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A Generalized Approach to Reservoir Material Balance Calculations

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Abstract

This work completes the search for a generalized material balance equation. No restrictions are placed on reservoir fluid compositions. Unlike the conventional material balance equation, the new equation specifically accounts for volatilized-oil and, therefore, is uniquely applicable to the full range of fluids, including volatile-oils and gas-condensates. Equally important, the new material balance equation retains the same simplicity with which the conventional material balance equation has become known. The new, generalized material balance equation is featured here by showing how it is used to predict reservoir performance and estimate reserves in example volatile-oil and gas-condensate reservoirs. In comparison, the conventional material balance equation is shown to lead to erroneous results. The new, generalized material balance equation leads to an improved method of reservoir performance analysis.

Introduction

Material balance calculations are a useful method of reservoir performance analysis. They are routinely used to estimate oil and gas reserves and predict future reservoir performance. Schilthuis[1], in 1936, was among the first to formulate and apply material balances. Inherent in his and others' use of material balances were the following assumptions: (1) at most, there are two hydrocarbon phases: oil and gas, (2) at most, there are two hydrocarbon components: stock-tank oil and separator (surface) gas, (3) the reservoir oil-phase consists of stock-tank oil and separator-gas, (4) the reservoir gas-phase consists of only separator-gas and no stock-tank oil, and (5) there are no compositional gradients within the system. Assumptions 1 and 2 define the popular two-hydrocarbon-component formulation. Assumption 3 effectively accounts for the effects of dissolved- or solution-gas. Assumption 4 ignores the possibility of volatilized-oil and, therefore, restricts application to black-oils and dry-gases and precludes application to volatile-oils, gas-condensates, or wet-gases. Volatilized-oil is the stock-tank oil content of the free reservoir gas-phase. Assumption 5 means that the system is effectively treated as a tank with no gradients; accordingly, Coats[2] and others have sometimes referred to this type of model as a tank or zero-dimensional model. Despite their apparent oversimplicity, early tank models found widespread use in reservoir engineering applications[3-16].

As time progressed, more sophisticated material balance models evolved, each striving for greater generality. Eventually, following the advent of digital computers, the material balance equations were discretized and the first generation of multi-dimensional, black-oil, finite-difference reservoir simulators were developed in the 1960's[17-19]. These models, like their zero-dimensional counterparts, could effectively simulate black-oil and dry-gas reservoirs but could not model volatile-oil and gas-condensate reservoirs. To address this limitation, Cook et al.[20] in 1974 introduced a volatile-oil, finite-difference simulator. Their formulation retained the simplicity of two hydrocarbon components, but allowed for volatilized-oil. Consequently, their formulation was applicable to the full range of reservoir fluids. The resulting formulation was applicable to the full range of reservoir fluids. Coats[21] later, in 1986, verified the applicability of the black- and volatile-oil approaches in reservoir simulation by showing agreement with a fully-compositional, equation-of-state (EOS) reservoir simulator.

Though finite-difference reservoir simulators have evolved from black- to volatile-oil models, zero-dimensional material balance models have not made commensurate advancements. Application of two-hydrocarbon-component, zero-dimensional material balance models is still restricted to black-oil or dry-gas reservoirs. As volatile-oil and gas-condensate reservoir exploitation increases, there is an increased need to address this limitation. This work addresses this limitation and this paper is the first to develop and apply a generalized material balance equation to treat the full spectrum of reservoir fluids. This new approach is more general than previous approaches[1,3-16], is simpler than alternative approaches[22-27], and leads to a deeper and more unified understanding of reservoir performance.

Mathematical Development

As derived in Appendix A, the generalized material balance equation ignoring the effects of water influx is

$$N\left[B_{oi}\left(1 - R_v R_s\right) - \left(B_g - R_v B_o\right) R_{si} - \left(B_o - R_s B_g\right)\right] + N_p\left[B_o\left(1 - R_v R_{ps}\right) + \left(R_{ps} - R_s\right) B_g\right] = 0 \quad \text{.....(1)}$$

where N and N_p are the original oil in-place (OOIP) and produced oil (expressed in stock-tank m^3 or stb), respectively, and R_{ps} is the cumulative produced gas-oil ratio (expressed in std m^3/stock-tank m^3 or scf/stb). See the nomenclature for a definition of the remaining variables. The oil saturation as a function of the fractional oil recovery N_p/N is given by:

$$S_o = \left(1 - S_{wi}\right)\left[\frac{\left(1 - \frac{N_p}{N}\right) B_o B_g - B_{oi} R_v B_o}{B_{oi}\left(B_g - R_v B_o\right)}\right] \quad \text{.....(2)}$$

TABLE 1: Fluid property data for example gas-condensate.

P MPa	B_o res m^3 per stock-tank m^3	B_g res m^3 per 10^3 std m^3	R_s std m^3 per stock-tank m^3	R_v stock-tank m^3 per 10^6 std m^3	μ_o mPa-s	μ_g mPa-s
40 0	4.382	4.07	1077	928	0.0612	0.0612
38.3	4.441	4.12	1077	928	0.0620	0.0620
36.5	2.366	4.15	490	879	0.1338	0.0554
33.1	2.032	4.26	379	640	0.1826	0.0436
29.6	1.828	4.43	308	499	0.2354	0.0368
26.2	1.674	4.77	253	366	0.3001	0.0308
22.7	1.554	5.33	210	271	0.3764	0.0261
19.3	1.448	6.11	171	196	0.4781	0.0222
15.9	1.360	7.35	138	140	0.6041	0.0191
12.4	1.279	9.42	108	107	0.7746	0.0166
9.0	1.200	13.02	79	84	1.0295	0.0148
5.5	1.131	20.70	52	76	1.3580	0.0135

where S_o is expressed in fractional pore volume (PV) units. For comparison, the conventional material balance counterparts to Equations (1) and (2) are given by Equations (A-22) and (A-23) in Appendix A or can be found in any standard reservoir engineering textbook. Note that Equations (1) and (2) are valid if and only if the reservoir pressure is less than the saturation pressure. If the reservoir pressure is greater than or equal to the saturation pressure, these equations can be simplified and the resulting simplifications are given in Appendix A. Equation (2) ignores the effect of a shrinking oil-leg due to either a lowering gas-oil contact caused by gas-cap expansion or a rising water-oil contact caused by water influx. Though these and other more complicated cases have been successfully treated, I opt not to consider these cases here for the sake of brevity and simplicity

Key to this development is the use of the volatile oil-gas ratio R_v, expressed in units of stock-tank m^3/std m^3 or stb/scf. This variable effectively describes the amount of volatilized-oil in the reservoir gas-phase. It has been introduced and used by others[20,21,28]. Cook et al.[20] referred to R_v as the "liquid content of the gas". Coats[21] referred to it as the "oil vapour in the gas." This variable is distinctly different from, but analogous to, the dissolved gas-oil ratio, R_s. The volatile oil-gas ratio varies as a function of the reservoir fluid composition. It also is a strong function of the separator configuration which seeks to maximize liquid dropout. For heavy- and black-oils, the volatile oil-gas ratio at the saturation pressure typically ranges from 0-60 stock-tank m^3/10^6 std m^3 (~0-10 stb/mmscf); for volatile-oils, it ranges from 60-1,200 stock-tank m^3/10^6 std m^3 (~10-200 stb/mmscf); for near-critical fluids, it reaches maximum values and ranges from 900-2,400 stock-tank m^3/10^6 std m^3 (~150-400 stb/mmscf); for gas-condensates, it ranges from 300-1,500 stock-tank m^3/10^6 std m^3 (~50-250 stb/mmscf); for wet-gases, it ranges from 120-600 stock-tank m^3/10^6 std m^3 (~20-100 stb/mmscf); and for dry-gases, it approaches zero.

The generalized material balance equation, like the conventional (black-oil) material balance equation, has broad applicability This paper illustrates two of its uses: (1) predicting future reservoir performance and (2) estimating oil and gas reserves.

Reservoir Performance Predictions

An important but less common application of material balance calculations is to predict future reservoir performance, e.g., oil and gas recoveries and producing GOR's as a function of time or reservoir depletion pressure. This application requires knowing the fluid properties B_o, B_g, R_s, and R_v and the phase viscosities μ_o and μ_g as a function of reservoir pressure and knowing the gas-oil relative permeability relationships. This type of application using the conventional (black-oil) material balance has been presented before[6,7,11,12]. Application using the generalized material balance differs only slightly.

The instantaneous producing GOR, R, is related to the oil saturation by

$$R = \frac{\dfrac{B_o k_{rg}(S_o)\mu_o}{B_g k_{ro}(S_o)\mu_g} + R_s}{\dfrac{B_o k_{rg}(S_o)\mu_o}{B_g k_{ro}(S_o)\mu_g} R_v + 1} \qquad (3)$$

where $k_{rg}(S_o)$ and $k_{ro}(S_o)$ are the gas and oil relative permeabilities

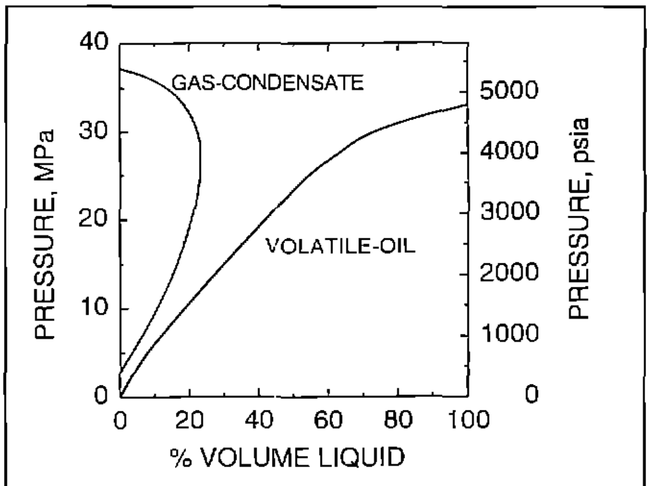

FIGURE 1: Constant composition expansions.

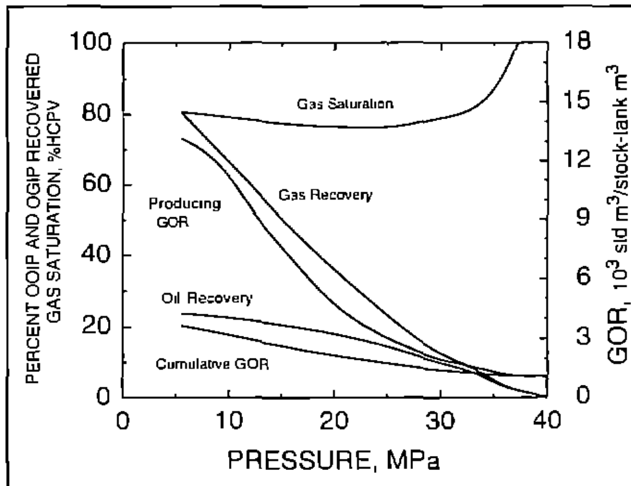

FIGURE 2: Gas-condensate performance predictions.

TABLE 2: Reservoir material balance calculations for example gas-condensate.

	PERFORMANCE PREDICTIONS					OOIP ESTIMATES	
Pressure, MPa	Oil Recovery % OOIP	Gas Recovery % OGIP	Producing GOR, R, 10^3 std m^3 per stock-tank m^3	Cumulative Producing GOR, R_{ps}, 10^3 std m^3 per stock-tank m^3	Gas Saturation, %HCPV	Eq. A-22 Estimated OOIP/Actual OOIP	Eq. 1 Estimated OOIP/Actual OOIP
40.0	0.0	0.0	1.08	1.08	100.0		
38.3	1.3	1.3	1.08	1.08	100.0	1.00	1.00
36.5	2.6	2.7	1.14	1.09	95.0	0.31	1.00
33.1	7.0	8.1	1.56	1.25	81.8	0.64	1.00
29.6	10.1	13.1	2.00	1.41	78.7	0.79	1.00
26.2	13.3	20.2	2.73	1.64	76.7	0.90	1.00
22.7	16.2	28.7	3.68	1.91	76.3	0.96	1.00
19.3	18.4	37.8	5.07	2.21	76.6	0.98	1.00
15.9	20.2	47.8	7.10	2.55	77.2	0.99	1.00
12.4	21.6	58.7	9.34	2.92	78.3	1.00	1.00
9.0	22.8	69.7	11.85	3.30	79.5	1.00	1.00
5.5	23.7	80.5	13.19	3.66	80.6	1.00	1.00

which are a function of the oil saturation. Equation (3) is derived in Appendix A. The producing GOR is expressed in surface units (std m^3/stock-tank m^3 or scf/stb) and is defined by

$$R = \frac{dG_p}{dN_p} \quad \text{.....(4)}$$

where G_p is the std m^3 or scf of cumulative produced gas and dG_p is differential change in the cumulative produced gas. The fractional gas recovery G_p/G is related to the fractional oil recovery by

$$\frac{G_p}{G} = \left(\frac{N_p}{N}\right) R_{ps} \left(\frac{N}{G}\right) \quad \text{.....(5)}$$

where G is the OGIP expressed in std m^3 or scf. If the differentials in Equation (4) are approximated by finite-differences, then Equations (1)-(5) represent a system of five equations and five unknowns (N_p, G_p, R_{ps}, R, and S_o) which can be solved iteratively. This system of equations is solved using Tarner's[11] solution procedure. Application of the generalized material balance is illustrated by presenting performance predictions for an example gas-condensate and volatile-oil.

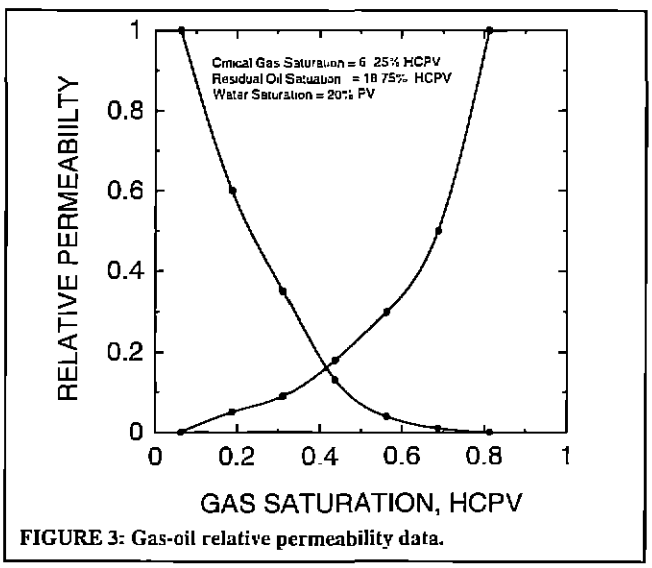

FIGURE 3: Gas-oil relative permeability data.

Gas-condensate Example

Table 1 shows the fluid properties as a function of pressure for a gas-condensate.* This data closely simulates a Western Overthrust Belt rich gas-condensate[29,30]. The fluid properties for this fluid were generated by developing a Redlich-Kwong equation-of-state (EOS) description for the reservoir fluid and then carrying out numerical experiments. The initial reservoir pressure is 40.0 MPa (5,800 psia) and temperature is 102°C (215°F). The dew point of the fluid is approximately 37.7 MPa (5,400 psia). Figure 1 shows the EOS-predicted constant composition expansion (CCE) for this fluid at 102°C (215°F). The CCE reveals retrograde condensation and shows a maximum 25% liquid volume at about 26.5 MPa (3,800 psia). The data in Table 1 were generated by carrying out a numerical differential vapourization experiment. The fluid properties derived from the differential vapourization experiment were then adjusted for the effects of multi-stage separation by carrying out numerical multi-stage separation experiments to optimize separator pressure. The initial producing GOR is 1,269 std m^3/stock-tank m^3 (7,117 scf/stb) using single-stage separators and 1,077 std m^3/stock-tank m^3 (6,042 scf/stb) using two-stage separation with a high pressure separator set at 4.1 MPa (600 psia). Notice that the oil-phase properties B_o, R_s and μ_o are identical or equivalent to the gas-phase properties B_g, R_v and μ_g at pressures greater than the dew point pressure. This is because a single hydrocarbon phase condition exists at these pressures. At these pressures B_o and R_s are related to B_g and R_v by the following equalities: $B_o = B_g/R_v$ and $R_s = 1/R_v$.

Figure 2 and Table 2 summarize the generalized material balance predictions. The results are given in terms of the oil and gas recoveries, instantaneous producing GOR, cumulative producing GOR, and gas saturation as a function of reservoir pressure. The gas saturation is expressed as a per cent of the hydrocarbon pore volume (HCPV). Given the necessary reservoir parameters, these production results can easily be converted to oil and gas rates as a function of time; however, for the sake of brevity, these results are purposely omitted. The calculations assume the gas-oil relative permeability in Figure 3 apply. The material balance equations predict a final oil recovery of about 24% and a final gas recovery of about 80% at the abandonment pressure of 5.5 MPa (800 psia). The initial producing GOR starts at 1,077 std m^3/stock-tank m^3 (6,042 scf/stb) and monotonically increases to about 13,191 std m^3/stock-tank m^3 (74,000 scf/stb). The gas saturation is initially 100% HCPV and, once below the saturation pressure, it decreases to a minimum of 76% HCPV at 22.7 MPa (3,300 psia) and then increases slightly to about 81% HCPV at abandonment. This saturation history reflects the retrograde condensation noted

* The reader is referred to the original preprint for a presentation of the figures and tables in Imperial units. See Paper No. CIM/AOSTRA 93-05, presented at the 44th Annual Technical Meeting of The Petroleum Society of CIM, Alberta, Canada, May 9-12, 1993.

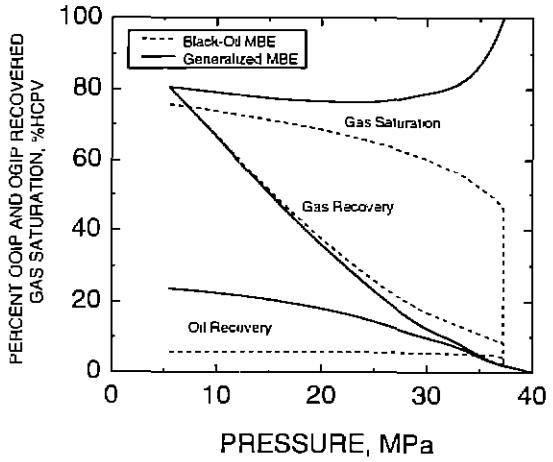

FIGURE 4: Comparison of generalized and conventional (black-oil) material balance calculations for gas-condensate.

in the constant composition expansion. A condition of simultaneous gas-oil flow is realized at pressures between about 9.0-29.6 MPa (1300-4300 psia). The fact that this work allows simultaneous gas-oil flow at reservoir conditions gives it greater generality than alternative approaches which neglect condensate mobility[31].

Figure 4 shows the predictions if the conventional (black-oil) material balance equations (MBE) [Equations (A-22) and (A-23)] are employed instead of using the generalized material balance equations [Equations (1) and (2)]. Using the conventional (black-oil) material balance equations is equivalent to using the generalized material balance equations and assuming R_v is identically zero. Figure 4 compares the generalized and conventional material balance predictions. At pressures greater than the dew point pressure, the generalized and conventional (black-oil) material balances predict identical results. This is expected inasmuch as the generalized and conventional material balance equations reduce to identical relations under these conditions. See Appendix A. However, at pressures less than the dew point, the latter yields erroneous results, as evidenced by: (1) the lack of additional oil recovery below the dew point, (2) the underpredicted ultimate oil recovery, and (3) the predicted non-physical discontinuity in the oil and gas recoveries and gas saturation at the saturation pressure.

Volatile-oil Example

Table 3 shows the fluid properties as a function of pressure for a volatile-oil. This data closely simulates a volatile-oil from the Smackover limestone in north-central Louisiana[25]. The fluid properties were generated by modeling the phase behaviour using an EOS. The initial reservoir pressure is approximately 34.5 MPa (5,000 psia) and temperature is 119°C (246°F). The bubble point of the fluid is approximately 33.1 MPa (4,800 psia). Figure 1 shows the EOS-predicted constant composition expansion (CCE) for this fluid at 119°C. The CCE shows a monotonically increasing volume per cent liquid. This behaviour is characteristic of oils. The gas-phase exhibits a volatile oil-gas ratio of 690 stock-tank $m^3/10^6$ std m^3 (123 stb/mmscf) at a pressure immediately below the saturation pressure. This value is much greater than that for black-oils but less than that for the gas-condensate in the previous example. The initial producing GOR is 596 std m^3/stock-tank m^3 (3,344 scf/stb) using single-stage surface separators and 519 std m^3/stock-tank m^3 (2,909 scf/stb) using two-stage separation with a high pressure separator set at 3.4 MPa (500 psia). The EOS-predicted stock-tank fluid gravity is 39.6° API.

Figure 5 and Table 4 summarize the material balance predictions. The calculations assume the gas-oil relative permeability in Figure 3 apply. The material balance equations predict a final oil recovery of about 23% and a final gas recovery of about 82% at the abandonment pressure of 4.1 MPa (600 psia). These values are very similar to those in the gas-condensate example. The initial producing GOR starts at 519 std m^3/stock-tank m^3 (2,909 scf/stb) and monotonically increases to about 5,760 std m^3/stock-tank m^3 (32,313 scf/stb) at 6.9 MPa (1,000 psia) and then decreases slightly at lower pressures. The decrease in the producing GOR is characteristic of and due to the low depletion pressure. The gas saturation monotonically increases below the saturation pressure and eventually reaches about 66% HCPV at abandonment. The final gas saturation for volatile-oil reservoirs is characteristically lower than that for gas-condensate reservoirs. Notice there is no retrograde condensation for the volatile-oil. A condition of simultaneous gas-oil flow is realized at virtually all pressures less than the

TABLE 3: Fluid property data for example volatile-oil.

P MPa	B_o res m^3 per stock-tank m^3	B_g res m^3 per 10^3 std m^3	R_s std m^3 per stock-tank m^3	R_v stock-tank m^3 per 10^6 std m^3	μ_o mPa-s	μ_g mPa-s
34.5	2.712	5.22	519	1924	0.0735	0.0735
33.1	2.739	5.27	519	1924	0.0716	0.0716
31.7	2.546	4.71	461	690	0.0769	0.0381
30.3	2.341	4.77	402	589	0.0845	0.0350
29.0	2.201	4.88	359	522	0.0912	0.0327
27.6	2.087	5.05	324	466	0.0977	0.0306
26.2	1.985	5.22	293	415	0.1050	0.0287
24.8	1.903	5.44	267	370	0.1118	0.0270
23.4	1.826	5.67	242	331	0.1194	0.0254
22.1	1.756	6.00	220	303	0.1274	0.0240
20.7	1.694	6.34	200	269	0.1358	0.0227
19.3	1.638	6.73	182	247	0.1445	0.0214
17.9	1.581	7.18	164	219	0.1555	0.0203
16.5	1.538	7.74	150	202	0.1642	0.0193
15.2	1.489	8.41	134	185	0.1763	0.0184
13.8	1.448	9.20	120	168	0.1878	0.0176
12.4	1.412	10.21	108	157	0.1988	0.0168
11.0	1.374	11.44	96	146	0.2128	0.0162
9.7	1.332	12.96	82	140	0.2307	0.0155
8.3	1.304	15.09	72	135	0.2432	0.0150
6.9	1.271	17.90	61	134	0.2606	0.0146
5.5	1.239	21.88	50	137	0.2798	0.0141
4.1	1.205	28.16	38	147	0.3013	0.0136

TABLE 4: Reservoir material balance calculations for example volatile-oil.

Pressure, MPa	Oil Recovery % OOIP	Gas Recovery % OGIP	Producing GOR, R, 10^3 std m^3 per stock-tank m^3	Cumulative Producing GOR, R_{ps}, 10^3 std m^3 per stock-tank m^3	Gas Saturation, % HCPV	Eq. A-22 Estimated OOIP/Actual OOIP	Eq. 1 Estimated OOIP/Actual OOIP
34.5	0.0	0.0	0.52	0.52	0.0		
33.1	1.0	1.0	0.52	0.52	0.0	1.00	1.00
31.7	3.0	2.9	0.49	0.51	14.2	0.80	1.00
30.3	5.3	5.2	0.53	0.51	25.6	0.80	1.00
29.0	7.4	7.6	0.62	0.53	32.5	0.84	1.00
27.6	9.5	10.3	0.80	0.57	37.5	0.88	1.00
26.2	11.1	13.3	1.05	0.62	41.5	0.91	1.00
24.8	12.6	16.6	1.35	0.68	44.4	0.93	1.00
23.4	13.7	19.8	1.62	0.75	46.9	0.95	1.00
22.1	14.9	23.8	1.91	0.83	49.2	0.97	1.00
20.7	15.8	27.6	2.25	0.90	51.1	0.98	1.00
19.3	16.7	31.5	2.61	0.98	52.8	0.98	1.00
17.9	17.4	35.6	3.09	1.06	54.4	0.99	1.00
16.5	18.1	39.8	3.51	1.14	55.7	0.99	1.00
15.2	18.7	44.3	3.95	1.23	57.2	0.99	1.00
13.8	19.3	48.7	4.38	1.31	58.4	1.00	1.00
12.4	19.8	53.3	4.73	1.40	59.5	1.00	1.00
11.0	20.3	58.0	5.12	1.48	60.6	1.00	1.00
9.7	20.7	62.7	5.45	1.57	61.9	1.00	1.00
8.3	21.2	67.4	5.64	1.65	62.8	1.00	1.00
6.9	21.6	72.2	5.76	1.73	63.8	1.00	1.00
5.5	22.1	77.0	5.69	1.81	64.9	1.00	1.00
4.1	22.5	82.2	5.40	1.89	66.0	1.00	1.00

saturation pressure.

Figure 6 compares the generalized and conventional material balance predictions. In general, the error incurred by the conventional material balance equations is not as great for volatile-oils as for gas-condensates. This is because volatile-oils do not exhibit retrograde condensation in their saturation history and they have lower volatile oil-gas ratios than gas-condensates. The conventional material balance equations slightly overpredict the oil recovery at pressures immediately below the saturation pressure because they overpredict the oil rate at reservoir conditions due to an underpredicted gas saturation. At lower pressures, the conventional material balance equations underpredict oil recovery because they neglect oil production due to liquid dropout from produced gas, i.e., they assume R_v=0. Although the error in the oil and gas recoveries by the conventional material balance equations is relatively small, the error in the producing gas-oil ratio is large. As shown in Figure 6, the conventional material balance equations greatly overpredict the GOR at moderate and low pressures.

In general, the error caused by using the conventional (black-oil) material balance equations tends to be greatest for rich gas-condensates and less for volatile-oils and wet-gases. Though the error may be less for volatile-oils and wet-gases, I do not, in general, recommend neglecting it.

Oil and Gas Reserve Estimation

A second and perhaps more common application of the material balance equations is to estimate oil and gas reserves based on early production data. In this application Equation (1) is solved for the OOIP (N) and then the OOIP (N) is computed for each pressure in which the fluid property data B_o, B_g, R_s, and R_v and production data N_p, G_p, and R_{ps} are known. If the proper material balance equation is used, this method will yield an accurate estimate of the OOIP (N) for each pressure. If, on the other hand, the conventional (black-oil) material balance equation is mistakenly used to estimate the OOIP and OGIP for either a volatile-oil, gas-condensate, or wet-gas reservoir, it will yield erroneous OOIP estimates. To illustrate the magnitude of this error, gas-condensate and volatile-oil examples are considered.

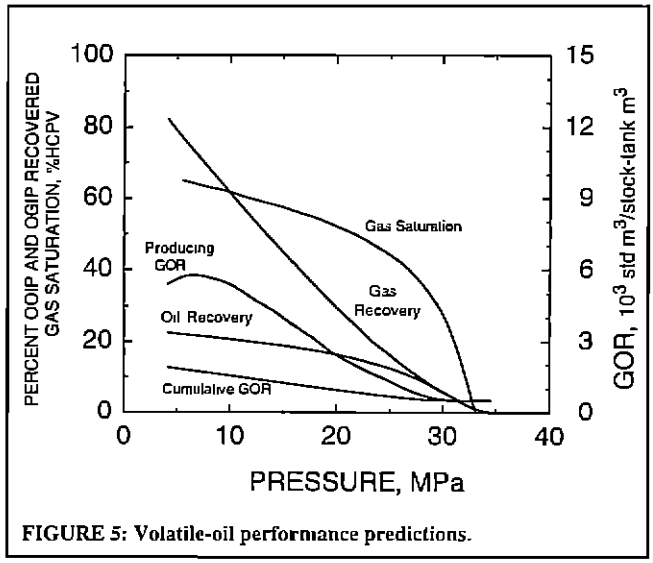

FIGURE 5: Volatile-oil performance predictions.

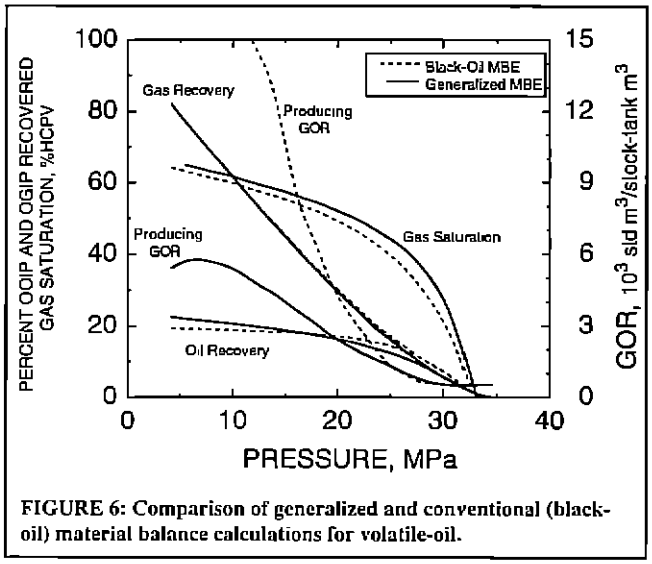

FIGURE 6: Comparison of generalized and conventional (black-oil) material balance calculations for volatile-oil.

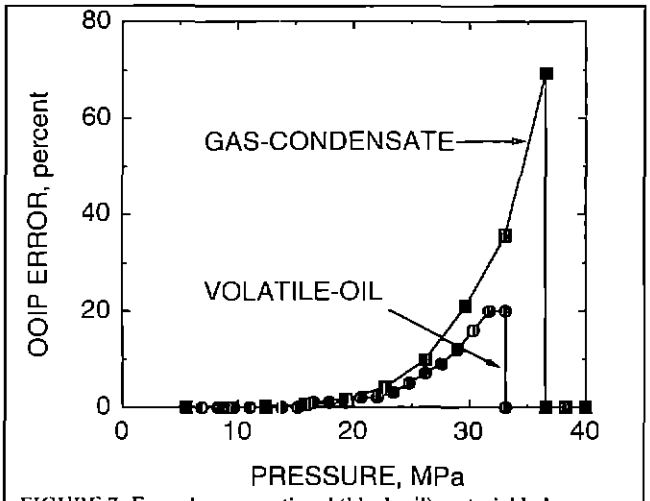

FIGURE 7: Error by conventional (black-oil) material balance equation in estimating OOIP.

Gas-condensate Example

This example uses the fluid property data B_o, B_g, R_s, and R_v given in Table 1 and assumes the oil recovery and cumulative produced gas-oil ratio data reported in Table 2 apply. If the generalized material balance equation, Equation (1), is used to estimate the OOIP (N), it will yield an accurate estimate of the OOIP. If, however, the conventional (black-oil) material balance equation, Equation (A-22), is used to estimate the OOIP, it will underpredict the OOIP at pressures less than the saturation pressure. Table 2 includes a summary the OOIP estimates as a function of pressure using the conventional material balance equation. The OOIP estimates are expressed as a fraction of the actual OOIP. Figure 7 shows the error as a function of pressure. Notice that there is no error at pressures greater than the saturation pressure. This is expected inasmuch as the conventional and generalized material balance equations are equivalent at pressures above the saturation pressure. However, at pressures below the saturation pressure, significant errors are incurred. For example, the error by the conventional material balance equation is as much as 69% at a pressure immediately below the saturation pressure. As the pressure is further reduced below the saturation pressure, the error magnitude decreases, until it is negligible at pressures below about 17.2 MPa (2,500 psia). Note, however, that the error in estimating the OOIP is greatest at early times (high pressures) when reservoir engineers are most interested in obtaining accurate OOIP estimates.

Volatile-oil Example

This example uses the fluid property data B_o, B_g, R_s, and R_v given in Table 3 and assumes the oil recovery and cumulative produced gas-oil ratio data reported in Table 4 apply. Table 4 and Figure 7 summarize the OOIP estimates as a function of pressure using the conventional material balance equation. Notice that, like the gas-condensate example, there is no error at pressures greater than the saturation pressure. However, at pressures less than the saturation pressure significant errors are incurred. The error magnitude is greatest at the saturation pressure and decreases as the pressure decreases. The error in estimating the OOIP is 20% at the saturation pressure. The error magnitude is less for volatile-oils than that for gas-condensates because the former act more like black-oils than the latter.

As an alternative to solving Equation (1) directly to estimate the OOIP, simple graphical methods akin to those popularized by Havlena and Odeh[4] are possible. See the recent work by Ansah, Walsh, and Raghavan[32,33].

Summary and Conclusions

This paper presents a new, generalized material balance equation which is applicable to the full range of reservoir fluids, including volatile-oils and gas-condensates. The new equation retains the simplicity of the two-hydrocarbon-component formulation popularized in earlier developments. Its application is illustrated for an example gas-condensate and volatile-oil. Misapplication of the conventional (black-oil) material balance equation is shown to lead to erroneous reservoir performance predictions and erroneous oil and gas reserves estimates. Application of the generalized material balance equation is recommended for reservoir fluids having a volatile oil-gas ratios greater than about 60 stock-tank $m^3/10^6$ std m^3 (~10 stb/mmscf).

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NOMENCLATURE

B_o = Oil formation volume factor (FVF), res m^3/stock-tank m^3 (rb/stb)
B_{oi} = Initial oil FVF, res m^3/stock-tank m^3 (rb/stb)
B_g = Gas FVF, res m^3/std m^3 (rb/scf)
B_{gi} = Initial gas FVF, res m^3/std m^3 (rb/scf)
B_w = Water FVF, res m^3/stock-tank m^3 (rb/stb)
G = Original gas in-place OGIP, std m^3 (scf)
G_p = Produced wellhead gas, std m^3 (scf)
G_f = Gas in free gas-phase, std m^3 (scf)
ΔG = Definition, see Appendix A
k_{ro} = Oil relative permeability
k_{rg} = Gas relative permeability
N = OOIP, stock-tank m^3 (stb)
N_p = Produced oil, stock-tank m^3 (stb)
N_f = Oil in free oil-phase, stock-tank m^3 (stb)
ΔN = Definition, see Appendix A
P = Pressure, MPa (psia)
q_g = Gas flow rate at reservoir (bottomhole) conditions, res m^3/day (rb/day)
q_{gs} = Gas flow rate at surface conditions, m^3/day (cf/day)
q_o = Oil flow rate at reservoir (bottomhole) conditions, res m^3/day (rb/day)
q_{os} = Oil flow rate at surface conditions, m^3/day (cf/day)
R_s = Solution gas-oil ratio, std m^3/stock-tank m^3 (scf/stb)
R_{si} = Initial solution gas-oil ratio, std m^3/stock-tank m^3 (scf/stb)
R_v = Volatile oil-gas ratio, stock-tank m^3/std m^3 (stb/scf)
R_{vi} = Initial oil-gas ratio, stock-tank m^3/std m^3 (stb/scf)
R_p = Cumulative produced wellhead gas-oil ratio, std m^3/stock-tank m^3 (stb/scf)
R_{ps} = Cumulative produced sales gas-oil ratio, std m^3/stock-tank m^3 (scf/stb)
r_g = Fraction of produced gas reinjected
S_{wi} = Initial water saturation, fraction PV
S_o = Oil saturation, fraction PV
S_g = Gas saturation, fraction PV
W_i = Influxed water, stock-tank m^3 (stb)
W_p = Produced water, stock-tank m^3 (stb)
ΔW = Definition, see Appendix A
V_p = Reservoir pore volume, res m^3 (rb)

Greek Symbols

μ_o = Oil viscosity, mPa•s (cp)
μ_g = Gas viscosity, mPa•s (cp)

Conversion Factors

1 m = 3.281 ft
1 m^3 = 6.299 bbl
1 m^3 = 35.34 cf
1MPa = 145.0 psi
1 mPa•s = 1 cp

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Appendix A: Derivation of the Generalized Material Balance Equation

Assumptions

The mathematical development in this paper is based on the following assumptions or idealizations:

1. The reservoir is an isothermal system.
2. The reservoir is comprised of, at most, four components: rock, water, stock-tank oil and surface-gas.
3. The reservoir is comprised of, at most, four phases: rock, water (aqueous), oil, and gas.
4. The surface gas component exists only in the oil- and gas-phases and does not partition into the water- or rock-phases. This assumption allows for dissolved-gas.
5. The stock-tank oil component exists only in the oil- and gas-phases and does not partition into the water- or rock-phases. This assumption allows for volatilized-oil.
6. The water component exists only in the water-phase and does not partition into either the oil-, gas-, or rock-phases.
7. The rock component exists only in the rock-phase.
8. The water- and rock-phases are incompressible. This assumption implies the reservoir pore volume is constant.
9. The reservoir pressure is uniform throughout the reservoir, i.e., no pressure gradients exist vertically or horizontally.
10. The reservoir fluids are in thermodynamic equilibrium.
11. Water may enter the reservoir, i.e., water influx may occur.
12. Water, stock-tank oil, and surface-gas components may be produced.
13. The produced gas component may be re-injected into the reservoir.
14. The porosity and initial water saturation are uniform throughout the reservoir.

Many of the above assumptions are routinely invoked in reservoir engineering calculations; I include them only for completeness. Note that Assumptions 1-14 make no statement as to the initial or later distribution of free oil- and gas-phases. Unique to this development is Assumption 5, which allows one to account for volatile-oil and gas-condensate systems as well as black-oil systems. To consider only black-oil systems, Assumption 5 is replaced by: the stock-tank oil component exists only in the oil-phase and does not partition into the water-, gas- or rock-phases.

Many of the listed assumptions can easily be relaxed to consider more general or realistic conditions. However, for the sake of brevity and simplicity, I purposely restrict myself to the above assumptions.

Volume and Material Balances

A volume balance demands:

V_p= (volume of free oil-phase) + (volume of free gas-phase) + (volume of water-phase) ..... (A-1)

where V_p denotes the reservoir pore volume. If Equation (A-1) is applied for some time after initial production, it yields

$$V_p = N_f B_o + G_f B_g + (W_i - W_p)B_w + S_{wi}V_p \quad \text{..... (A-2)}$$

where N_f is the stock-tank m³ or stb of remaining free oil-phase, G_f is the std m³ or scf of free gas-phase, and the remaining variables here and elsewhere are defined in the nomenclature. A material balance on the oil component demands:

(stock-tank m³ of oil component in oil-phase) =
(initial stock-tank m³ of oil) -
(produced stock-tank m³ of oil) -
(stock-tank m³ of oil component in gas-phase)

or

$$N_f = N - N_p - G_f R_v \quad \text{..... (A-3)}$$

where N is the stock-tank m³ or stb of original oil in-place (OOIP), N_p is the stock-tank m³ or stb of produced oil, and R_v describes the stock-tank m³ or stb of volatilized-oil in the free gas-phase per std m³ or scf of gas[20]. A material balance on the gas component demands

(std m³ of gas component in gas-phase) =
(initial std m³ of gas) - (produced std m³ of gas) -
(std m³ of gas component in oil-phase)

or

$$G_f = G - G_p(1 - r_g) - N_f R_s \quad \text{..... (A-4)}$$

where r_g is the fraction of the total produced gas which is reinjected. Substituting (A-3) into (A-4) and solving for G_f gives

$$G_f = \frac{\Delta G - \Delta N R_s}{(1 - R_v R_s)} \quad \text{..... (A-5)}$$

where the above expression is simplified by defining $\Delta G = G - G_p(1-r_g)$ and $\Delta N = N - N_p$. Substituting (A-3) into (A-2) gives

$$V_p = (\Delta N - G_f R_v) B_o + G_f B_g + \Delta W + V_p S_{wi} \quad \text{..... (A-6)}$$

where $\Delta W = (W_i - W_p)B_w$. Substituting (A-5) into (A-6) and simplifying gives

$$V_p(1 - S_{wi}) = \Delta W + \frac{(B_o - R_s B_g)\Delta N + (B_g - R_v B_o)\Delta G}{(1 - R_v R_s)} \quad \text{..... (A-7)}$$

For the sake of simplicity and brevity, I further assume:

15. The reservoir initially contains only one hydrocarbon phase.

This assumption implies the initial reservoir pressure is above the saturation pressure. It follows from Assumption 15 that the OOIP, N, is related to the system pore volume V_p by

$$N = \frac{V_p(1 - S_{wi})}{B_{oi}} = \frac{V_p(1 - S_{wi})R_{vi}}{B_{gi}} \quad \text{..... (A-8)}$$

where N is written either in terms of B_{oi} if the original in-place fluid is treated as an oil or in terms of B_{gi} and R_{vi} if the original in-place fluid is treated as a gas. The OGIP, G, is related to the OOIP, N, by

$$G = N R_{si} = \frac{N}{R_{vi}} \quad \text{..... (A-9)}$$

If the initial fluid is treated as an oil, and Equation (A-8) is solved for V_p, and this result and Equation (A-9) are substituted into Equation (A-7), one obtains

$$N\left[B_{oi}(1 - R_v R_s) - (B_g - R_v B_o)R_{si} - (B_o - R_s B_g)\right] + N_p\left[B_o(1 - R_v R_{ps}) + (R_{ps} - R_s)B_g\right] = \Delta W(1 - R_v R_s) \quad \text{..... (A-10)}$$

where I have used $G_p(1-r_g)=R_{ps}N_p$ and R_{ps} is the cumulative sales gas-oil ratio expressed in units of std m³/stock-tank m³ or scf/stb. Since gas reinjection may occur and not all the produced wellhead gas results in sales gas, the cumulative wellhead GOR, R_p, is related to R_{ps} by $R_{ps}=R_p(1-r_g)$. At pressures greater than the saturation pressure, Equation (A-10) reduces to

$$N(B_{oi} - B_o) + N_p B_o = \Delta W \quad \text{..... (A-11)}$$

Saturation Equation

The reservoir oil saturation can be evaluated if the following additional assumptions or idealizations are invoked:

16. If gas is re-injected, a secondary gas-cap does not form. This implies that the re-injected gas is dispersed uniformly throughout the oil-leg.
17. No water influx occurs.
18. The phase saturations are uniform throughout the oil-leg.

The oil saturation is related to the stock-tank m³ or stb of free oil-phase N_f by

$$N_f = \frac{V_p S_o}{B_o} \quad \text{..... (A-12)}$$

The gas saturation is related to the std m³ or scf of free gas-phase by

$$G_f = \frac{V_p S_g}{B_g} \quad \text{..... (A-13)}$$

where all saturations are expressed in fractional (reservoir) pore volume units. Substituting (A-12) and (A-13) into (A-3) gives

$$\frac{V_p S_o}{B_o} = N - N_p - \frac{V_p S_g R_v}{B_g} \quad \text{..... (A-14)}$$

If Equation (A-8) is solved for V_p, and this result and $S_g=1-S_{wi}-S_o$ are substituted into (A-14), and the resulting expression is solved for S_o, one obtains

$$S_o = (1 - S_{wi})\left[\frac{\left(1 - \frac{N_p}{N}\right)B_o B_g - B_{oi} R_v B_o}{B_{oi}(B_g - R_v B_o)}\right] \quad \text{..... (A-15)}$$

At pressures greater than the saturation pressure, the single hydrocarbon phase saturation is equal to $(1-S_{wi})$.

Equations (A-10), (A-11), and (A-15) have been arbitrarily written in terms of B_{oi} and R_{vi}. Alternatively, these equations can

be written in terms of B_{gi} and R_{vi} by substituting the following equalities: $B_{oi}=B_{gi}/R_{vi}$ and $R_{si}=1/R_{vi}$.

Producing GOR

The instantaneous producing GOR is defined by

$$R = \frac{q_{gs}}{q_{os}} \quad \text{(A-16)}$$

where q_{os} and q_{gs} are the instantaneous oil and gas volumetric flow rates at the surface. The produced stock-tank oil originates from either the stock-tank oil in the free reservoir oil-phase or the condensable liquids (stock-tank oil) in the free reservoir gas-phase. Accordingly, oil rate at the surface is given by

$$q_{os} = \frac{q_o}{B_o} + \frac{q_g}{B_g} R_v \quad \text{(A-17)}$$

where q_o and q_g are the instantaneous oil and gas flow rates at the reservoir (bottomhole) conditions. Similarly, the produced surface-gas originates from either the dissolved-gas in the free oil-phase or the surface-gas in the free gas-phase. Accordingly, the gas rate at the surface is given by

$$q_{gs} = \frac{q_o}{B_o} R_s + \frac{q_g}{B_g} \quad \text{(A-18)}$$

Substituting Equations (A-17) and (A-18) into (A-16) and simplifying gives

$$R = \frac{\frac{q_g B_o}{q_o B_g} + R_s}{\frac{q_g B_o}{q_o B_g} R_v + 1} \quad \text{(A-19)}$$

The reservoir oil and gas rates are given by Darcy's law and the ratio of q_g/q_o is given by

$$\frac{q_g}{q_o} = \frac{k_{rg}\mu_o}{k_{ro}\mu_g} \quad \text{(A-20)}$$

Substituting Equation (A-20) into (A-19) gives

$$R = \frac{\frac{B_o k_{rg}(S_o)\mu_o}{B_g k_{ro}(S_o)\mu_g} + R_s}{\frac{B_o k_{rg}(S_o)\mu_o}{B_g k_{ro}(S_o)\mu_g} R_v + 1} \quad \text{(A-21)}$$

Black-oil Reservoir

For the special case of a black-oil reservoir, i.e., $R_v = 0$. Equations (A-10) and (A-15) reduce to

$$N\left[B_{oi} - B_o - (R_{si} - R_s)B_g\right] + N_p\left[B_o + (R_{ps} - R_s)B_g\right] = \Delta W \quad \text{(A-22)}$$

$$S_o = (1 - S_{wi})\left(1 - \frac{N_p}{N}\right)\frac{B_o}{B_{oi}} \quad \text{(A-23)}$$

Equation (A-22) is the conventional (black-oil) material balance equation.

Dry-gas Reservoir

For the special case of a dry-gas reservoir, Equation (A-10) simplifies to:

$$B_{gi}\,G = B_g\,G - G_p B_g + \Delta W \quad \text{(A-24)}$$

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