The Effect of Water Influx on p/z-Cumulative Gas Production Curves

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Abstract

The relationship between p/z and cumulative gas production for typical gas reservoirs was studied by calculating pressure response to various modes of gas production and water encroachment. Water encroachment methods considered were Schilthuis, Hurst simplified and van Everdingen-Hurst. In the method, the assumptions normally made in water encroachment calculations were accepted. Normally, pressures are measured and the gas reserves and water encroachment found implicitly. Conversely, in this work various encroachment factors, reserves and reservoir-aquifer geometry were assumed and the pressures solved implicitly.

The results show the spectrum of p/z shapes that can be expected for real reservoirs. With normal encroachment rates for closed aquifers the p/z chart exhibits the typical inflection at early times. This has sometimes been interpreted as all measurement error. These studies have shown that a new look should be taken at interpretation. It is rather dangerous to extrapolate "straight-line" p/z charts if encroachment from an aquifer is suspected.

Introduction

A common method of predicting gas reserves is the graphical solution to the gas material balance equation. A special case of the material balance equation is linear in p/z with cumulative gas production (G_p) which predicts the initial in-place gas when p/z is extrapolated to zero. Derivation of this form is based on the equation of state, corrected for compressibility ($pV = znRT$), and, particularly, on the reservoir being closed (no water encroachment). A straight line on the p/z chart results when these conditions hold. However, an apparent straight line on the chart does not assure that the reservoir is closed. Many of the curves show a rapid decline in the early stages of production after which they flatten out. Confusion arises as to whether these characteristics are caused totally by pressure measurements. To answer this question in part, a series of controlled mathematical experiments was performed in which a typical gas field was produced subject to various forms of water encroachment. These runs were specifically designed to eliminate measurement errors by calculating pressures at the inner boundary of the aquifer. The resultant p/z charts were thus made available for study and direction in predicting reserves and to indicate the curvature that can be expected in addition to that caused by normal measurement error.

Solution of the Basic Equation for p/z

The basic equation solved for p and p/z is derived in Appendix A. It is

$$G_a = G_r + \frac{K_r S(p,t)}{B_g - B_{gi}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

G_a and G_r are the apparent and real values of original gas in place and are derived by assuming a closed reservoir for G_a, and one open to an aquifer for G_r. The function $S(p,t)$ is defined by three methods—Schilthuis, Hurst simplified or van Everdingen-Hurst.[1-4] The definitions of these functions are given in Appendix B.

Eq. 1 is the linear function that is commonly plotted (G_a vs $S(p,t)/B-B_i$) with the intercept predicting the original gas in place and slope predicting the water encroachment factor.[5,6] This is a graphical solution of Eq. 1 when histories on pressure and cumulative productions are known. In some cases the equation has been rearranged so a plot can be made such that the encroachment factor is predicted by the intercept and the reserve by the slope.[7]

In the calculations presented in this paper, in-place values, water encroachment factors, rock fluid properties, and cumulative production were set. Eq. 1 was solved implicitly for p/z.

The equivalent of Eq. 1 in terms of p/z is

$$p/z = \frac{p_i}{z_i}\left[\frac{G_r - G_p}{G_r - \dfrac{z_i\, T\, p_{sc}}{T_{sc}\, p_i\, K_r S(p,t)}}\right] \quad . \quad . \quad . \quad (2)$$

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[1]References given at end of paper.

Setting $K_e\,S\,(p,t) = 0$ (no water encroachment), produces the linear form. Obviously, whether the p/z curve is linear or not when $K_e\,S(p,t) \neq 0$ depends upon the $S(p,t)$ function.

The cumulative productions were determined from production rates calculated from wellhead operating curves subject to the maximum allowables.[8] The wellhead curve is defined by

$$q_g = C(p_{swh}^2 - p_{tf}^2)^n, \quad \ldots\ldots\ldots \quad (3)$$

where q_g = the production rate, Mscf/D
C = the performance coefficient
p_{swh} = the wellhead shut-in pressure, psia
p_{tf} = the tubing flowing pressure
n = the back-pressure exponent.

Shut-in wellhead pressures were determined after the reservoir pressure p was chosen by calculating the static head by the method of Cullender and Smith.[9] The static head was subtracted from p to give p_{swh}.

A general flow scheme of the calculation technique is given in Fig. 1, and the field conditions are given in Table 1.

Compressibilities were interpolated from the 1952 API tables. Tables 2 and 3 list conditions that were varied for individual runs.

Discussion of Results

The results of the calculations are shown in Figs. 2 through 9. All of the curves show p/z as a function of cumulative gas produced and are labeled with the numbers corresponding to the data in Tables 2 and 3. Each plotted point represents two years.

Fig. 2 gives the results when the aquifer was assumed to be unlimited, or when original aquifer pressure was assumed to remain constant at some outer boundary (Schilthuis). As the encroachment factor was increased the pressure was maintained at a higher and higher level. The dotted line at the bottom represents no encroachment and the top dotted line shows complete pressure maintenance by a very active water drive.

Fig. 3 shows the results of increasing the Hurst simplified encroachment factor from 2.5×10^4 to 2.5×10^6 (cu ft) ln (mo)/psi/year.

The van Everdingen-Hurst encroachment factors were assumed for runs shown in Figs. 4 through 9, and the aquifer was assumed closed and radial. Combinations of three variations in relative aquifer size, two water compressibilities and nine aquifer permeabilities were represented in the runs.

Curves with inflections, which have been observed in practice, were produced for the closed aquifers.

In most cases the families of curves appear to approach a common slope at zero time. At zero time this slope will represent the p/z line for no water encroachment.

Runs with the Schilthuis method and Hurst-simplified method converge at or near a horizontal line as water encroachment factors increase. This means that pressure drops in the aquifer are approaching zero.

In the van Everdingen-Hurst runs the curves respond to the mobility (k/μ) and compressibility of the water, and the relative size of the aquifer. For R_a/R_r of 1.5,

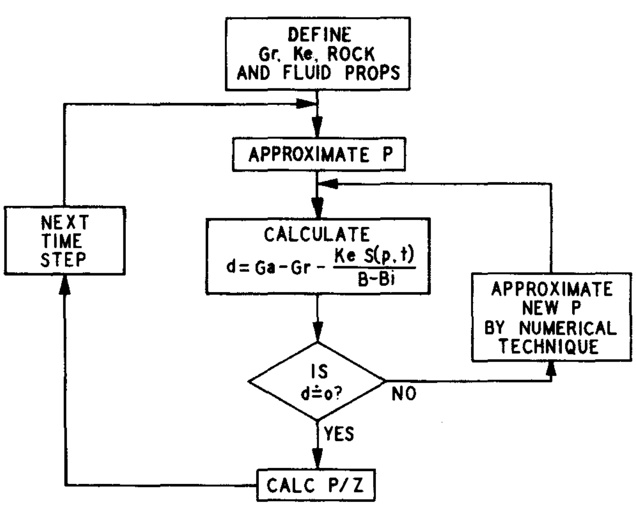

Fig. 1—Solution of Eq. A-12 for p/z.

TABLE 1—FIELD CONDITIONS

Area = 2,500 acres
Pay = 100 ft
Porosity = 0.25
Connate water = 0.3
Original BHP = 5,000 psia
Formation temperature = 250F
Depth = 10,000 ft
Gas Gravity = 0.68 (No N_2, CO_2 or H_2S)
(Radius)^2 = 34.7×10^6 sq ft
Initial wellhead shut-in pressure = 4,200 psia
Wellhead shut-in temperature = 100F
Back-pressure curve slope = 0.7
Open flow potential = 74.4 MM scf/D
Minimum wellhead flowing pressure = 100 psia
Maximum allowable field rate = 47.2 MM scf/D

TABLE 2—VARIABLE CONDITIONS FOR RUNS 1 THROUGH 14

Run No.	Type Encroachment Factor	Encroachment Factor
1	↑	5,900
2		18,000
3	Schilthuis	36,000
4	(cu ft/psi/year)	59,000
5	Radial Infinite	100,000
6	↓	200,000
7		590,000
8	↑	25,000
9		90,000
10	Hurst Simplified	150,000
11	(cu ft ln (month)/psi/year)	250,000
12	Radial Infinite	340,000
13	↓	610,000
14		2,500,000

TABLE 3—VARIABLE CONDITIONS FOR RUNS 15 THOUGH 38

Run No.	Type Encroachment Factor	Ratio of Aquifer Radius to Field Radius	Aquifer Permeability (md)	Dimensionless Time to Real Time Ratio (1/year)	Water Compressibility (1/psi)
15		1.5	1	.089	3.0×10^{-6}
16		1.5	10	.89	3.0×10^{-6}
17	↓	1.5	100	8.9	3.0×10^{-6}
18		5.0	1	.089	3.0×10^{-6}
19	van Everdingen	5.0	10	.89	3.0×10^{-6}
20	Hurst Radial Finite	5.0	100	8.9	3.0×10^{-6}
21		5.0	1000	89.	3.0×10^{-6}
22	16,350 cu ft/psi	10.0	1	.089	3.0×10^{-6}
23		10.0	10	.89	3.0×10^{-6}
24		10.0	100	8.9	3.0×10^{-6}
25	↓	10.0	1000	89.	3.0×10^{-6}
26		10.0	10000	890.	3.0×10^{-6}
27		1.5	10	.089	30×10^{-6}
28		1.5	100	.89	30×10^{-6}
29	↓	5.0	10	.089	30×10^{-6}
30		5.0	18	.16	30×10^{-6}
31	van Everdingen	5.0	39.3	.35	30×10^{-6}
32	Hurst Radial Finite	5.0	100	.89	30×10^{-6}
33		10.0	10	.089	30×10^{-6}
34	163,500 cu ft/psi	10.0	15.8	.141	30×10^{-6}
35		10.0	31.5	.28	30×10^{-6}
36		10.0	100	.89	30×10^{-6}
37	↓	10.0	1000	8.9	30×10^{-6}
38		10.0	10000	89.	30×10^{-6}

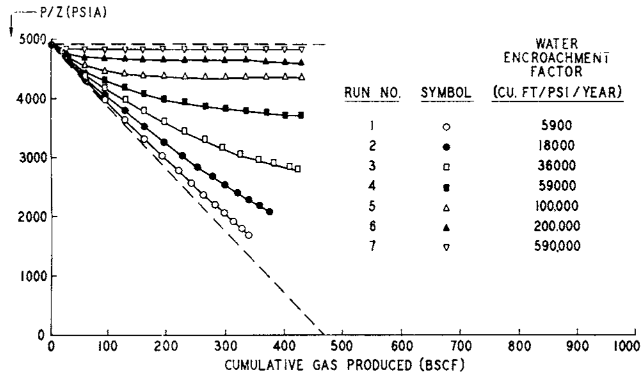

Fig. 2—Curves of p/z for gas reservoirs with water influx, Schilthuis method.

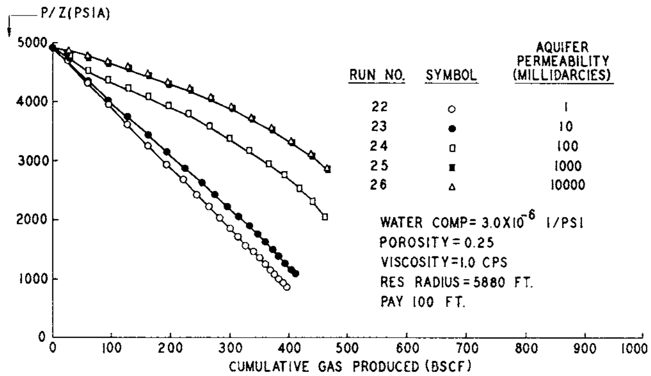

Fig. 6—Curves of p/z for gas reservoirs with water influx, van Everdingen-Hurst method, finite, $R_a/R_r = 10$.

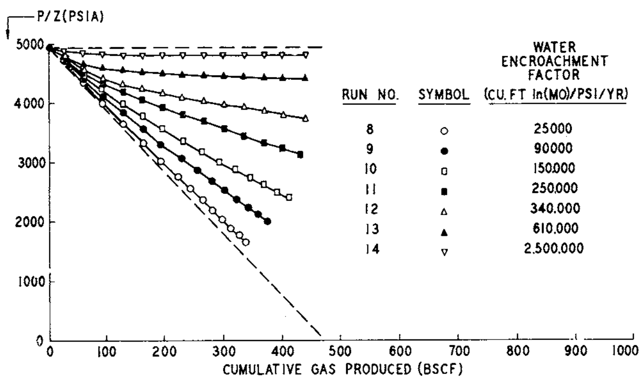

Fig. 3—Curves of p/z for gas reservoirs with water influx, Hurst simplified method.

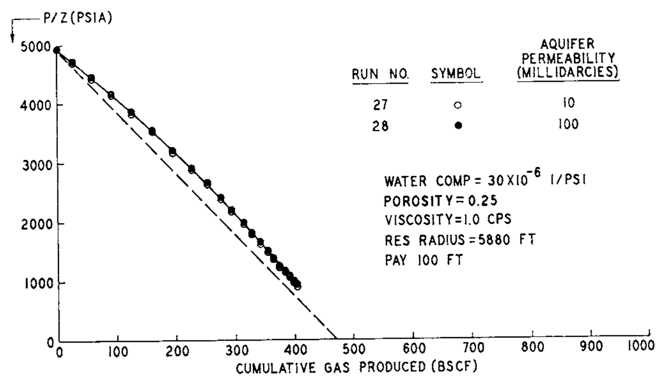

Fig. 7—Curves of p/z for gas reservoirs with water influx, van Everdingen-Hurst method, finite, $R_a/R_r = 1.5$.

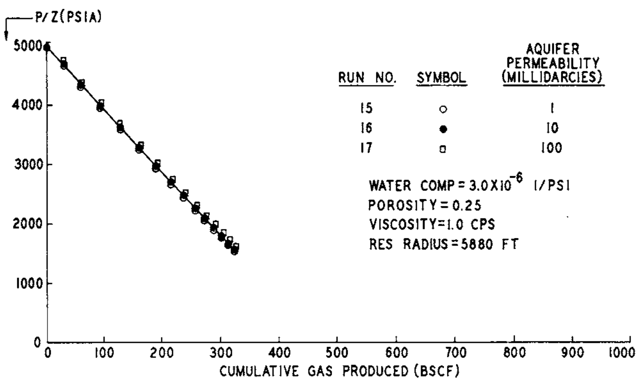

Fig. 4—Curves of p/z for gas reservoirs with water influx, van Everdingen-Hurst, finite, $R_a/R_r = 1.5$.

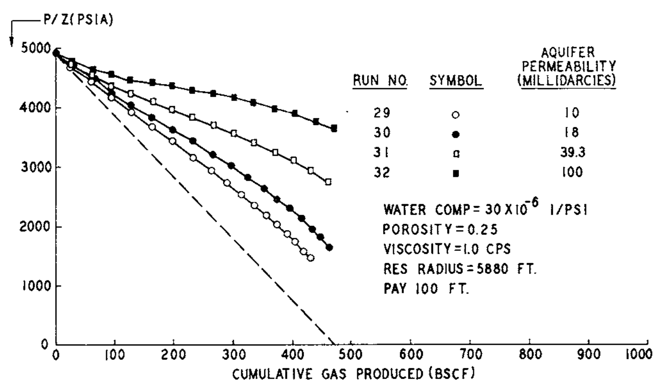

Fig. 8—Curves of p/z for gas reservoirs with water influx, van Everdingen-Hurst method, finite, $R_a/R_r = 5$.

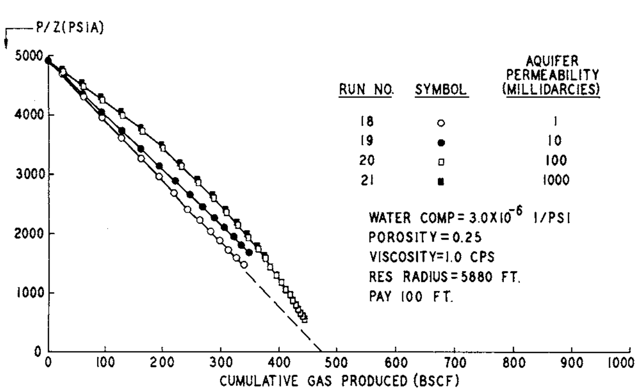

Fig. 5—Curves of p/z for gas reservoirs with water influx, van Everdingen-Hurst method, finite, $R_a/R_r = 5$.

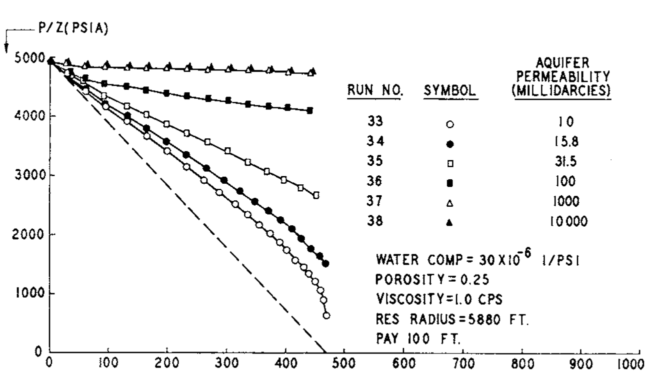

Fig. 9—Curves of p/z for gas reservoirs with water influx, van Everdingen-Hurst method, finite, $R_a/R_r = 10$.

the effect of the aquifer is negligible for a given water compressibility regardless of the permeability (Fig. 4). However, for a higher water compressibility an effect is felt for comparable mobilities (Fig. 7).

In general the pressure is maintained at higher levels as the water compressibility, aquifer size or water mobility is increased. Yet, even with increases in mobility an extreme curve was approached for the closed aquifers (Runs 25 and 26, Fig. 6). In these cases pressure drops in the aquifer were small and the shapes were controlled by the water compressibilities.

Conclusions

Fig. 10 illustrates the increasing error that occurs if a p/z curve is extrapolated with no regard for water encroachment. As the relative size of the aquifer increases from $R_a/R_r = 1.5$ to 10, the error increases from a negligible amount to an estimate of over 100 per cent of the actual initial gas in place. This estimate would be made after 65 per cent of the initial gas in place is produced.

This leads to the principal conclusion that it is dangerous to extrapolate p/z charts on a straight line without considering the possibility of water influx.

Runs performed here eliminated measurement error and the curved portions were produced under realistic production schedules. Thus, curved portions at the start of production history can be caused by the unsteady-state nature of the aquifer and not solely by measurement errors. So, these curved portions should not be neglected, but ought to be regarded as an indication of possible water encroachment.

These results make a case for accelerated early production so that the inflections will be accentuated, permitting better early estimates of gas in place.

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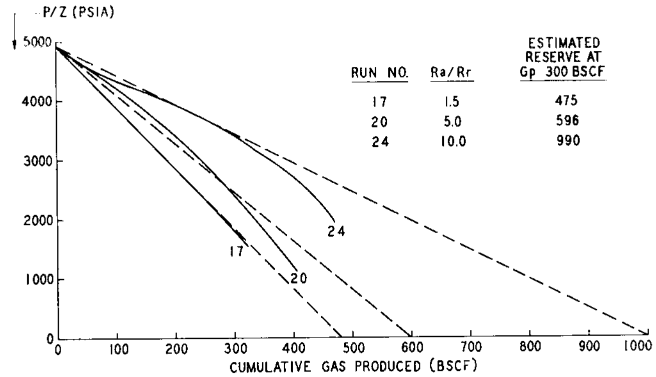


Fig. 10—Comparison of p/z curves for increasing aquifer sizes, van Everdingen-Hurst, radial finite.

APPENDIX A

Derivation of the Basic Equations

The apparent reserves for a gas field are those determined when no water encroachment is assumed, or,

$$V_p = V_{pi}, \quad \text{(A-1)}$$

where V_{pi} is the original pore volume, cu ft.

V_p is the pore volume containing gas at some later time.

$$V_p = (G_a - G_p)B_g \quad \text{(A-2)}$$

$$V_{pi} = G_a B_{gi}, \quad \text{(A-3)}$$

where G_a = the apparent original gas in place, scf
G_p = cumulative gas produced, scf

B_{gi} and B_g are the gas formation volume factors, cu ft/scf.

$$B_{gi} = \frac{p_{sc} T_i Z_i}{p_i T_{sc}} \quad \text{(A-4)}$$

$$B_g = \frac{p_{sc} T z}{p T_{sc}} \quad \text{(A-5)}$$

Substituting Eqs. A-2 and A-3 into Eq. A-1 and solving for G_a gives

$$G_a = G_p\left(\frac{B}{B_g - B_{gi}}\right) \quad \text{(A-6)}$$

When water encroachment is considered, Eq. A-1 is replaced by

$$V_p = V_{pi} - W_e \quad \text{(A-7)}$$

to account for the water influx W_e.

Under these conditions, G_a in Eqs. A-2 and A-3 is defined as G_r (real initial in place gas), or,

$$V_p = (G_r - G_p)B_g, \quad \text{(A-8)}$$

and

$$V_{pi} = G_r B_{gi} \quad \text{(A-9)}$$

Substitution of Eqs. A-8, A-9, and

$$W_e = K_e S(p,t) \quad \text{(A-10)}$$

into Eq. A-7 gives

$$G_r = \frac{G_p B}{(B_g - B_{gi})} - \frac{K_e S(p,t)}{(B_g - B_{gi})} \quad \text{(A-11)}$$

where K_e is the water encroachment factor and $S(p,t)$

TABLE 4

R_a/R_r	Middle Range of Δtd_i From	To	Equation B-10—A Constants A_1	A_2	A_3	A_4	A_5	Stabilized State Values Above the Middle Range
1.5	.1206	0.7	0.03255975	0.02485001	0.03179695	0.007970778	0.6164517	0.6235899
2.0	.418	2.5	0.3852062	0.09626595	0.05244533	−0.004754153	1.281475	1.509915
2.5	.815	6.0	0.7919653	0.05396428	−0.06348801	−0.01234595	1.534877	2.634689
3.0	1.33	11.0	1.046089	0.2388103	−0.08575203	−0.008445356	1.574667	3.994681
3.5	1.12	25.0	0.4178854	1.292179	−0.4404957	0.03704949	1.630682	5.650575
4.0	2.05	34.0	−2.231692	3.177286	−0.8411667	0.06444676	2.779082	7.499222
4.5	2.62	46.0	−6.108747	5.413047	−1.266439	0.09235701	4.890919	9.619498
5.0	3.06	60.0	−6.429505	4.823608	−0.8503674	0.03632684	5.599367	11.97866
6.0	5.85	110.0	−24.90336	12.44925	−2.042113	0.1044283	20.58242	17.48006
7.0	8.48	160.0	−43.33130	17.84979	−2.486980	0.1043855	39.95260	23.95055
3.0	9.29	240.0	−51.48727	19.26185	−2.365595	0.08228036	50.46776	31.66351
9.0	9.96	280.0	−31.97360	8.612722	−0.05276312	−0.08174990	38.52328	39.96676
10.0	14.52	360.0	−20.55106	0.6903652	1.759464	−0.2079732	37.02682	49.14654

is a function of pressure and time and describes the unsteady-state water influx.

Subtraction of Eq. A-11 from Eq. A-6 results in the basic equation

$$G_a = G_r + \frac{K_e\, S(p,t)}{(B_g - B_{gi})} \quad \ldots\ldots\ldots \quad \text{(A-12)}$$

APPENDIX B

Definition of $S(p,t)$

Schilthuis Method

$$S(p,t) = \sum_{i=2}^{n} \Delta p_i\, \Delta t_i, \quad \ldots\ldots\ldots \quad \text{(B-1)}$$

$$\text{where } \Delta p_i = p_i - \frac{(p_i + p_{i-1})}{2}, \quad \ldots\ldots\ldots \quad \text{(B-2)}$$

$$\text{and } \Delta t_i = t_i - t_{i-1} \quad \ldots\ldots\ldots \quad \text{(B-3)}$$

where n = number of pressure points

t_i = time in years

p = aquifer pressure (inner boundary) psia.

Hurst-Simplified Method

$$S(p,t) = \sum_{i=2}^{n} \frac{\Delta p_i\, \Delta t_i}{\ln(12 t_i)}, \quad \ldots\ldots\ldots \quad \text{(B-4)}$$

where Δp_i and Δt_i are still defined by Eqs. B-2 and B-3

van Everdingen-Hurst Method

$$S(p,t) = \sum_{i=1}^{n-1} \Delta p_i\, q_{\Delta t_{di}}, \quad \ldots\ldots\ldots \quad \text{(B-5)}$$

where

$$\Delta p_i = \frac{p_{i+1} - p_i}{2}, \quad \ldots\ldots\ldots \quad \text{(B-6)}$$

for $i = 1$, and

$$\Delta p_i = \frac{p_{i+1} - p_{i-1}}{2}, \quad \ldots\ldots\ldots \quad \text{(B-7)}$$

for $i = 2$ to $n - 1$.

$q\Delta t_{di}$ is the dimensionless flow rate and is a function of Δt_{di} (the dimensionless time increment) and aquifer geometry.

$$\Delta t_{di} = 2.309 \left(\frac{k \quad \Delta t}{\phi \mu C_w (R_r)^2} \right), \quad \ldots \quad \text{(B-8)}$$

with k in millidarcies, t in years, ϕ a fraction, μ in cp, C_w in 1/psi, R_r in ft.

$q\Delta t_{di}$ is defined under the following conditions. All $\Delta t_{di} <$ 0.01, or the linear system

$$q\Delta t_{di} = 2\sqrt{\Delta td_i / \pi}. \quad \ldots\ldots\ldots \quad \text{(B-9)}$$

Infinite Radial System

$$(0.01 < \Delta t_{di} \leq 10^8)$$

$$q\Delta t_{di} = e^{\,(A_1 \ln \Delta t_{di} + A_2) \ln \Delta t_{di})^2 + A_3 (\ln \Delta t_{di})^3 + A_4 (\ln \Delta t_{di})^4 + A_5)}$$

where $A_1 = 0.647692$

$A_2 = 0.0177318$

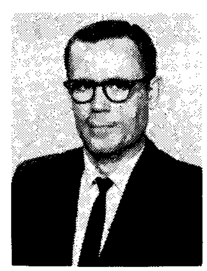
$A_3 = -0.0002737391$

$A_4 = -0.4318125 \times 10^{-5}$

$A_5 = 0.4506432.$

Finite Radial Systems

The finite radial systems are defined by the infinite radial Eq. B-10, where $\Delta t_{di} > 0.01$ and less than the middle range defined in Table 4. In the middle range the dimensionless flow is defined by an equation of the same form as Eq. B-10, but with constants shown in Table 4. Table 4 also gives the steady-state values applicable above the middle range. ★★★



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