NOTES ON RELATIVE PERMEABILITY RELATIONSHIPS

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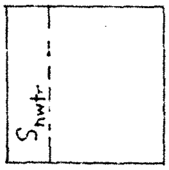
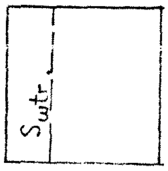
Page	Reference	Now Reads	Should Read
i	Par. 3, Line 2	there four	there are four
16	Line 2	= 0.413	= 0.413 k
16	Par. 3, Line 5	0.25	0.20
16	Par. 5, Table 2 Calculations . . .	$S_w =$	$S_w^* =$
18	Equation 15	$\left.\frac{k_o}{k_o}\right]_{S_L = 0}$	$\left.\frac{k_o}{k_o}\right]_{S_L^* = 1}$
25	Par. 2, Line 9	$S_L = 0.15$	$S_L = 0.85$
33	Figure 21		
34	Par. 1, lines 11 - 15 should read, "Continuing to desaturate to residual wetting phase saturation, S_{wtr}, and then resaturating back to the same relative permeability value (Point B) results in a greater non-wetting phase saturation because part of the non-wetting phase (saturation B minus saturation A) is trapped and does not contribute to k_{rnwt}."		
36	Equation 26	$\left[1 - S_w^{*2}\right]$	$\left[1 - S_w^{*\frac{2 + \lambda}{\lambda}}\right]$

December 1, 1974

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PREFACE

One of the primary functions of reservoir and production engineers is to predict, by means of valid engineering relationships, results of simultaneous flow of gases and liquids through reservoir rock. The rates of flow into or away from wells and the fraction of oil and gas that will be recovered are very important factors that the engineer is constantly concerned with. Of course, in addition to using valid engineering relationships (in contrast to looking into crystal balls) there is an implied requirement that the prediction be reasonable accurate.

Both flow and recovery of gas and oil involve relative permeability values as a function of fluid saturation. In many instances the relative permeability curve selected to represent the subsurface flow behavior has more effect on the ultimate answer than any other parameter in the equations used. Thus, it is important that the engineer have a good understanding of relative permeability behavior.

What about the sources of relative permeability data ? Basically there are four sources:

1. Guess. Take a piece of graph paper and draw curved lines simulating the shapes seen in text books, technical articles, etc. The results will be of unknown (and generally poor) accuracy and subject to argument by other engineers.

2. Analogy. Select relative permeability - saturation curves from the literature and assume your system has the same characteristics. A very favorite correlation is that of Arps and Roberts (Trans. AIME 204(1955) 120) that is reproduced on pages 386-387 of Craft and Hawkins. These results may be just as inaccurate as those mentioned above but will be more acceptable to other engineers.

3. Use measured capillary pressure-saturation data to characterize the pore structure of the reservoir rock.

Use this characteristic in empirical relationships that relate relative permeability to pore structure, saturation history, saturation and other pertinent parameters. In many instances this approach will yield fairly accurate results. Furthermore, the empirical relationships can often be used to extrapolate and average measured data in a consistant manner.

4. Laboratory measured values. These are generally believed to be the most accurate values. Yet, in my opinion they can be fairly inaccurate if the laboratory measurements are not carefully performed. However, measured values are least apt to be questioned by other engineers.

The subject of these notes is the empirical relationships that tie to capillary pressure. I have found these to be very useful in day-to-day engineering, primarily because of the scarcity of measured relative permeability data. Furthermore, an understanding of the theory behind these relationships makes the engineer much more capable of handling and using relative permeability data.

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August 6, 1974

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Introduction

Equations concerned with fluid flow in reservoir rocks make use of effective permeabilities, k_g, k_o, and k_w. Effective permeabilities are functions of:

1. pore size
2. pore size distribution
3. wettability
4. saturation
5. saturation history

Relative perms are the result of normalizing effective permeability values. Reservoir units of similar pore size, geometry, and wettability should have characteristic relative permeability relationships when plotted against saturation and saturation history.

Relative permeabilities may be expressed in terms of any specified base permeability. The three most common base values are (1) dry air permeability, k_a, measured at atmospheric pressure, (2) absolute permeability, k, and (3) effective hydrocarbon permeability at irreducible water saturation, S_{iw}. For example, consider a core sample in which the effective oil permeability at a particular saturation is 50 md. ($k_o\rfloor_{S_o,\ S_w}$ = 50 md.) and the three base permeabilities are:

k_a = 115 md, k = 102 md, $k_o\rfloor_{S_{iw}}$ = 85 md.

The relative permeability values could be either

k_{ro} = 50/115 = 0.43; k_{ro} = 50/102 = 0.49; k_{ro} = 50/85 = 0.59

Be careful to understand which base is used!

Saturation history is indicated by two terms; drainage and imbibition. Drainage relative permeability curves apply to processes in which the wetting phase is, or has been decreasing in magnitude. Imbibition relative permeability curves apply to processes in which the wetting phase is, or has been increasing in magnitude. The way of indicating drainage and imbibition values are

$k_{ro}\rfloor_{dr}$ = drainage $k_{ro}\rfloor_{imb}$ = imbibition and by use of arrows pointing the direction of wetting phase saturation change on plots.

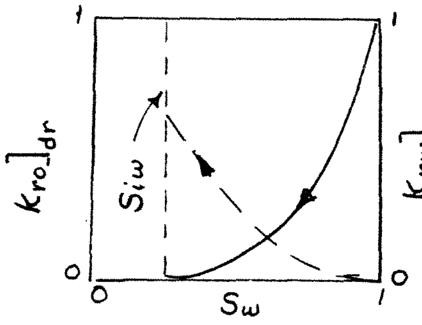


Figure 1a

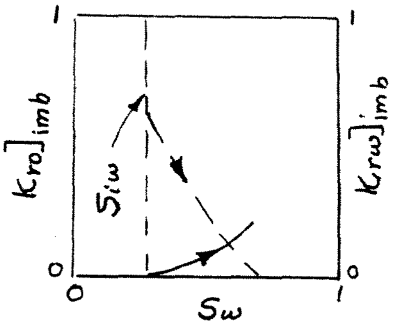


Figure 1b

— — — oil
——— water

Figure 1a illustrates drainage oil and water relative permeability curves while Figure 1b illustrates imbibition curves. Water is the wetting phase in both sets of curves. Figure 2a and 2b illustrate gas and oil relative permeability curves in the presence of irreducible water, S_{iw}. Note that in this instance oil is the wetting phase and that the abcissa value is total liquid saturation, S_L. (Total liquid saturation $S_L = S_o + S_{iw}$.)

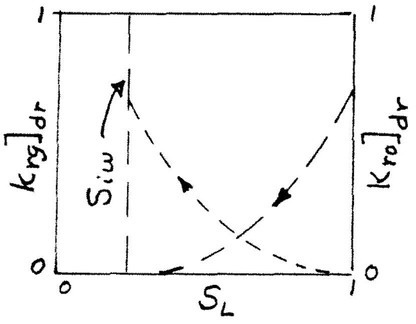


Figure 2a

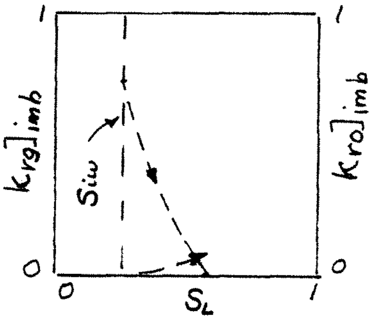


Figure 2b

------ gas
— — — oil

Application of drainage and imbibition curves to reservoir processes are usually as follows:

Drainage Curves

1. Tarner or Muskat sol'n gas drive calculations. (gas displacing oil)
2. Gravity drainage calc's (gas replacing oil)
3. Gas drive calculations (gas displacing oil)
4. Oil or gas displacing water

Imbibition Curves

1. Waterflood calculations (water displaces oil & gas)
2. Water influx calculations (water displaces oil or gas)

Fundamental Concepts

a) Fluids in pore structure are under capillary control. For "water wet" systems water preferentially fills smallest pores, gas fills largest pores, and oil fills what is left.

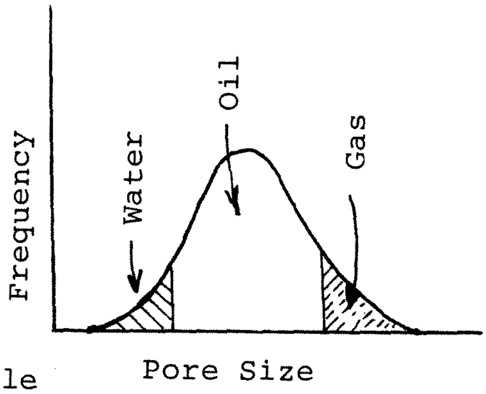


Figure 3

(1) k_{rw} depends only on amount of mobile water, $(S_w - S_{iw})$. Does not depend on whether hydrocarbon phase is oil, gas, or both.

(2) k_{rg} depends on amount (saturation) of gas present, s_g Does not depend on proportions of oil and water.

(3) k_{ro} depends on amount (saturation) of oil, s_o, and range of pore size in which it lies. k_{ro} for $s_o = 0{,}55$, $s_w = 0{,}40$, $s_g = 0.05$ will be larger than k_{ro} for $S_o = 0.55$, $s_w = 0.30$, $S_g = 0{,}1$ because oil will be distributed in smaller size pores in the second case.

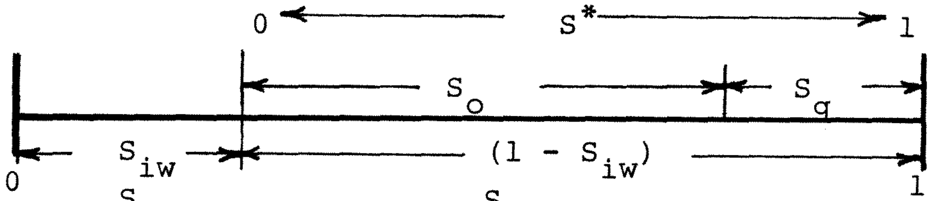
b) Each fluid moves through separate groups of pores. Two or three fluids do not flow in the same pore. Saturation changes cause redistribution of pore size range occupied by the individual fluids.

c) Because of pore sizes being distributed throughout the rocks, fluids tend to "block" flow of other fluids. This requires that flow-path length change as saturation changes. (This refered to as tortuosity effect). Relative perm curves reflect average pore size of pores containing fluid and tortuosity.

Effective (Normalized) Saturations

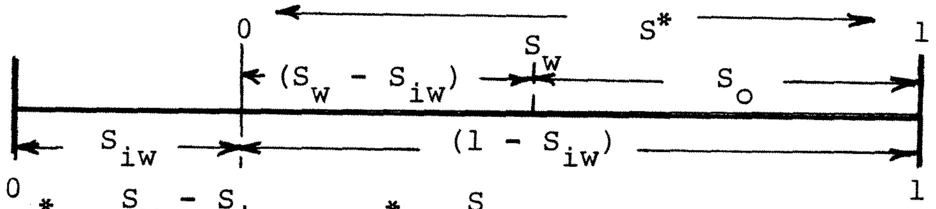
Relative permeability relationships can be expressed most easily in terms of effective saturations, S_g^*, S_o^*, S_w^*. The effective saturation is the saturation expressed as a fraction of the pore space not occupied by irreducible (non-mobile) water. The bars below illustrate effective saturations in different reservoir systems.

Irreducible water + oil + gas



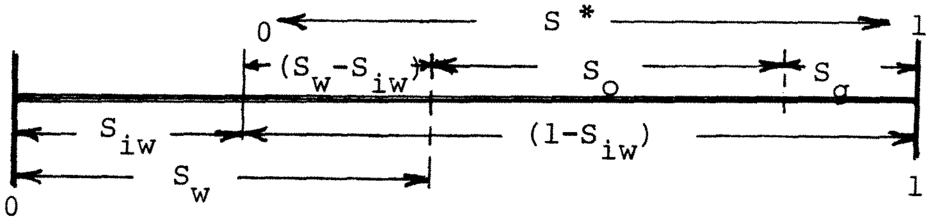
$$S_o^* = \frac{S_o}{(1 - S_{iw})} \; ; \quad S_g^* = \frac{S_g}{(1 - S_{iw})}$$

Irreducible water + mobile water + oil



$$S_w^* = \frac{S_w - S_{iw}}{(1 - S_{iw})} \; ; \quad S_o^* = \frac{S_o}{(1 - S_{iw})}$$

Irreducible water + mobile water + oil + gas



$$S_w^* = \frac{S_w - S_{iw}}{(1 - S_{iw})} \; ; \quad S_o^* = \frac{S_o}{(1 - S_{iw})} \; ; \quad S_g^* = \frac{S_g}{(1 - S_{iw})}$$

Theory of Two-Phase Drainage Relative Permeabilities

It was pointed out in the introduction that effective permeabilities are a function of pore size and pore size distribution. When the effective permeability is normalized to absolute permeability, yielding the relative permeability, the dependency on pore size is eliminated. Relative permeability, when expressed as a function of saturation, becomes strongly dependent on pore size distribution. Wettability, and saturation history are, of course, important parameters also.

The discussions that follow are aimed at developing relative permeability relationships for wetting and nonwetting fluids in rocks having some definable pore size distribution. Later the results will be applied to pore systems containing gas, oil, and water. The relative permeability and effective wetting phase saturation units that will be used are defined as follows:

$$k_{rwt} = k_{wt} / k_{wt}\Big]_{S_{wt}^* = 1} \qquad (1a)$$

$$k_{rnwt} = k_{nwt} / k_{nwt}\Big]_{S_{wt}^* = 0} \qquad (1b)$$

$$S_{wt}^* = \frac{S_{wt} - S_{wtr}}{1 - S_{wtr}} \qquad (1c)$$

where

k_{wt}, k_{nwt} = effective permeability of wetting and nonwetting phase at a given wetting phase saturation.

$k_{wt}\Big]_{S_{wt}^*= 1}$ = effective permeability of wetting phase at 100% wetting phase saturation (Point A in Figure 4)

$k_{nwt}\Big]_{S_{wt}^*= 0}$ = effective permeability of non-wetting phase at residual wetting phase saturation (Point B in Figure 4)

Figure 4 shows effective wetting and non-wetting phase permeabilities plotted against wetting phase saturation. Figure 5 shows the results of normalizing the effective permeabilities to their end point values (points A & B) and expressing the normalized, or relative permeability values as a function of effective wetting phase saturation, S_{wt}^*.

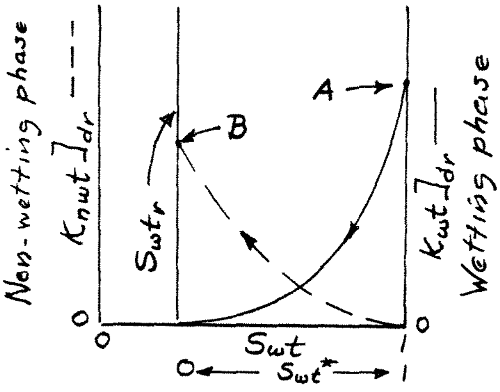


Figure 4

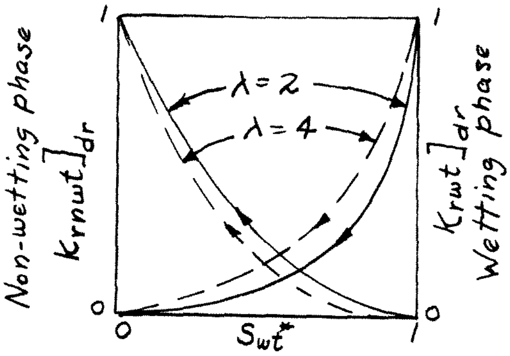


Figure 5

Figure 5 also illustrates the effect of pore size distribution on the resulting relative permeability curves. Lamda, λ, is called the pore size distribution index. The solid curves, $\lambda = 2$, are for a wide range of pore sizes, while the $\lambda = 4$ dashed curves represent a medium range of pore sizes. The larger the value of λ, the more uniform is the pore size distributions. An index of $\lambda = \infty$ represents a uniform pore size. Natural sandstones and limestones usually can be represented by pore size distribution indexes between about 0.5 and 4.

The pore size distribution index, λ, can be obtained from the shape of a capillary pressure-saturation curve, or, for a group of curves, from the shape of the Leverett J function-saturation curve. Brooks and Corey(1) (2) on the basis of a large amount of experimental data have shown that the ratio of capillary pressure to capillary entry pressure, (P_c/P_e) and effective wetting phase saturation, S_{wt}^*, can often be represented by the relationship

$$S_{wt}^* = (P_c/P_e)^{-\lambda} \tag{2}$$

or

$$\log P_c = \log P_e - {}^1/\lambda \log S_{wt}^* \tag{3}$$

Equation 3 is a straight line on $\log P_c$ vs $\log S_{wt}^*$ coordinates the slope of the straight line defines λ. Figure 6 is such a plot for air-water capillary pressure data on two Berea and two Boise Sandstone samples. It illustrates how pore size distribution indexes can be obtained. Water was, of course, the wetting phase in these tests.

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(1) Brooks, R.H. and Corey, A.T. "Hydraulic Properties of Porous Media." Hydraulic Paper Number 3, Colorado State University, 1964.

(2) Brooks, R.H. and Corey A.T. "Properties of Porous Media Affecting Fluid Flow." Journal of the Irrigation and Drainage Division, Proc. of ASCE (1966), vol. 92, No. IR2, pages 61-88.

The early work of Burdine(3) and others associated with Gulf Research and Development Company lead to the following two relative permeability relationships in terms of the effective wetting phase saturation.

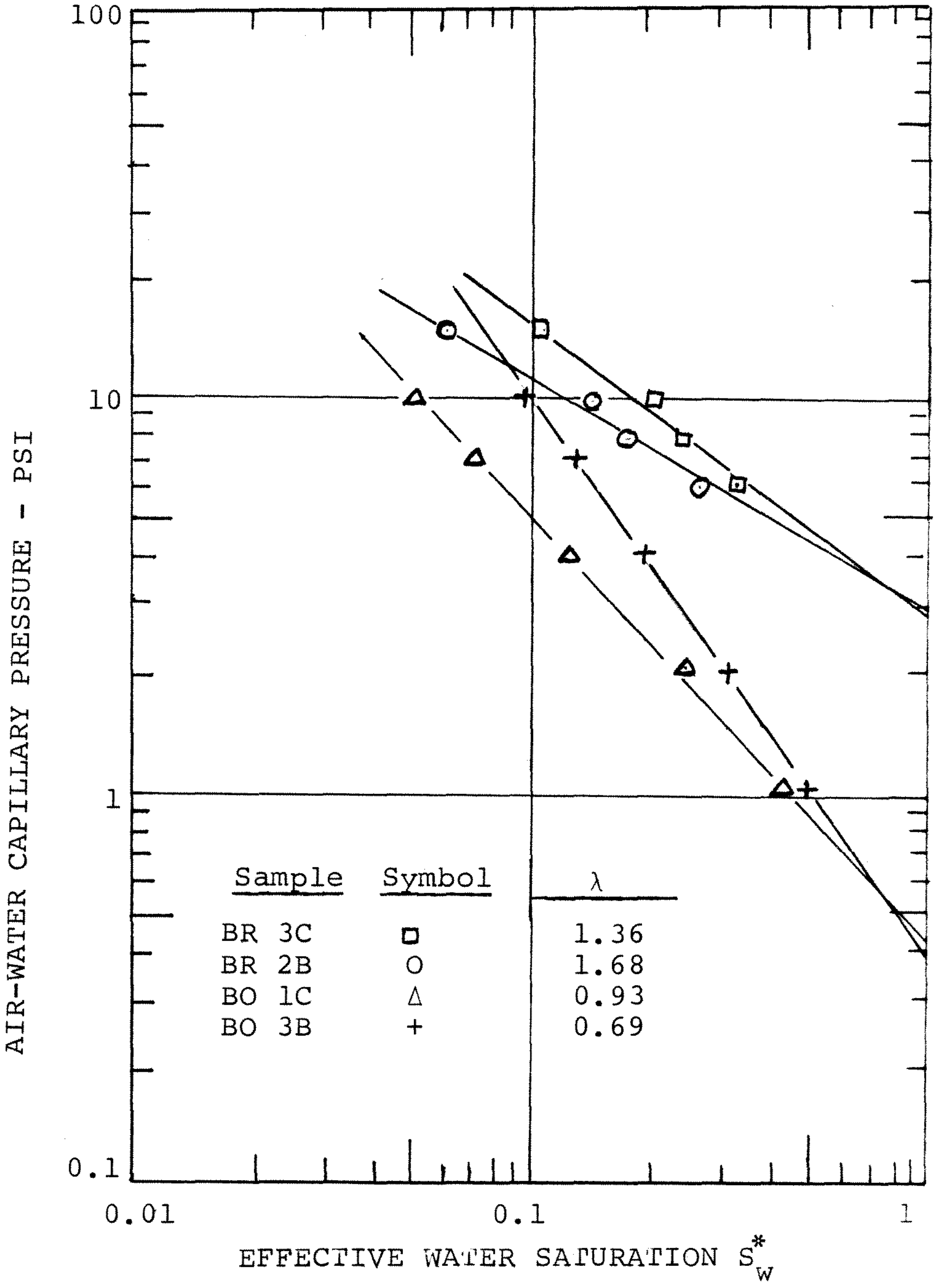

Figure 6. Log-Log Relationship of Capillary Pressure and Effective Water Saturation.
Boise and Berea Sandstone Samples.

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(3) Burdine, N.T. "Relative Permeability Calculations from Pore Size Distribution Data" Trans.AIME 198 (1953), 71-78.

For the wetting phase:

$$k_{rwt}\Big]_{dr} = (S_{wt}^{*})^{2} \frac{\int_{0}^{S_{wt}^{*}} \frac{dS_{wt}^{*}}{P_c^{2}}}{\int_{0}^{1} \frac{dS_{wt}^{*}}{P_c^{2}}} \tag{4}$$

For the non-wetting phase:

$$k_{rnwt}\Big]_{dr} = (1 - S_{wt}^{*})^{2} \frac{\int_{S_{wt}^{*}}^{1} \frac{dS_{wt}^{*}}{P_c^{2}}}{\int_{0}^{1} \frac{dS_{wt}^{*}}{P_c^{2}}} \tag{5}$$

The integrals in Equations 4 and 5 can be solved in either of two ways. Where the pore size distribution index, λ, is known the solutions become:

For the wetting phase:

$$k_{rwt}\Big]_{dr} = (S_{wt}^{*})^{\frac{2+3\lambda}{\lambda}} \tag{6}$$

For the non-wetting phase:

$$k_{rnwt}\Big]_{dr} = (1 - S_{wt}^{*})^{2} \left[1 - (S_{wt}^{*})^{\frac{2+\lambda}{\lambda}}\right] \tag{7}$$

Where the index, λ, is not known, or where it is not constant within the saturation range of interest, one can use graphical integration methods to get a solution. This is illustrated by the following sketches.

For the wetting phase:

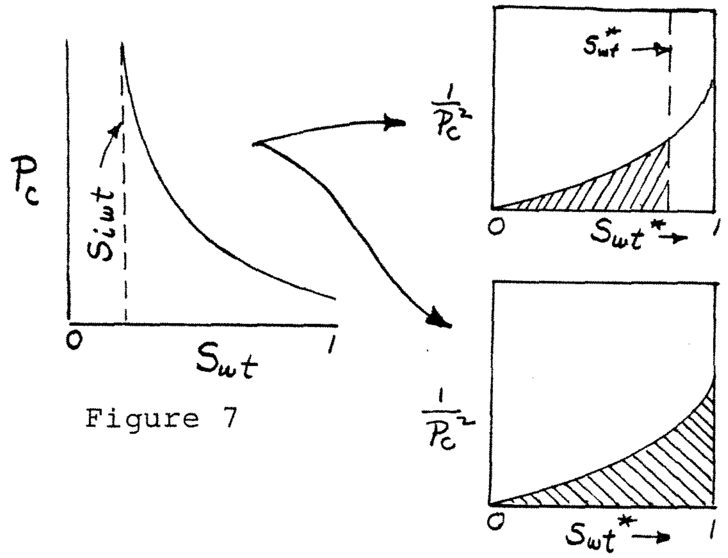

Figure 7

Shaded area under $^{1}/P_c^{2}$ curve equals $\int_{0}^{S_{wt}^{*}} \frac{dS_{wt}^{*}}{P_c^{2}}$ of Equation 4

Shaded area under $^{1}/P_c^{2}$ curve equals $\int_{0}^{1} \frac{dS_{wt}^{*}}{P_c^{2}}$ of Equation 4

For the non-wetting phase:

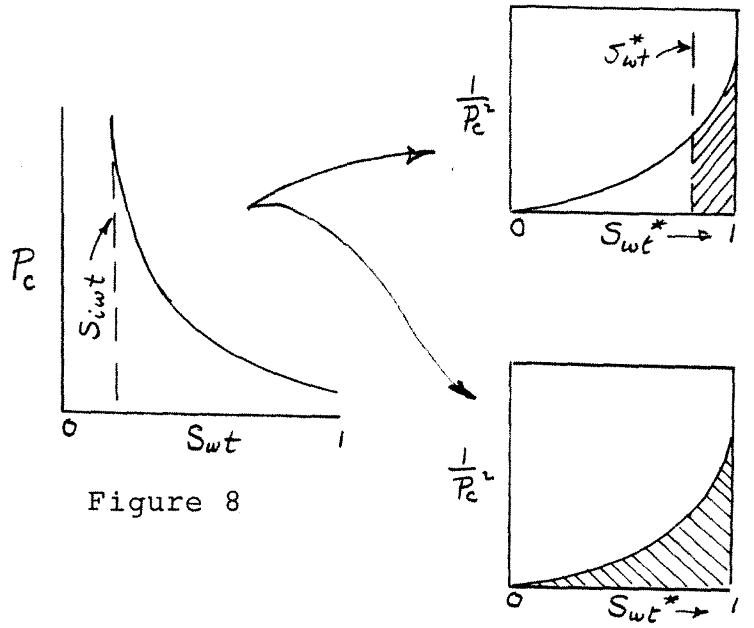

Figure 8

Shaded area under $^1/p_c{}^2$ equals $\int_{S_{wt}^*}^{1} \frac{dS_{wt}^*}{P_c^2}$ of Equation 5

Shaded area under $^1/p_c{}^2$ curve equals $\int_{0}^{1} \frac{dS_{wt}^*}{P_c^2}$ of Equation 5

Note that it is not necessary to perform four graphical integrations as indicated by the sketches above. Two suffice.

The table below shows drainage relative permeability equations for several typical pore size distribution indexes.

TABLE I

Two-Phase Drainage Relative Permeability Equations

Porous Media	Dist. Index λ	k_{rwt}	k_{rnwt}
Very wide range of pore size	0.5	$(S_{wt}^*)^7$	$(1 - S_{wt}^*)^2 \left[1 - (S_{wt}^*)^5\right]$
Wide range of pore size	2	$(S_{wt}^*)^4$	$(1 - S_{wt}^*)^2 \left[1 - (S_{wt}^*)^2\right]$
Medium range of pore size	4	$(S_{wt}^*)^{3.5}$	$(1 - S_{wt}^*)^2 \left[1 - (S_{wt}^*)^{1.5}\right]$
Uniform pore size	∞	$(S_{wt}^*)^3$	$(1 - S_{wt}^*)^3$

Note: In Table I

$$k_{rwt} = k_{wt} \Big/ k_{wt}\Big]_{S_{wt}^* = 1}$$

$$k_{rwnt} = k_{nwt} \Big/ k_{nwt}\Big]_{S_{wt}^* = 0}$$

The pore size distribution indexes of 2, 4, and ∞ produce the equations proposed by Wyllie(4) to be used for cemented sandstones and oolitic and small-vugular limestones ($\lambda = 2$); poorly sorted unconsolidated sandstones ($\lambda = 4$); and well sorted unconsolidated sandstones ($\lambda = \infty$).

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(4) Wyllie, M.R.J. "Relative Permeability" Petroleum Production Handbook, Chapter 25, vol. II. McGraw-Hill Publishers, 1962.

A study of many types of reservoirs has lead to the conclusion that a pore size distribution index greater than 6 should not be common for reservoirs containing hydrocarbons. This shows that most reservoir formations are likely to be in the very poorly to reasonable sorted range. The use of the pore size distribution index of 2 leads to the so-called Corey Equations, which are the best known forms of Equations 6 and 7. Corey Equations, therefore, are strictly valid only for a particular pore size distribution. They are often used however, to calculate relative permeability values when direct information on the pore structure is not known.

Referring back to Equations 1a, 1b, and to Figures 4 and 5 it will be seen that the two effective permeabilities were normalized to different base values when defining the relative permeabilities. Wetting phase permeabilities were normalized to the wetting phase permeability at 100% wetting phase saturation. If the wetting fluid is "non reactive", that is, does not react with rock components, the base permeability is, by definition, the absolute permeability of the rock, k. Thus, we can say that Equation 6 expresses a relative permeability relationship that is based on absolute permeability. On the other hand, Equation 7 is an expression in terms of effective permeability at partial wetting phase saturation, S_{wtr}, which is different than absolute permeability. Therefore, to get the non-wetting phase relative permeability expression onto an absolute permeability base it is necessary to introduce a relationship between $\left.k_{nwt}\right]_{S_{wtr}}$ and absolute permeability, k. This is the subject of the next section.

Non-wetting Phase Permeability at Residual Wetting Phase Saturation. Consider a pore structure containing a wetting phase at residual saturation and a non-wetting phase. Figure 8 illustrates the pore size ranges that contain the two fluids. Conceptually, the absolute permeability, k, is proportional to the total area under curve. Likewise, the effective permeability to the non-wetting phase, k_{nwt}, can be said to be proportional to the area designated to contain non-wetting phase. On this basis, it is easy to see that $k_{nwt}\big]_{S_{wtr}}$ will decrease as S_{wtr} increases. This can be expressed by the relationship

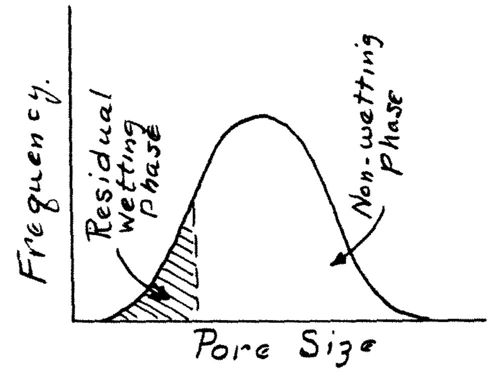

Figure 8

$$k_{nwt}\Big]_{S_{wtr}} = k \cdot f\,(S_{wtr}) \qquad (8)$$

where

$f\,(S_{wtr})$ = a function of residual wetting phase saturation

It would be expected that the saturation function in Equation 8 would depend to some degree on the pore size distribution index, λ. This has not been tested to the author's knowledge. However, results of many tests made at Chevron Oil Field Research Company lead to a general relationship that can be used until something better is available. This relationship is shown in Figure 9. An equation for the curve between saturation units of 0.2 and 0.5 is

$$k_r^o = \frac{k_{nwt}\Big]_{S_{wtr}}}{k} = 1.08 - 1.11\,S_{wtr} - 0.73\,(S_{wtr})^2 \qquad (9)$$

When the values of k_{rnwt} calculated from Equation 7 are multiplied by the value of k_r^o calculated from Equation 9, we have the relative permeability of the non-wetting phase in terms of the absolute permeability of the rock.

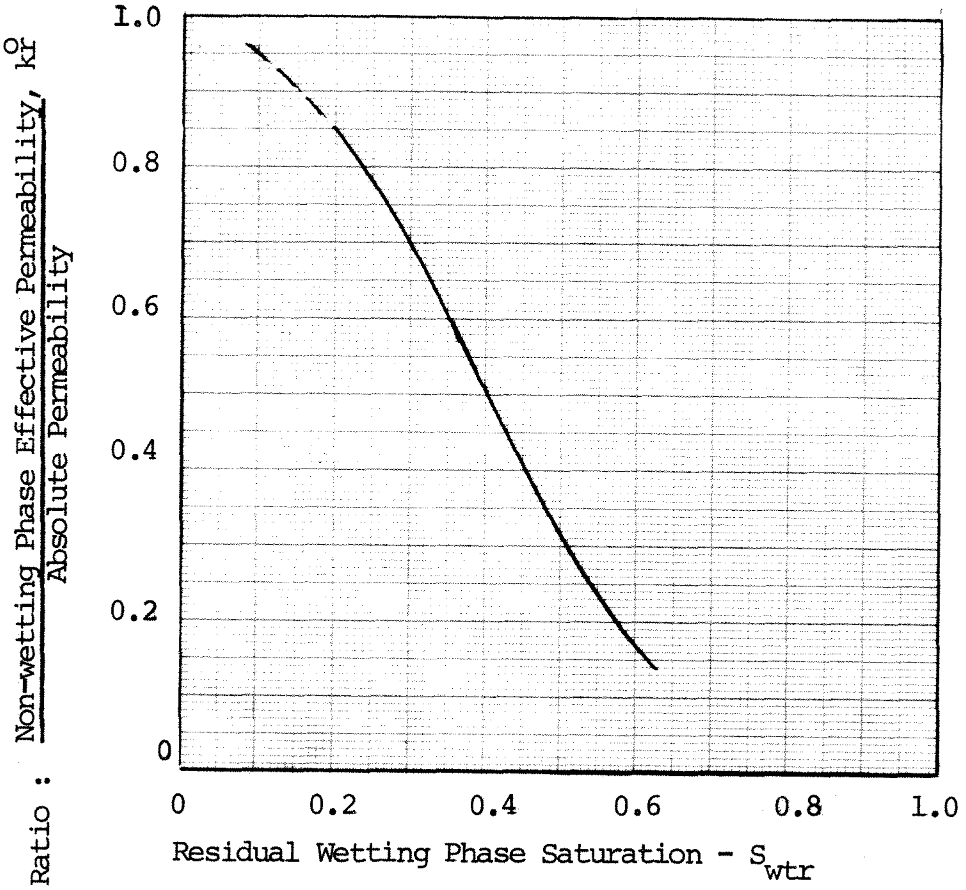

Figure 9. Average curve showing effect of residual wetting phase saturation on effective permeability of non-wetting phase.

Before considering application of the above relationships to reservoir systems is is necessary to account for so-called "critical non-wetting phase (gas) saturation" effects. This will be discussed next.

Critical Non-wetting Phase Effects. It is generally conceded that the non-wetting phase must have some finite saturation before k_{nwt} can have a non-zero value. The idea that at least one connected channel of pores must be full of non-wetting fluid before the fluid can flow. The saturation of non-wetting fluid necessary to permit flow is called "critical saturation." Most often one hears the term, "critical gas saturation", in connection with the flow of gas in a system of gas, oil, and water. However, critical saturation behavior applies to oil in an oil-water system equally well and it is best to think of the effect as being particular to the non-wetting fluid.

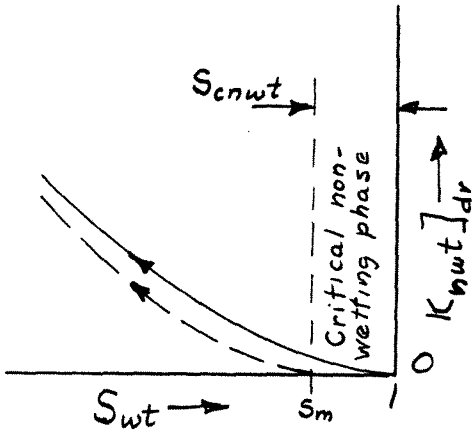

Figure 10

The requirement of a critical non-wetting phase saturation simply means that the non-wetting curve start from $S_{wt} = S_m$ in Figure 10 rather than from $S_{wt} = 1$. (Figure 10 is a blow-up of the lower right hand corner of Figure 4.) S_m is defined as the wetting phase saturation that marks the start of the non-wetting phase permeability curve. In terms of the critical non-wetting phase saturation, S_{cnwt}

$$S_m = 1 - S_{cnwt} \qquad (10)$$

The method of taking critical saturation into account in Equation 7 and Table I is to change the quantity $(1 - S_{wt}^*)^2$ so that it will have zero value at the critical saturation value. The result is

$$\left.k_{rnwt}\right]_{dr} = \left[1 - \frac{S_{wt} - S_{wtr}}{S_m - S_{wtr}}\right]^2 \left[1 - \left(S_{wt}^*\right)^{\frac{2+\lambda}{\lambda}}\right] \qquad (11)$$

Equation 11 is, of course, still on the basis of $\left.k_{nwt}\right]_{S_{wtr}}$. To place it on an absolute permeability basis only requires multiplication by k_r^o from Equation 9.

Corey and Rathjens(5) have shown that S_m values determined by back extrapolating laboratory determined relative permeability curves to $k_{rnwt} = 0$ are effected by stratification within the core. A given core sample always showed a higher value of S_m when fluid flow was parallel to bedding planes than when flow was perpendicular to bedding. Furthermore, cores that were highly stratified yielded S_m vlaues greater than unity. Within the concept that S_m is numerically equal to $(1 - S_{cnwt})$ (See Equation 10), having a value of S_m greater than unity is impossible. On the other hand, if S_m is viewed simply as a saturation variable that is

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(5) Corey, A.T. and Rathjens, C.H. "Effect of Stratification on Relative Permeability" Trans AIME 207 (1956), 353.

dependent on both stratification and critical saturation effects, and is used to limit the range of relative permeability values, then it makes sense for S_m to be greater than unity.

It should be noted that only the non-wetting phase permeabilities are affected by S_m. We do not apply the correction to the wetting phase relationships of Equation 6.

This section of the notes may appear to be complicated expressions having little use in practical engineering calculations. This is not so as will be illustrated in the section on application that follows.

Application of Two-Phase Drainage Relative Permeability Relationships

When using the relationships given so far one must keep in mind the conditions to which they apply. To reitterate, these conditions are

1. Two-phases. In petroleum reservoirs these would normally be gas-water and oil-water. Thus the relationships could apply to gas-cap conditions and to oil-zone conditions where only two phases are present.

2. Drainage. Saturation changes previous to the time of calculation and/or during the calculation time must be in the direction of decreasing wetting phase saturation. For example, it is generally believed that hydrocarbons migrate into and displace original water from petroleum bearing structures. Thus, drainage conditions would apply to calculations concerned with initial conditions found in the reservoir. A second example of drainage is injection of gas into an aquifer for gas storage purposes.

3. Wettability. One of the two phases in the pore structure must wet the rock matrix preferentially to the other. In the system gas-water, water is always the wetting phase. In oil-water systems, water is usually the wetting phase. However, some reservoir rocks appear to be preferentially oil wet, in which case Equation 6 would be used to calculate oil relative permeability values and Equation 7 would apply to water relative permeabilities. Note, however, that to use drainage relationships for oil-wet

reservoirs means that water saturation must increase—such as would occur under water flooding or aquifer influx conditions.

In preferentially water-wet reservoirs the wetting phase residual saturation, S_{wtr}, is analogous to irreducible water saturation, S_{iw}. Irreducible water saturation is approximated in most reservoir systems by the saturation corresponding to 50 psi capillary pressure in the gas-water system.

Two examples follow to illustrate the use of the relative permeability relationships presented to this point.

Example A. Calculation of Possible Water/Oil Ratio

Given: Conditions at a potential completion interval in a discovery well are believed to be as follows:

	Oil	Water
Fluid saturations, s	0.55	0.45
Irreducible saturation, S_{iw}	-	0.30
Fluid viscosities, μ	5 cp.	0.5 cp.
Form. vol. factor, B	1.5	1.05

If the producing water/oil ratio is greater than 1, it might not be profitable to complete the well in this interval. What will be the possible magnitude of the water/oil ratio?

Solution: The water/oil ratio can be obtained from radial flow equations for water and oil separately. That is,

$$q_w/q_o = \frac{k_w \ \mu_o \ B_o}{k_o \ \mu_w \ B_w}$$

Calculation of k_w and k_o yield

$$S_w^* = \frac{S_w - S_{iw}}{1 - S_{iw}} = \frac{0.45 - 0.30}{1 - 0.30} = 0.214$$

Assuming Corey equation with $\lambda = 2$

$$k_w = k \cdot k_{rw}\Big]_{dr} = k \left[S_w^*\right]^4 = 0.00021k$$

$$k_o = k \cdot k_r^o \cdot k_{ro}\Big]_{dr} = k \cdot k_r^o \cdot \left[1 - S_w^*\right]^2 \left[1 - (S_w^*)^2\right]$$

From Figure 9, $k_r^o = 0.70$

$$k_o = k \cdot 0.7 \left[1 - 0.214\right]^2 \left[1 - (0.214)^2\right] = 0.413\,k$$

$$q_w/q_o = \frac{0.0021\ k \cdot 5 \cdot 1.5}{0.413\ k \cdot 0.5 \cdot 1.05} = \underline{\underline{0.073}}$$ answer

Comments: Note the low effective water permeability (0.0021 k) even when mobile water saturation amounts to 15% of total pore space. This illustrates the asymtotic-to-zero shape of the water relative permeability curve when approaching the irreducible water saturation.

Example B. Calculation of Relative Gas Permeabilities

Given: A study of gas storage in an aquifer is being made. Requirement is for values of realtive gas permeabilities in the gas saturation range between 5 and 20 per cent. Capillary pressure tests indicate average pore size distribution index of 1.20 and irreducible water saturation of 0.20.

Solution: Method is to use Equations 7 and 9 in order to calculate relative gas permeabilities in terms of absolute permeability. Table 2 shows calculations. (Two sets of calculations are left for the student to do.)

TABLE 2 Calculation of Gas Relative Permeabilities--Example B

$S_{iw} = 0.20$; $S_w^* = (S_w - S_{iw})/(1 - S_{iw}) = (1.25\ S_w - 0.25)$; $S_m = 0.95$

$\lambda = 1.20$; $(2 + \lambda)/\lambda = 2.67$: $k_r^o = 1.08 - 1.11S_{iw} - 0.73(S_{iw})^2 = 0.83$

$$k_{rg} = \frac{k_g}{k} = k_r^o \left[1 - \frac{S_w - S_{iw}}{S_m - S_{iw}}\right]^2 \left[1 - (S_w^*)^{\frac{2+\lambda}{\lambda}}\right]$$

①	②	③	④	⑤	⑥	⑦
S_g	S_w^*	$\frac{S_w - S_{iw}}{S_m - S_{iw}}$	$\left[1 - \frac{S_w - S_{iw}}{S_m - S_{iw}}\right]^2$	$(S_w^*)^{2,67}$	$\left[1 - (S_w^*)^{2,67}\right]$	k_{rg}
0.05	0.938	1.000	0	0.843	0.157	0.000
0.08						
0.11	0.862	0.920	0.0064	0.673	0.327	0.00174
0.14	0.825	0.880	0.0144	0.598	0.402	0.00480
0.17						
0.20	0.750	0.800	0.040	0.465	0.535	0.0178

Three-Phase Drainage Relative Permeability Relationships

The theory outlined for two-phase drainage relative permeability relationships in a previous section can be extended to cover three-phase behavior. The resulting three-phase relationships find application in many reservoir engineering calculations that concern simultaneous flow of gas and oil in the presence of water.

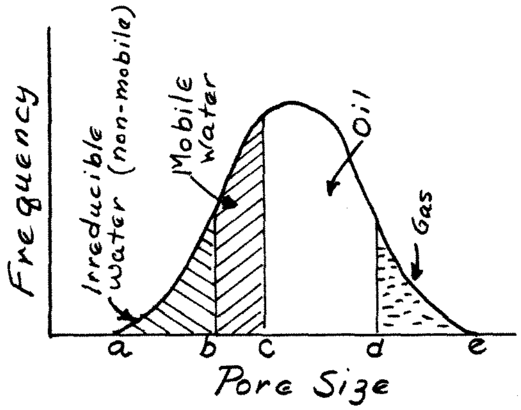

Figure 11

Figure 11 illustrates the basic concept of fluid location during flow. Irreducible water, which is considered to be the wetting phase, occupies pores of size range (a→b). Mobile water (free to move) occupies pore size range (b→c). Oil and gas occupy pore size ranges (c→d) and (d→e). As pointed out on page 3, k_{rw} depends on the amount of mobile water present, $(S_w - S_{iw})$; k_{rg} depends on the amount of gas present, S_g; but k_{ro} depends on both the amount of oil and pore range size in which it is located.

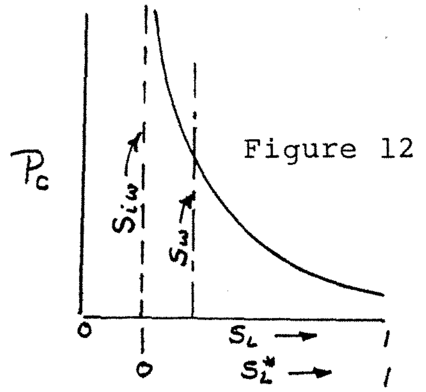

Figure 12

Figure 12 illustrates a curve of capillary pressure plotted against total liquid saturation. Total liquid is water plus oil phases. The water phase consists of irreducible (non-mobile) water and mobile water. The saturation equation can be written as

$$S_{iw} + (S_w - S_{iw}) + S_o + S_g = 1 \qquad (11)$$

The three mobile fluid saturations in Equation 11 can be converted to effective saturations as illustrated by the lowermost bar on page 4. These effective saturations are:

$$S_w^* = \frac{S_w - S_{iw}}{1 - S_{iw}} \; ; \quad S_o^* = \frac{S_o}{1 - S_{iw}} \; ; \quad S_g^* = \frac{S_g}{1 - S_{iw}}$$

A fourth effective saturation for total liquid is

$$S_L^* = \frac{S_L - S_{iw}}{1 - S_{iw}} = S_w^* + S_o^* \qquad (13)$$

Extending the ideas developed by Burdine(3) (see pages 7, 8, and 9) to the three mobile phases yields equations similar to Equations 4 and 5—the major difference being that total effective saturation, S_L^*, is the independent variable. These are:

For the mobile water phase:

$$\frac{k_w}{k_w\big]_{S_w=1}} = k_{rw} = (S_w^*)^2 \frac{\int_0^{S_w^*} \frac{1}{P_c^2}\, dS_L^*}{\int_0^1 \frac{1}{P_c^2}\, dS_L^*} \qquad (14)$$

For the oil phase:

$$\frac{k_o}{k_o\big]_{S_o^*=1}} = k_{ro} = (S_o^*)^2 \frac{\int_{S_w^*}^{S_L^*} \frac{1}{P_c^2}\, dS_L^*}{\int_0^1 \frac{1}{P_c^2}\, dS_L^*} \qquad (15)$$

For the gas phase:

$$\frac{k_g}{k_g\big]_{S_g^*=1}} = k_{rg} = (S_g^*)^2 \frac{\int_{S_L^*}^{1} \frac{1}{P_c^2}\, dS_L^*}{\int_0^1 \frac{1}{P_c^2}\, dS_L^*} \qquad (16)$$

Note that the differences in Equations 14, 15, and 16 are the effective fluid saturations squared and the limits of integration of the upper integral expression. Note also the mobile water relative permeability is in terms of absolute permeability while the two hydrocarbon relative permeabilities are in terms of k_r^o.

To obtain solutions to the above equations requires an important assumption. It is that the capillary pressure-total liquid saturation curve obtained when gas displaces oil in the presence of water will be the same as when gas displaces water with no oil present. This means, in effect, that there will be zero residual oil phase remaining when capillary pressure is sufficient to get to irreducible water saturation. Undoubtedly this will not occur, but apparently the effect is small enough that useable relationships are obtained. At least the data presented by Corey, et al(6) on measurements on Berea sandstone samples bear this out.

As outlined previously, when the pore size distribution index, λ, is known (See page 6-7) Equations 14, 15, and 16, can be integrated directly. The following integrated expressions are on the basis of absolute permeability, k, for all three phases. The gas relationship in Equation 19 includes S_m as a variable.

For the mobile water phase:

$$k_{rw}\Big]_{dr} = \frac{k_w}{k} = \left(\frac{S_w - S_{iw}}{1 - S_{iw}}\right)^{\frac{2+3\lambda}{\lambda}} \tag{17}$$

For the oil phase:

$$k_{ro}\Big]_{dr} = \frac{k_o}{k} = k_r^o \left(\frac{S_o}{1 - S_{iw}}\right)^2 \left[\left(\frac{S_o + S_w - S_{iw}}{1 - S_{iw}}\right)^{\frac{2+\lambda}{\lambda}} - \left(\frac{S_w - S_{iw}}{1 - S_{iw}}\right)^{\frac{2+\lambda}{\lambda}}\right] \tag{18}$$

For the gas phase:

$$k_{rg}\Big]_{dr} = \frac{k_g}{k} = k_r^o \left(\frac{S_g + S_m - 1}{S_m - S_{iw}}\right)^2 \left[1 - \left(\frac{S_o + S_w - S_{iw}}{1 - S_{iw}}\right)^{\frac{2+\lambda}{\lambda}}\right] \tag{19}$$

Note that if $S_w = S_{iw}$ (no mobile water) Equation 17 reduces to zero and Equations 18 and 19 are simplified somewhat. Figure 13 illustrates the shape of the k_{ro} and k_{rg} curves for S_{iw} values of 0.2 and 0.4 calculated from Equations 18 and 19.

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(6) Corey, A.T., Rathjens, C.H., Henderson, J.H., and Wyllie, M.R.J. "Three-Phase Relative Permeability" Trans. AIME 207 (1956) page 349.

Note that the gas curve is not affected by the water saturation but the oil curve is affected drastically.

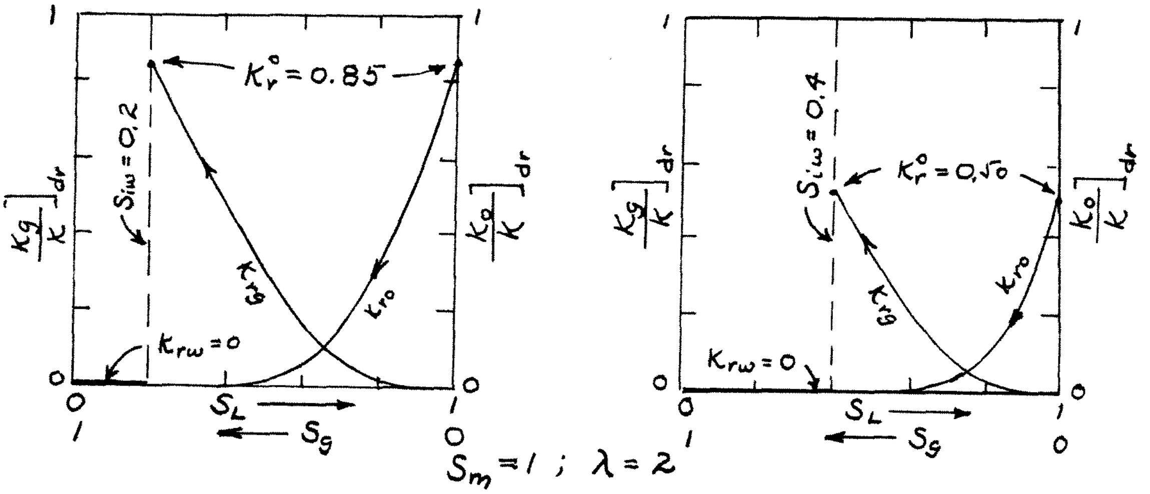


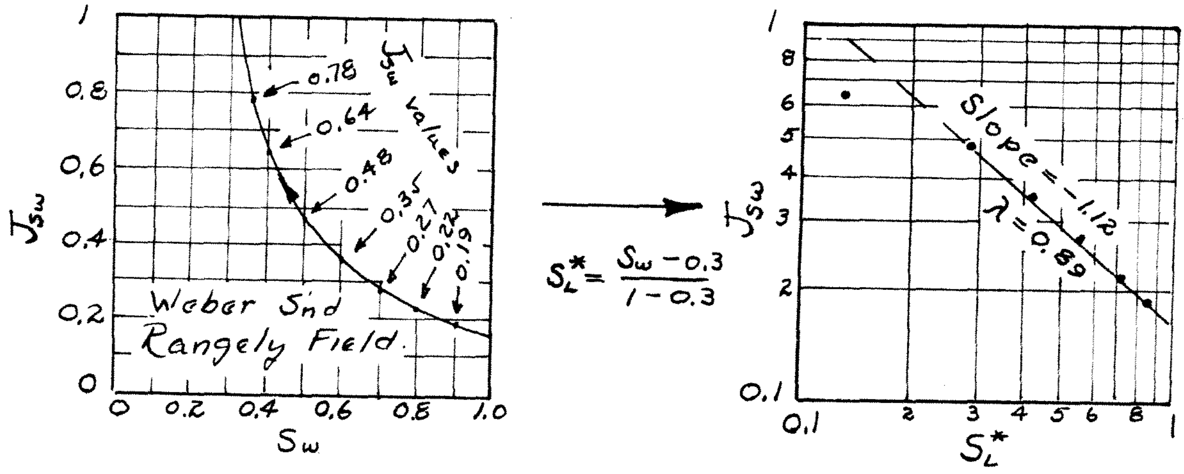
Figure 13 Three-phase drainage relative permeability curves

Example C that follows illustrates the calculation of drainage k_g/k_o vs. S_g data such as used in gas depletion type reservoirs. The basic data for the calculation is the Leverette J function curve for the Rangley Field shown on page 156 of Amyx, Bass, and Whiting.

Example C. Calculation of Drainage k_g/k_o vs S_g Relationship. Weber Sandstone, Rangely Field, Colorado

Given: Leverett J_{SW} vs S_w curve, page 156, ABW.
$S_{iw} = 0.30$; $S_w = 0.36$; $S_m = 1$; k_g/k_o required for gas saturations $0.01 < S_g < 0.11$

Solution:



$\lambda = 0.89 \; ; \quad \frac{2+\lambda}{\lambda} = 3.25 \; ; \quad \frac{2+3\lambda}{\lambda} = 5.25$

$$\left(\frac{S_w - S_{iw}}{1 - S_{iw}}\right) = \left(\frac{0.36 - 0.30}{1 - 0.30}\right) = 0.086$$

$$k_{rw}\Big]_{dr} = \frac{k_w}{k} = \left(\frac{S - S_{iw}}{1 - S_{iw}}\right)^{\frac{2+3\lambda}{\lambda}} = (0.086)^{5.25} = 2.5(10^{-6})$$

$$k_{ro}\Big]_{dr} = \frac{k_o}{k} = k_r^{\circ}\left(\frac{S_o}{1-S_{iw}}\right)^2\left[\left(\frac{S_o + S_w - S_{iw}}{1 - S_{iw}}\right)^{\frac{2+\lambda}{\lambda}} - \left(\frac{S_w - S_{iw}}{1 - S_{iw}}\right)^{\frac{2+\lambda}{\lambda}}\right] \quad \text{Eq 18}$$

$k_r^{\circ} = 0.70$ (Figure 9)

S_g	S_o	$\left(\frac{S_o}{1-S_{iw}}\right)^2$	$\left(\frac{S_o+S_w-S_{iw}}{1-S_{iw}}\right)$	$\left(S_o + S_w - S_{iw}\right)^{3.25}$	$\left(\frac{S_w-S_{iw}}{1-S_{iw}}\right)^{3.25}$	$\frac{k_o}{k}$
0.01	0,63	0,81	0,986	0,955	3.5(10⁻⁴) ↓	0,542
0.03	0,61	0,76	0,957	0,867		0,461
0.05	0,59	0,71	0,929	0,787		0,391
0.07	0,57	0,66	0.900	0,710		0,328
0.09	0,55	0,62	0,871	0.639		0,277
0.11	0,53	0,57	0,842	0,575		0,229

$$k_{rg}\Big]_{dr} = \frac{k_g}{k} = k_r^{\circ}\left(\frac{S_g + S_m - 1}{S_m - S_{iw}}\right)^2\left[1 - \left(\frac{S_o + S_w - S_{iw}}{1 - S_{iw}}\right)^{\frac{2+\lambda}{\lambda}}\right] \quad \text{Eq. 19}$$

$k_r^{\circ} = 0.70 \; ; \quad S_m = 1$

S_g	$\left(\frac{S_g}{1-S_{iw}}\right)^2$	$\left[1-\left(\frac{S_o+S_w-S_{iw}}{1-S_{iw}}\right)^{3.25}\right]$	$\frac{k_g}{k}$
0.01	0.0002	0.045	6.30(10⁻⁶)
0.03	0.0018	0.133	1.67(10⁻⁴)
0.05	0.0051	0.213	7.60(10⁻⁴)
0.07	0.0100	0.290	2.03(10⁻³)
0.09	0.0165	0.361	4.17(10⁻³)
0.11	0.0247	0.426	7.21(10⁻³)

S_g	$\frac{k_g}{k_o}$
0.01	1.16(10⁻⁵)
0.03	3.64(10⁻⁴)
0.05	1.94(10⁻³)
0.07	6.19(10⁻³)
0.09	1.51(10⁻²)
0.11	3.09(10⁻²)

answer.

Use of Corey Type Equations in Averaging and Extrapolating Laboratory Measured k_{rg}/k_{ro} Data

The petroleum engineer is often faced with the problem of adjusting results of laboratory measured k_{rg}/k_{ro} data to conditions other than those measured. For example, he may have k_{rg}/k_{ro} data taken with 20 per cent water saturation but needs to make calculations for a reservoir condition of 30 per cent water. How does he adjust the measured values to correctly account for the additional water? A second example is that he has measured k_{rg}/k_{ro} data at values from 0.1 to 100 but finds that he needs values in the region of 0.005. How does he extrapolate the measured data to lower ratios? (0.05 represents about the lowest value of k_{rg}/k_{ro} that can be determined in the laboratory.) A final example is that the engineer has k_{rg}/k_{ro} data on, say, five core samples, each of which contained a different amount of water. How can he use the data to get an average set of curves that can be used for any given water saturation?

The crux of the above is that while measured data are sometimes (not too often, however) available, the data often must be adjusted to the conditions being calculated. Methods of making such adjustments are the subject of this section.

Averaging k_{rg}/k_{ro} Data. Laboratory measured k_{rg}/k_{ro} ratio data are most often reported as a function of gas saturation, S_g. Water saturation for the conditions of measurement will be given and usually will be close to the irreducible water saturation, S_{iw}. Where a number of cores were used in the measurement program it is probable that each core contained a different amount of water. As pointed out in the previous section, k_{rg} depends primarily on the amount of gas present but k_{ro} depends primarily on the amount of oil and water.

Figure 14 illustrates three k_{rg}/k_{ro} curves, plotted on semi-logarithmic coordinates. Water saturation for each curve is different. The first step in getting an average curve is to remove the effect of the different water saturations

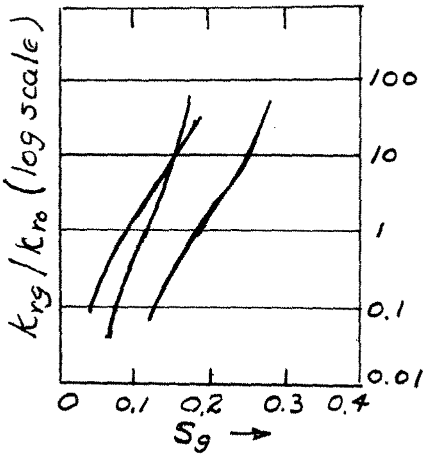


Figure 14

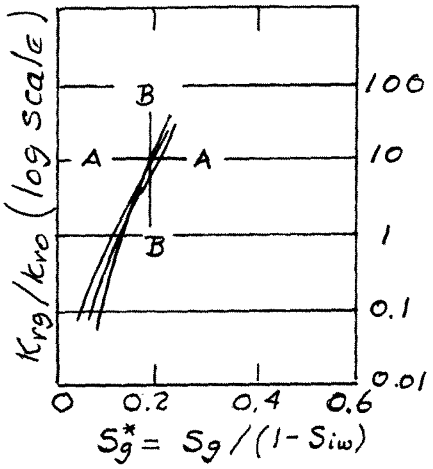


Figure 15

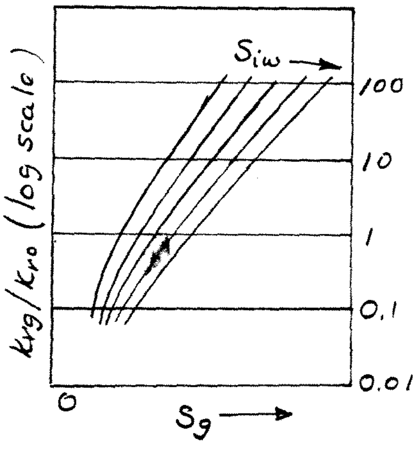


Figure 16

by normalizing gas saturation to the hydrocarbon pore volume, $(1 - S_{iw})$. This places the curves on a more common base of effective gas saturation, S_g^*

$$S_g^* = \frac{S_g}{1 - S_{iw}} \qquad (20)$$

Values of k_{rg}/k_{ro} are then plotted against S_g^* as illustrated in Figure 15. This preserves the shape of the curves but groups them closer together.

The average curve is constructed through average values k_{rg}/k_{ro} and S_g^*. The easiest way is to calculate the arithmetic average of S_g^* values at given values of k_{rg}/k_{ro} such as 0.1, 0.3, 1, 3, 10, etc. and to use these as the control points for the average curve. This is illustrated in Figure 15 as averaging along the line A-A. The other way is to select a number of S_g^* values and calculate the geometric average of the k_{rg}/k_{ro} values. This is illustrated in Figure 15 as averaging along the B-B line.

Having determined the average curve of k_{rg}/k_{ro} vs S_g^* it is a simple matter to construct a smooth curve of k_{rg}/k_{ro} vs S_g for various constant values of S_{iw}. The plot will have the appearance of Figure 16.

Gas depletion drive calculations make use of k_{rg}/k_{ro} vs S_g data. Such calculations are usually called Tarner calculation or Muskat calculations. For most reservoir systems the Tarner and Muskat calculations will not need k_{rg}/k_{ro} data at values greater than 5. Therefore, when averaging laboratory data spend most effort on averaging the low values of k_{rg}/k_{ro} and little effort on the high values.

A second method of averaging k_{rg}/k_{ro} vs S_g data will be presented after the method of extrapolating relative permeability ratios has been presented.

Extrapolating k_{rg}/k_{ro} Data. It is difficult to measure k_{rg}/k_{ro} on core samples in the laboratory where the value is less than about 0.05. Most laboratory data fall between values of 0.1 and 100. Most reservoir calculation require values between 0.001 and 1. The problem is, how to extrapolate to lower k_{rg}/k_{ro} values in a consistent, reproducible manner that has some scientific basis.

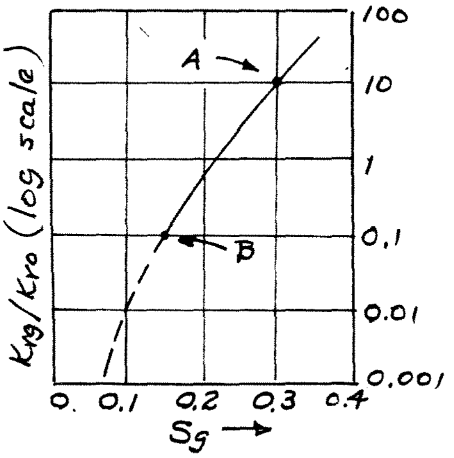


Figure 17

One method of extrapolating is graphical—use a french curve and extend the line. This method is satisfactory if the right french curve is used and the right extrapolation is made. The method is not considered to be a consistent and reproducible one.

The method outlined below is based on the work of C.E. Johnson(7) of the Chevron Oil Field Research Company, and allows one to make the extrapolation mathematically. Thus, it is consistent and reproducible, and, in addition, has some scientific basis.

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(7) Johnson, C.E. "A Two-Point Graphical Determination of the Constants S_{Lr} and S_m in the Corey Equation for Gas-Oil Relative Permeability Ratio" Journal of Petroleum Technology, October 1968.

As mentioned previously, the so-called Corey Equations are general expressions for k_{rg} and k_{ro} for a pore size distribution index, λ, of 2. An equation for the ratio, k_{rg}/k_{ro} written in general terms is,

$$k_{rg}/k_{ro} = \frac{\left[1-\left(\frac{S_L - S_{Lr}}{S_m - S_{Lr}}\right)\right]^2 \left[1-\left(\frac{S_L - S_{Lr}}{1 - S_{Lr}}\right)^2\right]}{\left(\frac{S_L - S_{Lr}}{1 - S_{Lr}}\right)^4} \qquad (21)$$

The reader will recognize the similarity of this equation and Equations 18 and 19.

In using Equation 21, S_m and S_{Lr} are considered simply as two variables in the Corey ratio equation that relates the ratio k_{rg}/k_{ro} to total liquid saturation S_L. (Of course, $S_L = 1 - S_g$). For example, the point A in Figure 17 has a value of k_{rg}/k_{ro} of 10, and a corresponding value of S_L of 0.70. This combination of k_{rg}/k_{ro} and S_L could be satisfied by any number of combinations of S_m and S_{Lr} and fill the requirements of Equation 21. Similarly, point B in Figure 17 ($k_{rg}/k_{ro} = 0.1$; $S_L = 0.85$) can be fit by Equation 21 and many combinations of S_m and S_{Lr}. However, there is only one combination of S_m and S_{Lr} that will fit both points A and B. In effect, two unknowns in Equation 21 can be determined by having two solutions of the equation.

Johnson prepared three charts that are used to determine constants S_m and S_{Lr}. These are given as Figures 18, 19, and 20. In Figure 18, the gas saturation at which $k_{rg}/k_{ro} = 10$ is compared against the gas saturation at which $k_{rg}/k_{ro} = 0.1$. S_m and S_{Lr} values that fit these conditions are read from the grid. For example, values given by Points A and B on Figure 17 yield $S_m = 0.95$; $S_{Lr} = 0.5$. Similar comparisons are made for k_{rg}/k_{ro} ratios of 1 and 0.01, and 0.1 and 0.001 by Figures 19 and 20. If essentially the same values of S_m and S_{Lr} are indicated over the whole data range,

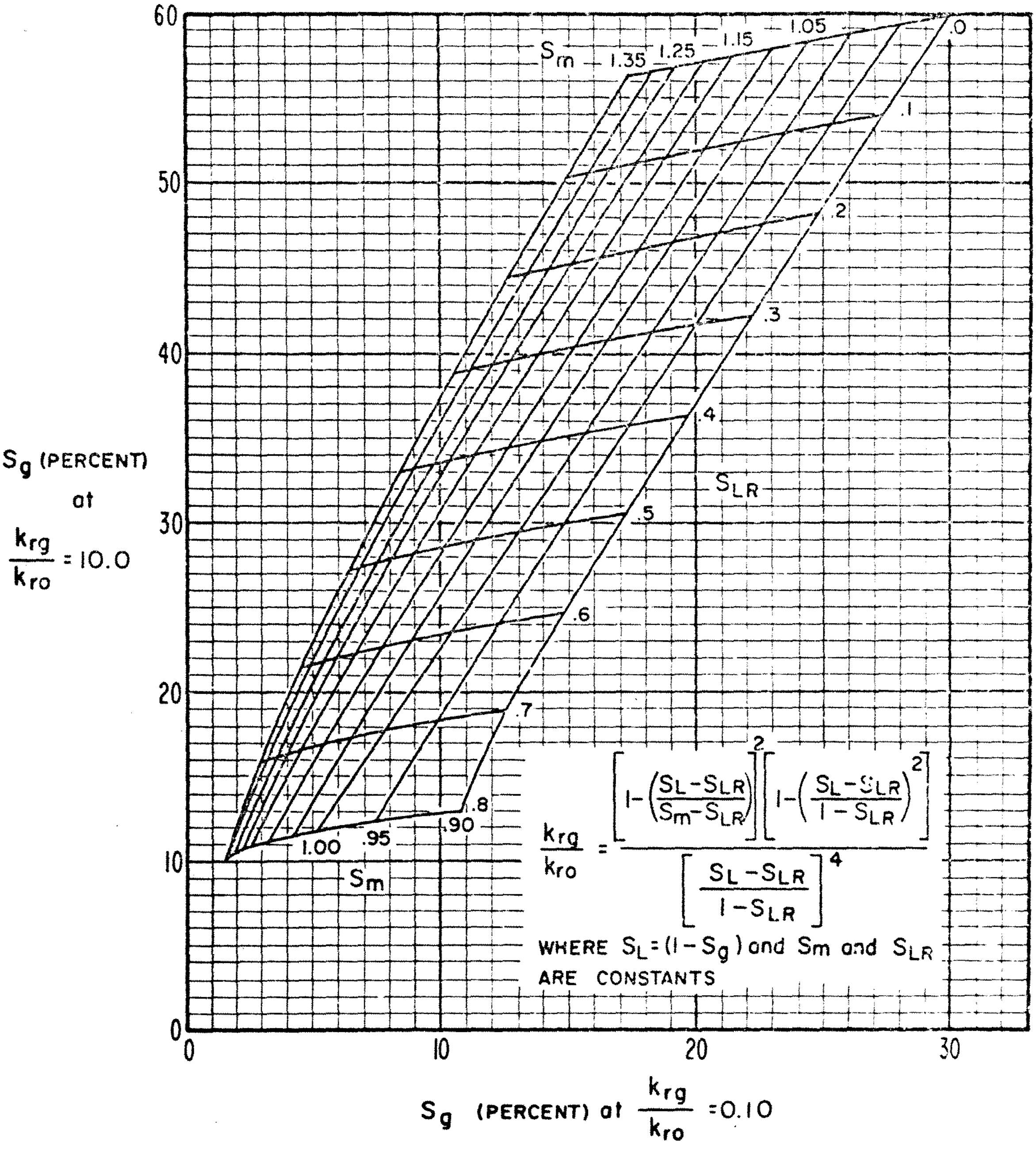


Figure 18. Chart for Calculating the Constants S_{Lr} and S_m in the Corey Relative Permeability Equation When k_{rg}/k_{ro} Have the Value of 10 and 0.1.

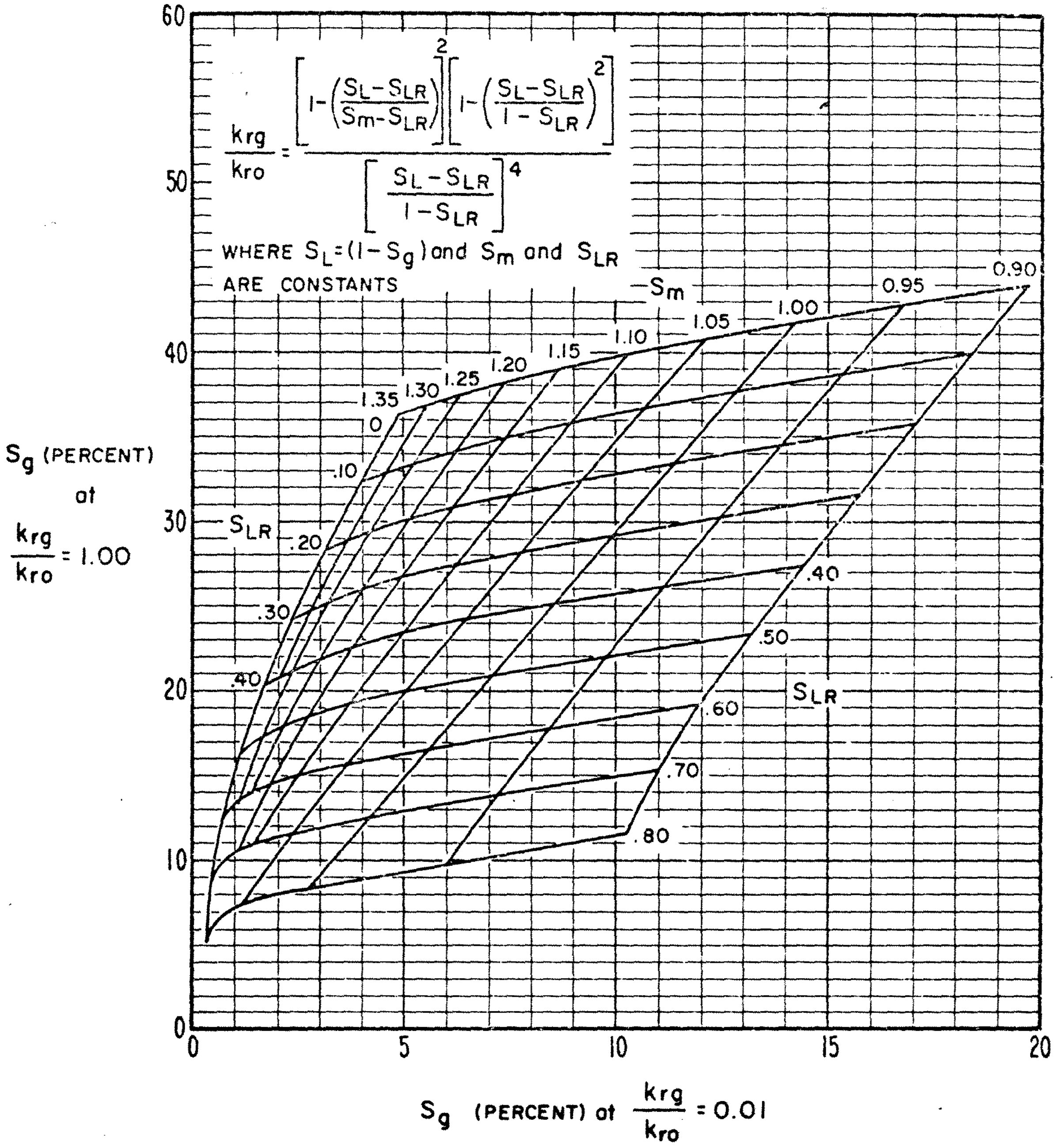


Figure 19. Chart for Calculating the Constants S_{Lr} and S_m in the Corey Relative Permeability Equation When k_{rg}/k_{ro} Have the Value of 1 and 0.01.

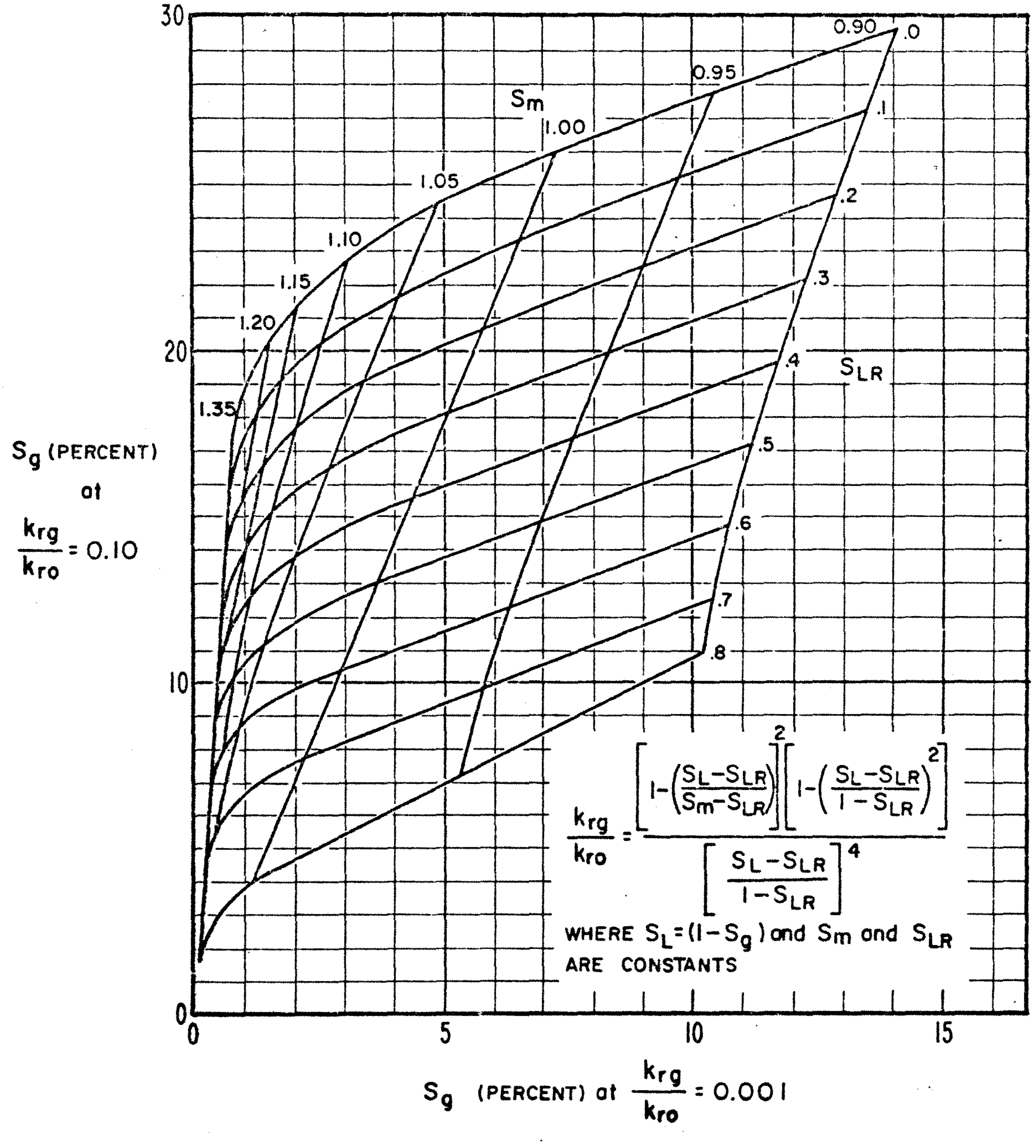


Figure 20. Chart for Calculating the Constants S_{Lr} and S_m in the Corey Relative Permeability Equation When k_{rg}/k_{ro} Have the Value of 0.1 and 0.001.

the values may be averaged and used to calculate the extrapolated part of the curve. This procedure is illustrated by the following example.

Example D. Extrapolation of k_{rg}/k_{ro} vs S_g. Data of C.R. Knopp (Trans AIME, 234 (1965) 1111)

Given: k_{rg}/k_{ro} vs S_g values taken from his Figure 5A. These are, for $S_{iw} = 0.10$.

k_{rg}/k_{ro}	S_g	k_{rg}/k_{ro}	S_g
10	0.435	0.1	0.155
1	0.300	0.01	0.070

Solution:

k_{rg}/k_{ro}	S_g		S_m	S_{Lr}
10	43.5	Fig 18 →	1.14	0.24
1	30.0			
0.1	15.5	Fig 19 →	1.10	0.24
0.01	7.0	Avg.	1.12	0.24

Using $\bar{S}_m = 1.12$; $\bar{S}_{Lr} = 0.24$ and Equation 21 yield the following calculated values of k_{rg}/k_{ro}

$$k_{rg}/k_{ro} = \frac{\left[1-\left(\frac{S_L-0.24}{0.88}\right)\right]^2\left[1-\left(\frac{S_L-0.24}{0.76}\right)^2\right]}{\left(\frac{S_L-0.24}{0.76}\right)^4}$$

S_g	S_L	$\frac{S_L-0.24}{0.76}$	$\frac{S_L-0.24}{0.88}$	$\left(1-\frac{S_L-0.24}{0.88}\right)^2$	$1-\left(\frac{S_L-0.24}{0.76}\right)^2$	$\left(\frac{S_L-0.24}{0.76}\right)^4$	Calc. k_{rg}/k_{ro}	Knopp k_{rg}/k_{ro}
0.435	0.565	0.428	0.369	0.398	0.817	0.0336	9.68	10
0.300	0.700	0.605	0.523	0.224	0.634	0.134	1.06	1
0.155	0.845	0.796	0.688	0.0973	0.366	0.401	0.089	0.1
0.070	0.930	0.908	0.784	0.0467	0.176	0.680	0.0121	0.01
0.060	0.940	0.921	0.795	0.0420	0.152	0.720	0.0089	—
0.040	0.960	0.947	0.818	0.0331	0.103	0.804	0.0042	—
0.020	0.980	0.974	0.841	0.0253	0.0513	0.900	0.0014	—
0.010	0.990	0.987	0.852	0.0219	0.0258	0.949	0.0006	—

Two things are apparent from the calculations in Example D. First, the two values of S_{Lr} of 0.24 are the same for the high and low portion of the k_{rg}/k_{ro} curve. Note, however, that S_{Lr} is 2.4 times the value of S_{iw}. Do not interpret the difference of S_{Lr} and S_{iw} as being residual oil, S_{or}. In this use of the Corey ratio equation, S_{Lr} is a variable obtained when fitting a particular equation to set of data and should not be interpreted in a physical sense.

The second thing to notice is that two values of S_m were obtained. This means that a single Corey ratio equation does not fit the full range of data. Taking the average value of S_m of 1.12 yields a "best fit" curve to the data over the full range. Extrapolated values of k_{rg}/k_{ro} at gas saturations less than 0.06 were calculated using the average S_m of 1.12. An equally valid choice would have been to extrapolate the measured data by using an S_m value of, say, 1.10 to represent the lower curve. Also, one could have assigned weighting factor to the S_m values in order to get a "weighted" curve. For example, the engineer may wish to give twice the weight to the lower end of the k_{rg}/k_{ro} curve than to the upper end. The average value of S_m would then be

$$\bar{S}_m = \frac{1.14 \cdot 1 + 1.10 \cdot 2}{3} = 1.113.$$

Averaging k_{rg}/k_{ro} Data. Method 2. A method of developing an average k_{rg}/k_{ro} vs S_g^* relationship was discussed earlier. The method that follows is an alternate method that makes use of the Corey ratio equation as in the last section. This method is particularly useful where one wishes to simultaneously average, smooth, and extrapolate a group of laboratory k_{rg}/k_{ro} data.

The basis of the method is to determine S_m and S_{Lr} values for each k_{rg}/k_{ro} vs S_g curve by use of Johnson's charts (Figures 18, 19, 20). The arithmetic average of all S_m values is then determined and used to calculate the final curve. The ratio $\bar{S}_{Lr}/S_{iw}$ is determined for each curve ($\bar{S}_{Lr}$ being the average of the S_{Lr} values obtained for a given curve and S_{iw} being the water saturation in the core at the time of testing) and plotted against S_{iw} . The trend of $\bar{S}_{Lr}$ / S_{iw} vs S_{iw} is used to obtain a value of S_{Lr} at the desired water saturation. Values of k_{rg}/k_{ro} vs S_g at the desired water saturation are then calculated using Equation 21 or Johnson's charts.

Example E illustrates the steps outlined above. In this example, k_{rg}/k_{ro} vs S_g curves were determined in the laboratory using three core samples from the formation of interest. The water saturation in each core at the time of testing corresponded to the irreducible saturation for that core. The average water saturation in the reservoir, however, is different than any of the tested values. The object is to produce a single composit k_{rg}/k_{ro} curve, adjusted to reservoir water saturation, that is usable in reservoir calculations.

Example E. Determination of Average Reservoir k_{rg} /k_{ro} Curve from Laboratory Data

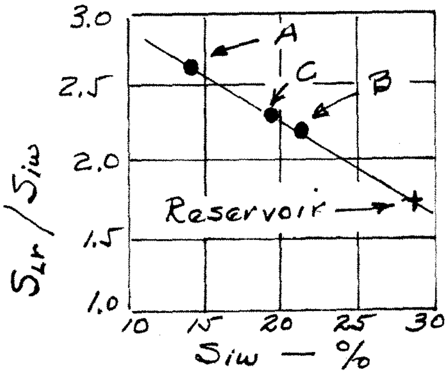
Given: 1. Reservoir water saturation = 0.28

2. Following data read from laboratory measurement of k_{rg}/k_{ro} vs S_g

		Core		
		A	B	C
S_{iw} at test		0.14	0.21	0.19
S_g at k_{rg}/k_{ro} ratio of				
	10	0.380	0.315	0.340
	1	0.265	0.238	0.252
	0.1	0.165	0.168	0.180
	0.01	0.090	0.110	0.125
	0.001	0.040	0.070	0.090

Solution:

	Core					
	A		B		C	
k_{rg}/k_{ro} comparisons	S_m	S_{Lr}	S_m	S_{Lr}	S_m	S_{Lr}
10 / 0.1 (Fig. 18)	1.02	0.35	0.92	0.48	0.92	0.43
1 / 0.01 (Fig. 19)	1.00	0.37	0.94	0.47	0.92	0.43
0.1 / 0.001 (Fig. 20)	1.02	0.35	0.96	0.42	0.93	0.41
Avg $\bar{S}_m$	1.01		0.94		0.92	
Avg $\bar{S}_{Lr}$		0.357		0.457		0.423
Avg S_{Lr} /S_{iw}		2.55		2.18		2.23



From trend plot of $\overline{S}_{Lr}/S_{iw}$ vs S_{iw}, reservoir ratio of S_{Lr}/S_{iw} = 1.75

For Reservoir

$$\overline{\overline{S}}_m = \frac{1.01 + 0.94 + 0.93}{3} = 0.96$$

$$\overline{S}_{Lr} = 1.75 \cdot 0.28 = 0.49$$

Calculation, Average Curve:

$\overline{\overline{S}}_m = 0.96$; $\overline{S}_{Lr} = 0.49$: $S_{iw} = 0.28$

From Figures 18 - 20

k_{rg}/k_{ro}	S_g
10	0.305
1	0.222
0.1	0.147
0.01	0.095
0.001	0.066

This concludes the notes on drainage relative permeability relationships. The next section will consider imbibition relative permeabilities relationships.

Theory of Two - Phase Imbibition Relative Permeabilities

Imbibition relative permeabilities apply when the wetting phase is, or has been, increasing in magnitude. The most important use of imbibition values is in waterflood calculations where water (wetting phase) is displacing oil (non-wetting phase). A similar application of imbibition values occurs in calculations concerned with influx of aquifer water into gas reservoirs.

The most important early work on imbibition relationships was that of Naar and Henderson(8). More recently C.S. Land(9) (10) of the U.S. Bureau of Mines has extended the earlier work. The notes that follow essentially reproduce Land's work.

The very earliest laboratory work (1950) of measuring relative permeabilities showed that direction of saturation change has an important bearing on the value of the relative permeabilities at a given saturation. This is illustrated by the two non-wetting phase curves of Figure 21.

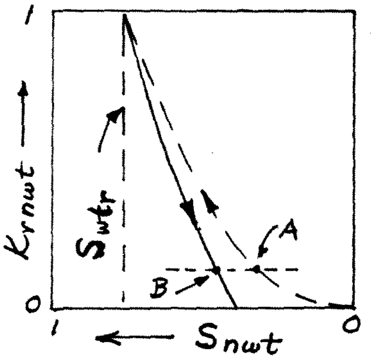


Figure 21

The reason for this behavior results from the sequence in which pores of given sizes are desaturated and resaturated during the saturation changes involved. Under drainage operations capillary forces and viscous forces both operate in the direction to promote desaturation of the largest pores first, followed by progressive desaturation of smaller and smaller pores. Under imbibition operations capillary forces and viscous forces operate, in effect,

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(8) Naar, J. , and Henderson, J.H. "An Imbibition Model--Its Application to Flow Behavior and the Prediction of Oil Recovery" Trans AIME 222 (1961) 61.

(9) Land, C.S. "Calculation of Imbibition Relative Permeability for Two- and Three-Phase Flow from Rock Properties" Trans AIME 251 (1971) II, 149.

(10) Land, C.S., "Comparison of Calculated with Experimental Imbibition Relative Permeability" Trans AIME 251 (1971) II, 419.

in opposite directions. Capillary forces tend to cause resaturation of the smallest pores first while viscous forces favor resaturation of largest pores first. The net effect is that during the imbibition process a portion of the non-wetting phase becomes trapped within the pore structure and is unable to move. The result can be seen by referring to the dashed horizontal line in Figure 21. Point A represents the non-wetting phase saturation required to yield the relative permeability value indicated by the dashed line when the non-wetting phase saturation has changed from $S_{nwt} = 0$ to $S_{nwt} = A$. Continuing to desaturate to residual wetting phase saturation, S_{wtr}, and then resaturating back to the same relative permeability value (Point B) results in a greater non-wetting phase saturation because part of the non-wetting phase (saturation B minus saturation A) is trapped and does not contribute to k_{rnwt}.

The crux of Land's method of calculating two- and three-phase relative permeabilities is to correct total non-wetting phase saturation for the amount of trapped phase. The resulting "free" saturation is than used to calculate relative permeability using the same basic equations discussed previously for drainage conditions. The results seem to be quite good as indicated by the data in Reference 10.

The material that follows is presented in terms of a gas-water two-phase system in which water is the wetting phase. The reasons for doing this is that the number of subscripts are reduced (over wetting and non-wetting) and the equations are easier to compare with Land's equations. However, it should be remembered that the gas relationships apply vis-a-vis to oil in a two-phase oil-water system provided the oil-water system is strongly water wet also.

Trapped Gas Saturation. A number of technical papers have shown that initial gas saturation, S_{gi}, and residual gas saturation, S_{gr}, left after imbibition are related. The general shape of the relationship is shown in Figure 22, where

$$S^*_{gi} = S_{gi}/(1 - S_{iw}) \qquad (22)$$

$$S^*_{gr} = S_{gr}/(1 - S_{iw}) \qquad (23)$$

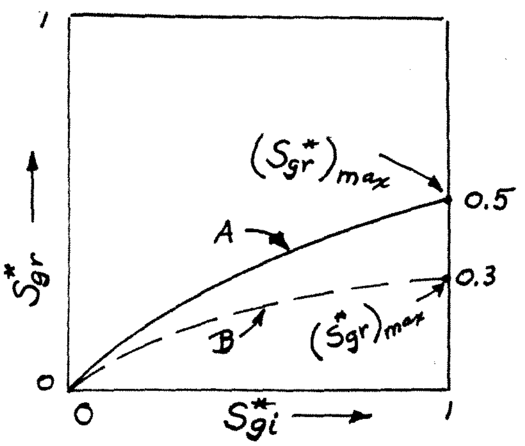


Figure 22

Land(9) found that a general equation for the relationship is

$$1/S^*_{gr} - 1/S^*_{gi} = C \qquad (24)$$

In equation 24, C is a "trapping constant" and is a function of the particular rock involved. (It probably has something to do with pore size distribution.) The value of C can be determined by simple laboratory drainage and imbibition experiments. If $(S^*_{gr})_{max}$ represents the effective residual gas saturation in the rock after imbibing water from irreducible water saturation, S_{iw}, the value of C is, from Equation 24

$$C = 1/(S^*_{gr})_{max} - 1 \qquad (25)$$

By way of illustration, the two curves of Figure 22 have the following trapping constants:

Curve	(S^*_{gr})	C
A	0.5	1.0
B	0.3	2.33

Note that if no trapping occurs (This will be $S^*_{gr} = 0$) the value of C becomes infinity.

In the absence of laboratory results to determine the trapping constant it is probably best to use a value between 1 and 3. The higher values correspond to an often used rule of thumb in waterflood calculations that the residual oil left in a core after many pore volumes of water throughput will be about 20 per cent of the initial oil at the start of displacement.

The trapping constant, C, is an important parameter in the imbibition relative permeability relationships developed in the next section. When one specifies the value of C to be used he automatically fixes the relative permeability curve limits.

Imbibition Relationships, Non-Wetting Phase. Referring back to Equations 5 and 7 onpage 8, the *drainage* gas relative permeability, written in terms of wetting phase saturation units is (changing to gas terms)

$$k_{rg}\Big]_{dr} = (1 - S_w^*)^2 \left[1 - (S_w^*)^{\frac{2+\lambda}{\lambda}}\right] \qquad (26)$$

Recognizing that in a two-phase system $S_w^* + S_g^* = 1$, Equation 26 can be rewritten as

$$k_{rg}\Big]_{dr} = (S_g^*)^2 \left[1 - (1 - S_g^*)^{\frac{2+\lambda}{\lambda}}\right] \qquad (27)$$

The expression for the *imbibition* gas relative permeability is similar to Equation 27 except the gas saturation must be expressed in terms of "free" gas saturation units. This leads to

$$k_{rg}\Big]_{imb} = (S_{gF}^*)^2 \left[1 - (1 - S_{gF}^*)^{\frac{2+\lambda}{\lambda}}\right] \qquad (28)$$

To make use of Equation 28 requires that the "free" gas saturation be known as a function of total gas saturation. To do this we can say, first, that total gas saturation is equal to free gas plus trapped gas saturation. An expression for this is

$$S_g^* = S_{gF}^* + S_{gt}^* \qquad (29)$$

where S_{gt}^* is the trapped gas saturation.

A second relationship is that the trapped gas saturation, at any total gas saturation value, is equal to the residual gas saturation present when $k_{rg}\big]_{imb} = 0$ minus the amount of free gas that gets trapped during saturation change from S_g^* to S_{gr}^*. The equation for this behavior is

$$S_{gt}^* = S_{gr}^* - \frac{S_{gF}^*}{CS_{gF}^* + 1} \qquad (30)$$

Eliminating S_{gt}^* between Equations 29 and 30 results in a quadratic in S_{gF}^*, from which

$$S_{gF}^* = \tfrac{1}{2}\left[(S_g^* - S_{gr}^*) + \sqrt{(S_g^* - S_{gr}^*)^2 + \frac{4}{C}(S_g^* - S_{gr}^*)}\right] \qquad (31)$$

The third and final equation that is required evaluates the amount of residual gas saturation in terms of the starting, or initial gas saturation, S^*_{gi}. It is

$$S^*_{gr} = S^*_{gi}/(C\ S^*_{gi} + 1) \qquad (32)$$

Before working an example problem to illustrate the use of Equations 28, 31, and 32 in calculating an imbibition curve for a non-wetting phase it is of interest to try and show graphically what the equations imply. Figure 23 shows three imbibition gas relative permeability curves (marked A, B, and C) and a drainage curve (dashed line) that starts at $S^*_g = 0$ and goes to $S^*_g = 1$. The starting point for any imbibition curve is a point on the drainage curve defined by the value of S^*_{gi}. At the starting point there is zero trapped gas (because the imbibition process hasn't started yet) so the value of $k_{rg}]_{imb}$ can be calculated from Equation 28 by letting $S^*_{gF} = S^*_{gi}$. Since $k_{rg}]_{imb} = k_{rg}]_{dr}$ at the starting point, the value may also be calculated from Equation 7 by noting that $S^*_{wt} = 1 - S^*_{gi}$.

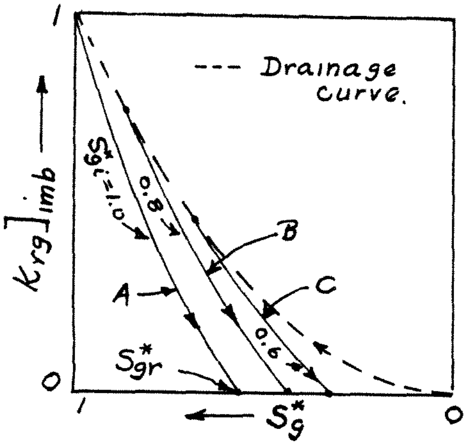


Figure 23

The bottom end of the imbibition curves is fixed by the starting gas saturation S^*_{gi} and the trapping constant, C, in accordance with Equation 32. This defines the residual gas saturation, S^*_{gr}, the saturation at which $k_{rg}]_{imb} = 0$. Note that as C in Equation 32 increases the value of S^*_{gr} decreases, or in effect moves further to the right in Figure 23. For a value of C equal to infinity, S^*_{gr} will be equal to zero, and the imbibition curve will lie exactly on the drainage curve.

The shape of the imbibition curve between the two limits is controlled by Equations 28 and 31. In general, the lower the value of C the straighter will be the $k_{rg}]_{imb}$ curve.

The notes on imbibition relative gas (non-wetting phase) permeability so far have followed Land's (9) treatment of the theory. All of Land's work considers that the drainage curve starts from $S_g^* = 0$, as illustrated in Figure 23. However, should the drainage curve start from so-called "critical" saturation. S_{gc}^*, it may be appropriate to introduce a modification into the imbibition curve equations to account for this. Otherwise, at large values of C, the computed value of $k_{rg}]_{imb}$ may become greater than $k_{rg}]_{dr}$ The imbibition relative permeability value must always be less than the drainage value.

The method of handling "critical" non-wetting phase saturation in the drainage relationship was shown by Equation 11 on page 13 of these notes. This was to introduce the parameter, S_m, defined as the wetting saturation at which the non-wetting phase relative permeability starts. A similar modification of the imbibition relative permeability relationship (11) leads to

$$k_{rg}]_{imb} = \left[\frac{(S_m - 1)}{(S_m - S_{iw})} + S_{gF}^* \frac{(1 - S_{iw})}{(S_m - S_{iw})}\right]^2 \left[1 - (1 - S_{gF}^*)^{\frac{2+\lambda}{\lambda}}\right] \qquad (33)$$

---

(11) This is an unpublished development by M.R. Monroy of Chevron Oil Field Research Company.

The following illustration of calculating non-wetting phase imbibition relative permeability values makes use of the data on the Rangely Field, Colorado given in Example C, page 20. Pertinent information from Example C is that $\lambda = 0.89$, $S_{iw} = 0.30$, and $S_{wi} = 0.36$. The trapping constant, C, will be assumed to be 1.71.

Example F. Calculation of Imbibition k_{rg} vs S_g Relationship, Weber Sandstone, Rangely Field, Colorado.

Given: $\lambda = 0.89$, $S_{iw} = 0.30$, $S_{wi} = 0.36$, $S_m = 1$, $C = 1.71$

Solution:

$$S^*_{gi} = \frac{1-0.36}{1-0.30} = 0.914 \qquad \frac{2+\lambda}{\lambda} = 3.25$$

$$S^*_{gr} = \frac{S^*_{gi}}{C \cdot S^*_{gi} + 1} = \frac{0.914}{1.71 \cdot 0.914 + 1} = 0.357 \tag{32}$$

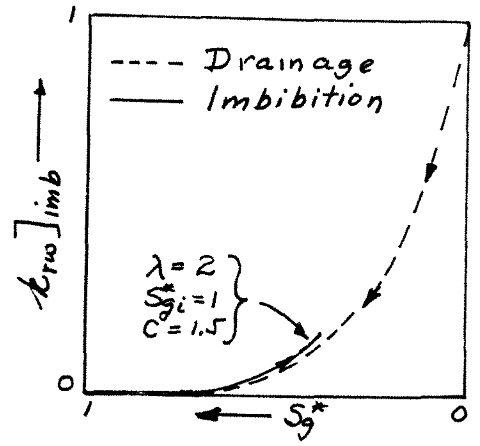
$$S^*_{gF} = \frac{1}{2}\left[(S^*_g - S^*_{gr}) + \sqrt{(S^*_g - S^*_{gr})^2 + \frac{4}{C}(S^*_g - S^*_{gr})}\right] \tag{31}$$

$$k_{rg}\Big]_{imb} = (S^*_{gF})^2 \left[1 - (1 - S^*_{gF})^{\frac{2+\lambda}{\lambda}}\right] \tag{28}$$

S_g	S^*_g	$(S^*_g - S^*_{gr})$	$(S^*_g - S^*_{gr})^2$	S^*_{gF}	$(S^*_{gF})^2$	$(1-S^*_{gF})^{3.25}$	$k_{rg}\Big]_{imb}$	(1) $k_{rg}\Big]_{dr}$
0.64	0.914	0.557	0.310	0.914	0.835	0.000	0.835	0.835
0.60	0.857	0.500	0.250	0.846	0.715	0.0023	0.713	0.733
0.50	0.714	0.357	0.127	0.669	0.448	0.0280	0.435	0.501
0.40	0.571	0.214	0.046	0.477	0.228	0.122	0.200	0.305
0.30	0.429	0.072	0.005^2	0.245	0.060°	0.401	0.036	0.154
0.25	0.357	0.000	0.000	0.000	0.000	0.000	0.000	0.097

Note (1) Drainage values calculated from Equation 27 for comparison with $k_{rg}\Big]_{imb}$

Imbibition Relationships, Wetting Phase. In two-phase systems, the entire water phase remains mobile. As water saturation increases the water invades increasingly larger size pores trapping some gas in the invaded pores. Because of the trapping of gas, at any particular water saturation value some water must occupy pores of larger size than it would occupy if gas had not been trapped. As a consequence of the increased pore size occupied by the water, $k_{rw}\big]_{imb}$ values are always greater than $k_{rw}\big]_{dr}$ values, for the same value of saturation. Figure 24 illustrates the difference in the imbibition and drainage curves. It is to be noted that the difference is small. For greater values of C and lesser values of S^*_{gi} the difference becomes even less. While equations for pore size distribution index other than 2 have not been worked out it would be expected that the differences would also be less than illustrated. For these reasons it is usual to use the same equation for imbibition and drainage. The equation can be written in simplest form as



$$k_{rw}\big]_{imb} = k_{rw}\big]_{dr} = (S^*_w)^{\frac{2+3\lambda}{\lambda}} \tag{34}$$

Equation 34 may also be written in terms of effective gas saturation units as

$$k_{rw}\big]_{imb} = k_{rw}\big]_{dr} = (1 - S^*_g)^{\frac{2+3\lambda}{\lambda}} \tag{35}$$

Example G illustrates the calculation of a $k_{rw}/k_{ro}\big]_{imb}$ vs S_o curve for the Weber Sandstone in the Rangely Field, Colorado. Note that when working with ratio data that the base of both curves must be the same. For this reason, the k_{ro} values (which were calculated as k_{rg} values in Example F) have been changed to be on an absolute permeability base by introduction of k_r^o. The values of k_{rw} are already on the absolute permeability base.

Example G. Calculation of Imbibition k_{rw}/k_{ro} vs S_o Curve. Weber Sandstone, Rangely Field, Colorado.

Given: $\lambda = 0.89$; $S_{iw} = 0.30$; $S_{wi} = 0.36$; $S_m = 1$; $C = 1.71$; $k_r^o = 0.70$

Solution:

$$k_{ro}\big]_{imb} = k_{rg}\big]_{imb} = \frac{k_g}{k_g\big]_{S_g^*=1}} \cdot k_r^o = \frac{k_o}{k}$$

$$k_{rw}\big]_{imb} = \frac{k_w}{k_w\big]_{S_g^*=0}} = \frac{k_w}{k} = (1 - S_g^*)^{\frac{2+3\lambda}{\lambda}}$$

$$\frac{2 + 3\lambda}{\lambda} = 5.25$$

S_g	S_g^*	$(1-S_g^*)$	$k_{rw}\big]_{imb} = (1-S_g^*)^{5.25}$	$k_{ro}\big]_{imb}$	$k_{rw}/k_{ro}\big]_{imb}$	S_w
0.64	0.914	0.086	$2.54(10^{-6})$	0.584	$4.35(10^{-6})$	0.36
0.60	0.857	0.143	$3.68(10^{-5})$	0.500	$7.36(10^{-5})$	0.40
0.50	0.714	0.286	0.0014	0.305	$4.59(10^{-3})$	0.50
0.40	0.571	0.429	0.0115	0.140	$8.21(10^{-2})$	0.60
0.30	0.429	0.571	0.0530	0.0252	2.10	0.70
0.25	0.357	0.653	0.108	0.000	∞	0.75

Values of $k_{rw}/k_{ro}\big]_{imb}$ in the sixth column of the above calculations are plotted against water saturation (Column 7) in Figure 25. Note that the curve becomes asymtotic to the $S_w = 0.3$ (irreducible water saturation) and the $S_w = 0.75$ (one minus residual oil saturation).

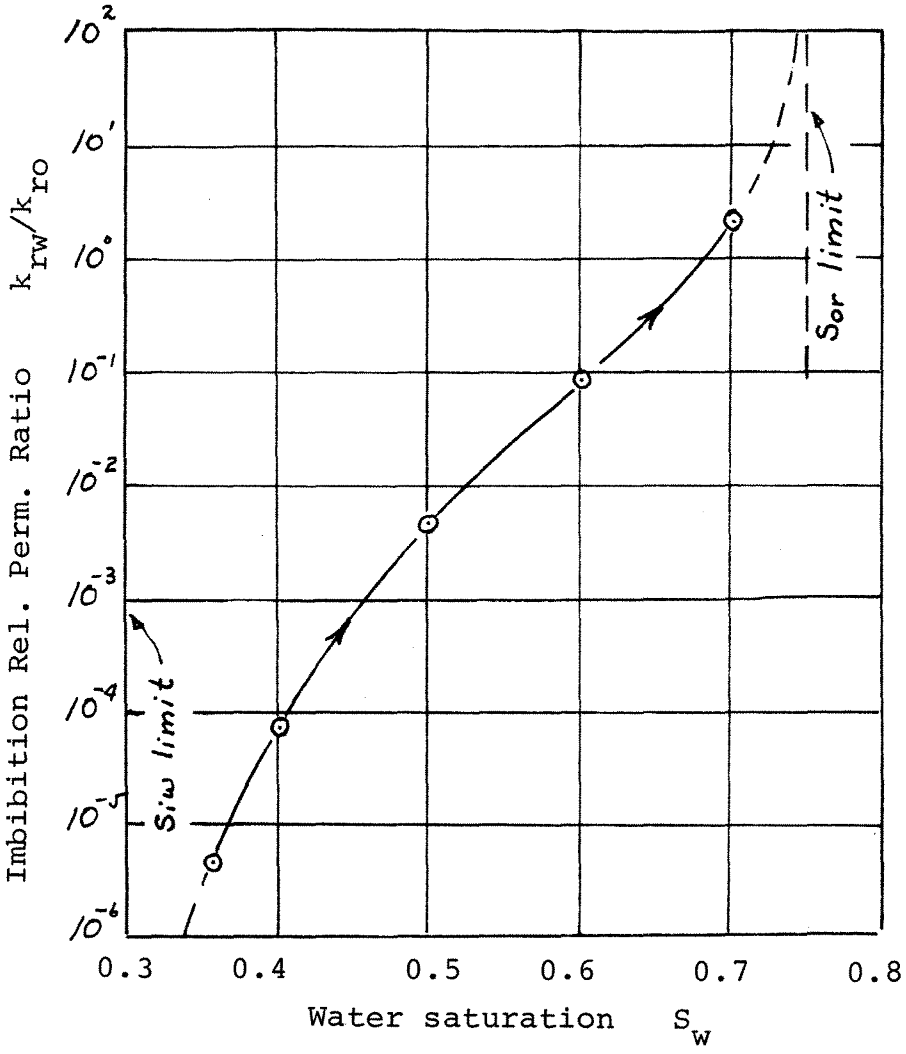


Figure 25. Plot of imbibition k_{rw}/k_{ro} values, Example G, Weber Sandstone, Rangely Field, Colo.

Averaging Imbibition k_{rw}/k_{ro} Data

Laboratory data on the imbibition relative permeability ratios are usually reported as function of water saturation. The shape of data plots will be similar to that of Figure 25. The starting water saturation will usually be near the irreducible water saturation, S_{iw}. As with drainage values, there is often need to intercorrelate the data of a number of cores and obtain an average relationship that can be used for reservoir calculations.

One procedure of averaging the data is the same as outlined on page 23 for drainage data. Convert the water saturation parameter to effective saturation units, S_w^*, by the relationship

$$S_w^* = (S_w - S_{iw})/(1 - S_{iw}) \qquad (36)$$

This requires a value of S_{iw}. If a value is not given in the report, use the value of the lowest water saturation reported as S_{iw}. Plot the $k_{rw}/k_{ro}]_{imb}$ values on log scale against S_w^* on arithmetic scale. This will usually bunch the data so that an average curve can be easily obtained. (See page 23 for method of obtaining average values.)

A second method is as follows:

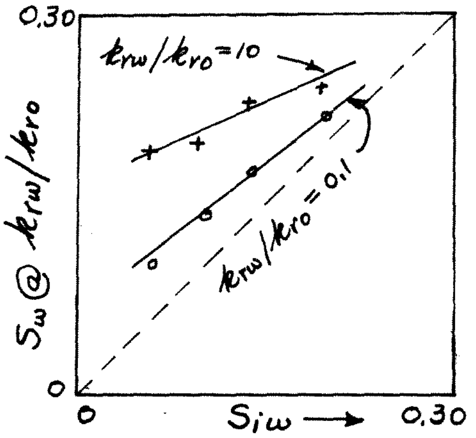


Figure 26

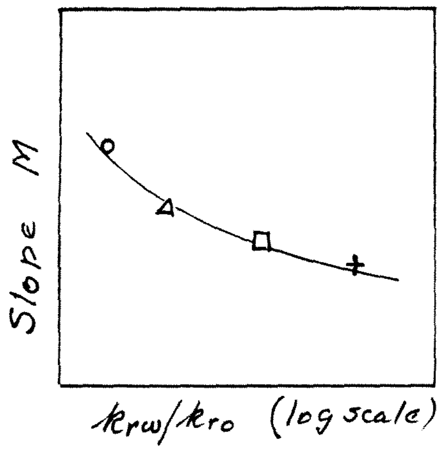


Figure 27

1. Prepare a plot of S_w on the y axis against S_{iw} on the x axis. Read values of S_w from the laboratory data at several selected values of k_{rw}/k_{ro}. Plot values of S_w vs S_{iw} as illustrated in Figure 26. Do this for all data (cores). Construct what appear to be the best average straight lines through the point. There should be one line for each $k_{rw}/k_{ro}]_{imb}$ ratio selected.
2. Plot the slope of the straight lines obtained by the above procedure against the logarithm of the selected k_{rw}/k_{ro} value and construct the best smooth curve through the points. The plot should have the appearance of Figure 27.
3. Using values from the smoothed slope vs log k_{rw}/k_{ro} curve (Figure 27) go back and adjust the slopes of the straight lines in the first plot. Do the adjusting at about mid value of S_{iw} of the original data.

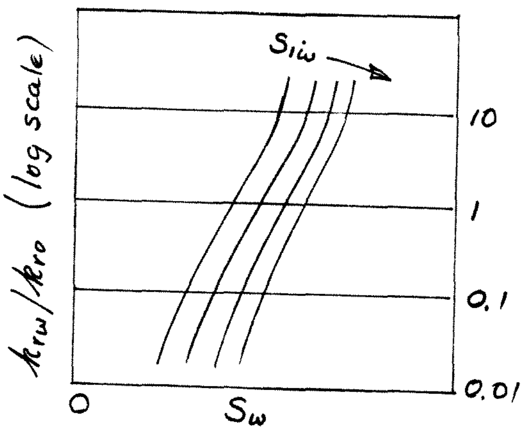


Figure 28

4. From the adjusted plot of S_w vs S_{iw} read values of S_w at selected values of S_{iw} and k_{rg}/k_{ro}. Plot these values on semilog paper of k_{rw}/k_{ro} vs S_w for lines of constant S_{iw}. The final smoothed and averaged data will have the appearance of Figure 28.

M.B. Standing
August 1974

NOMENCLATURE

C	a trapping characteristic constant of each porous media
C	$= \frac{1}{(S^*_{nwr})_{max}} - 1$
J_{sw}	Leverett function
k	absolute permeability
k_a	air permeability
k_g	effective permeability to gas
k_o	effective permeability to oil
k_w	effective permeability to water
k_{wt}	effective permeability to wetting phase
k_{nwt}	effective permeability to non-wetting phase
k_{rg}	relative permeability to gas
k_{ro}	relative permeability to oil
k_{rw}	relative permeability to water
k_{rwt}	relative permeability to wetting phase
k_{rnwt}	relative permeability to non-wetting phase
k_r^o	$= k_{nwt}\big]_{S_{iw}} \big/ k$
λ	(lamda) pore size distribution index, exponent in equation $\frac{P_e}{P_c} = (S^*_{wt})^{\lambda}$
P_e	entry pressure
P_c	capillary pressure
ϕ	porosity
S	saturation
S_{gr}	residual gas saturation
S_{gF}	"free" (mobile) gas saturation
S_{gt}	trapped gas saturation

NOMENCLATURE (Continued)

S_o	oil saturation
S_{or}	residual oil saturation
S_w	water saturation
S_{wi}	initial water saturation (connate water saturation)
S_{iw}	irreducible water saturation
S_{Lr}	total residual liquid phase saturation
S_L	total liquid phase saturation
S_m	$= 1 - S_{cnwt}$
S_{cnwt}	critical nonwetting phase saturation
S_{wt}	wetting phase saturation
S_{wtr}	residual wetting phase saturation after complete drainage
S^*	effective saturation
S^*_g	$= S_g/(1 - S_{iw})$
S^*_o	$= S_o/(1 - S_{iw})$
S^*_w	$= (S_w - S_{iw})/(1 - S_{iw})$
S^*_L	$= S_L/(1 - S_{iw})$
S^*_{wt}	$= S_{wt}/(1 - S_{iw})$
S^*_{gF}	$= S_{gF}/(1 - S_{iw})$ = effective "free" (mobile) gas saturation
σ	(sigma) interfacial tension

Subscripts

F	free or mobile
dr	drainage
imb	imbibition
wt	wetting phase
nwt	non-wetting phase