An Analysis of a Volumetric Balance Equation for Calculation of Oil in Place and Water Influx

D.H. Tehrani,* SPE, Heriot-Watt U.

Summary

When fluids are withdrawn from a petroleum reservoir, the space left behind is filled partly by the expansion of the remaining fluids and rock and partly by the influx of water from a contiguous aquifer, if it exists. The volumetric balance equation (VBE) is an expression of this same statement. Its simplified form is

$$Z=N_aX+e_{wc}Y. \quad (1)$$

When sufficient historical data on X and Z are available, various functions for Y can be tried; using the technique of least squares, a set of values can be calculated for N_a and e_{wc}.

It has been convenient to write Eq. 1 in the following form:

$$Z/X=N_a+e_{wc}(Y/X). \quad (2)$$

The advantages of this form are that it has only two variables, so least-squares calculations are easier for it, and that the values of Y/X and Z/X can be plotted graphically, so that a linear trend can be visually examined. The disadvantage is that the equation has a low resolving power and can produce erroneous answers. Nevertheless, most authors use the form of Eq. 2.

Eq. 1 may be written in many different forms, all of which are algebraically equivalent to each other; however, when the method of least squares is applied to them, they will produce different results.

This paper shows that the best form of the VBE for calculation of the original active oil-in-place (OIP) and water influx constant is the form of Eq. 1. It is also shown that the least-squares calculation based on minimizing the sum of the squared deviations of the calculated oil pressures from the observed pressures is equivalent to carrying out the least-squares method on Eq. 1.

Introduction

The Volumetric Balance Equation. The VBE was first introduced in general form by Schilthuis in 1935.[1] It can be written as follows:

$$N_p[B_t+B_g(R-R_{si})]+W_p=NB_{ti}[(B_t/B_{ti}-1)$$

$$+m(B_g/B_{gi}-1)+(1+m)(c_f+S_wc_w)(p_{oi}-p_o)$$

$$\div(1-S_w)]+e_w. \quad (3)$$

*Currently with Britoil plc.

The left side is the reservoir volume of the total withdrawals, shown here by Z. The first term on the right side (NB_{ti}) is the original OIP in reservoir barrels multiplied by the unit expansion $[(B_t/B_{ti}-1)+\ldots]$. The former is shown here by N_a and the latter by X. The second term on the right side is the water influx, which can be represented by an influx constant, e_{wc}, multiplied by a water influx function, Y. Eq. 3 can thus be written in the simple form of Eq. 1:

$$Z=N_aX+e_{wc}Y.$$

Dependent on the shape, type, and flow characteristics of the aquifer, various equations have been introduced for the Y term in Eq. 1.[1-5]

Least-Squares Method. Let us indicate one set of data points by X_i, Y_i, and Z_i, in which the subscript i varies from 1 to n. If we move all terms of Eq. 1 to the right side and replace X, Y, and Z by X_i, Y_i, and Z_i, we get

$$z_i=(N_aX_i+e_{wc}Y_i)-Z_i. \quad (4)$$

The least-squares method defines the best set of values of N_a and e_{wc} as the set that corresponds to the minimum of sum square of z—i.e., the minimum of

$$\mathrm{SSD}=\sum_{i=1}^{i=n} z_i^2, \quad (5)$$

where SSD is the sum squared deviation.

Hazards in Reducing the Number of Variables From Three to Two Before Applying Least-Squares Method to VBE.

The VBE (Eq. 1) could be written in many different forms; Eqs. 6 through 11 are examples that are mathematically equivalent to Eq. 1. (Eq. 6 is the conventional form.)

$$Z/X=N_a+e_{wc}Y/X. \quad (6)$$

$$Y/X=Z/e_{wc}X-N_a/e_{wc}. \quad (7)$$

$$Z/Y=N_aX/Y+e_{wc}. \quad (8)$$

$$X/Y=Z/N_aY-e_{wc}/N_a. \quad (9)$$

$$X/Z=1/N_a-e_{wc}Y/N_aZ. \quad (10)$$

$$Y/Z=1/e_{wc}-N_aX/e_{wc}Z. \quad (11)$$

However, when the least-squares method is applied to each of these equations, different results will be obtained for each form because of the nature of the errors in the basic data. If there were absolutely no discrepancies between measured and calculated values, the results of all forms would, of course, be identical. With the real field data, the answers obtained from these forms can deviate considerably from the true answers.

Eq. 1 represents a plane in X, Y, and Z dimensions. Eqs. 6 through 11, however, represent a straight line in two dimensions. This paper shows that reducing the problem from three to two dimensions magnifies the errors, reduces the resolving power of the equation, and results in more erratic answers.

Summary of Previous Work on VBE

The work of Schilthuis,[1] Old,[6] Muskat and Woods,[7] Brownscombe and Collins,[8] van Everdingen *et al.*,[9] Carter and Tracy,[10] Tracy,[11] McEwen,[12] Havlena and Odeh,[13] Wall and Craven-Walker,[14] and Dake[15] were carefully studied. Except for Brownscombe and Collins[8] and McEwen,[12] all investigators evidently used the form of Eq. 2; i.e., a two-variable, straight-line equation for the least-squares analysis of the VBE.

Of the papers examined, only Muskat and Woods[7] analyzed the resolving power of the VBE. They used the form of Eq. 2 but wrote the water influx term in the form of

$$e_w=e_{wc}Y+e_2$$

and the VBE as

$$\frac{Z}{X}=N_a+e_{wc}\frac{Y}{X}+e_2\frac{1}{X}. \quad (12)$$

Then they calculated the mean square deviation (MSD) of N_a from its mean N_a to find best values of N_a, e_{wc}, and e_2, corresponding to a minimum MSD. They applied the method to two sets of reservoir data, the Monroe reservoir Reed City pool and the Jones sand of Schuler pool.

Their main conclusion was that the fluctuations in the calculated volumes (of OIP) are very insensitive to the values chosen for the rate of water intrusion and initial gas-cap volume and, hence, cannot be used safely in discriminating between spurious values for these parameters. They concluded that the material-balance method did not provide a satisfactory criterion for determining the basic physical unknowns of a producing reservoir or in making conclusive decisions regarding the production mechanism.[7]

Muskat and Woods provided theoretical interpretation for the low resolving power of the VBE as follows. First, they wrote

$$Z_i=N_aX_i+e_{wc}Y_i+e_2, \quad (13)$$

$$\frac{Z_i}{X_i}=N_a+e_{wc}\frac{Y_i}{X_i}+\frac{e_2}{X_i}, \quad (14)$$

and

$$z_i=N_a-\frac{Z_i}{X_i}+e_{wc}\frac{Y_i}{X_i}+\frac{e_2}{X_i}. \quad (15)$$

Let

$$\frac{Z_i}{X_i}=Q_{ai},$$

$$\frac{Y_i}{X_i}=\gamma_i, \text{ and}$$

$$\frac{1}{X_i}=\alpha_i, \quad (16)$$

to arrive at

$$z_i=N_a-Q_{ai}+e_{wc}\gamma_i+e_2\alpha_i. \quad (17)$$

To obtain an idea of the rate of variation with respect to e_{wc}, which controls the water intrusion term, they set $e_2=0$ and let

$$\overline{\Delta_i^2}=\frac{\Sigma z^2{}_i}{n} \quad (18)$$

and

$$\Delta_{ai}=Q_{ai}-\bar{Q}_a. \quad (19)$$

Then by squaring, expanding, summing, and simplifying Eq. 18, they derived

$$\overline{\Delta_i^2}=\overline{\Delta_{ai}^2}+2e_{wc}(\bar{\gamma}\,\overline{Q_a}-\overline{\gamma Q_a})$$
$$+e_{wc}{}^2(\overline{\gamma^2}-\bar{\gamma}^2)+\bar{z}_2. \quad (20)$$

(The bars indicate mean values.)

Then they discussed the coefficients of e_{wc} and $e_{wc}{}^2$, which are merely the differences between two types of averages of similar terms and will be relatively small. They concluded that the mean square deviation basically would be insensitive to the variations in the magnitude of e_{wc} and that great caution should be used in applying it as a criterion for determining the true value of e_{wc} and, ultimately, the magnitude of the volume of OIP.

Indeed, this interesting interpretation explains why Eq. 2 has low resolving power. Now let us examine, using the same approach, the sensitivity of the mean square deviations of Z, which is the criterion for determining N_a and e_{wc} when the form of Eq. 1 is used.

$$\overline{\Delta_i^2}=\sum z_i^2/n=\frac{1}{n}\sum(Z_i-N_aX_i-e_{wc}Y_i)^2. \quad (21)$$

TABLE 1—HISTORICAL BASIC DATA OF AN IDEALIZED RESERVOIR

Time (year)	Oil Pressure (psi)	Withdrawals (Z) (million bbl)	X	Y
1	2,750	0.0	0.0	0.0
2	2,735	0.3327	0.0033	7.33
3	2,720	0.7160	0.0069	29.31
4	2,690	1.4919	0.0142	73.21
5	2,655	2.4395	0.0229	148.74
6	2,620	3.4432	0.0318	258.30
7	2,585	4.5056	0.0410	401.82
8	2,550	5.6296	0.0505	579.26
9	2,475	7.9949	0.0718	809.94
10	2,420	9.9586	0.0886	1,103.47
11	2,360	12.2452	0.1079	1,452.37
12	2,275	15.6419	0.1377	1,870.92
13	2,225	18.0190	0.1566	2,354.09
14	2,150	21.6216	0.1872	2,896.77
15	2,085	25.1138	0.2161	3,505.90
16	2,000	29.9195	0.2573	4,185.89
17	1,920	34.9611	0.3002	4,943.45
18	1,860	39.2728	0.3351	5,766.33
19	1,810	43.2460	0.3661	6,640.07
20	1,770	46.7720	0.3922	7,555.06

By expansion and summation of Eq. 21, it is readily found that

$$\overline{\Delta_i^2}=\overline{(Z-N_aX)^2}-2e_{wc}\overline{(Z-N_aX)Y}+e_{wc}{}^2\,\overline{Y^2}. \qquad (22)$$

The coefficients of e_{wc} and $e_{wc}{}^2$ are $-2\overline{(Z-N_aX)Y}$ and $\overline{Y^2}$, which are not normally small; thus, $\overline{\Delta_i^2}$ is sensitive to variation in e_{wc}.

If we consider that $(Z-N_aX)$ is very close to $e_{wc}Y$, the sum of the last two terms is then approximated by $e_{wc}{}^2\overline{Y^2}$, which again indicates that the mean square deviation is sensitive to e_{wc}. This sensitivity toward e_{wc} becomes low only when Y is very small (i.e., when the water drive is very weak) and this is expected. Therefore, the resolving power of Eq. 1 is much greater than the resolving power of the conventional form, Eq. 2.

Method of Investigation

Idealized Reservoir. First an idealized reservoir very similar to the Douglas field in the U.S.[16] is constructed. The original OIP of this reservoir is 100 million res bbl [16 million res m^3], and its water influx constant is 1,000 RB/psi-yr [2306 res m^3/kPa·a].

Then a pressure/production history is constructed so that the VBE is satisfied thoroughly for all years of the history.

TABLE 2—CALCULATED OIP AFTER INTRODUCING RANDOM ERRORS IN PRESSURE AND PRODUCTION DATA

(True OIP = 100)

Run No.	Eq. 1	MSSDP*	Eq. 6**	Eq. 7	Eq. 8	Eq. 9	Eq. 10	Eq. 11
1	101.72	101.64	93.77	88.59	88.37	88.94	93.03	87.15
2	102.90	103.22	111.13	90.74	109.99	110.65	111.01	94.29
3	100.81	100.73	95.87	83.89	83.95	86.22	94.48	83.03
4	95.96	96.04	97.19	85.69	83.78	86.10	95.85	83.41
5	101.58	101.66	136.50	211.69	175.33	185.13	131.34	244.77
6	99.86	100.18	97.99	95.87	97.10	97.34	97.76	95.14
7	95.56	95.88	95.24	91.90	94.34	94.83	94.90	90.64
8	96.33	96.33	95.04	72.24	75.65	80.29	92.34	71.98
9	102.26	102.34	94.83	89.04	86.85	87.27	94.02	88.18
10	94.37	94.69	88.61	79.91	73.05	75.59	86.48	75.34
11	103.51	103.83	113.09	96.77	119.02	119.37	113.18	97.59
12	100.19	100.59	136.83	336.19	184.93	222.00	123.04	−167.89
13	103.46	103.54	127.75	167.11	145.40	147.13	126.62	183.71
14	100.92	101.00	101.36	99.87	101.89	102.00	101.32	99.62
15	104.45	104.37	117.50	240.24	132.49	133.89	117.01	−679.23
16	102.82	102.90	104.28	94.00	98.51	99.52	103.80	94.61
17	98.38	98.30	85.13	77.47	68.02	70.40	82.49	71.85
18	100.40	100.48	114.02	26.74	122.29	122.98	114.13	56.89
19	95.58	95.50	89.49	85.99	79.46	80.45	88.38	83.54
20	98.72	98.72	93.84	98.63	100.74	102.47	93.36	85.70
Maximum	104.5	104.4	136.8	336.1	184.9	222.0	131.3	244.8
95% upper confidence level	101.4	101.5	117.7	148.4	121.2	127.5	109.3	134.0
Mean	100.0	100.1	104.5	115.1	106.1	109.6	102.7	47.0
95% lower confidence level	98.6	98.7	97.3	81.8	90.9	91.8	96.1	−40.0
Minimum	94.4	94.7	85.1	26.7	68.0	70.4	82.5	−679.2
Range	10.1	9.7	51.7	309.4	116.9	151.6	3.8	924.0
Standard deviation	3.1	3.1	15.4	71.2	32.3	38.1	14.1	185.9

*Minimized sum square deviation of calculated pressures from the measured pressures.
**This is the form of VBE conventionally used (also Eq. 2).

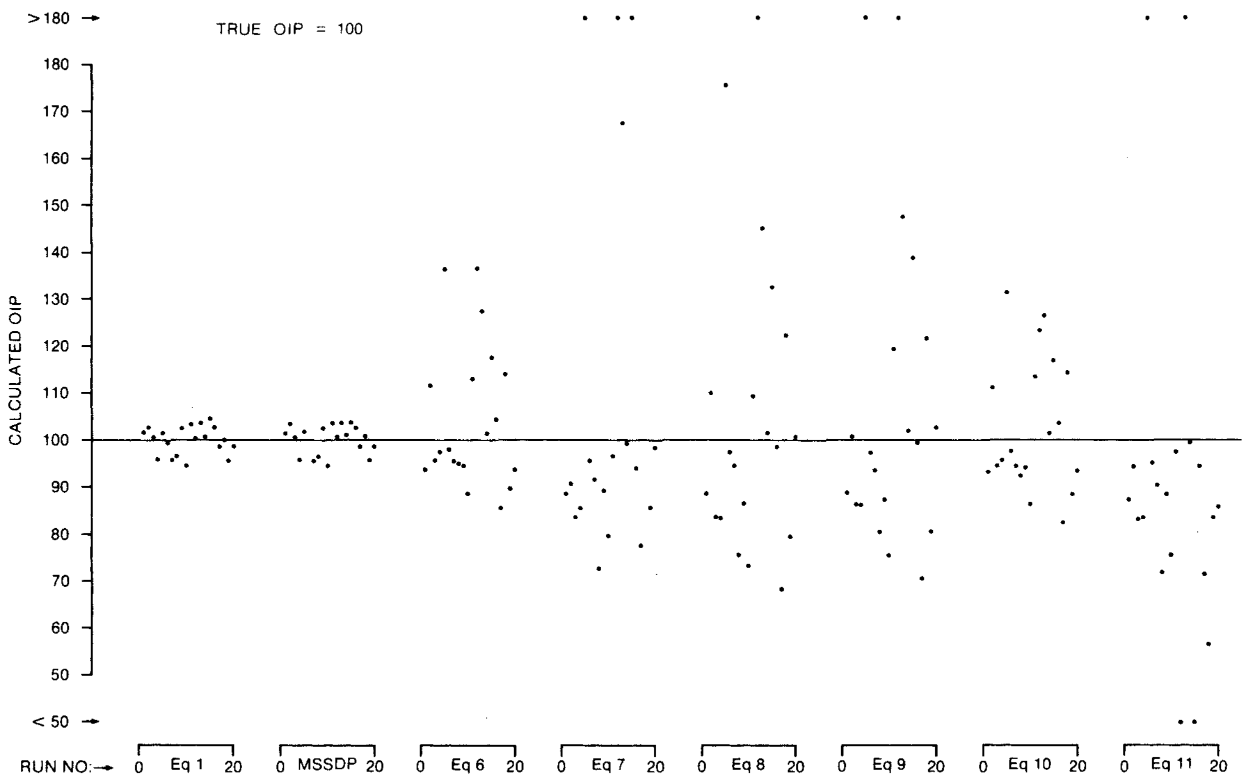

Fig. 1—Calculated OIP values using different forms of VBE.

That is, the production (Z) was back-calculated from Eq. 1, a fixed relationship between X and p_o, and a fixed pressure history. The results are presented in Table 1. There are no errors in the pressure, production, and fluid data of this hypothetical model. Therefore, if any pair of data points is used in Eq. 1, the values of $N_a = 100$ and $e_{wc} = 1{,}000$ will be calculated. Any form of VBE would also give the same answers.

Simulation Model. A mathematical simulation model is then developed, (1) to introduce reasonable errors in a completely random manner and to simulate an actual reservoir history, and (2) to calculate the original OIP and the water influx constant, using eight different forms of the VBE and printing out the results for comparison.

Answers tabulated in Table 2 and plotted on Fig. 1 illustrate that reduction of VBE from three to two dimensions (straight-line) produces a wide scatter in answers, depending on the data errors.

Weighting the Data Points. Suppose that one wishes to assign different weights to different data points. Let the weight for Data Point i be represented by W_i. Then application of the least-squares technique means minimizing the weighted sum of the squared deviations; i.e., Eq. 5 becomes

$$\mathrm{SSD} = \sum_{i=1}^{n} W_i z_i^2. \quad \ldots\ldots (23)$$

We can investigate the effect of multiplying both sides of Eq. 1 by a variable, U:

$$UZ = N_a UX + e_{wc} UY. \quad \ldots\ldots (24)$$

Assuming that for every data point (X_i, Y_i, Z_i) there is one value of U_i and carrying out least-squares analysis without specifying any weight, we derive

$$\mathrm{SSD} = \sum [(N_a U_i X_i + e_{wc} U_i Y_i) - U_i Z_i]^2, \quad \ldots\ldots (25)$$

or factorizing U_i^2,

$$\mathrm{SSD} = \sum U_i^2 [(N_a X_i + e_{wc} Y_i) - Z_i]^2 = \sum U_i^2 z_i^2. \quad \ldots\ldots (26)$$

Comparison of Eqs. 23 and 26 reveals that multiplying both sides of Eq. 1 by the variable U before carrying the least square is equivalent to giving a weight factor of $W_i = U_i^2$ to each data point i.

Now, suppose that we divide both sides of Eq. 1 by X before carrying out the least square and write it in this form:

$$\frac{Z}{X} = N_a + e_{wc}\left(\frac{Y}{X}\right), \quad \ldots\ldots (2)$$

as is done conventionally. Based on the foregoing reasoning, this is equivalent to giving a weight factor of $(1/X_i)^2$ to each data point i.

Is it justified to give a weight of $(1/X_i)^2$ to each data point i? X is the unit expansion. Its value grows from zero in the first year to some value, normally less than one, later on. In our example (Table 1) it grows to 0.3922 after 20 years. Since division by zero is not allowed, the first data point has to be eliminated. The values of X in early years are very small. In our example X is 0.0033 in the second year. The value of $(1/X)^2$ for the second year is, therefore, about 90,000; for the last year it is only about 6.3. This means giving a weight factor of 90,000 to the second data point and decreasing the weight factor gradually to 6.3 for the last data point.

Why should we give higher weight to a particular data point? Usually we do this only when that data point is more reliable than the remaining points, but are the earlier data more reliable than the later ones? The answer is generally no. The earlier data are generally less reliable for several reasons, the three most important of which are as follows.

1. The expansion term, X, and influx term, Y, are both functions of the pressure drop. A small error in pressure in the second year (for example, 5 psi [35 kPa] in Table 1) is equivalent to a large percentage (30%) of the total pressure drop (only 15 psi [103 kPa]), whereas the same error in the last pressure would mean a small value (5 psi [35 kPa]) over a large (980 psi [6757 kPa]) pressure drop, which is a small percentage. Therefore, the expansion and influx data of earlier years are less reliable.

2. In calculation of the values of Y for water influx, the pressure drop of each year has an influence on all the following years because of the application of Duhamel's theorem (principle of superposition). For example, the pressure drop of the first year has implicit influence on the water influx of all years, whereas the pressure drop of the last year has only some influence in the last year's water influx. Also, the production data are cumulative; i.e., the last year's withdrawal is only a part of the total withdrawals. Therefore, if any explicit weighting is to be given, it should be the least weight for the first data point and the highest weight for the last one.

3. The technology of measurements improves with time. Therefore, the pressure and production data of earlier times generally are less accurate than those of later times.

Looking at Eqs. 6 through 11 in relation to the previous three points, we see that all the forms are equivalent to implementation of high weight factors for earlier data.

Physical vs. Mathematical Relationships. When a physical relationship between several variable properties of a natural system is written, care must be taken that the effect of all individual significant variables be included independently. If we divide both sides of Eq. 1 by variable X, we are, in effect, reducing the number of variables from three to two, (Z/X) and (Y/X), which are related to each other not only by VBE but also by variable X.[16] Although Eqs. 1 and 6 are mathematically equivalent, they are not physically and statistically equivalent.

Division of variables before any regression or functional analysis can be hazardous.[17] Suppose we choose three sets of completely random numbers and assign them to three variables X, Y, and Z so that there is absolutely no correlation between these variables. If you have n random numbers in each set (i.e., X_i, Y_i, Z_i, with $i=1$ to n) and if you divide all values of Z_i by the corresponding values of X_i and all values of Y_i by the corresponding values of X_i and then plot $(Z/X)_i$ vs. $(Y/X)_i$, we will observe a considerable correlation. When X is very small, both (Z/X) and (Y/X) are large; when X is very large, both (Z/X) and (Y/X) are small. Thus, a superficial and spurious correlation can be created between the two new variables.[17] Now if a true correlation between variables X, Y, and Z existed, it could be masked and completely distorted by a spurious correlation created because of division of two of these variables by the third one.

The simulation model developed in this paper is designed to introduce random errors into the data of an idealized reservoir and to calculate the OIP and the water influx constant by application of the eight different forms mentioned earlier. This exercise demonstrates very clearly that Eq. 1 and minimized sum square deviations of pressures represent the basic physical relationship much more closely than any of the other six forms of the VBE.

Results of the Simulation Model Analysis. To check the data of the idealized reservoir, least-squares calculations were carried out using all eight cases mentioned previously. The answers to all eight cases were perfect (i.e., 100 for N_a and 1,000 for e_{wc}). One can conclude that, when the errors in the input data are really very small, it does not matter which form is used for OIP and influx calculations. However, this is generally not the case because errors in average reservoir pressure of up to ± 5 psi [35 kPa] are not uncommon, and in many instances the pressure errors are even greater. Moreover, there are some errors in production and PVT data that are not negligible.

To examine the effect of errors, random errors within ± 5 psi [35 kPa] are imposed on the idealized pressures and within ± 1% on the annual withdrawals. Least-squares calculations were then carried out for all eight forms of the VBE.

The results of a single run cannot be very conclusive. Results depend on what the set of random numbers happens to be. It is necessary, therefore, to make numerous runs and to analyze the results statistically.

Twenty sets of random numbers were taken from the first 20 columns of the table of random numbers in Abramowitz and Stegun's handbook.[18] Using these, 20 sets of runs were made.

To examine the cases more carefully, an OMNITAB[19] computer program was used for statistical analysis. The results are summarized at the bottom of Table 2. By looking at the mean values, one can see that the answers to Eq. 1 and Case MSSDP* are very close to the true OIP, whereas other forms of VBE produce results considerably different from the true OIP of 100. (Calculations for Case MSSDP are very laborious because the method used is iterative.)

The water-influx constants exhibit exactly the same conclusions. The only difference is that, when the OIP value is too high, the influx constant is too low and vice versa. To visualize the spread of data, all the calculated OIP values from the previously mentioned 20 runs were plotted in Fig. 1. Each point represents one value of OIP

TABLE 3—STANDARD DEVIATION OF CALCULATED RESERVOIR OIL PRESSURES FROM TRUE RESERVOIR OIL PRESSURES AND STANDARD DEVIATION OF CALCULATED PRESSURES FROM THE OBSERVED PRESSURES, AVERAGE FOR 20 RUNS*

Equation	Equation	SDT**	SDO†	Remarks
1	$Z = N_a X + e_{wc} Y$	1.90	3.59	good
—	MSSDP	1.86	3.57	good but laborious
6	$Z/X = N_a + e_{wc}(Y/X)$	8.49	9.32	conventional but poor
7	$Y/X = (1/e_{wc})(Z/X) - N_a/e_{wc}$	36.46	36.60	unsuitable
8	$Z/Y = N_a(X/Y) + e_{wc}$	37.27	37.72	unsuitable
9	$X/Y = (1/N_a)(Z/Y) - e_{wc}/N_a$	41.24	41.74	unsuitable
10	$X/Z = 1/N_a - (e_{wc}/N_a)(Y/Z)$	8.48	9.20	poor
11	$Y/Z = 1/e_{wc} - (N_a/e_{wc})(X/Z)$	37.61	37.70	unsuitable

*Actual errors in reservoir pressure were random errors between −5 and +5 psi.
**Standard deviation of calculated reservoir oil pressures from true reservoir oil pressures.
†Standard deviation of calculated pressures from the observed pressures.

calculated for one set of data. All points with OIP less than 50 or greater than 180 are shown on the extreme lines.

The evident conclusions are that Eqs. 6 through 11 have low resolving power and that Eq. 1 is superior.

Pressure Matches

Once the OIP and influx constant are known, the reservoir oil pressure can be calculated for any value of reservoir withdrawals. To compare pressure matches of results obtained from Eqs. 1 and 2, the historical pressures were calculated for all 20 runs in which random errors were present. The standard deviation between the calculated pressures and the true pressures (SDT) and the standard deviations between the calculated pressures and the observed reservoir pressures (SDO) were calculated for all 20 runs and for each form of VBE. The averages of all SDT's and of all SDO's for 20 runs are presented in Table 3. The runs that result in better values of OIP also result in better pressure match.

One typical run was chosen to compare the actual pressure matches. The deviations between the true pressures and calculated and observed pressures are plotted in Fig. 2. The following important conclusions can be drawn from Fig. 2.

1. Eq. 1 produces a better pressure match than Eq. 2 (conventional).

2. The pressures calculated by Eq. 1 are closer to the true pressures than the observed pressures, although the observations are the original basis for calculations.

3. The deviations of observed pressure from calculated pressures are very large in Year 2. However, as can be seen in Fig. 2, the observed pressure is very close to the true pressure. Therefore, the discarding or adjusting of observed values simply because they do not match closely with the calculated values should be avoided. For example, discarding the observed values of Years 2 and 5 would improve the match between the calculated and observed pressures. However, the calculated pressures obtained in this manner would be further away from the true values.

4. Because Eq. 2 (conventional method) gives greater weight to earlier data points and much smaller weight to later data points (see the section on Weighting the Data Points), its pressure match is good for early years, but very poor for later years of history.

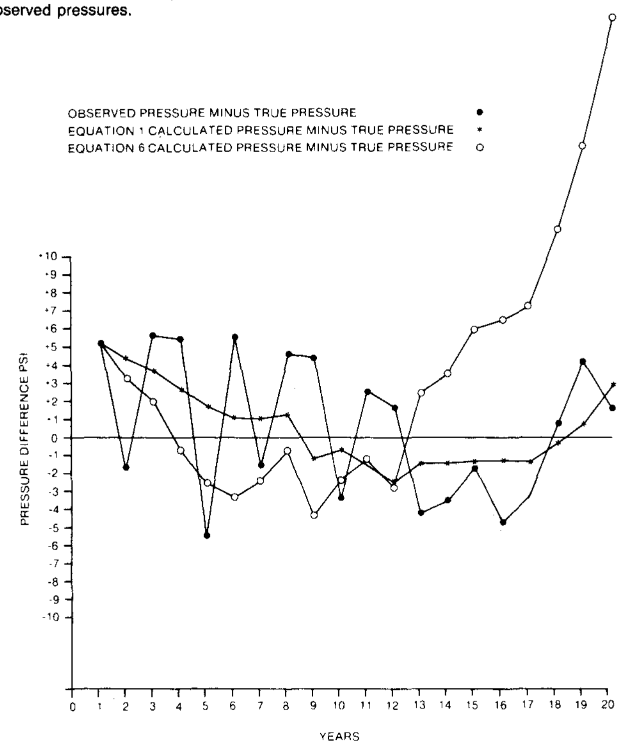

Fig. 2—Comparison of pressure matches.

Conclusions and Recommendations

1. The conventional form of VBE as a straight line (Eq. 2),

$$\frac{Z}{X} = N_a + e_{wc}\frac{Y}{X},$$

has low resolving power and should not be used for calculation of OIP (N_a) and influx constant (e_{wc}).

2. The best form recommended by this paper is Eq. 1.

3. Minimizing the sum of the squared deviations of the calculated oil pressures from the observed pressures is equivalent to least-squares calculations with the form of Eq. 1.

4. Other forms of VBE examined in this paper (Eqs. 7 through 11) have low resolving power and are not recommended.

5. Standard deviation between the true reservoir oil pressures and their calculated values is normally less than the standard deviation between the observed reservoir oil pressures and their calculated values when Eq. 1 is used.

6. Before least-squares analysis is carried out on any physical relationship, the independent variables should be maintained preferentially as they appear in the original equation. The number of variables should not be reduced indiscriminately by combining or eliminating any of them.

7. The discarding or adjusting of observed values simply because they do not match closely with the calculated values should be avoided. Experiments with the simulation model presented in this paper have shown that observed values that deviate significantly from their corresponding calculated values sometimes are more valuable to the final objective of the problem than those that produce smaller deviations. However, if there is a positive reason for a data point to be spurious other than its large deviation from its calculated value, it may be discarded.

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Nomenclature

B_g = gas formation volume factor, RB/scf [res m^3/std m^3]
B_{gi} = initial gas formation volume factor, RB/scf [res m^3/std m^3]
B_o = oil formation volume factor, RB/STB [res m^3/stock-tank m^3]
B_{oi} = initial oil formation volume factor, RB/scf [res m^3/stock-tank m^3]
B_t = total (two-phase) formation volume factor, RB/STB [res m^3/stock-tank m^3]
B_{ti} = initial value of B_t, RB/STB [res m^3/stock-tank m^3]
c_f = effective aquifer fluid compressibility, psi^{-1} [kPa^{-1}]
c_w = connate water compressibility, psi^{-1} [kPa^{-1}]
e_w = water influx, res bbl [res m^3]
e_{wc} = water influx constant, RB/psi-yr [res m^3/kPa·a]
e_2 = second influx constant, res bbl [res m^3]
m = ratio of initial gas cap to original OIP, dimensionless
n = number of data points
N = original OIP, STB [stock-tank m^3]
N_a = active original OIP, res bbl [res m^3]
N_i = initial estimate of N by geological and petrophysical data, STB [stock-tank m^3]
N_p = produced oil, STB [stock-tank m^3]
p_o = average reservoir oil pressure, psig [kPa]
p_{oi} = initial value of p_o, psig [kPa]
p_w = average reservoir pressure at original water/oil level, psig [kPa]
p_{wi} = initial value of p_w, psig [kPa]
q_g = gas production, scf [m^3]
R = producing GOR, scf/STB [std m^3/stock-tank m^3]
R_{si} = initial solution GOR, scf/STB [std m^3/stock-tank m^3]
S_w = connate water saturation
t = time, years
U = any variable
W_p = produced water, res bbl [res m^3]
X = unit expansion; expansion of fluids and rock when pressure drops from p_{oi} to p_o, corresponding to 1 res bbl of original OIP, dimensionless
Y = influx function (in Schilthuis equation), yr×psi×10^{-6} [1×kPa×10^{-6}]
z = deviation of a calculated value from the observed value
Z = reservoir withdrawals, res bbl [res m^3]

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SI Metric Conversion Factor

psi × 6.894 757 E+00 = kPa

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