#### NOTES ON

### CALCULATING SOLUTION GAS DRIVE RESERVOIR PERFORMANCE

#### BY TARNER'S METHOD

The terms solution gas drive, internal gas drive, and dissolved gas drive are used interchangeably to designate the process of oil production that results from desolution and expansion of gas in the reservoir. The process starts at the bubble point pressure of the reservoir hydrocarbon system and continues during pressure depletion of the reservoir.

The general concept of the physical behavior in the reservoir is that saturated reservoir oil and free gas flow simultaneously when the gas saturation is greater than the critical gas saturation. The relative rates of subsurface flow are determined by the  $k_g/k_o$  vs S relationship of the rock, the gas-oil viscosity ration, and the oil-gas formation volume ratio. These factors control the relative rates at which the oil and gas phases will be produced from the reservoir.

The following assumptions are often made to simplify the calculation procedure:

- a) The reservoir rock is homogeneous with respect to permeability, porosity, fluid saturation, and relative permeability.
- b) Uniform pressure exists throughout the reservoir.
- c) Gravity forces are negligible.
- d) Equilibrium exists at all times between saturated oil and gas that has been evolved from saturated oil. The PVT properties used in the calculation were determined by the manner in which the reservoir operates.
- e) Reservoir hydrocarbon pore volume remains constant. This means there is no water influx into the reservoir nor compaction of the reservoir rock.

### Gas-Oil Ratio Equation

Equations for subsurface flow of free gas and saturated oil can be written in the usual manner. The instantaneous surface producing gas-oil ratio obtained from these equations is:

$$R = \frac{k_g}{k_o} \cdot \frac{\mu_o B_o}{\mu_g B_g} \cdot 5.615 + R_g$$

(1)

where:

 $\begin{array}{l} R = \text{instantaneous surface ratio, scf/STB} \\ B_{o}, B_{g} = \text{saturated oil and free gas formation volumes, respectively} \\ \mu_{o}, \mu_{g} = \text{phase viscosities, cp} \\ k_{g}, k_{o} = \text{gas and oil effective permeabilities, respectively, md} \\ R_{s} = \text{solution gas-oil ratio of saturated oil, scf/STB} \end{array}$ 

The effective permeability ratio,  $k_g/k_o$ , is saturation dependent for a given rock. All other factors are pressure dependent.

#### Saturation Equations

Because  $k_g/k_o$  is saturation dependent, an equation is required to keep account of gas or liquid saturation in the reservoir during the production process. This can be developed as follows:

Let: N = initial stock tank barrel of oil in place

Then:

 $NB_{oi}$  = initial reservoir barrels of oil in place (2)

$$\frac{NB_{oi}}{(1-S_w)} = \text{initial reservoir pore volume}$$
(3)

The reservoir oil volume remaining after N barrels of stock tank oil has been produced is  $(N-N_p)B_0$  barrels. Therefore, the reservoir oil saturation is:

$$S_{o} = \frac{(N-N_{p})B_{o}}{\frac{NB_{oi}}{(1-S_{w})}} = \left[1 - \frac{N_{p}}{N}\right] \frac{B_{o}}{B_{oi}} (1-S_{w})$$
(4)

The corresponding reservoir gas saturation is:

$$S_{g} = 1 - S_{w} - S_{o} = 1 - S_{w} - \left[1 - \frac{N}{N}\right] \frac{B_{o}}{B_{oi}} (1 - S_{w})$$
 (5)

Notice that in deriving Eqs. 4 and 5, nothing has been said about the reservoir pore volume or interstitial water volume changing. As they stand, Eqs. 4 or 5 are justifiable simplifications of the real situation--justifiable because, at the pressure where gas and oil coexist, volume changes caused by pore volume compressibility and interstitial water compressibility are small compared to the hydrocarbon system compressibility. They can be neglected. This is not true at pressures above the bubble point pressure of the hydrocarbon system, where one must consider volume changes of all system components.

Note that the term (1-N/N) in Eqs. 4 and 5 is the fraction of original stock tank oil that is still in the reservoir.

### Material Balance Equation

The following development employs the Tarner method of predicting solution gas drive performance using equation forms proposed by Tracy (<u>Trans</u>., AIME [1955], <u>204</u>, 243). The equations will be developed for the simplest case: a constant volume oil band initially at its bubble point without water influx or production.

The fundamental precept of a volumetric material balance is that at every pressure below the initial pressure, the volume of products removed from the reservoir (expressed at the pressure) must equal the expansion of the original system between the initial pressure and that pressure. Thus the key wording <u>voidage = expansion</u> and the following mathematical expressions for voidage and expansion:

voidage (res. bbl), 
$$F = N_{po} + (G_{p} - N_{ps} R_{s}) \frac{B_{g}}{5.615}$$
 (6)

expansion (res. bb1), 
$$E_0 = N \left[ B_0 + (R_{sb} - R_s) \frac{B_g}{5.615} \right] - NB_{ob}$$
 (7)

Equating Eqs. 6 and 7 and solving for N yields:

$$N = \frac{N_{p} \left[ B_{o} - \frac{R_{s}B_{g}}{5.615} \right] + \frac{G_{p}B_{g}}{5.615}}{(B_{o} - B_{o}) + (R_{sb} - R_{s}) \frac{B_{g}}{5.615}}$$
(8)

If we let:

$$\frac{B_{o} - \frac{R_{s}B_{g}}{5.615}}{(B_{o} - B_{o}) + (R_{sb} - R_{s}) \frac{B_{g}}{5.615}} = \Phi_{n}$$
(9)

and:

$$\frac{\frac{B_{g}}{5.615}}{(B_{o}-B_{ob})+(R_{sb}-R_{s})\frac{B_{g}}{5.615}} = \Phi_{g}$$
(10)

we can write Eq. 8 as:

$$N = N_{p} \Phi_{n} + G_{p} \Phi_{g}$$
(11)

Notice that  $\Phi_n$  and  $\Phi_g$  are grouped fluid property parameters that are functions of pressure. Precalculating these parameters and tabulating them at several pressures facilitates the calculations. Also, it is common practice to make the calculation for a unit volume of original stock tank oil in place; i.e., N = 1. In this case, N becomes a fraction of N.

# Auxiliary Equations

I will now write four auxiliary equations to allow us to make a material balance between any two pressures. In the following equations, the subscript j indicates the present pressure and the subscript k indicates the next pressure state to be calculated. Normally, the calculations are toward decreasing pressure.

The average surface producing gas-oil ratio,  $\overline{R}$ , in the pressure interval between  $p_j$  and  $p_k$  will be:

$$\overline{R}_{jk} = \frac{R_{j}+R_{k}}{2}$$
(12)

The cumulative stock tank oil production at pk will be:

$$N_{P_{k}} = N_{P_{j}} + \Delta N_{P_{jk}}$$
(13)

where  $\Delta N_{\ p_{ik}}$  is the interval oil production.

Similarly, the cumulative surface gas production at  $p_k$  will be:

$$G_{p_{k}} = G_{p_{j}} + \Delta G_{p_{jk}}$$
(14)

where  $\Delta G$  is the interval gas production.  $${}^{P}{}_{jk}$$ 

During the interval between  $\boldsymbol{p}_{j}$  and  $\boldsymbol{p}_{k},$ 

$$\Delta G_{p_{jk}} = \Delta N_{p_{jk}} \cdot \overline{R}_{jk} = \Delta N_{p_{jk}} \left( \frac{R_j + R_k}{2} \right)$$
(15)

If the relationships of the four auxiliary equations are substituted into Eq. 11, we have, at pressure  $p_k$ :

$$N = (N_{p_{j}} + \Delta N_{p_{j}k}) \Phi_{nk} + (G_{p_{j}} + \Delta G_{p_{j}k}) \Phi_{gk}$$
(16)

$$N = N_{p_{j}} \Phi_{nk} + \Delta N_{p_{jk}} \Phi_{nk} + G_{p_{j}} \Phi_{gk} + \Delta N_{p_{jk}} \left(\frac{R_{j} + R_{k}}{2}\right) \Phi_{gk}$$
(17)

If we let N = 1 so that N becomes the fraction of original stock tank oil produced at pressure j, the j-k interval production,  $\Delta N$ , becomes:

$$\Delta N_{p} = \frac{1 - N_{p_{j}} \Phi_{nk} - G_{p_{j}} \Phi_{gk}}{\Phi_{nk} + \left(\frac{R_{j} + R_{k}}{2}\right) \Phi_{gk}}$$
(18)

Use of Eq. 18 involves trial-and-error solutions in that we do not know what  $R_k$  will be at the next pressure point,  $p_k$ . It is customary to make the calculations at predetermined pressures (usually 200-500 psi differences) to minimize looking up  $\Phi_n$  and  $\Phi_g$  values. The following calculation procedure is often recommended.

### Calculation Procedure

At pressure p<sub>i</sub>:

- 1) Select pressure  $\mathbf{p}_k$  to make the next calculation. Calculate  $\Phi_{nk}$  and  $\Phi_{gk}$ .
- 2) Estimate what  $R_k$  will be. The best way is to keep a running plot of R vs p and extrapolate it to  $p_k$ .
- 3) Calculate  $\Delta N$  from Eq. 18.  $p_{jk}$
- 4) Calculate N from Eq. 13.  $P_k$
- 5) Calculate S from Eq. 14. If necessary, calculate S from Eq. 15.
- 6) From S<sub>o</sub> (or S<sub>g</sub>), obtain  $k_g/k_o$  and calculate R<sub>k</sub> from Eq. 1.
- 7) If  $R_k$  from step 6 does not agree fairly well with  $R_k$  assumed in step 2, go back through steps 2 through 6 using the value calculated in step 6.
- 8) After satisfactory agreement is obtained, calculate  $\Delta G$  from Eq. 15 and G from Eq. 14. Also calculate N from Eq. 13.
- 9) Use calculated values of N and G in Eq. 11 to check that  $P_k$   $P_k$   $P_k$  $N_{p_k} \Phi + G \Phi = 1$ . You should check within  $\pm 0.001$ .

### Producing Rate Equations

As pressure declines in a reservoir producing under solution gas drive, it becomes increasingly difficult for oil to flow from far regions of the well's drainage volume to the wellbore. This is because the effective oil permeability,  $k_0$ , becomes lower as a result of increasing gas saturation in the reservoir rock and because the reservoir oil becomes more viscous as gas is released from solution. On the other hand, the reservoir oil formation volume factor,  $B_0$ , decreases, which is, in effect, an opposite effect. The rate of stock tank oil production,  $q_0$ , from a nondamaged well can be calculated by the relationship:

$$q_{o} = \frac{k_{o}h\overline{p} \left[ 1 - 0.2(p_{wf}/\overline{p}) - 0.8(p_{wf}/\overline{p})^{2} \right]}{254.2 \,\overline{\mu}_{o}\overline{B}_{o}(\ln \ 0.47 \ r_{e}/r_{w})}$$
(19)

where:

- k = effective oil permeability, md, at the average fluid saturation of the drainage volume
- p = average pressure of the drainage volume, psia
- $\overline{\mu}_{0}\overline{B}_{0}$  = viscosity and formation volume factor at the average pressure of the drained volume, p
  - p<sub>wf</sub> = wellbore pressure, psia
  - r<sub>e</sub> = equivalent exterior circular radius, ft

The corresponding oil productivity equation is:

$$J_{o} = \frac{q_{o}}{\overline{p} - p_{wf}} = \frac{k_{o}h(1 + 0.8 \ p_{wf}/\overline{p})}{254.2 \ \overline{\mu}_{o}\overline{B}_{o}(\ln \ 0.47 \ r_{o}/r_{w})}$$
(20)

When the well is damaged (plus or minus skin factor), it is necessary to use the dimensionless IPR curves for damaged wells developed by the author (J. Pet. Tech. [Nov. 1970], 1399) to calculate the oil production rate as a function of wellbore flowing pressure.

Gas producing rates generally increase as pressure falls in solution gas drive reservoirs. However, if the gas relative permeability curve used provides for a critical gas saturation to be developed before gas flow commences in the reservoir, the resulting surface gas flow rate may decrease slightly or remain essentially constant for a brief period while pressure is still near the bubble point pressure. The rate of gas production can be most easily expressed through the oil production rate and the producing gas-oil ratio. That is, at any pressure:

$$q_{g} = q_{o}R$$
(21)

where  $q_0$  and R were previously calculated as a function of average reservoir pressure,  $\overline{p}$ , or of the cumulative oil produced,  $N_p$ .

# Production-Time Equation

In order to handle economic and production scheduling calculations, it is necessary to express oil and gas producing rates as a function of time. The easiest way to do this is through the increments of oil production between successive pressure points,  $p_j$  and  $p_k$ , and the arithmatic average oil producing rate during the pressure interval. That is:

$$\Delta_{t_{jk}}(days) = \frac{\Delta_{N}}{\overline{q}_{o_{jk}}}$$
(22)  
$$t_{k}(days) = \sum_{N_{p}=0}^{N_{p}} \Delta_{t_{jk}}$$
(23)

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### CSD Petroleum Industry Course

#### Material Balance Problem 4

[This problem teaches the calculating procedure used to predict oil and gas recovery factors of reservoirs that produce by solution gas drive (depletion drive) mechanism. The general method is referred to as a Tarner prediction, using the Tracy modification.]

- (1) Tarner, J.: "How Different Size Gas Caps and Pressure Maintenance Programs Affect Amount of Recoverable Oil," <u>Oil Weekly</u> (June 12, 1944), p. 32.
- (2) Tracy, G.W.: "Simplified Form of the Material Balance Equation," Trans., AIME (1955), 204, 243.

The Cook Zone of the S.E. Stanford Field produces  $35.1^{\circ}API$  stock tank oil from depths ranging from -5217' (zone top) to -5854' (O/W contact).



Datum level in this zone is -5700'. Laboratory tests on bottomhole samples indicate the reservoir oil was saturated in the crest of the structure at discovery. However, the reservoir oil gets progressively more undersaturated as one moves away from the crest.

The PVT data listed below were determined by Marsden Laboratories, Inc., on a

Laboratories, Inc., on a bottomhole sample from Well Luntzel No. 4. From early well tests, it appears that fluid in the region of this well had a bubble point pressure of 2500 psia at the regional temperature of 180°F. Calculations based on resistivity logs indicate the interstitial water in the region of this well to average 20%. Average porosity (of 67 cores) is 25.5% (arithmetic).

# CSD Petroleum Industry Course

# Laborial Balance Problem 4 Cont.

SUMMARY OF RESERVOIR FLUID PROPERTIES, WELL LUNTZEL #4, COOK ZONE

Pressure psia	R ft <sup>3/</sup> STB	B <sub>o</sub> RB/STB	ft <sup>3</sup> /ft <sup>3</sup>	<sup>µ</sup> о ср
2500	650	1.325	0.00447	0.90
2300	618	1.311	0.00473	0.97
2100	586	1.296	0.00509	1.05
1900	553	1.281	0.00562	1.14
1700	520	1.266	0.00638	1.26
1500	486	1.250	0.00750	1.39
1300	450	1.233	0.00907	1.54

Pressure psia	μ cp	5.615 $\frac{\mu_o}{\mu_g} \frac{B_o}{B_g}$		ф <sub>в</sub>
2500	0.0160	93.62(10 <sup>3</sup> )	8	$\infty$
2300	0.0158	95.84	61.00	0.0650
2100	0.0156	96.23	26.36	0.0312
1900	0.0154	94.74	13.70	0.0189
1700	0.0152	92.36	7.61	0.0128
1500	0.0150	86.72	4.17	0.00927
1300	0.0148	79.43	2.19	0.00699

# ASSIGNMENT

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- Using the attached relative permeability data published by C.R. Knoff (<u>Trans</u>., AIME, 1965), calculate the percent of initial gas and oil in the region of the Luntzel No. 4 well that would be produced at reservoir pressures of 2300, 2100, 1900, and 1700 psia.
- (2) Calculate the instantaneous producing gas-oil ratio at the above pressures.

# GUIDE ANSWERS

p psia	N P % IOIP	G P % IGIP		
2300	0.956	0.985		
1700	4.41	7.99		



The following values for solution of Problem, 4 were developed by SR-52 program "Sol's Zae Drive-Tariner" of April 1 1977 Boi (2500 psic) = 1.325 5w=0,20

pia	PK	Gpj SCF	Npj Bbi	ØgK	Øn K	Rsk ft3/661	Box	Rj ft <sup>3</sup> /661	J. 6N LIO BO LLS PS )K
2500	2300	C	0	0.0650	61,00	618	1.311	650	95.84 (103)
2300	2100	6,401	0,00956	0,0312	26,36	586	1,296	690	96,23
2100	1900	14.918	0,02025	0,0189	13,70	553	1.281	905	94,74
1900	1700	29,377	0,03246	0.0128	7,61	520	1.266	1462	92,36
1700	1500	51.912	0,04409	0,00927	4.17	486	1,250	2413	86.72
Noo	1300	83.402	0,05440	0.00699	2,19	450	1,233	3695	79.43

PK	RK Est	Ss	kstko	Recent	GPK FT3	NPR BUI	Test.
2300	690	0.0/60	-75 (103)	690	6,401	0,00956	0,99996
2100	908	0.0333	3,3(10)	903	14.918	0,02025	0.9992
1900		0,0516	9.6 (10)	1462	29.377	0,03246	1.0000
1700		0,0693	20,5(103)	2413	51,912	0,04409	1,0000
1500		0,0863	37.0(10)	3695	83.402	0,05440	1,000
1300		0,1026	61,0 (10)	5295	123,24)	0,06326	1,0000

Solution Material Balance Problem 4

Bridar