CRITERIA FOR DISPLACEMENT BY GAS VERSUS WATER IN OIL RESERVOIRS

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SUMMARY

Numerical models were used to study conditions in oil reservoirs under which gravity drainage (up-dip gas injection) is more efficient than down-dip water injection. A wide range of field operating conditions was studied including dip angles between zero and ninety degrees and different reservoir dimensions. The conditions did not include different capillary pressure and relative permeability functions or variations in well spacing.

Results are presented as correlations between oil recovery and dimensionless rate and time. For the range of conditions investigated, these correlations may be used to estimate gravity drainage performance or compare recovery by gravity drainage with recovery by water injection. The correlations extend Richardson and Blackwell's correlation to different reservoir geometries and give Dykstra's correlation an interpretation of dimensionless time.

Recovery by gravity drainage is found to vary directly with reservoir dimensions, dip, vertical permeability, and fluid density difference; and inversely with the total flow rate and oil viscosity. When the dimensionless rate is less than the dimensionless critical rate, recovery by gravity drainage is found to be greater in most cases than recovery by water injection.

Field examples are presented to illustrate the use of the correlations. The examples indicate recovery is extremely sensitive to the value of residual oil saturation.

INTRODUCTION

Gravity drainage performance, a subject which has long been of concern to reservoir engineers, has received considerable attention in the petroleum literature. Muskat[1] discusses different aspects of the problem and presents field examples where gravity drainage is an important oil producing mechanism. Additionally, several case histories highlight its importance.[2-4] Predicting gravity drainage performance as well as other problems dealing with gravity effects have been studied by a number of authors using a variety of methods.[5-14] These problems usually arise in evaluating alternative recovery methods.

Of particular interest, Cardwell and Parsons[15] developed an approximate theory on gravity drainage in 1949, but only recently has evidence appeared on its application. In 1970, Richardson and Blackwell[16] presented a simplified gravity drainage model which combined parts of the Buckley-Leverett[5] (as extended by Welge[17]), Dietz[7] and Cardwell-Parsons theories. They obtained excellent agreement between predicted recoveries and field and numerical model results. More recently (1978) Dykstra[18] expanded the Cardwell-Parsons theory by accounting for residual oil saturation and including a recovery equation. He, too, obtained excellent agreement between calculated and observed gravity drainage performance in a number of field studies.

Since mobility ratios are unfavorable in gas-oil displacements, it is known that gravity drainage is the dominant displacement mechanism when gas is injected up-dip at rates below the critical rate

(rate below which a stable interface will develop). Such a process is very efficient resulting in a high oil recovery. Richardson and Blackwell[16] correlated recovery calculated by their simplified gravity drainage model with a dimensionless group for a particular reservoir. Dykstra[17] also correlated recovery with a dimensionless group. In water-oil displacements, there are two cases. If the mobility ratio is unfavorable and the displacement is carried out at rates less than the critical rate, gravity effects counterbalance viscous effects and promote a gravity stable displacement which increases oil recovery. If the mobility ratio is favorable, gravity effects which tend to decrease recovery can be diminished by increasing the rate.

The theories which explain the phenomena under study are well developed and generally well-known techniques can be applied in study of a particular reservoir. The methods recently presented by Richardson and Blackwell[16] and by Dykstra[17] are relatively straightforward and inexpensive to apply. More detailed analyses can be made using generally available two-dimensional numerical models which account for gravity effects. There is a need, however, for approximate methods such as correlations over a range of conditions and reservoir geometries.

The present work is a simulation study using numerical models of a vertical section to determine conditions under which gravity drainage, i.e., displacement by low rate up-dip gas injection, is more efficient than displacement by down-dip water injection. The problem is studied by varying the production rate, dip, oil viscosity, permeability, density difference and reservoir dimensions while observing displacement efficiency. Well placement was optimum for the process being studied, i.e., along the top of the section for water injection and along the bottom for gas injection. The objective of the study is to present information that may be used as a first approximation by the engineer faced with the problem of screening a number of reservoirs for those where gravity drainage might be important or of evaluating alternative recovery methods. If these approximate methods appear promising, he should then be able to use detailed methods more effectively. Subsequent sections of this paper discuss procedures used in the study, results, application of the results, and conclusions.

PROCEDURE

Two-dimensional two- and three-phase numerical models were used in the study.* Both models accounted for gravity and capillary effects. Detailed descriptions of the two- and three-phase models are given by Morse and Whiting[19] and Strickland,[20] respectively.

A 5x9 computing grid was used for all simulations. Up-dip gas injection was simulated by injection at a constant rate in cell(1,1) while producing at a constant pressure from cell(5,9). Figure 1 is a diagram of this procedure. Down-dip water injection was simulated by a reverse procedure.

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*A two-phase model would have been sufficient, two models were used to permit broadening the work to include effects of producing near an oil-water contact while injecting gas up-dip. This effort was not undertaken in the study.

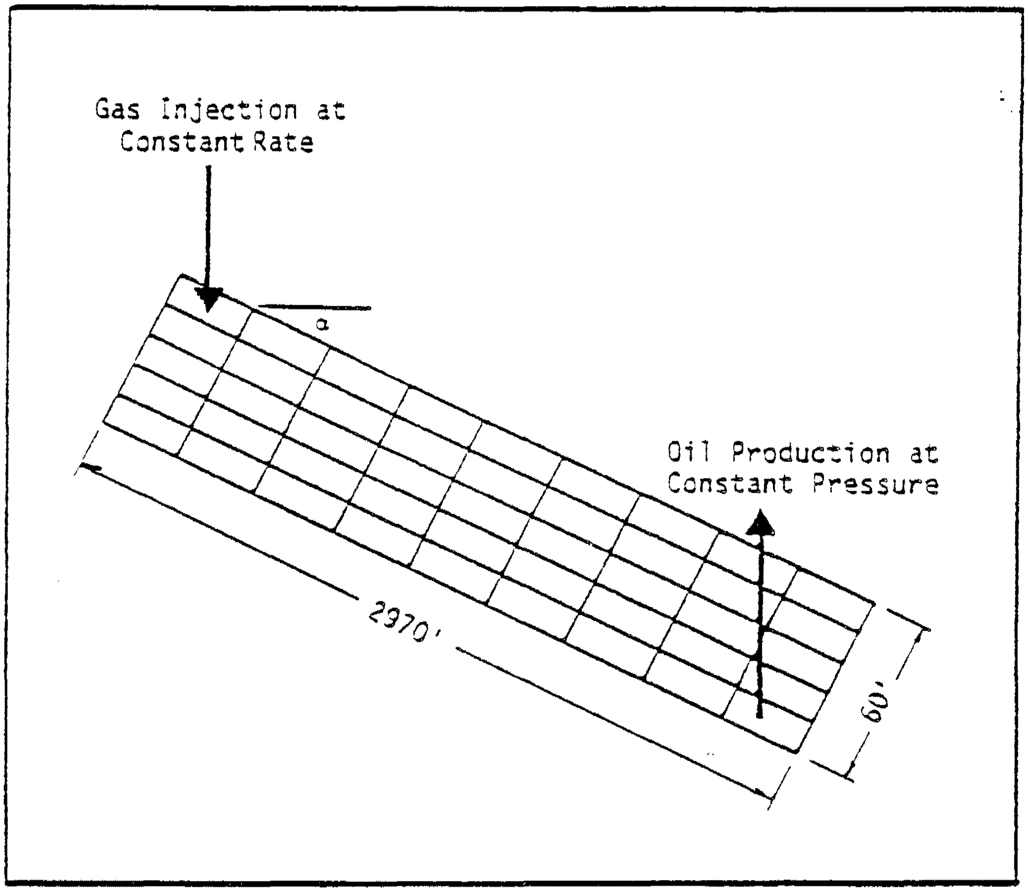

Figure 1. Diagram of Model for Base
Case Gas Injection Runs

Model parameters that were held constant are shown in Table 1. The time-step size was controlled by a six percent limit on the saturation change in any cell. Material balance errors, defined as a percentage of the fluids in place (original plus injected volumes), were less than 0.1 percent. As indicated in Table 1, fluid compressibility and gas solubility effects were not simulated. Capillary pressure and relative permeability saturation functions used are presented at Figures 2-4.

Model parameters which were varied are shown in Table 2. As indicated, these parameters were varied over a representative range of reservoir conditions. Base case runs for up-dip gas injection and down-dip water injection were made with constant reservoir dimensions and rock properties to establish the effect of varying dip angles, injection rates, and oil viscosity. Two sets of additional runs were then made. First, the density difference and permeability were varied. The height to length ratio was then varied by three methods--by keeping the height-length product constant, by doubling the height while keeping the length constant, and vice versa. In all cases, the 5x9 grid break-up was retained. While certain of these variations were combined to insure applicability over the range of conditions investigated, most were made for up-dip gas injection runs only.

The critical rate as defined by Terwilliger, et al[6] was used in the study. This definition can be written in practical units as

$$q_c = 1.1271 \frac{A\, k_o\, \Delta\rho\, \sin\alpha}{\mu_o}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Table 1. Model Parameters Held Constant

Computation Parameters	
Grid	5x9
Max Saturation Change per time step	6 percent
Convergence criterion	0.001
Fluid Properties	
Original Pressure	2000 psi
Oil Formation Volume Factor	1.25 RB/STB
Gas Volume Factor	1.25 RB/MSCF
Water Volume Factor	1.0 RB/STB
Gas in Solution	0.0
Gas Viscosity	0.02 cp
Water Viscosity	1.0 cp
Gas Density	0.038 psi/vertical ft.
Water Density	0.468 psi/vertical ft.
Rock Properties	
Porosity	20 percent
Irreducible Water Saturation	20 percent
Residual Oil Saturation	
to Gas	0.0
to Water	32 percent
Model Dimensions	
Width	660 ft.

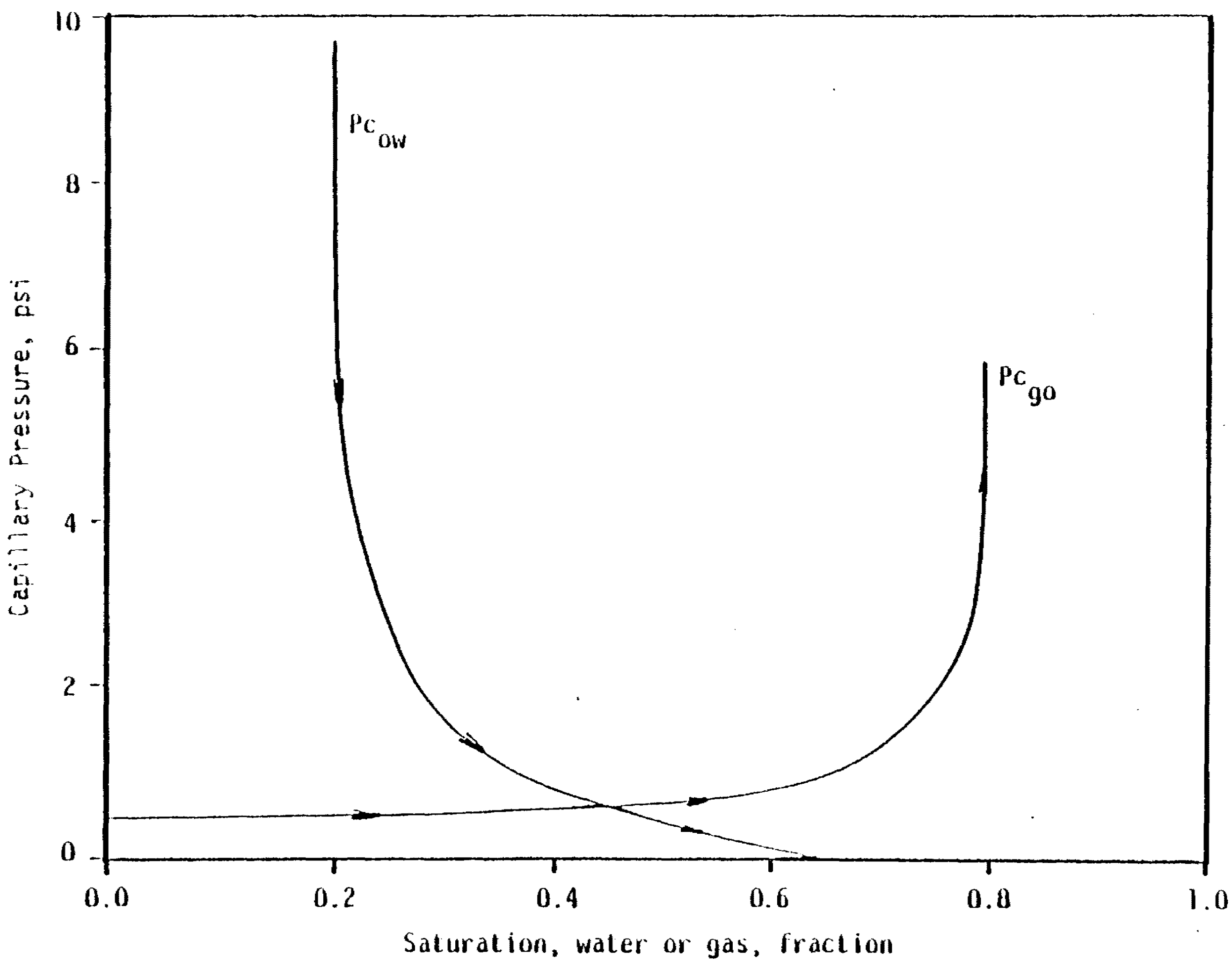

Figure 2. Capillary Pressure Functions

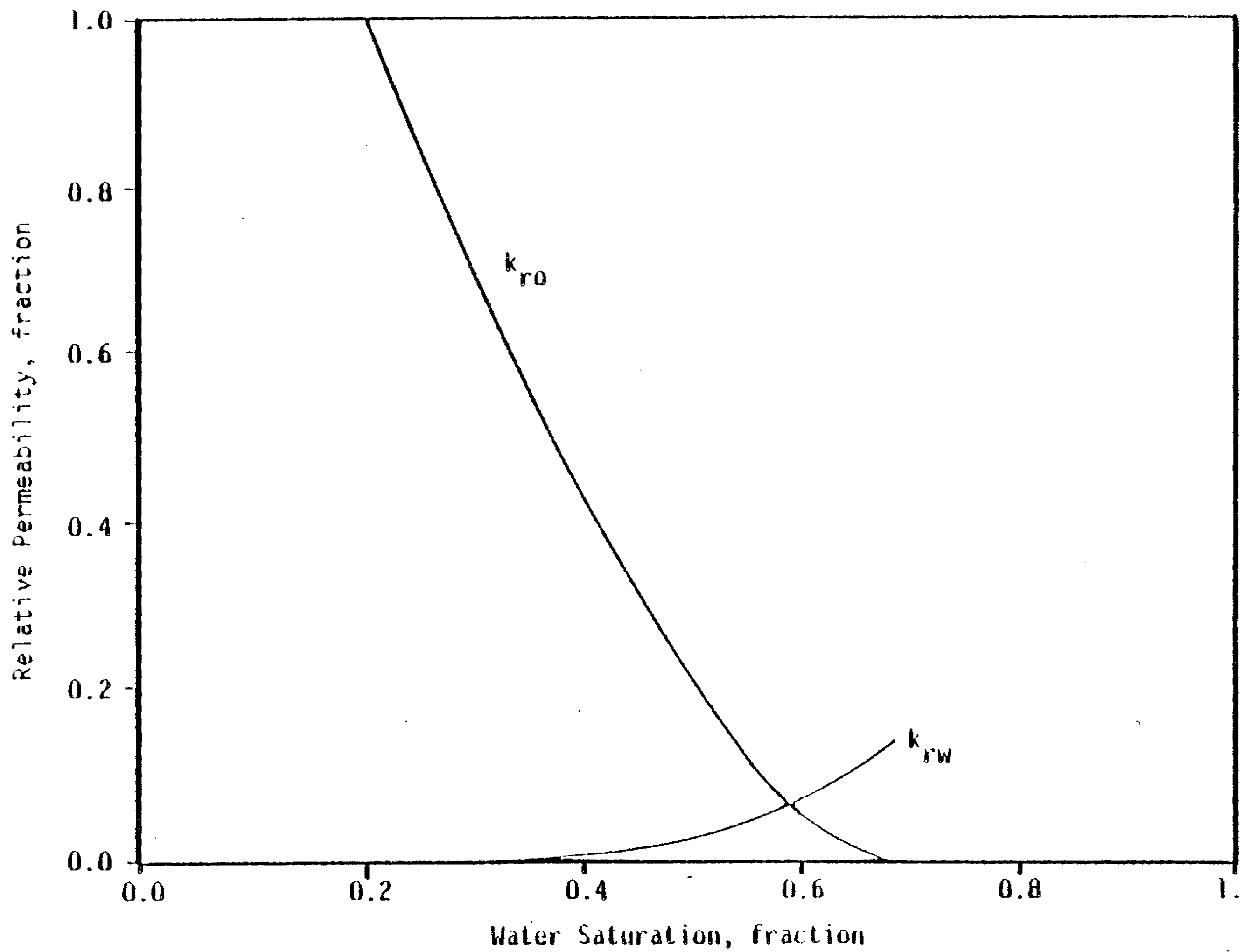

Figure 3. Oil-Water Relative Permeability

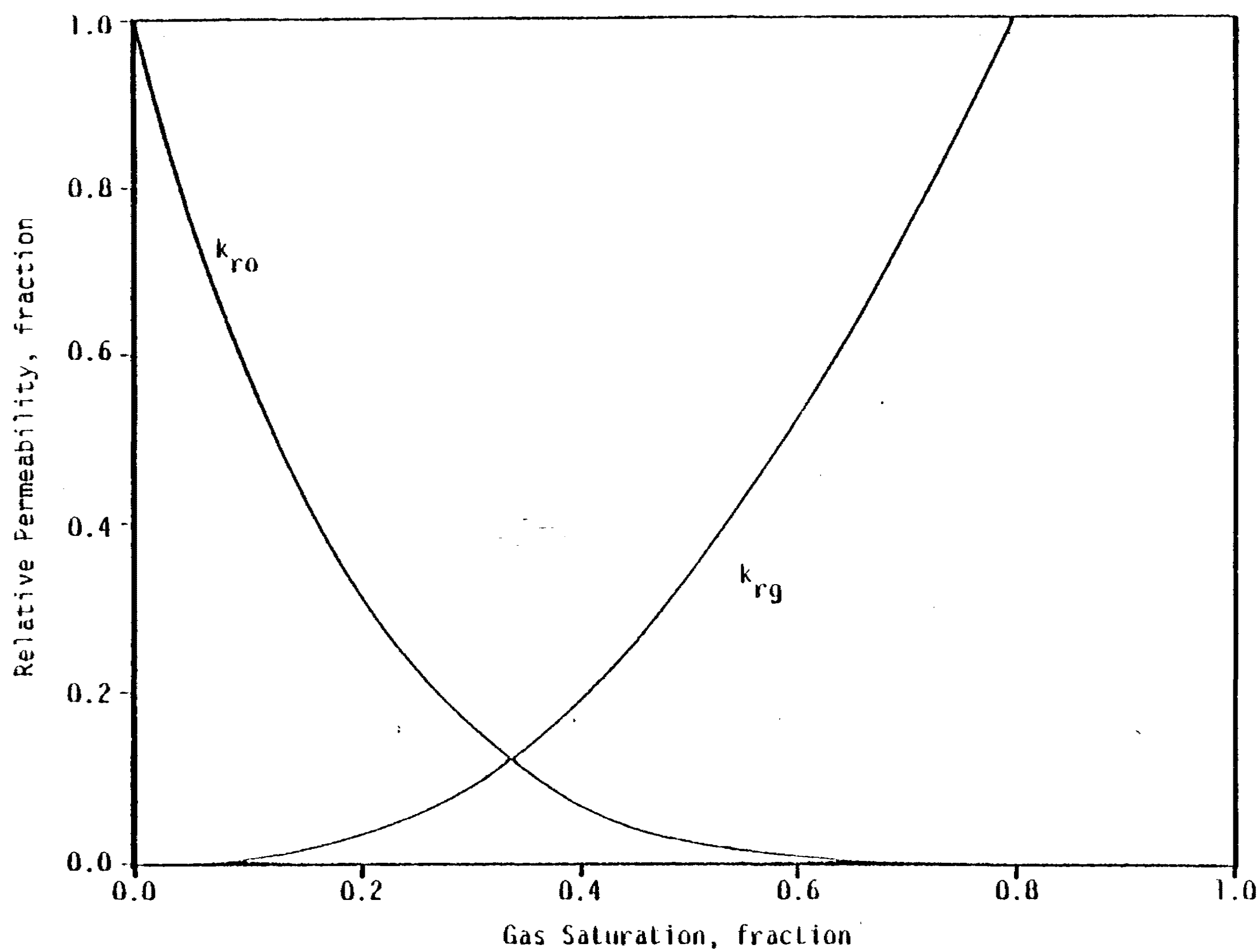

Figure 4. Gas-Oil Relative Permeability

Table 2. Model Parameters Varied

	Base Case Runs	Other Runs
Operational		
Injection Rates	1-500 percent of critical rate at 90° (7)*	2.5-700 percent of critical rate at 90° (11)
Fluid Properties		
Oil Viscosity	1, 10, 100 cp	1, 100 cp
Oil Density	0.368 psi/ft.	.338-.458 psi/ft. (4)
Density Differences		
oil-water	0.1 psi/ft.	0.01, 0.1 psi/ft.
gas-oil	0.33 psi/ft.	0.3, 0.33 psi/ft.
Rock Properties		
Permeability	0.25 darcy	0.025, 0.25 darcy
Vertical to Horizontal Permeability Ratio	1.0	0.5, 1.0
Reservoir Dimensions		
Dip	0°-90° (7)	5°-85° (4)
Length	2970 ft.	422-5940 ft. (6)
Height	60 ft.	60-422 ft. (6)
Height to Length Ratio	0.02	0.01-1.0 (6)

*The number of values included in the range is shown in parentheses.

where

A = area of cross section normal to bedding plane, sq. ft.

k_o = effective permeability to oil, darcy

$\Delta\rho$ = density difference, psi/ft.

α = dip angle

μ_o = oil viscosity, cp.

Alternatively, Dietz's[7] definition, given by

$$q_c' = 1.1271 \frac{A \ \Delta\rho \ \sin \alpha}{(\frac{\mu_o}{k_o} - \frac{\mu_d}{k_d})} = 1.1271 \frac{A \ k_o \ \Delta\rho \ \sin \alpha}{\mu_o(1 - \frac{1}{M})},$$

where M is the mobility ratio, might have been used. The former was selected since the principle conditions under study involve unfavorable mobility ratios and it is the limiting form of the latter.

Oil recovery as a percentage of the oil in place at various values of the displacing fluid-to-oil ratio (DFOR)* was used as the measure of displacement efficiency. This procedure was used for two reasons. First, economic limits may be established based on DFOR. Recovery at various DFOR should be useful in this regard. Secondly, numerical dispersion affects the length of the stabilized zone, making breakthrough determination almost impossible using a uniform grid such as used in this study. Coats[21] studied computational aspects of the effect of gravity on saturation dispersion in a flat vertical cross section. Although his main emphasis was the real saturation

*Units used in this study are MSCF/STB for gas and STB/STB for water.

dispersion (i.e., underrunning or overriding) caused by gravity forces, he showed that use of coarse grids such as used in this study introduce numerical dispersion resulting in an elongated front. While the effect of using a finer grid was not investigated, the effect of taking recovery at various OFOR is clarified in the next section which discusses results.

RESULTS

A review of the scaling laws[22,23] for modeling petroleum reservoirs suggested recovery should correlate with some form of the ratio of $q\mu_o$ to $k\Delta\rho$. Indeed, percentage of the critical rate is such a ratio. Richardson and Blackwell[16] correlated recovery with a dimensionless parameter which can be written in practical units as

$$q_D^* = 0.8872 \frac{q_T\, \mu_o (h/\ell)}{A\, k_v\, \Delta\rho\, \cos\alpha}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where

q_D = dimensionless rate

q_T = total flow rate through area A, RBPD

h/ℓ = height to length ratio, dimensionless

k_v = permeability in the vertical direction, darcy.

Richardson and Blackwell[16] derived this parameter by dividing the time required for vertical drainage by the time required for flow along the bedding plane. It is similar to the ratio of two of the dimensionless groups (Rd/Ra) derived by Craig, et al[13] using inspectional analysis.

Recoveries obtained from the simulations of this study could be correlated with q_D^* only if the systems under study had the same dimensions. Systems having different dimensions could be correlated using

$$q_D = 0.8872 \frac{q_T \mu_o (h/\ell)^{1/2}}{A k_v \Delta\rho \cos \alpha}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where $0 \leq \alpha < 90°$. At $\alpha = 90°$, $\frac{h/\ell}{\cos \alpha}$ was defined as 1.0.

For completeness, two other dimensionless groups used in the discussion which follows are presented now. Combining Eqs. 1 and 3, an expression for dimensionless critical rate as a function of dip is obtained, for $0 \leq \alpha < 90°$,

$$q_{DC} = (h/\ell)^{1/2} \tan \alpha. \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

After Dykstra[18] and making use of the Richardson and Blackwell[16] expression for the time required for vertical drainage and the "correction" found for different geometries, a useful expression for dimensionless time is obtained, for $0 \leq \alpha < 90°$,

$$t_D = 411.7 \frac{k_v \Delta\rho \cos \alpha (h/\ell)^{1/2} t}{h \mu_o}, \quad \ldots\ldots\ldots\ldots\ldots (5)$$

where t is the time in years.

Figure 5 presents the main result of the study. Oil recovery by gravity drainage (up-dip gas injection at rates less than the critical rate) is compared with recovery by down-dip water injection at DFOR between 5 and 50. The left half of Figure 5 shows recovery correlated with dimensionless rate (Eq. 3) for dip angles between 5 and 85 degrees. Recovery by water injection is displayed for both a favorable viscosity ratio (μ_o/μ_w = 1) and an unfavorable viscosity

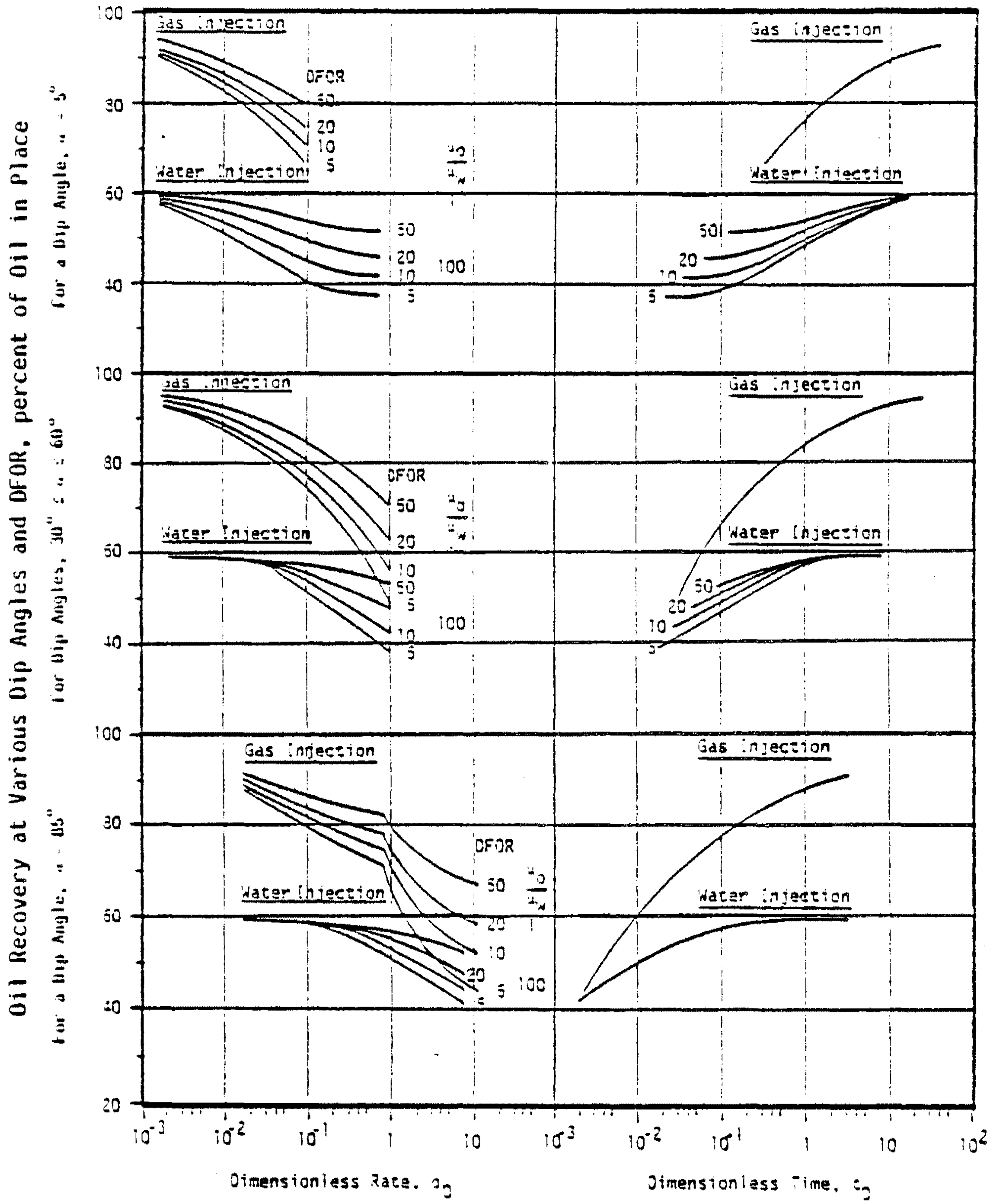

Figure 5. Correlation of Oil Recovery with Dimensionless Rate and Time

[For Gas Injection: $q_D < q_{DC} = (h/\ell)^{1/2} \tan \alpha$, $0.01 \leq h/\ell \leq 1.0$]

ratio (μ_o/μ_w = 100). As noted, the correlations for recovery by gravity drainage are applicable for $q_D < q_{DC}$ (Eq. 4) and $0.01 \leq h/\ell \leq 1.0$. The right half of the figure shows recovery correlated with dimensionless time (Eq. 5). In most cases, a single correlation is presented for all DFOR. These correlations extend Richardson and Blackwell's correlation to other reservoir geometries (dips and dimensions) and give Dykstra's correlation an interpretation of dimensionless time.

Some qualitative observations can be made from Figure 5. The effect of rate and dip on recovery by gravity drainage is clear--recovery increases as the rate decreases and the dip increases. Also, it is apparent that relatively high recoveries by gravity drainage can be obtained from reservoirs with small dip angles, but at comparatively longer recovery times. The time element aside, it appears that when the dimensionless rate is less than the dimensionless critical rate, recovery by gravity drainage will normally be greater than recovery by water injection.

The correlations are discussed in detail in the subsections which follow. Application of the correlations is discussed in the next section.

Effect of Rate and Oil Viscosity

Figure 6 is a plot of oil recovery versus q_D^* (Eq. 2) for base case gas injection runs at a dip of five degrees compared with oil recovery data calculated by Richardson and Blackwell[16] along with recovery data calculated using the Buckley-Leverett frontal advance

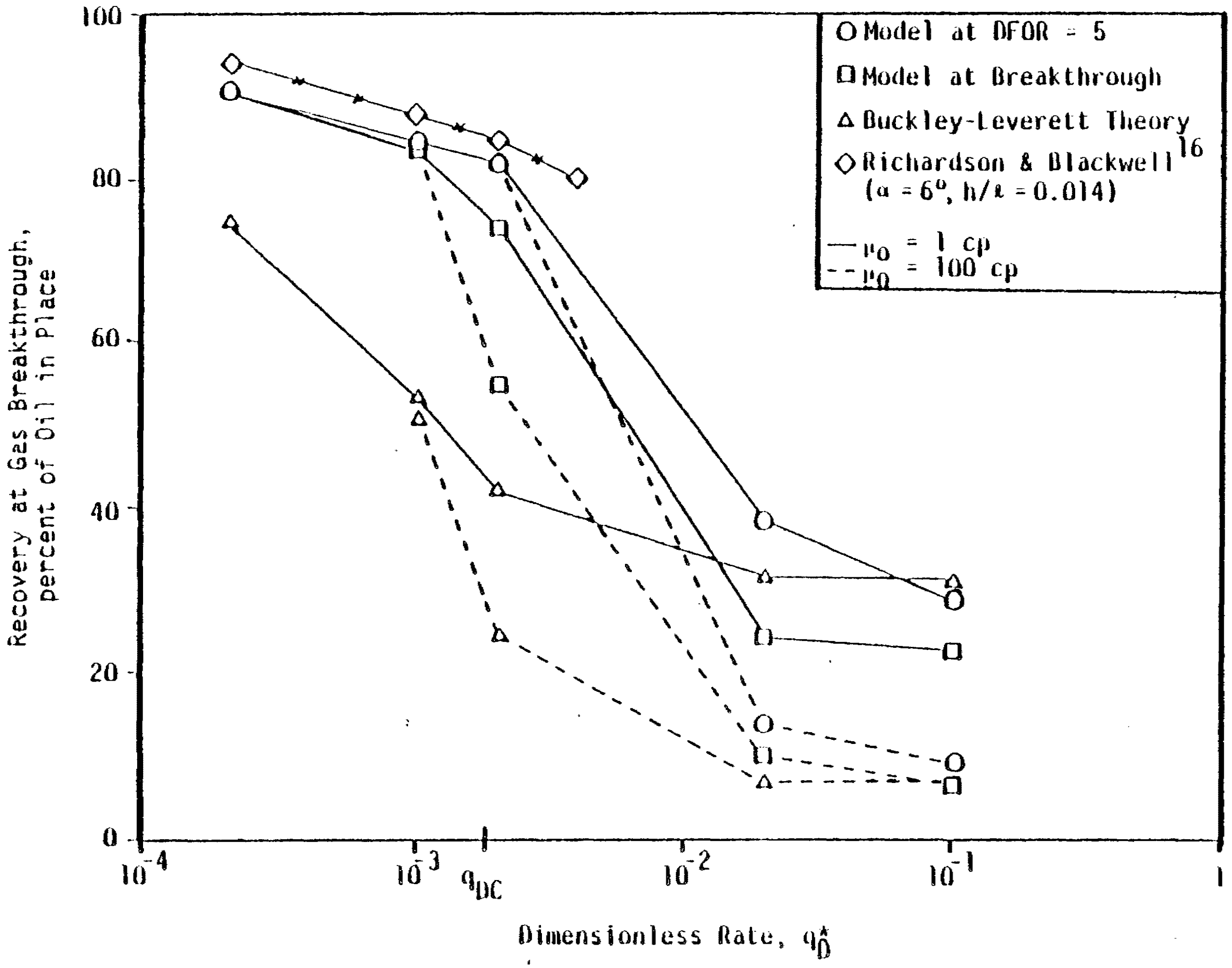

Figure 6. Comparison of Oil Recovery from Base Case Gas Injection Runs at a Dip of 5 Degrees

theory. These data are at breakthrough except where noted, i.e., the model data at a DFOR of five. Breakthrough recoveries for the model data were taken at breakthrough DFOR calculated using the Buckley-Leverett method.

Several observations can be made from Figure 6. As expected, recovery at rates below the critical rate varied inversely with the rate-viscosity product. Varying the oil viscosity from 1 to 100 cp had little effect at rates below the critical rate. At higher rates, the difference became pronounced. Next, note the difference between recovery predicted from the one-dimensional Buckley-Leverett theory and the two-dimensional model except at relatively high rates. The difference at rates below the critical rate can be explained by cross flow due to gravity effects. As may be seen, this difference is significant even at relatively small dip angles. The effect of taking recovery at a DFOR of five can also be gauged. There is a small difference between recovery at a DFOR of five and breakthrough recovery at the slowest rates which increases as the critical rate is approached and becomes quite large at the highest rates. The latter is of no concern to the objectives of this study, however, the difference in the vicinity of the critical rate requires further consideration.

Finally, note the good agreement with the data calculated by Richardson and Blackwell[16] using their simplified gravity drainage model, replotted here through use of Eq. 2 which differs from their dimensionless group only by a scaling constant. The dip angle and height-to-length ratio are slightly different making q_{DC} = .0015 for their data as opposed to q_{DC} = .0018 for the data from this study. The fact that there is not a sharp decrease in the slope of their curve at this point may partially explain their caution that the simplified model is applicable only when $q_T < 0.5\ q_C$.

Figure 7 presents a similar comparison for oil recoveries from base case gas injection runs for a dip of 90 degrees. In this figure, the comparison is made with experimental data from Terwilliger, et al.[6] Their recovery data have been replotted here as a percentage of the movable water to correspond with the recoveries obtained from this study. As noted in their paper, "At higher rates (last 5 points) stabilized zones did not develop for the reason that the experimental column (13 ft.) was of shorter length than the stabilized zone length would have been." This might explain why the last part of the plot dips below the theoretical plot. Observations similar to those made from Figure 6 can be made from this plot. There is an unexplained difference in recovery at and below the critical rate due to differences in oil viscosity. A difference of this magnitude was not noted at other dip angles. As expected, there is good agreement between model and Buckley-Leverett calculations since a vertical two-dimensional system behaves very much like a one-dimensional system. The difference that does exist can probably be attributed to numerical dispersion. Finally, note that the effect of taking recovery at a DFOR of five is generally as described above.

Figure 8 compares oil recoveries from base case gas injection runs for a dip of 45 degrees with recoveries calculated using Buckley-Leverett theory. Again, notice the difference between the latter

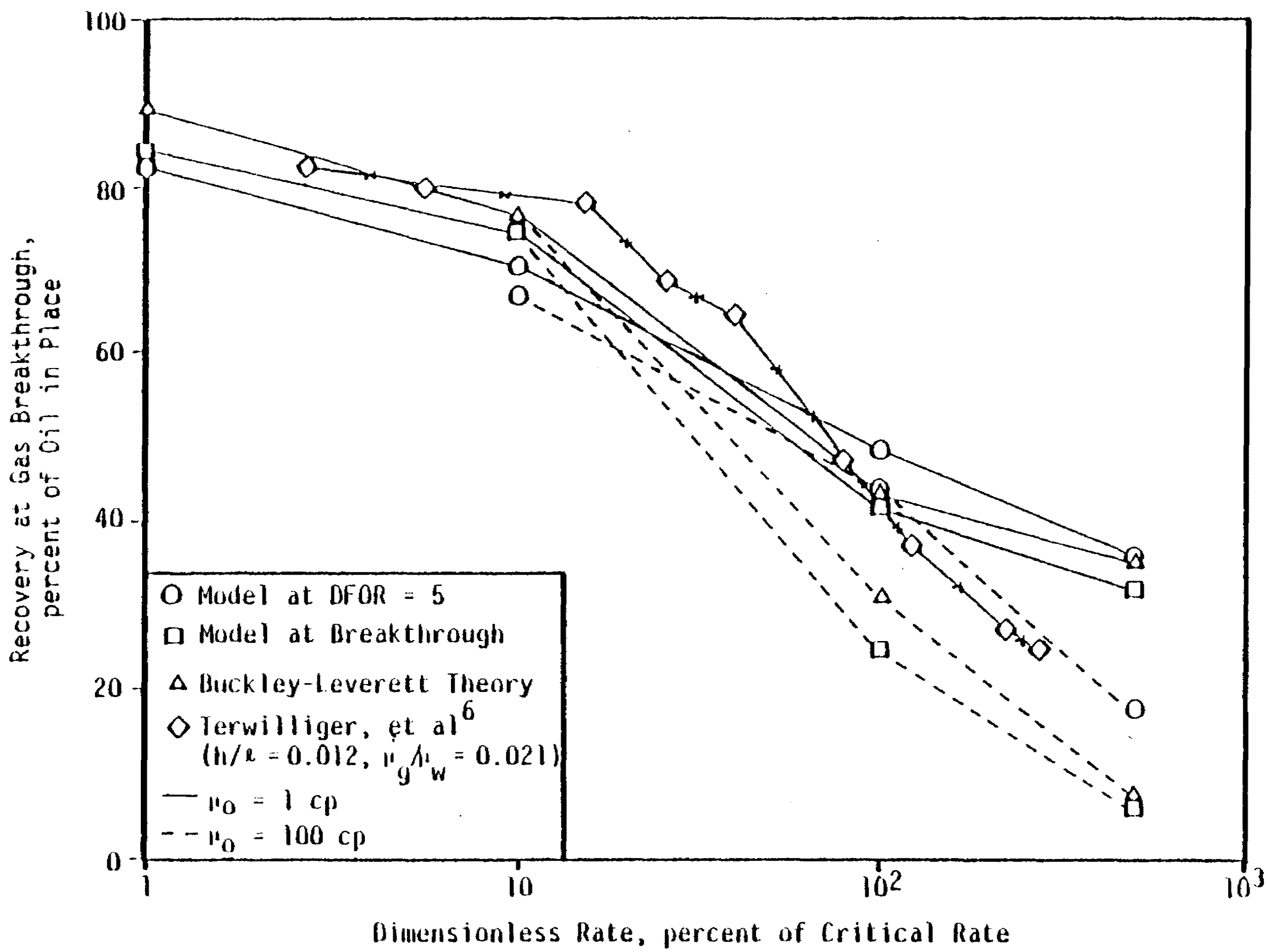

Figure 7. Comparison of Oil Recovery from Base Case Gas Injection Runs at a Dip of 90 Degrees

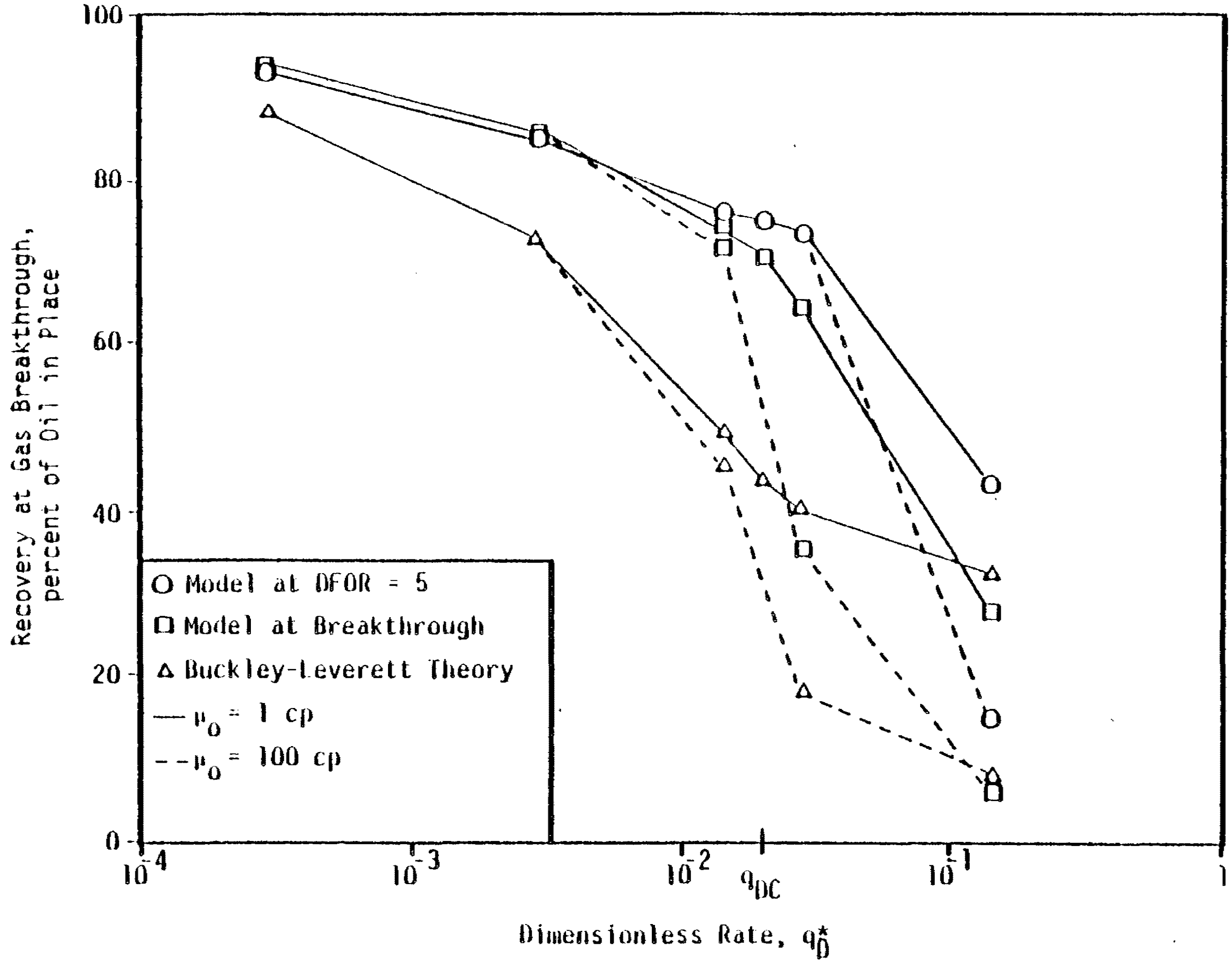

Figure 8. Comparison of Oil Recovery from Base Case Gas Injection Runs at a Dip of 45 Degrees

and the model data. As expected, it is intermediate to the extremes noted above. Also, the effect of taking recovery at a DFOR of five is clear and can be summarized. In essence, the recovery curve is smoother, tending to average recovery across rates and suppress oil viscosity differences. Near the critical rate, these recoveries represent post-breakthrough recoveries while at the slowest rates, they represent pre-breakthrough recoveries. They are sufficient for comparative purposes.

Interestingly, the maximum oil recovery from up-dip gas injection at a given rate occurred at a dip of 45 degrees. Whether this observation is real or a result of dispersion is unknown. However, the general form of the mathematical relationship between recovery and dip, $f(\alpha) = b(\alpha) - m \log \left(\frac{C}{\cos \alpha}\right)$, admits the possibility of a maximum at α = 45 degrees. While the relationship was not investigated in detail, if real, it may be analogous to a phenomenon that has been observed in the flow of gas bubbles in inclined tubular goods.[24*]

Figure 9 is a similar comparison for base case water injection runs at a dip of 45 degrees and oil viscosities of 1, 10, and 100 cp. In contrast with the data from base case gas injection runs, there is only a slight difference between recovery from the water injection runs and recovery calculated using Buckley-Leverett theory. Likewise, small differences were observed for water injection runs at dips of 5 and 85 degrees. It appears that for the range

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*This phenomenon was brought to the authors' attention by S. H. Neuse.

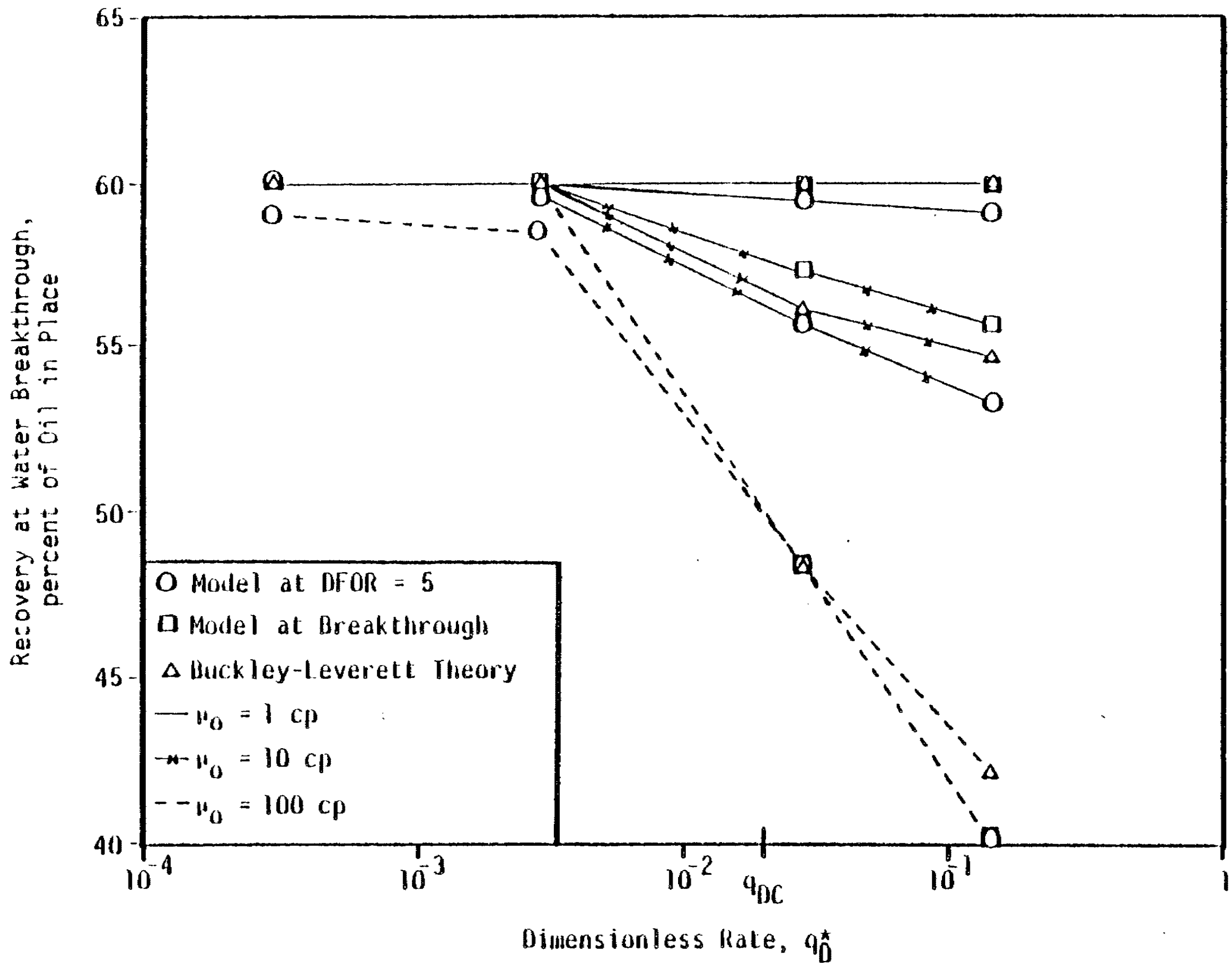

Figure 9. Comparison of Oil Recovery from Base Case Water Injection Runs at a Dip of 45 Degrees

of rates studied, the Buckley-Leverett method yields acceptable results for recovery from water injection; whereas, as noted earlier, recovery by gas injection varied significantly. Note again that the value of oil viscosity has little effect on recovery at rates less than the critical rate. At rates higher than the critical rate, the effect of oil viscosity is very pronounced. In fact, it appears that recovery decreases exponentially as oil viscosity and q_D^* increase. This observation agrees qualitatively with data reported by Croes and Schwarz.[12] Their Figure 2 indicates a logarithmic relationship between viscosity ratio and recovery over much of the range of their investigation.

Effect of Permeability and Density Difference

Figure 10 is a plot of oil recovery at a DFOR of five for selected gas and water injection runs at 45 degrees. Two data points on the plot depict variations in permeability and/or density difference (the point shown by a triangle is actually two points--one in which the permeability was varied by a factor of ten, in the other the density difference was varied by a factor of ten). As expected, these points correlated with good agreement, i.e., recovery varied directly with vertical permeability and density difference. Similar agreement was obtained at the other dip angles.

It is emphasized that the large difference between recovery from gas injection as compared to water injection results from the difference between the residual oil saturation to water, S_{orw}, and the residual oil saturation to gas, S_{org}. As may be seen in Table 1,

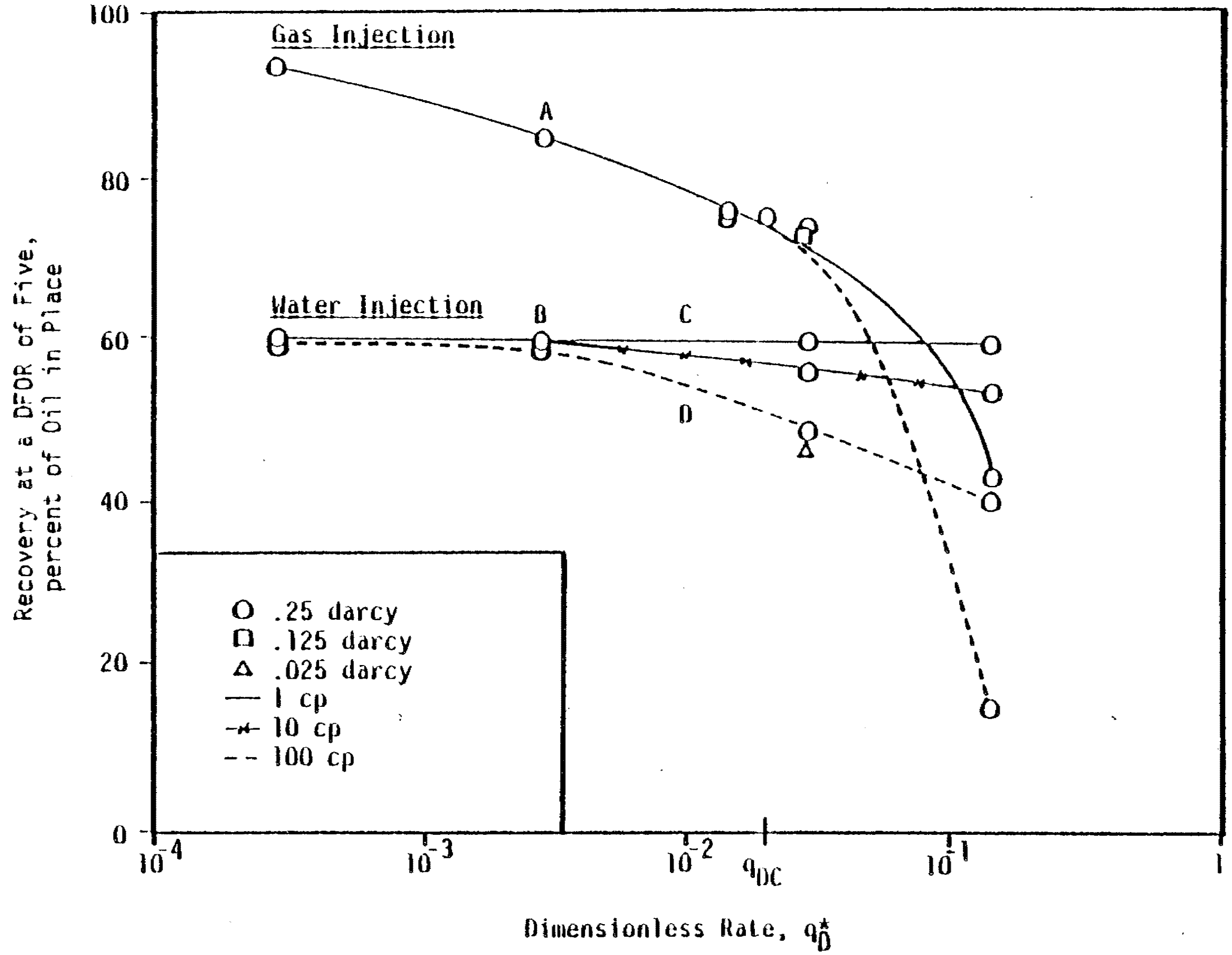

Figure 10. Comparison of Oil Recovery from Gas and Water Injection Runs at a Dip of 45 Degrees for Various Values of Vertical Permeability

values used in the study were 0.32 and 0.0, respectively. It is also emphasized that points on the gas and water recovery curves at the same value of q_D^* reflect vastly different conditions due to density differences. For example, A and B, both plotted at $q_D^* = 0.003$, reflect $q_{TA} = 3.3\ q_{TB}$. The comparable rate point is at C to D depending on the value of the oil viscosity.

Effect of Dip

Figure 11 presents a plot of oil recovery at a DFOR of five for base case gas and water injection runs at various dip angles. As indicated, recovery was found to vary directly with dip. For clarity, all points on this plot reflect displacement efficiency under unfavorable mobility conditions--data from water injection runs at a favorable mobility ratio were omitted. Data from gas injection runs at a rate greater than the critical rate were also omitted. This means the gas injection curves reflect gravity drainage performance. The dashed lines are the loci of the dimensionless critical rates with points above these lines representing stable displacements. These regions may be referred to as gravity stable regions.

Effect of Reservoir Dimensions

Figure 12 is a plot of oil recovery at a DFOR of five versus dimensionless rate as defined by Eq. 3 for gas and water injection runs at 45 degrees and different values of the height-to-length ratio, h/ℓ. Note that recovery varies directly with $w\cdot(h\ell)^{1/2}$. To see this, rewrite Eq. 3 with $A = hw$. Note also that the size of

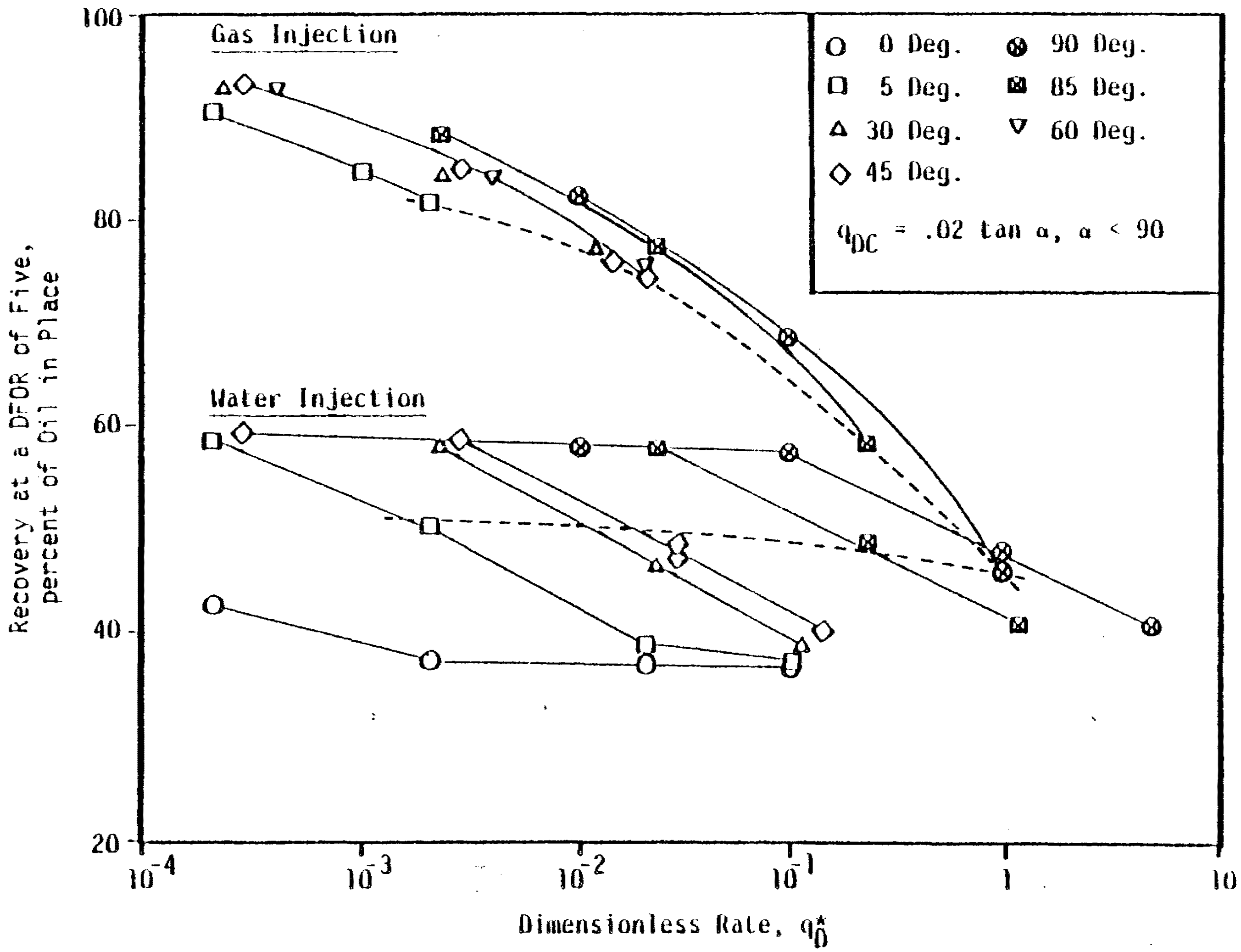

Figure 11. Comparison of Oil Recovery from Base Case Gas and Water Injection Runs at Various Dip Angles

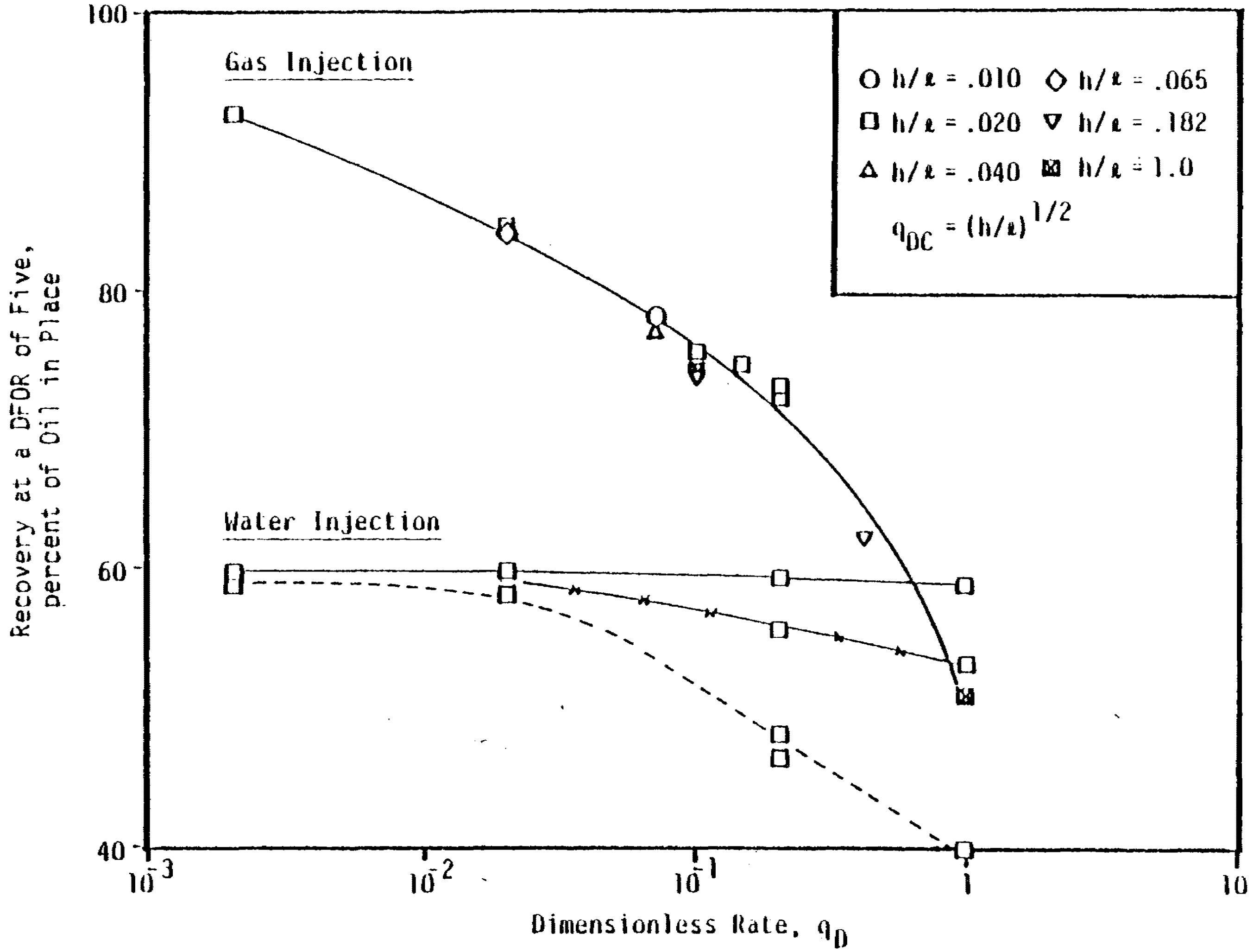

Figure 12. Comparison of Oil Recovery from Gas and Water Injection Runs at a Dip of 45 Degrees for Various Values of the Height to Length Ratio

the gravity stable region increases with h/ℓ, i.e., in thicker reservoirs, a stable displacement is possible over a wider range of conditions. This is not surprising in view of the length of the region observed at 90 degrees and confirms the experimental observation of Terwilliger, et al[6] quoted above. The fact that the region increases with h/ℓ means one must restrict $q_D \leq q_{DC}$ if he is interested in a gravity stable displacement. Similar agreement was obtained at other dip angles.

Correlation with Dimensionless Rate and Time

Figure 13 presents more detail on the correlation of oil recovery wi h dimensionless rate for dip angles between 30 and 60 degrees. The relationship between recovery, DFOR, q_D and α has the general form, $f(DFOR, q_D, \alpha) = b(DFOR, \alpha) - m(\log q_D - C)$ where C is the shift factor and m is the slope of the shifted straight line plot. Data from 28 gas and seven water injection runs at three dip angles are represented in the plot. Many of the data points were nearly coincident, e.g., gas runs at different oil viscosity, making it impossible to show all 140 of the data points. Average recoveries are plotted in these cases. The recovery curve shown for water injection at a favorable viscosity ratio ($\mu_o/\mu_w = 1$) is the theoretical curve rather than the model data. Likewise, Figure 14 presents more detail on the correlation of oil recovery with dimensionless time for dip angles between 30 and 60 degrees. The remarks above concerning data representation also apply to this plot. Similar procedures in data representation were followed in constructing the cor-

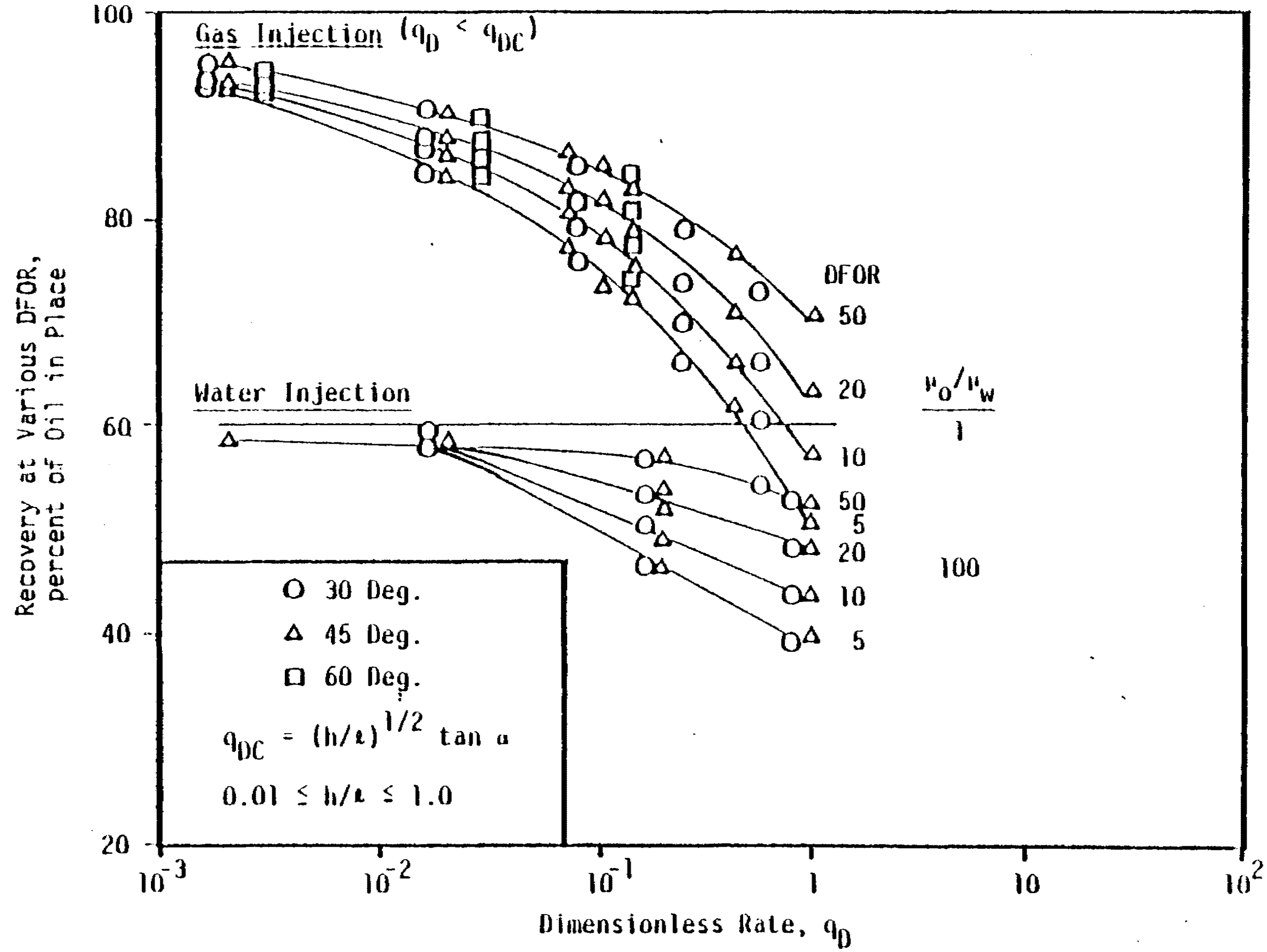

Figure 13. Correlation of Recovery with Dimensionless Rate for Dip Angles Between 30 and 60 Degrees

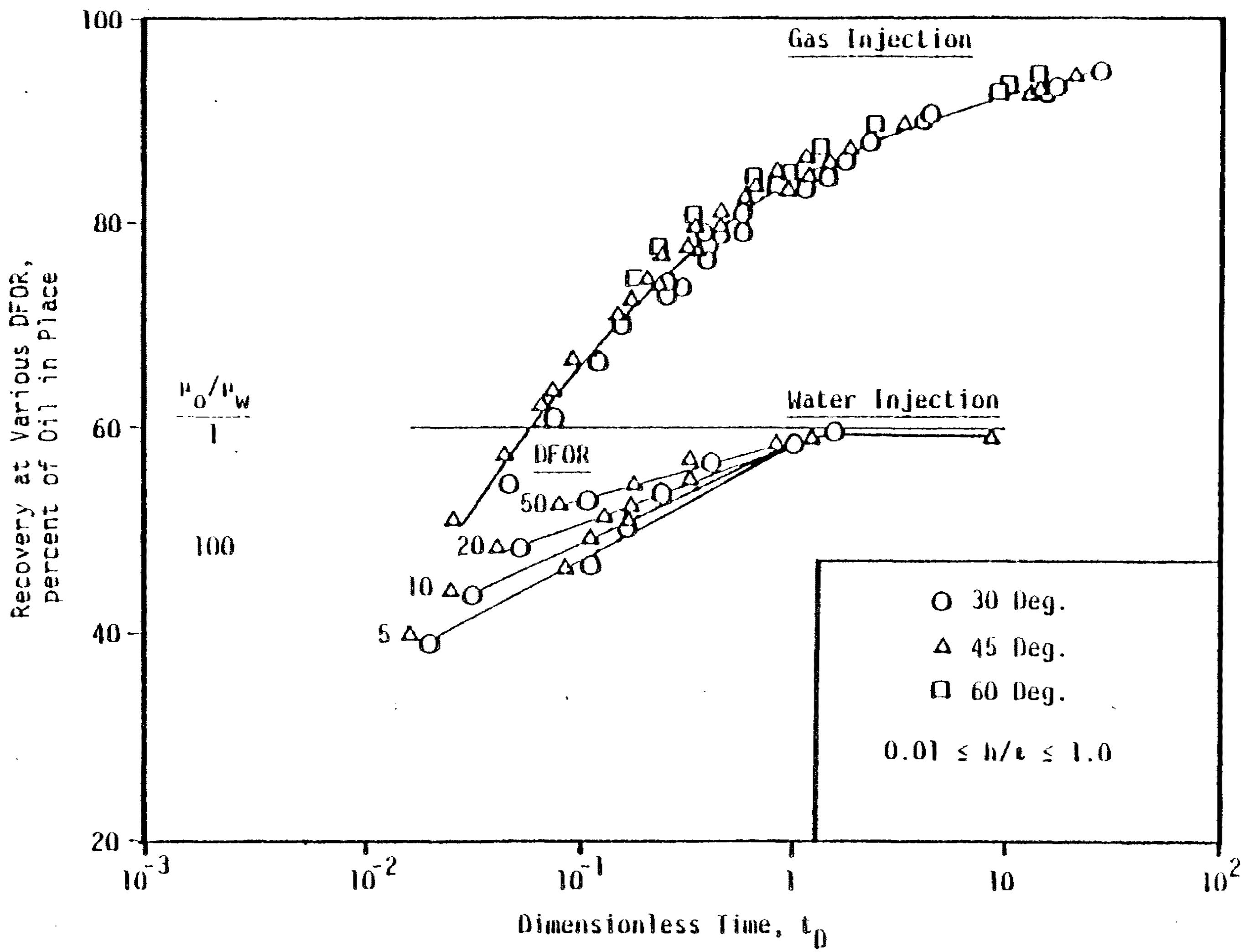

Figure 14. Correlation of Recovery with Dimensionless Time for Dip Angles Between 30 and 60 Degrees

relations of Figure 5 for dips of 5 and 85 degrees. Of possible interest, data from 11 gas and four water injection runs was used in the former and data from nine gas and three water injection runs was used in the latter.

APPLICATION

For the range of conditions investigated, Eqs. 3-5 and Figure 5 may be used to estimate recovery by gravity drainage or to compare recovery by gravity drainage with recovery by water injection. One proceeds as follows:

1. Estimate pertinent reservoir parameters. Average dimensions, fluid, and rock properties may be used. Dimensions should be those of the oil zone.

2. For the displacement rates of interest, calculate q_D using Eq. 3 and q_{DC} using Eq. 4. A different form of Eq. 3 may at times be more useful. For example, Eq. 3 can be rewritten as

$$q_D = 2.0368 \times 10^{-5} \frac{q_T' \mu_o}{k_v \ \Delta\rho \ (h/\ell)^{1/2}}, \quad \ldots\ldots\ldots\ldots\ldots (6)$$

where q_T' has units of RBPD per productive surface acre.

3. If $q_D > q_{DC}$ and one is interested in a stable displacement, choose $q_T < q_C$ (Eq. 1).

4. Find estimated recovery at the selected DFOR and corresponding t_D from Figure 5. For water displacements, if $q_D > q_{DC}$ logarithmic interpolation may be used as an approximation for viscosity ratios between 1 and 100. If fluid saturations differ from those used in this study (S_{wi} = 0.2, S_{org} = 0.0, and S_{orw} = 0.32) adjust

recovery as appropriate. For example, in this study 60 percent recovery by water injection was 100 percent of the recoverable oil by water injection, that is, $\frac{1 - 0.2 - 0.32}{1 - 0.2} \times 100 = 60$ percent of the oil in place.

5. Alternatively, in comparing recovery by gravity drainage with recovery by water injection, the Buckley-Leverett method may be used to estimate the latter.

6. Compute recovery time using Eq. 5.

Two examples from published field data further illustrate application in the subsections that follow.

Elk Basin (Tensleep) Field

This example illustrates estimation of recovery by gravity drainage. Data for this reservoir have been reported by Stewart, et al[4] and other authors.[25-27] The reservoir is a large anticline originally having a productive area of 6,300 acres, as much as 2,300 ft. of productive oil closure and approximately 500 million barrels of oil in place. Injection of inert gas along the crest of the structure began in 1949. The latest information in the literature indicated that, as of 1973, crestal gas injection continued and had been supplemented by water injection on the West flank. While drawing on more recent information, the recovery estimate presented below is based on operating conditions existing as of April 1954 for comparison with the Stewart, et al estimate. Average reservoir properties are in Table 3 where the last two parameters have been estimated from structure data presented in Ref. 4.

Table 3. Average Reservoir Properties for the Elk Basin Field

Average permeability, darcy	0.112
Average porosity, fraction	0.11
Average initial water saturation, fraction	0.08
Average dip, degrees	30
Average net thickness, ft.	130
Average viscosity of oil, cp	2.24
Average viscosity of gas, cp	0.0177
Average density of oil, psi/ft.	0.351
Average density of gas, psi/ft.	0.026
Average total production rate (1951-53), RBPD	24,200
Average gas injection rate (1951-53), RBPD	26,500
Productive oil area (1954), acres	5,200 est.
Average length of oil column along bedding planes (1954), ft.	4,600 est.

From these data, $h/l = 0.028$, $q_D = 0.035$ (Eq. 6) and $q_{DC} = 0.097$. Since $q_D < q_{DC}$, a stable displacement should occur. Assuming that $S_{org} = 0.0$, Figure 5 indicates a recovery of approximately 82 percent at DFOR of five. Using $S_{org} = 0.12$ (the laboratory data reported in Ref. 4 indicated $k_o = 0.0$ at $S_g = 0.8$), the recovery from Figure 5 is adjusted to 71 percent of the oil in place. Stewart, et al[4] calculated a 64 percent recovery using the Terwilliger, et al[6] method and 66 percent from field performance. This example clearly indicates the sensitivity of ultimate recovery to the value of the residual saturation. Finally, $t_D = 0.65$ implies time for recovery is 87 years.

Hawkins (Woodbine) Field

Reservoir data for the Hawkins Field have recently been reported [illegible] is a large, complexly faulted anticline covering about 10,000 acres

Average porosity, fraction	0.279

and having about 1000 ft. of hydrocarbon closure ranging from a multi-tiered gas cap to a partially underlying asphalt layer. Table 1 from Ref. 16 is reproduced below (in the units of this study) as Table 4.

Table 4. Average Reservoir Properties for the Hawkins Field

Average permeability, darcy	3.40
Average vertical permeability, darcy	2.38
Average initial water saturation, fraction	0.08
Angle of dip, degrees	6
Average thickness (in vertical direction), ft.	49
Average length (along bedding planes), ft.	3,500
Average reservoir pressure, psi	1,500
Average viscosity of oil, cp	4.45
Average viscosity of gas, cp	0.0185
Average density of oil, psi/ft.	0.359
Average density of gas, psi/ft.	0.037
Average total flow rate per unit area, RBPD/sq. ft.	0.0065

With these data and the relative permeability-saturation relation for the Hawkins Field, Richardson and Blackwell[16] calculated a recovery of 91.5 percent of the oil in place using their simplified gravity drainage model. Time to gas breakthrough was 33 years. Applying the results of this study, $h/\ell = 0.0139$, $q_D = 0.004$ (Eq. 3) and $q_{DC} = 0.012$. Using Figure 5 at a dip of five degrees and assuming $Sor_g = 0.0$ gives a recovery of 87 percent of the oil in place at a DFOR of five by gravity drainage. Taking $Sor_g = 0.035$ as reported by King and Lee[28] gives a recovery of 84 percent. While this number is lower than that calculated by Richardson and Blackwell using their model, it closely agrees with the recovery observed from field performance (87 percent) and the recovery they obtained from a two-dimensional numerical model (87.2 percent). Also from Figure 5, t_D = 5.0 from which the time is calculated at 29.3 years.

For comparison, recovery by water injection may also be estimated. Assuming the same displacement rate, a density difference of 0.1 psi/ft., an oil-water viscosity ratio of 10 and a Sor_w of 0.2 (Ref. 28 reports an average of 0.15 from 35 pressure cores taken from watered out zones), one obtains q_D = 0.013. Since $q_D > q_{DC}$ the displacement is not stable. From Figure 5, the recovery by logarithmic approximation for μ_o/μ_w = 10 is about 55 percent or 91.7 percent of the recoverable oil which at Hawkins saturation conditions is 72 percent of the oil in place at a DFOR of five. This number is 8 to 33 percent higher than any of the several estimates at breakthrough and 1-PV throughput reported in Refs. 16 and 28; however, rates used were not reported. From Figure 5, t_D = 1.5 from which t = 28.3 years.

CONCLUSIONS

1. For the range of conditions investigated, correlations between oil recovery and dimensionless rate and time have been found which permit estimation of the recovery by gravity drainage (up-dip gas injection) and comparison between recovery by gravity drainage and down-dip water injection. These correlations extend the use of Richardson and Blackwell's correlation to different reservoir geometries (dips and dimensions) and give Dykstra's correlation an interpretation of dimensionless time.

2. If the dimensionless rate is less than the dimensionless

critical rate, recovery by gravity drainage normally will be greater than recovery by water injection.

3. Recovery by gravity drainage varies directly with the square root of the product of the reservoir height and length, the dip angle, the vertical permeability and the fluid density difference. It varies inversely with the total flow rate and the oil viscosity.

4. Acceptable agreement with published recovery estimates was obtained for two field examples illustrating use of the correlations. The examples indicate recovery is extremely sensitive to the value of residual oil saturation.

NOMENCLATURE

1. Capital Letters

A - area of cross section normal to bedding plane, sq. ft.

C - constant

DFOR - displacing fluid-to-oil ratio, MSCF/STB for gas and STB/STB for water

M - mobility ratio, $k_{rd}\mu_o/k_{ro}\mu_d$

P_c - capillary pressure, psi

PV - pore volume

Ra,Rd - dimensionless parameters derived by Craig, et al[13]

Sor - residual oil saturation, fraction

Swi - irreducible water saturation, fraction

2. Lower Case Letters

h - height of formation normal to bedding plane, ft.

k - permeability, darcy

ℓ - length of formation along bedding plane, ft.

q - flow rate through area A, RBPD

q_C - critical rate defined by Eq. 1

q_D^* - dimensionless rate defined by Eq. 2

q_D - dimensionless rate defined by Eq. 3

q_{DC} - dimensionless critical rate defined by Eq. 4

t - time, years

t_D - dimensionless time defined by Eq. 5

w - width of formation, ft.

3. Greek Letters

Δ - finite difference operator

α - dip angle, degrees

μ - viscosity, cp

ρ - density, psi/ft.

4. Subscripts

T - total

d - displacing fluid

g - gas

o - oil

r - relative

w - water

v - vertical

5. Abbreviations

cp - centipoise

deg - degrees

ft - feet

MSCF - thousands of standard cubic feet

psi - pounds per square inch

RB - reservoir barrels

RBPD - reservoir barrels per day

STB - stock tank barrels

sq - square

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