Note Title 10/15/2018

WATER INFLUX:

Encroachment of external water into the HC reservoir pore volume from "Aquifer" (AQ)

(+) ⇒ Reduction of the HCPV ⇒ slows the average pressure decline during depletion

(-) ⇒ Gas reservoirs, may lead to water production leading to the "death" of producers

(+) ⇒ Oil reservoirs, displacement of oil that otherwise would not be recovered by SGD (expansion) depletion

"EOR" from mother nature

SGD 15% → 50 → 9x %

Pot Aquifer Model gives in the MAXIMUM, FASTEST encroachment of water for a finite aquifer

$$p_{R(HC)}(t) = p_{AQ}(t) = \text{"}p_R\text{"}$$

W_e = cumulative water volume encroachment from an aquifer into the HC reservoir

$$W_e^{POT} = V_{AQ}(c_f + c_w)(p_{Ri} - p_R) = W_{e,max}$$

$$G(B_g - B_{gi}) + \frac{GB_{gi}}{1-S_{wi}}\left[S_{wi}\left(\frac{B_{tw}-B_{twi}}{B_{twi}}\right) + \bar{c}_f(p_i - p)\right.$$

$$\left. + M\left(\frac{B_{tw}-B_{twi}}{B_{twi}}\right) + M\,\bar{c}_f(p_i - p)\right] \quad \ldots\ldots \quad (A25)$$

$$= (G_p - W_p R_{sw} - G_{inj})B_g + 5.615\left(W_p - W_{inj} - \frac{W_e}{B_w}\right)B_w$$

The p/z-cumulative plot including all terms would consider $(p/z)[1-\bar{c}_e(p_i - p)]$ versus the entire production/injection term Q

$$(p/z)[1-\bar{c}_e(p_i - p)] = (p/z)_i - \frac{(p/z)_i}{G}Q \quad \ldots\ldots\ldots\ldots\ldots \quad (A31)$$

with

$$Q = G_p - G_{inj} + W_p R_{sw} + \frac{5.615}{B_g}(W_p B_w - W_{inj} B_w - W_e) \quad \ldots \quad (A32)$$

where the intercept is given by $(p/z)_i$ and the slope equals $(p/z)_i/G$.

The water encroachment term calculated by superposition is expressed,

$$W_e = B\sum_j Q_D(\Delta t_j)_D\,\Delta p_j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (A36)$$

where $Q_D(t_D)$ is the dimensionless cumulative influx given as a function of dimensionless time t_D and aquifer-to-reservoir radius $r_D = r_{AQ}/r_R$. Δp_j is given by $p_j - p_{j-1}$ (in the limit for small time steps), and $\Delta t_j = t - t_{j-1}$.

Radial Aquifers

The water influx equation for radial aquifers is:

$$W_e = 1.119\ \phi ch\ r_w^{\ 2} \cdot \frac{\Theta}{360} \sum_{0}^{n} \Delta p\ Q_D \qquad (11)$$

where

Θ = angle subtended by the reservoir circumference, degrees.

r_w = radius of the aquifer inner boundary, ft.

Q_D = radial efflux functions, dim.

ϕch = aquifer storage number, ft. $\cdot$ psi^{-1}.

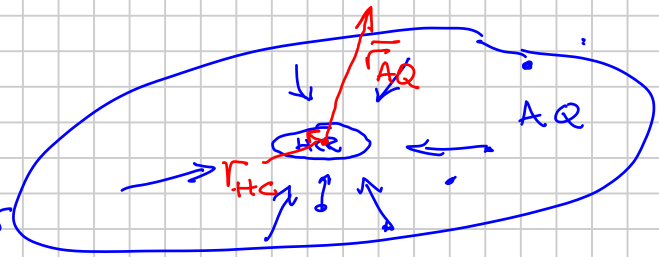
Values of Q_D for infinite and limited outer boundaries are available in equation, chart, and tabular form as a function of dimensionless wellbore time, t_{Dw}. Chart 48 in Volume 4 gives Q_D vs. t_{Dw} curves for several limited no-flow aquifers. Tabulated values can be found in Craft and Hawkin's, "Applied Petroleum Reservoir Engineering", pages 212-217.

PE/RE Books & Papers & PetroWiki

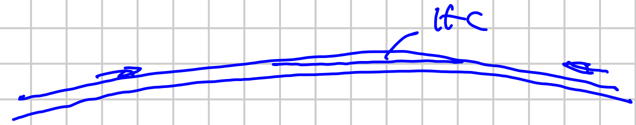
Estimation of $W_e(t)$

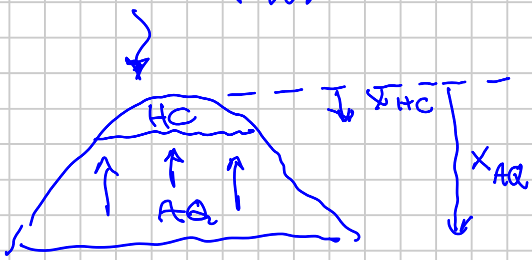
① GEOMETRY

- Radial Flow Geometries



- Linear Flow — " —





$$r_D = \frac{r_{AQ}}{r_{HC}} = 1.x - 10^{(+)}$$

Dimensionless Length L_D

$$x_D = \frac{x_{AQ}}{x_{HC}}$$

② $k_{AQ} \propto v_w$ in AQ

③ $p_{RHC}(t)$

Pot Aquifer : $p_{RHC}(t)$ only time dependency

$k_{AQ} \sim \infty$
$L_D \sim$ "small"
} Instantaneous Encroachment

$$W_{e,max}(t) = V_{AQ}(c_f + c_w)\left(p_{Ri} - \underline{\underline{p_{RHC}(t)}}\right)$$

How fast you empty the HC's

Water Encroachment is modeled EXACTLY as single phase fluid flow in "Well Testing" ("PTA" Pressure Transient Analysis and/or "RTA" Rate Transient Analysis)

Solves PDE using continuity Eq. (material balance) & Darcy's eq.

: Geometry (Radial, Cylindrical)
(Linear)

: Boundary Conditions:

PTA: Well Testing

$r = r_w$ $\quad q = \text{constant} = \nu$

(a) $r = r_e$ $\quad (dp/dr) = 0$ $\quad$ No Flow $(q = 0)$

(b) $\quad p = p_e = \text{const.}$

(c) No outer boundary ("infinite" $r_e = \infty$)

$\Rightarrow$ $\quad p(t, r)$ $\qquad p_{wf}(t, r = r_w)$

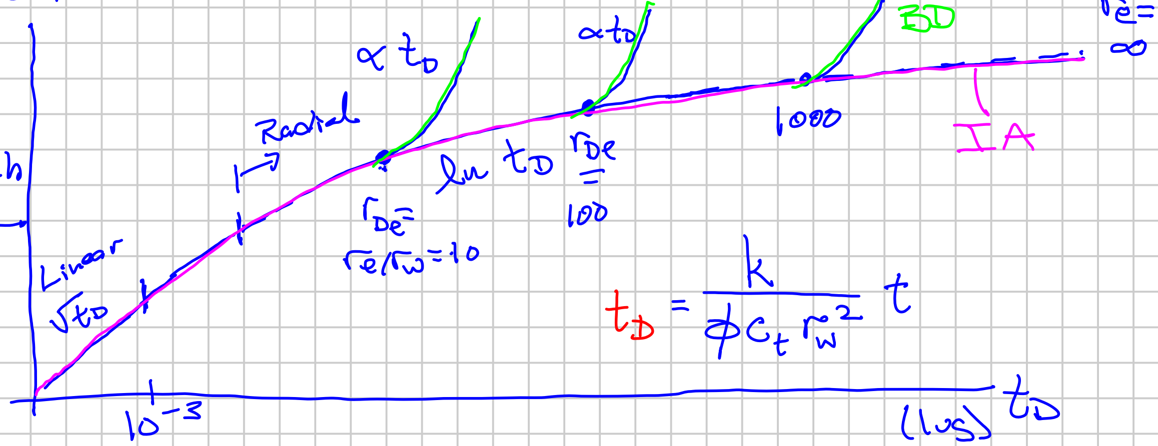
$p_D(t_D, r_D)$ $\quad$ General Dimensionless Solution

log-log plot; $\qquad$ Any r_w, r_e, k, c, h, q, ϕ

$$p_D = \frac{[(p_{Ri} - p_{wf})]kh}{q\mu B}$$

$f(t)$

const



$$t_D = \frac{k}{\phi c_t r_w^2} t$$

RTA (Rate Transient Analysis)
"Time"

"p_{RHC}"

B.C. $r = r_w$: $p = p_{wf}$ = constant

$\Rightarrow q(t) \;|\; \underbrace{q_D(t_D)}$

B.C. Used for Water Influx Calculations $W_e(t)$

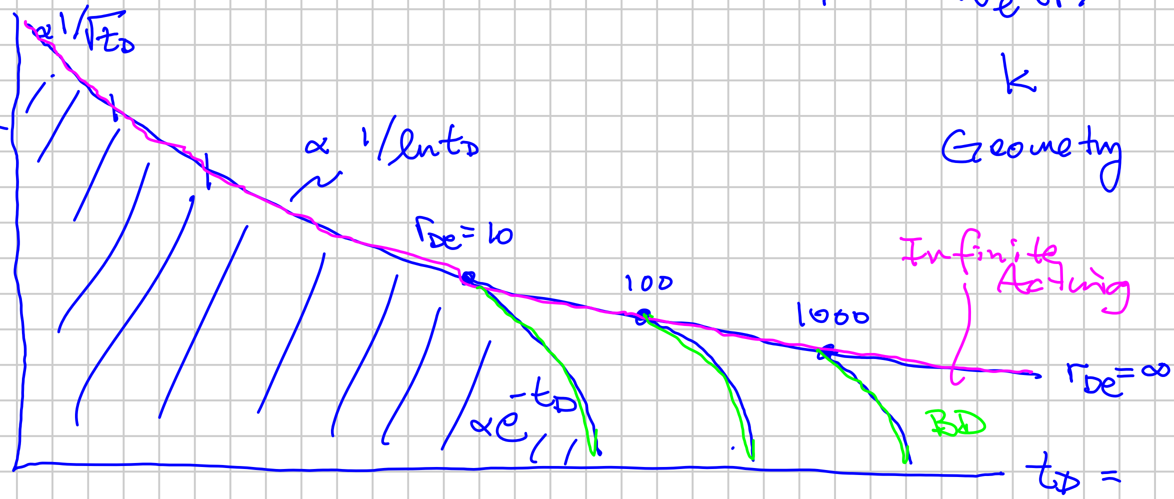
k

Geometry

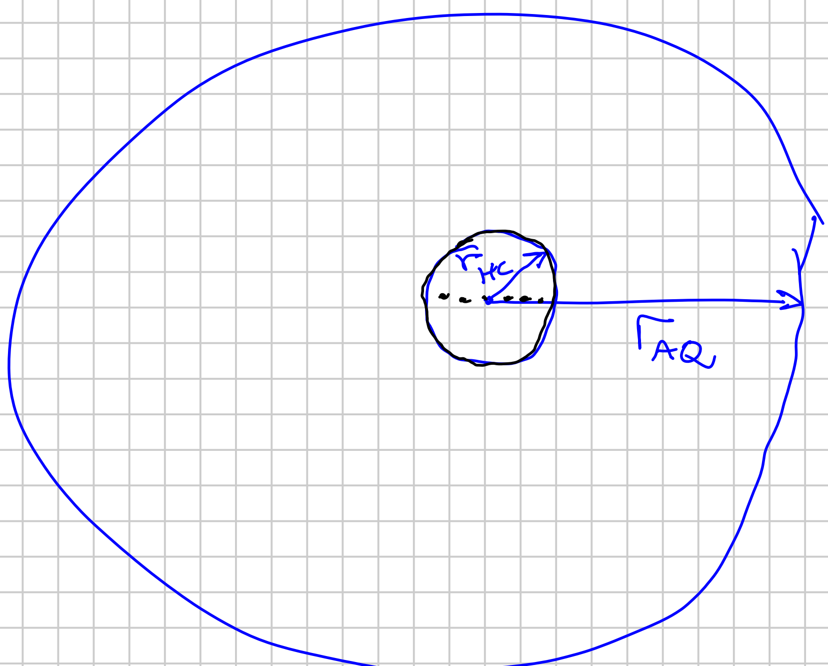
$f(t)$

$$q_D = \frac{q\mu B}{(p_{Ri} - p_{wf})\,kh}$$

ϕ const

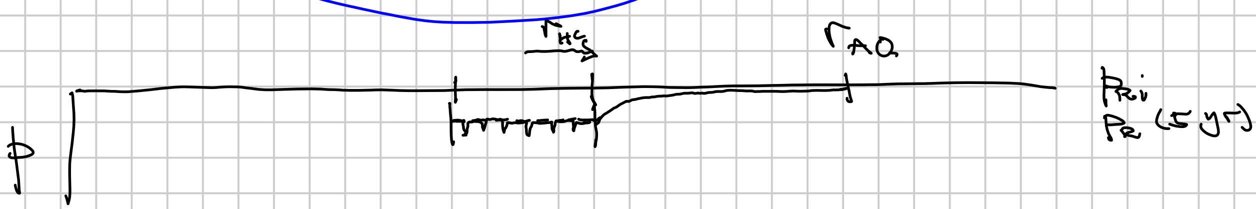


$$t_D = \frac{k}{\phi\mu c_t}\,t$$



Aquifer Influx

p_{wf} = "p_{RHC}" (t)

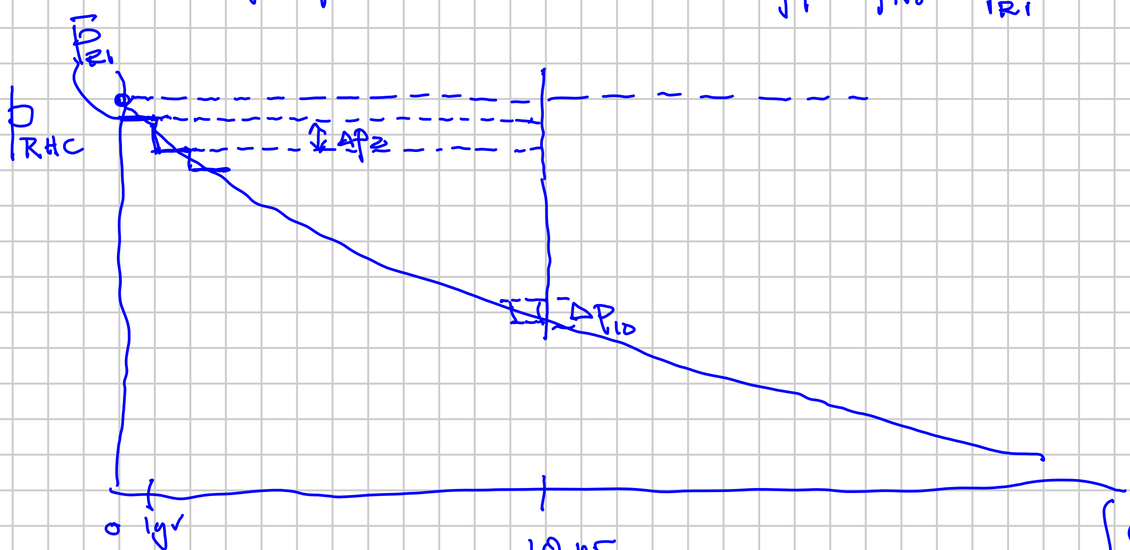


Analogy to Well Behavior	r_w	r_e
Aquifer	r_{HC}	r_{AQ}
	r_R	

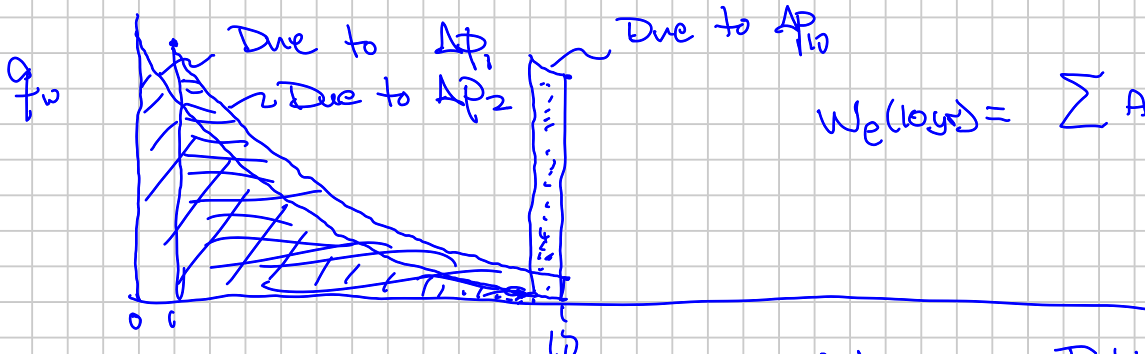
Because inner BC "p_{wf}" "p_{RHC}" (t)

use "Superposition"

$$\Delta p_1 = p_{Ri} - \bar{p}_{R1} \Rightarrow q_w(t) \text{ for } 10 \text{ yrs}$$



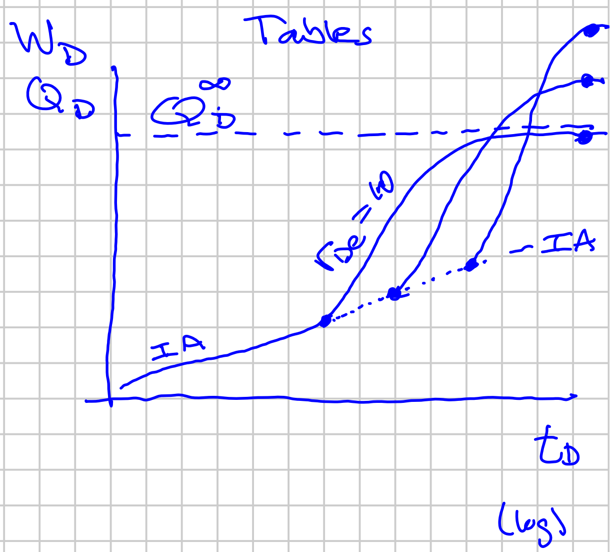
$\int q(t)$



$$W_e(10\text{ yrs}) = \sum \text{Areas}$$

Cumulative Volume =

$$W_D = Q_D = \int_0^{t_D} q_D(t_D)\, dt_D$$



$$\boxed{W_e = U \sum \Delta p_k \, W_D(\Delta t_{D_k})}$$

$$\Delta p_k = p_{k-1} - p_k$$

$$Q_D^{\infty}(r_{De})$$

$$\Delta t_{D_k} = t_D - t_{D_{k-1}}$$

$$\Delta t_{D_1} = 10 - 0$$

$$\Delta t_{D_2} = 10 - 1$$

$$W_e = u \sum \Delta p_{RHC,k}\, Q_D(\Delta t_{D_k})$$

$$W_{e,max} = u \sum \Delta p_{RHC,k}\, Q_D^{\infty}$$

$$= u\,(p_{RHi} - p_{RHC,k})\, Q_D^{\infty} \qquad \text{Pot Aq.}$$

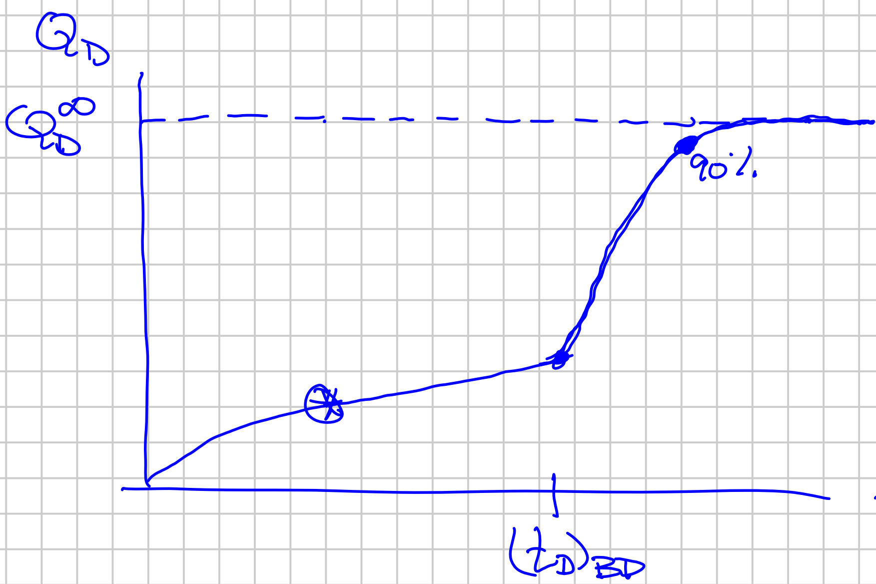
$$= (c_w + c_f)\, V_{AQ}\, (p_{RHCi} - p_{RHC,k})$$

$$\Rightarrow u = \frac{V_{AQ}(c_w + c_f)}{Q_D^{\infty}}$$

$$\Rightarrow \boxed{W_e = \underbrace{V_{AQ}(c_w + c_f)} \sum_{k=1}^{N} \Delta p_k \left[\underbrace{\frac{Q_D(\Delta t_k)}{Q_D^{\infty}}} \right]}$$

fraction of the total influx achieved in Δt_k for Δp_k

0 — 1



units

$$t_D = \frac{k\,(1\ \text{year})}{\phi\, \mu\, c_t\, r_{HC}^2}$$

$c_f + c_w$

$$\frac{Q_D}{Q_D^{\infty}} > 90\% \quad \text{for incremental "}\Delta t\text{" (e.g. 1 year)}$$

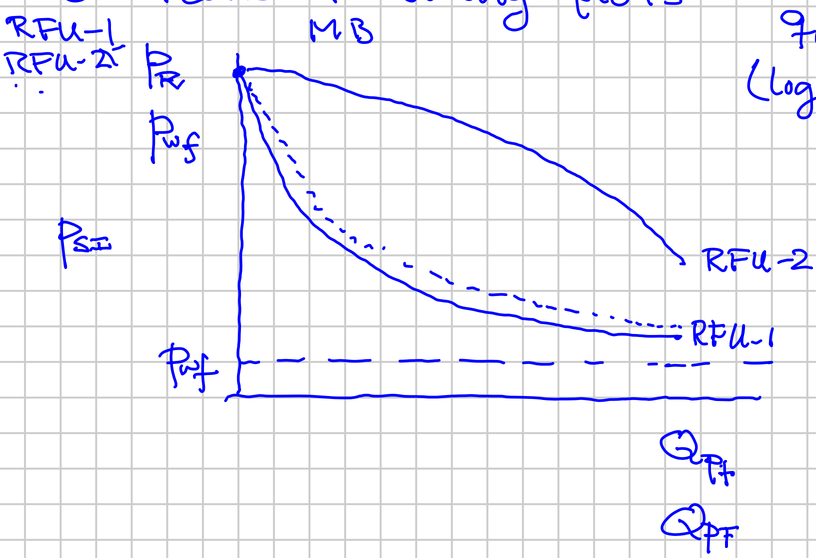
$\Rightarrow$ Pot Aq. assumption is <u>valid</u>

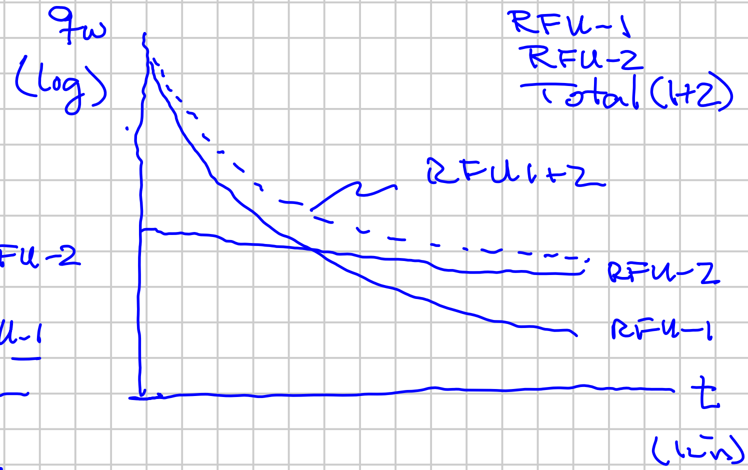
Note Title 10/16/2018

Excel

Q&A on MB-IPR Solution for q(t) Forecasting and understanding of LNX reservoir behavior

① Create following plots





SI : P_{SI} =

SI $\quad q_w = 0 \quad q_1 = -q_2$ @ P_{wf} $\quad P_{R1} < P_{wf} < P_{R2}$

$$J_1 (P_{wf} - P_{R1}) = -J_2 (P_{R2} - P_{wf})$$

$$\Rightarrow \quad P_{wf} = \frac{J_1 P_{R1} + J_2 P_{R2}}{J_1 + J_2}$$

Note Title 10/17/2018

* New Required Reading paper:

1973 – Fetkovich – DCA (JPT – 1980) {supplement to lectures}

* IPR Eq. for "Wellbore Backflows" in LNK systems during a shut-in period $q_t = 0 = \underset{<0}{q_1} + \underset{>0}{q_2}$

(1)
$$q = J(p_R - p_x) + J_x(p_x^2 - p_{wf}^2) \quad \Big| \quad p_{wf} < p_x < p_R$$

(2)
$$q = J(p_R - p_{wf}) \quad \Big| \quad p_{wf} > p_x$$

(3)
$$q = J_x(p_R^2 - p_{wf}^2) \quad \Big| \quad p_R < p_x$$

How to handle $p_R < p_{wf}$?

$$q < 0$$

$$J_x = \frac{J}{2p_x}$$

DECLINE CURVE ANALYSIS (DCA)

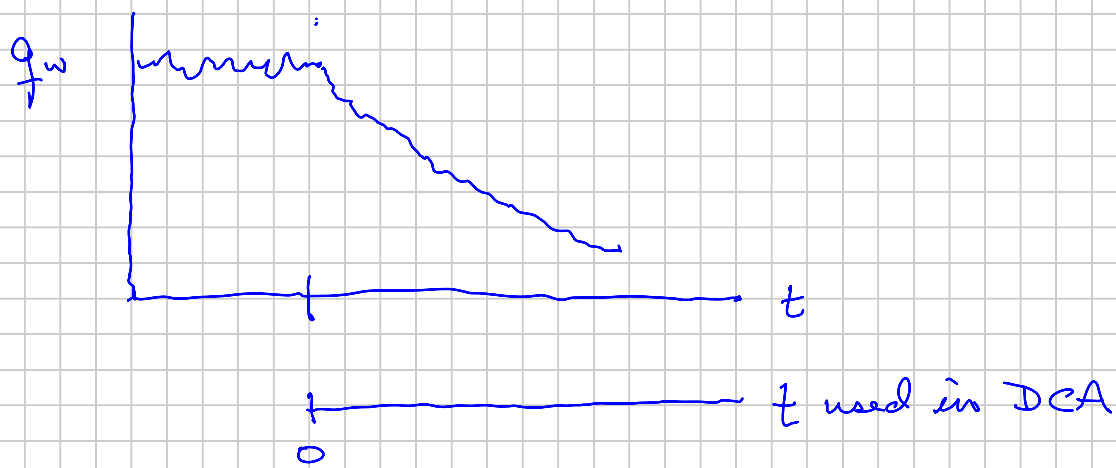
(RTA)

① Arp's Eq

$$q = \frac{q_i}{[1 + bDt]^{1/b}} \qquad 0 < b < 1$$

$$q = q_i \cdot e^{-Dt} \qquad b = 0$$

Applies after a well goes on decline



② Fetkovich (1973)

(a) Arps 3 parameters q_i D b are expressed in physical terms

(i) $q_i = q$ for PSS (BD) flow with a constant FBHP at the start of decline ($\bar{p}_R \le \bar{p}_{Ri}$)

e.g. $q = J(\bar{p}_R - p_x) + J_x(p_x^2 - p_{wf}^2)$

$k \quad h \quad r_e \quad r_w \quad s \quad \mu \quad B \quad p_x$

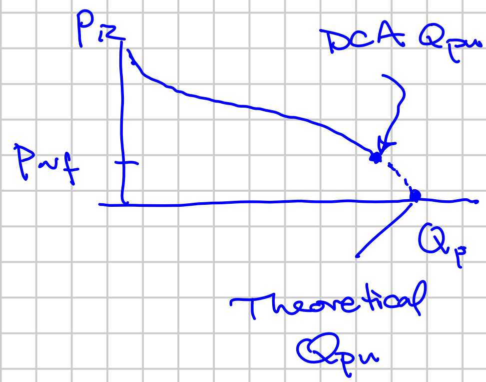
(ii) D = "decline constant"

$$D \equiv \frac{q_i}{Q_{pu}^{DCA}} \cdot \frac{1}{1-b}$$

$$Q_{pu} = \int_0^{\infty} q \, dt$$

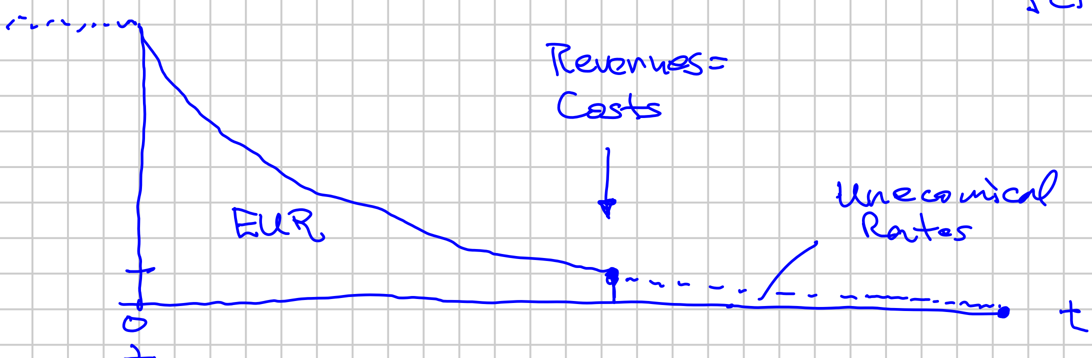
$$p_R^{\infty} = p_{wf}$$

$$q^{\infty} = 0$$



EUR = Est. Ultimate Recovery

Sm^3
STB
Scf



$$Q_{pu}^{DCA} = EUR + \text{Non-Econ}$$

$$EUR \le Q_{pu}^{DCA} \le Q_{pu}$$

$p_R \to p_{wf,min}$ $\quad\quad$ $p_R \to 0$

(iii) b : reflects the shapes of IPR & MB

$q(p_{wf})$ $\bar{p}_R(Q_p)$

Single-phase vs Multiphase Flow

$p_R > p_b$

SGP

$p_R < p_b$

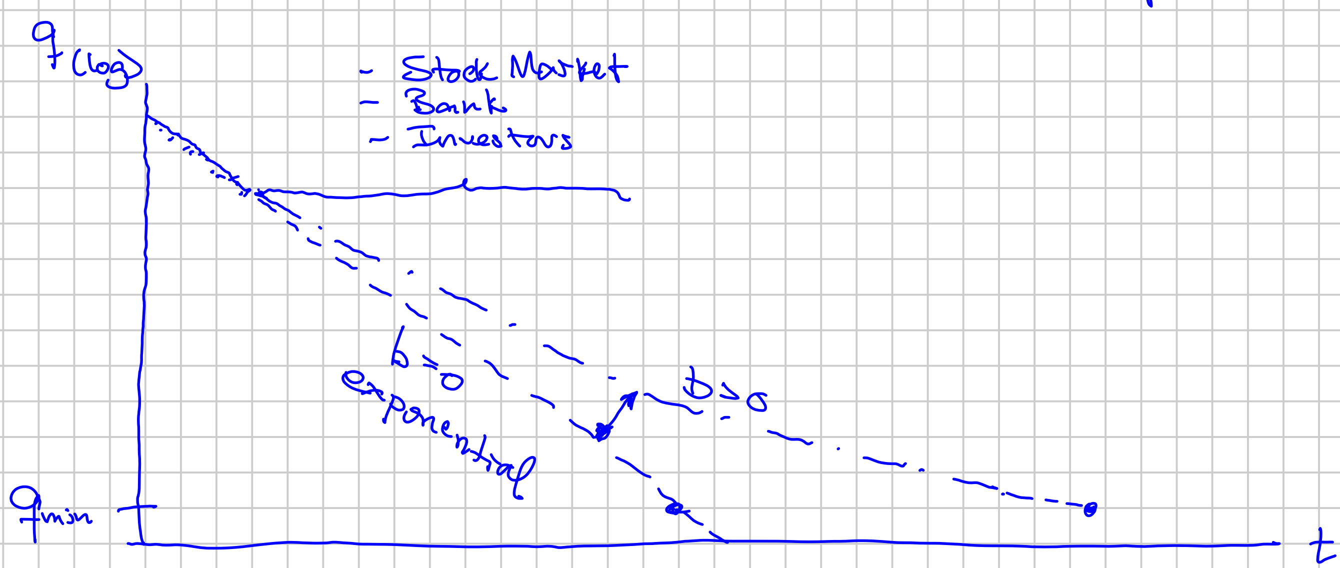
$p_{wf} > p_x$

Reservoir Flow and PVT behavior

1973 Single RFU : $0 < b < 0.5$

1990 Multiple RFUs (LNX) : $0.x \le b < 1$

RFUs have ~ same $D \Rightarrow q_w$ "b" ~ q_{RFW} "b"

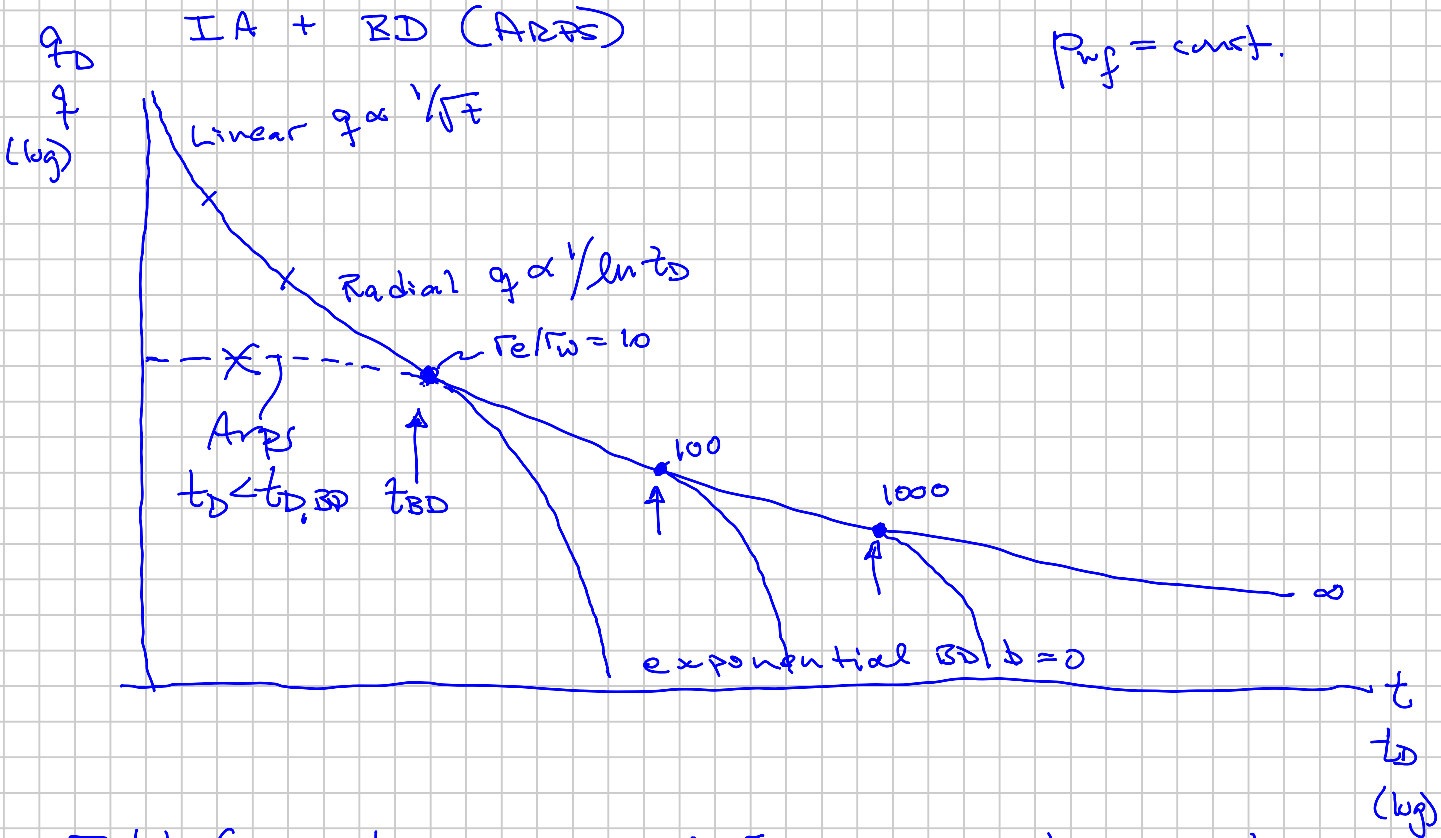


Fetkovich DCA

(b) Coupled the general flow theory in porous media (i.e. not just PSS / BD flow) which is both "Infinite Acting" (IA) & BD behavior.

IA + BD (ARPS)

$p_{wf} = $ const.



Tight Gas $\quad t_{BD} \sim 0.5\ yr \rightarrow 5\ yr \qquad p_{wf} \sim$ const

$\uparrow$

$\underline{\underline{k}} \quad r_e$

New Dimensionless variables

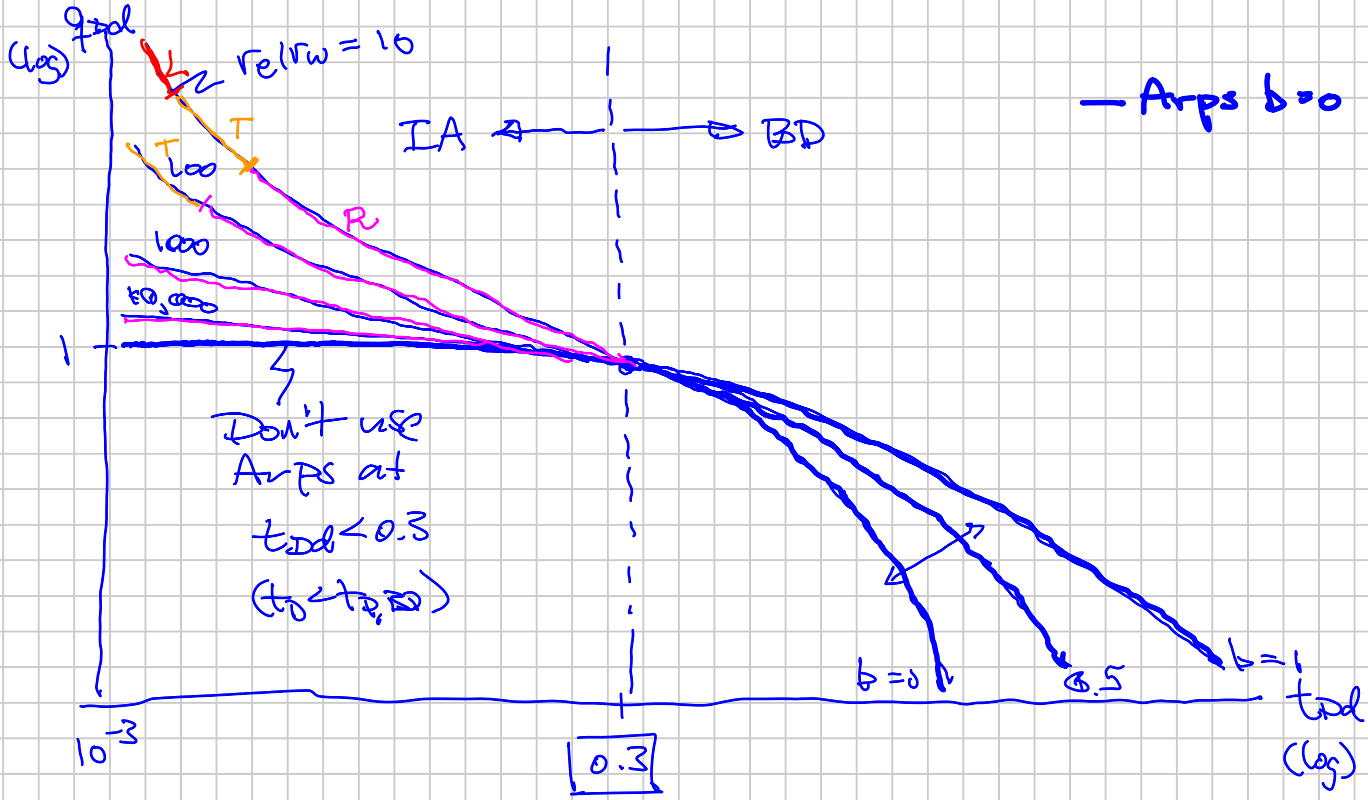
$$t_{Dd} \approx \frac{t_D}{t_{D,BD}(r_e/r_w)} \quad (0.3)^{\pi/16}$$

$$q_{Dd} \approx \frac{q_D(t_D)}{q_D(t_{D,BD})}$$

Collapses all $q_D(t_D > t_{D,BD})$ to a single exponential curve

$$q_{Dd} = e^{-t_{Dd}}$$

— Arps b=0



Generalized Fetkovich Decline Type Curve (IA & BD)

$$\frac{q}{q_i} = q_{Dd} = \frac{1}{[1+b\,t_{Dd}]^{1/b}}$$

$$\frac{q}{q_i} = q_{Dd} = e^{-t_{Dd}}$$

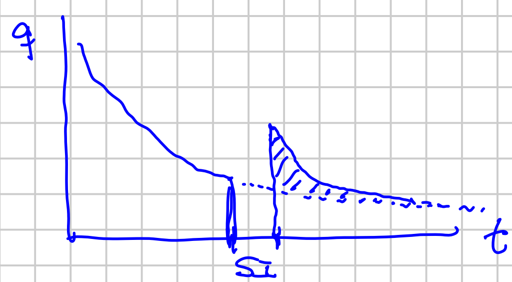
$t_{Dd} \equiv D t$

(c) $p_{wf}(t)$

(i) Rigorous Superposition – analogous to water influx

– Any $p_{wf}(t)$ variation

e.g. q (t > shut-in)



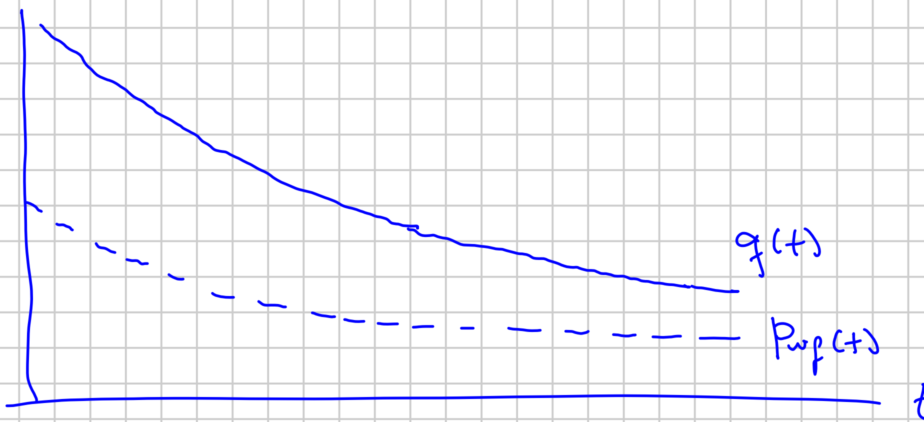
(ii) Smoothly Varying $p_{wf}(t)$:

"Rate Normalization" (Winestock & Colpitts)

$\boxed{\text{IA}}$: $q_{tD}(t_D) \simeq p_D^{-1}(t_D)$

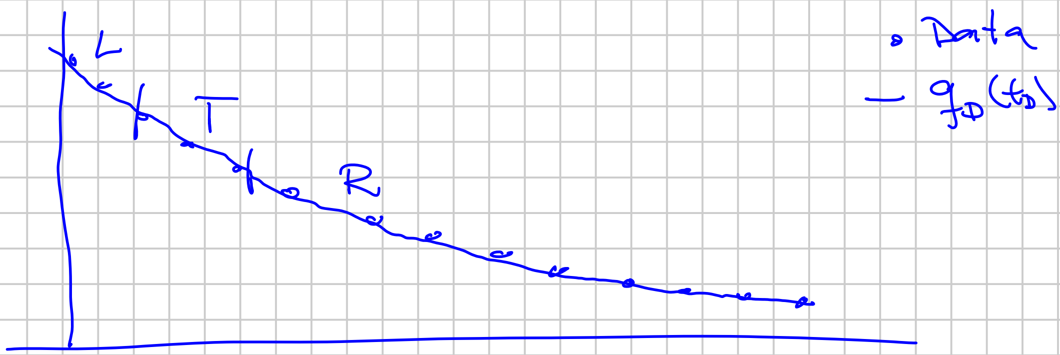
p_{wf} = const q = const

$$p_{wf}(t) \Rightarrow \frac{q(t)}{p_{Ri}-p_{wf}(t)} \simeq \frac{q(t)}{p_{Ri}-\underset{\text{const.}}{p_{wf}}} : \frac{q_D(t_D)}{T}$$



$$\left[\frac{q(t)}{P_i-P_{wf}(t)}\right]$$

$q_{tD}(t_D)$



Rate Normalization

BD: DOES NOT WORK

Arp's

$$q_D(t_D > t_{D,BD}) \propto e^{-t_D} \qquad b = 0$$

$$p_D^{-1}(t_D > t_{D,BD}) \propto \frac{1}{1+t_D} \qquad b = 1$$

③ Post Fetkovich

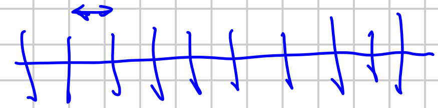
- nothing much new > 1990s LNX

~ 2005-ish "(Unconventional 'Shales') NA

$k \sim$ 10 – 1000 md

$10^{-5} - 10^{-3}$ md

L_{ff} - 500 ft → 20 ft



$t_{BD} \sim \underbrace{2 \text{ yr} \to 20 \text{ yr}}$

(k, L_{ff})

$10^{-5} - 10^{-3}$ md

$t < t_{BD}$

"LIA" $q \propto \frac{1}{\sqrt{t}}$

↓

BD

$$q = \frac{q_i}{[1 + bDt]^{1/b}}$$

$b = 2$

$1 \ll bDt$

$$q \sim \frac{1}{\sqrt{t}}$$

Use Arps' $b = 2$