Effect of Permeability Stratification in Cycling Operations

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Abstract

A general theory has been developed for the effect of permeability stratification on the efficiency of the gas-injection phase of cycling operations. It has been applied to three special types of permeability variation; namely, exponential, probability, and linear. In the case of the exponential permeability distribution the effect of areal pattern sweep efficiency was also taken into account.

The exponential permeability distribution can be characterized by the ratio of the maximum to minimum permeability, which has been termed the stratification constant. Curves were calculated for the variation in total wet gas recovery and total gas throughflow, to give that recovery to various abandonment limits of the wet gas content in the produced gas, as a function of the stratification constant. The cumulative wet gas recovery decreases monotonically as the stratification constant increases and is generally higher at the lower values of the wet gas content abandonment limits. The total gas throughflow first rises to a maximum as the stratification increases, and then ultimately declines. The effect of the areal sweep pattern efficiency is relatively minor as compared to that of the stratification constant, except in the region of low values of the latter where the formation is substantially uniform.

The probability distribution can be characterized by a "variation" parameter varying from 0 to 1 as the formation changes from strict uniformity to extreme variability. The curves of total wet gas recovery and total gas throughflow to different abandonment limits of wet gas content versus the variation parameter have the same general characteristics as for the exponential permeability distribution.

In the linear permeability distribution the ratio of maximum to minimum permeability also serves as a stratification constant index defining the distribution. The curves of total wet gas recovery and gas throughflow to fixed abandonment limits versus the stratification constant are similar to those for the exponential permeability distribution. However, for the higher values of the stratification constant the recoveries and throughflows do not asymptotically fall to 0 as in the latter, but approach constant values determined by the abandonment limit of wet gas content in the produced gas.

Introduction

It is becoming generally recognized that one of the most important factors determining the economic feasibility of cycling operations is the areal continuity and permeability distribution of the producing formation. While the effects of areal variations in permeability, porosity, thickness, and well pattern on the sweep efficiency can be evaluated by electrical model studies,[1] those due to permeability stratification require separate treatment. Several studies have been reported[2,3] on the influence of permeability variations on the wet gas recovery by cycling. In these, however, discontinuous permeability variations either have been assumed explicitly, or the analysis has been carried through as if they were discontinuous. While, in fact, the actual permeability profiles will undoubtedly be discontinuous, recent develop-

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[1] References are at the end of the paper.

ments[4] in the statistical analysis of such profiles suggest that often they can be approximated by continuous functions. Such representations are especially convenient because they can be characterized by simple parameters in the basic functional expressions. The sweep efficiency analysis for several such basic types of permeability functions will be given in this paper.

As in all analytical studies of the type to be considered here, the composite producing formation being cycled will be treated as a parallel superposition of the individual strata of different permeability and thickness.* Each, however, will be assumed to have constant effective permeability, thickness, and net porosity over the producing area. It will be assumed further that all the wells, both production and injection, completely penetrate all strata. Accordingly, there will be no crossflow as long as differences in mobility and density of the injected and displaced fluids are neglected, and the various strata can be re-arranged so as to give monotonic variations in the permeability with the depth.

General Theory

Assuming the actual formation strata to be rearranged to give a monotonic continuous permeability distribution $k(z)$,* where z is measured along the well bore from the layer of lowest permeability, the rate of reservoir throughflow, per unit thickness in the lamina at depth z, may be expressed as:

$$Q(z) = ck(z) \qquad [1]$$

where c is a constant determined by the areal geometry of the reservoir, the well distribution, and the relative injection and producing rates. Now for a fixed cycling pattern and operating plan the composition of the produced gas in a uniform zone will be a function only of the total gas throughflow,† expressed as a fraction of the hydrocarbon pore volume. The rate of wet gas production from a unit thickness lamina at z at the time t will therefore be:

$$Q_w(z,t) = ck(z)F\left(\frac{ctk(z)}{A\bar{f}(z)}\right) \qquad [2]$$

where F denotes the variation of the wet gas fraction in the produced gas with the total gas throughflow for a uniform stratum, as determined by the well pattern and flux distribution, and its argument is the cumulative gas throughflow divided by the displaceable hydrocarbon volume available at z. A is the reservoir area, and $\bar{f}(z)$ the net hydrocarbon displacement porosity‡ in the

* This idealized representation of perfect stratification and areal continuity will, of course, never occur in practice and lead to maximum by-passing effects. In actual reservoirs, where the permeability will vary laterally, with its own statistical distribution, at least the initial breakthrough sweep efficiencies will be higher than calculated here. The analytical treatment of such heterogeneous systems, however, would be extremely difficult even for the simplest types of permeability distributions. It is also assumed throughout this paper that gross steady state conditions obtain during the cycling operations in the sense that the rate of total gas injection, in reservoir measure, is the same as the rate of total gas withdrawals. This implies that make-up gas is provided to replace the losses due to shrinkage and use of some of the produced gas for fuel.

* In the formal theory of this section the re-arrangement is actually assumed to provide a monotonic variation of $k(z)/\bar{f}(z)$. In the treatment of specific cases, however, as will be done in the following sections, it is necessary to specify $k(z)$ and $\bar{f}(z)$ separately, and for convenience the latter will be taken as constant.

† In actual practice, of course, the effective well pattern and rate distributions will change during the cycling life as producing wells are abandoned or converted to injection wells when their wet gas production becomes too low. While such effects could be formally included by proper choice of the function F, no such detailed interpretation will be attempted here.

‡ $\bar{f}$ represents the net porosity occupied by the invading fluid, and may be taken simply as the net hydrocarbon porosity, since there is little evidence of an important degree of microscopic mixing in gas displacement processes.

rearranged system defined by $k(z)$. For brevity the argument of F will be denoted by u. The fraction of wet gas in the total effluent from the stratified formation, at the time t, will then be:

$$R_w(t) = \frac{\int_o^H k(z)F(u)\,dz}{\int_o^H k(z)\,dz}; \qquad u = \frac{ctk(z)}{A\bar{f}(z)} \qquad [3]$$

where H is the total thickness of the permeable pay.

Eq 3 defines the composition history of the production as a function of time. It can be related implicitly to the total fractional reservoir sweep by noting that the total wet gas produced at time t is:

$$\left.\begin{aligned} \bar{Q}_w(t) &= \int_o^t dt \int_o^H Q_w(z,t)dz \\ &= c\int_o^t dt \int_o^H k(z)F(u)dz \\ &= Q_o \int_o^t R_w(t)dt \end{aligned}\right\} \qquad [4]$$

where Q_o is the throughflow rate, assumed constant, from the composite formation. The fractional reservoir sweep is, then,

$$\bar{V}(t) = \frac{\bar{Q}_w(t)}{A\int_o^H \bar{f}dz} \qquad [5]$$

To proceed further, it is noted from the definition of the function F, and its argument u, that while it will vary with each cycling pattern, it must always satisfy the relations:

$$\int_o^\infty F(u)du = 1; \qquad \int_S^\infty F(u)du = 1 - S; \qquad F(u) = 1; \qquad u \leq S \qquad [6]$$

where S is the areal sweep efficiency, or the fractional displaceable reservoir volume of a uniform stratum swept out by the time of first gas breakthrough. Then if the permeability range in the producing formation has a nonvanishing lower bound and a finite maximum value, as will generally obtain except in the ideal probability distribution,* the composition time history can be divided into three segments as follows. For such values of t before any breakthrough has developed, that is, for:

$$t \leq \frac{AS}{c}\left(\frac{\bar{f}}{k}\right)_{z=H} \equiv t_b:$$

$$F = 1; \quad R_w(t) = 1; \quad \bar{V} = \frac{Q_o t}{A\int_o^H \bar{f}dz} \qquad [7]$$

At times t between t_b and the time for breakthrough in the tightest lamina, t_m, that is, for:

$$t_b \leq t \leq t_m = \frac{AS}{c}\left(\frac{\bar{f}}{k}\right)_{z=0}:$$

$$R_w(t) = \frac{c\int_o^{z_0} k(z)dz + c\int_{z_0}^H k(z)F(u)dz}{Q_o}: \qquad u = \frac{ct\,k(z)}{A\bar{f}(z)} \qquad [8]$$

where:

$$\frac{k(z_o)}{\bar{f}(z_o)} = \frac{AS}{ct}$$

The cumulative fractional reservoir sweep will be:

$$\bar{Q}_w(t) = \left[ct\int_o^{z_0} k(z)dz + SA\int_{z_0}^H \bar{f}dz + c\int_{z_0}^H k(z)dz \int_{\frac{AS\bar{f}(z)}{ck(z)}}^t F\left(\frac{c\tau k(z)}{A\bar{f}(z)}\right)d\tau\right] \Big/ A\int_o^H \bar{f}dz \qquad [9]$$

Finally, after breakthrough in the tightest layer, that is:

$$t \geq t_m$$

$$R_w(t) = \frac{c\int_o^H k(z)F(u)dz}{Q_o} \qquad [10]$$

The cumulative wet gas recovery and reservoir sweep will be given by the general expressions of Eq 4 and Eq 5.

* In this case, as will be seen below, the intermediate segment ($t_b \leq t \leq t_m$) comprises the whole composition history.

Eq 7 to 10 provide the basis for treating any type of permeability distribution and the effects of incomplete areal sweep efficiency. They will be illustrated in the following sections by a detailed analysis of the exponential, probability, and linear permeability distributions, including the effects of the areal sweep efficiency for the first of these.

Exponential Permeability Variation

Although there is little statistical evidence that actual condensate reservoir formations have an exponential type of permeability variation, the exponential function is convenient for graphical approximation, and arbitrary distributions can often be resolved into approximate exponential segments. Moreover, with the exponential distribution it is possible to derive closed expressions for the cycling history even when account is taken of the incompleteness of the areal pattern sweep efficiency and the declining wet gas content of the produced gas following the initial dry gas breakthrough.

The exponential permeability variation may be defined by:

$$k(z) = ae^{bz/H} \qquad [11]$$

The constant a evidently represents the minimum permeability ($z = 0$). The maximum permeability is ae^b. As the absolute permeability is of no importance with respect to the effects of stratification, the parameter defining the exponential distribution for the present purposes may be conveniently chosen as the ratio, $r = e^b$, of the maximum to minimum permeability. This "stratification constant" will hereafter be used as an index of the exponential distribution.

As indicated previously, the displacement porosity $\bar{f}$ will be taken as constant.*

The function F will, of course, depend on the well pattern and the areal characteristics of the reservoir. It will have no universal form, although it will be subject to Eq 6. Since a specific form must be chosen in order to apply the general equations of the last section, it will be assumed that:

$$F(u) = 1: \quad u \leq S: \quad F(u) = e^{\frac{S-u}{1-S}}: \quad u \geq S \qquad [12]$$

This form satisfies Eq 6, and roughly approximates the calculated variation of F in special cases. While not quantitatively accurate in general, it should provide a fair approximation of the effects due to the failure of the well pattern to give 100 pct sweep efficiency.

Introducing now the notation:

$$\bar{t} = t/t_b: \quad r = \frac{t_m}{t_b}: \quad b = \log r \qquad [13]*$$

where, by Eq 8, $t_b = \dfrac{AS\bar{f}}{acr}$, it is found by applying Eq 7 that for:

$$t \leq 1:$$

$$R_w(\bar{t}) = 1; \quad \bar{V}(\bar{t}) = \frac{S(r-1)\bar{t}}{rb} = \bar{Q}(\bar{t}) \qquad [14]$$

$$1 \leq \bar{t} \leq r:$$

$$R_w(\bar{t}) = \frac{1}{r-1}\left[\frac{r}{S\bar{t}} - 1 - \frac{(1-S)r}{St} e^{\frac{S}{1-S}(1-\bar{t})}\right] \qquad [15]$$

$$\bar{V}(\bar{t}) = \frac{1}{b}\log \bar{t} + \frac{S}{b}(1 - \bar{t}/r) - \frac{(1-S)}{b} e^{\frac{S}{1-S}} \left[Ei\left(\frac{-S\bar{t}}{1-S}\right) - Ei\left(\frac{-S}{1-S}\right)\right] \qquad [16]$$

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* The assumed constancy of $\bar{f}$, for the specific permeability distributions treated in detail, further tends to give maximal values for the stratification effects. Aside from the possibility of general trends of decreasing total porosity with decreasing permeability, the $\bar{f}$ should decrease with k because of higher connate water saturations in the low permeability strata. By-passing of the latter therefore should not be as serious from a volumetric standpoint as would be indicated by the calculations based on strictly uniform values of $\bar{f}$.

* The expression for r as the ratio t_m/t_b is, in the light of Eq 7 and 8, evidently equivalent to the previously indicated definition of r as the ratio of the maximum to minimum permeability.

$$\bar{t} \geq r:$$

$$R_w(\bar{t}) = \frac{r(1 - S)e^{\frac{S}{1-S}}}{S(r - 1)\bar{t}} \left[e^{-S\bar{t}/(1-S)r} - e^{-S\bar{t}/(1-S)}\right] \quad [17]$$

$$V(\bar{t}) = 1 - \frac{(1 - S)}{b} e^{\frac{S}{1-S}} \left[Ei\left(\frac{-S\bar{t}}{1 - S}\right) - Ei\left(\frac{-S\bar{t}}{(1 - S)r}\right)\right] \quad [18]$$

In all cases the total throughput at the time $\bar{t}$, as a fraction of the net reservoir pore volume, is:

$$\bar{Q}(\bar{t}) = \frac{S(r - 1)\bar{t}}{rb} \quad [19]$$

It will be readily verified that Eq 14 to 18 are continuous at their mutual contact points. In particular, at the time of breakthrough in the tightest zone, $\bar{t} = r$, Eq 15 and 17 give for $R_w(\bar{t})$:

$$R_w(r) = \frac{1 - S}{S(r - 1)} \left(1 - e^{\frac{S}{1-S}(1-r)}\right) \quad [20]$$

which reduces to the coefficient for $r \geq 1$. And for $\bar{V}(r)$, Eq 16 and 18 give:

$$\bar{V}(r) = 1 - \frac{(1 - S)}{b} e^{\frac{S}{1-S}} \left[Ei\left(\frac{-Sr}{1 - S}\right) - Ei\left(\frac{-S}{1 - S}\right)\right] \quad [21]$$

which has the asymptotic value:

$$\bar{V}(r) = 1 - \frac{(1 - S)^2}{Sb} \quad [22]$$

In the limiting case of a uniform formation, $b \to 0$, $r \to 1$, Eq 14 to 19 reduce to:

$$\left.\begin{array}{l} \bar{t} \leq 1: \\ R_w(\bar{t}) = 1: \quad \bar{V}(\bar{t}) = (S\bar{t}) = \bar{Q}(\bar{t}) \\ \bar{t} \geq 1: \\ R_w(\bar{t}) = e^{\frac{S}{1-S}(1-\bar{t})} = F(S\bar{t}); \\ V(\bar{t}) = 1 - (1 - S)e^{\frac{S}{1-S}(1-\bar{t})} \\ \qquad = 1 - (1 - S)F(S\bar{t}); \\ \bar{Q}(\bar{t}) = S\bar{t}, \text{ for all } \bar{t} \end{array}\right\} \quad [23]$$

In the limit of 100 pct areal sweep efficiency, $S = 1$, Eq 14 to 19 reduce to:

$$\left.\begin{array}{l} \bar{t} \leq 1: \\ R_w(\bar{t}) = 1; \quad \bar{V}(\bar{t}) = \frac{(r - 1)\bar{t}}{rb} = \bar{Q}(\bar{t}) \\ 1 \leq \bar{t} \leq r: \\ R_w(\bar{t}) = \frac{1}{r - 1}\left(\frac{r}{\bar{t}} - 1\right) \\ \bar{V}(\bar{t}) = \frac{1}{b}\left[1 - \frac{\bar{t}}{r} + \log \bar{t}\right]; \\ \bar{t} \geq r: \\ R_w(\bar{t}) = 0; \quad V(\bar{t}) = 1; \\ \bar{Q}(\bar{t}) = \frac{(r - 1)\bar{t}}{rb}, \text{ for all } \bar{t} \end{array}\right\} \quad [24]$$

For the intermediate time interval, $\bar{Q}$ and $\bar{V}$ can be expressed directly as functions of R_w as:

$$\left.\begin{array}{l} \bar{Q} = \dfrac{r - 1}{b[1 + (r - 1)R_w]}, \\ \bar{V} = 1 - \dfrac{1}{b}\left[\log\{1 + (r - 1)R_w\} - \dfrac{(r - 1)R_w}{1 + (r - 1)R_w}\right] \end{array}\right\} \quad [25]$$

To illustrate these general relationships, the wet gas content and cumulative wet gas recovery have been plotted in Fig 1 versus the total gas throughflow for $S = 0.60, 0.75, 0.90$, and for $r = 1, 10$, and 100. $r = 10$ corresponds to a ratio of the maximum to minimum permeability equal to 10, and this ratio is 100 for $r = 100$. $r = 1$ represents the strictly uniform reservoir. The abscissa values $\bar{Q}$ represent the total gas injection, or production, divided by the total hydrocarbon pore volume. $\bar{Q}$ is related to the argument $\bar{t}$ of Eqs 14 to 18 by Eq 19. The crosses denote the states of first dry gas breakthrough in the most permeable zone, and the circles indicate breakthrough in the tightest layer. The curves for $r = 1$ simply reflect the functional form assumed for F, as required by Eq 23.

It will be noted from Fig 1 that whereas in a uniform formation the dry gas breakthrough will develop after a total throughflow equal to the sweep efficiency S, dry gas will first appear in the producing wells for $r = 10$ after a throughflow of only 23.4, 29.3, and 35.2 pct of the total hydrocarbon pore volume, for $S = 0.60$, 0.75, and 0.90, respectively. These throughflows also represent the fractions of total displaceable reservoir wet gas content produced by the time of first dry gas breakthrough. And for $r = 100$, the corresponding breakthrough periods will represent recoveries of 12.9, 16.1, and 19.4 pct of the original wet gas content.

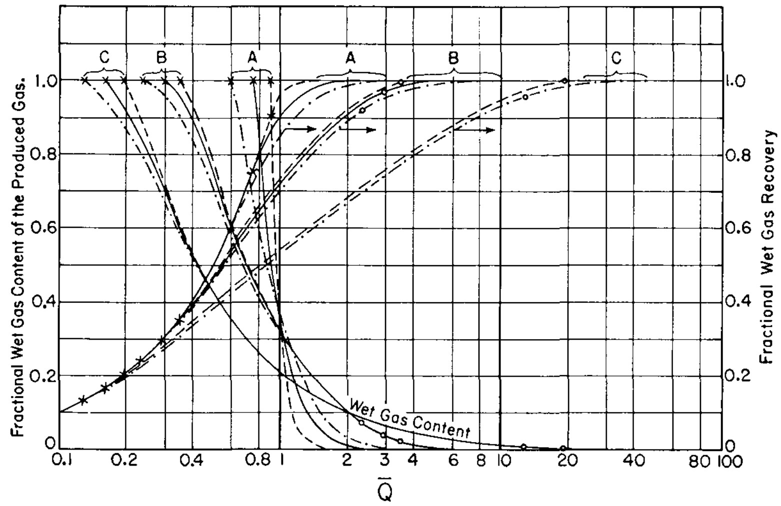

FIG 1—THE CALCULATED VARIATIONS OF THE FRACTIONAL WET GAS CONTENT OF THE PRODUCED GAS AND OF THE TOTAL FRACTIONAL WET GAS RECOVERY VERSUS THE TOTAL GAS PROCESSED IN CYCLING OPERATIONS, IN FORMATIONS WITH EXPONENTIAL PERMEABILITY DISTRIBUTIONS.

$\bar{Q}$ = total gas processed in units of the initial displaceable hydrocarbon pore volume. S = areal sweep efficiency; r = ratio of maximum to minimum permeability, = 1, 10, and 100 for curves A, B, and C. ———: $S = 0.90$; ———·: $S = 0.75$; —·—·: $S = 0.60$. Crosses denote states of first dry gas breakthrough; circles represent states of breakthrough in the tightest strata.

For dry gas breakthrough in the tightest layers, with $r = 10$, the total gas processed will correspond to 2.34, 2.93, and 3.52 times the reservoir hydrocarbon pore volume,* for $S = 0.60$, 0.75, and 0.90. By that time the wet gas content of the produced gas will be 7.41, 3.70, and 1.23 pct, respectively. And the total wet gas recovery will be 92.2, 97.2, and 99.56 pct the initial reservoir content. For $r = 100$, the volumes of gas processed before breakthrough in the tightest layer will be 12.90, 16.12, and 19.35 times the reservoir hydrocarbon pore volume, for $S = 0.60$, 0.75, and 0.90. The produced gas will then have wet gas contents equal to 0.68, 0.33, and 0.11 pct, respectively. And the total wet gas recoveries will be 96.1, 98.6, and 99.99 pct of the initial wet gas content of the reservoir.

As shown in Fig 1, the total gas processed and wet gas recovery by the time of first dry gas breakthrough are directly proportional to the sweep efficiency. And the cumulative wet gas recovery curves remain somewhat higher for all values of $\bar{Q}$ after breakthrough for the higher values of S. The wet gas content curves, however, first tend to merge and ultimately cross, al-

* The reservoir hydrocarbon volume used as a unit for expressing the abscissa variable in Fig 1 and in these comparisons is gas volume under reservoir conditions equal to the absolute displaceable net pore volume, and not the volume of the reservoir gas in surface measure. The recoveries are also expressed as fractions of the reservoir content displaceable by cycling. The latter, however, will be equivalent to the actual hydrocarbon content if f is taken as the net hydrocarbon porosity.

though the latter divergence is so slight for $r = 10$, and 100 that it could not be shown on the scale of Fig 1. For $r = 1$ the crossing point lies at $\bar{Q} = 1$, by virtue of the functional form assumed for $F(u)$.

effects of stratification may be far more serious in limiting the total condensate recoveries than the areal sweep efficiency. Thus for $r = 100$, which does not represent an abnormally high degree of stratification

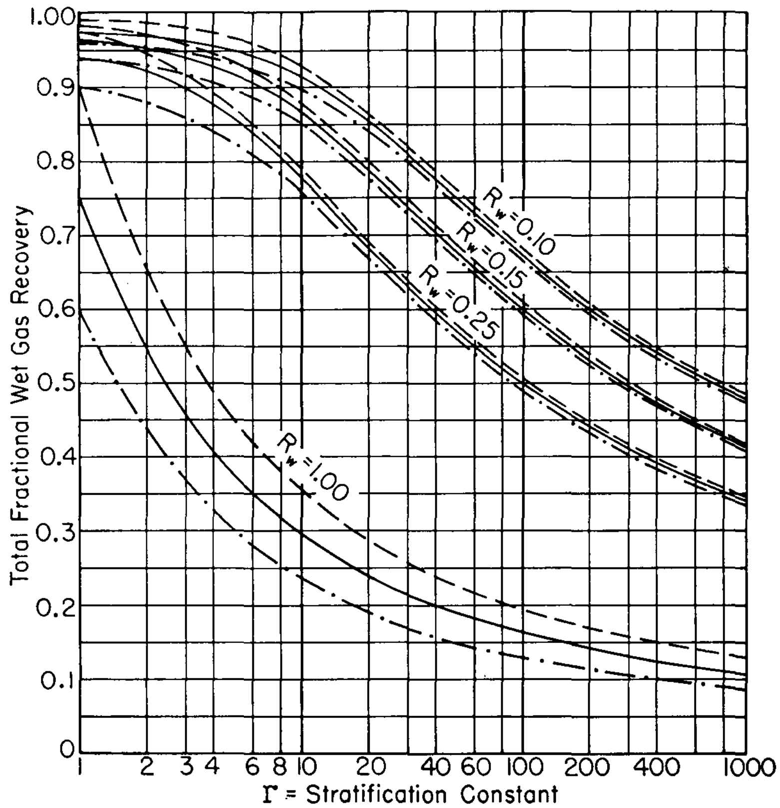

FIG 2—THE CALCULATED VARIATIONS OF THE TOTAL FRACTIONAL WET GAS RECOVERY VERSUS THE STRATIFICATION CONSTANT, r, IN CYCLING OPERATIONS IN FORMATIONS WITH EXPONENTIAL PERMEABILITY DISTRIBUTIONS.

r = ratio of maximum to minimum permeability. R_w = fractional wet gas content of the produced gas at abandonment of cycling. — — —: $S = 0.90$; ———: $S = 0.75$; —·—·: $S = 0.60$; S = areal sweep efficiency.

The variation of the total wet gas recovery versus r by the time the wet gas content falls to fixed limits, at which further injection may become unprofitable, is plotted in Fig 2. As is to be expected, the recovery curves decrease continually with increasing values of r or degree of stratification. For high values of r the recovery assumes an approximately logarithmic decline with increasing r. Fig 2 shows that the as compared to those often observed, the total recovery at an abandonment limit of 15 pct wet gas content will be only 61 pct even if the areal sweep efficiency is 90 pct. The curves for $S = 1$ are not plotted in Fig 2 because they would overlap completely those for $S = 0.90$ for $r > 10$, and would be interlaced with the other curves plotted for lower values of r.

The curves in Fig 2 for $R_w = 1$ represent the fractional recoveries at the time of first dry gas breakthrough. They are given by:

$$\bar{V}(R_w = 1) = \frac{S(r - 1)}{r \log r} \qquad [26]$$

and may be considered as the composite sweep efficiency resulting both from the well pattern and permeability stratification. It will be seen that even for $r = 100$ the permeability stratification will reduce the composite sweep efficiency almost by a factor of 5 compared to the areal sweep efficiency, S. It is evidently because of the continued cycling operation to rather low wet gas contents after the initial dry gas breakthrough that the total wet gas recoveries in practice will represent significant fractions of the original reservoir contents, as indicated by the upper curves of Fig 2. As implied by Eq 26, the recovery to the time of initial breakthrough is directly proportional to S, in contrast to the much slower variation with S at low values of R_w.

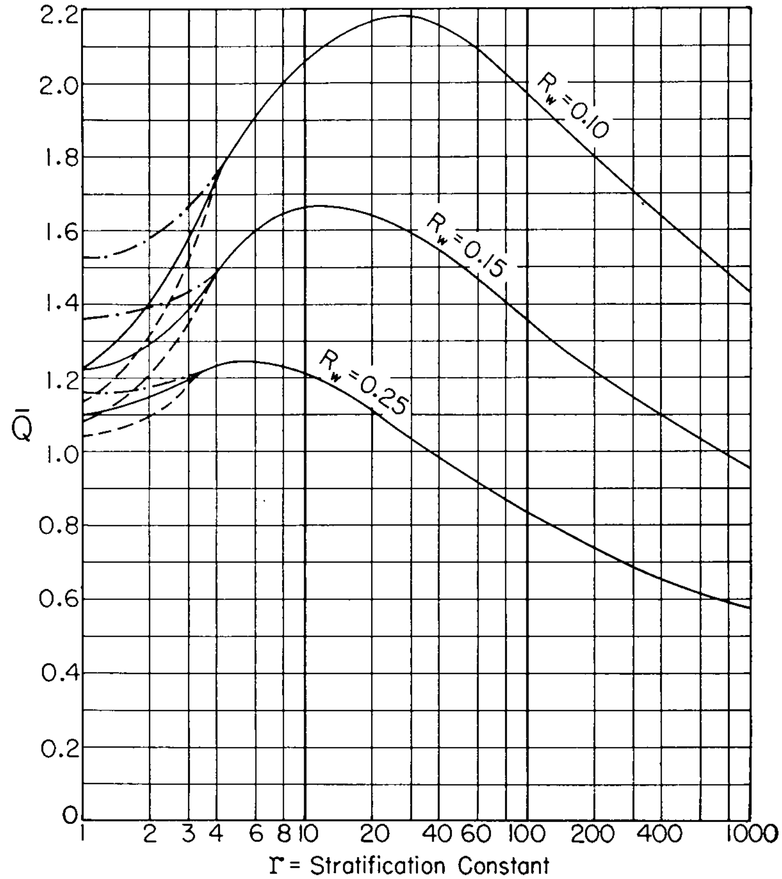

FIG 3—THE CALCULATED VARIATION OF THE TOTAL GAS THROUGHFLOW, $\bar{Q}$, IN UNITS OF THE INITIAL DISPLACEABLE HYDROCARBON PORE VOLUME, VERSUS THE STRATIFICATION CONSTANT, r, IN CYCLING OPERATIONS IN FORMATIONS WITH EXPONENTIAL PERMEABILITY DISTRIBUTIONS.

r = ratio of maximum to minimum permeability. R_w = fractional wet gas content of the produced gas at abandonment of cycling. ———: S = 0.90; – – –: S = 0.75; —·—·: S = 0.60; S = areal sweep efficiency.

The total volumes of gas throughflow or processed, in reservoir measure, and as fractions of the total reservoir hydrocarbon volume, are plotted versus r in Fig 3, for various abandonment limits for the wet gas content.* It will be observed that these are affected by the areal sweep efficiency only at the lower values of r. In fact $r > 5$, the total throughflow to abandonment is, for practical purposes, independent of S. Moreover, the curves all show maxima in the range of r of 5–30, and then decline as r is still further increased. The initial rise in the curves of Fig 3 is due to the increasing throughflow required to give the approximately constant total wet gas displacements indicated by Fig 2 as the formation becomes increasingly nonuniform. The ultimate declines reflect the corresponding reductions in total wet gas recovery, shown in Fig 2 at high stratification ratios, which can be swept out by relatively small volumes of injected gas. It will be noted also that the volumes of gas processed will vary more rapidly with the abandonment limit of wet gas content than the total wet gas recovery.

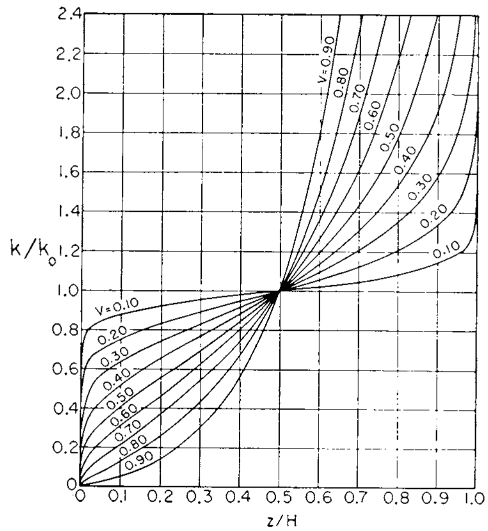

FIG 4—CARTESIAN PLOTS OF THE PERMEABILITY, k, VERSUS THE DFPTH, z, IN IDEAL PROBABILITY DISTRIBUTIONS.

k_o = most probable value of k. H = total net thickness of productive section. V = "variation" of the probability distribution.

* The total gas throughflows for $R_w = 1.00$ are evidently equal to the total wet gas recoveries for $R_w = 1.00$ plotted in Fig 2.

PROBABILITY DISTRIBUTION

The probability distribution in permeability may be defined by the equation:

$$\frac{dz}{d\Psi} = \frac{H}{\sigma\sqrt{2\pi}} e^{-\Psi^2/2\sigma^2}; \qquad \Psi = \log k/k_o \quad [27]$$

For convenience in subsequent analysis the logarithm to the base e has been used in defining Ψ, rather than $\sqrt{2}$, as introduced in previous work.[4] k_o is the permeability of maximum probability. σ is the standard deviation, and is related to the "variation" V by[3]:

$$\sigma = -\log(1 - V); \qquad V = 1 - e^{-\sigma} \quad [28]$$

H is the total thickness of the productive section. If the probability distribution is strictly satisfied it includes the complete permeability range of 0 to ∞. While this is not to be expected from both physical and practical standpoints, the actual volumetric and flow capacity contributions of the regions of very low and very high permeabilities will be extremely small because of the inherent characteristics of the probability distribution. A Cartesian coordinate plot of the permeability distribution for various values of V, as implied by Eq 27, is given in Fig 4.

It may then be shown that:

$$\left.\begin{aligned} R_w(t) &= \frac{1}{2}\left[1 + I\left(\frac{\bar{\Psi}}{\sqrt{2}\,\sigma} - \frac{\sigma}{\sqrt{2}}\right)\right]: \quad \bar{\Psi} > \sigma^2 \\ &= \frac{1}{2}\left[1 - I\left(\frac{\sigma}{\sqrt{2}} - \frac{\bar{\Psi}}{\sqrt{2}\,\sigma}\right)\right]: \quad \bar{\Psi} < \sigma^2 \end{aligned}\right\} \quad [31]$$

where I is the probability integral.

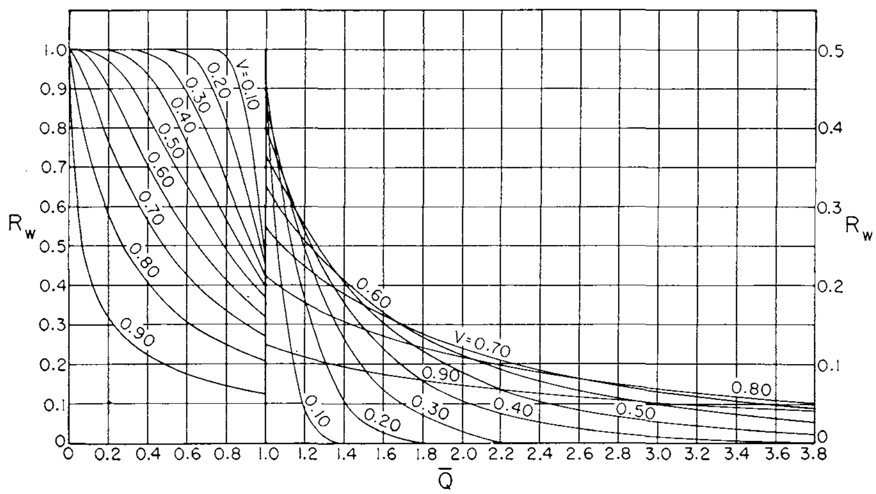

FIG 5—THE CALCULATED COMPOSITION HISTORIES FOR CYCLING SYSTEMS WITH PROBABILITY DISTRIBUTIONS IN PERMEABILITY.

R_w = fractional wet gas content of the produced gas. $\bar{Q}$ = total gas throughflow in units of the initial displaceable hydrocarbon pore volume. V = "variation" of the probability distribution. For $\bar{Q} > 1$, R_w scale is that on right. Areal sweep efficiency assumed = 1.00.

The total flow capacity of the section with the permeability distribution of Eq 27 will be:

$$Q_o = cHk_oe^{\sigma^2/2} \quad [29]$$

where c is essentially the same constant as introduced in Eq 2. To determine the composition history of the produced gas, it is noted that at time t there will develop breakthrough* in all strata for which Ψ exceeds $\bar{\Psi}$, given by:

$$\bar{\Psi} = \log\frac{A\bar{f}}{ck_ot} \quad [30]$$

The cumulative fractional wet gas recovery, as a function of t or $\bar{\Psi}$ is given by:

$$\left.\begin{aligned} \bar{V}(t) &= \bar{Q}R_w(t) + \frac{1}{2}\left[1 - I\left(\frac{\bar{\Psi}}{\sqrt{2}\,\sigma}\right)\right]: \quad \bar{\Psi} > 0 \\ &= \bar{Q}R_w(t) + \frac{1}{2}\left[1 + I\left(\frac{-\bar{\Psi}}{\sqrt{2}\,\sigma}\right)\right]: \quad \bar{\Psi} < 0 \end{aligned}\right\} \quad [32]$$

where $\bar{Q}$ is again the cumulative throughput, expressed in units of the total net displaceable pore volume, and is related to $\bar{\Psi}$ by:

$$\bar{Q}(t) = e^{-\bar{\Psi}+\sigma^2/2} \quad [33]$$

* It is assumed throughout this treatment of the probability distribution that the areal sweep efficiency, S, is 100 pct.

Thus, on choosing σ or V, through Eq 28, $\bar{\Psi}$ can be computed as a function of $\bar{Q}$ from Eq 33. Eqs 31 and 32 will then give R_w and $\bar{V}$ as functions of $\bar{Q}$.

complete R_w curves must all be equal to unity, they ultimately cross in Fig 5, as they do in Fig 1, at higher values of $\bar{Q}$. To minimize the complex overlapping in the region

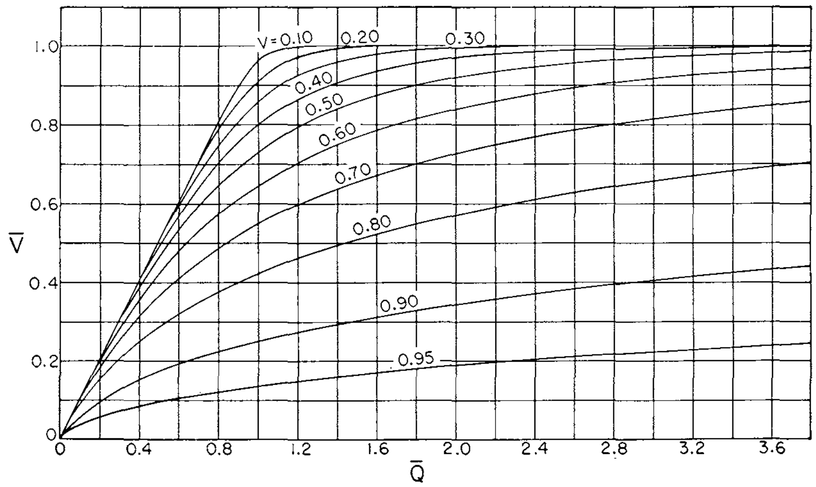

FIG 6—THE CALCULATED CUMULATIVE WET GAS RECOVERY HISTORIES FOR CYCLING SYSTEMS WITH PROBABILITY DISTRIBUTIONS IN PERMEABILITY.

$\bar{V}$ = cumulative wet gas recovery in units of the initial total displaceable wet gas reservoir content. $\bar{Q}$ = total gas throughflow in units of the initial displaceable hydrocarbon pore volume. V = "variation" of the probability distribution.

The composition histories so calculated for fixed values of the variation, V, are plotted in Fig 5. Although a Cartesian scale is used here for the total fractional throughflow, $\bar{Q}$, it will be seen that qualitatively the curves are of the same character as those plotted in Fig 1 for an exponential permeability variation. As is to be expected, the period of undiluted* wet gas production increases as V decreases, that is, as the formation becomes more uniform. Moreover, after breakthrough the decline in wet gas content falls more sharply as V decreases. Finally, since the areas under the

* Strictly speaking there will necessarily be an immediate dry gas breakthrough for all values of V in a strict and complete probability distribution system. However, as is clear from Fig 5, there is always an initial segment for which $R_w = 1$ from a practical standpoint. Moreover, the R_w versus $\bar{Q}$ curves always begin with zero slope at $\bar{Q} = 0$, although this could not be shown for the higher values of V.

of crossing, the R_w scale in Fig 5 has been expanded by a factor of 2 for $\bar{Q} > 1$.

The cumulative wet gas recovery for fixed variation parameters, V, is plotted versus the total gas throughflow in Fig 6. These curves, similar to those of Fig 1 for the exponential permeability distribution, show the increasingly rapid completion of the displacement of the wet gas as the variation decreases, that is, as the formation becomes more uniform. Thus with a variation of only 0.10, 96 pct of the wet gas content of the reservoir would be displaced after a throughflow of only 1 reservoir net displaceable pore volume, for 100 pct areal sweep efficiency. On the other hand, if the variation were 0.95, only 22 pct of the displaceable reservoir wet gas content would be displaced even after a throughflow of 3 reservoir pore volumes.

Analogous to Fig 2, the cumulative wet gas recovery to limiting values of wet gas content in the produced gas, R_w, is plotted versus the variation V in Fig 7. It will be seen that for variations less than 0.5 more than 80 pct of the displaceable wet gas content of the reservoir will be recovered during the cycling operations even if the latter be abandoned at a wet gas content of the produced gas of 25 pct. These recoveries would be reduced somewhat in actual systems where the areal sweep efficiencies are less than 100 pct. At high variations, however, the areal sweep efficiency will be of smaller importance, as may be inferred from Fig 2.

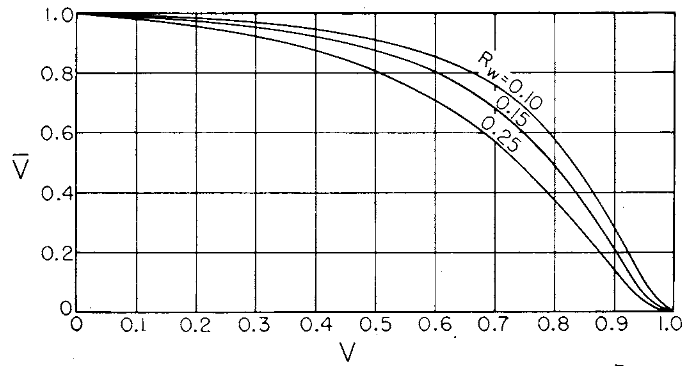

FIG 7—THE CALCULATED CUMULATIVE WET GAS CYCLING RECOVERIES, $\bar{V}$, IN UNITS OF THE INITIAL TOTAL DISPLACEABLE WET GAS RESERVOIR CONTENT, FROM FORMATIONS WITH PROBABILITY DISTRIBUTIONS IN PERMEABILITY VERSUS THE "VARIATION," V, OF THE DISTRIBUTION.

R_w = fractional wet gas content of the produced gas at abandonment of cycling.

The total gas throughflow, as a function of the variation, for fixed limiting values of R_w, is plotted in Fig 8. These curves show maxima similar to those of Fig 3 for the exponential permeability distribution, and for the same reasons. Moreover, the values of these maxima are only slightly greater than those of Fig 3.

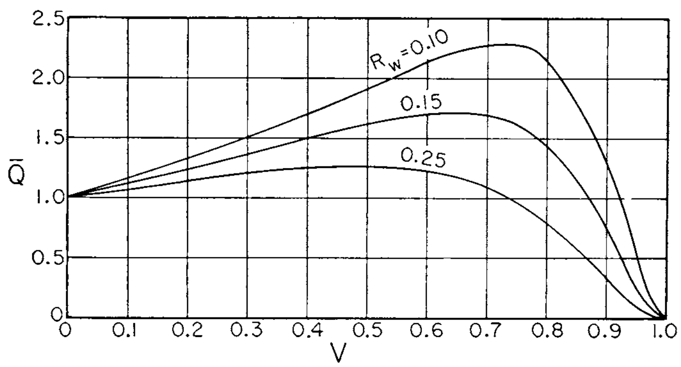

FIG 8—THE CALCULATED TOTAL GAS THROUGHFLOW, $\bar{Q}$, IN UNITS OF THE INITIAL DISPLACEABLE HYDROCARBON PORE VOLUME, IN CYCLING OPERATIONS WITH PROBABILITY DISTRIBUTIONS IN PERMEABILITY VERSUS THE "VARIATION," V, OF THE DISTRIBUTION.

R_w = fractional wet gas content of the produced gas at abandonment of cycling.

LINEAR PERMEABILITY VARIATION

As a final type of a continuous permeability distribution, the results to be expected for a linear variation will be briefly outlined. This, too, can be defined, by analogy with the exponential distribution, by a parameter r, representing the ratio of the maximum to minimum permeability, that is, by:

$$k(z) = k_o[1 + (r - 1)z/H] \qquad [34]$$

where k_o is the minimum permeability, at $z = 0$.

Although closed expressions can also be obtained for this case for the composition history with imperfect areal sweep patterns ($S < 1$), using an F function of the type of Eq 12, the study of the exponential case indicates that the exact value of S is of minor importance as long as it is of the order of 0.70 or greater. Assuming then for simplicity that $S = 1$, it is readily found, by the methods used for the exponential and probability distributions, that the composition histories can be explicitly expressed by the equations:

$$\left.\begin{array}{c} \bar{Q} \leq \dfrac{1 + r}{2r}: \\ R_w = 1; \quad \bar{V} = \bar{Q}; \\ \dfrac{1 + r}{2r} \leq \bar{Q} \leq \dfrac{1 + r}{2}: \\ R_w = \dfrac{1}{r^2 - 1}\left[\dfrac{(r + 1)^2}{4\bar{Q}^2} - 1\right]; \\ \bar{V} = \dfrac{r}{r - 1} + \bar{Q}R_w - \dfrac{(1 + r)}{2(r - 1)\bar{Q}}; \\ \bar{Q} \geq \dfrac{1 + r}{2}: \\ R_w = 0: \quad \bar{V} = 1 \end{array}\right\} \qquad [35]$$

The total recovery and throughflow curves versus the stratification parameter, for fixed values of the limiting fractional wet gas content of the produced gas, as calculated by Eq 35, are plotted in Fig 9 and 10. Comparing Fig 9 with Fig 2, it will be seen that especially for the higher values of the stratification constant the total recoveries fall off much slower for the linear than for the exponential permeability distribution. In fact, whereas in the latter $\bar{V}$ asymptotically approaches 0 with increasing r, $\bar{V}$ approaches $1 - \sqrt{R_w}/2$ in the limit of indefinitely increasing r for the linear distribution. That is, there will be a definite nonvanishing wet gas recovery, to pre-assigned abandonment limits of R_w, regardless of the degree of stratification, as long as the permeability distribution is linear. The total throughflows, as plotted in Fig 10, also approach nonvanishing limits—$\dfrac{1}{2\sqrt{R_w}}$—with increasing r for a linear permeability distribution, although the curves of $\bar{Q}$ versus r show maxima similar to those of Fig 3. This is, of course, associated with the nonvanishing total recoveries implied by Fig 9.

The reason for the differences between the behavior of linear and exponential distribution systems, with the same ratios of extreme permeabilities, evidently lies in the nature of the permeability variations between the limits. Thus whereas in the linear distribution the mean (integrated) permeability will always lie at the midpoint of the thickness, it moves toward greater depths as r increases for an exponential distribution. The fraction of the section having a permeability exceeding the mean decreases as $\frac{1}{b}\log b$ as b and r increase. In the latter case the high permeability "half" of the pay becomes an infinitesimal part of the section volumetrically, while it still contributes half of the flow capacity.

Conclusions

As would have been expected from elementary considerations, the gross composite efficiency of cycling operations continually decreases as the degree of stratification or variation in permeability increases. The effect of such stratification may be far more serious in limiting the total recoveries and operating life of cycling operations than the areal or pattern efficiency, except in quite uniform formations.

If the permeability distribution is exponential, the stratification may be defined by the ratio of the maximum to minimum permeability. The wet gas recoveries to initial dry gas breakthrough decrease rapidly with increasing values of this ratio, being only 35 pct, at a value of 10 even if the areal sweep efficiency is 90 pct. At a the order of 50 pct in an exponential permeability distribution.

The total gas throughflow to various abandonment limits first increases as the

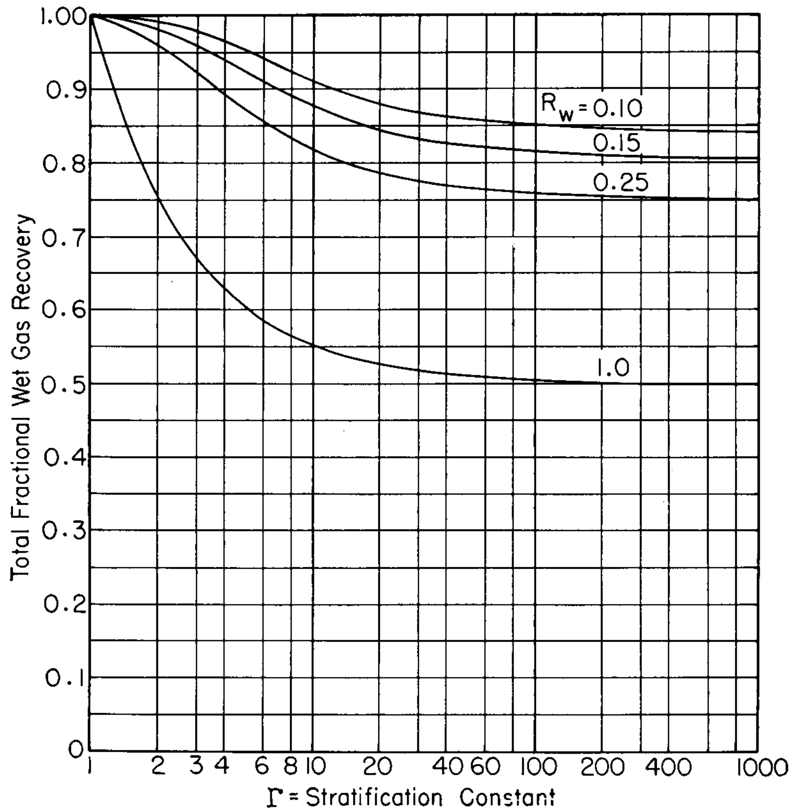

FIG 9—THE CALCULATED VARIATION OF THE TOTAL FRACTIONAL WET GAS RECOVERY IN CYCLING OPERATIONS IN LINEARLY STRATIFIED FORMATIONS VERSUS THE STRATIFICATION CONSTANT, r = RATIO OF MAXIMUM TO MINIMUM PERMEABILITY.

R_w = fractional wet gas content of the produced gas at abandonment of cycling.

permeability ratio of 1000 the initial breakthrough recovery for a 90 pct areal sweep efficiency will be only 13 pct. However, if the cycling operations are continued to low values of wet gas content in the produced gas, the total recoveries are very materially increased. At an abandonment limit of 10 pct wet gas, the total recoveries may exceed 90 pct of the initial reservoir content if the stratification ratio is 10; and even when the latter is 1000, the recoveries will be of stratification ratio increases, reaches a maximum, and finally decreases again. The maximum values are rather moderate, and equals only 2.18 times the hydrocarbon pore volume even if the cycling is abandoned at a limit of only 10 pct wet gas content.

In ideal probability distributions of permeability the degree of uniformity may be expressed by the value of the "variation," which increases from 0 to 1 from a strictly uniform section to one of maximum variability. In this distribution the permeability theoretically would vary from 0 to ∞, but the extreme limits represent infinitesimal volumetric contributions. Thus while theoretically there would be some dry gas breakthrough immediately for all values of the variation, the dry gas dilution will be so slow, especially for low values of the variation, that the formations will simulate those with finite upper limits in permeability. The total wet gas recoveries decrease monotonically with increasing values of the variation, but will exceed 75 pct, for 100 pct areal sweep efficiency and 10 pct limiting wet gas content for abandonment, if the "variation" is less than 0.70. The corresponding total gas throughflows versus the variation show maxima as in the case of the exponential distribution, and the values of the maxima are only slightly higher than in the latter.

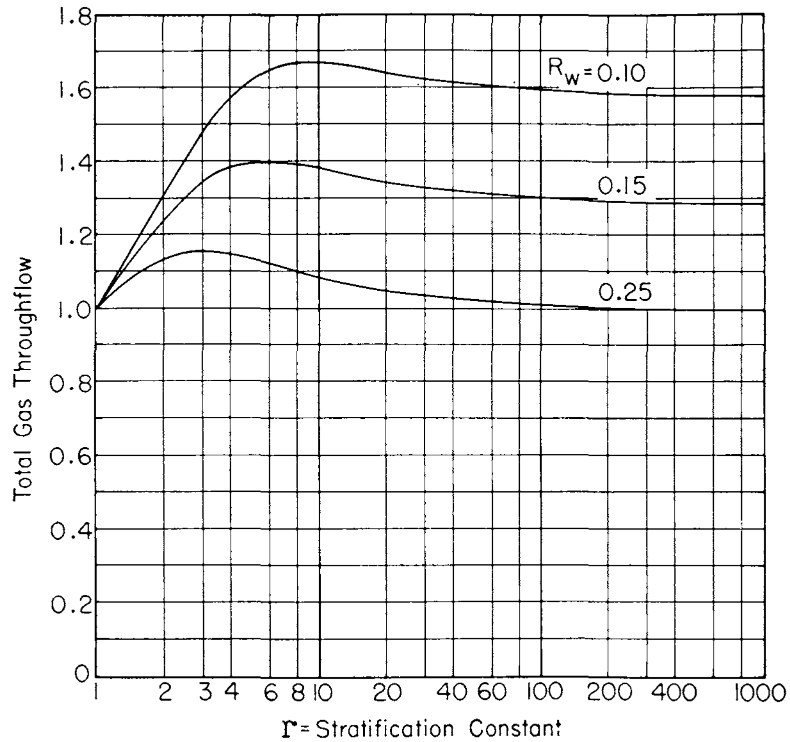

FIG 10—THE CALCULATED VARIATION OF THE TOTAL GAS THROUGHFLOW, IN UNITS OF THE INITIAL DISPLACEABLE HYDROCARBON PORE VOLUME, IN CYCLING OPERATIONS IN LINEARLY STRATIFIED FORMATIONS VERSUS THE STRATIFICATION CONSTANT, r = RATIO OF MAXIMUM TO MINIMUM PERMEABILITY.

R_w = fractional wet gas content of the produced gas at abandonment of cycling.

The linear permeability distribution can also be defined by the ratio of maximum to minimum permeability. The general trends of the curves of total wet gas recovery and throughflow with the stratification ratio are similar to those for the exponential distribution. In contrast to the latter, however, nonvanishing asymptotic limits of total recovery and throughflow are approached as the stratification ratio is indefinitely increased in the case of the linear distribution. In particular, such limits are 84.2 pct for total recovery at an abandonment wet gas content of 10 pct, and 75 pct even if the cycling is abandoned at a wet gas content of 25 pct. Moreover, the initial dry gas breakthrough will never develop until at least 50 pct of the reservoir volume is swept out.

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