SPE
Society of Petroleum Engineers of AIME

SPE 12248

A Consistent Method for Calculating Transmissibilities in Nine-Point Difference Equations

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ABSTRACT

In the past few years there has been increased interest in modeling miscible and thermal recovery processes by the use of reservoir simulation models. The prediction of sharp saturation or temperature fronts resulting from these processes has shown that these simulators can in many cases be very sensitive to grid orientation. One method for minimizing the effect of grid sensitivity is to describe the difference equations with a nine-point rather than the standard five-point finite difference approximations. Inherent in the use of the nine-point approximations is the necessity for calculating consistent transmissibility coefficients. Previously published methods for calculating these coefficients do not have general applicability to heterogeneous reservoirs and/or various irregular grid spacings. This paper presents a method for calculating nine-point transmissibilities for a general heterogeneous system with unequal grid spacing.

INTRODUCTION

Recent interest in recovering oil from heavy oil reservoirs has led to the development of reservoir models that can accurately model such processes as miscible displacement, steam flooding and in-situ combustion. These processes are often characterized by two phenomena. First there exists a high viscosity ratio between the displaced fluid (oil) and the displacing fluid (carbon dioxide, steam, miscible gas, etc.). Second, the residual oil saturation behind the displacement front can be quite low. This results in an unfavorable mobility ratio piston-type displacement.

Reservoir simulation models utilizing standard five-point differencing techniques can exhibit severe grid orientation effects when applied to unfavorable mobility ratio displacement processes. A number of investigators have studied grid orientation effects using five-point difference methods such as Todd et al[1] and Coats[2]. One method proposed to minimize the effect of grid orientation is to use nine-point difference equations. Yanosik and McCracken[3] proposed a method of applying nine-point difference equations to reservoir simulation problems and showed results that indicate this approach is helpful in minimizing grid orientation effects. Coats and Ramesh[4] applied the procedure proposed by Yanosik and McCracken to simulating thermal recovery processes and obtained a similar reduction of grid orientation effects.

The application of nine-point difference equations to homogeneous systems with uniform grid spacing is straightforward. However the extension to non-uniform grids with heterogeneous permeability is not as simple as it might first appear. This paper presents a consistent method for calculating the nine-point transmissibilities that may be used in a reservoir model of arbitrary heterogeneity and irregular or non-uniform grid spacing. Examples are included that show the differences between this proposed method and that of Yanosik and McCracken.

BACKGROUND

A five-point difference formulation only considers flow between a grid point and the four blocks that are adjacent to its boundaries. Figure 1 shows a portion of a typical two dimensional grid system. The five-point formulation considers flow between points (i, j), (i, j-1), (i+1, j), (i-1, j) and (i, j+1). A nine-point difference formulation considers these points plus the flow between point (i, j) and its four corner points at (i-1, j-1), (i+1, j-1), (i-1, j+1) and (i+1, j+1).

Yanosik and McCracken published an excellent paper describing a method for formulating the nine-point difference equations describing flow in porous media. The equation that they proposed to describe two-dimensional flow of phase p was given as,

$$\left(\frac{T\,k_{rp}}{\mu_p B_p}\right)_{i+\frac{1}{2},j} (\Phi_{i+1,j} - \Phi_{i,j})$$

$$+\left(\frac{T\,k_{rp}}{\mu_p B_p}\right)_{i-\frac{1}{2},j} (\Phi_{i-1,j} - \Phi_{i,j})$$

$$+\left(\frac{T\,k_{rp}}{\mu_p B_p}\right)_{i,j+\frac{1}{2}} (\Phi_{i,j+1} - \Phi_{i,j})$$

References and illustrations at end of paper.

$$+ \left(\frac{T\,k_{rp}}{\mu_p B_p}\right)_{i,j-\frac{1}{2}} (\Phi_{i,j-1} - \Phi_{i,j})$$

$$+ \left(\frac{T\,k_{rp}}{\mu_p B_p}\right)_{i+\frac{1}{2},j-\frac{1}{2}} (\Phi_{i+1,j-1} - \Phi_{i,j})$$

$$+ \left(\frac{T\,k_{rp}}{\mu_p B_p}\right)_{i-\frac{1}{2},j-\frac{1}{2}} (\Phi_{i-1,j-1} - \Phi_{i,j})$$

$$+ \left(\frac{T\,k_{rp}}{\mu_p B_p}\right)_{i+\frac{1}{2},j+\frac{1}{2}} (\Phi_{i+1,j+1} - \Phi_{i,j})$$

$$+ \left(\frac{T\,k_{rp}}{\mu_p B_p}\right)_{i-\frac{1}{2},j+\frac{1}{2}} (\Phi_{i-1,j+1} - \Phi_{i,j})$$

$$+ \quad q = V \frac{\partial}{\partial t} \frac{\phi S_p}{B_p} \tag{1}$$

In their paper the authors propose a method for calculating the transmissibilities T that may be used in equation (1). This paper follows that of Yanosik and McCracken in that it begins with equation (1) and develops another procedure for calculating the transmissibilities.

DEVELOPMENT OF EQUATIONS

Consider a finite difference grid system with variable spacing in both x and y directions as shown in Figure 2. Throughout this paper the term uniform grid spacing will mean a rectangular grid with all Δx values constant and all Δy values constant but not necessarily equal. The square grid case is a subset of the uniform grid with Δx and Δy constant and equal. The term non-uniform or irregular grid will mean a rectangular grid where Δx and/or Δy are not constant. The non-uniform grid is also often referred to as variable grid spacing in the literature. Only the four grid points shown in the shaded area will be considered from here on as only this section is required for the following development. This section of the grid is shown in more detail in Figure 3. Note that the permeabilities within each of the four areas defined by the grid boundaries are in general unequal; however, within any individual area, the permeability is constant. In order to simplify the nomenclature and minimize the subscripting needed in this development, these same four points are shown in Figure 4 with an easier numbering convention. The four grid points are numbered 1 through 4, the intersection of the grid block boundaries and this region boundary are numbered 5, 7, 8 and 9 and the intersection of the grid block boundaries inside the region is point number 6. The permeability of each of the sections is k_1 through k_4 and the dimensions are Δx_1, Δx_2, Δy_1 and Δy_2. The thickness of the missing dimension is taken to be unity. The reason for simplifying the nonmenclature is because transmissibilities must be defined between all these points and it is easier for example to refer to the x direction permeability between points (i, j) and (i+1, j) $T_{x\,i+1/2,j)}$ simply as T_{12} and a diagonal transmissibility between points (i, j) and (i+1, j+1) $T_{xy\,i+1/2,\,j1/2}$ simply as T_{14}.

One point of clarification must be made about the following development that is a result of only considering the region defined by the four points. The edge transmissibilities which correspond to the normal x and y direction transmissibilities are computed only for the region interior to the four points and do not include the contribution from the section of the grid block outside the defined region. For example the paper by Yanosik and McCracken would calculate the x-direction transmissibility T_x from

$$T_x = T_{x1} + T_{x2} - T_{xy1} - T_{xy2} \tag{2}$$

as shown in Figure 5. The approach used here is only to calculate the portion of the T_x derived from the region of four points, i.e.

$$T_{x\,(4\text{ point region})} = T_{x1} - T_{xy1} \tag{3}$$

The total T_x would have to be calculated by adding the value T_{x2} from the missing part of the grid block and subtracting off its diagonal contribution T_{xy2}. This has no effect on the anaylsis which is presented only for the four-point region to simplify the equations. Final value of x and y transmissibilities may be easily computed using this method in a simple program. Note that even though the edge transmissibilities are only one component, the diagonal transmissibility T_{xy1} is the full complete value.

Now referring again to the simplified numbering scheme in Figure 4, the four-point region transmissibility equations as presented by Yanosik and McCracken (hereafter called the Amoco Method) may be written as

$$T_{12} = \frac{1}{2}\, k_{12} \frac{\Delta y}{\Delta x} - T_{xy} \tag{4}$$

$$T_{34} = \frac{1}{2}\, k_{34} \frac{\Delta y}{\Delta x} - T_{xy} \tag{5}$$

$$T_{13} = \frac{1}{2}\, k_{13} \frac{\Delta x}{\Delta y} - T_{xy} \tag{6}$$

$$T_{24} = \frac{1}{2}\, k_{24} \frac{\Delta x}{\Delta y} - T_{xy} \tag{7}$$

where

$$k_{mn} = \frac{1}{2} (k_m + k_n) \tag{8}$$

$$T_{xy} = T_{14} = T_{23} = \frac{1}{3}\, k_{xy} \frac{\Delta x\ \Delta y}{\Delta x^2 + \Delta y^2} \tag{9}$$

$$k_{xy} = \frac{1}{4} \quad (k_1 + k_2 + k_3 + k_4) \tag{10}$$

$$\Delta x = \quad \Delta x_1 + \Delta x_2 \tag{11}$$

$$\Delta y = \quad \Delta y_1 + \Delta y_2 \tag{12}$$

Several problems arise when applying these equations to heterogeneous systems. First, areas of zero porosity present a problem. Second, anisotropy is not considered and finally the equations are independent of individual grid block sizes. The equations depend only on the total Δx and Δy. The individual Δx's and Δy's do not appear so the location of block boundaries is not accounted for.

The development from here on relates to the new proposed method for calculating the nine-point transmissibilities. First, to treat the problem of anisotropy, let

k_{1x} = permeability in block 1 in x direction

k_{1y} = permeability in block 1 in y direction

then assuming a harmonic average, for block 1

$$k_{xy\,1} = \frac{(\Delta x_1 + \Delta y_1)\ k_{1x}\ k_{1y}}{k_{1x}\ \ \Delta y_1 + k_{1y}\ \ \Delta x_1} \tag{13}$$

with similar expressions for the other three blocks to get k_{xy2}, k_{xy3} and k_{xy4}. The value of k_{xy1} is the absolute permeability in the diagonal direction which will be used in calculating the diagonal transmissibility only. Note that while the use of equation (13) is arbitrary and intuitive, it does reduce to k_1 if there is no anisotropy and it yields k_{xy1} equal to zero if either k_{1x} or k_{1y} is zero, which should also be true. Furthermore, it weights the individual directional permeabilities by distance in both directions. Now within each of the four regions a diagonal transmissibility is calculated from the Amoco Method as

$$T_{16} = \quad \omega\, k_{xy1} \quad \frac{\Delta x_1\ \ \Delta y_1}{\Delta x_1^2 + \Delta y_1^2} \tag{14}$$

$$T_{26} = \quad \omega\, k_{xy2} \quad \frac{\Delta x_2\ \ \Delta y_1}{\Delta x_2^2 + \Delta y_1^2} \tag{15}$$

$$T_{36} = \quad \omega\, k_{xy3} \quad \frac{\Delta x_1\ \ \Delta y_2}{\Delta x_1^2 + \Delta y_2^2} \tag{16}$$

$$T_{46} = \quad \omega\, k_{xy4} \quad \frac{\Delta x_2\ \ \Delta y_2}{\Delta x_2^2 + \Delta y_2^2} \tag{17}$$

where ω is a constant to be determined. A value of ω equal to 4/3 will give results identical to the Amoco Method for a uniform (square or non-square) grid spacing with homogeneous isotropic permeability.

Now the problem of calculating the edge or "normal" transmissibilities T_{12}, T_{13}, T_{34} and T_{24} will be illustrated for the T_{12} coefficient. The edge x direction transmissibility T_{12} must contain four elements, the value from point 1 to point 8, plus the value from point 2 to point 8 and must be reduced by the diagonal transmissibilities from both points 1 and 2 to point 6. Therefore

$$T_{18} = k_{1x} \quad \frac{\Delta y_1}{\Delta x_1} \tag{18}$$

$$T_{28} = k_{2x} \quad \frac{\Delta y_1}{\Delta x_2} \tag{19}$$

Now using a harmonic average of the transmissibilities,

$$T_{12} = \frac{T_{18}\ T_{28}}{T_{18} + T_{28}} - \frac{T_{16}\ T_{26}}{T_{16} + T_{26}} \tag{20}$$

Equation (20) insures that the sum of the two parallel flow paths through points 1-8-2 plus 1-6-2 will give the exact x direction flow capacity for the layer of thickness. Δy_1. Similar expressions may be derived for the remaining three edge transmissibilities T_{13}, T_{34} and T_{24}.

Now assuming steady-state incompressible flow, mass balance equations may be written for each corner point and the center point as,

$$-(T_{12} + T_{13} + T_{16})p_1 + T_{12}p_2 + T_{13}p_3 + T_{16}p_6 = Q_1 \tag{21}$$

$$T_{12}p_1 - (T_{12} + T_{24} + T_{26})p_2 + T_{24}p_4 + T_{26}p_6 = Q_2 \tag{22}$$

$$T_{13}p_1 - (T_{13} + T_{34} + T_{36})p_3 + T_{34}p_4 + T_{36}p_6 = Q_3 \tag{23}$$

$$T_{24}p_2 + T_{34}p_3 - (T_{34} + T_{24} + T_{46})p_4 + T_{46}p_6 = Q_4 \tag{24}$$

$$T_{16}p_1 + T_{26}p_2 + T_{36}p_3 + T_{46}p_4 - (T_{16} + T_{26} + T_{36} + T_{46})p_6 = Q_6 \tag{25}$$

Where p_j is the pressure and Q_j is the production rate at point j. The contraints on the rates Q_j are that the sum at the rates must be zero and that the rate at the center point, Q_6, must also be zero. Note that the equations do not contain pressures at points 5, 7, 8 and 9 since the transmissibilities T_{56}, T_{76}, T_{86} and T_{96} are all zero because the thickness of the flow path assigned between these points is zero. The left hand side of equations (21) through (25) is a (5 x 5) matrix of the form

	p_1	p_2	p_3	p_4	p_6	
equation 1	x	x	x	0	x	
equation 2	x	x	0	x	x	
equation 3	x	0	x	x	x	(26)
equation 4	0	x	x	x	x	
equation 5	x	x	x	x	x	

If the variable p_6 is eliminated from equation (26) using the element (5,5) as the pivot element the result is a (4,4) matrix involving only pressures at the four corner points. If the (4,4) matrix is then manipulated so that the right hand side Q's reappear as the original, unmodified values, the resulting (4,4) matrix must have the following form:

$$\begin{matrix} x & T_{12} & T_{13} & T_{14} \\ x & x & T_{23} & T_{24} \\ x & x & x & T_{34} \\ x & x & x & x \end{matrix} \tag{27}$$

where the T's represent the six transmissibilities for the nine-point difference equations. One serious problem occurs if the development stops here for both this proposed method or in the Amoco Method; it is possible to use heterogeneity such that at least one of the edge transmissibilities T_{12}, T_{13}, T_{34} or T_{24} can turn out to be negative. If any negative value is set to zero, the total system will not match the overall or correct fluid conductivity so merely zeroing a negative value is not correct.

In order to eliminate the problem of zero edge values and remove the inconsistency, first note the diagonal transmissibilities as defined by equations (14) through (17) are of the form

$$T_{16} = \omega t_{16} \tag{28}$$

$$T_{26} = \omega t_{26} \tag{29}$$

$$T_{36} = \omega t_{36} \tag{30}$$

$$T_{46} = \omega t_{46} \tag{31}$$

where

$$t_{16} \equiv k_{xy1} \frac{\Delta x_1 \ \Delta y_1}{\Delta x_1^2 + \Delta y_1^2} \tag{32}$$

with similar expressions for the t_{26}, t_{36} and t_{46}. Second note that equation (20) for the edge transmissibility T_{12} is of the form

$$T_{12} = \tau_{12} - \frac{T_{16} \ T_{26}}{T_{16} + T_{26}} \tag{33}$$

where

$$\tau_{12} \equiv \frac{T_{18} \ T_{28}}{T_{18} + T_{28}} \tag{34}$$

Therefore combining equations (28), (29) and (33) the result is

$$T_{12} = \tau_{12} - \frac{\omega t_{16} \ t_{26}}{t_{16} + t_{26}} \tag{35}$$

with similar expressions for the other edge values. If equation (35) and its analogs are substituted into the mass balances and the partial elimination to the (4x4) matrix is performed an expression for T_{12} is obtained,

$$T_{12} = \tau_{12} - \frac{\omega t_{16} \ t_{26}}{t_{16} + t_{26}} \left[\frac{t_{36} + t_{46}}{t_{16} + t_{26} + t_{36} + t_{46}} \right] \tag{36}$$

Now going back to the Amoco Method as described by equation (3)

$$T_{12} = \frac{1}{2} k_{12} \frac{\Delta y}{\Delta x} - T_{xy} = \frac{1}{2} T_x - T_{xy} \tag{37}$$

and requiring that T_{12} must be non-negative, then from equation (37) for the Amoco Method

$$\frac{T_{xy}}{T_x} \leq 2 \tag{38}$$

Note that the result expressed in equation (38) that the diagonal transmissibility cannot be greater than twice the edge value is a direct result of using the equations given by the Amoco Method. The imposition of this constraint is arbitrary and could be relaxed by treating the transmissibility ratio as another adjustable parameter, i.e.

$$\frac{T_{xy}}{T_x} \leq \beta \tag{39}$$

The parameter β could possibly be estimated by running a simulator and observing the effect of a varying on the calculated results. The following development does not contain this adjustable parameter but uses β equal to 2 as in the Amoco Method.

Rearranging equation (38) the result is

$$T_x \geq 1/3 \quad (T_x + T_{xy}) \tag{40}$$

Now applying equation (40) which insures a non-negative edge transmissibility to the expression for T_{12} as given in equation (36) the result is

$$T_{12} \geq 1/3 \quad \tau_{12} \tag{41}$$

Now combining equations (36) and (41) and solving for and calling the result ω_1 the answer is

$$\omega_1 \leq \frac{2/3 \quad \tau_{12}}{\frac{t_{16}\ t_{26}}{t_{16}+t_{26}}\left[\frac{t_{36}+t_{46}}{t_{16}+t_{26}+t_{36}+t_{46}}\right]} \tag{42}$$

Equation (42) was derived for the edge transmissibility T_{12} and therefore used properties from regions 1 and 2. Similar expressions may be derived for the other three edge transmissibilities using properties from regions 1-3, 3-4 and 2-4. The results give a value of ω for each region.

$$\omega_2 \leq \frac{2/3 \quad \tau_{13}}{\frac{t_{16}\ t_{36}}{t_{16}+t_{36}}\left[\frac{t_{26}+t_{46}}{t_{16}+t_{26}+t_{36}+t_{46}}\right]} \tag{43}$$

$$\omega_3 \leq \frac{2/3 \quad \tau_{34}}{\frac{t_{36}\ t_{46}}{t_{46}+t_{36}}\left[\frac{t_{16}+t_{26}}{t_{16}+t_{26}+t_{36}+t_{46}}\right]} \tag{44}$$

$$\omega_4 \leq \frac{2/3 \quad \tau_{24}}{\frac{t_{26}\ t_{46}}{t_{26}+t_{46}}\left[\frac{t_{16}+t_{36}}{t_{16}+t_{26}+t_{36}+t_{46}}\right]} \tag{45}$$

In addition to these four possible values of ω given by equations (42) - (45), one more value must be considered. A value of ω equal to 4/3 will give results identical to the Amoco Method for all transmissibilities in a homogeneous, uniform grid spacing model. The Amoco paper shows a value of 2/3 in their final paper which is equivalent to the SSI Method value of 4/3. Therefore, to insure that no negative transmissibilities will arise,

$$\omega = \text{minimum of} \quad 4/3,\ \omega_1, \omega_2, \omega_3, \omega_4 \tag{46}$$

Therefore, the procedure used to calculate all final nine point transmissibilities is:

1. Calculate the four values of t_{16}, t_{26}, t_{36} and t_{46} from equation (32) and its analogs.
2. Calculate the four τ values τ_{12}, τ_{13}, τ_{34} and τ_{24} from equation (34) and its analogs.
3. Calculate the four ω values from equations (42) - (45).
4. Choose the minimum ω as expressed by equation (46).
5. Calculate the four uncorrected edge value transmissibilities T_{12}, T_{13}, T_{34}, and T_{24}, from equation (20) and its analogs.
6. Calculate the four interior diagonal transmissibilities T_{16}, T_{26}, T_{36}, and T_{46} from equations (14) - (17).
7. Substitute all these eight transmissibilities into the mass balances, equations (21) - (25).
8. Reduce the matrix to the (4x4) matrix and obtain the final six transmissibilities as shown in equation (27).
9. Move to the next four point region and then augment the edge transmissibilities by their component from the other part of each grid cell.

Once this is complete the final result is a set of nine-point transmissibilities that insure no negative values, preserve the total flow capacity at each point regardless of heterogeneity and grid spacing, and are harmonically averaged as series resistance to flow in all directions. Note that zero values or missing grid blocks are valid in any direction.

The value of ω chosen as the minimum of the five possible choices does not preclude values less than this calculated minimum. For example, a given system may yield a minimum ω value of 1.0, however, a value of 0.5 is still valid in that no negative transmissibilities occur for values less than the minimum. Therefore, it is possible to use the ω as an adjustable parameter. Apriori selection of this value would appear to be difficult in that no theoretical value seems evident, and the value chosen for any particular heterogeneous grid system may not be transportable to another system. Possibly further research and analysis may allow a more general method for selecting ω values that best reduce grid orientation effects.

DISCUSSION

Comparing this procedure with the Amoco Method illustrates that the two methods can give quite different results. Figure 6 shows transmissibility coefficients for a square grid system calculated by both methods. Note these coefficients are not true transmissibilities for use in a simulator as they contain no conversion constants but only include the geometry and permeability and the edge values are only for the four point region as discussed previously. Regardless of these two considerations, the ratio between these values and true transmissibilities is valid. Also when examining these drawings it should be noted that both diagonal values are labeled in the SSI Method only when the two values are unequal and both diagonal values are always equal in the Amoco Method. The top drawing in Figure 6 shows that for a square grid with uniform permeability both methods are identical. The lower three drawings show the difference between the two methods for heterogeneous permeability square grid systems. Note the appearance of negative values in the Amoco Method even for the square grid case.

Figure 6 shows the comparison between the two methods for non-uniform spacing grid system. As in the square grid cases, certain permeability distributions can yield negative transmissibilities as shown in the middle two drawings. It is significant to note that even for the homogeneous permeability distribution shown in the top drawing of Figure 7, the two methods give different results. The importance of this fact may be observed if one considers the grid to be vertical and a well producing at the left hand side of this element. For simplicity assume the pressures are equal in the vertical direction so the well should be producing according to the Kh product from each layer. Thus the rate from the top layer should be five times the rate from the lower layer. For the Amoco Method, since the edge transmissibilities are both equal to 0.058 and both diagonals are equal to 0.917, the rate from each layer would be identical, not five to one. For the SSI Method the rate would be proportional to the edge plus the diagonal transmissibility values for each layer or,

$$\text{Ratio} = \frac{.21667 + .0333}{.01667 + .0333} = \frac{5}{1}$$

Therefore, the SSI Method would preserve the correct production distribution.

An additional feature of the SSI Method is that since all permeabilities are "harmonicized" to include the effects of permeability heterogeneity and variable grid spacing, this method will give results consistent with the series resistance to flow concept. Such a treatment is necessary to calculate correct pressure distributions in any stratified system.

CONCLUSIONS

1. A simple method has been developed for calculating transmissibility coefficients for inclusion in nine-point difference equations that gives consistent results for heterogeneous permeability distributions.

2. The method is applicable to nonuniform or variable grid spacings.

3. Coefficients calculated by this method will preserve the concept of series flow in resistance.

NOMENCLATURE

B = formation volume factor

k = absolute permeability

k_r = relative permeability

p = pressure

Q = production rate

q = production or injection rate

S = saturation

T = transmissibility

t = time

t_{mn} = permeability-geometry factor for point m,n

V = grid block volume

Δx = x direction distance between grid point and grid block boundary

Δy = y direction distance between grid point and grid block boundary

β = maximum ratio of diagonal to edge transmissibility

μ = viscosity

Φ = flow potential

ϕ = porosity

ω = transmissibility weighting coefficient

τ = harmonic averaged transmissibility

SUBSCRIPTS

i = x direction grid block index

j = y direction grid block index

m,n = point numbers in four point grid system

p = phase index

x = x direction value

y = y direction value

xy = diagonal value

REFERENCES

1. Todd, M. R., O'Dell, P.M., and Hirasaki, G. J.: "Methods for Increased Accuracy in Numerical Simulation", Soc. Pet. Eng. J. (Dec 1972) 515-530.

2. Coats, K. H., George, W. D., and Marcum, B.E.: "Three-Dimensional Simulation of Steamflooding", Soc. Pet. Eng. J. (Dec 1974) 573-592.

3. Yanosik, J. L., and McCracken, T. A.: "A Nine-Point, Finite-Difference Reservoir Simulator for Realistic Prediction of Adverse Mobility Ratio Displacements", Soc. Pet. Eng. J. (Aug 1979) 253-262.

4. Coats, K. H., Ramesh, A. B.: "Effects of Grid Type and Difference Scheme on Pattern Steamflood Simulation Results", SPE Paper 11079 presented at Fall Meeting of SPE, New Orleans, LA, Sept 26-29, 1982.

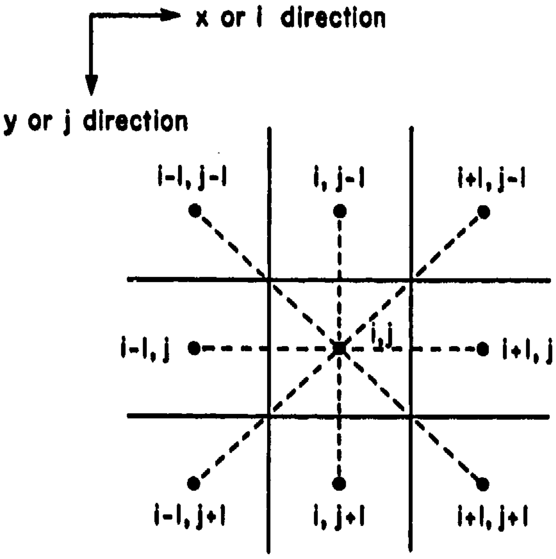

FIGURE 1

NINE POINT FINITE DIFFERENCE SYSTEM

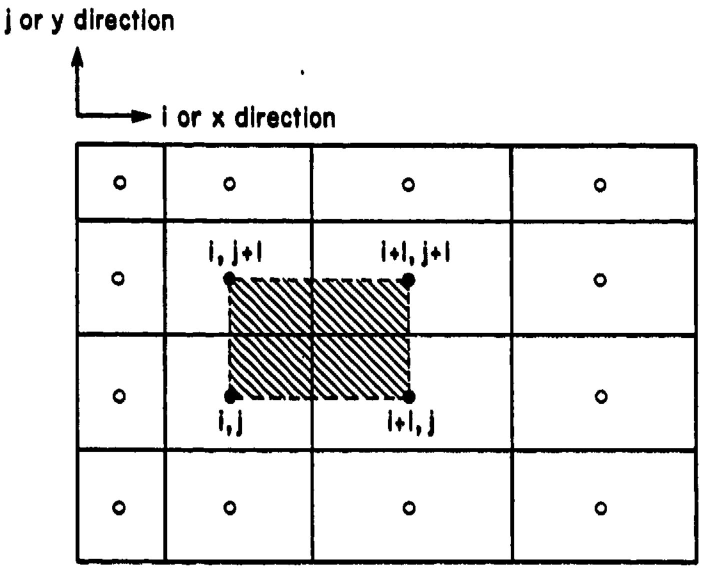

FIGURE 2

FINITE DIFFERENCE GRID SYSTEM

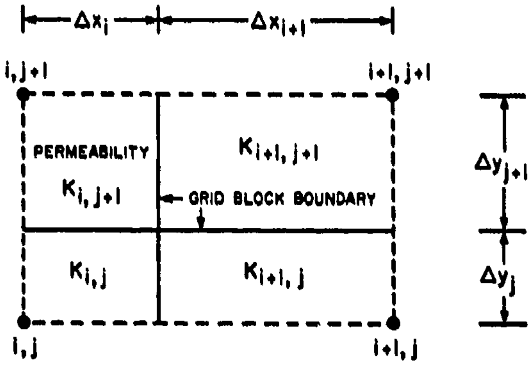

FIGURE 3

FOUR POINTS WITHIN GRID SYSTEM

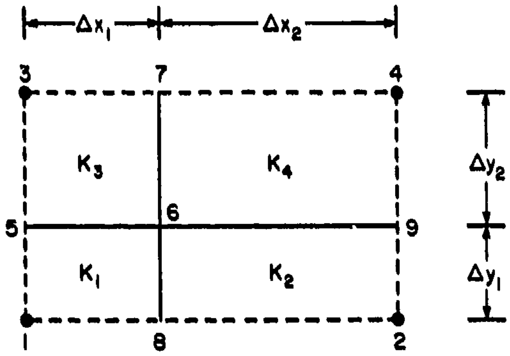

FIGURE 4

FOUR POINTS WITH SIMPLIFIED NOTATION

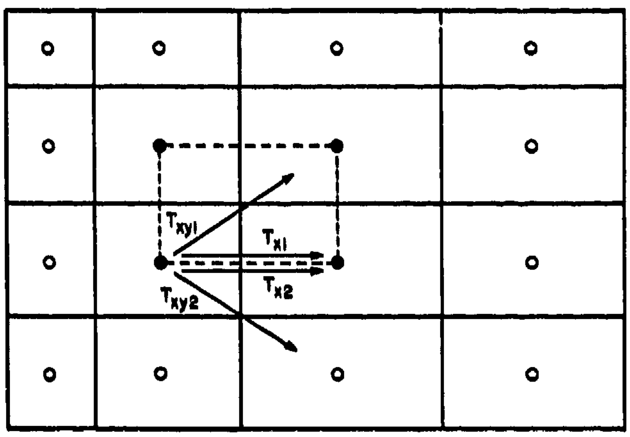

FIGURE 5

X-DIRECTION TRANSMISSIBILITIES

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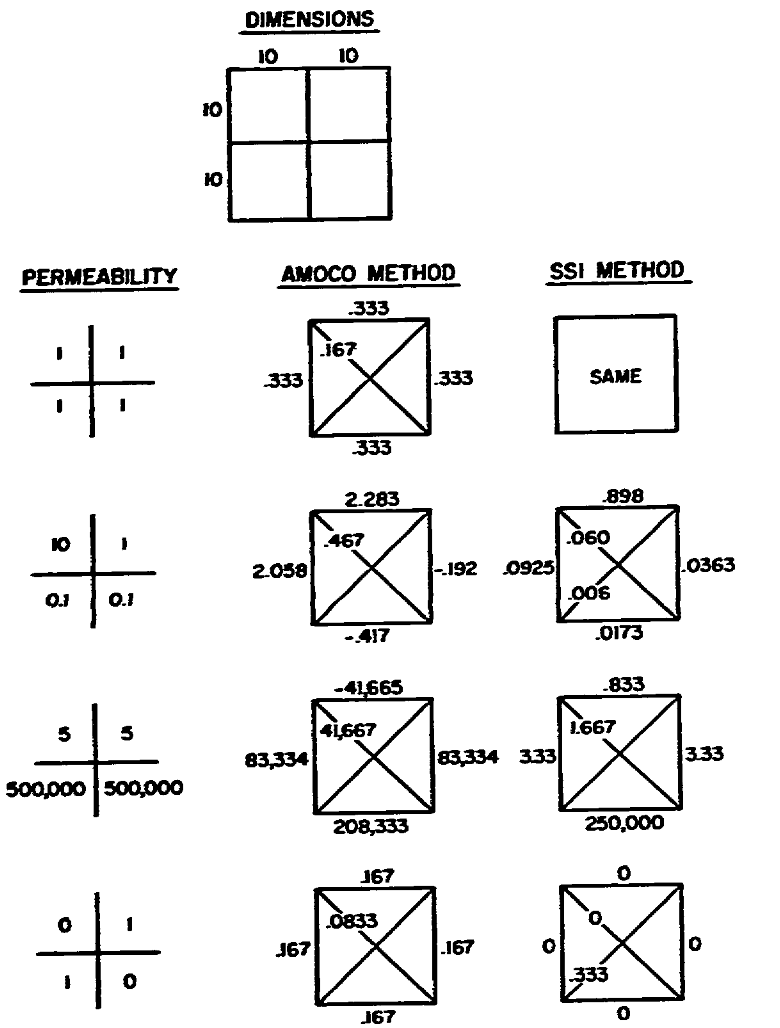

FIGURE 6

COMPARISON OF TRANSMISSIBILITIES WITH SQUARE GRIDS

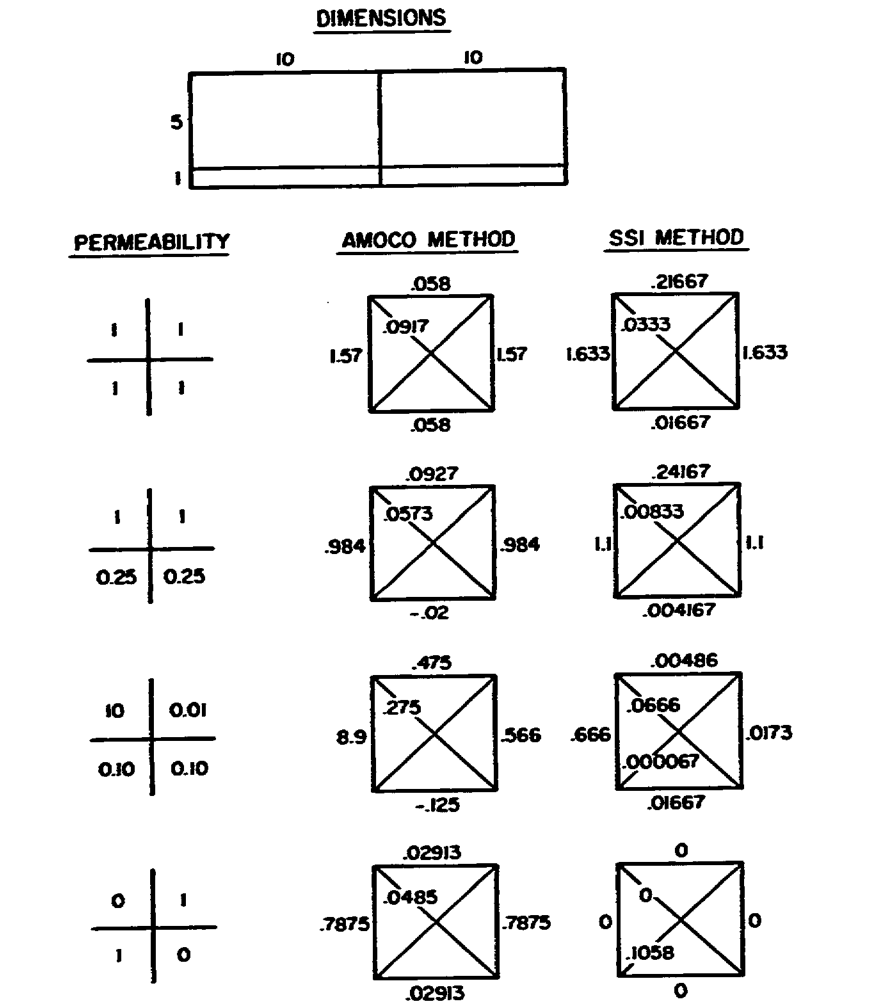

FIGURE 7

COMPARISON OF TRANSMISSIBILITIES WITH RECTANGULAR GRIDS

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