USE OF PERMEABILITY DISTRIBUTION IN WATER FLOOD CALCULATIONS

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ABSTRACT

A method is presented for predicting the performance of water flooding operations in depleted, or nearly depleted, petroleum reservoirs. The method makes use of permeability variations and the vertical distribution of productive capacity. From these two parameters can be calculated the produced water cut versus the oil recovery. Derivations of the mathematical analogy is shown and sample calculations and curves of prediction are presented. Comparison is made of the predicted and actual performance of a typical 5-spot in an Illinois water flood.

INTRODUCTION

The use of water as a flooding medium in both depleted and "flush" oil reservoirs is gaining greater recognition and acceptance. Many of the shallower fields, depleted by primary production, have been and are being subjected to water injection in order to obtain some part of the large volume of oil remaining after primary production. Some of the earlier water flood installation proved highly discouraging and the value of water flooding was often questioned. Many of these earlier floods were haphazardly selected and developed as little was known of the physical characteristics and contents of the producing formations. The prior evaluation of the flood performance was impossible.

During the past decade the development of the required reservoir engineering tools—core analysis, reservoir fluid analysis, electric logs, fluid flow formulae, etc.—has allowed the engineer to construct and apply the methods which are presently being used to evaluate the economic and mechanical susceptibility of a reservoir to flooding.

This discussion will present a method for taking into account the effect of permeability variations in predicting the performance of water floods in depleted reservoirs.

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PERMEABILITY AND CAPACITY DISTRIBUTION

It is generally agreed by most investigators that in a single phase system fluid will flow in a porous and permeable medium in proportion to the permeability of the medium.

Producing formations are usually highly irregular in permeability, both vertically and horizontally. However, zones of higher or lower permeability are often found to exhibit lateral continuity. Thus, while structurally comparable stringers in adjacent wells may differ several fold in permeability values, they usually bear resemblance as being part of a general continuous higher or lower permeability section. It is generally agreed that where such stratification of permeability exists, injected water sweeps first the zones of higher permeability, and it is in these zones that "break-through" first occurs in the producing well. It is a basic assumption of the presently described method that penetration of a water front follows the individual permeability variations as if such variations were continuous from input to producing well. This is admittedly not rigorously true, but can be justified as making possible a simplifying mathematical approach to an otherwise extremely complicated three dimensional flow problem.

As a basis for study of the lateral flow of fluids in formations of irregular permeability, the irregularities may be conveniently represented by a permeability distribution curve and a capacity distribution curve. In obtaining these curves, the permeability values, regardless of their structural position in the formation, are rearranged in order of decreasing permeability.

If these permeability values so arranged are plotted against the cumulative thickness, a permeability distribution curve is obtained. This curve may then be likened to a "smoothed" permeability profile of the formation.

In making comparison between different distribution curves it is convenient to state the permeabilities in terms of the ratio of the actual permeability values to the average permeability of the formation. These ratios termed "dimensionless permeabilities", are used in this paper rather than the permeabilities in terms of millidarcys.

The capacity distribution curve is a plot of the cumulative capacity (starting with the highest permeabilities) versus the cumulative thickness. The capacity and thickness are given as fractions of the total capacity and thickness. Mathematically, the capacity distribution is the intergration of the permeability distribution curve.

In practice it is convenient to first obtain the capacity distribution curve and derive from it a smoothed dimensionless permeability curve.

The method of obtaining the capacity distribution curve is illustrated in the successive column of Table 1, in which capacity and thickness are derived as fractions of their respective totals. If only a small number of permeability values are available, it is generally desirable to smooth the resultant curve. This has been done to give the capacity distribution curve shown in Figure 1.

The differentiation of the capacity distribution curve to obtain the permeability distribution curve is shown in Table 2. Here, the capacity values are read from the smoothed curve at intervals of cumulative thickness, and the increments of capacity are divided by the increments of thickness to obtain the dimensionless permeability, K'. Due to this stepwise procedure of calculation these premeability values must be plotted at the midpoints of the successive increments of thickness. The curve so obtained from these data is shown in Figure 1. The total area under the K' curve is equal to unity.

TABLE 1

Calculation of Capacity Distribution

Cumulative Thickness; Feet	h = Fraction of Cumulative Thickness	K = Permeability; Millidarcys	ΔC = Increment of Total Capacity	C = Cumulative Capacity; Fraction
1	.0345	776	.153	.153
2	.0690	454	.089	.242
3	.1034	349	.069	.311
4	.1380	308	.061	.372
5	.1724	295	.058	.430
6	.2070	282	.056	.486
7	.2414	273	.054	.540
8	.2759	262	.052	.592
9	.3103	228	.045	.637
10	.3448	187	.037	.674
11	.3793	178	.035	.709
12	.4138	161	.032	.741
13	.4483	159	.031	.772
14	.4828	148	.029	.801
15	.5172	127	.025	.826
16	.5517	109	.021	.847
17	.5862	88	.017	.864
18	.6207	87	.017	.881
19	.6552	87	.017	.898
20	.6897	77	.015	.913
21	.7241	71	.014	.927
22	.7586	62	.012	.939
23	.7931	58	.011	.950
24	.8276	54	.011	.961
25	.8621	50	.010	.971
26	.8966	47	.009	.980
27	.9310	47	.009	.989
28	.9655	35	.007	.996
29	1.0000	16	.004	1.000
		ε 5,075		

Derivation of Water Cut and Recovery Equations

The derivation of the water cut and the recovery equations is based on two principal assumptions: (1) fluid flow is linear and (2) the distance of penetration of the flood front is proportional to permeability.

With these assumptions, a cross section of the flood front would show penetration proportional to the permeability distribution. At the time when all permeabilities greater than a given value, K′, have "broken through" to water at the producing well, a schematic diagram of the water penetration would be as shown in Figure 2.

In the diagram of Figure 2, the intake well exposed to water injection is represented by the line *ab* and the producing well by line *cd*. The rectangle *abcd* represents the floodable volume of the formation (total acre-feet of formation times the unit water flood recovery per acre-foot).

The curve *gfb* represents the water front and the enclosed area *agfba* is the permeability distribution curve. Since dimensionless permeability is used, the area under this curve is equal to unity.

Area $abcd$ $= X+Y+Z=1$

$=$ floodable formation $=$

$ab \times fe = ab \times K'$

Area $agfba = W+X+Y=1$

Since the capacity distribution curve is the integration of the permeability curve, $W+X=C=$ capacity corresponding to the formation thickness h and permeability K′, also:

$Y=1-(W+X)=1\text{-}C$, and

$X=fe\times ac=K'h$

Areas X and Y represent the portion of the formation from which oil has been displaced by the encroaching water. The oil recovery expressed as a fraction of the total floodable formation is then:

$$\text{Recovery}=\frac{X+Y}{X+Y+Z}=\frac{K'h+(1\text{-}C)}{K'}$$

In the producing well, it has been assumed that all permeabilities greater than K′ are flowing only water. The capacity flowing water is therefore C, and the capacity flowing oil is (1-C).

The water and oil production rates are calculated from these capacities by including relative permeability and viscosity terms, and in the case of oil, a formation volume factor, *u*.

$$\text{Rate of water production}=C\left(\frac{Krw}{\mu w}\right)$$

Rate of stock tank oil production $=$

$$(1\text{-}C)\left(\frac{Kro}{\mu o}\right)\left(\frac{1}{u}\right)$$

Rate of total fluid production $=$

$$C\left(\frac{Krw}{\mu w}\right)+\left(1\text{-}C\right)\left(\frac{Kro}{\mu o}\right)\left(\frac{1}{u}\right)$$

Thus, water cut $=$

$$\frac{C\left(\dfrac{Krw}{\mu w}\right)}{C\left(\dfrac{Krw}{\mu w}\right)+\left(1\text{-}C\right)\left(\dfrac{Kro}{\mu o}\right)\left(\dfrac{1}{u}\right)}$$

$$=\frac{C\left(\dfrac{Krw}{\mu w}\cdot\dfrac{\mu o}{Kro}\cdot u\right)}{C\left(\dfrac{Krw}{\mu w}\cdot\dfrac{\mu o}{Kro}\cdot u\right)+\left(1\text{-}C\right)}$$

Assigning the term A to

$$\left(\frac{Krw}{\mu w}\cdot\frac{\mu o}{Kro}\cdot u\right)$$

$$\text{Water Cut}=\frac{CA}{CA+(1\text{-}C)}$$

The water cut is expressed as a fraction, the ratio of the water production rate to the total fluid production rate.

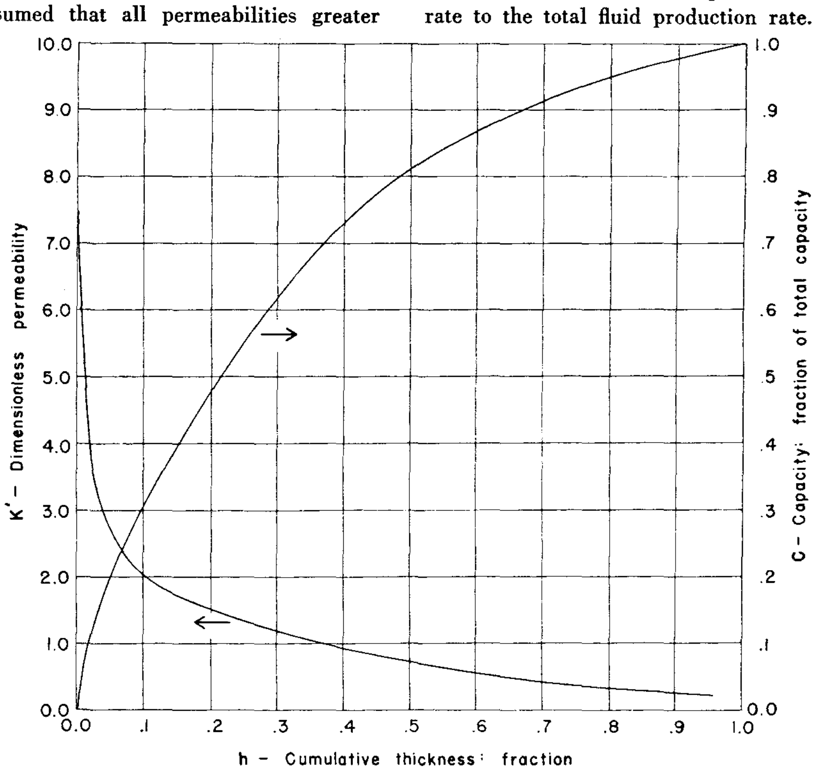

FIG. 1 — PERMEABILITY AND CAPACITY DISTRIBUTION.

TABLE 2

Calculation of Permeability Distribution

h=Fraction of Cumulative Thickness	Δh=Increment of Cumulative Thickness	C=Cumulative Capacity; Fraction	ΔC=Increment of Cumulative Capacity	$K'=\frac{\Delta C}{\Delta h}$ Dimensionless Permeability	h'=Average Cumulative Thickness Fraction
.01	.01	*.065	.065	6.50	.005
.02	.01	.110	.045	4.50	.015
.05	.03	.200	.090	3.00	.035
.10	.05	.308	.108	2.16	.075
.20	.10	.476	.168	1.68	.150
30	.10	.620	.144	1.44	.250
.40	.10	.731	.111	1.11	.350
.50	.10	.812	.081	.81	.450
.60	.10	.870	.058	.58	.550
.70	.10	.917	.047	.47	.650
.80	.10	.952	.035	.35	.750
.90	.10	.980	.028	.28	.850
.95	.05	.991	.011	.22	.925
1.00	.05	1.000	.009	.18	.975

*From capacity distribution curve.

APPLICATION OF THE WATER CUT AND RECOVERY EQUATIONS

Sample calculations for recovery and water cut based on the previous permeability distribution data are shown stepwise in Tables No. 3 and 4.

In these tables water cut and recovery are calculated independently of each other, but as functions of the thickness *h* as a parameter.

A plot of the resultant water cut versus recovery data is shown in Figure 3.

The percentage recovery values can be converted to barrels of oil per acre foot by multiplying by the unit recovery at 100 per cent water cut and the appropriate flood coverage factor.

The unit recovery is calculated as shown in the following example:

Connate water saturation=24 per cent of pore space.

Reservoir oil saturation (after primary depletion) =59 per cent of pore space

Residual oil saturation after complete flushing=21 per cent of pore space

Porosity—19 per cent

Formation Volume Factor=1.073 barrels/barrel

Unit Recovery=

$$7758\ (.19)\ \left(\frac{.59-1.073(.21)}{1.073}\right) =$$

501 barrels of stock tank oil per acre foot

The total liquid saturation in this example is the sum of the 59 per cent oil saturation and the 24 per cent water saturation, or 83 per cent. The remaining 17 percent of pore space is occupied by the free gas remaining after primary depletion.

In order to increase reservoir pressure sufficiently to attain the desired production rate it is necessary to compress this gas space with injected water.

RECOVERY EQUATION—

$$\text{Recovery} = \frac{X+Y}{X+Y+Z} = \frac{K'h+(1-C)}{K'} \qquad (1)$$

WATER CUT EQUATION—

$$\text{Water Cut} = \frac{(W+X)\left(\frac{Krw}{\mu w}\cdot\frac{\mu o}{Kro}\cdot u\right)}{(W+X)\left(\frac{Krw}{\mu w}\cdot\frac{\mu o}{Kro}\cdot u\right)+\left(1-(W+X)\right)}$$

since $W+X=C$, and $\left(\frac{Krw}{\mu w}\cdot\frac{\mu o}{Kro}\cdot u\right) = \text{constant} = A$

$$\text{Water Cut} = \frac{C\cdot A}{C\cdot A+(1-C)} \qquad (2)$$

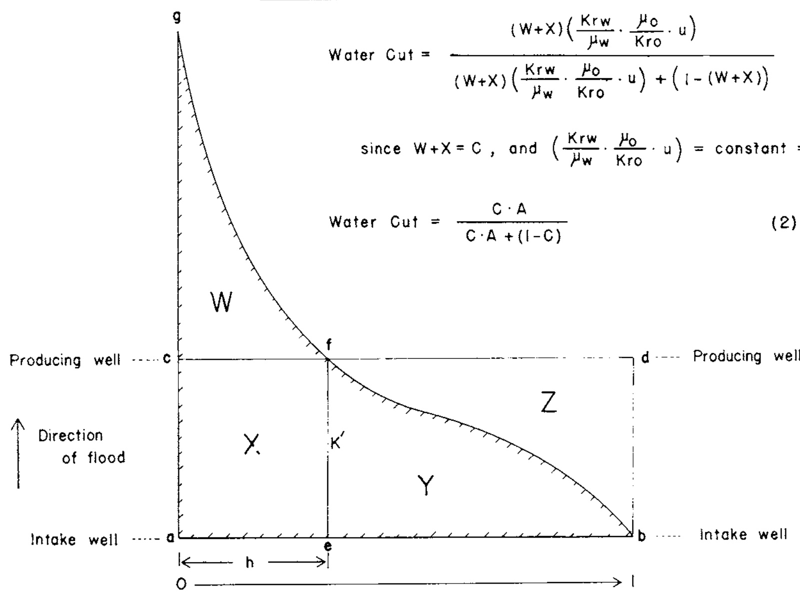

FIG. 2—BASIC EQUATIONS AND SCHEMATIC ILLUSTRATION OF FLOOD FRONT.

TABLE 3

Calculation of Recovery

h=Fraction of Cumulative Thickness	K'=Dimensionless Permeability	C=Cumulative Capacity; Fraction	K'•h	K'h+(1-C)	$\frac{(K'h+(1-C))}{K'}$ =Recovery Fraction of Total Recovery
.00	*7.50	*.000	.000	1.000	.133
.01	5.32	.065	.053	.988	.186
.02	3.83	.110	.076	.966	.252
.05	2.69	.200	.135	.935	.348
.10	2.03	.308	.203	.895	.441
.20	1.55	.476	.310	.834	.538
.30	1.19	.620	.357	.737	.619
.40	.92	.731	.368	.637	.692
.50	.71	.812	.355	.543	.765
.60	.55	.870	.330	.460	.836
.70	.41	.917	.287	.370	.902
.80	.31	.952	.248	.296	.955
.90	.25	.980	.225	.245	.980
.95	.20	.991	.190	.199	.995
1.00	.00	1.000	0.000	0.000	1.000

*From permeability and capacity distribution curves.

Since reservoir pressure is built up as this gas is compressed, a "kick" in production rate is obtained somewhat before an amount of water equivalent to the gas space has been injected. However, for purposes of predictions the volume of water required to fill up is generally assumed equal to this gas space.

For example, using the above 17 per cent gas space, a 5-spot unit pattern containing 100 acre feet of formation and a porosity of 19 per cent or 1474 barrels per acre foot, the amount of water required for fill up is 100 (1474x.17), or 25,000 barrels. The time required at an injection rate of

100 barrels per day per 5-spot is, therefore, 250 days.

It should be noted that thus far the calculations have not included a flood coverage factor which even under the most favorable conditions may limit the actual recovery to some 70 to 90 per cent of the calculated maximum value at 100 per cent water cut. (The term "water cut", as used in the text, is intended to be synonymous with the phrase "water per cent by volume".) The flood coverage factor may be estimated from experience or electrical model studies.

After the flood coverage factor and the unit recovery to 100 per cent water cut have been determined they may be applied to the previously obtained water cut versus recovery data to convert to recovery in terms of barrels. These data and the assumed water injection rate furnish the necessary information for determining the time behavior of the flood unit. Illustrated in Table 5 are the stepwise calculations for determining the cumulative oil recovery, oil rate, and cumulative water injected versus time.

From these basic results and other derived information, such as total injection water requirements and time to reach economic limit, can be determined the economic feasibility of a water flood project.

TABLE 4

Calculation of Water Cut

h=Fraction of Cumulative Thickness	C=Cumulative Capacity; Fraction	C•A[1]	C•A+(1-C)	$\frac{C \bullet A}{C \bullet A+(1-C)}$ =Wat:r cut Fraction
.00	*.000	.000	1.000	—
.01	.065	.092	1.027	.090
.02	.110	.156	1.046	.149
.05	.200	.284	1.084	.262
.10	.308	.437	1.129	.387
.20	.476	.676	1.200	.563
.30	.620	.880	1.260	.698
.40	.731	1.038	1.307	.794
.50	.812	1.153	1.341	.860
.60	.870	1.235	1.365	.905
.70	.917	1.302	1.385	.941
.80	.852	1.352	1.400	.966
.90	.980	1.392	1.412	.986
.95	.991	1.408	1.417	.994
1.00	1.000	1.420	1.420	1.000

*From capacity distribution curve.

$$^{1}A = \frac{Krw}{Kro} \times \frac{\mu o}{\mu w} \times \text{Ų} = \frac{.20}{.80} \times \frac{4.34}{.82} \times 1.073 = 1.42$$

COMPARISON OF ACTUAL AND PREDICTED BEHAVIOR

The presently described method of taking into account the variations in permeability has been used in a number of engineering studies of water flood projects. Figure 4 shows a comparison of the predicted and the actual recovery versus water cut relationship in one of these earlier projects which has now progressed sufficiently to make possible such comparison. The behavior of this project, a Benoist Sand flood in Illinois, was calculated by the above method prior to the start of injection.

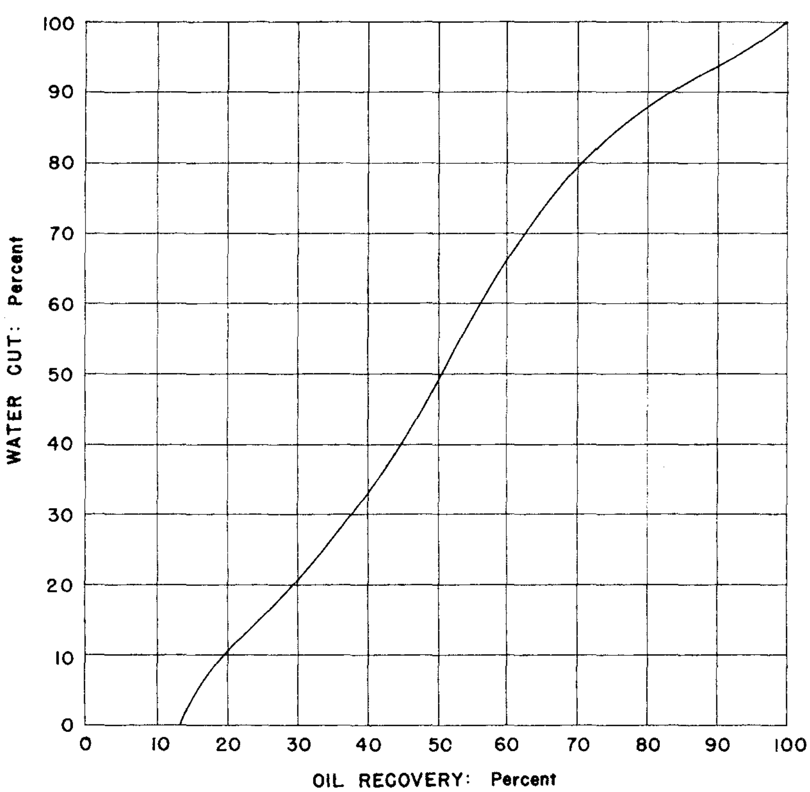

FIG. 3 — CALCULATED WATER CUT VS. RECOVERY

LIMITATIONS

The limitations of the above method of calculating water flood behavior should be pointed out. In particular, this method should not be applied where there is present a gas zone or water zone immediately above or below the oil zone under consideration. In this event there would be by-passing of the oil zone by injected water, or there would be coning and the oil recovery to any given water cut would be less than the calculated recovery. However, in the case of gas or water zones of known permeability certain modifications can be made in the basic equations to adjust for those conditions.

The water cut recovery curve should not be interpreted as a prediction of the behavior of any individual well, since structural consideration may make individual recoveries greater or less than the calculated value. Instead, the water cut recovery curve must be considered an average relationship for an entire field assuming a uniformly spaced flood is established therein. The water cut-recovery relationships should be based on the permeability and capacity distribution of a large number of permeability measurements from many wells in the area to be flooded.

This method does not take into account all factors which may influence the production history, such as the presence of gas or water zones, distance from fluid contacts, rate of production, structural position of the individual wells, lateral versus upward encroachment, shape of field, spacing pattern effect, etc. As more data and experience is obtained the effect of these factors will be better understood. In

TABLE 5

Predicted Performance of 5-Spot at 100 Bbls/Day Injection Rate

Cumulative Recovery; Fraction of Total	Cumulative Recovery; Barrels	Δ Recovery; Barrels	Water cut; Fraction	Average Water cut; Fraction	Average Oil Rate; bbls†day = 100(1-W)	Days to Produce Δ Recovery at Average Oil Rate	Cumulative Days after Fill-up	Cumulative Water Injected; Barrels
.000	*0	—	.000	.000	100.0	—	0.0	25,100 (fill-up)
.133	5,664	5,664	.000	.000	100.0	56.6	56.6	30,760
.200	8,517	2,853	.102	.051	94.9	30.1	86.7	33,770
.250	10,646	2,129	.147	.125	87.5	24.3	111.0	36,200
.300	12,776	2,130	.204	.176	82.4	25.8	136.8	38,780
.350	14,905	2,129	.264	.234	76.6	27.8	164.6	41,560
.400	17,034	2,129	.328	.296	70.4	30.2	194.8	44,580
.450	19,163	2,129	.397	.363	63.7	33.4	228.2	47,920
.500	21,293	2,130	.485	.441	55.9	38.1	266.3	51,730
.600	25,551	4,258	.667	.576	42.4	100.4	366.7	61,770
.700	29,810	4,259	.799	.733	26.7	159.5	526.2	77,720
.800	34,068	4,258	.882	.841	15.9	267.8	794.0	104,500
.850	36,197	2,129	.911	.897	10.3	206.7	1,000.7	125,170
.900	38,327	2,130	.938	.925	7.5	284.0	1,284.7	153,570
.950	40,456	2,129	.964	.951	4.9	434.5	1,719.2	197,020
.970	41,307	851	.977	.970	3.0	283.7	2,002.9	225,390
.990	42,159	852	.991	.984	1.6	532.5	2,535.4	278,640

*Total recoverable oil in 5-spot = 100 acre-feet × 501 bbls†ac-ft. × 85% coverage = 42,585 bbls.

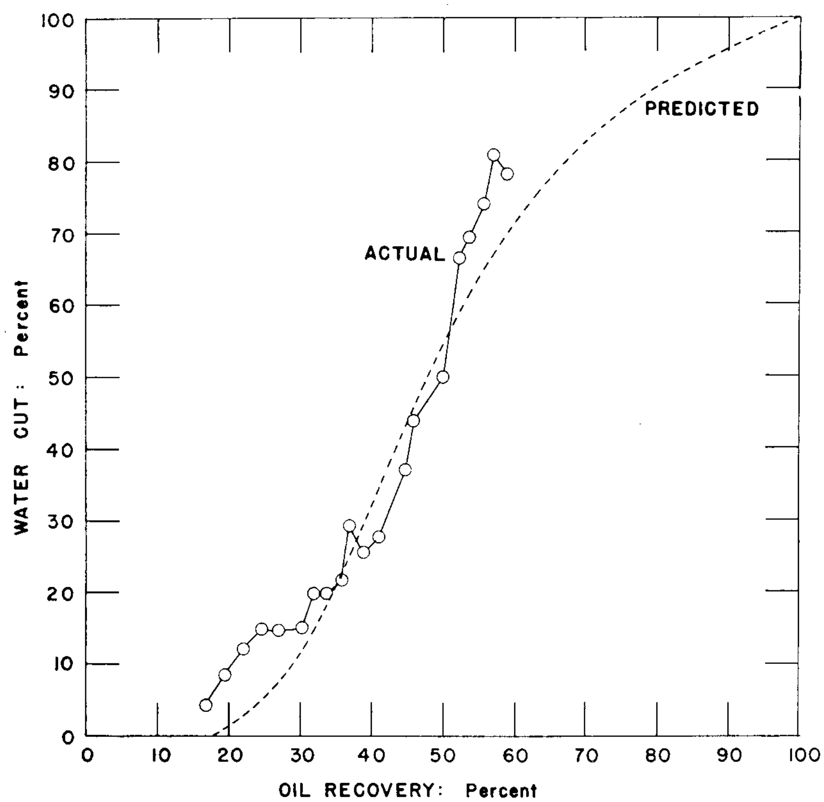

FIG. 4 — COMPARISON OF PREDICTED AND ACTUAL FLOOD PERFORMANCE.

the meanwhile the method herein presented must be considered an approach towards a mathematical treatment of the effect of permeability distribution in water flood performance.

SUMMARY

It has been shown that permeability variations in a reservoir may be represented by a permeability distribution curve and a capacity distribution curve.

Equations have been derived to incorporate mathematically the effect of the permeability distribution in the calculation of water flood recoveries.

Examples are presented to show the application of the equations to predictions of water flood performance in a depleted reservoir; however, the method may be applied to studies in "flush" fields.

A comparison of predicted and actual water cut-recovery relationship is shown.

Certain limitations of the method are presented and it is pointed out that the method is essentially an approximation intended to take into account principally the distribution of permeability in a reservoir.

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