

Radial Aquifers

The water influx equation for radial aquifers is:

$$W_e = 1.119 \phi ch r_w^2 \cdot \frac{\theta}{360} \sum_0^n \Delta p Q_D \quad (11)$$

where

θ = angle subtended by the reservoir circumference, degrees.

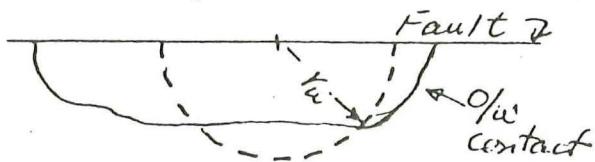
r_w = radius of the aquifer inner boundary, ft.

Q_D = radial efflux functions, dim.

ϕch = aquifer storage number, ft. \cdot psi $^{-1}$.

Values of Q_D for infinite and limited outer boundaries are available in equation, chart, and tabular form as a function of dimensionless wellbore time, t_{Dw} . Chart 48 in Volume 4 gives Q_D vs. t_{Dw} curves for several limited no-flow aquifers. Tabulated values can be found in Craft and Hawkin's, "Applied Petroleum Reservoir Engineering", pages 212-217.

Quite often a petroleum reservoir lies against or close to a fault so water entry is limited as to direction.



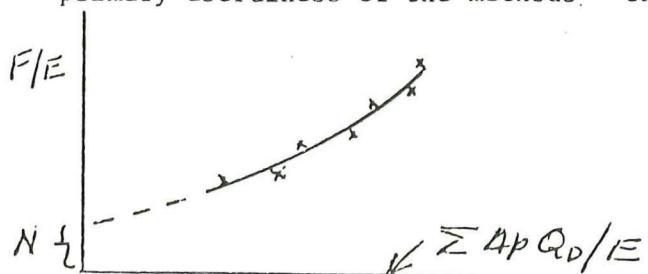
This is illustrated by the sketch. This type of situation is handled by defining the "expose angle", θ , (180° in the sketch) and computing an r_w consistent with the length of oil-water contact.

To use the radial influx model in the material balance equation, one plots F/E against $\sum \Delta p Q_D/E$. Should a straight line result, the intercept defines the original oil in place, N , and the slope defines the aquifer storage, $1.119 \phi ch r_w^2 \theta/360$.

General Comments

The several "type" aquifers described above are not the only ones for which solutions are available. They are, however, the most common ones. There is, of course, no assurance that the petroleum reservoir and aquifer will act such that a satisfactory straight line material balance solution is obtained for any of the "type" aquifers' responses. This may be because of poor reservoir data (noteable pressure data) or it may be because the aquifer does not respond in accordance with the "type" solutions.

The fact that a straight line is not achieved does not destroy the primary usefulness of the methods - the determination of original oil in place.



The adjacent sketch illustrates the situation where a non-linear plot of F/E vs. $\sum \Delta p Q_D / \epsilon$ is obtained.

Extrapolation of the best smooth curve to the $X=0$ axis will yield as good a value of N as possible to get. Note that it would be incorrect procedure to place a straight line through the points and extrapolate it to $X=0$.

Having developed a satisfactory straight line (or curved line for that matter) material balance relationship, the relationship can be used as a prediction tool for future reservoir behavior by extrapolation. However, note that in making the extrapolation there is an assumption that all reservoir and aquifer responses in the future have been manifested in past behavior. This is not always the situation as, for example, sometimes other petroleum reservoirs in the same aquifer are to be put on production and effect the influx into the original reservoir. A second example is where a reservoir is against a sealing fault and the fault develops "leaks" when the reservoir pressure drops to a low threshold value.

M.B.Standing

Reservoarteknikk II

Notes On Water Influx Equations in Material

Balance Calculations

Many petroleum reservoirs are in contact with aquifers capable of supplying quantities of water into the reservoir as a result of pressure reduction. The actual shape, size, and ability of the aquifer to supply water generally is not known, although one can often make use of regional geology and seismology studies to develop ideas of the general aquifer properties to be expected.

From an analytical point of view, the aquifer is best considered as an independent unit which supplies water to the petroleum reservoir in response to the time variation of pressure at the inner boundary of the aquifer. The inner aquifer boundary is normally taken to be the location of the original hydrocarbon-water contact, although one could select some other location to define as the inner boundary. While water encroachment (influx into the petroleum reservoir, but efflux from the aquifer) usually is calculated using boundary pressures, it often is satisfactory to use average reservoir pressure in the calculations in place of boundary pressure.

Havlena and Odeh demonstrated (see reprint in Section 2) how the material balance equation can be arranged to indicate the type of water influx being experienced by the reservoir. Essentially, their method is to assume a type of aquifer, calculate the response of the aquifer to the observed change in boundary pressure, and test whether the assumed aquifer type is correct by determining whether a straight line is obtained when certain material balance parameters are plotted against each other. As pointed out by Havlena and Odeh, the straight line method of solving the material balance makes use of the dynamic sequence of the plotted points and the shape of the resulting plot to develop the proper aquifer characteristics, and differs from the more common material balance methods that rely mainly on averaging techniques.

From what has been said above, it is apparent that finding the proper aquifer characteristics to fit the observed field pressure behavior takes on a trial and error approach. Consequently, it makes good sense to try the simplest aquifer characteristic first and then, if not successful, proceed to aquifer characteristics requiring more complicated calculations. This is the approach that will be used in presenting the several standard aquifer responses - the easiest to handle first, followed by the more complex.

Small Aquifers

A small, limited aquifer can be represented by the water influx relationship:

$$W_e = C(p_i - p) \quad (1)$$

where

W_e = cumulative water entering the reservoir, bbl.

C = $V_a \cdot \phi (c_w + c_f)$

V_a = aquifer bulk volume, bbl.

ϕ = aquifer porosity, fraction

c_w, c_f = water and rock compressibilities, psi^{-1}

Using this influx relationship yields a material balance equation of

$$\frac{F}{E} = N + C \cdot \frac{(p_i - p)}{E} \quad (2)$$

where

F = net voidage volume

E = reservoir system expansion

N = original stocktank oil in place

To use this relationship, values of F/E are plotted as a Y coordinate against $(p_i - p)/E$ as an X coordinate. The slope becomes the value of C and the intercept becomes the value of N . Havlena and Odeh point out that the points will plot backward as indicated by their Figure 3B on page 5.

Steady State Aquifers

Experience has shown that the Shilthuis steady state relationship will satisfy more than 50 per cent of material balance calculations. It is the second easiest to apply. If we represent the cumulative amount of water that has crossed the inner boundary at time t , the Shilthuis relationship is expressed by the rate equation:

$$\frac{dw_e}{dt} = k(p_i - p) \quad (3)$$

where

p_i = initial inner boundary pressure

p = inner boundary pressure at time t

The cumulative influx is then,

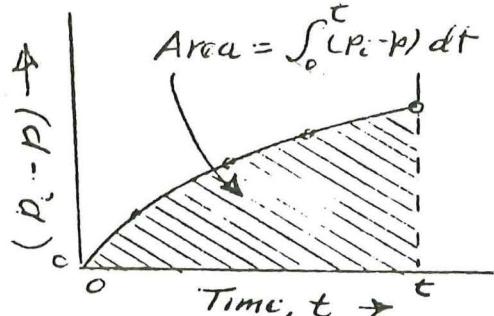
$$w_e = k \int_{t=0}^t (p_i - p) dt \quad (4)$$

Using this influx relationship in the material balance relationship yields

$$\frac{F}{E} = N + \frac{k \int_0^t (p_i - p) dt}{E} \quad (5)$$

The straight line plot is prepared by plotting F/E as a Y coordinate against

$\int_0^t (p_i - p) dt/E$ as an X coordinate.



Values of $\int_0^t (p_i - p) dt$ at specified

time values are obtained by graphical integration of the area under a $(p_i - p)$ vs. t curve as illustrated in the sketch. As in the previous example, value of N and k are determined from the intercept and slope of the resulting straight line.

Infinite Linear Aquifers

An infinite linear aquifer has a response characteristic that is easy to handle. This is because the transient influx function reduces to a simple \sqrt{t} function and does not require estimation of dimensionless time in order to get the influx function. This is explained in the following.

For linear aquifers, the transient response to pressure changes at the inner boundary is represented by:

$$w_e = \frac{bh\phi c}{5.615} \sum_0^n \Delta p \cdot Q_D \quad (6)$$

where

b, h, L = width, thickness, and length of the aquifer, ft.

ϕ = porosity, fraction

c = compressibility, psi^{-1}

Q_D = linear influx function, $f(t_{DL})$

Δp = pressure change, psi

For an infinitely long linear aquifer

$$Q_D = 2 \sqrt{\frac{t_{DL}}{\pi}} \quad (7)$$

where

$$t_{DL} = \frac{6.33(10^{-3})kt}{\phi\mu cL^2} \quad (8)$$

Solving equations 6, 7, and 8 yields

$$w_e = \left(0.016 \sqrt{b^2 \cdot \frac{kh}{\mu} \cdot \phi ch} \right) \cdot \sum_0^n \Delta p \cdot \sqrt{t} \quad (9)$$

Using this influx relationship yields a material balance equation of:

$$\frac{F}{E} = N + B \cdot \sum_0^n \frac{\Delta p \sqrt{t}}{E} \quad (10)$$

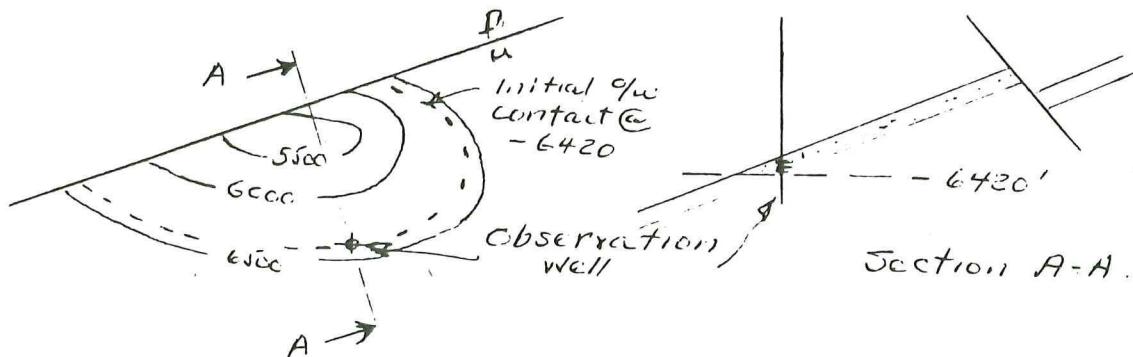
where B is the group in front of the summation sign in Equation 9 and is the slope of the straight line when F/E is plotted against $\sum \Delta p \sqrt{t}/E$. Note that $\sum \Delta p \sqrt{t}$ involves superposition of the pressure changes and square root of time.

Limited Linear Aquifers

Influx from limited or restricted aquifers is calculated by use of Equation 6. Nabor and Barham (Trans AIME 231 (1964) 561) gives efflux (Q_D) function values for constant pressure and no flow (sealed) outer boundary conditions in both chart and tabular form. Superposition principles must be used in applying these aquifer responses to the material balance.

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Water Influx Problem 1



(The object of this problem is to illustrate the calculation of cumulative water entering an oil field, given the pressure history at an observation well near the original oil-water contact and an assumed aquifer behavior)

The two sketches illustrate an elliptically shaped anticlinal reservoir located against a sealing fault. One of the early development wells was apparently completed close to the original oil-water contact (now believed to have been at -6420 feet) as it quickly watered out. Since being abandoned as an oil producer it has been used as a pressure observation well.

Table 1 shows dates and water pressures (corrected to -6400 ft) measured in the observation well. Assuming that the aquifer response could have been that of the Schilthuis steady-state form with a constant, k , of 2 bbl/psi-day, calculate the amount of water that has entered the reservoir at each date.

Table 1. Pressure History - Observation Well #1

Date	Cum. Days Since First Production	-6400 Datum Pressure psig
March 13, 1970	0	3000
October 1, 1970	202	2995
February 20, 1971	344	2982
July 5, 1971	479	2965
December 8, 1971	635	2940
June 1, 1972	811	2900
October 15, 1972	947	2865
January 30, 1973	1054	2841

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Water Influx Problem 2

(The purpose of this problem is to make you adept at applying superposition principles to water influx calculations.)

The problem calls for the calculation of the cumulative water influx into the Nabisco Oil Field at one-half year intervals over the first four years of its life. (In case you are wondering, the name Nabisco Field comes from its biscuit-like shape.) Fortunately, the Nabisco Field lies at the exact center of an infinitely large aquifer. Figure 1 illustrates the shape of the field.

Average fluid and rock properties in the oilfield and the aquifer are as follows:

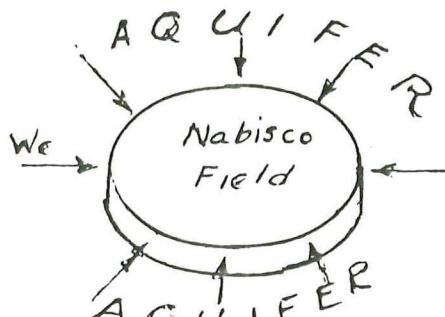


Figure 1. Illustration
of Nabisco Field Shape
(Not to scale)

1980

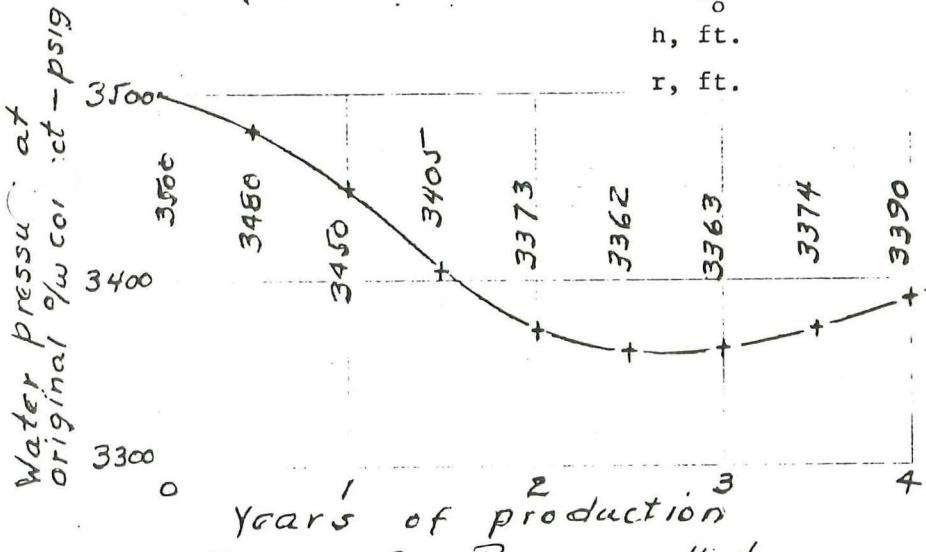


Figure 2 Pressure History -
Nabisco Field.

if the average water saturation behind the water-oil front is 50 per cent?

* From Hall's correlation, pg 132, Craft & Hawkins.

Guide Answers:

t_D for 1/2 year = 6.2 ; Q_D for 1/2 year = 5.3 ; W_e for 1 year = 368,000 bbl.

	<u>Oilfield</u>	<u>Aquifer</u>
$k(\text{eff})$, md	237	100
ϕ , fr.	0.21	0.20
μ , cp. $^{-1}$.	1.67 oil	0.50 water
c , psi $\cdot 10^6$		
c_w	3.0	3.0
c_o	11.0	-
c_f , fr.	3.8	3.7
S ,		
S_w	0.31	1.00
S_o	0.69	--
h , ft.	40	40
r , ft.	5280 (r_e)	5280 (r_w)
	∞ (r_w)	e

Figure 2 shows water pressure at the location of the original oil-water contact location vs time. The inflection at about 1½ years time reflects a major reduction in off-take at that time.

ASSIGNMENT

(1) Calculate the barrels of water that have entered the oil reservoir at each ½ year time.

(2) What fraction of the oil reservoir volume has been invaded at the end of 4 years

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Water Influx Problem 3

The Nairobi Zone of the Palo Alto Field is anticlinal in nature and covers approximately 2900 acres in T3W, R7E, Stanford Base and Meridian. As illustrated by the structure map on the sand top (Fig.1) the zone extends from -7292 to -7376 feet subbay. An oil-water contact was identified on electric logs in the first well at the later depth. Because of the small dip of the flanks (less than 1 degree) most of the oil zone was underlaid by water when the field was discovered.

Various maps have been made and data assembled for the purpose of making engineering studies of this zone. Several of these are attached. Pertinent information regarding the maps is:

Figure 1 shows the depth to the top of the oil sand at various parts of the field. The area enclosed by each contour was found by planimetering to be:

Contour	Area -acres
7300	193
7320	789
7340	1648
7360	2266
7376	2880 (oil-water contact)

Figure 2 shows contours of constant sand thickness. Thinning of the sand occurs to the south-east.

Figure 3 shows oil isobars at datum (-7350 ft.) corresponding to January 1, 1977. The initial datum oil pressure was 2056 psig.

Figure 4 is a blank sheet to be used as you see fit.

Production from the Nairobi zone is 30° API.

What's Wanted:

Calculations are to be made to evaluate the water influx into the zone, assuming various size/shape aquifers. Calculations that will involve the Hurst-vanEverdingen transient efflux behavior of the aquifer require equivalent values for r_w , h , and Δp_w for use in the appropriate equations. Please calculate values for the above parameters. Should you need to make assumptions of one kind or another, don't forget to list what they are.

NAIROBI ZONE - PALO ALTO FIELD

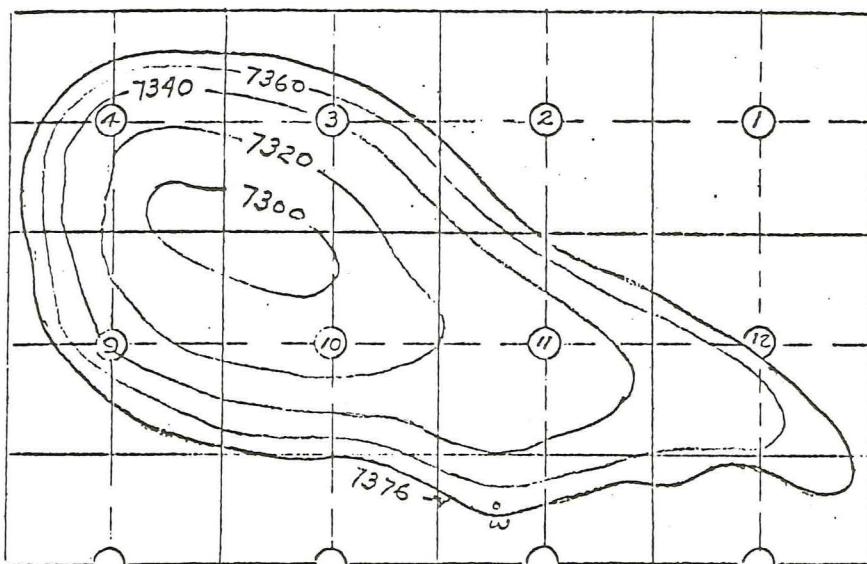


Fig. 1 Contours of top of oil zone

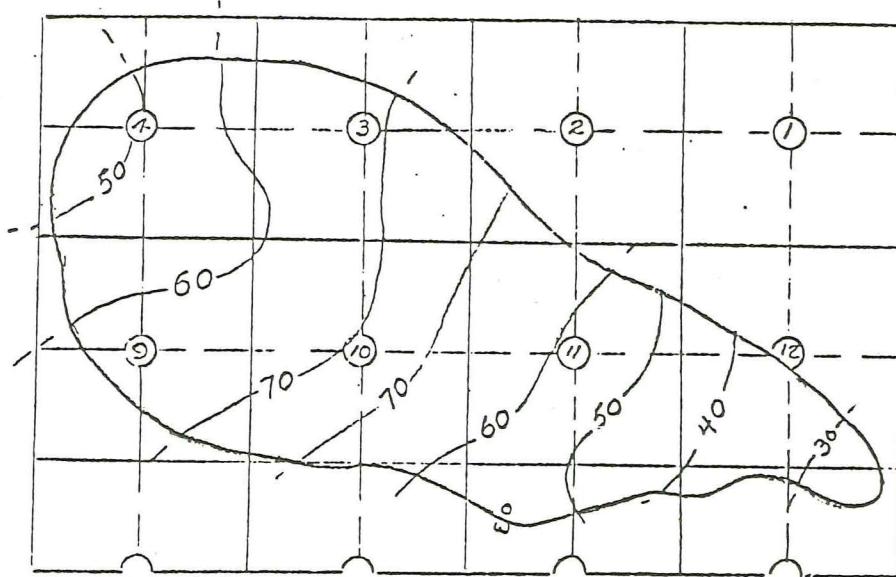


Fig. 2 Contours of sand thickness.

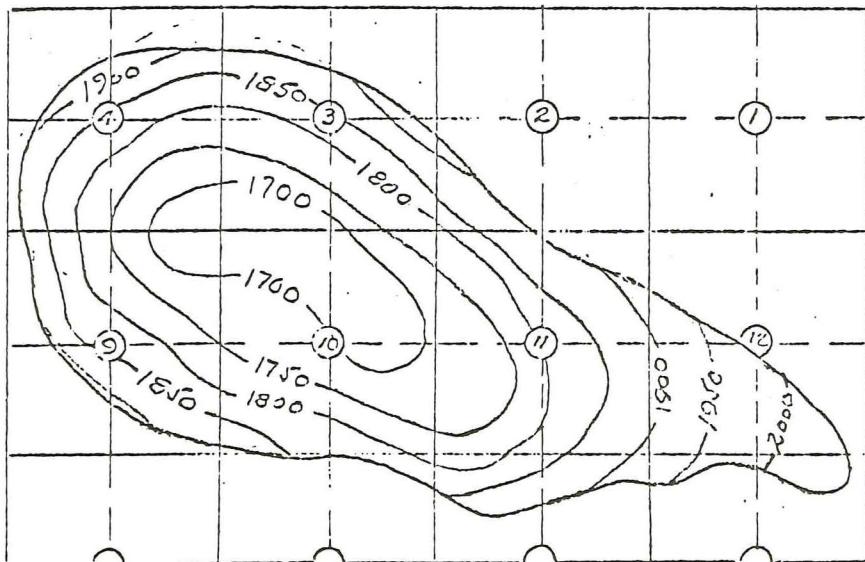


Fig. 3 Datum oil pressure as of 1/1/1977

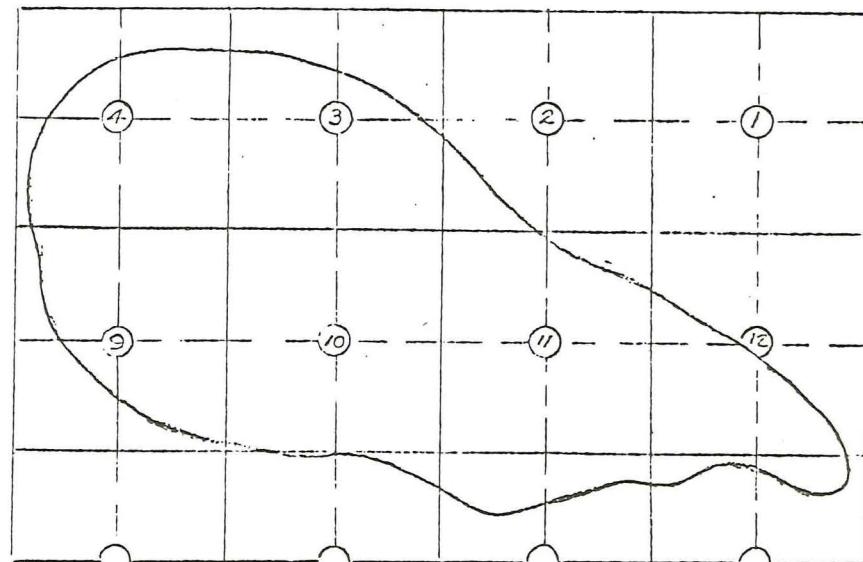


Fig. 4.

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Solution Water Influx Problem 1

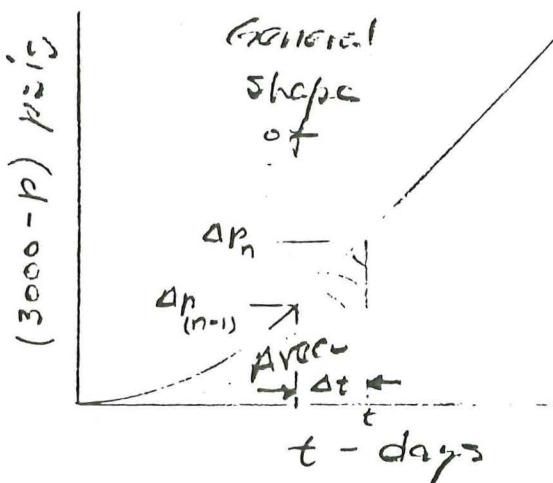
Given: Schilthuis form, $k = 2 \text{ bbl/psi-yr}$.

$$w_e = k \int_0^t (P_i - p) dt \quad P_i = 3000 \text{ psig}$$

Cum. Days <u>t</u>	<u>(3000 - p)</u>	<u>Cumulative Area Under $(3000 - p) \text{ vs } t$ curve psi.yr.</u>	<u>w_e bbl.</u>
0	0	0	0
202	5	505 *	1000
344	18	2138	4300
479	35	5715	11400
635	60	13125	26250
811	100	27205	54400
947	135	43185	86400
1054	159	58914	117800

* Trapezoidal integration

$$\text{Area} = \sum \frac{1}{2} (\Delta P_{n-1} + \Delta P_n) \cdot \Delta t$$



miss

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Solution Water Influx Problem 2

A. Calc. of dimensionless units.

Per 1/2 year real time;

$$t_D = \frac{6.33(10^3) \cdot 100 \cdot 182.5 \cdot t}{0.20 \cdot 0.50 (3.0 + 3.7) 10^6 (5280)} = 6.185 \cdot t$$

$$B = 1.12 \rho c h r_w^2 = 1.12 \cdot 0.20 \cdot 6.7(10^{-6}) \cdot 40 \cdot (5280)^2 = 1673 \text{ Bbl/psi.}$$

i. Number of
1/2 units

	t_D	Q_D
0	0	0
1	6.19	5.3
2	12.37	8.7
3	18.56	11.7
4	24.74	14.5
5	30.9	17.2
6	37.1	19.8
7	43.3	22.3
8	49.5	24.7

From Craft & Hawkinson

B. Calc. of W_e .

$$W_e = 1673 \cdot \sum Q_D \cdot \Delta P$$

	0	1	2	3	ΔP	4	5	6	7	8	Σ	Bbl
t.	10	25	37.5	38.5	21.5	5	-6	-13.5	-8			0
0	0											
1	53	0										$8.87(10^4)$
2	57	133	0									$3.68(10^5)$
3	117	218	199	0								$8.93(10^5)$
4	145	293	326	204	0							$1.62(10^6)$
5	172	363	439	335	114	0						$2.38(10^6)$
6	198	430	544	450	187	27	0					$3.07(10^6)$
7	223	495	645	558	252	44	-32	0				$3.66(10^6)$
8	247	558	743	662	312	59	-52	-72	0			$4.11(10^6)$

Ans (1)

(2) At end of 4 years $W_e = 4.11(10^6)$ bbl.

45_w in effected volume $= (0.50 - 0.31) = 0.19$

ft^3 of effected reservoir $= \frac{4.11(10^6)}{0.19} \cdot \frac{5.615}{0.21} = 5.78(10^8)$

Fraction of res. effect. $= \frac{5.78(10^8)}{\pi \cdot (5280) \cdot 40} = 0.165$

16.5%

Oil Recovery

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Arcs of $\frac{1}{2}$ mile (2640 feet) were laid off around the original oil-water contact contour. The average value of P and h of each arc was determined by inspection. This gave the following values:

<u>Segment, j</u>	<u>\bar{P}_j</u>	<u>\bar{h}_j</u>	<u>Segment</u>	<u>\bar{P}_j</u>	<u>\bar{h}_j</u>
1	1940	47	11	1990	30
2	1910	60	12	1940	37
3	1880	61	13	1880	48
4	1930	71	14	1850	55
5	1900	72	15	1825	63
6	1880	64	16	1870	71
7	1920	50	17	1900	70
8	1970	41	18	1910	64
9	2020	30	19	1900	56
10	2040	25	20*	1900	46

* Segment 20 is $\frac{3}{4} \cdot 2640$ feet long.

The average sand thickness at the original oil-water contact on the sand top (-7376 contour) is

$$\bar{h}_{ow} = \frac{\sum h_j \cdot l_j}{\sum l_j} = \frac{1053 \cdot 2640 \text{ ft}^2}{19.75 \cdot 2640 \text{ ft}} = 53.3 \text{ ft}$$

Not knowing how much water has entered the reservoir as of Jan 1, 1977 it will be assumed that the change in oil pressure at the original oil-water contact level (-7376) will be the same as the change in oil pressure at the -7350 foot datum. An assumption of zero capillary pressure ($P_o = P_w$) at the oil-water contact is also involved.

Solution Water Influx Problem 3 Cont.

For these conditions:

$$\Delta P_w)_{0/w} = \Delta P_o = 2056 - \frac{\sum p_j \ell_j}{\sum \ell_j}$$

$$= 2056 - \frac{1,000 (10^8)}{19,75 \cdot 2640} \text{ psif per foot} = 2056 - 1918$$

$$\Delta P_{0/w} = \underline{\underline{138 \text{ psi}}}$$

The equivalent r_w of the initial oil-water contact can be calculated by two means. The method proposed by Craft and Hawkins is

$$r_w' = \sqrt{\frac{\text{Area}}{\pi}} = \sqrt{\frac{2880 \text{ acres} \cdot 43560 \text{ ft}^2}{\pi}} = \underline{\underline{6319 \text{ ft}}}$$

The method proposed by Kucik & Brigham is

$$a = 60 \text{ mm} \cdot \frac{43560 \text{ ft}}{116 \text{ mm}} = 10924 \text{ ft}$$

$$\text{Area, } A = 2880 \cdot 43560 = 1,255 (10^8) \text{ ft}^2$$

For an ellipse $A = \pi ab$

$$\therefore b = \frac{1,255 (10^8)}{\pi \cdot 10924} = 3657 \text{ ft}$$

From Kucik & Brigham

$$\frac{r_w'}{a} = \frac{(a+b)}{2a} \quad \text{or} \quad r_w' = \frac{a+b}{2}$$

$$\therefore r_w' = \frac{(10924 + 3657)}{2} = \underline{\underline{7290 \text{ ft}}}$$

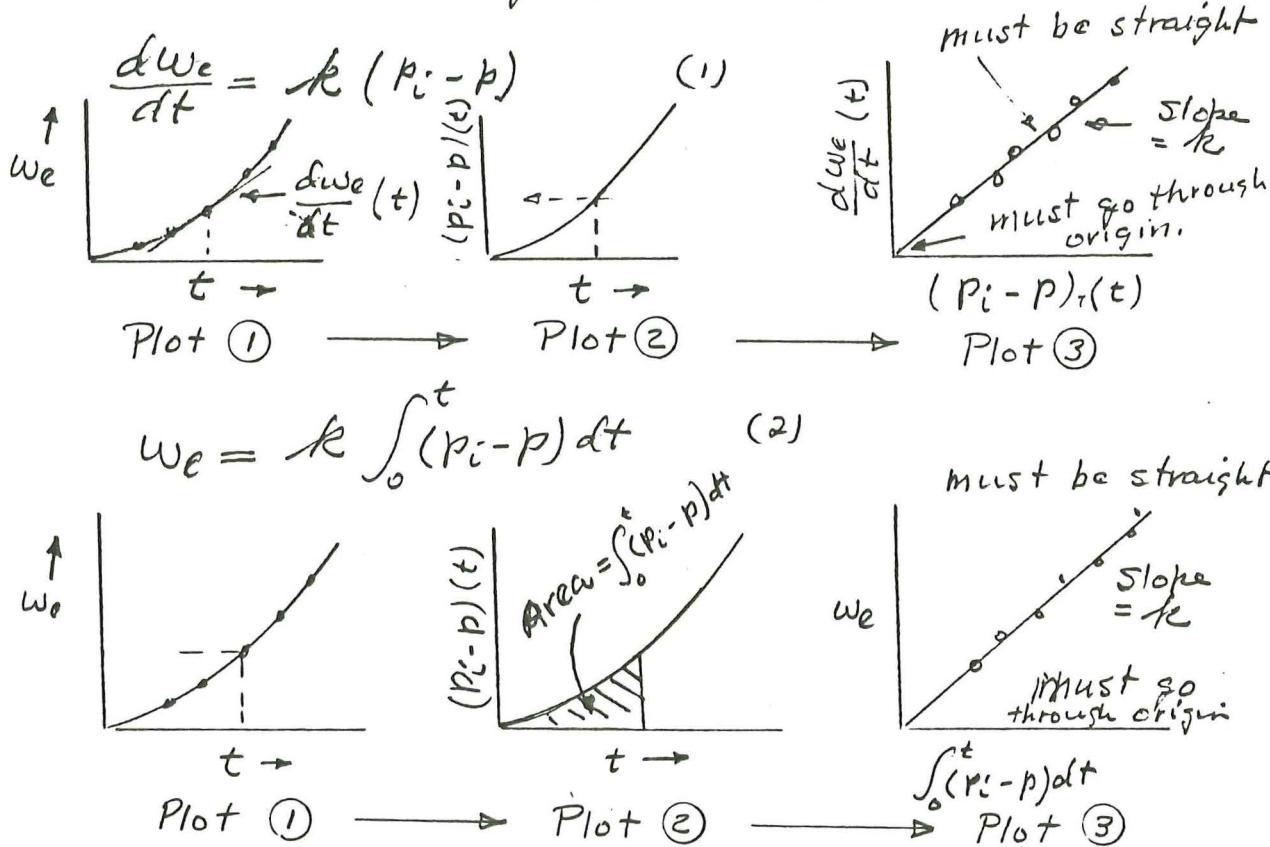
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Notes on Determining Steady-State Influx Constants When Influx is Known From Material Balance Calculations.

Ref: Craft & Hawkins pgs 224 - 230

- (1) Cumulative water entering reservoir is known as a function of time, $w_e = f_1(t)$
- (2) Field water pressure at location of original equilibr location is known as a function of time, $p = f_2(t)$.

1. Schilthuis Steady-State Form.



Schilthuis form valid only if suitable straight line of Plot 3 is obtained that goes through origin. Curved trend of Plot 3 indicates form of water influx is not Schilthuis steady-state.

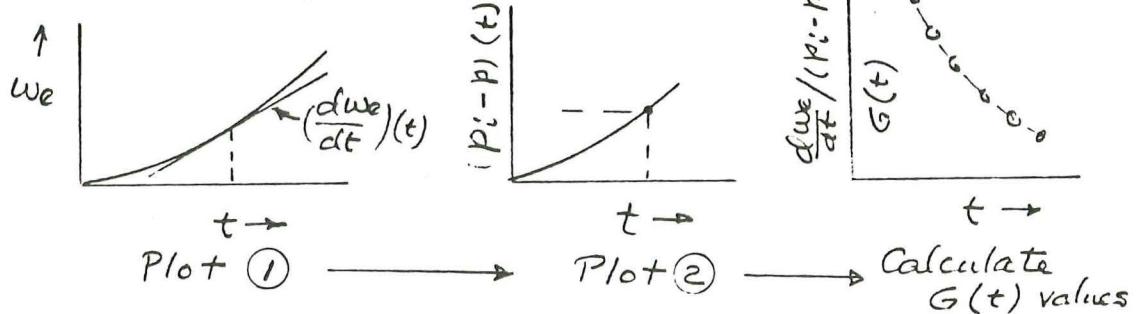
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2. Hurst Modified Steady-State Form

$$\frac{dW_e}{dt} = \frac{c(P_i - p)}{\log at} \quad (3)$$

$$W_e = c \int_0^t \frac{(P_i - p) dt}{\log at} \quad (4)$$

$$\text{Let } \frac{\frac{dW_e}{dt}(t)}{(P_i - p)(t)} = G(t) \quad (5)$$



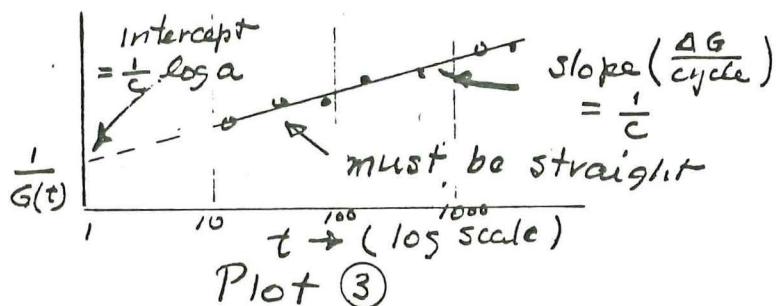
From equation 3

$$G(t) = \frac{c}{\log at} \quad (6)$$

$$\frac{1}{G(t)} = \underbrace{\frac{1}{c} \log a}_{b} + \underbrace{\frac{1}{c} \log t}_{m} \quad (7)$$

$$y = b + mx$$

Eq 7 is a straight line when $\frac{1}{G(t)}$ is plotted vs $\log t$.



Hurst Modified form valid only if suitable straight line on Plot 3 is obtained.

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Linear Aquifer Equations

$$\frac{W_e}{\Delta p} = \frac{\phi c_t h b L}{5,615} F(t_D) \quad \frac{\Delta p}{8} = \frac{5,615 B_w \mu L}{k h b} F(t_D)$$

$$t_D = \frac{6.33(10^3) k t}{\phi \mu c_t L^2}$$

Outer Boundary Condition

Infinite

Closed
(No Flow)

Constant Pressure

Constant Pressure Inner Boundary

$$F(t_D) = F_{1/2}(t_D)$$

$$F(t_D) = F_0(t_D)$$

$$F(t_D) = F_i(t_D)$$

Constant Rate Inner Boundary

$$F(t_D) = F_{1/2}(t_D)$$

$$F(t_D) = F_i(t_D)$$

$$F(t_D) = F_0(t_D)$$

$$\text{For } t_D \leq 0.25 \quad F_0(t_D) = F_{1/2}(t_D) = F_i(t_D) = 2 \sqrt{\frac{t_D}{\pi}}$$

$$\text{For } t_D > 2.0 \quad F_0(t_D) = 1 ; F_{1/2}(t_D) = 2 \sqrt{\frac{t_D}{\pi}} ; F_i(t_D) = t_D + \frac{1}{3}$$

$$\text{For } 0.25 < t_D < 2.0 \quad F_0(t_D) \neq F_i(t_D) \text{ Use chart} ; F_{1/2}(t_D) = 2 \sqrt{\frac{t_D}{\pi}}$$

W_e = cum. bbls

Δp = psi

ϕ = fraction

b = width, ft.

h = thickness, ft

L = length, ft

t = time, days

c_t = total compressibility,
psi⁻¹

μ = viscosity, cp.

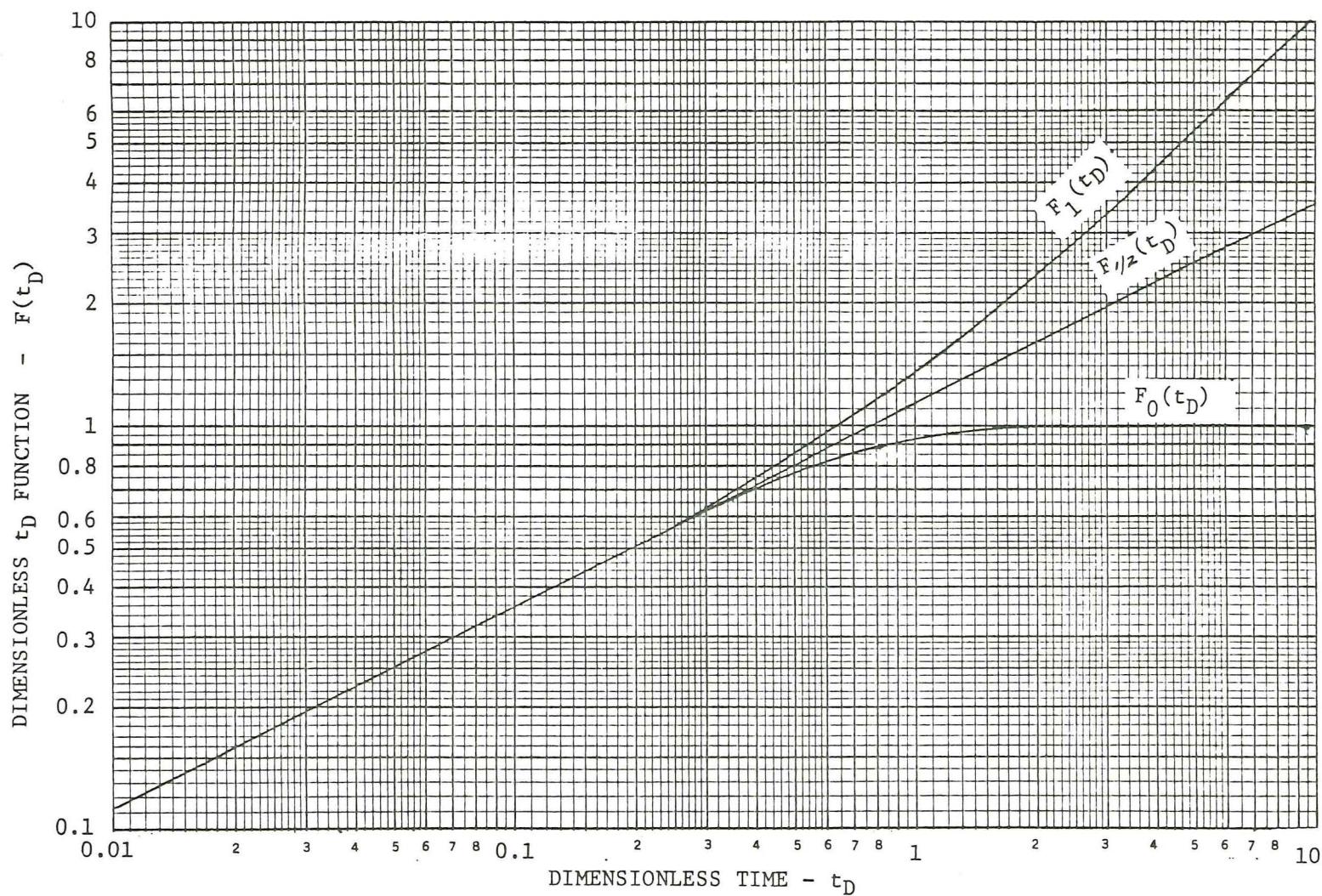
g = surface rate, bbl/day

k = permeability, md

B = form. vol. fact., dim.

Reference: Nabor & Barham
Trans. AIME (1964) 231, 155-161

Standing
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DIMENSIONLESS PRESSURE CHANGE and EFFLUX FUNCTIONS - LINEAR AQUIFERS

Ref: Nabor and Barham
 Trans AIME (1964) 231, 561

TABLE I—PRESSURE DROP AND CUMULATIVE INFLUX FUNCTIONS FOR
LINEAR AQUIFERS

t_D	$F_0(t_D)$	$F_{12}(t_D)$	$F_1(t_D)$	t_D	$F_0(t_D)$	$F_{12}(t_D)$	$F_1(t_D)$
1.00(10 ⁻²)	1.128379(10 ⁻¹)	1.128379(10 ⁻¹)	1.128379(10 ⁻¹)	3.10(10 ⁻¹)	6.226824(10 ⁻¹)	6.282549(10 ⁻¹)	6.338276(10 ⁻¹)
1.10(10 ⁻²)	1.183454(10 ⁻¹)	1.183454(10 ⁻¹)	1.183454(10 ⁻¹)	3.50(10 ⁻¹)	6.581891(10 ⁻¹)	6.675581(10 ⁻¹)	6.769283(10 ⁻¹)
1.25(10 ⁻²)	1.261566(10 ⁻¹)	1.261566(10 ⁻¹)	1.261566(10 ⁻¹)	4.00(10 ⁻¹)	6.978819(10 ⁻¹)	7.136496(10 ⁻¹)	7.294231(10 ⁻¹)
1.40(10 ⁻²)	1.335116(10 ⁻¹)	1.335116(10 ⁻¹)	1.335116(10 ⁻¹)	4.50(10 ⁻¹)	7.329537(10 ⁻¹)	7.569398(10 ⁻¹)	7.809461(10 ⁻¹)
1.60(10 ⁻²)	1.427299(10 ⁻¹)	1.427299(10 ⁻¹)	1.427299(10 ⁻¹)	5.00(10 ⁻¹)	7.639503(10 ⁻¹)	7.978846(10 ⁻¹)	8.318760(10 ⁻¹)
1.80(10 ⁻²)	1.513880(10 ⁻¹)	1.513880(10 ⁻¹)	1.513880(10 ⁻¹)	5.60(10 ⁻¹)	7.964332(10 ⁻¹)	8.444016(10 ⁻¹)	8.925272(10 ⁻¹)
2.00(10 ⁻²)	1.595769(10 ⁻¹)	1.595769(10 ⁻¹)	1.595769(10 ⁻¹)	6.20(10 ⁻¹)	8.244456(10 ⁻¹)	8.884866(10 ⁻¹)	9.528875(10 ⁻¹)
2.25(10 ⁻²)	1.692569(10 ⁻¹)	1.692569(10 ⁻¹)	1.692569(10 ⁻¹)	7.00(10 ⁻¹)	8.558930(10 ⁻¹)	9.440697(10 ⁻¹)	10.331309(10 ⁻¹)
2.50(10 ⁻²)	1.784124(10 ⁻¹)	1.784124(10 ⁻¹)	1.784124(10 ⁻¹)	8.00(10 ⁻¹)	8.874029(10 ⁻¹)	10.092530(10 ⁻¹)	11.332578(10 ⁻¹)
2.80(10 ⁻²)	1.888139(10 ⁻¹)	1.888139(10 ⁻¹)	1.888139(10 ⁻¹)	9.00(10 ⁻¹)	9.120229(10 ⁻¹)	10.704744(10 ⁻¹)	12.333052(10 ⁻¹)
3.10(10 ⁻²)	1.986717(10 ⁻¹)	1.986717(10 ⁻¹)	1.986717(10 ⁻¹)	1.00	9.312597(10 ⁻¹)	1.128379	1.333323
3.50(10 ⁻²)	2.111004(10 ⁻¹)	2.111004(10 ⁻¹)	2.111004(10 ⁻¹)	1.10	9.462902(10 ⁻¹)	1.183454	1.433329
4.00(10 ⁻²)	2.256758(10 ⁻¹)	2.256758(10 ⁻¹)	2.256758(10 ⁻¹)	1.25	9.629049(10 ⁻¹)	1.261566	1.583332
4.50(10 ⁻²)	2.393654(10 ⁻¹)	2.393654(10 ⁻¹)	2.393654(10 ⁻¹)	1.40	9.743799(10 ⁻¹)	1.335116	1.733333
5.00(10 ⁻²)	2.523133(10 ⁻¹)	2.523133(10 ⁻¹)	2.523133(10 ⁻¹)	1.60	9.843590(10 ⁻¹)	1.427299	1.933333
5.60(10 ⁻²)	2.670232(10 ⁻¹)	2.670232(10 ⁻¹)	2.670232(10 ⁻¹)	1.80	9.904512(10 ⁻¹)	1.513880	2.133333
6.20(10 ⁻²)	2.809641(10 ⁻¹)	2.809641(10 ⁻¹)	2.809641(10 ⁻¹)	2.00	9.941705(10 ⁻¹)	1.595769	2.333333
7.00(10 ⁻²)	2.985411(10 ⁻¹)	2.985411(10 ⁻¹)	2.985411(10 ⁻¹)	2.25	9.968541(10 ⁻¹)	1.692569	2.583333
8.00(10 ⁻²)	3.191537(10 ⁻¹)	3.191538(10 ⁻¹)	3.191539(10 ⁻¹)	2.50	9.983024(10 ⁻¹)	1.784124	2.833333
9.00(10 ⁻²)	3.385133(10 ⁻¹)	3.385138(10 ⁻¹)	3.385141(10 ⁻¹)	2.80	9.991902(10 ⁻¹)	1.888139	3.133333
1.00(10 ⁻¹)	3.568234(10 ⁻¹)	3.568248(10 ⁻¹)	3.568262(10 ⁻¹)	3.10	9.996137(10 ⁻¹)	1.986717	3.433333
1.10(10 ⁻¹)	3.742370(10 ⁻¹)	3.742410(10 ⁻¹)	3.742451(10 ⁻¹)	3.50	9.998560(10 ⁻¹)	2.111004	3.833333
1.25(10 ⁻¹)	3.989280(10 ⁻¹)	3.989423(10 ⁻¹)	3.989566(10 ⁻¹)	4.00	9.999581(10 ⁻¹)	2.256758	4.333333
1.40(10 ⁻¹)	4.221615(10 ⁻¹)	4.222008(10 ⁻¹)	4.222401(10 ⁻¹)	4.50	9.999878(10 ⁻¹)	2.393654	4.833333
1.60(10 ⁻¹)	4.512368(10 ⁻¹)	4.513517(10 ⁻¹)	4.514665(10 ⁻¹)	5.00	9.999964(10 ⁻¹)	2.523133	5.333333
1.80(10 ⁻¹)	4.784617(10 ⁻¹)	4.787307(10 ⁻¹)	4.789997(10 ⁻¹)	5.60	9.999992(10 ⁻¹)	2.670232	5.933333
2.00(10 ⁻¹)	5.040878(10 ⁻¹)	5.046265(10 ⁻¹)	5.051652(10 ⁻¹)	6.20	9.999998(10 ⁻¹)	2.809641	6.533333
2.25(10 ⁻¹)	5.341424(10 ⁻¹)	5.352372(10 ⁻¹)	5.363320(10 ⁻¹)	7.00	10.000000(10 ⁻¹)	2.985411	7.333333
2.50(10 ⁻¹)	5.622335(10 ⁻¹)	5.641896(10 ⁻¹)	5.661456(10 ⁻¹)	8.00	10.000000(10 ⁻¹)	3.191538	8.333333
2.80(10 ⁻¹)	5.936127(10 ⁻¹)	5.970821(10 ⁻¹)	6.005516(10 ⁻¹)	9.00	10.000000(10 ⁻¹)	3.385138	9.333333

Dimensionless Efflux Functions, Φ_D

Radical Infinite Aquifer.

$$\epsilon_D = \frac{D t}{r_w^2} = \frac{6.33(10^3) k t}{\phi_{\text{rec}} r_w^2}$$

$$Q_D = f(t_D)$$

For $t_D < 0.01$

$$Q_D = 2 \sqrt{t_D / \pi}$$

For $0.01 < t_D < 200$

$$Q_D = \frac{1.12838 \sqrt{t_D} + 1.19328 t_D + 0.269872 t_D \sqrt{t_D} + 0.008 \sqrt{294} t_D^2}{1 + 0.616599 \sqrt{t_D} + 0.0413008 t_D}$$

precision 0.02 % ±

For $t_D > 200$

$$Q_D = \frac{-4.29881 + 2.02566 t_D}{\ln t_D}$$

precision 0.02 % ±

For $0.05 < t_D < 7$ (Standing)

$$Q_D = b t_D^m$$

$$b = 1.577 - 0.023 \log t_D - 0.111 (\log t_D)^2$$

$$m = 0.624 + 0.0824 \log t_D + 0.0187 (\log t_D)^2$$

precision 1 % ±

For $7 < t_D < 1000$ (Standing)

$$Q_D = b t_D^m$$

$$b = 1.844 - 0.477 \log t_D + 0.04107 (\log t_D)^2$$

$$m = 0.612 + 0.1236 \log t_D - 0.0139 (\log t_D)^2$$

precision 1 % ±

Craft & Hawkins

Q_D for Limited Radial Graders

Table 5.2. LIMITED AQUIFER VALUES OF DIMENSIONLESS WATER INFLUX $Q(t)$ FOR VALUES OF DIMENSIONLESS TIME t_D AND FOR SEVERAL RATIOS OF AQUIFER-RESERVOIR RADII r_e/r_w

$r_e/r_w = 1.5$	$r_e/r_w = 2.0$	$r_e/r_w = 2.5$	$r_e/r_w = 3.0$	$r_e/r_w = 3.5$	$r_e/r_w = 4.0$	$r_e/r_w = 4.5$
Dimensionless time t_D	Dimensionless fluid influx $Q(t)$	Dimensionless time t_D	Dimensionless fluid influx $Q(t)$	Dimensionless time t_D	Dimensionless fluid influx $Q(t)$	Dimensionless time t_D
5.0(10) ⁻¹	0.276	5.0(10) ⁻¹	0.278	1.0(10) ⁻¹	0.408	3.0(10) ⁻¹
6.0	0.304	7.5	0.345	1.5	0.509	4.0
7.0	0.330	1.0(10) ⁻¹	0.404	2.0	0.599	5.0
8.0	0.354	1.25	0.458	2.5	0.681	6.0
9.0	0.375	1.50	0.507	3.0	0.758	7.0
1.0(10) ⁻¹	0.395	1.75	0.553	3.5	0.829	8.0
1.1	0.414	2.00	0.597	4.0	0.897	9.0
1.2	0.431	2.25	0.638	4.5	0.962	1.00
1.3	0.446	2.50	0.678	5.0	1.024	1.25
1.4	0.461	2.75	0.715	5.5	1.083	1.50
1.5	0.474	3.00	0.751	6.0	1.140	1.75
1.6	0.486	3.25	0.785	6.5	1.195	2.00
1.7	0.497	3.50	0.817	7.0	1.248	2.25
1.8	0.507	3.75	0.848	7.5	1.299	2.50
1.9	0.517	4.00	0.877	8.0	1.348	2.75
2.0	0.525	4.25	0.905	8.5	1.395	3.00
2.1	0.533	4.50	0.932	9.0	1.440	3.25
2.2	0.541	4.75	0.958	9.5	1.484	3.50
2.3	0.548	5.00	0.983	1.0	1.526	3.75
2.4	0.554	5.50	1.028	1.1	1.605	4.00
2.5	0.559	6.00	1.070	1.2	1.679	4.25
2.6	0.565	6.50	1.108	1.3	1.747	4.50
2.8	0.574	7.00	1.143	1.4	1.811	4.75
3.0	0.582	7.50	1.174	1.5	1.870	5.00
3.2	0.588	8.00	1.203	1.6	1.924	5.50
3.4	0.594	8.00	1.253	1.7	1.975	6.00
3.6	0.599	1.00	1.295	1.8	2.022	6.50
3.8	0.603	1.1	1.330	2.0	2.106	7.00
4.0	0.606	1.2	1.358	2.2	2.178	7.50
4.5	0.613	1.3	1.382	2.4	2.241	8.00
5.0	0.617	1.4	1.402	2.6	2.294	9.00
6.0	0.621	1.6	1.432	2.8	2.340	10.00
7.0	0.623	1.7	1.444	3.0	2.380	11.00
8.0	0.624	1.8	1.453	3.4	2.444	12.00
2.5	1.487	4.2	2.525	16.00	3.993	17
3.0	1.495	4.6	2.551	18.00	3.997	18
4.0	1.499	5.0	2.570	20.00	3.999	20
5.0	1.500	6.0	2.599	22.00	3.999	25
8.0	2.619				35	5.624
9.0	2.622				40	5.625
10.0	2.624					50
						7.497

Table 5.2. (Cont'd.)

$r_e/r_w = 5.0$		$r_e/r_w = 6.0$		$r_e/r_w = 7.0$		$r_e/r_w = 8.0$		$r_e/r_w = 9.0$		$r_e/r_w = 10.0$	
Dimensionless time t_D	Fluid influx $Q(t)$										
3.0	3.195	8.0	5.148	9.00	6.861	9	6.861	10	7.417	15	9.965
3.5	3.542	6.5	5.440	9.50	7.127	10	7.398	11	9.945	20	12.32
4.0	3.875	7.0	5.724	10	7.389	11	7.920	12	12.26	22	13.22
4.5	4.193	7.5	6.002	11	7.902	12	8.431	13	13.13	24	14.09
5.0	4.499	8.0	6.273	12	8.397	13	8.930	14	13.98	26	14.95
5.5	4.792	8.5	6.537	13	8.876	14	9.418	15	14.79	28	15.78
6.0	5.074	9.0	6.795	14	9.341	15	9.895	16	15.59	30	16.59
6.5	5.345	9.5	7.047	15	9.791	16	10.361	17	16.35	32	17.38
7.0	5.605	10.0	7.293	16	10.23	17	10.82	18	17.10	34	18.16
7.5	5.854	10.5	7.533	17	10.65	18	11.26	19	17.82	36	18.91
8.0	6.095	11	7.767	18	11.06	19	11.70	20	18.52	38	19.65
8.5	6.325	12	8.220	19	11.46	20	12.13	20	19.19	40	20.37
9.0	6.547	13	8.651	20	11.85	22	12.95	22	19.85	42	21.07
9.5	6.760	14	9.063	22	12.58	24	13.74	24	20.48	44	21.76
10.0	6.965	15	9.456	24	13.27	26	14.50	44	21.09	46	22.42
11	7.350	16	9.829	26	13.92	28	15.23	46	21.69	48	23.07
12	7.706	17	10.19	28	14.53	30	15.92	48	22.26	50	23.71
13	8.035	18	10.53	30	15.11	34	17.22	50	22.82	52	24.33
14	8.339	19	10.85	35	16.39	38	18.41	52	23.36	54	24.94
15	8.620	20	11.16	40	17.49	40	18.97	54	23.89	56	25.53
16	8.879	22	11.74	45	18.43	45	20.26	56	24.39	58	26.11
18	9.338	24	12.26	50	19.24	50	21.42	58	24.88	60	26.67
20	9.731	25	12.50	60	20.51	55	22.46	60	25.36	65	28.02
22	10.07	31	13.74	70	21.45	60	23.40	65	26.48	70	29.29
24	10.35	35	14.40	80	22.13	70	24.98	70	27.52	75	30.49
26	10.59	39	14.93	90	22.63	80	26.26	75	28.48	80	31.61
28	10.80	51	16.05	100	23.00	90	27.28	80	29.36	85	32.67
30	10.98	60	16.56	120	23.47	100	28.11	85	30.18	90	33.66
32	11.26	70	16.91	140	23.71	120	29.31	90	30.93	95	34.60
34	11.46	80	17.14	160	23.85	140	30.08	95	31.63	100	35.48
36	11.61	90	17.27	180	23.92	160	30.58	100	32.27	120	38.51
38	11.71	100	17.36	200	23.96	180	30.91	120	34.39	140	40.89
40	11.99	150	17.49	240	24.00	200	31.12	140	35.92	160	42.75
42	12.00	160	17.49	240	31.34	160	37.04	180	44.21		
44	12.00	180	17.50	280	31.43	180	37.85	200	45.36		
46	12.00	200	17.50	280	31.43	180	37.85	200	45.36		
48	12.00	220	17.50	280	31.43	180	37.85	200	45.36		
50	12.00	240	17.50	280	31.43	180	37.85	200	45.36		
52	12.00	260	17.50	280	31.43	180	37.85	200	45.36		
54	12.00	280	17.50	280	31.43	180	37.85	200	45.36		
56	12.00	300	17.50	280	31.43	180	37.85	200	45.36		
58	12.00	320	17.50	280	31.43	180	37.85	200	45.36		
60	12.00	340	17.50	280	31.43	180	37.85	200	45.36		
62	12.00	360	17.50	280	31.43	180	37.85	200	45.36		
64	12.00	380	17.50	280	31.43	180	37.85	200	45.36		
66	12.00	400	17.50	280	31.43	180	37.85	200	45.36		
68	12.00	420	17.50	280	31.43	180	37.85	200	45.36		
70	12.00	440	17.50	280	31.43	180	37.85	200	45.36		
72	12.00	460	17.50	280	31.43	180	37.85	200	45.36		
74	12.00	480	17.50	280	31.43	180	37.85	200	45.36		
76	12.00	500	17.50	280	31.43	180	37.85	200	45.36		
78	12.00	520	17.50	280	31.43	180	37.85	200	45.36		
80	12.00	540	17.50	280	31.43	180	37.85	200	45.36		
82	12.00	560	17.50	280	31.43	180	37.85	200	45.36		
84	12.00	580	17.50	280	31.43	180	37.85	200	45.36		
86	12.00	600	17.50	280	31.43	180	37.85	200	45.36		
88	12.00	620	17.50	280	31.43	180	37.85	200	45.36		
90	12.00	640	17.50	280	31.43	180	37.85	200	45.36		
92	12.00	660	17.50	280	31.43	180	37.85	200	45.36		
94	12.00	680	17.50	280	31.43	180	37.85	200	45.36		
96	12.00	700	17.50	280	31.43	180	37.85	200	45.36		
98	12.00	720	17.								

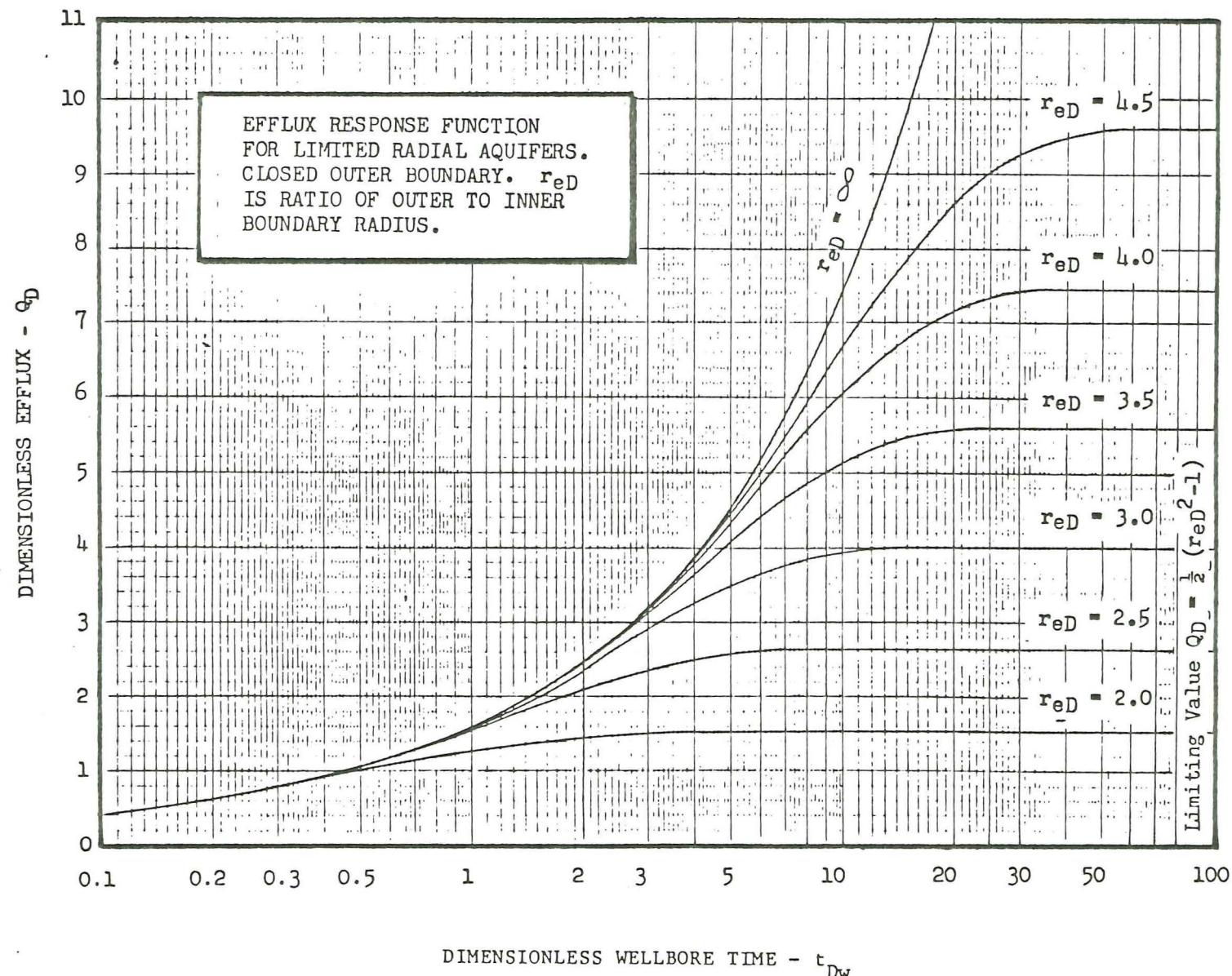


Table 5.1. INFINITE AQUIFER VALUES OF DIMENSIONLESS WATER INFLUX $Q(t)$ FOR VALUES OF DIMENSIONLESS TIME t_D

Dimensionless time t_D	Dimensionless influx $Q(t)$	Dimensionless time t_D	Fluid influx $Q(t)$										
0.00	0.000	79	35.697	455	150.249	1190	310.843	3250	816.090	35,000	6780.2C		
0.01	0.112	80	36.058	460	151.640	1200	343.308	3300	827.088	40,000	7650.06		
0.05	0.278	81	36.418	465	153.029	1210	345.770	3350	838.067	50,000	933.06		
0.10	0.404	82	36.777	470	154.416	1220	348.230	3400	849.028	60,000	11,047.2B		
0.15	0.520	83	37.136	475	155.801	1225	349.460	3450	859.974	70,000	12,708.2A		
0.20	0.606	84	37.494	480	157.184	1230	350.688	3500	870.903	75,000	13,531.4S		
0.25	0.689	85	37.851	485	158.565	1240	353.144	3550	881.816	80,000	14,350.1Z		
0.30	0.758	86	38.207	490	159.945	1250	355.597	3600	892.712	90,000	15,975.2B		
0.40	0.898	87	38.563	495	161.322	1260	358.048	3650	903.594	100,000	17,586.2A		
0.50	1.020	88	38.919	500	162.698	1270	360.496	3700	914.459	125,000	21,560.7D		
0.60	1.140	89	39.272	510	165.444	1275	361.720	3750	925.309	1.5(10) ⁵	2,538.1D		
0.70	1.251	90	39.626	520	168.183	1280	362.942	3800	936.144	2.0	3,308		
0.80	1.359	91	39.979	525	169.549	1290	365.386	3850	946.966	2.5	4,066		
0.90	1.469	92	40.331	530	170.914	1300	367.828	3900	957.773	3.0	4,817		
1	1.569	93	40.684	540	173.639	1310	370.267	3950	968.566	4.0	6,267		
2	2.447	94	41.034	550	176.357	1320	372.704	4000	979.344	5.0	7,699		
3	3.202	95	41.385	560	179.069	1325	373.922	4050	990.108	6.0	9,113		
4	3.893	96	41.735	570	181.774	1330	375.139	4100	1000.858	7.0	10,51(10) ⁴		
5	4.539	97	42.084	575	183.124	1340	377.572	4150	1011.595	8.0	11,89		
6	5.153	98	42.433	580	184.473	1350	380.003	4200	1022.318	9.0	13,326		
7	5.743	99	42.781	590	187.166	1360	382.432	4250	1033.028	1.0(10) ⁵	1,462		
8	6.314	100	43.129	600	189.852	1370	384.859	4300	1043.724	1.5	2,126		
9	6.869	105	44.858	610	192.533	1375	386.070	4350	1054.409	2.0	2,781		
10	7.411	110	46.574	620	195.208	1380	387.283	4400	1065.082	3.0	3,427		
11	7.940	115	48.277	625	196.544	1390	389.705	4450	1075.743	4.0	4,064		
12	8.457	120	49.968	630	197.878	1400	392.125	4500	1086.390	5.0	5,313		
13	8.964	125	51.648	640	200.542	1410	394.543	4550	1097.024	6.0	6,544		
14	9.461	130	53.317	650	203.201	1420	396.959	4600	1107.646	7.0	7,761		
15	9.949	135	54.976	660	205.854	1425	398.167	4650	1118.257	7.0	8,965		
16	10.434	140	56.625	670	208.502	1430	399.373	4700	1128.854	8.0	1,016(10) ⁴		
17	10.913	145	58.265	675	209.825	1440	401.786	4750	1139.439	9.0	1,134		
18	11.386	150	59.895	680	211.145	1450	404.197	4800	1150.012	1.0(10) ⁵	1,252		
19	11.855	155	61.517	690	213.784	1460	406.606	4850	1160.574	1.5	1,828		
20	12.319	160	63.131	700	216.417	1470	409.013	4900	1171.125	2.0	2,398		
21	12.778	165	64.737	710	219.046	1475	410.214	4950	1181.666	2.5	2,901		
22	13.233	170	66.336	720	221.670	1480	411.418	5000	1192.198	3.0	3,517		
23	13.684	175	67.928	725	222.980	1490	413.820	5100	1213.222	4.0	4,610		
24	14.131	180	69.512	730	224.289	1500	416.220	5200	1234.203	5.0	5,689		
25	14.573	185	71.090	740	226.904	1525	422.214	5300	1255.141	6.0	6,758		
26	15.013	190	72.661	750	229.514	1550	428.196	5400	1276.037	7.0	7,816		
27	15.450	195	74.226	760	232.120	1575	434.168	5500	1296.893	8.0	8,866		
28	15.883	200	75.785	770	234.721	1600	440.128	5600	1317.709	9.0	9,911		
29	16.313	205	77.338	775	236.020	1625	446.077	5700	1338.486	1.0(10) ⁵	1,095(10) ⁴		
30	16.742	210	78.886	780	237.318	1650	452.016	5800	1359.225	1.5	1,604		
31	17.167	215	80.428	790	239.912	1675	457.945	5900	1379.927	2.0	2,108		
32	17.590	220	81.965	800	242.501	1700	463.863	6000	1400.593	2.5	2,607		
33	18.011	225	83.497	810	245.086	1725	469.771	6100	1421.224	3.0	3,100		
34	18.429	230	85.023	820	247.668	1750	475.669	6200	1441.820	4.0	4,071		
35	18.845	235	86.545	825	248.957	1775	481.558	6300	1462.383	5.0	5,032		
36	19.259	240	88.062	830	250.245	1800	487.437	6400	1482.912	6.0	5,984		
37	19.671	245	89.575	840	252.819	1825	493.307	6500	1503.408	7.0	6,928		
38	20.080	250	91.084	850	255.388	1850	499.167	6600	1523.872	8.0	7,865		
39	20.488	255	92.589	860	257.953	1875	505.019	6700	1544.305	9.0	8,797		
40	20.894	260	94.090	870	260.515	1900	510.861	6800	1564.706	1.0(10) ⁵	9,725		
41	21.298	265	95.588	875	261.795	1925	516.695	6900	1585.077	1.5	1,429(10) ⁴		
42	21.701	270	97.081	880	263.073	1950	522.520	7000	1605.418	2.0	1,880		
43	22.101	275	98.571	890	265.629	1975	528.337	7100	1625.729	2.5	2,328		
44	22.500	280	100.057	900	268.181	2000	534.145	7200	1646.011	3.0	2,771		
45	22.897	285	101.540	910	270.729	2025	539.945	7300	1666.265	4.0	3,645		
46	23.291	290	103.019	920	273.274	2050	545.737	7400	1686.490	5.0	4,510		
47	23.684	295	104.495	925	274.545	2075	551.522	7500	1706.688	6.0	5,368		
48	24.076	300	105.968	930	275.815	2100	557.299	7600	1726.859	7.0	6,220		
49	24.466	305	107.437	940	278.353	2125	563.068	7700	1747.002	8.0	7,064		
50	24.855	310	108.904	950	280.888	2150	568.830	7800	1767.120	9.0	7,909		
51	25.244	315	110.367	960	283.420	2175	574.585	7900	1787.212	1.0(10) ⁵	8,747		

Craft & Hawkes

WATER INFLUX

Q_D for Infiltration of Radial Aquifers

Table 5.1. (Cont'd.)

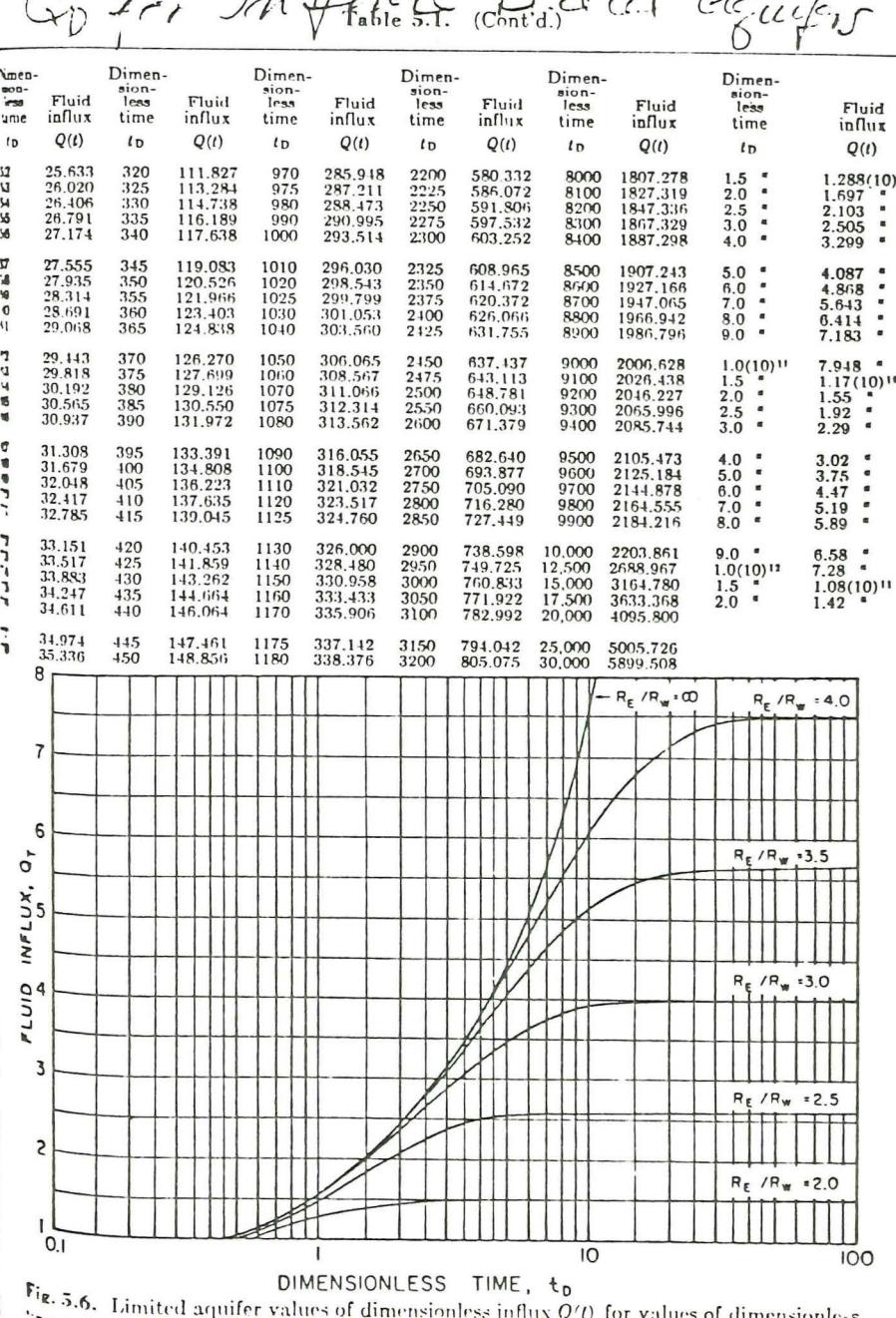


Fig. 5.6. Limited aquifer values of dimensionless influx Q/D for values of dimensionless time t_D and aquifer limits given by the ratio R_E/R_w .