV remained marginally positive.

Conclusions

> "The meek shall inherit the earth, but not the mineral rights." J. Paul Getty
>
> "No one can possibly achieve any real and lasting success or "get rich" in business by being a conformist." J. Paul Getty
>
> "Oil is like a wild animal. Whoever captures it has it." J. Paul Getty

- Risk and uncertainty are ever present and must be incorporated into the decision making process. Although not gambling in the sense of risk prone we are gambling risk averse.
- The decision making process of determining a single robust injection / production strategy across the entire uncertainty range in the WAG flood models proved to be difficult but attainable.
- Generated proxy models were able to adequately represent the oil recovery, NPV, and utility of the WAG process. The proxy model allowed the WAG process to be optimized even under uncertainty.
- Smart well technology offers various benefits over conventional wells when appropriate robust control algorithms are applied. The WAG process showed significant improvements in NPV.
- Optimizing over the entire design space using the response surface proxy model proved to be robust. Before breakthrough the method is extremely robust and post breakthrough works though is more challenging.

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Tables

Number Parameters	Regression Parameters	3-level full factorial
4	15	81
5	21	243
6	28	729
8	45	6561
10	66	59049

Table 1: Growth in simulation requirements for full factorial design.

	Scaled Setting		
	A	B	C
run 1	1	1	1
run 2	1	1	-1
run 3	1	-1	1
run 4	1	-1	-1
run 5	-1	1	1
run 6	-1	1	-1
run 7	-1	-1	1
run 8	-1	-1	-1

Table 2: A sample coded 2 level 3 parameter full factorial screening design.

	Scaled Setting		
	A	B	C
run 1	1	0	1
run 2	0	1	-1
run 3	1	-1	1
run 4	1	0	-1
run 5	-1	1	0
run 6	-1	1	-1
run 7	0	-1	0
run 8	-1	-1	0

Table 3: A part of the 3 level optimum design.

Relative Range	Low	Medium	High
Coded Value	-1	0	1
WAG Ratio	1:3	3:3	3:1
Arial Perm. Multiplier	2	4	6
Mobility Oil	0.9	1.0	1.1
Well Completion i prod	shut		open
Well Completion i inj	shut		open

Table 4: Coded parameters with values used in the WAG pattern flood simulations.

Means	Fully Optimized	Opt. Comp	Opt. WAG	Conv. Well
Recovery	0.39	0.35	0.30	0.15
NPV MM$	21.35	16.78	13.51	7.78
Utility	1.33	1.22	1.12	0.89

Table 5: Results of the analysis.

Figures

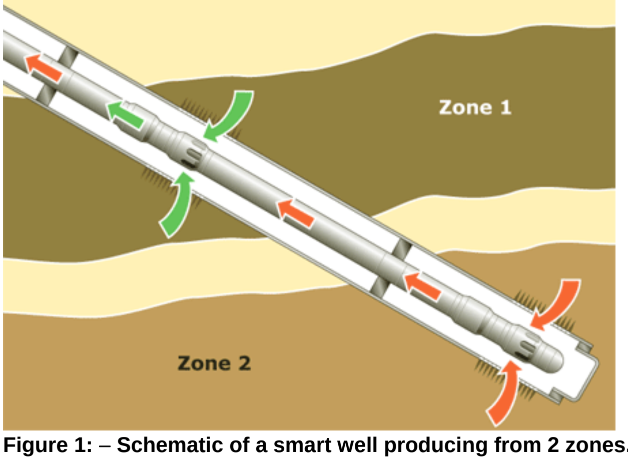

Figure 1: – Schematic of a smart well producing from 2 zones.

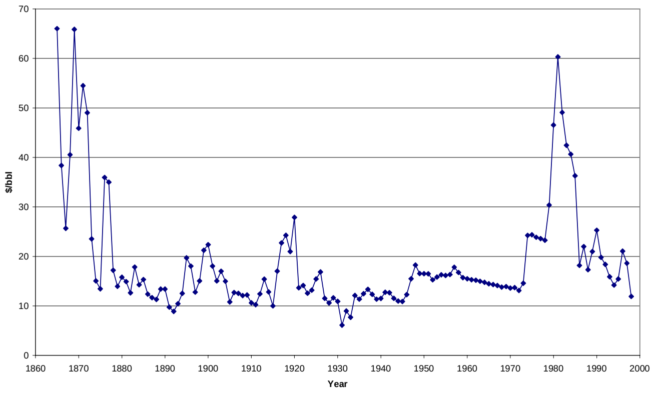

Figure 2: Historic Oil Prices.

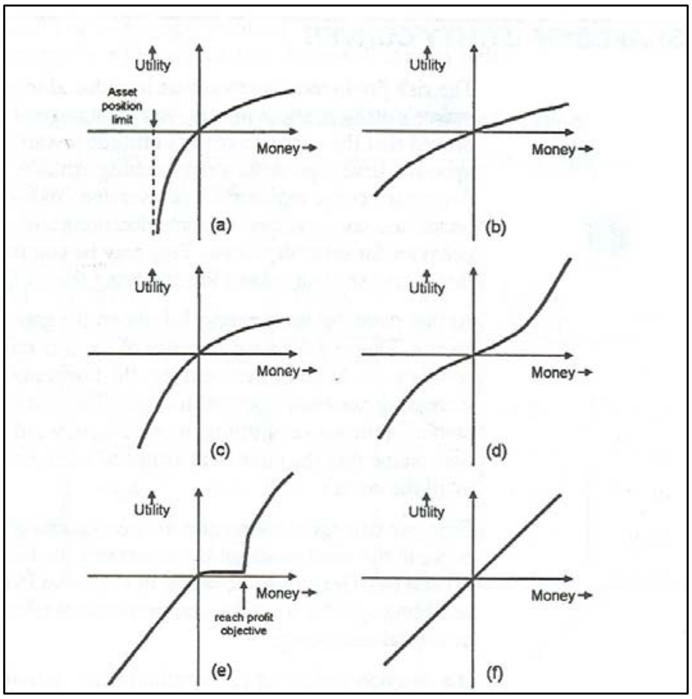

Figure 3: Sketches of various types of utility functions. Shape (a) is bounded on the downside by an asset position or other exposure limit. (b) does not imply a limit in the range of money shown. (c) represents a conservative, or risk averse attitude for taking risk. (d) is conservative about negative amounts yet risk-seeking for positive outcomes. (e) represents a step up in value when a profits target is reached. (f) is a utility curve for a risk-neutral decision maker, i.e., an EMV decision policy.

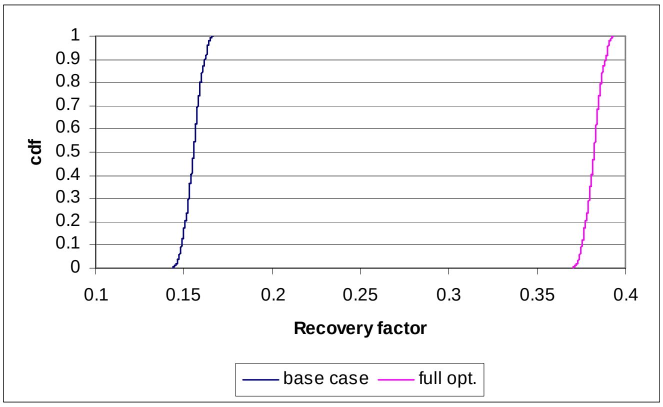

Figure 4: Recovery distribution

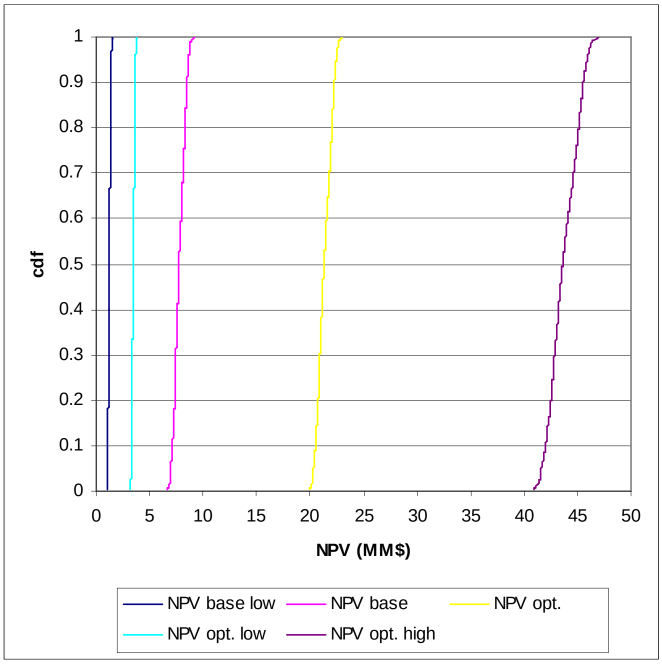

Figure 5: NPV distribution

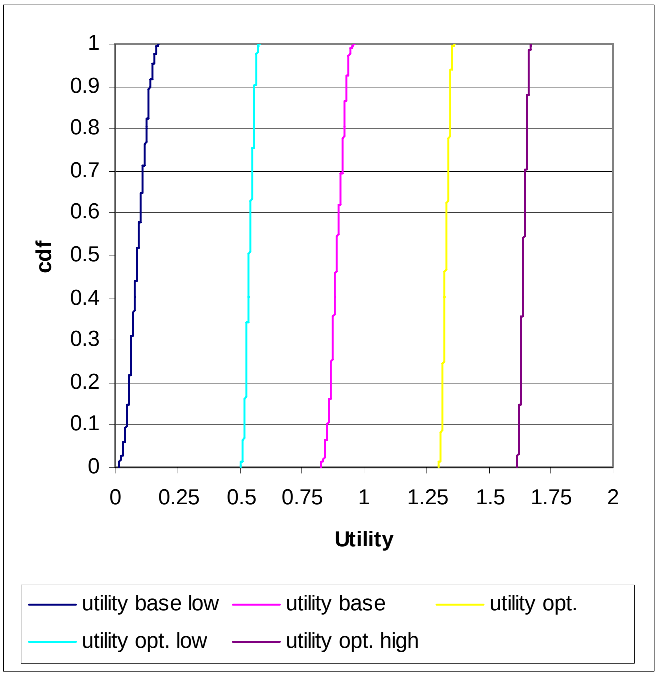

Figure 6: Utility distribution

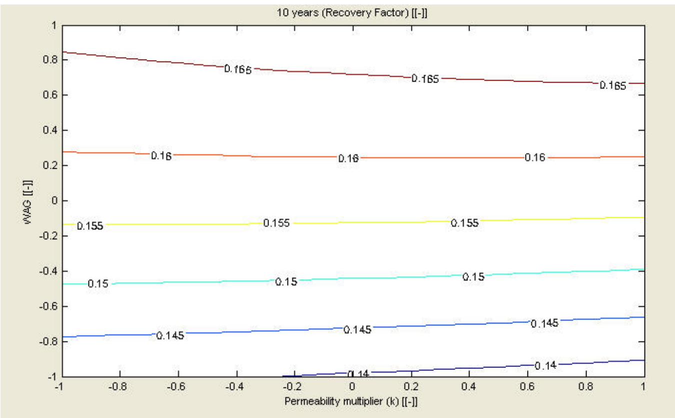

Figure 7: Contour map of recovery for WAG setting and the permeability at an early time step. (only conventional well)

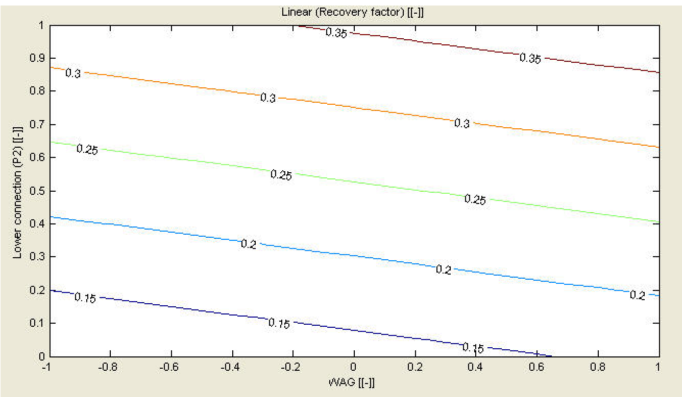

Figure 8: Contour map of recovery for WAG setting and the lower completion interval of the producer at an early time step.