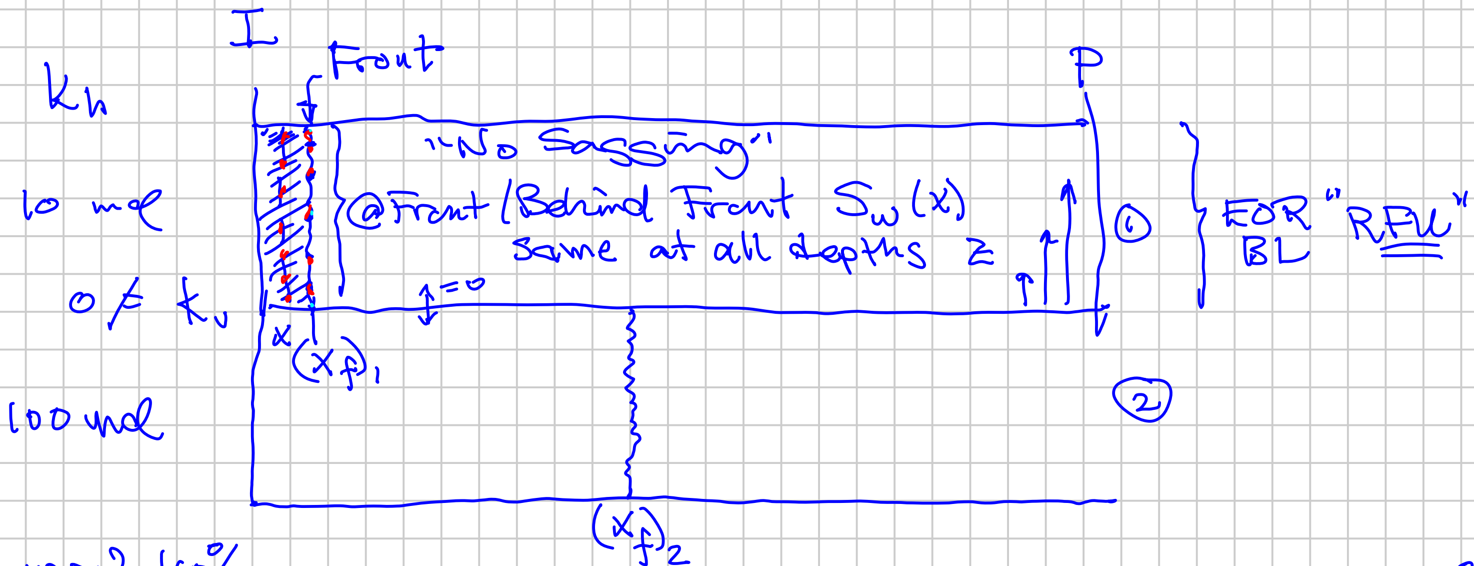
Note Title 10/29/2018

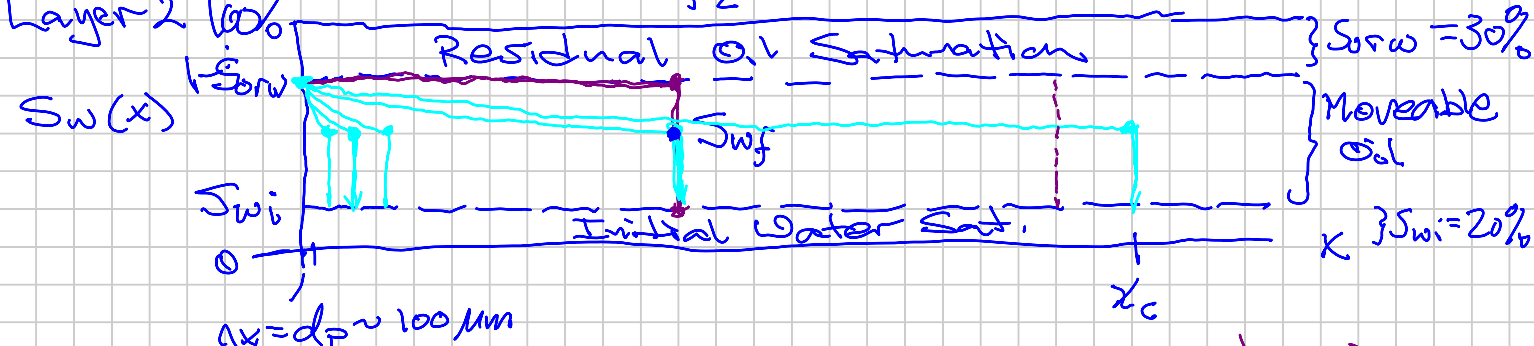
Water-Oil (Gas-Oil) Immiscible Displacement

- Buckley-Leverett Theory (1942)
  - Muskat Comments (1950)
  - Standing notes (197x) "Fractional Flow"
  - Mathematical verification (?) using 'Method of Characteristics'
  - One-dimensional flow theory (H | I | V) $0 < \alpha \le 90^\circ$

    $q, k, \phi, h, k_r, \Delta\rho \ldots \} \to 1DV$

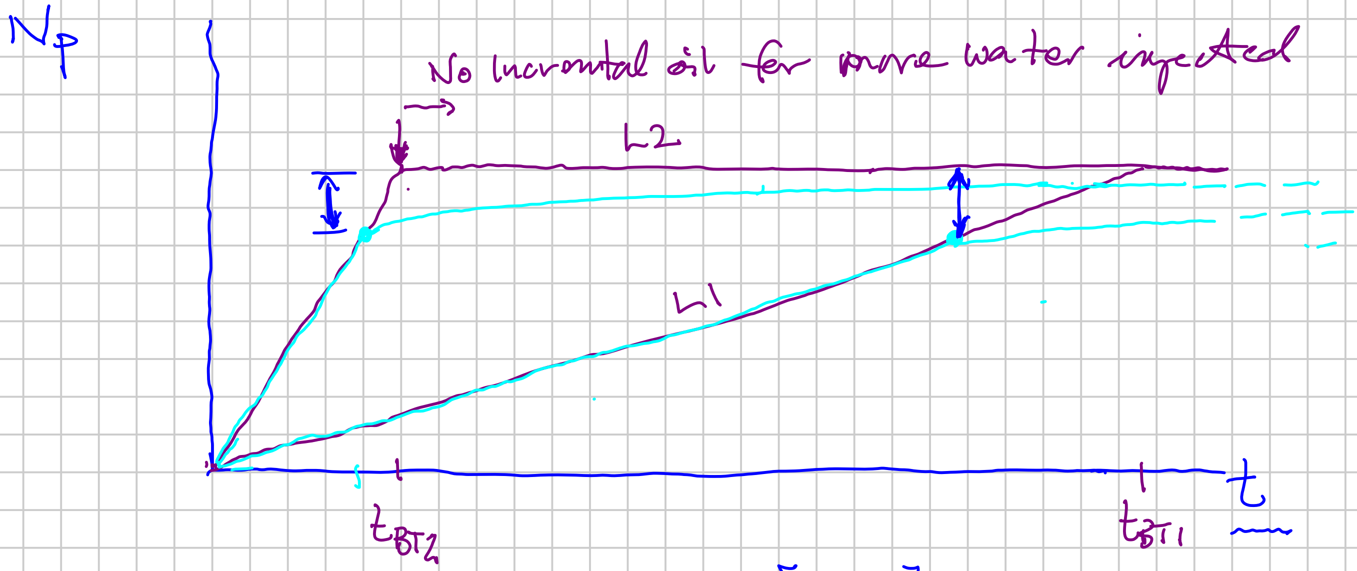
- Alternative : "Leaky Piston" (Muskat)





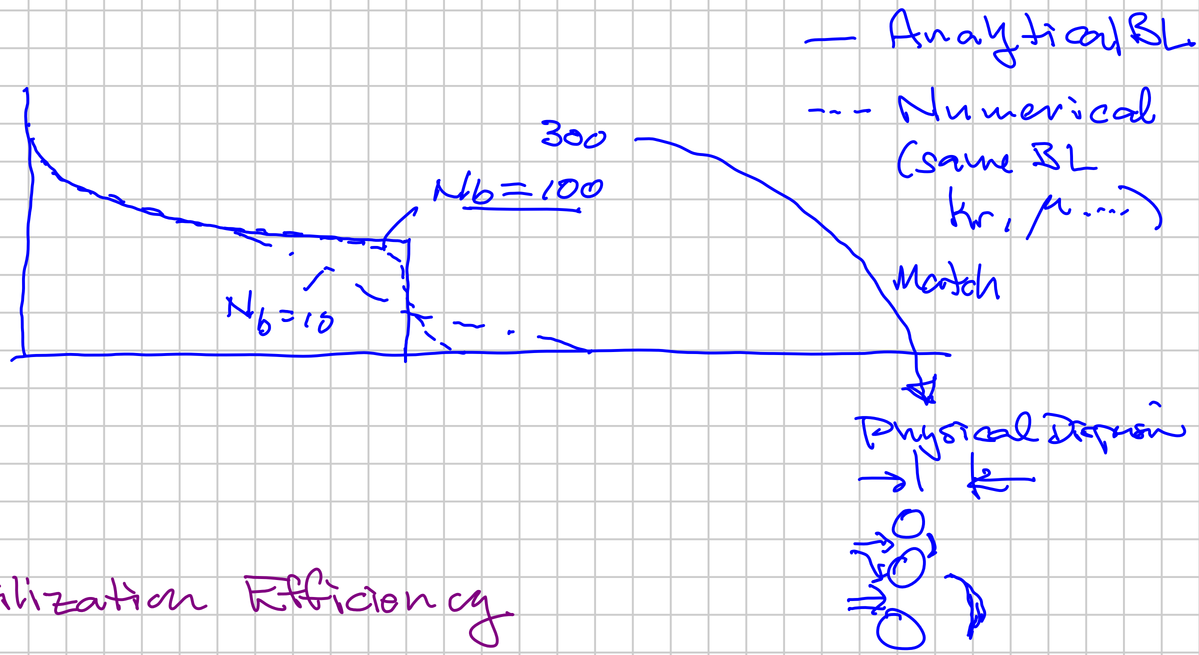
— Leaky Piston (Optimistic, RF_o & t_{BT})

— BL : $S_w \to 1 - S_{orw}$ requires "infinite" amount of water passing the pore



$[q_i(t)]_\ell$ for BL vs LP

Assuming the same water rate injected into L1 & L2 prior to BT



Injectant Utilization Efficiency

$$\frac{\$80/\text{bbl} \quad \Delta q_{o}}{\$10/\text{bbl} \quad \Delta q_{iw}} = \underbrace{\text{constant (max)}}_{\text{before BT i.e. RFW}} \sim 1 \longrightarrow \underset{\text{BL}}{0.x} \longrightarrow \underset{\text{Leaky Piston}}{0}$$

Oil Stripping After BT

Standing Fraction Flow Notes

① Fractional Flow Eqs.

② Frontal Advance Eq. (BL) $S_w(x, t)$

$\downarrow$

PVI

"dimensionless time"

③ Average Saturation ($\bar{S}_w \Rightarrow \bar{S}_o$)
Behind Front

④ Injection Volume (PVI | V_{iD}) - Recovery $\frac{N_p}{N}$
Relationship

Note Title 10/30/2018

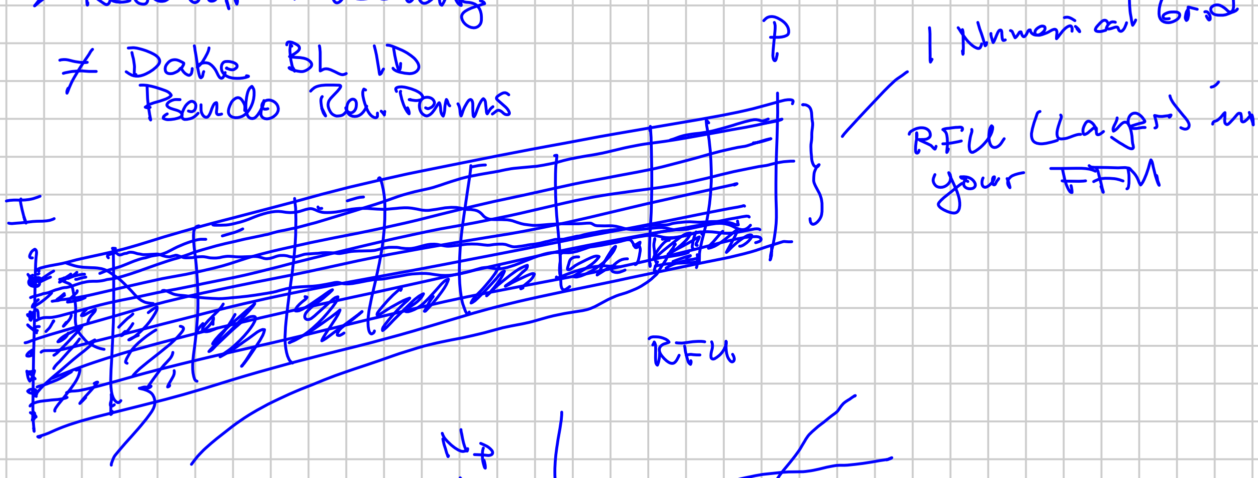
Previous Unresolved Discussion Topics
Related to Applicability of BL Theory

① Residual Oil Saturation to Water Displacement in the Presence of Initial Gas Saturation

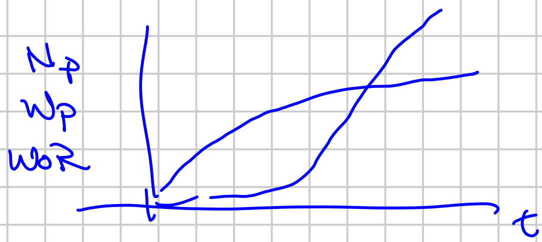
② When 1D BL theory "breaks down" because of gravitational segregation – and its impact on swept volume recovery, E_p

⇒ Reservoir Modeling

≠ Dake BL 1D Pseudo Rel. Perms

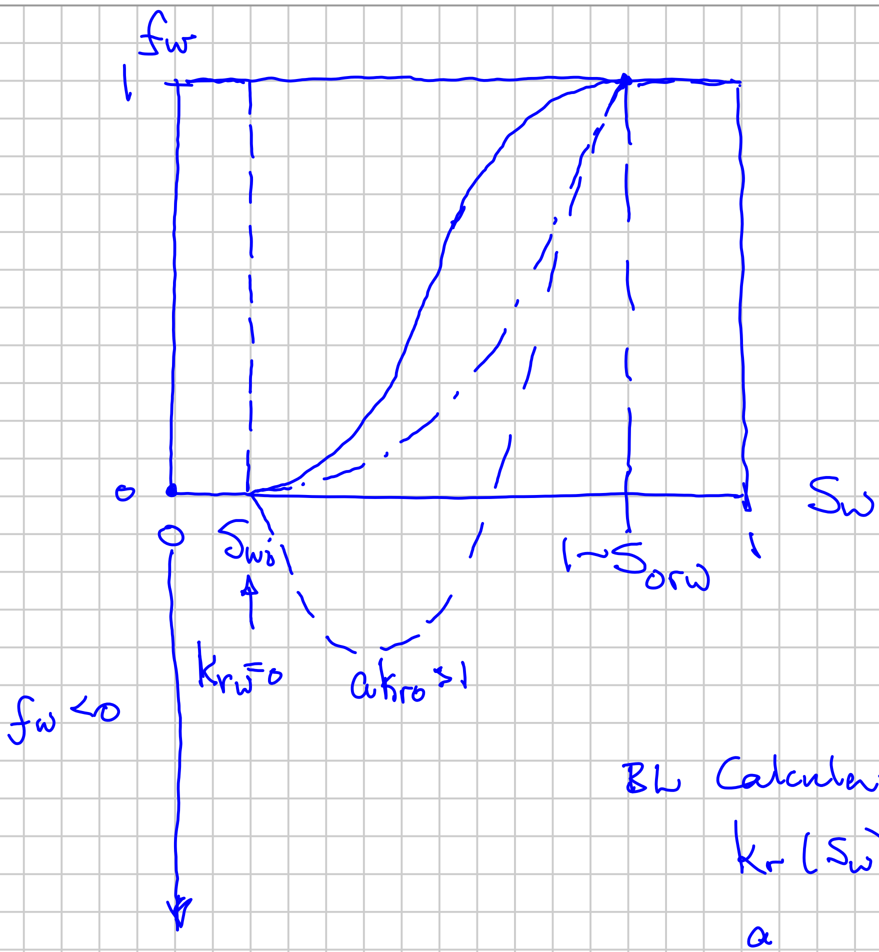


RFU



HAPPY HALLOWEEN

Note Title 10/31/2018



— H

-- $a>0$

In this example
--- $(a>0) \Rightarrow$
Leaky Piston
No stripping

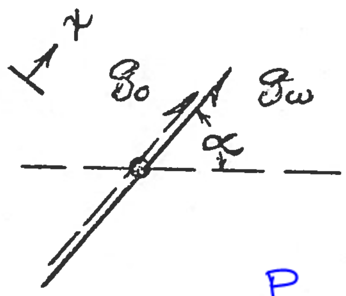
BL Calculations:

$$\left.\begin{array}{l} k_r(S_w) \\ a \\ \mu_w/\mu_o \end{array}\right\} f_w(S_w)$$

$S_w(x,t)$: Frontal Advance Eq.

Rate $q = \frac{q_R}{A_\perp} = v_D$ in Standing notes

FRACTIONAL FLOW EQUATION, I



P_c

$$q_o = -\frac{k_o}{\mu_o}\left[\frac{\partial p_o}{\partial x} + \rho_o g \sin\alpha\right]$$

$$q_w = -\frac{k_w}{\mu_w}\left[\frac{\partial p_w}{\partial x} + \rho_w g \sin\alpha\right]$$

Darcy 1D
H | I | V
$0 \le \alpha \le 90$

TOTAL FLOW, $q_t = q_o + q_w$

CAPILLARY PRESSURE, $P_c = P_o - P_w$

$P_o \neq P_w$

$$\frac{\partial P_c}{\partial x} = \frac{\partial P_o}{\partial x} - \frac{\partial P_w}{\partial x}$$

$$\frac{\partial P_c}{\partial x} = -\frac{(q - q_w)\mu_o}{k_o} - \rho_o g \sin\alpha + \frac{q_w \mu_w}{k_w} + \rho_w g \sin\alpha$$

LET $\Delta\rho_{wo} = \rho_w - \rho_o$

$$q_w\left[\frac{\mu_o}{k_o} + \frac{\mu_w}{k_w}\right] = \frac{q\mu_o}{k_o} + \frac{\partial P_c}{\partial x} - \Delta\rho\, g \sin\alpha$$

FRACTIONAL FLOW EQUATION, II

DIVIDING BY $\frac{q\mu_o}{k_o}$;

$$\frac{q_w}{q}\left[1+\frac{k_o}{\mu_o}\cdot\frac{\mu_w}{k_w}\right]=1+\frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right]$$

$$\frac{v_{wD}}{v_{wD}+v_{oD}}$$

$$\boxed{\frac{q_w}{q}=f_w}=\frac{1+\frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right]}{1+\frac{k_o}{k_w}\cdot\frac{\mu_w}{\mu_o}}$$

AS $\boxed{k_o = k\cdot k_{ro}}$ AND IGNORING $\frac{\partial P_c}{\partial x}$ ~~BECAUSE~~ SMALL

$$\boxed{f_w=\frac{1-ak_{ro}}{1+\frac{k_{ro}}{k_{rw}}\cdot\frac{\mu_w}{\mu_o}}}$$

WHERE $k_r(S_w)$ | $\frac{\mu_w}{\mu_o}$ = constant

$\Delta\rho$ = constant

$$a=\frac{0.488\,k\,\Delta\rho\sin\alpha}{q\mu_o}$$

IF $\Delta\rho$ = gm/cc

k = DARCY

q = BBL/DAY/FT^2

μ_o = cp

μ_o = constant

k = constant

$a\left(\frac{1}{q}\right)$

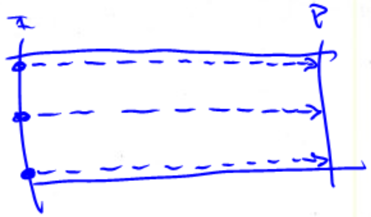
$a(q)$

12.

H: $\alpha=0 \Rightarrow a=0$

q = constant (normal assumption)

$q > q^*$



$\alpha = 0$

FRACTIONAL FLOW EQUATION, III

$$a = \frac{\boxed{7.84(10^{-6})}\, k\, \Delta\rho \sin\alpha}{q\, \mu_o}$$

IF $\Delta\rho = \dfrac{\text{lbs}}{\text{ft}^3}$, k = md., q = BBL/DAY/FT^2, μ_o = cp. — SPE Field Units

No rate dependance

"SPE Metric"

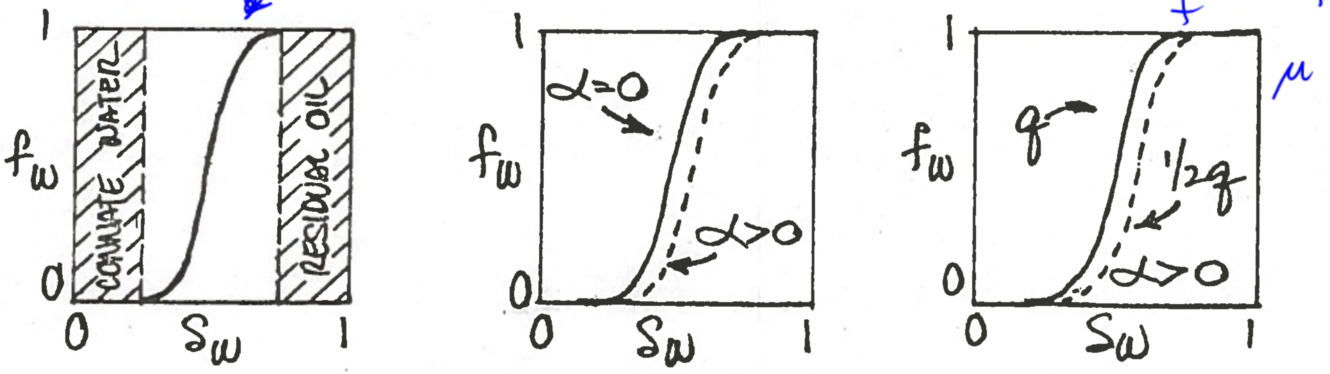
$\Delta\rho$	kg/m^3
k	md
q	m^3/d/m^2 = m/d
μ	cp ≡ mPa·s

BL ff Eq.

$f_w(S_w, \underbrace{q > q^*}_{BL \checkmark})$

WHEN $\alpha = 0$ (H):

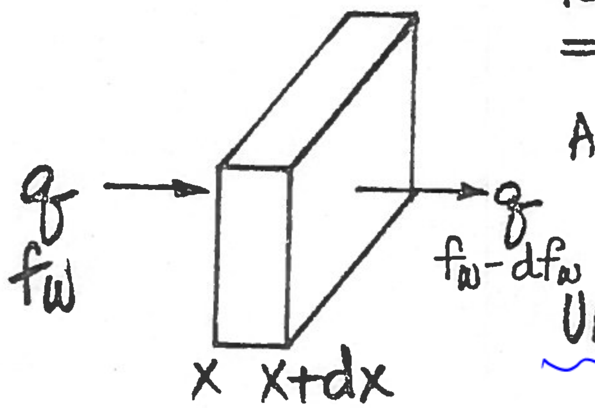
$$\boxed{f_w = \frac{1}{1 + \dfrac{k_{ro}}{k_{rw}} \cdot \dfrac{\mu_w}{\mu_o}}}$$



H: $\alpha = 0$

$\alpha > 0$ q $1/2\, q$

FRONTAL ADVANCE EQUATION, I



RATE OF WATER ACCUMULATION
= WATER IN MINUS WATER OUT

$$\text{ACCUM. RATE} = q f_w - q(f_w - df_w)$$
$$= q\, df_w$$

$$\text{UNIT PORE VOL.} = \frac{A \cdot dx \cdot \phi}{5.615}$$

RATE OF WATER SATURATION CHANGE, $\frac{dS_w}{dt}$, IS RATE OF WATER ACCUMULATION DIVIDED BY PORE VOLUME.

$$\frac{dS_w}{dt} = \frac{q\, df_w}{A\, dx\, \phi} \cdot 5.615$$

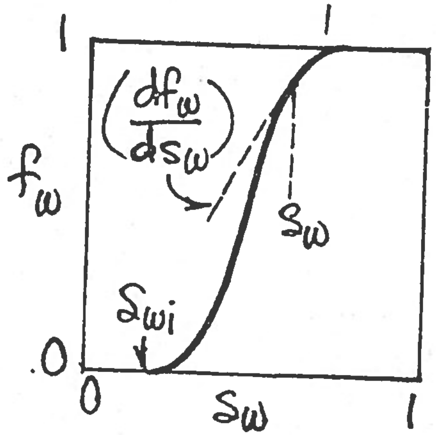
$$dx = \frac{5.615\, q}{A\phi} \left(\frac{df_w}{dS_w}\right) dt$$

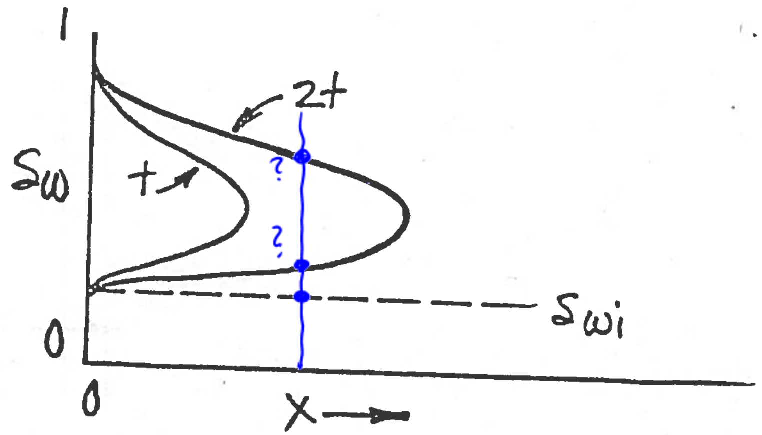
$$\int_{x_0}^{x_{S_w}} dx = 5.615 \cdot \frac{q}{A\phi} \left(\frac{df_w}{dS_w}\right) \int_{t=0}^{t} dt$$

FRONTAL ADVANCE EQUATION, II

$$(x)_{s_w} - X_o = \frac{5.615}{A\phi}\, q t \underbrace{\left(\frac{df_w}{ds_w}\right)}$$

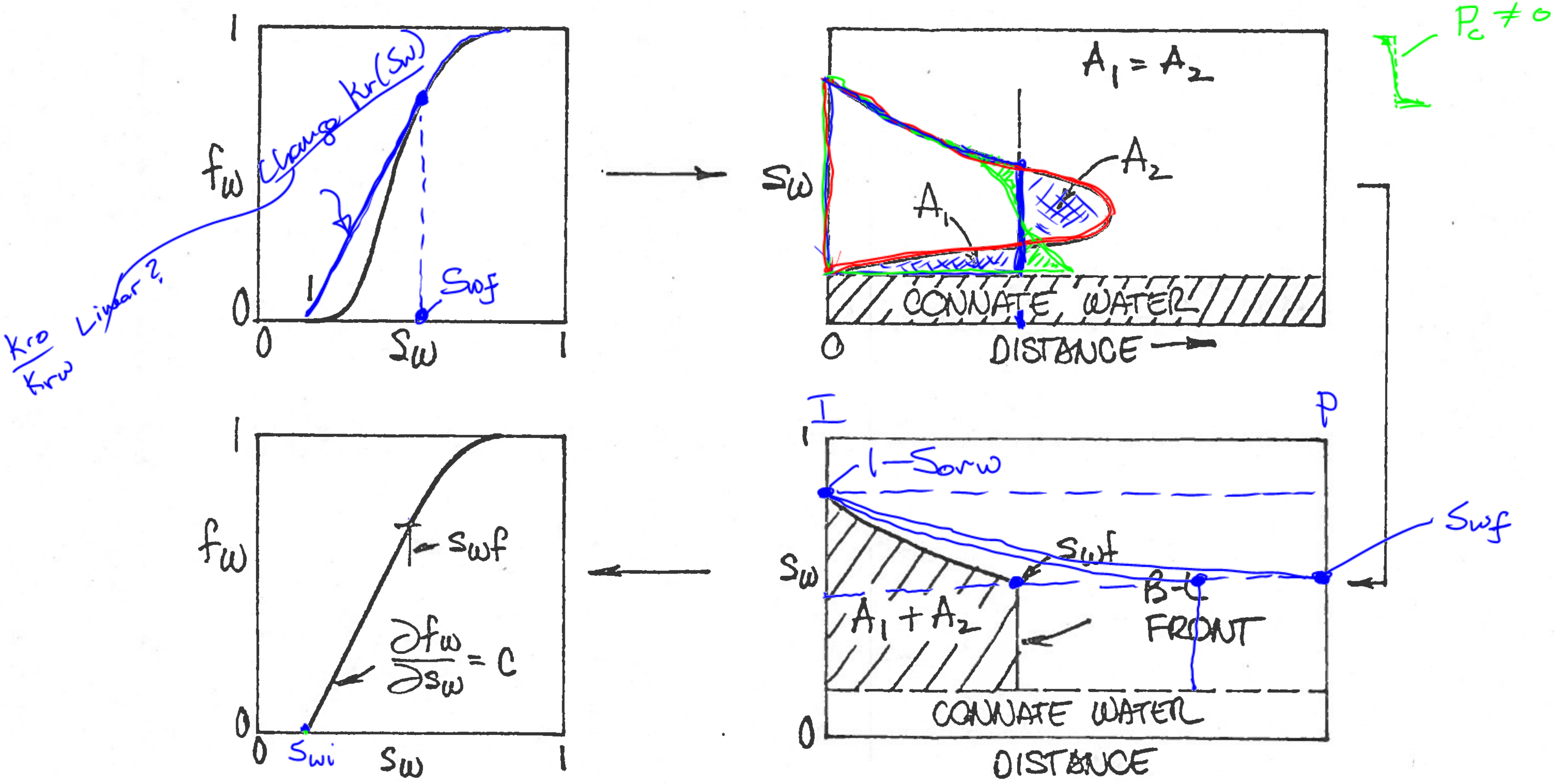
THIS IS THE BUCKLEY-LEVERETT FRONTAL ADVANCE EQUATION. IT GIVES THE DISTANCE, $[(x)_{s_w} - X_o]$, THAT A GIVEN SATURATION, s_w, MOVES IN TIME t.





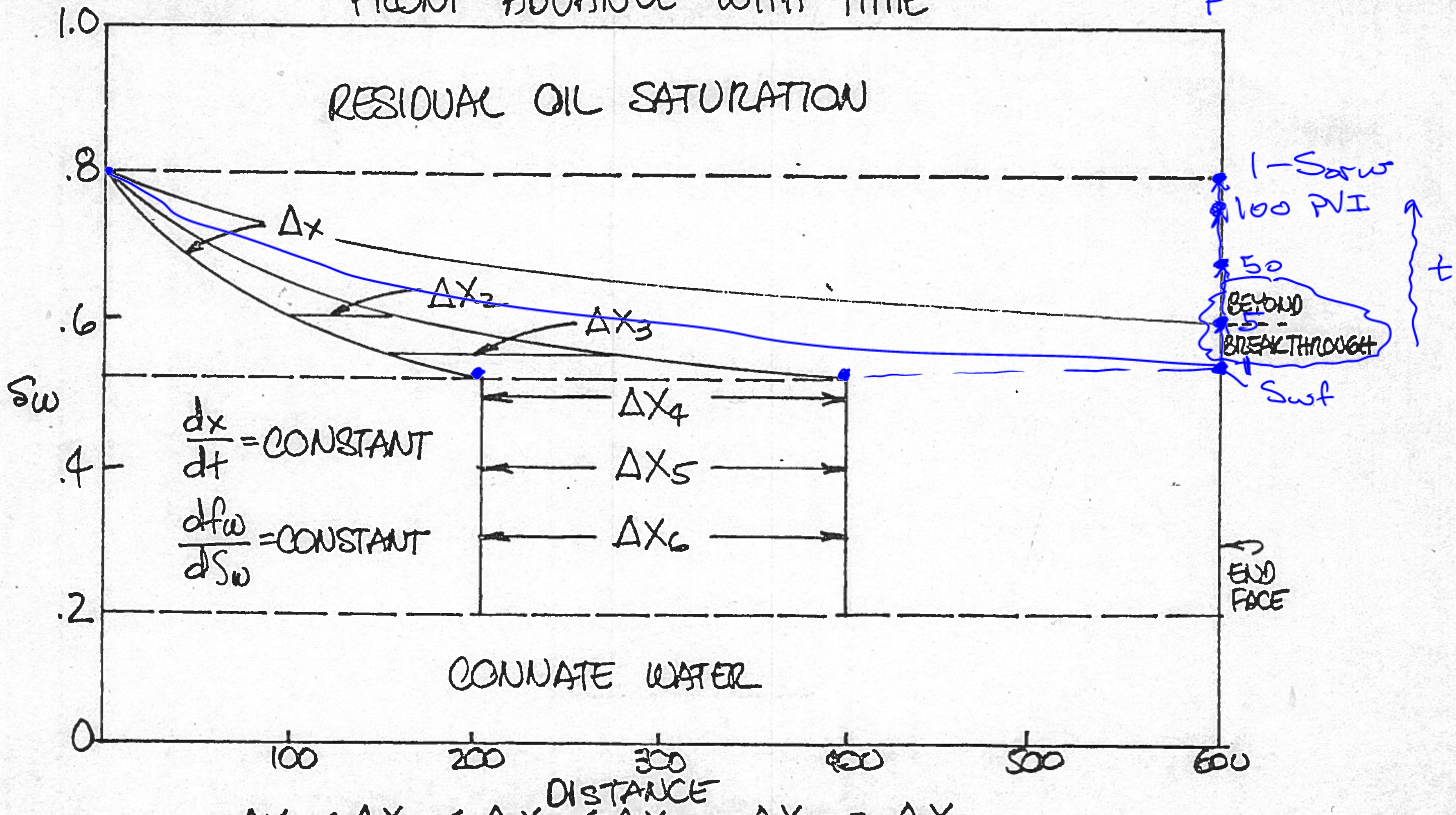
FRONTAL ADVANCE EQUATION, III

MODIFICATION OF B-L RELATIONSHIP TO AVOID TRIPLE SATURATION VALUES.



FRONTAL ADVANCE EQUATION, IV

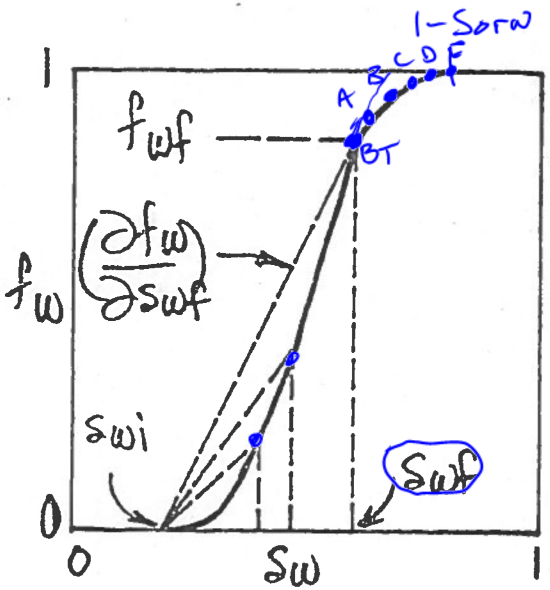
FRONT ADVANCE WITH TIME



$$\Delta X_1 < \Delta X_2 < \Delta X_3 < \Delta X_4 = \Delta X_5 = \Delta X_6$$

FRONTAL ADVANCE EQUATION V

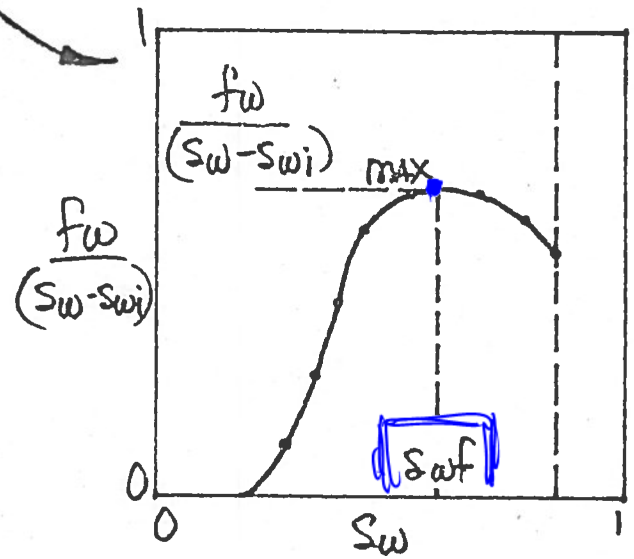
DETERMINATION OF SATURATION AND FRACTIONAL FLOW AT THE FRONT.



ALSO CAN BE OBTAINED GRAPHICALLY

CALCULATE $\frac{f_w}{(S_w - S_{wi})}$ AND PLOT AGAINST S_w.

FRONT CONDITIONS ARE WHERE THIS HAS THE MAXIMUM VALUE.

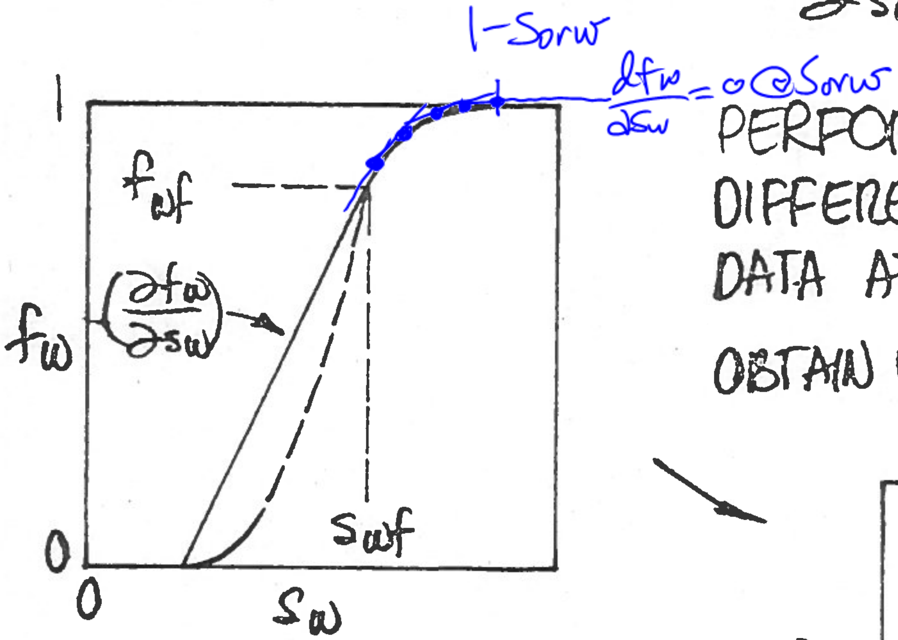


Define inflection Point ⇒ S_{wf}

18.

FRONTAL ADVANCE EQUATION VI

DETERMINATION OF $\frac{\partial f_w}{\partial s_w}$



PERFORM GRAPHICAL DIFFERENTIATION OF $f_w - s_w$ DATA AT $S_w > S_{wf}$ · to OBTAIN VALUES OF $\frac{\partial f_w}{\partial s_w}$ VS s_w

$(S_w)_p = f(PVI)$

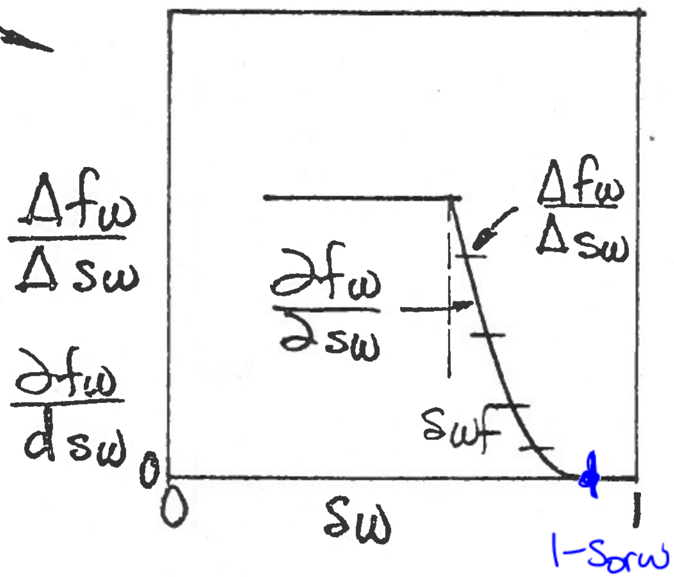
or

$$\boxed{(\bar{S}_w)_{I-P} = f(PVI)}$$

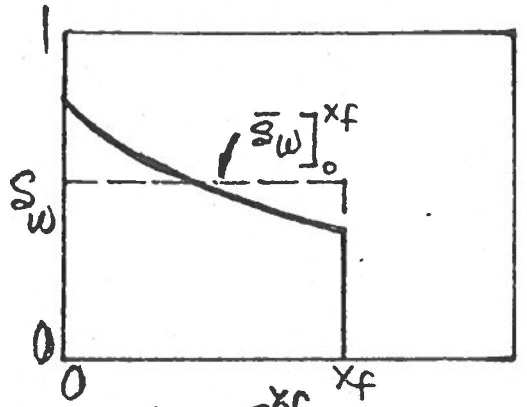
$\Downarrow$

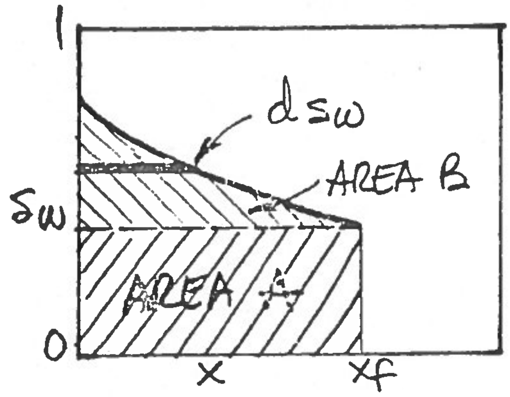
$RF_o\ (PVI)_{I-P}$

AND



AVERAGE SATURATION BEHIND FRONT - I





$$\bar{S}_w\Big]_{0}^{x_f} = \frac{\int_0^{x_f} S_w \, dx}{x_f} = \frac{\text{AREA A} + \text{AREA B}}{x_f} \qquad \mathrm{I}$$

FROM BUCKLEY-LEVERETT

$$x_f = \frac{5.615}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_f \int_0^t q \, dt$$

$$= \frac{5.615 \; Q}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_f$$

$$\text{AREA A} = S_{wf} \cdot x_f$$

$$\text{AREA B} = \int_{S_{wf}}^{1} x \, dS_w$$

$$\text{AS} \quad x = \frac{5.615 Q}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_x$$

$$\text{AREA B} = \frac{5.615 \; Q}{A\phi}\int_{S_{wf}}^{1}\frac{df_w}{dS_w} \cdot dS_w$$

AVERAGE SATURATION BEHIND FRONT, II

$$\text{AREA } B = \frac{5.615\,Q}{A\phi} \int_{f_w @ s_{wf}}^{f_w @ s_w=1} df_w$$

$$= \frac{5.615\,Q}{A\phi}\left[(f_w @ s_w=1) - (f_w @ s_{wf})\right]$$

BUT: $(f_w @ s_w=1) = 1$; $(f_w @ s_{wf}) = f_{wf}$

THEREFORE

$$\text{AREA } B = \frac{5.615\,Q}{A\phi}\left[1 - f_{wf}\right]$$

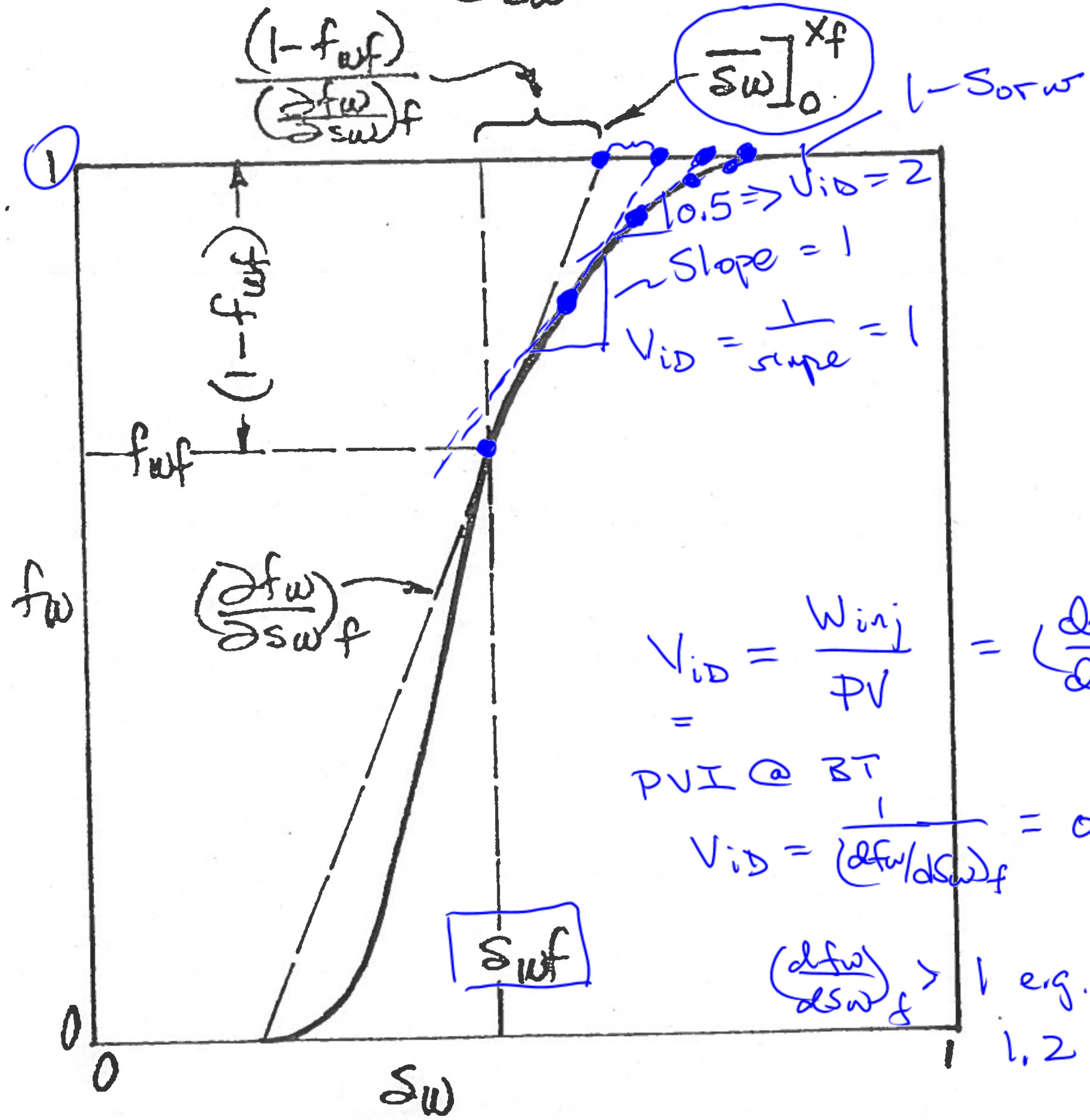
$$\text{AS } \left.\bar{S}_w\right]_0^{x_f} = \frac{\text{AREA } A + \text{AREA } B}{x_f}$$

$$\left.\bar{S}_w\right]_0^{x_f} = \frac{\dfrac{5.615\,Q}{A\phi}\left[S_{wf}\left(\dfrac{\partial f_w}{\partial s_w}\right)_f + (1 - f_{wf})\right]}{\dfrac{5.615\,Q}{A\phi}\left(\dfrac{\partial f_w}{\partial s_w}\right)_f}$$

$$\boxed{\left.\bar{S}_w\right]_0^{x_f} = S_{wf} + \frac{(1 - f_{wf})}{\left(\dfrac{\partial f_w}{\partial s_w}\right)_f}}$$

AVERAGE SATURATION BEHIND FRONT III

$$\bar{S}_w\Big]_0^{x_f} = S_{wf} + \frac{(1 - f_{wf})}{\left(\frac{\partial f_w}{\partial S_w}\right)_f}$$



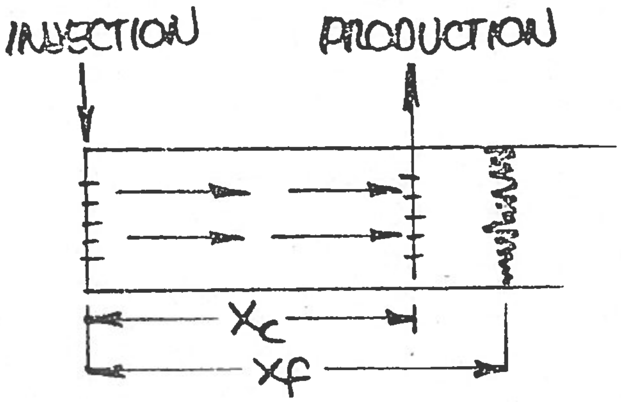
$V_{iD} = \frac{W_{inj}}{PV} = \left(\frac{dfw}{dS_w}\right)^{-1}$

= PVI @ BT

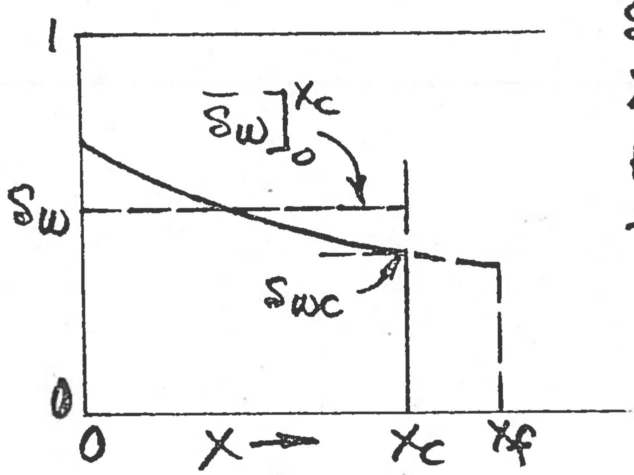
$V_{iD} = \frac{1}{(dfw/dS_w)_f} = 0.8$

$\left(\frac{dfw}{dS_w}\right)_f > 1$ e.g. 1.2

AVERAGE SATURATION BEHIND FRONT - IV



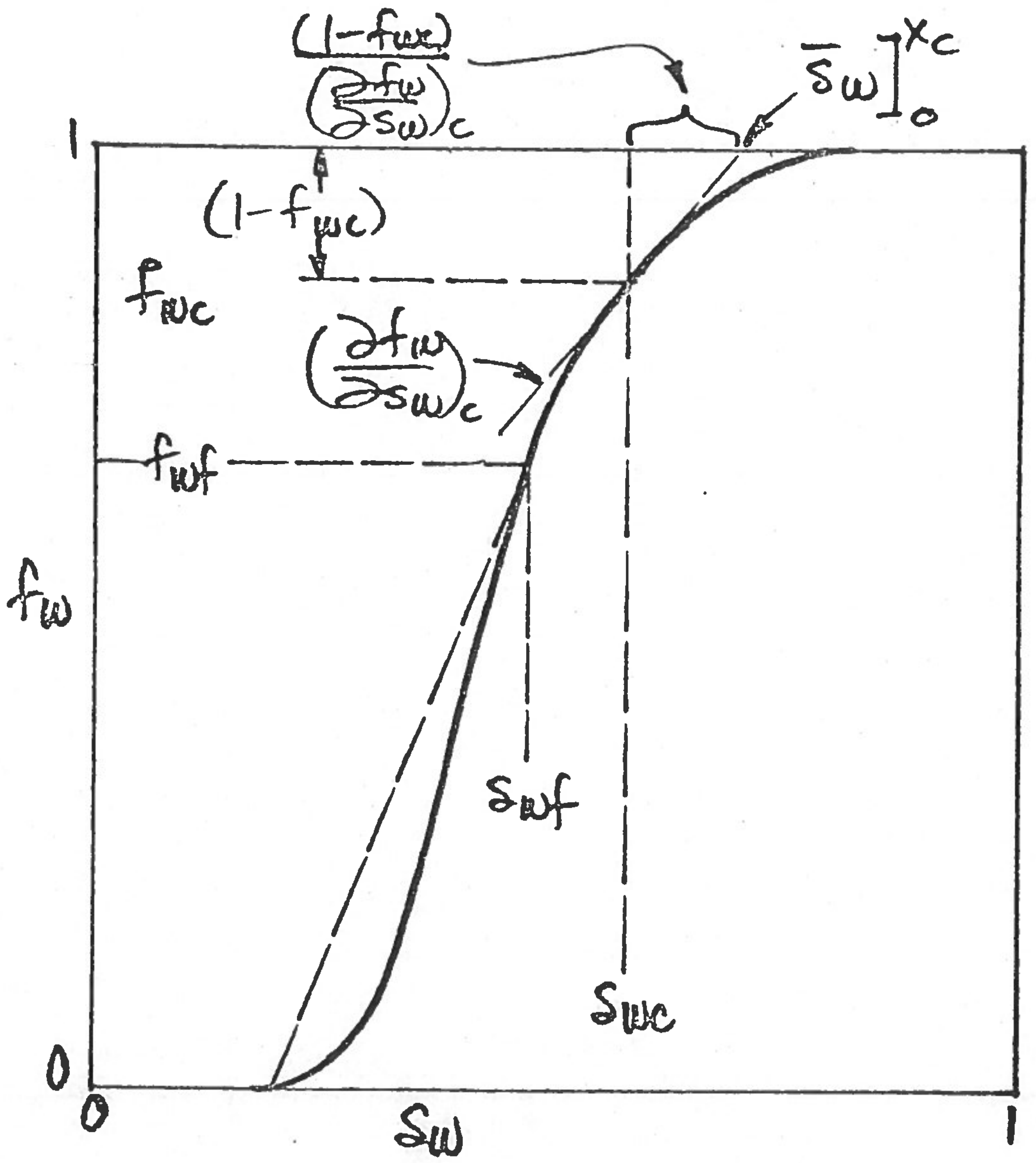
WHEN THE FRONT LINE HAS PASSED POSITION x_c, THE AVERAGE WATER SATURATION BEHIND x_c CAN BE DEVELOPED BY A SIMILAR APPROACH. THE FINAL EQUATION IS



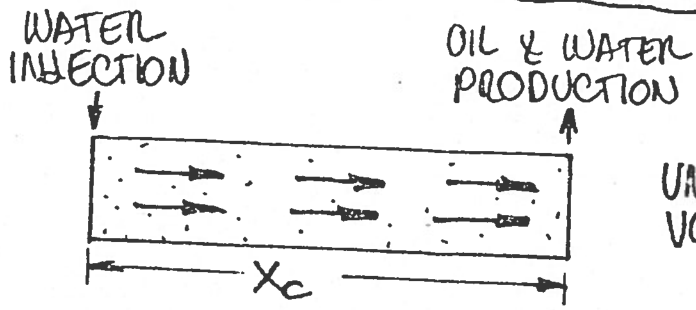
$$\bar{S}_w\Big]_0^{x_c} = S_{wc} + \frac{(1 - f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$

AVERAGE SATURATION BEHIND FRONT - II

$$\bar{S}_w \Big]_0^{x_c} = S_{wc} + \frac{(1 - f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$



INJECTION VOLUME VS. RECOVERY RELATIONS, I



UNIT PORE VOLUME, $V_p = \dfrac{A \phi x_c}{5.615}$

DIMENSIONLESS PORE VOLUMES INJECTED

$$V_{iD} = \frac{W_i}{V_p} = \frac{W_i}{\frac{A\phi x_c}{5.615}} = \frac{5.615\, W_i}{A \phi x_c}$$

FROM BUCKLEY - LEVERETT

$$x_c = \frac{5.615\, W_i}{A\phi} \left(\frac{\partial f_w}{\partial s_w}\right)_c$$

OR

$$\frac{5.615\, W_i}{A \phi x_c} = 1 \Big/ \left(\frac{\partial f_w}{\partial s_w}\right)_c$$

THEREFORE THE PORE VOLUMES INJECTED TO REACH A GIVEN WATER SATURATION AT THE OUTFLOW FACE, S_{wc} IS,

$$\boxed{V_{iD} = 1 \Big/ \left(\frac{\partial f_w}{\partial s_w}\right)_c}$$

$\Rightarrow (\bar{S}_w)$ I-P @ "c"

INJECTION VOLUME VS. RECOVERY RELATIONS II

RESERVOIR OIL DISPLACED $= V_p (\Delta \bar{S}_w)_o^{x_c}$

$$= \frac{A \phi x_c}{5.615} \left[(\bar{S}_w)_o^{x_c} - S_{wi} \right]$$

STOCK TANK OIL PRODUCED $= \dfrac{\text{RES. OIL DISP.}}{B_o}$

$$N_p = \frac{A \phi x_c}{5.615 B_o} \left[(\bar{S}_w)_o^{x_c} - S_{wi} \right]$$

FLOWING WATER-OIL RATIO AT OUTFLOW FACE;

$$WOR = \frac{f_{wc}}{1 - f_{wc}}$$

SURFACE PRODUCING WATER-OIL RATIO,

$$F_{wo} = \frac{f_{wc} \cdot B_o}{(1 - f_{wc}) B_w}$$

SURFACE WATER CUT $= \dfrac{\dfrac{f_{wc}}{B_w}}{\dfrac{f_{wc}}{B_w} + \dfrac{(1 - f_{wc})}{B_o}} = \dfrac{1}{1 + F_{wo}}$

TIME, $t = \dfrac{W_i}{i_w}$

$$= \frac{V_{iD} \cdot A \phi x_c}{5.615 i_w} \text{ (DAYS)}$$

WHERE i_w IS BBL PER DAY