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A Numerical Model Study of Gravitational Effects and Production Rate on Solution Gas Drive Performance of Oil Reservoirs

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Introduction

The relationship between ultimate oil recovery and rate of oil production has been the subject of many investigations since 1924 when Cutler[1] published a study of field data. Cutler concluded that higher oil production rates yielded higher ultimate oil recoveries. Subsequent studies[2-4] of performance from a variety of fields showed that factors other than producing rate caused so much variation in total oil recovered that no conclusion could be reached concerning the effects attributable to this single factor. Because of the impossibility of imposing different production histories on a given reservoir to observe the difference in ultimate oil produced, confirmation of any such relationship had to depend on analytical studies and laboratory model tests. Analytical approaches using simplified models, in which effects of capillary pressure and gravity forces were ignored,[6,7] led to the conclusion that any effect of production rate upon ultimate recovery was negligible. They did show gross differences in horizontal saturation distributions imposed by production rate variations.

In 1953 and 1954 the foundation was laid for solution of the unsteady-state fluid flow equations by use of difference approximations and high-speed computers.[8,9] The development of these techniques led to numerous mathematical model investigations of fluid displacement in one and two dimensions.[10-15] These works treated a range of fluid systems in one and two dimensions; generally they were confined to incompressible fluids, and included gravity and capillary effects. When simulating compressible fluid systems, gravity and capillary forces were ignored. Levine *et al.*[16] and Heuer *et al.*[5] studied the effects of production rate on ultimate oil recovery by solution gas drive, using these more sophisticated mathematical techniques. Both efforts neglected the effect of gravity, but Levine included capillary forces. Both concluded that producing rate had a negligible effect on ultimate recovery. Ridings *et al.*[17] showed that performance of long horizontal laboratory systems coincided closely with performance calculated by numerical modeling techniques, with one exception. Noticeably increasing recovery was observed with increasing production rate in the laboratory system, whereas the numerical model showed no rate effect. Gravity was not a factor in either system. Since the production rate sensitivity was less on very long laboratory systems than for the shorter ones, it was concluded that the rate effect observed on the short laboratory system was, for reasons not specified, typical only of short systems.

It has been recognized for some time that, under certain conditions, gravitational forces can be very important in the displacement of oil from natural reservoirs. Muskat[18,19] treats exhaustively the performance of wells and reservoirs in which gravity is the sole driving mechanism. Also a considerable amount of attention has been given to methods of

Ultimate oil recovery by solution gas drive can be greatly affected by variations in production rates — in general, the higher the rate the higher the recovery. However, the quantitative relationship for a given field will be influenced by factors other than rate and vertical permeability: reservoir thickness, gas solubility, shrinkage factor, oil viscosity, relative permeability, and capillary pressure.

investigating the effect of gravity on performance of reservoirs where gravity is a dominating factor — i.e., where high formation dips and high permeability cause gravity drainage effects to overshadow solution gas drive.[20-23] It is generally conceded that, in reservoirs where gravity is the predominant drive mechanism, oil recoveries will be higher than those attainable by any other process and that the solution gas drive effects are almost negligible.

Martin[24] and Cook[25] investigated, by analytical techniques, the effects of gravity forces on fluid movements in oil reservoirs subject to gas injection, water injection and a mechanism combining solution gas drive and gravity drainage. Capillary pressure effects were ignored in their work. Their efforts were largely directed toward explaining the performance of reservoirs in which solution gas drive effects were incidental to effects of gravity. Nevertheless, Cook observed that even in substantially horizontal, low-permeability reservoirs, there should be a tendency for gas to flow to the top of the productive section. This would cause a zone of high gas saturation at the top and provide a channel for flow in the horizontal direction. Further, he observed that the gas saturation in the lower part of the section should remain at a relatively low value throughout the depletion period. Thus was explained the migration of gas between producing areas even in shallow dipping reservoirs. Also explained was the observed low gas-oil ratio (GOR) production from wells recompleted in the base of the producing section after producing at high GOR's from higher in the section.

That gravity effects can cause at least some tendency for vertical segregation during depletion of an oil reservoir by solution gas drive is certain. In over 90 percent of the reservoir volume, the vertical pressure gradients resulting from the difference in densities of oil and gas are generally much greater than the horizontal pressure gradients imposed by flow. The only exception to this is in the immediate vicinity of producing wells, where horizontal pressure gradients can exceed those in the vertical direction. The question that remains is the quantitative significance of gravity forces on solution gas drive. Intuitively it would seem that the accumulation of gas at the top of the producing section, whence it could flow easily to the producing well, would result in less oil recovery than if the gas remained uniformly distributed. Also, it seems reasonable that the degree of vertical segregation should be sensitive to producing rate. At very low rates vertical segregation should be almost complete, and at very fast rates oil production would cease before any substantial vertical segregation could occur.

Recently, a variety of mathematical models and solution techniques have been developed allowing calculation of fluid flow in three dimensions and accounting for all known factors affecting flow in systems flowing one, two, or three phases.[26-34] Our aim here is to investigate the quantitative effect of oil production rate on ultimate oil recovery by solution gas drive mechanism from horizontal, thin reservoirs in which regional migration of gas will be negligible and well completion effects will be minimized.

Mathematical Model

Except for local effects around the producing wells, flow in an oil reservoir can be simulated by a linear system in which the total pressure differential from production point to reservoir boundary is less than 10 to 20 percent of total pressure. The limitation on pressure differential is imposed by the radial nature of the producing wells and the immediately adjacent reservoir volume. In this zone, pressure gradients are very high but the fraction of total reservoir volume is small. The essentially linear nature of flow in a radial system beyond about 100 ft from a producing well, and the impossibility of imposing a pressure differential of more than 20 percent of the reservoir pressure from 100 to 1,000 or more ft, even at capacity production rates, is well illustrated by the work of Muskat[18,19] and Levine.[16]

For this study the physical model chosen for mathematical representation was a linear reservoir segment 25 ft high, 1,400 ft long, and 1 ft wide.

Following are the equations describing the flow of fluids in a two-dimensional system of constant porosity, flowing gas and oil.[19]

Oil Equations

$$u_{ox} = -k\frac{k_{rox}}{\mu_o}\left(\frac{\delta p_o}{\delta x}\right) \quad . . . . \quad (1)$$

$$u_{oz} = -k\frac{k_{roz}}{\mu_o}\left(\frac{\delta p_o}{\delta z} - \rho_o\right) \quad . . . . \quad (2)$$

Gas Equations

$$u_{gx} = -k\frac{k_{rgx}}{\mu_g}\left(\frac{\delta p_g}{\delta x}\right) \quad . . . . \quad (3)$$

$$u_{gz} = -k\frac{k_{rgx}}{\mu_g}\left(\frac{\delta p_g}{\delta z} - \rho_g\right) \quad . . . . \quad (4)$$

The continuity equations for each phase

$$\frac{\delta}{\delta x}(u_{ox}) + \frac{\delta}{\delta z}(u_{oz}) - q_o = -\frac{\delta S_o}{\delta_t} \quad . \quad (5)$$

$$\frac{\delta}{\delta x}(u_{gx}) + \frac{\delta}{\delta z}(u_{gz}) - q_g = -\frac{\delta S_g}{\delta_t} \quad . \quad (6)$$

As illustrated by Breitenbach,[28] West,[9] and Rachford,[11] combining these equations with other relationships between saturations and pressures in each phase results in simulation equations for each block of the form

$$O^-_x\Delta p^-_x + O^-_z\Delta p^-_z + O^+_x\Delta p^+_x +$$
$$O^+_z\Delta p^+_z + A9 = A8\frac{p_{t_1} - p_{t_0}}{t_1 - t_0} \quad . \quad (7)$$

where the subscripts x and z designate horizontal and vertical directions and the superscripts + and − designate positive and negative directions. This nomenclature is that of Breitenbach *et al.*[29] The coefficients designated as O^+_y, O^-_y, O^+_x and O^-_x represent the coefficients of flow in the various directions, $A9$ represents gravitational, capillary, production and injection terms. For this study the pressures were determined at each step by an implicit solution of

Eq. 7 and saturations were then determined explicitly from the oil equation (combination of Eqs. 1 and 5) as outlined by Breitenbach.[28] The only significant variations from this reference are indicated in the paragraph that follows.

For the next calculation interval, Breitenbach used compressibilities, relative permeabilities, etc., as determined by pressure and saturation conditions at the start of that interval. In this investigation an attempt was made to use a better value for all factors by determining them at the average pressure and saturation for the calculation interval, with the throught that this would allow the use of larger time intervals. Our technique was to estimate the oil pressure and saturation at the end of the next time interval by extrapolating from the values calculated at the end of each of the three preceding time steps. The pressure and saturation used to calculate the coefficients for the interval were the arithmetic average of those at the start of the interval and those estimated for the end. These average properties were used in both the pressure and saturation calculations, with the exception that the estimated densities for gas and oil used in the pressure equation were those for the end of the interval. In the saturation calculation the values used for pressure differentials between blocks and those used for fluid densities were averages for the time interval.

If the calculated pressures and saturations checked within the desired tolerance with those estimated, the calculated values were accepted. If such agreement did not occur, the calculation procedure was repeated substituting the calculated values for those estimated on the first cycle. Recycling of the calculation for an interval was continued until a satisfactorily small difference was observed between results of two successive cycles or until a preset maximum number of cycles had been finished. A variable time step was used that was automatically increased or decreased, depending on the check between initially estimated values for pressure and saturation and those finally accepted. For this particular problem, a block-by-block check was not made for each of these factors. Rather, the producing GOR was checked against that estimated from parabolic extrapolation of ratios calculated for the three previous time intervals. The net effect of the above system for this study was that a complete depletion of a one-dimensional horizontal system could be done in less than 40 steps. Recycling was necessary on only two or three steps. The final oil material balance checked within 0.015 percent. By tolerating larger material balance errors the calculations could be reduced to less than 20 steps. For one-dimensional vertical systems or two-dimensional systems, it was still possible to calculate a complete depletion by solution gas drive with very little recycling and to maintain good final material balances. In general, more steps were required for these systems than for horizontal systems, the steps varying from almost the same number at very high rates, to 20 times as many for the lowest production rate.

The physical reservoir that was modeled is shown in Fig. 1. Exploratory calculations were made in which the 25-ft-thick reservoir was represented by three, four, five and six segments of various thickness. Though subdivision beyond four segments caused minor variations in the details of performance, such as slight shifting of the GOR peak, ultimate oil produced was negligibly affected. Hence, we decided that all comparative runs would be made with the system divided vertically into only four blocks as shown in Fig. 1. Similarly, horizontal subdivisions in excess of five yielded little discernible beneficial detail to the calculated performance. Early test runs with the two-dimensional model showed that the effect of horizontal permeability — in the range where total pressure differential across the system is held to less than 20 percent of the absolute pressure — is negligible. Therefore, for the few two-dimensional runs, the horizontal permeability was arbitrarily fixed at a value that would cause the pressure differential to be in the desired range for the producing rate chosen.

Basic capillary pressure and relative permeability characteristics of the reservoir rock for all runs are shown in Figs. 2 and 3. As indicated in Fig. 3, hysteresis in gas relative permeability was included. At gas saturations below 30 percent of hydrocarbon pore space, any decrease in gas saturation causes a very rapid decline in gas relative permeability. Though there are only a few representative curves in the region of decreasing gas permeability, the program provides for continuous representation of relative permeabilities in the region. Including relative permeability hysteresis is necessary only when the oil rate is declining. As long as a constant oil rate is maintained during a solution gas drive, the gas saturations at all levels in the reservoir are increasing. As soon as

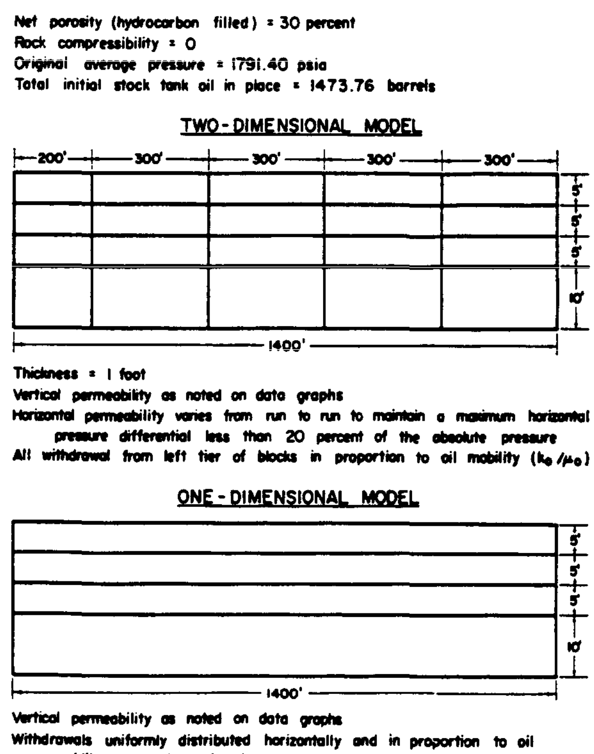

Fig. 1—Diagrams of model reservoirs.

TABLE 1—FLUID CHARACTERISTICS

	Reservoir Voume Factor		Density Gradient		Gas Solubility	Viscosity	
Pressure (psia)	Oil (reservoir bbl/ STB)	Gas (Mcf/ reservoir bbl)	Gas (psi/ ft height)	Oil (psi/ ft height)	(Mcf*/ STB)	Oil (cp)	Gas (cp)
1800	1.270	1.1071	0 0724	0.3198	0.5800	1.350	0.0170
1500	1.235	0.9167	0.0599	0.3245	0.4980	1.400	0.0158
1200	1.202	0.7161	0.0468	0.3290	0.4160	1.450	0.0145
900	1.170	0.5093	0.0333	0.3334	0.3340	1.550	0.0137
600	1.136	0.3135	0.0205	0.3387	0.2520	1.750	0.0126
300	1.095	0.1409	0.0092	0.3459	0.1600	2.187	0.0116
100	1.050	0.0414	0.0027	0.3555	0.0760	3.050	0.0111
25	1.014	0.0098	0.0006	0.3646	0.0218	4.379	0.0105

* Mcf = 1,000 cu ft at standard temperature and pressure (60°F and 14.7 psia).

the oil production rate starts to decline, some of the lower part of the producing section starts to increase in oil saturation at the expense of the upper part of the section. If gas relative permeability hysteresis is not considered during this time, the resaturation of the lower part of the producing section will increase too fast and go too far. For example, with hysteresis (and no pressure increase) gas saturation can increase only 0 to 3 percent of pore volume in the range of 0 to 30 percent gas saturation; without hysteresis, resaturation with oil could be complete. This would result in gross differences in performance during the period of declining oil rates. In general, calculated GOR's will be higher during this period if gas relative permeability hysteresis is ignored. The basic relative permeability and capillary characteristics were calculated by the formulas suggested by Wyllie[36] and Corey.[37] Only the hydrocarbon pore space was considered. It was assumed that water saturation was uniform and at the minimum value attainable by flow. Hence, the irreducible oil saturation in the system should approach zero and the "effective saturation" used by Corey will equal the actual saturation (expressed as a fraction of the hydrocarbon pore space instead of the total pore space). The equations for relative permeabilities, then, become

$$k_{rg} = (1 - S_o)^2[1 - (S_o)^2] \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$k_{ro} = (S_o)^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The oil and gas properties used for all calculations are shown in Table 1. From this information all data needed for the reservoir simulation program were obtained by parabolic interpolation to calculate every

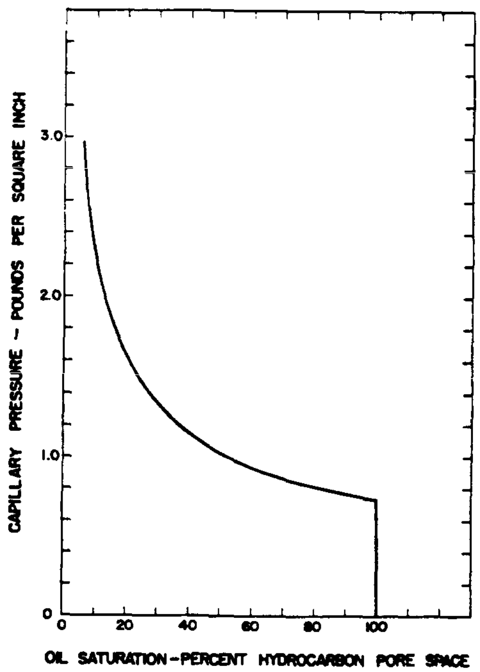

Fig. 2—Relationship between capillary pressure and oil saturation.

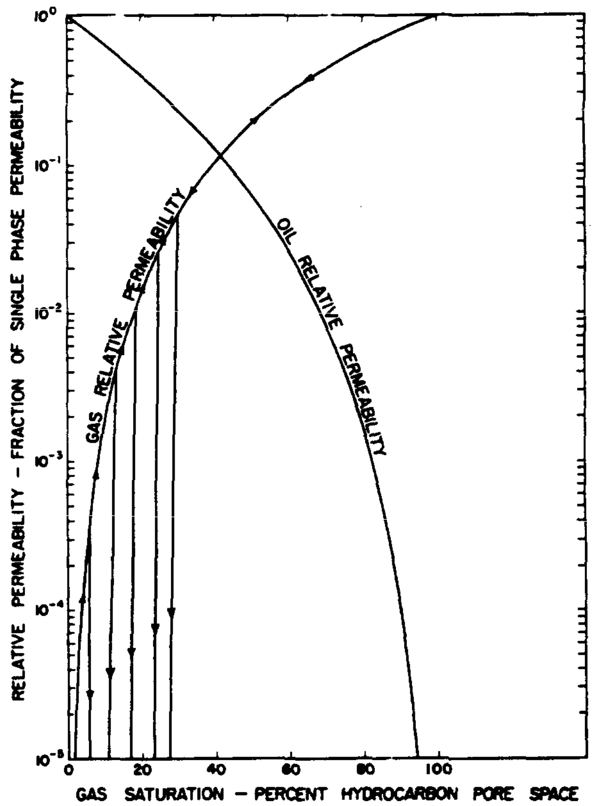

Fig. 3—Relationship between relative permeability and gas saturation.

pressure variable needed in the program at fixed pressure intervals of 25 psi. The simulation program used linear interpolation between the data at these fixed points to obtain required values for any specific pressure. Initial calculations were made with linear interpolation between data at pressure intervals of 50 psi. Reducing the pressure interval to 25 psi caused no significant improvement in results.

Two solution techniques, Gauss-Seidel iteration and Gaussian elimination, were used for calculating pressures by the equation system represented by Eq. 7. For the iterative method, iteration was started from the pressure estimated for each block by extrapolation from the last three values. The number of iterations was greatly reduced from those necessary if iteration were started from the pressure values at the end of the last interval. In fact, for this problem the solution times using the ordinary Gauss-Seidel technique were only slightly different from those using an efficient Gaussian elimination technique.[38] Since the small size of the model resulted in very small round-off error, the Gaussian elimination method was used for most of the calculations.

Final material balance errors on all runs were less than 0.015 percent of fluid originally present for oil and less than 0.4 percent for the gas. By accepting longer calculation times these errors could be reduced even further, but there was no significant change in the details of performance or in the ultimate oil recovered. Total solution times varied from less than 10 seconds for high-production-rate, one-dimensional runs to 2.6 minutes for the lowest-rate, two-dimensional runs. An IBM model 360-65 computer was used for the calculations.

Discussion of Results

Even for the thin (25-ft) horizontal reservoir section being simulated, early calculations showed that a significant difference in performance resulted, depending on whether production was withdrawn near the top or near the bottom of the section. For this study, one objective was to eliminate well effects; so the computer program was modified to distribute production, at every step, from each vertical section in such a way as to minimize the disturbance to saturation distribution. This was effected by distributing the total oil production in proportion to oil mobility in each vertical block at the end of the preceding time step. That this technique resulted in negligible disturbance to vertical saturation distribution is illustrated by the data of Tables 2 and 3. These tables show several pressure and saturation distributions calculated for the two-dimensional model producing at rates of 0.1 and 0.5 STB/D. When the horizontal pressure differential was limited to less than 20 percent of the total pressure, the saturation gradients in the horizontal direction were very small. We concluded, therefore, that for this pressure differential limitation a one-dimensional vertical model should yield identical over-all performance. Tables 2 and 3 also include the vertical pressure and saturation for a one-dimensional model at about the same stage of depletion as for the two-dimensional system, using the same production rates. The data for the two models are not for exactly the same depletion stage because of time-step variations between the calculations. The results show very close correspondence between details of saturation and pressure distribution. Figs. 4 and 5 show the complete GOR and production histories for these two comparative runs. The precise check between results from one- and two-dimensional models led to the conclusion that two-dimensional models were unnecessary for the problem under study. Also, these and other runs, in which horizontal permeability was varied over a wide range below that necessary to maintain the desired maximum horizontal pressure differential, showed that only the vertical permeability is of significance to solution gas drives under the conditions of this investigation. A few depletion calculations were made, using two-dimensional models, in which horizontal pressure differentials were allowed to increase to as much as 90 percent of the total pressure. Under these conditions some details of performance were altered, mainly because of declining production rates. (This in turn causes increasing oil saturations in the lower part of the section.) Except for the points noted on Figs. 4 and 5, all the remaining calculations were made using the one-dimensional model.

Effect of Production Rate

Maintaining all other factors constant, a series of calculations was made in which the oil production rate was varied in six steps from 0.1 to 50 B/D. Gross pressure performance of the system at each of these conditions is shown in Fig. 6 as a function of oil recovery. Also shown on this figure is the pressure performance of a horizontal, one-dimensional system producing at a rate of 0.1 B/D. The horizontal system performance is identical with that of the 50 B/D system in which gravitational effects were included. This indicates that, at this high rate, fluid segregation was a negligible factor in over-all performance. However, at lower production rates, the oil recovery obtained at any pressure decreased substantially. Assuming an abandonment pressure of 100 psi, the ultimate oil recovery ranged from 13.5 percent at 0.1 B/D to 20.9 percent at 50 B/D — a ratio of approximately 1.55. This production rate range is, of course, much wider than the possible range of rates in the exploitation of any commercial reservoir. It can be seen from the curves that significant differences in ultimate recovery result from production rate ratios as small as 2 to 4.

Fig. 7 is a graph of the GOR vs oil recovery data from the same series of calculations. Data for the 12.5-B/D run were omitted to avoid undue crowding. The reason for the more rapid pressure depletion at lower production rates is clear from the GOR curves. Increasing GOR's start progressively earlier at lower production rates and result in higher GOR peaks. Thus production of all the solution gas results in lower oil recoveries as production rate is decreased.

The reason for such variation in performance must be, of course, the tendency of gas to migrate to the upper part of the producing section and of oil to flow to the lower part as production proceeds. Figs. 8

through 10 show the saturation history at various levels as a function of oil recovery from the system for high, intermediate, and low production rates. Fig. 8 shows that even at the highest rate of 50 B/D there is some vertical variation in oil saturation during depletion. Saturations at all levels were identical until about 6 percent of the oil in place was recovered (pressure decline from original 1,791.4 psia to about 1,400 psia), after which some differences developed. The maximum saturation difference between the top (5-ft-thick section) and the bottom (10-ft section) was approximately 5 percent and was relatively constant over the last half of the depletion life. Although not shown on the figure, the saturations of the other blocks fell between the two curves shown. This difference in oil saturation was not enough to cause a detectable difference in performance or in ultimate oil recovery from the horizontal system in which there was no fluid segregation. At 5 B/D (Fig. 9), the oil saturation difference between the top and the bottom

TABLE 2—CALCULATED PERFORMANCE OF TWO-DIMENSIONAL AND ONE-DIMENSIONAL MODELS FOR AN OIL PRODUCTION RATE OF 0.1 STB/D

Horizontal Permeability = 1,000 md
Vertical Permeability = 101 md

Vertical Distance from Top of Model (ft)	Horizontal Distance from Left End of Model (ft): Two-Dimensional Model 100	350	650	950	1,250	One-Dimensional Model 700
	DATA FOR AVERAGE OIL PRESSURE OF ABOUT 1,500 PSIA					
	Oil Pressure, psia					
2.5	1498.57	1500.23	1501.73	1502.74	1503.23	1500.21
7.5	1500.18	1501.84	1503.34	1504.34	1504.84	1501.82
12.5	1501.79	1503.46	1504.96	1505.96	1506.45	1503.44
20.0	1504.22	1505.88	1507.38	1508.39	1508.88	1505.86
	Oil Saturation, Percent Hydrocarbon Pore Space					
2.5	80.28	80.50	80.69	80.78	80.77	80.29
7.5	96.67	96.71	96.72	96.71	96.72	96.74
12.5	97.10	97.14	97.12	97.14	97.13	97.16
20.0	97.48	97.52	97.50	97.51	97.52	97.52
	DATA FOR AVERAGE OIL PRESSURE OF ABOUT 1,000 PSIA					
	Oil Pressure, psia					
2.5	1035.28	1037.27	1039.05	1040.24	1040.83	1052.49
7.5	1036.77	1038.74	1040.52	1041.72	1042.31	1053.99
12.5	1038.42	1040.39	1042.17	1043.37	1043.95	1055.63
20.0	1040.89	1042.87	1044.65	1045.84	1046.43	1058.11
	Oil Saturation, Percent Hydrocarbon Pore Space					
2.5	48.80	48.41	48.38	48.40	48.40	49.14
7.5	94.64	94.62	94.62	94.64	94.67	94.77
12.5	95.74	95.76	95.77	95.77	95.77	95.80
20.0	96.31	96.32	96.32	96.32	96.32	96.34
	DATA FOR AVERAGE OIL PRESSURE OF ABOUT 500 PSIA					
	Oil Pressure, psia					
2.5	560.65	563.19	565.46	566.97	567.73	563.50
7.5	561.49	564.03	566.31	567.82	568.57	564.35
12.5	563.17	565.71	567.99	569.50	570.25	566.02
20.0	565.70	568.24	570.52	572.03	572.78	568.56
	Oil Saturation, Percent Hydrocarbon Pore Space					
2.5	29.50	29.40	29.39	29.40	29.41	29.38
7.5	87.40	87.38	87.40	87.43	87.45	87.40
12.5	94.35	94.37	94.38	94.39	94.40	94.38
20.0	95.11	95.13	95.14	95.15	95.15	95.14
	DATA FOR AVERAGE OIL PRESSURE OF ABOUT 100 PSIA					
	Oil Pressure, psia					
2.5	95.91	101.06	105.44	108.20	109.67	99.88
7.5	96.61	101.76	106.15	108.90	110.39	100.59
12.5	98.25	103.38	107.74	111.16	112.02	102.24
20.0	100.88	106.00	110.39	113.73	114.87	104.88
	Oil Saturation, Percent Hydrocarbon Pore Space					
2.5	20.75	20.73	20.75	20.76	20.76	20.69
7.5	60.18	60.08	60.60	61.97	58.50	60.49
12.5	90.93	91.15	91.37	90.00	95.45	91.24
20.0	92.71	92.86	92.99	93.28	92.49	92.82

of the producing section increased to slightly over 28 percent. At 0.5 B/D (Fig. 10), oil saturation ranged from 22 percent in the top of the section to 84 percent in the bottom near the end of production.

Summarized in Fig. 11 are the oil saturations vs height in the reservoir for all production rates studied and at the time of almost complete depletion of solution gas. Final pressure was in the range of 30 to 40 psia. For the 0.1 B/D calculation, the final oil saturation ranges from 20 percent at the top to over 90 percent at the bottom of the producing section. Such very low oil saturation in the top of the section provides a bypassing channel so gas can escape without displacing oil and must result in poorer ultimate oil recovery than if no segregation occurred. At first glance it seems remarkable that the ultimate oil produced is not more greatly affected by gravity segregation than these results show. The saving feature is

TABLE 3—CALCULATED PERFORMANCE OF TWO-DIMENSIONAL AND ONE-DIMENSIONAL MODELS FOR AN OIL PRODUCTION RATE OF 0.5 STB/D

Horizontal Permeability = 10,000 md
Vertical Permeability = 101 md

Vertical Distance from Top of Model (ft)	Horizontal Distance from Left End of Model (ft)					
	Two-Dimensional Model					One-Dimensional Model
	100	350	650	950	1,250	700
	DATA FOR AVERAGE OIL PRESSURE OF ABOUT 1,500 PSIA					
	Oil Pressure, psia					
2.5	1477.82	1478.70	1479.49	1480.01	1480.28	1486.74
7.5	1470.38	1480.26	1481.05	1481.57	1481.84	1488.31
12.5	1480.97	1481.85	1482.64	1483.17	1483.43	1489.90
20.0	1483.38	1484.26	1485.04	1485.57	1485.83	1492.31
	Oil Saturation, Percent Hydrocarbon Pore Space					
2.5	83.43	83.87	83.94	84.01	84.01	84.11
7.5	94.46	94.55	94.56	94.56	94.56	94.72
12.5	95.12	95.19	95.20	95.19	95.20	95.34
20.0	95.74	95.80	95.82	95.81	95.81	95.94
	DATA FOR AVERAGE OIL PRESSURE OF ABOUT 1,000 PSIA					
	Oil Pressure, psia					
2.5	1009.25	1010.37	1011.37	1012.04	1012.37	1012.64
7.5	1010.47	1011.58	1012.58	1013.25	1013.59	1013.86
12.5	1012.08	1013.20	1014.20	1014.86	1015.20	1015.47
20.0	1014.53	1015.64	1016.64	1017.31	1017.64	1017.92
	Oil Saturation, Percent Hydrocarbon Pore Space					
2.5	52.59	52.54	52.54	52.54	52.54	52.55
7.5	90.58	90.58	90.58	90.59	90.59	90.64
12.5	92.89	92.89	92.90	92.90	92.90	92.93
20.0	93.83	93.84	93.84	93.84	93.84	93.86
	DATA FOR AVERAGE OIL PRESSURE OF ABOUT 500 PSIA					
	Oil Pressure, psia					
2.5	506.29	507.89	509.33	510.28	510.76	504.09
7.5	506.96	508.56	509.99	510.95	511.42	504.75
12.5	508.55	510.15	511.59	512.54	513.02	506.35
20.0	511.04	512.64	514.07	515.03	515.50	508.84
	Oil Saturation, Percent Hydrocarbon Pore Space					
2.5	37.33	37.29	37.29	37.29	37.29	37.15
7.5	79.31	79.29	79.29	79.30	79.31	79.22
12.5	90.16	90.18	90.20	90.21	90.22	90.20
20.0	91.64	91.66	91.67	91.68	91.68	91.67
	DATA FOR AVERAGE OIL PRESSURE OF ABOUT 50 PSIA					
	Oil Pressure, psia					
2.5	45.12	49.39	52.95	55.21	56.32	48.25
7.5	45.63	49.90	53.45	55.71	56.81	48.77
12.5	46.91	51.18	54.78	47.07	58.18	50.22
20.0	49.32	53.59	57.25	59.59	60.72	52.79
	Oil Saturation, Percent Hydrocarbon Pore Space					
2.5	28.34	28.35	28.37	28.38	28.38	28.24
7.5	56.81	56.57	56.51	56.48	56.51	56.76
12.5	80.95	81.43	81.94	82.30	82.45	81.77
20.0	85.28	85.64	86.02	86.28	86.38	85.65

Note: At average pressure of about 50 psi, production rate in the two-dimensional model had declined from 0.5 B/D, whereas the one-dimensional model still produced at that rate.

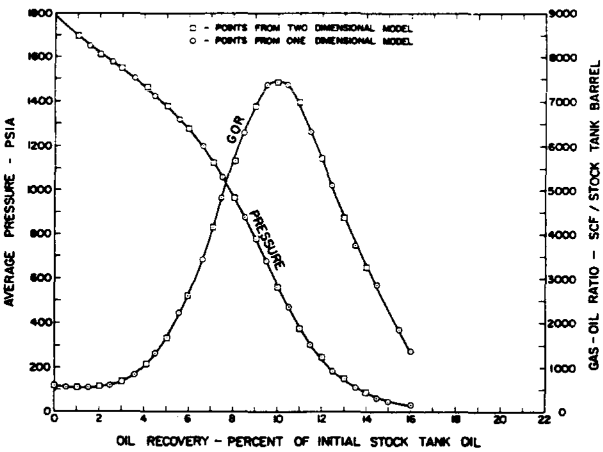

Fig. 4—Comparison of performance data from one- and two-dimensional models (oil production rate = 0.1 B/D; vertical permeability = 101 md).

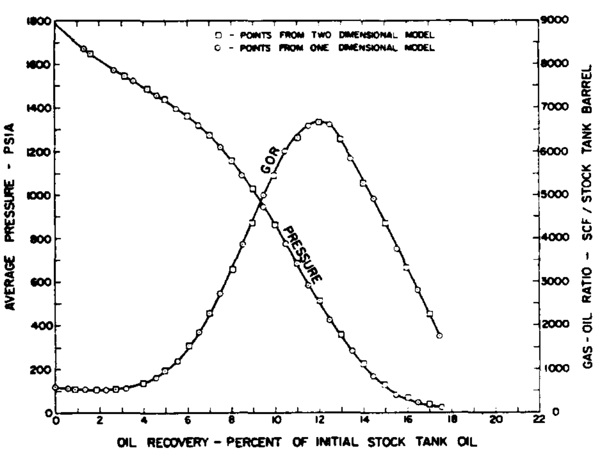

Fig. 5—Comparison of performance data from one- and two-dimensional models (oil production rate = 0.5 B/D; vertical permeability = 101 md).

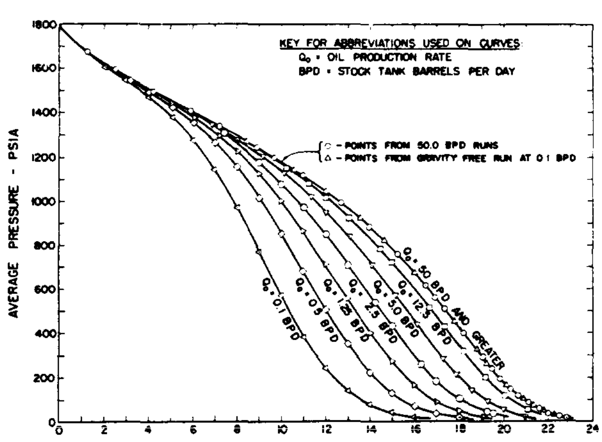

Fig. 6—Relationship between pressure and oil recovery (vertical permeability = 101 md).

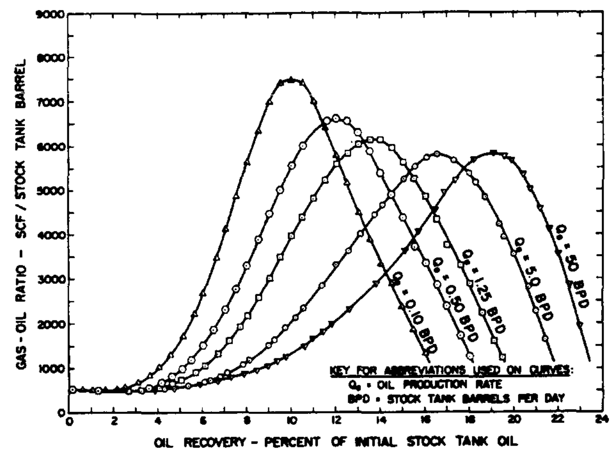

Fig. 7—GOR performance (vertical permeability = 101 md).

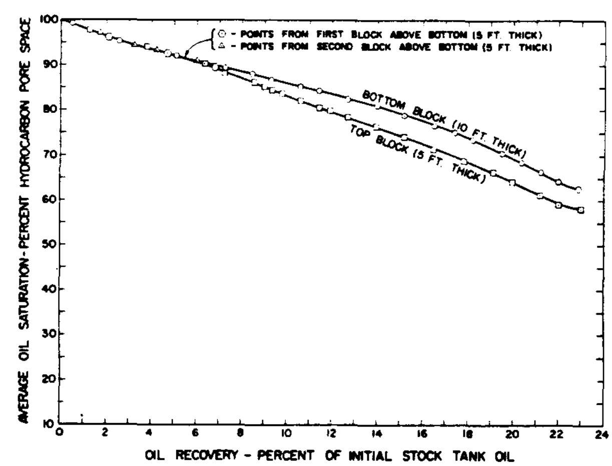

Fig. 8—Relationship between oil saturation and oil recovery for various levels in the model (oil production rate = 50 B/D; vertical permeability = 101 md).

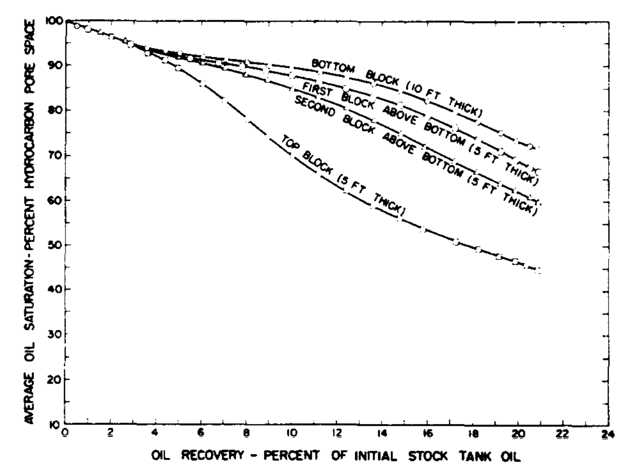

Fig. 9—Relationship between oil saturation and oil recovery for various levels in the model (oil production rate = 5.0 B/D; vertical permeability = 101 md).

that, although gas can flow much easier in the segregated system, oil flow is also easier because of the high oil saturation in the base of the section. Unfortunately, in the regime of solution gas drive, the ease of gas flow increases faster with gravity segregation than does the ease of oil flow. The net effect is poorer performance at low production rates in systems where gravity is a factor.

Relative Permeability Ratios Calculated from Performance

Basic to the prediction of reservoir performance by solution-gas drive or external gas drive is knowledge of the relationship between oil saturation and the ratio of gas permeability to oil permeability (k_g/k_o). Knowing the amount of oil in place, fluid pressure-volume relationships, and the amount of oil produced to a given time, the remaining average oil saturation can be calculated. Also the "apparent k_g/k_o" can be calculated at any time using the producing GOR, reservoir pressure, and pressure-volume relationships for the reservoir fluids. This apparent k_g/k_o vs oil saturation relationship is used by reservoir engineers,[39, 36] extrapolated by various techniques to predict reservoir performance. Laboratory measurements of the relative permeability relationship to oil saturation seldom give results approximating the field observations. Usually the apparent k_g/k_o from field performance is two to 10 times the laboratory-measured value at a given oil saturation. The usual reservoir engineering procedure for a performance prediction is to use a curve extrapolated from field performance in a direction generally parallel to the laboratory data. To determine the effect of observed vertical segregation of oil and gas, during depletion by solution gas drive, calculations were made of the apparent k_g/k_o at frequent intervals during the depletion. Fig. 12 summarizes these calculations. The basic rock flow characteristics derived using the relationships of Fig. 3 are shown for reference. Apparent k_g/k_o values calculated for the 50-B/D run including gravity, and the 0.1-B/D run with no gravity, coincide exactly with the true rock characteristics. Values

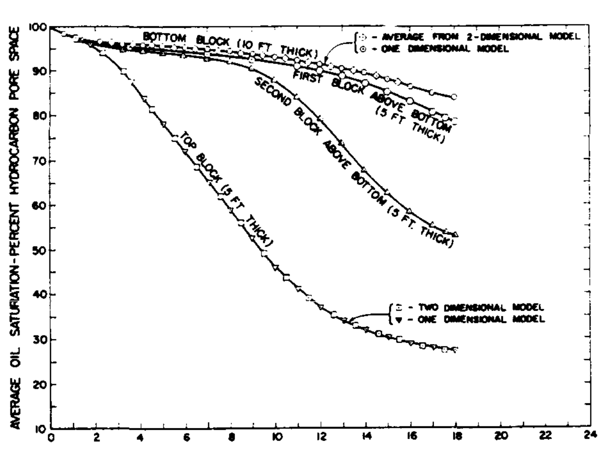

Fig. 10—Relationship between oil saturation and oil recovery for various levels in the model (oil production rate = 0.5 B/D; vertical permeability = 101 md).

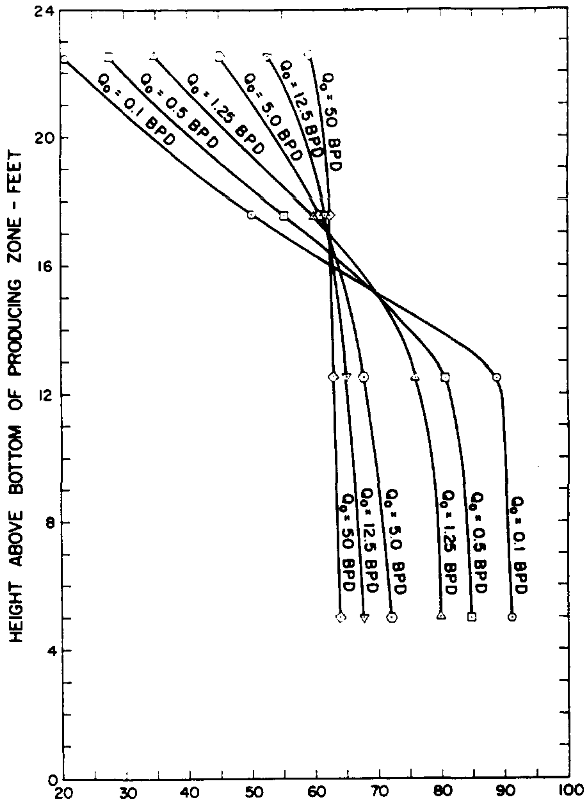

Fig. 11—Vertical distribution of oil saturation after depletion by solution gas drive (final pressure = 30 to 40 psia; vertical permeability = 101 md).

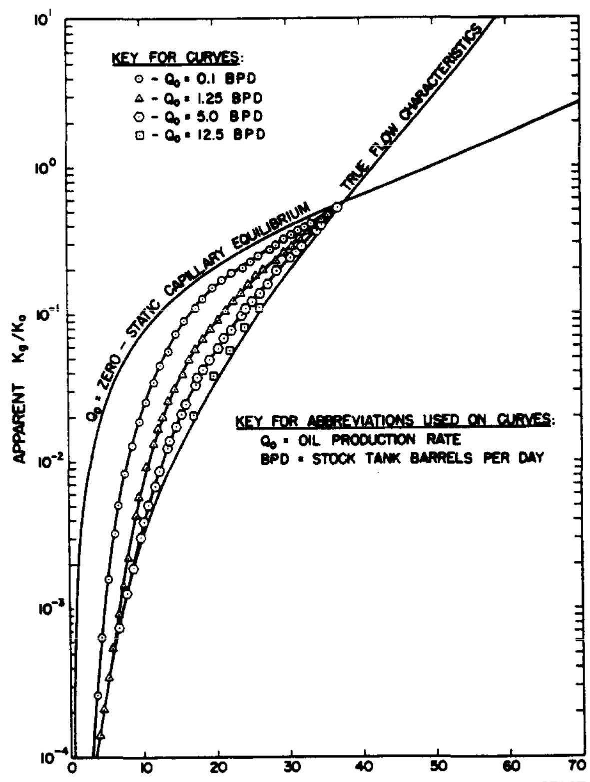

Fig. 12—Gas-oil permeability ratio calculated from performance data for various oil production rates (vertical permeability = 101 md).

calculated for the 12.5-B/D run were very slightly above the true flow properties curve. Note that this comparison provides a much less sensitive measure of performance than pressure or GOR history. The apparent k_g/k_o curve for the other rates diverged progressively more from the true characteristics curve at lower production rates, except that all converge to a common point at about 37.5 percent gas saturation. At the farthest point from the basic rock characteristics curve, the apparent k_g/k_o for the 0.1-B/D rate was approximately 10 times the true value. Hence it is readily apparent that fluid segregation is a major factor in the lack of coincidence of laboratory- and field-derived relative permeability ratios. Also it is clear that it would be highly coincidental if any scheme for extrapolation of any of the low production rate curves from field performance should accurately predict future performance.

At production rates approaching zero the system should be at capillary equilibrium at all times. Complete oil saturation should exist below the gas-oil contact, and saturations above the contact will be determined by the static capillary pressure curve of Fig. 2. Under these conditions a calculation of the over-all k_g/k_o is simply a matter of summing the gas and oil relative permeabilities at each level at various stages of depletion, and hence, at various average oil saturations. The apparent k_g/k_o curve shown for zero gas production rate was thus calculated. The apparent k_g/k_o relationships for all producing rates should lie between this curve (where fluid distribution is completely controlled by gravity-capillary pressure equilibrium) and the true flow characteristics curve (where no gravitational effects are present). Fig. 12 shows this to be true. Also the zero rate curve intersects the common point of all other curves at 37.5 percent gas saturation. It is important to note that, beyond this intersection, performance with gravity segregation should be better (lower k_g/k_o and hence lower GOR's) than would be indicated by the true rock characteristics and uniform saturations. This improved behavior is not observed during depletion by solution gas drive because all the solution gas is produced before this point is reached. However there is ample evidence of such low k_g/k_o performance over wide ranges of oil saturation from reservoirs in which low-pressure gas drive has been successful. A notable example of such performance is that of the Cook Ranch field.[20]

Generalization of Calculated Performance

Studies of the criteria for scaling laboratory models to simulate oil reservoirs[40] indicate that for the system under study the performance of the system should be controlled by the ratio of oil production rate to vertical permeability. To demonstrate the validity of this hypothesis, three sets of calculations were made in which the oil production rates were 12.5, 1.25, and 0.125 B/D and the vertical permeability of the system was set at 100, 10, and 1 md to maintain a constant ratio of production rate to vertical permeability. The results of this series of calculations are shown in Fig. 13. Every detail of the performance was identical. This allowed extension of the applicability of the results from the system having 101 md permeability to systems of any permeability. Fig. 14 presents curves of the previously discussed data, showing the relationship between fraction of oil in place produced per year per millidarcy vertical permeability vs oil recovered by solution gas drive from the original pressure to various final pressures. For example, if a reservoir (having the rock and fluid characteristics of the model) with a vertical permeability of 10 md were produced at an oil rate of 10 percent of the oil in place per year, recovery to 30 psi reservoir pressure would be 20.5 percent of the oil in place. Total producing life would be over 2.0 years. Producing that same field at one-tenth the rate (total field life just over 20 years), would result in an ultimate recovery of 17 percent, or 82 percent of that produced at the higher rate. Similar comparisons can be made for any combination of producing rate and vertical permeability. For most combinations likely to occur in real fields, even twofold changes in producing rate will result in significant changes in the oil ultimately recovered by solution gas drive.

It must be emphasized at this point that Fig. 14 *is not* quantitatively applicable to all reservoirs but is only a qualitative guide. The exact relationship between rate and oil recovery by solution gas drive will be affected to a marked degree by fluid properties — such as gas solubility, viscosity, and shrinkage — as well as by the relative permeability and, probably to a lesser degree, by the capillary pressure characteristics of the reservoir rock. It is conceivable that any particular reservoir could exhibit either greater or less sensitivity to producing rate in a given range than the reservoir modeled in this investigation.

Summary and Conclusions

Numerical model simulations have been made of solution gas drive performance covering a 500-fold range of oil production rates. By scaling theory, confirmed by calculation, these results have been extended to all levels of vertical permeability. Two-dimensional simulations were made in which the maximum total horizontal pressure drop was held to less than 20 percent of the average absolute pressure. These tests showed such small horizontal saturation

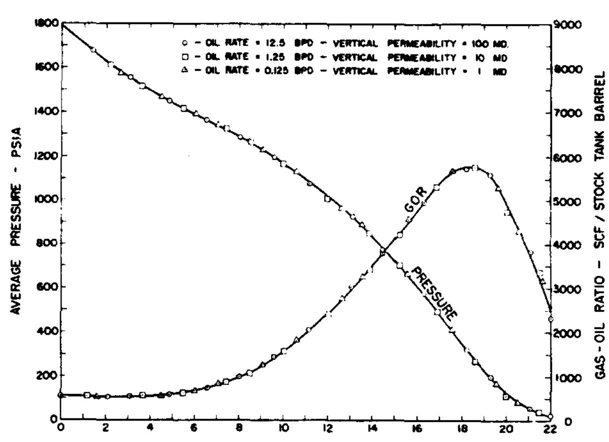

Fig. 13—Performance of systems of constant ratio between oil production rate and vertical permeability.

gradients that comparative one-dimensional tests were made for the same producing rates. In every detail of performance the one-dimensional model results were almost identical with the two-dimensional runs. Hence, most of the results reported are from one-dimensional simulations. The model simulated was a linear reservoir segment 1,400 ft long, 25 ft high and 1 ft thick. For the 101-md model the oil production rates ranged from 0.1 B/D to 50.0 B/D. Horizontal permeability does not affect performance as long as the horizontal pressure differential is limited to 20 percent of the average reservoir pressure. The production rates simulated correspond to a range of 2.48 to 1,280.0 percent of the oil in place produced per year. These same production rates would correspond to 0.248 to 128 percent of oil in place produced per year for a system having 10 md vertical permeability. At the lower production rates the gas and oil show marked vertical segregation during production. This gravity segregation very significantly affects the GOR and pressure performance, resulting in large differences in ultimate oil recovery by solution gas drive. At high production rates there is some slight gravity segregation, but the effect on performance is negligibly different from that of a gravity-free system.

Conclusions of this work are:

1. Ultimate oil recovery by solution gas drive can be greatly affected by variations in production rates within the range of rates possible in normal field operations. In general, higher production rates result in higher oil recoveries. The quantitative relationship for a given field will be influenced by factors other that rate and vertical permeability — namely, reservoir thickness, gas solubility, shrinkage factor, oil viscosity, relative permeability, and capillary pressure characteristics. Also it should be emphasized that selective well completion can importantly affect performance in some field situations.

2. The underlying reason for the variation in oil ultimately produced by solution gas drive is the vertical segregation of oil and gas during depletion.

3. Although vertical fluid segregation causes generally poorer performance (higher k_g/k_o) than does uniform saturation in the range of solution gas performance, the performance of the segregated system is much better than the uniform system at high gas saturations. This is confirmed by very high recoveries obtained in fields where low-pressure gas drive has been economically feasible.

4. In horizontal producing reservoirs one-dimensional numerical models are adequate for determining the relationship between production rate and performance by solution gas drive.

Nomenclature

$A8$ = factor including all compressibility terms
$A9$ = factor including terms for production, injection, gravity and capillary pressure forces
g = gas
k = single-phase permeability
k_{rg} = relative permeability to gas, fraction of single-phase permeability
k_{rgx} = relative permeability to gas in the x direction
o = oil
O = coefficient of pressure differential between blocks
O^{+}_{x} = coefficient of pressure differential between the element under consideration and the adjacent block in the direction of increasing x
p = pressure
Δp = pressure differential between adjacent elements
p_o = oil pressure
q = flow rate at reservoir conditions
q_g = gas flow rate
S = saturation, fraction of total hydrocarbon pore space
S_g = gas saturation
S_o = oil saturation
t_o = time at start of calculation interval
t_1 = time at end of calculation interval
u = fluid flow rate per unit area
u_o = oil flow rate
μ = viscosity
μ_g = gas viscosity
ρ = fluid density
ρ_o = oil density

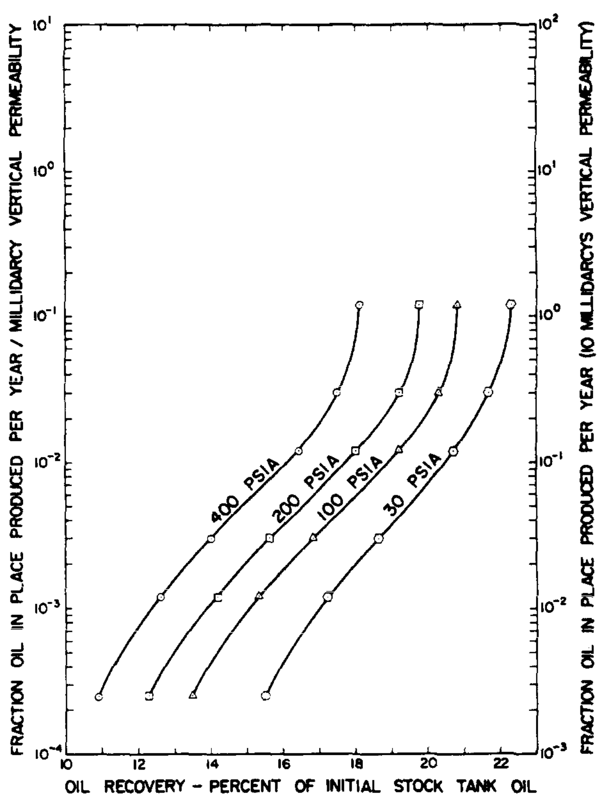

Fig. 14—Relationship between oil recovery by solution gas drive from 1,791 psi to various pressure levels and oil production rates.

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