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The Material Balance as an Equation of a Straight Line

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ABSTRACT

The material balance equation used by reservoir engineers is arranged algebraically, resulting in an equation of a straight line. The straight line method of analysis imposes an additional necessary condition that a successful solution of the material balance equation should meet. In addition, this algebraic arrangement attaches a dynamic meaning to the otherwise static material balance equation.

The straight line method requires the plotting of one variable group vs another variable group. The sequence of the plotted points as well as the general shape of the resulting plot is of utmost importance. Therefore, one cannot program the method entirely on a digital computer as is usually done in the routine solution of the material balance equation. If this method is applied, then plotting and analysis are essential.

Only the appropriate equations and the method of analysis and interpretation with comments and discussion are presented in this paper. Illustrative field examples for the various cases treated are deferred to a subsequent writing.

INTRODUCTION

One of the fundamental principles utilized in engineering work is the law of conservation of matter. The application of this principle to hydrocarbon reservoirs for the purpose of quantitative deductions and prediction is termed "the material balance method of reservoir analysis". While the construction of the material balance equation (MBE) and the computations that go with its application are not difficult tasks, the criteria that a successful solution of the MBE should fulfill have always been a problem facing the reservoir engineer.

True and complete criteria should embody necessary and sufficient conditions. The criteria which the reservoir engineer uses possess a few necessary but no sufficient conditions. Because of this, the answers obtained from the MBE are always open to question. However, the degree of their acceptability should increase with the increase in the number of the necessary conditions that they should satisfy.

Generally, the necessary conditions commonly used are (1) an unspecified consistency of the results and (2) the agreement between the MBE results and those determined volumetrically.

This second criterion is usually overemphasized. Actually, the volumetrically determined results are based on geological and petrophysical data of unknown accuracy. In addition, the oil-in-place obtained by the MBE is that oil which contributes to the pressure-production history,[1] while the volumetrically calculated oil-in-place refers to the total oil, part of which may not contribute to said history. Because of this difference, the disagreement between the two answers might be of paramount importance, and the concordance between them should not be overemphasized as the measure of correctness of either one.

In this paper, a third necessary condition of mathematical as well as physical significance is discussed. It is not subject to any geological or petrophysical interpretation, and as such, it is probably the most important necessary condition. It consists essentially of rearranging the MBE to result in an equation of a straight line. This straight line method of the MBE solution has invalidated a few long time accepted concepts. For instance, it has always been advocated that if a water drive exists, but one neglects to take it into account in the MBE, the calculated oil-in-place should increase with time. The straight line method shows that in some cases, depending on the size of the neglected aquifer, the calculated oil-in-place might decrease with time.

The straight line method requires the plotting of a variable group vs another variable group, with the variable group selection depending on the mechanism of production under which the reservoir is producing. The most important aspect of this method of solution is that it attaches a significance to the sequence of the plotted points, the direction in which they plot, and to the shape of the resulting plot. Thus, a dynamic meaning has been introduced into the picture in arriving at the final answer. Since the emphasis of this method is placed on the interpretation of the sequence of the points and the shape of the plot, one cannot completely automate the whole sequence to obtain "the best value" as normally done in the routine application of the MBE. If one uses the straight line method, then plotting and analysis *are musts.*

The straight line method was first recognized by van Everdingen, et al,[2] but for some reason it was never fully exploited. The advantages and the elegance of this method can be more appreciated after a few cases are carefully treated and worked out by it.

SOLUTION OF THE MATERIAL BALANCE EQUATION

SATURATED RESERVOIRS

The MBE for saturated reservoirs written in AIME symbols is

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[1]References given at end of paper.

$$N_p [B_t + B_g (R_p - R_{si})] + W_p - W_i - Gi\,Big = N\left[(B_t - B_{ti}) + \frac{B_{ti}}{1 - S_w}(c_f + S_w c_w)\,\Delta p + \frac{m\,B_{ti}}{B_{gi}}(B_g - B_{gi})\right] + W_e \quad . \ . \ . \ . \ . \ . \ . \quad (0)$$

The left hand side of Eq. 0 represents the net production in reservoir barrels and will be denoted by F. On the right hand side, the first term includes, respectively, the expansion of the oil E_o, the rock and connate water $E_{f,w}$, and the free gas E_g. The second term represents the water influx which is given by[3,4]

$$W_e = C\Sigma\Delta pQ(\Delta t_D)$$

For saturated reservoirs, one normally neglects the rock and water expansion $E_{f,w}$. Thus, Eq. 0 becomes

$$F = NE_o + Nm\frac{B_{ti}}{B_{gi}}E_g + C\Sigma\Delta pQ(\Delta t_D) \quad . \ . \quad (0a)$$

Eq. 0a is the expanded form of the MBE, where the three mechanisms of production, i.e., oil expansion, $E_o = (B_t - B_{ti})$, gas expansion, $E_g = (B_g - B_{gi})$ and water drive are included. Absence of one or two of the above mechanisms requires deletion of the appropriate terms from the equation.

In the figures that follow, the sequence of the individual plotted points, calculated for increasing cumulative production, will be indicated by an arrow.

No Water Drive, No Original Gas Cap

$$F = NE_o \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

A plot of F vs E_o should result in a straight line going through the origin with N being the slope, Fig. 1. It should be noted that the origin is a *must* point; thus, one has a fixed point to guide the straight line plot.

No Water Drive, A Known Gas Cap

$$F = N(E_o + m\frac{B_{ti}}{B_{gi}}E_g) \quad . \ . \ . \ . \ . \ . \ . \ . \quad (1a)$$

A plot of F vs $(E_o + m\frac{B_{ti}}{B_{gi}}E_g)$ should result in a straight line going through the origin with a slope of N.

No Water Drive, N and m are Unknown

The appropriate MBE is written in two forms so as to result in two methods of solution, Eqs. 2a and 2b.

$$\frac{F}{E_o} = N + G\frac{E_g}{E_o} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2a)$$

where $G = Nm\frac{B_{ti}}{B_{gi}}$ = the original gas-cap gas in scf. A plot of $\frac{F}{E_o}$ vs $\frac{E_g}{E_o}$ should result in a straight line with N being the Y intercept and G being the slope, Fig. 2a.

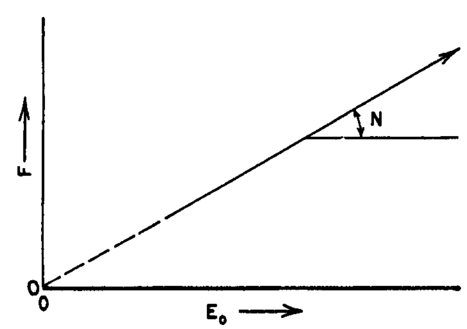

FIG. 1—F vs E_o

$$F = N\left(E_o + m\frac{B_{ti}}{B_{gi}}E_g\right) \quad . \ . \ . \ . \ . \ . \ . \ . \quad (2b)$$

Assume an m and plot F vs $\left(E_o + m\frac{B_{ti}}{B_{gi}}E_g\right)$. If the assumed m is correct, the plot will be a straight line going through the origin with N being the slope. If the assumed m is too small the line will go through the origin but will curve upward. If the assumed m is too large the line will go through the origin but will curve downward (Fig. 2b). Several values of m are assumed until the straight line going through the origin plot is satisfied.

As the reader will appreciate, the solution (Eq. 2b) is a more powerful method than the one in Eq. 2a since it specifies that the line must go through the origin. However, for checking purpose it is recommended that both methods be used in every case.

Water Driven Reservoirs, Two Unknowns

Water Drive, No Original Gas Cap:

$$\frac{F}{E_o} = N + C\frac{\Sigma\Delta pQ(\Delta t_D)}{E_o} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (3a)$$

Assume an aquifer configuration, an $\frac{r_e}{r_w}$ and a dimensionless time Δt_D. Calculate $\Sigma\Delta pQ(\Delta t_D)$ and plot $\frac{F}{E_o}$ vs $\frac{\Sigma\Delta pQ(\Delta t_D)}{E_o}$. If the assumed aquifer and dimensionless

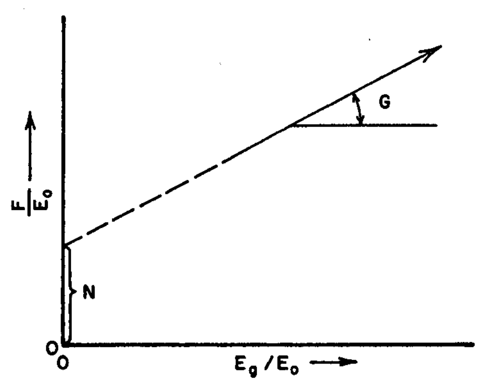

FIG. 2A—$\frac{F}{E_o}$ vs $\frac{E_g}{E_o}$

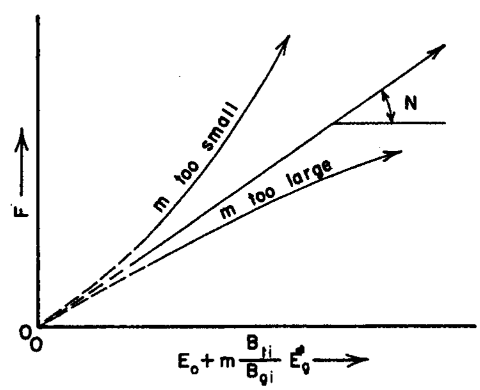

FIG. 2B—F vs $(E_o + m\frac{B_{ti}}{B_{gi}}E_g)$

time are correct, the plot will be a straight line with N being the Y intercept and C being the slope.

Four other different plots beside the straight line may result. These are a complete scatter, a line curved upward, a line curved downward, and an S-shaped curve (Fig. 3a).

Complete random scatter of the individual points indicates that the calculations and/or the basic data are in error. A systematically upward or downward curved line suggests that the $\Sigma\Delta pQ(\Delta t_D)$ is too small or too large, respectively. This means that the assumed $\frac{r_e}{r_w}$ and/or the Δt_D are, respectively, too small or too large. An S-shaped curve indicates that a better fit could be obtained if a linear water influx is assumed.

The sequence of the plotted points as indicated by the arrow of Fig. 3a will persist as long as the aquifer behaves like an infinite one. This is particularly applicable for infinite or fairly large aquifers. In this case, non-steady state water influx calculations are a must. On the other hand, if one suspects the presence of a small aquifer, in which steady-state depletion type flow would obtain in a short time after production commences, then, it is better to start with the case shown in Eq. 3b.

After satisfactory values for $\frac{r_e}{r_w}$ and for Δt_D are chosen, the results can be refined by applying the standard deviation test suggested by van Everdingen, et al.[2] The most probable values for N and C will be those corresponding to the dimensionless time which gives the minimum standard deviation σ min.

In some reservoirs the standard deviation σ plotted vs log Δt_D will not give a sharp minimum but will be "dish-shaped". This phenomenon usually results from the fact that the particular reservoir is insensitive to the changes of Δt_D. The establishment of the most probable value of Δt_D becomes, in such a case, only of academic interest.

An additional criterion used to judge the most probable values for N and C is called the consistency test, which is described in the following. Several Δt_D values around the minimum point of the standard deviation plot are read. For every chosen Δt_D, N and C as functions of real time are calculated. Plots of N vs real time and C vs real time are constructed, and by means of the least square method, the best straight line is drawn through the points of every plot. The slopes of the N and C straight lines are then calculated and plotted vs their corresponding Δt_D values on a common graph paper. The intersection of the two plots gives the most probable value for the Δt_D. Theoretically, the two plots should intersect at a value of zero slope. This is true because if the correct $\frac{r_e}{r_w}$ and Δt_D are chosen, and if the field data are correct, then N and C should not vary with time, i.e., the N-time plot as well as the C-time plot should result in a zero slope.

As it is evident from the foregoing, there are two basic sources of errors, systematic and random, which could prevent the obtention of a straight line when Eq. 3a is applied. Proper statistical analysis could indicate which source causes the linearity of the plot predicted by Eq. 3a not to be satisfied. In addition, statistical methods[7,8] could be used in the consistency test to determine for a pre-assigned degree of probability the confidence band for the calculated values of N and C.

In many large fields it is often found that an infinite linear water drive satisfactorily describes the production-pressure behavior of the said fields. For a unit pressure drop, the cumulative water influx in an infinite linear case is simply proportional to $\sqrt{t}$ and does not require the estimation of a dimensionless time. Thus, the summation term in Eq. 3a becomes $\Sigma\Delta p_n\sqrt{t - t_n}$. Because of this, it is suggested to try first the infinite linear case to determine if a successful solution could be obtained. However, even in such a case, the confidence band should be evaluated as a numerical aid in judging the acceptability of N and C.

Very Small Aquifer: In this case the water influx W_e could be represented by either

$$W_e = \Sigma\Delta pQ(\Delta t_D)$$

or by the approximate but simpler equation

$$W_e = C'\Delta p'$$

where $\Delta p' = p_i - p$, $C' = Wc_w$, W is the water volume in the aquifer and the assumption is made that a steady-state depletion condition obtains. The MBE becomes

$$\frac{F}{E_o} = N + C'\frac{\Delta p'}{E_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3b)$$

A plot of $\frac{F}{E_o}$ vs $\frac{\Delta p'}{E_o}$ should result in a straight line with N being the Y intercept and C' being the slope. The points will plot *backwards* as shown in Fig. 3b.

The reversal in the sequence of points is based on the fact that E_o increases faster than $\Delta p'$. Thus, $\frac{\Delta p'}{E_o}$ decreases as the pressure decreases. Since C', the water influx constant, is always positive and is given by the slope of the

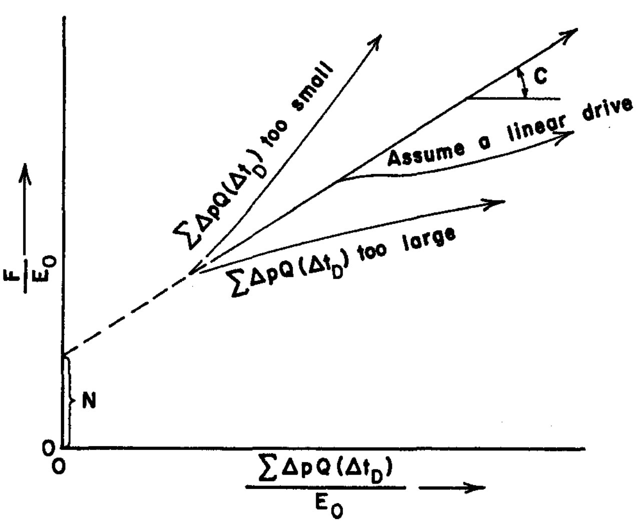

FIG. 3A—$\frac{F}{E_o}$ vs $\frac{\Sigma\Delta pQ\ (\Delta t_D)}{E_o}$

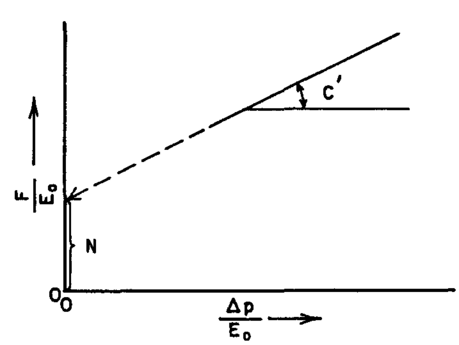

FIG. 3B—$\frac{F}{E_o}$ vs $\frac{\Delta p}{E_o}$

straight line plot, then of necessity $\frac{F}{E_o}$ should also decrease as the pressure decreases. Therefore, the points *must move in a backward sequence.*

Thus, in this case, if one neglects to take into account the water influx when performing the MBE calculations, the resulting $\frac{F}{E_o}$ which is equal to the apparent N will decrease with time.

In practical application it is often found that such a steady-state water influx sets in after a certain period of time, the length of which depends mainly on the size of the aquifer. In such a case, the plotted points, representing the early period of reservoir history during which the non-steady state water influx prevails, will plot in a forward sequence as in Fig. 3a. However, when the effect of the boundary becomes appreciable, the plotted points will reverse the sequence and plot backwards.

Sometimes, an appreciable change in the exploitation policy of the reservoir might temporarily reverse the sequence. Even in such a case the points must remain on a straight line if the correct parameters were assumed.

Having determined C', one can calculate the amount of water W contained within the aquifer since $C' = Wc_w$.

Water Drive, A Known Gas Cap:

$$\frac{F}{E_o + m\frac{B_{ti}}{B_{gi}}E_g} = N + C\frac{\Sigma\Delta pQ(\Delta t_D)}{E_o + m\frac{B_{ti}}{B_{gi}}E_g} \quad . \quad . \quad (3c)$$

A plot of the left hand side of Eq. 3c vs the variable term of the right hand side should result in a straight line if the correct aquifer and dimensionless time are assumed. If the line is not straight, then what was discussed in Eq. 3a under saturated reservoirs section applies also here.

Very Small Aquifer, A Known Gas Cap:

$$\frac{F}{E_o + m\frac{B_{ti}}{B_{gi}}E_g} = N + C'\frac{\Delta p'}{E_o + m\frac{B_{ti}}{B_{gi}}E_g} \quad . \quad . \quad (3d)$$

A plot of the left hand side of Eq. 3d versus the C'—term should result in a straight line. The points will plot backwards as shown in Fig. 3b.

Before closing the water drive section, it must be pointed out that it is not necessary to know the dimensionless time and/or the $\frac{r_e}{r_w}$ of the system. Any assumed values that satisfy the linearity of the plot are acceptable solutions. Thus, it is possible, at least theoretically, to find more than one set of aquifer properties which give a solution. However, the N's and W_e's evaluated for such cases would be identical.

In addition to the fact that too large $\frac{r_e}{r_w}$ or Δt_D will bend the line downward, interference between the reservoirs will result in the same effect. Thus, if interference is suspected, one must correct for it before applying the straight line criteria. The straight line equation to be plotted in such a case is

$$\frac{F + \text{Correction for interference}}{E_o} = N + C\frac{\Sigma\Delta pQ(\Delta t_D)}{E_o}$$

Refs. 5 and 6 outline a method for calculating the interference correction factor.

Water Drive, Original Gas Cap and N Are Unknown

Eq. 0a is differentiated with respect to pressure and the resulting equation is used with Eq. 0a to elminate m. The final equation is rearranged to give

$$\frac{Fb' - F'b}{E_ob' - E_o'b} = N + \frac{C}{E_ob' - E_o'b}\left[b'\Sigma\Delta pQ(\Delta t_D) - b\left(\Sigma\Delta pQ(\Delta t_D)\right)'\right] \quad . \quad . \quad . \quad (4)$$

where $b = \frac{B_{ti}}{B_{gi}}E_g$. The primes denote derivatives with respect to pressure.

Thus, a plot of the left hand side of Eq. 4 vs the C—term of the right hand side should result in a straight line with N being the Y intercept and C being the slope, provided the correct aquifer is chosen. When N and C are determined, then Eq. 0a is used to solve for m as a function of real time. The best value of m is then calculated by least squares.

For greater accuracy the derivatives of the summation term $\Sigma\Delta pQ(\Delta t_D)$ should be evaluated by using the derivatives of the $Q(t_D)$ function with the corresponding pressure drops.[3]

UNDERSATURATED RESERVOIRS

No Water Drive

$$N_pB_o = NB_{oi}\frac{(S_oc_o + S_wc_w + c_f)\Delta p'}{1 - S_w} \quad . \quad . \quad . \quad . \quad (5)$$

A plot of N_pB_o vs $\frac{B_{oi}\Delta p'}{1 - S_w}(S_oc_o + S_wc_w + c_f)$ should result in a straight line going through the origin similar to Fig. 1 with N being the slope. $\Delta p' = p_i - p$.

With Water Drive

$$\frac{N_pB_o + W_p - W_i}{\frac{B_{oi}\Delta p'}{1 - S_w}(S_oc_o + S_wc_w + c_f)} = N + c\frac{\Sigma\Delta pQ(\Delta t_D)}{\frac{B_{oi}\Delta p'}{1 - S_w}(S_oc_o + S_wc_w + c_f)} \quad . \quad . \quad . \quad . \quad (6)$$

The procedure is similar to that given in Eq. 3a under saturated reservoirs section. A plot of the left-hand side of Eq. 6 vs the C—term of the right-hand side should result in a straight line with N being the Y intercept and C being the slope. If the plot is not straight, refer to the discussion under Eq. 3a.

GAS RESERVOIRS

No Water Drive

$$G_pB_g = GE_g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

A plot of G_pB_g vs E_g should result in a straight line going through the origin, similar to Fig. 1 with G being the slope.

With Water Drive

$$\frac{G_pB_g + W_p - W_i}{E_g} = G + C\frac{\Sigma\Delta pQ(\Delta t_D)}{E_g} \quad . \quad . \quad (8)$$

A plot of $\frac{G_pB_g + W_p - W_i}{E_g}$ vs $\frac{\Sigma\Delta pQ(\Delta t_D)}{E_g}$ should result in a straight line with G being the Y intercept and C being the slope. The procedure of the analysis is identical with that advanced in Eq. 3a of the saturated reservoirs section. If the aquifer is very small, then Eq. 3b applies.

DISCUSSION

The straight line method of solving the material balance equation differs from the commonly used one, in that it imparts a dynamic meaning to the individual points. The usual method considers each calculated point separately or some averaging technique, whereas the straight line method stresses the dynamic sequence of the plotted points and the shape of the resulting plot. Because of this, plotting and analyzing the calculated points are of utmost importance for an intelligent interpretation.

Although it is theoretically possible to solve by the straight line method for all the cases treated in this paper, the authors have met only limited success in Cases 2 and 4 under the saturated reservoirs section. This is so, because whenever a gas cap is to be solved for, an exceptional accuracy of basic data, mainly pressures, is required. Furthermore, the presence of the derivatives with respect to pressure in Case 4 adds more to the necessity of exceptionally accurate data.

The rest of the cases, especially when water drive exists, have been tested on many field examples with remarkable success. The shape of the resulting plot and usual sequence of the plotted points have been of great help in gaining understanding to the problem at hand.

Often it is found that the points calculated for the early history do not conform with the latter points. This is caused either by inaccuracy of the early average production-pressure-PVT data or because pressure-production effect has not yet been felt by all the active oil-in-place. In such cases these early points should not be considered in drawing the best straight line. Moreover, once the points to be excluded are decided upon, the same points must be excluded from all subsequent analyses.

In conclusion, it should be stressed that the straight line requirement does not suffice to prove the uniqueness of the solution, but is only one of the conditions that a satisfactory solution should meet. The quantity and quality of the derived information will depend on the quantity and quality of the data; and last but not least, on the experience, judiciousness, and ingenuity of the analyst.

ACKNOWLEDGEMENT

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