A Statistical Approach to the Interstitial Heterogeneity of Sand Reservoirs

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Problems of oil recovery are attacked from the approaches dictated by the two strikingly dissimilar complexes that comprise an oil reservoir—the fluid complex and the interstitial complex. Knowledge of the fluid complex, due to labors in the fields of fluid mechanics and physical chemistry, has far outdistanced knowledge of the interstitial characteristics of the sand complex. This paper is concerned with the latter and applies the technique of statistics to petrographic facts—namely, porosity and permeability—dealing with what might be called petrometry.

Statisticians have long worked within the problem of reconstruction of a universe of variables by means of samples that comprise a small percentage of the universe from which the samples were drawn. Core-analysis data comply with the requisite of random sampling as stipulated by theoretical statistics. A basic concept of statistics employs the following substitution:

1. The results of sampling are arranged as a frequency distribution.
2. The observed frequency distribution is compared with a mathematically precise distribution.
3. If a sufficiently good agreement is found between the two, the precise distribution is substituted in all subsequent operations involving characteristics of the sampled material.

This mathematically precise distribution is also known as the normal curve, curve of error, the probability, the Gaussian, and the bell-shaped curve and is describable with but two figures. Further, these two figures, the arithmetic average and the standard deviation, are subject to mathematical manipulation.

This paper demonstrates that, in some cases in the Dominguez field of Southern California, if sufficient core-analysis samples are taken over not too great a thickness of sediments, the resultant permeability and porosity assemblages give satisfactory agreement with normal curves.

It is shown that the indications are strong that oil wells wherein a satisfactory agreement is found are predictable as to productivity. Since the predictions are made by employing the laws of chance that are operative within sedimentation, it follows that the mechanism by which the predictions were made is a basic concept of reservoir mechanics. Therefore, tentative examples are given of the application of normal curves to the solution of problems of primary and secondary recovery.

Nature of Permeability Frequency Distributions

Core-analysis data are made available as a tabulated depth vs. permeability profile. The nature of the distribution of permeability values within a given stratigraphic interval may be approximated from an array constructed with samples plotted in the order of their appearance with depth (accession number) against their permeability value in millidarcys.

A wide variation in the nature of the distribution of K (permeability) values for

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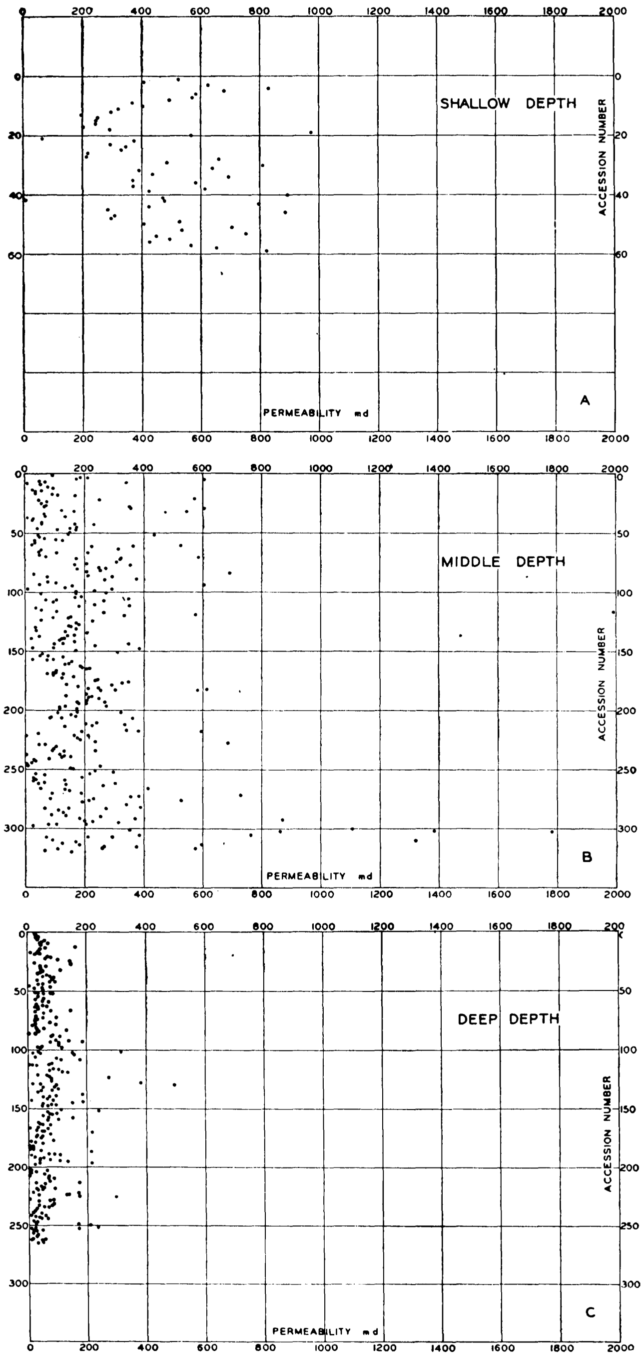

FIG. 1.—ARITHMETIC PERMEABILITY ARRAY.

a shallow, a middle depth, and a deep interval is shown in Fig. 1.

The qualitative measure of distribution available by inspection from these figures is available quantitatively from a tabulation of K by class intervals (Table 1).

TABLE 1.—*Example of Arithmetic Frequency Distribution in Shallow Zone*

	Class Limits, Millidarcys					
	100–300	300–500	500–700	700–900	900–1100	
	292	370	523	799	975	
	191	320	585	707	937	
	251	377	575	756		
	248	346	565	832		
	241	353	558	810		
	203	390	530	888		
	294	370	539	883		
	294	370	568	824		
	217	305	625			
	214	407	680			
	282	497	660			
	299	402	640			
		484	696			
		439	615			
		425	650			
		466				
		477				
		426				
		403				
		447				
		498				
		424				
Totals........	12	22	15	8	2	59
Percentages...	20.34	37.29	25.42	13.56	3.39	100

Comparable tabulations and percentages were obtained for the three intervals and plotted as frequency distributions in Fig. 2.

The frequency distributions as shown in Fig. 2 are asymmetrical about the central class or, in statistical language, skewed to the right. Mathematical manipulation of badly skewed curves is unsatisfactory. Also, it was found necessary to vary the class interval from zone to zone in order to obtain curves that reveal the distribution. This variation of class interval renders direct mathematical comparison zone to zone impossible.

An array constructed on the basis of accession number vs. the logarithims of K gives a symmetrical distribution of values about a central trend of greatest density of points, as shown in Fig. 3.

A frequency distribution that would be symmetrical can be derived from this array by laying off equally spaced intervals across the K scale, such as ½ in. or 1 in., and counting the number of times that K values fall within the individual limits laid down.

Such a procedure would be difficult to standardize. The author proposes a mathematical expression to standardize this operation, which was suggested by Krumbein.[1] The expression relates the logarithm of K to the size of the class interval.

Tests were made to determine the applicability of class intervals that result from the expressions:

$$\phi = \mathrm{Log}_{\sqrt{2.5}} \frac{K}{10}$$

$$\phi = \mathrm{Log}_{\sqrt{2}} \frac{K}{10}$$

$$\phi = \mathrm{Log}_{2} \frac{K}{10}$$

Phi in these cases is merely a scale value. The $\log_{\sqrt{2}}$ was found to give the most symmetrical curves and was then used to accumulate K values in new class intervals as shown in Table 2.

The percentages derived from Table 2 were calculated for the three intervals and Figs. A, B and C of Fig. 4 were constructed by tying the percentage values together with straight dashed lines.

Since there are chance variations attributable to sampling, it is desired to determine the approximation of this sampled distribution to a pure probability distribution.

The values that describe the sampled distribution were calculated and are shown in Table 3. The ordinates of the normal curve that has the same values of X_ϕ and σ_ϕ as the observed curve as calculated are shown in Table 4. π_{ϕ_3} and α_{ϕ_4} being small are considered to represent the amount of variation due to sampling—not characteristics of the distribution of the material from which the samples were taken.

[1] References are at the end of the paper.

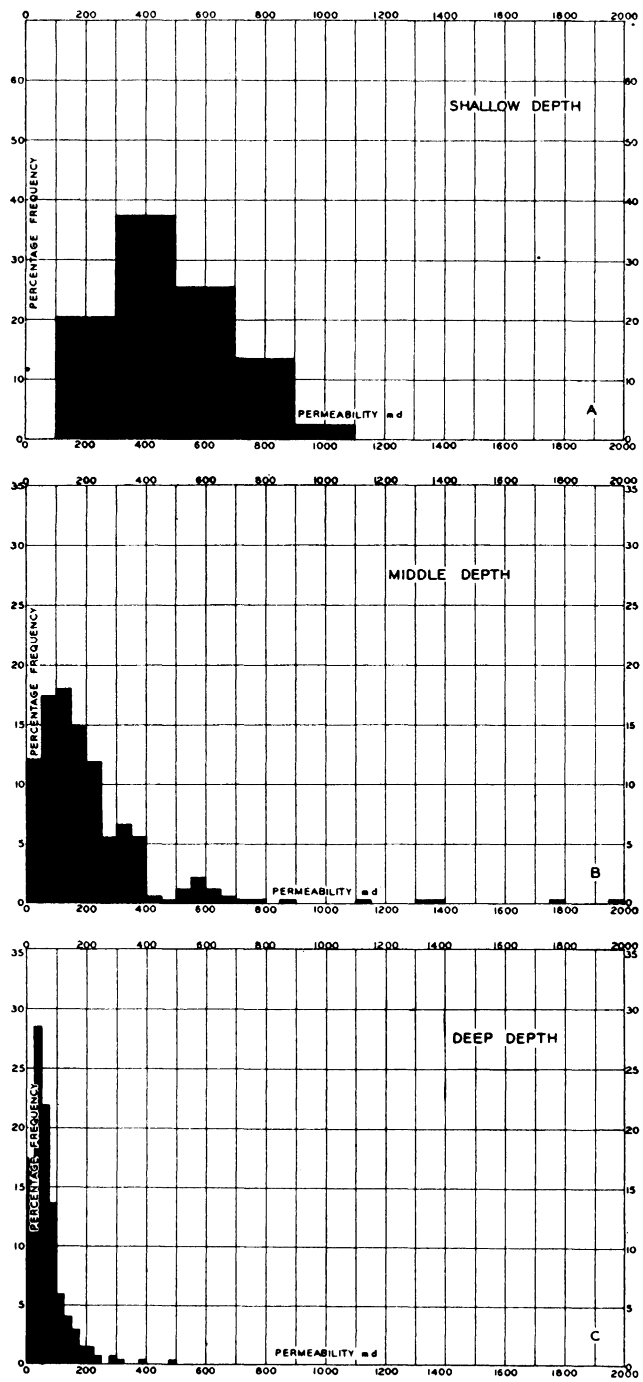

FIG. 2.—ARITHMETIC PERMEABILITY HISTOGRAM.

The solid-line curves of Fig. 4 were obtained by joining with a smooth line the ordinate heights calculated in Table 4.

The answer to this test is given in terms of probability P. For this example, if the material being sampled had a normal distribution, 40 times out of 100 as poor a fit or worse than that obtained would have occurred, because of the chance variations attributable to sampling.

Hence hereafter a substitution of the normal curve for the observed curve is made for all operations involving the distribution of permeability in the interval.

TABLE 2.—*Example of Logarithmic Frequency Distribution in Shallow Zone*
Class Limits

Phi scale	8–9	9–10	10–11	11–12	12–13	13–14	
Millidarcy scale	160–224	224–320	320–448	448–640	640–896	896–1280	
	191	292	407	523	832	975	
	203	251	370	625	680	937	
	217	248	402	585	660		
	214	241	320	575	810		
		294	377	497	696		
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		299	439	558	707		
			370	615	756		
			425	477	650		
			426	530	824		
			403	539			
			447	498			
			424	568			
			370	466			
Totals	4	9	16	16	12	2	59
Percentages	6.78	15.25	27.12	27.12	20.34	3.39	100.00
Accumulated percentages	6.78	22.03	49.15	76.27	96.61	100.00	

A test was then made to determine the chances that the distribution of the material being sampled is normal.

Areas under the normal curve were computed and compared with areas under the sampled curve by means of the chi-squared test for goodness of fit. These computations are given in Tables 5 and 6.

TABLE 3.—*Example of Calculation of Distribution Characteristics by Moment Method, Shallow Zone*
PER CENT

Phi Class Limits	f	d	fd	d^2	fd^2	d^3	fd^3	d^4	fd^4
8–9	6.78	−2	−13.56	4	27.12	−8	−54.24	16	108.48
9–10	15.25	−1	−15.25	1	15.25	−1	−15.25	1	15.25
10–11	27.12	0	0.00	0	0.00	0	0.00	0	0.00
11–12	27.12	+1	+27.12	1	27.12	+1	+27.12	1	27.12
12–13	20.34	+2	+40.68	4	81.36	+8	+162.72	16	325.44
13–14	3.39	+3	+10.17	9	30.51	+27	+91.53	81	274.59
Algebraic totals			+49.16		+181.36		+211.88		+750.88

$V_1 = +0.49 \quad \pi_1 = 0 \quad X_\phi = 10.5 + 0.49 = 10.99$

$V_2 = +1.81 \quad \pi_2 = 1.57 \quad \sigma_\phi = (1.57)^{1/2} = 1.25$

$V_3 = +2.12 \quad \pi_3 = -0.422 \quad \pi_{3\phi} = \quad = -0.422$

$V_4 = +7.51 \quad \pi_4 = +5.79 \quad \alpha_{4\phi} = \quad = +1.77$

$V_1, V_2 \ldots V_4$ to $\pi_1, \pi_2 \ldots \pi_4$ is translation of axis.

X_ϕ is the arithmetic mean in phi, or the abscissa value of the center of gravity of the curve.

σ_ϕ is the standard deviation in phi, or the radius of gyration of the distribution about a vertical axis through the center of gravity.

$\left.\begin{matrix}\pi_{3\phi}\\ \alpha_{4\phi}\end{matrix}\right\}$ are, respectively, in phi, the absolute skewness and the relative kurtosis and are derived from the third and fourth moments of the distribution.

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10–11	27.12	0	0.00	0	0.00	0	0.00	0	0.00
11–12	27.12	+1	+27.12	1	27.12	+1	+27.12	1	27.12
12–13	20.34	+2	+40.68	4	81.36	+8	+162.72	16	325.44
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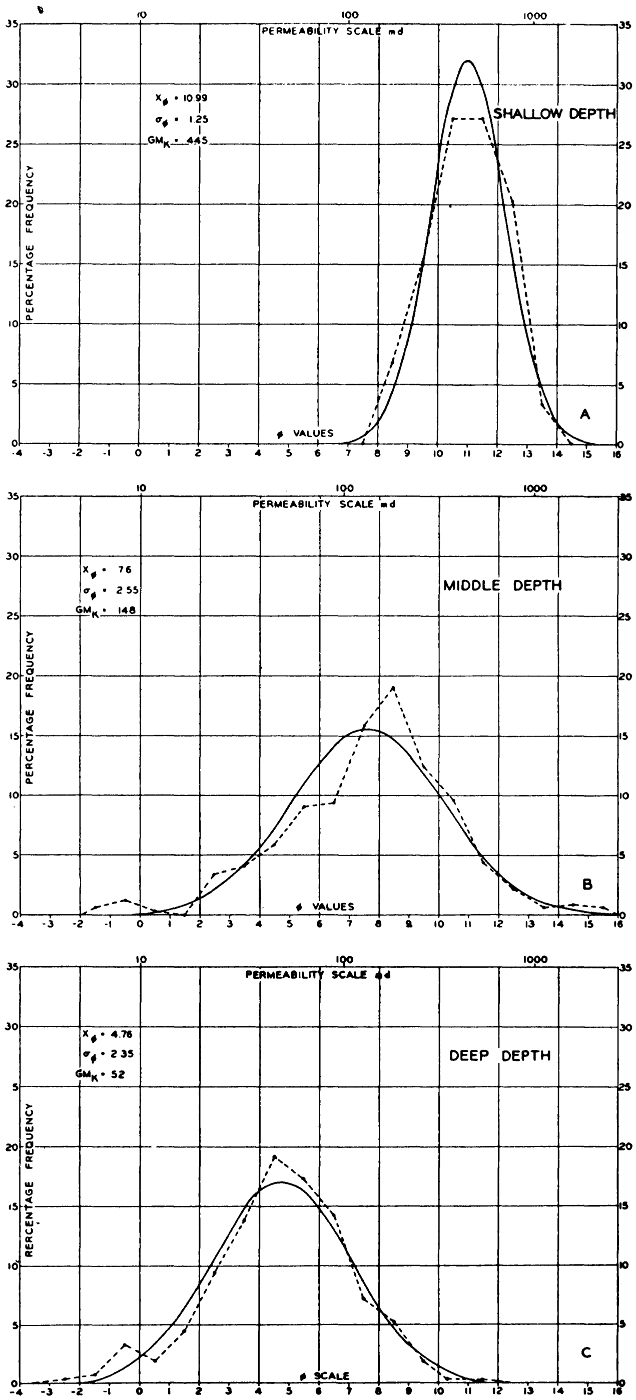

FIG. 4.—LOGARITHMIC PERMEABILITY DISTRIBUTION.

From a petrographic sense a precise mathematical measure of the magnitude and spread of permeability values is available.

TABLE 4.—*Example of Calculation of Fitted Curve, Shallow Zone*
$X_\phi = 11.00$ and $\sigma_\phi = 1.25$

Limit Phi	X	$\frac{X}{\sigma_\phi}$	$-\frac{X^2}{e^{2\sigma^2}}$	Ordinate Height
6	5	4.00	0.0003	0.01
7	4	3.20	0.006	0.19
8	3	2.40	0.056	1.78
9	2	1.60	0.278	8.90
10	1	0.80	0.726	23.20
11	0	0.00	1.000	32.00
12	1	0.80	0.726	23.20
13	2	1.60	0.278	8.90
14	3	2.40	0.056	1.78
15	4	3.20	0.006	0.19
16	5	4.00	0.0003	0.01

$$Y_0 = \frac{100}{\sigma\sqrt{2\pi}} e^{-\frac{X^2}{2\sigma^2}} = 32.00 \text{ where } X = 0$$

A rapid method is available for the determination of X_ϕ and σ_ϕ and to determine approximately the fit obtained. The procedure is described by Otto.[2] The percentages determined in the calculation of the phi frequency distribution are accumulated and plotted on arithmetic probability paper. The results are shown in Fig. 5.

TABLE 5.—*Example of Calculations of Areas under the Normal Curve, Shallow Zone*

Phi		X	$\frac{X}{\sigma}$	Per Cent Area[a]	Per Cent Area in Class	Observed Per Cent Area
Lower Limits	Upper Limits					
6		5	4.00	0.4999	0.06	0.00
7		4	3.20	0.4993	0.75	0.00
8		3	2.40	0.4918	4.66	6.78
9		2	1.60	0.4452	15.71	15.25
10		1	0.80	0.2881	28.81	27.12
11		0	0.00			
	12	1	0.80	0.2881	28.81	27.12
	13	2	1.60	0.4452	15.71	20.34
	14	3	2.40	0.4918	4.66	3.39
	15	4	3.20	0.4993	0.75	0.00
	16	5	4.00	0.4999	0.06	0.00

[a] Per cent area $= \int_0^X \frac{100}{\sigma\sqrt{2\pi}} e^{-\frac{X^2}{2\sigma^2}}$

The intercept of the line drawn between the 15.9 per cent and the 84.1 per cent values of the curve and the 50 per cent line is the arithmetic average in phi.

The arithmetic average in phi subtracted from the 84.1 per cent value is the σ_ϕ.

Departures of the observed curve from the line joining the 84.1 per cent and the 15.9 per cent intercepts and its prolongation indicate the approximate amount of skewness and kurtosis present in the observed data. If the departures are sufficient, caution is indicated in the use of the normal curve to describe the observed assemblage.

TABLE 6.—*Examples of Chi-squared Test for Goodness of Fit, Shallow Zone*

Limits	Observed f	Calculated fc	$f - fc$	$(f - fc)^2$	$\frac{(f - fc)^2}{fc}$
6–7	0.00	0.06			
7–8	0.00	0.75	+1.31	1.72	0.38
8–9	6.78	4.66			
9–10	15.25	15.71	−0.46	0.21	0.13
10–11	27.12	28.81	−1.69	2.85	0.10
11–12	27.12	28.81	−1.69	2.85	0.10
12–13	20.34	15.71	+4.63	21.40	1.36
13–14	3.39	4.66			
14–15	0.00	0.75	−2.08	4.31	0.92
15–16	0.00	0.06			
Total..					2.99

Chi-squared = 2.99
$N = 6 - 3 = 3$
$P = \quad = 0.40$

A Demonstration, That Oil Wells Perform in Accordance with Their Permeability Distributions

In dealing with permeability, one is dealing with the resistance to the flow of fluids in the rock. If the composite effect of the numerous scattered values of permeability encountered in a well can be evaluated, oil-well performance can be put on a rational basis.

The most direct proof that intervals are describable in terms of a normal distribution of K lies in the prediction of initial productivity.

An attempt was made to predict initial productivity by the following procedure:

1. The Norris Johnston[3] adaptation of the Pyle-Sherborne[4] curve of specific productivity index vs. permeability was set at

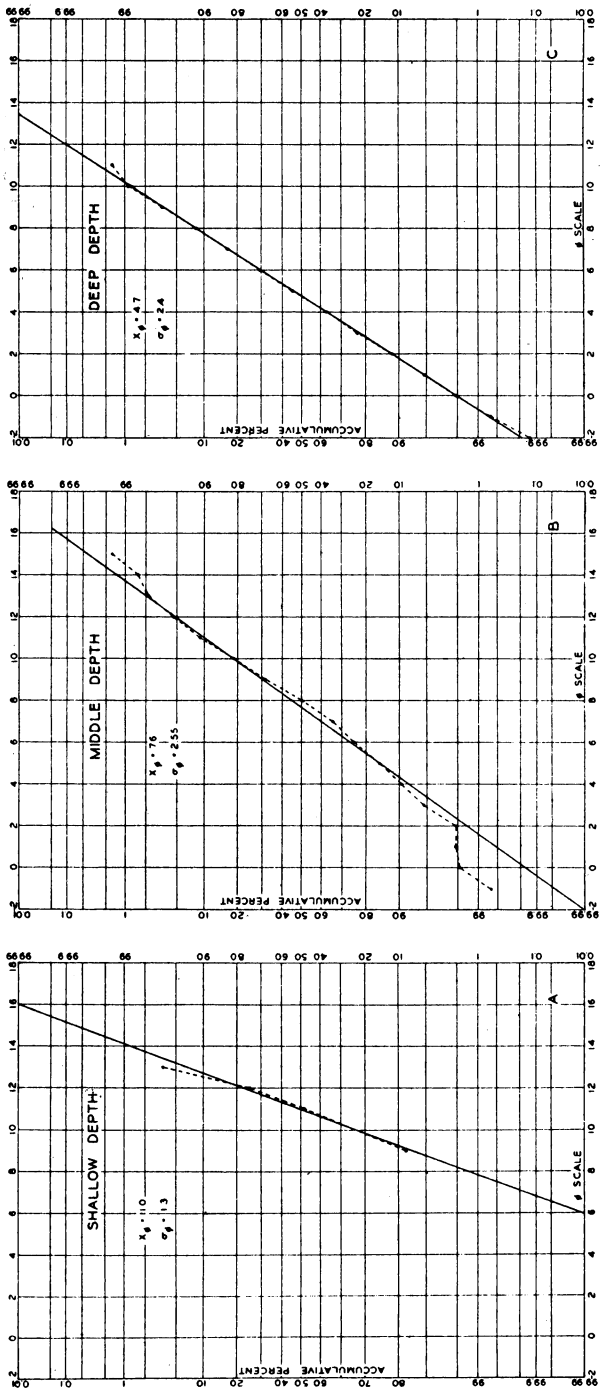

FIG. 5.—ARITHMETIC PROBABILITY PLOTS OF PERMEABILITY DISTRIBUTION.

a slope of 2 and shifted to higher values of specific productivity index for equivalent

TABLE 7.—*Type Calculation of Specific Productivity Index*
$X_\phi = 5$ and $\sigma_\phi = 1$

Class Limits	*f*, Per Cent	Mid-point Sp. P.I.	P.I. (for 100 Ft. Sand)	Productivity Index: Per Cent	Productivity Index: Accumulated, Per Cent
1–2	0.13	0.00037	0.000048		
2–3	2.14	0.00067	0.001430	0.4	0.4
3–4	13.60	0.00118	0.016100	4.5	4.9
4–5	34.13	0.00210	0.072000	20.0	24.9
5–6	34.13	0.00410	0.140000	38.7	63.6
6–7	13.60	0.00740	0.101000	28.0	91.6
7–8	2.14	0.01300	0.027800	7.5	99.1
8–9	0.13	0.02500	0.003250	0.9	100.0
Totals......			0.361638	100.0	
Sp. P.I......			0.0036		

Variation of Specific Productivity Index with σ_ϕ

X_ϕ	σ_ϕ	Sp. P.I.	Increase, Per Cent
5	0	0.0031	0
5	1	0.0036	16
5	2	0.0062	100
5	3	0.0132	325

K values. The curve was adopted because it was felt that it expressed correctly the effect of increasing values of permeability but was inadequate to express the spread of permeability encountered. The shift was necessitated because entry is made in these predictions with the geometric average permeability, which is always below that of the arithmetic average. The amount of shift was governed by trial and error, using one well as a criterion.

The next three steps are illustrated in Table 7.

2. The mid-point value of each phi class was determined in terms of equivalent specific productivity index.

3. Percentages under the curve between class limits were determined for a set of normal curves of which X_ϕ was held constant and σ_ϕ varied between 1 and 3. This process was repeated for other values of X_ϕ.

4. The resultant areas were multiplied by their corresponding specific productivity index values, the resultant productivity indexes added and divided by 100.

5. The results of such computations, together with the curve of specific productivity index vs. permeability, were plotted on Fig. 6.

6. Twelve wells in a single zone at Dominguez were employed for a test of predictability. Their permeability values were gathered in phi, percentages taken, and accumulated. From a plot of these percentages on arithmetic-probability paper

TABLE 8.—*Tabulation of Prediction of Productivity of Twelve Wells*

Well No.	X_ϕ	σ_ϕ	Completion Pressure as Per Cent Initial Pressure	First Approx. Sp. P.I.	Produced Sp. P.I.	Produced Sp. P.I. as Percentage of First Approx. Sp. P.I.	Predicted Sp. P.I.
T- 1	4.7	2.3	89	0.0053	0.0067	126	0.0066
T- 2	4.86	3.23	89	0.0160	0.0193	120	0.0190
T- 3	6.42	2.05	97	0.0128	0.0142	114	0.0224
T- 4	2.2	2.4	77	0.0015	0.0015	100	0.0010
T- 5	5.4	2.25	75	0.0081	0.0036	44	0.0048
T- 6	4.0	2.25	75	0.0036	0.0029	57	0.0022
T- 7	2.4	3.10	52	0.0033	0.0007	21	0.00065
T- 8[a]	5.75	1.65	83	0.0075	0.0015	20	0.0057
T- 9	6.6	2.2	55	0.0185	0.0035	19	0.0042
T-10[a]	2.6	3.1	63	0.00325	0.0002	6	0.0011
T-11	5.8	2.75	72	0.0117	0.0062	53	0.0062
T-12	5.5	2.2	50	0.0095	0.0016	17	0.0017

[a] Cemented liners, gun perforated.

the X_ϕ and σ_ϕ (Table 8) for each well determined.

With these values in hand, the specific productivity indexes of the wells were computed graphically from Fig. 6.

effective permeability should decrease as the zone pressure is decreased and experience is common that productivity indexes do so decrease in practice with depletion. Babson[5] has calculated a theoretical de-

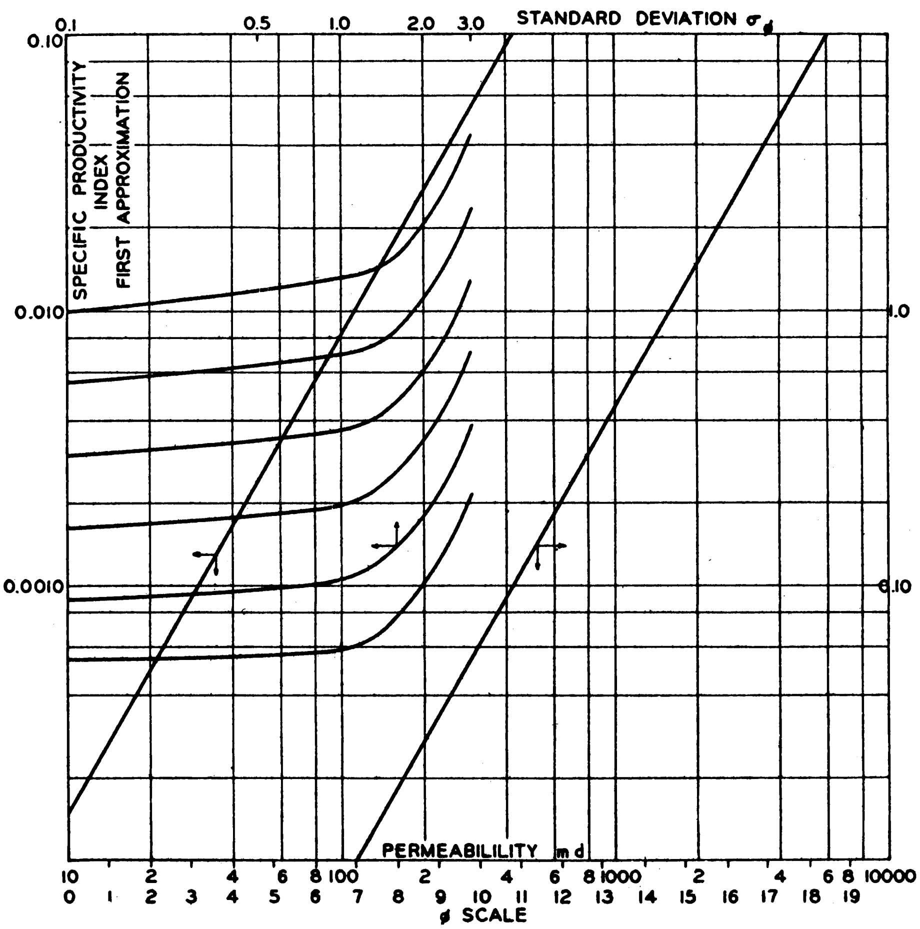

FIG. 6.—CALCULATION OF FIRST APPROXIMATION OF SPECIFIC PRODUCTIVITY INDEX.

7. The computed specific productivity indexes were compared with the measured initial values. The measured values were seen to fall below the computed values as successively larger well numbers were encountered. Since the wells were drilled in the order of their numbers, it followed that successively lower formation pressures were being encountered with each new completion. Standard works on saturation and formation viscosities indicate that the

cline in productivity index as a function of recovery and has obtained an approximately straight line.

8. Therefore a plot was made of the measured initial specific productivity indexes expressed as a percentage of the calculated indexes and plotted against the initial well pressure expressed as a percentage of the original zonal pressure. The results are plotted on Fig. 7. Values in excess of 100 per cent of the calculated

specific productivity index presumably result because the well that was used as a criterion to shift the Pyle-Sherborne curve was not completed at original zonal pressure.

9. On Fig. 8 45° lines were laid off through points that represented the intercepts of the even 10 per cent pressure values and their corresponding percentages of specific productivity index from Fig. 7.

10. The calculated indexes by the graphic method of Fig. 6 were entered on the ordinate scale of Fig. 8, moved horizontally across the page until the line of percentage pressure, corresponding to the pressure of the well being determined, was reached, then the value on the abscissa scale was read.

11. The values of specific productivity index read on the abscissa scale, therefore, are the predicted specific productivity indexes for the 12 wells in question.

For comparative purposes, the calculated specific productivity indexes are plotted against the measured initial indexes on Fig. 9. Except for the two wells with cemented liners, the accuracy of prediction is that there is a 6 to 10 chance of a correct answer within ±10 per cent, and a certainty of a correct answer within ±35 per cent. The deviations of the two gun-perforated wells from the trend is in partial accord with Muskat.[6]

Fig. 7 is sufficiently compatible with both theory and oil-field experience, and the mechanism of the increase in specific productivity index with increased σ_ϕ is sufficiently reasonable so that the demonstration is strong that oil wells perform in accordance with their permeability distributions.

Employment of the Normal Distribution of Permeability to Problems of Recovery

The effectiveness of a gas or water drive is a function of the uniformity of the sands being flushed. The σ_ϕ appears to be the proper criterion of uniformity. If normality is not present, the best sand to flood will approach minimum σ_ϕ, maximum negative $\pi_{3\phi}$ and minimum $\alpha_{4\phi}$.

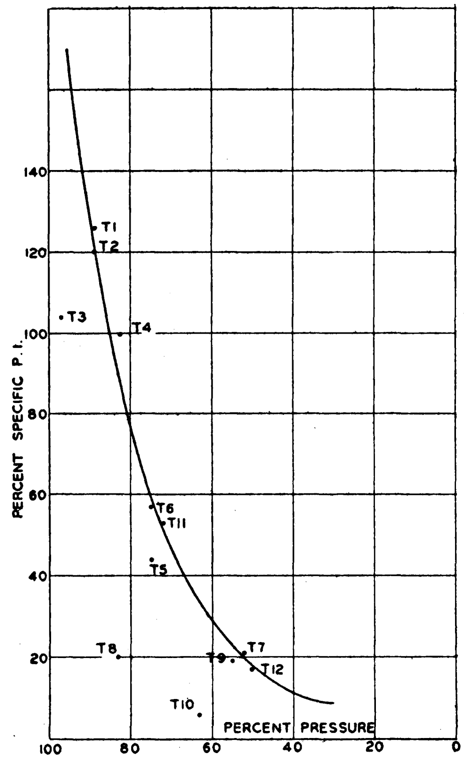

Fig. 7.—Effect of decrease in pressure on initial produced specific productivity index.

The sharply increasing specific productivity indexes of K assemblages as a func-

Table 9.—*Type Calculation of Cumulative Per Cent Curve*

$X_\phi = 5$, $\sigma_\phi = 1$ (values from Table No. 7)

Limits		Sand		P.I.	
Phi	Md.	Per Cent	Acc. Per Cent	Per Cent	Acc. Per Cent
1–2	14–20	0.13	0.13	0.0	0.0
2–3	20–28	2.14	2.27	0.4	0.4
3–4	28–40	13.60	15.87	4.5	4.9
4–5	40–56	34.13	50.50	20.0	24.9
5–6	56–80	34.13	84.13	28.7	63.9
6–7	80–112	13.60	97.73	28.0	91.6
7–8	112–160	2.14	99.87	7.5	99.1
8–9	160–224	0.13	100.00	0.9	100.0

tion of the σ_ϕ indicates that a major portion of the early production of a well comes from a minor portion of the exposed sand.

These proportions may be calculated as is done in Table 9 and displayed for the shallow, middle depth and deep intervals in Fig. 10.

By choosing each even 10 per cent of the sand and following downward lines of constant permeability, the corresponding percentage of productivity index is available. From such operations the curves of Fig. 11 are constructed. These are known as Lorenz curves and are used in economics to show the concentration of production.

Assuming complete horizontal uniformity of permeability, the Lorenz curve depicts schematically the shape of an advancing edge water under conditions of equivalent effective permeability and no interface effect. Expressed quantitatively, if the Lorenz curve shows that 60 per cent of the productivity of a zone comes from 10 per cent of the sand, it follows that in a well close to original edge water that at present has a water cut of 60 per cent, of the total sand exposed 10 per cent has gone to water.

If there is complete horizontal nonuniformity of permeability, the schematic face would be vertical, and under conditions of complete nonuniformity of horizontal per-

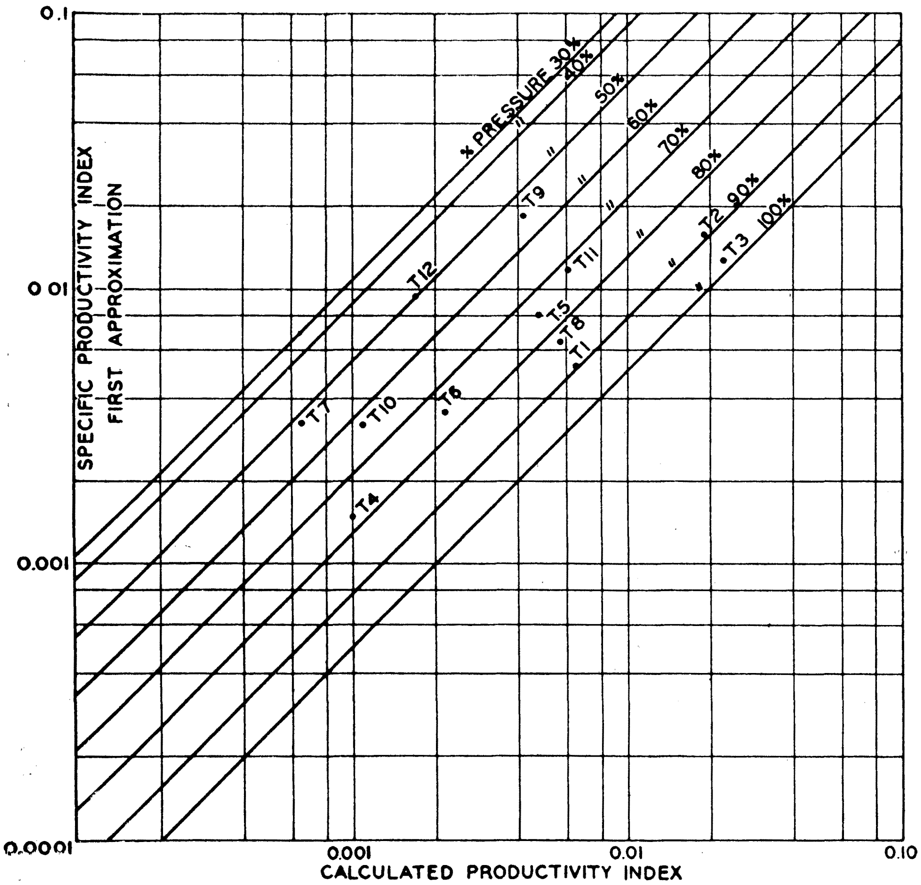

FIG. 8.—GRAPHIC SOLUTION FOR EFFECT OF DECREASE IN PRESSURE OF SPECIFIC PRODUCTIVITY INDEX.

meability, if the well cut is 60 per cent the proportion of sand gone to water is 60 per cent.

If gas or water is injected into a well highest permeability, are subject to demonstration by gas injection into a wet well followed by gas withdrawal and return to production.

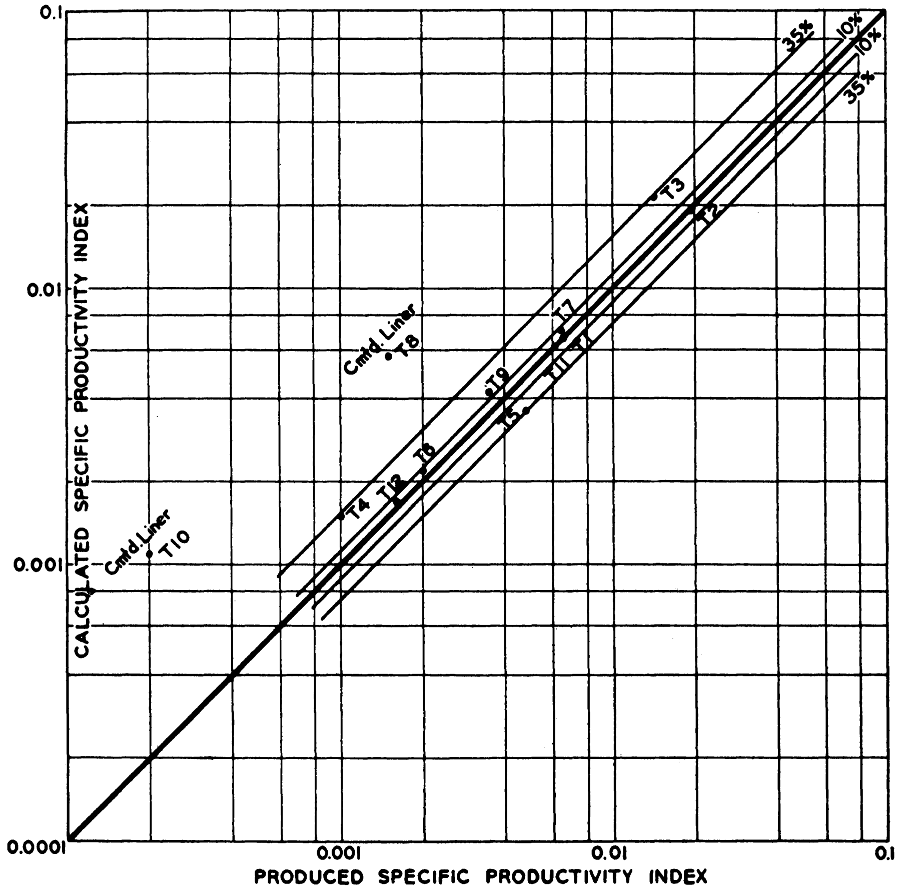

FIG. 9.—COMPARATIVE RESULTS CALCULATED VS. PRODUCED SPECIFIC PRODUCTIVITY INDEX.

with a 60 per cent P.I. and 10 per cent sand relationship, 60 per cent of the induced fluid will enter 10 per cent of the sand. This process will be initiated regardless of horizontal variation in permeability. The process will continue until the sand that is permeable at the well bore is packed to the outer reaches of relatively high permeability.

The two postulates; first, that water will enter the well via the sand of highest permeability and, second, that gas in the well bore will also selectively enter the sand of

Fig. 12 is such an example. The relationship here shown between injected gas and water cut is in accord with the postulates.

A Proposed Method by Which the Degree of Horizontal Uniformity of a Sand Might Be Evaluated

Since individual K samples span approximately ¾ in. of vertical distance within the well bore, apparently it is impossible to correlate K values well to well as stratigraphic equivalents. If on the other hand a section of sand were sampled every 1 in.

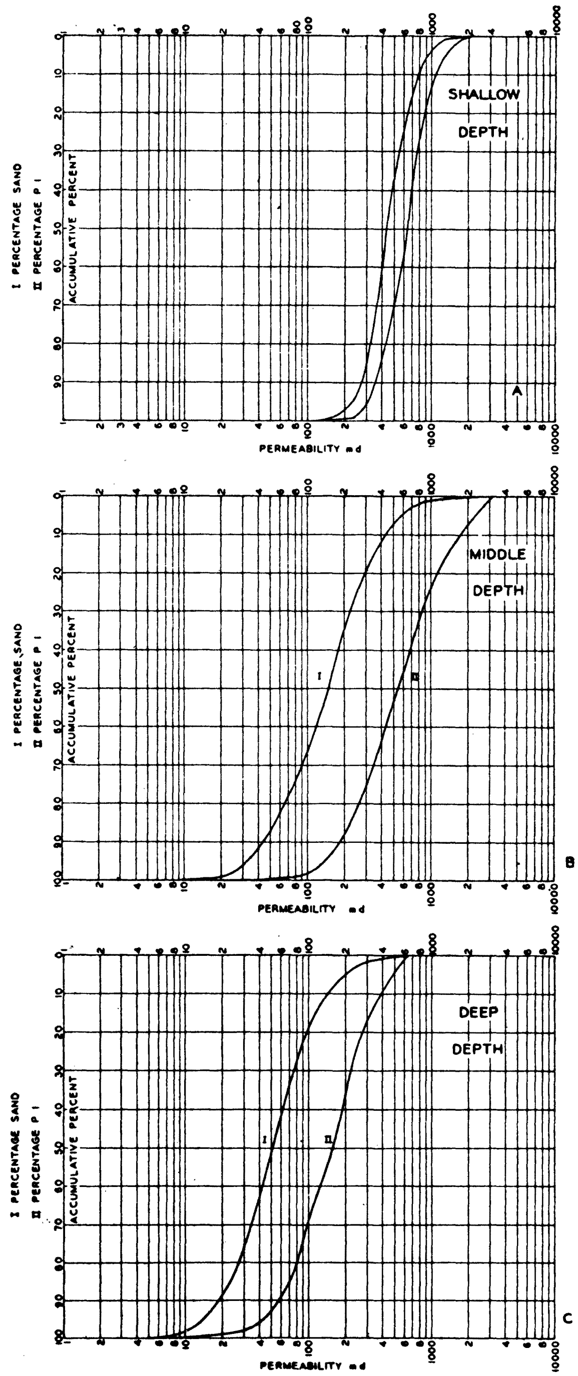

FIG. 10.—ACCUMULATIVE PER CENT OF SAND AND PRODUCTIVITY INDEX.

and the resultant permeability profile were sufficiently regular to allow interpolation, assurance of proper stratigraphic permeability correlation would be possible.

The permeability profile of Fig. 13 is the result of such a study made on a 4-ft. sand capped and based with shales and containing a 1-in. streak of shale through the center. The profile is sufficiently random so that the validity of interpolation is in doubt. Therefore a screen analysis was made of the K samples for stations 24 through 38. The grain-size profile of Fig. 13 resulted.

Since the screen-analysis profile and the permeability profile are similar, and the screen-analysis profile delineates a stable variation in sedimentation, it follows that the amount of interpolation necessary to compare two permeability profiles taken of this stratigraphic interval in two offsetting wells could be made with some assurance.

Until another well is drilled in which all the hazards of acquiring such a comprehensive analysis as performed on the 4-ft. section are met fortuitously, this matter must remain unanswered.

Porosity Distributions

Porosity distributions give symmetrical arithmetic normal curves. The standard deviations encountered by the author show a small range. As seen in Fig. 14, the arithmetic average decreases with depth.

The distributions give a precise affirmation to the use of the arithmetic average of porosity for reservoir calculations.

Porosity-permeability Correlation

The correlation table of Fig. 15 warrants thoughtful consideration. Much that is statistically unsatisfactory is believed to be due to the inability to always take K values parallel to bedding and to the measurement of porosity values as total rather than effective.

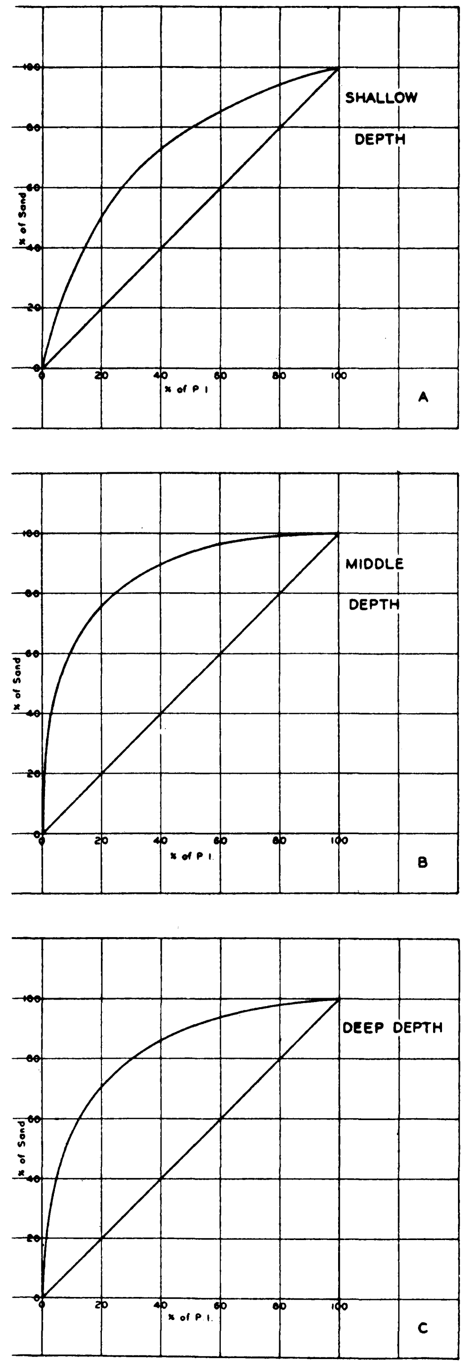

Fig. 11.—Lorenz curve of sand and productivity index.

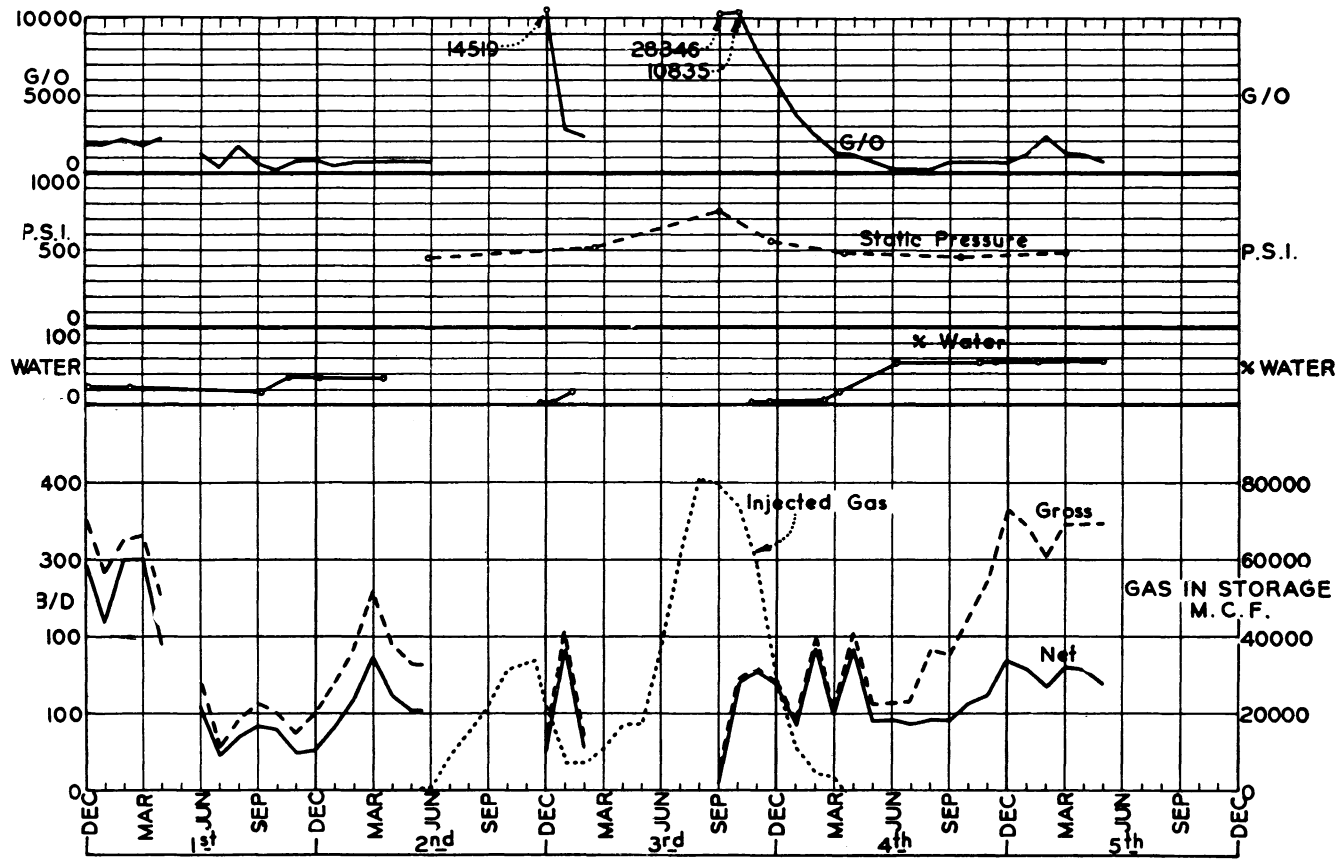

FIG. 12.—EFFECT OF GAS INJECTION ON WATER PRODUCTION.

It is believed that this table presents an approach to interstitical heterogeneity on an intensive basis.

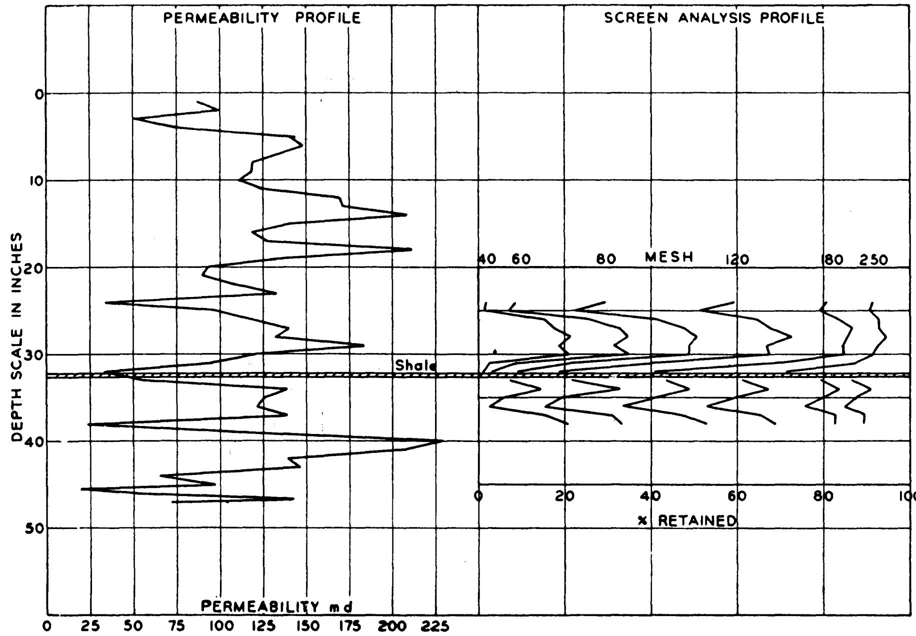

FIG. 13.—DETAILED PERMEABILITY PROFILE.

CONCLUSIONS

It has been desired that this paper portray the salient features of distributions. Therefore no space has been given to either the refinements indicated or the features of speculation aroused. The major conclusions to be derived from the data are believed to be:

1. With some exceptions permeability and porosity assemblages give respectively satisfactory logarithmic and arithmetic normal frequency distributions.
2. Statistical technique appears to offer the reservoir analyst opportunities for the solution of oil-field problems.
3. The solution of the problem of the anatomy of sand reservoirs is a problem common to the sedimentary geologist and the reservoir analyst. The recognition of permeability distributions further provides that the efforts of one should complement the other.

ACKNOWLEDGMENTS

The author wishes to thank the Dominguez Oil Fields Co. for release of this material; the Union Oil Co. for cooperation in certain phases of this work performed by its core-analysis laboratory outside of routine operations, and the Union Oil Co. and the Shell Oil Co. for release of Dominguez core data.

If this paper is a contribution, it is due to the early faith of H. C. Pyle and J. E. Sherborne that if sufficient core data could be gathered a comprehensive analysis would be forthcoming. If their faith had not been followed by perseverance, the paper could not have been written.

The work of Krumbein on analysis of sand grain size was but slightly altered by the author, and applied to permeability assemblages.

The criticism of E. C. Babson, Milan Arthur, and Roy Wagoner, all of the Union Oil Co., has provided whatever degree of coherency the paper may have attained.

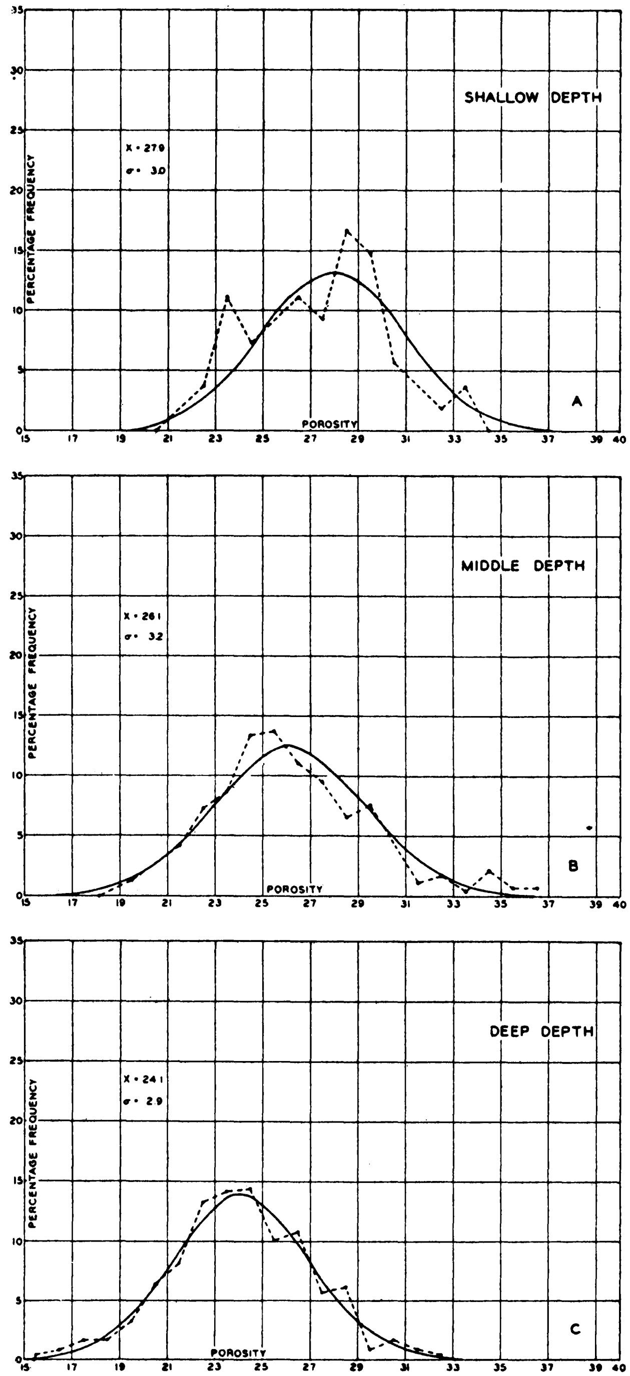

FIG. 14.—POROSITY DISTRIBUTIONS.

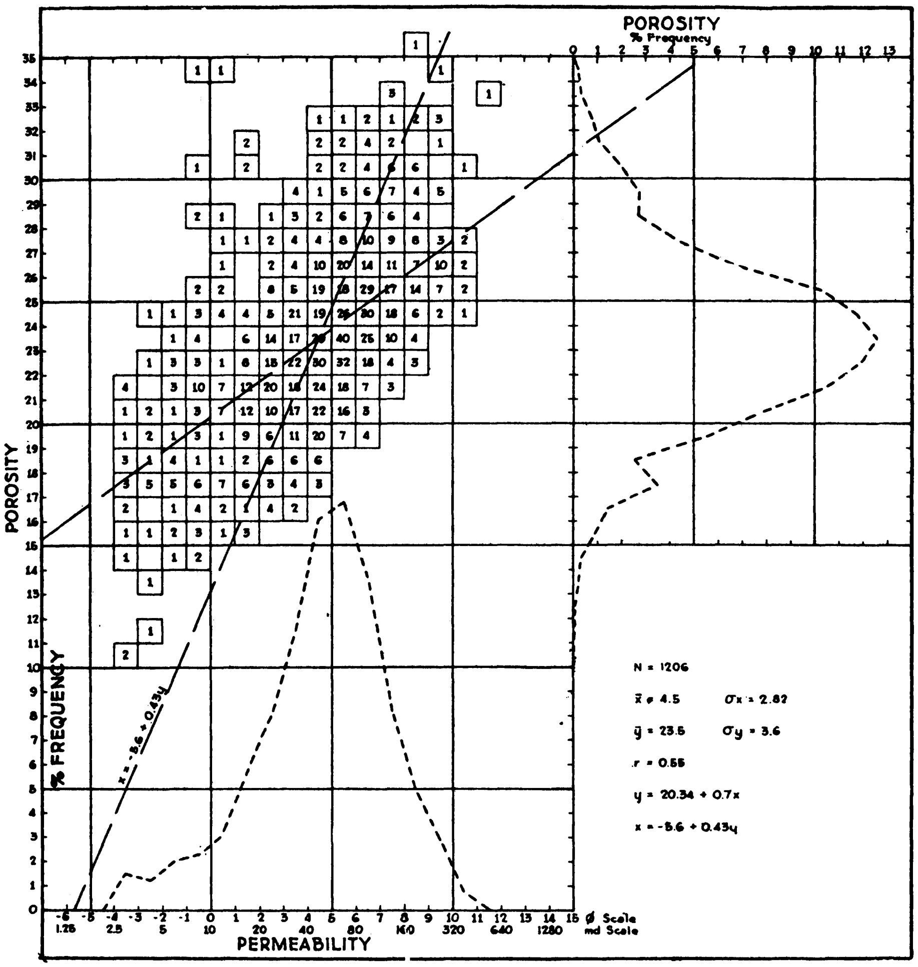

FIG. 15.—POROSITY VS. PERMEABILITY, DEEP INTERVAL.

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DISCUSSION

N. JOHNSTON,* Los Angeles, Calif.—Mr. Law has made a new and useful application of an old and well-known method of analysis. He comes to the conclusion that the graphical relation between permeability and productivity

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index is subject to an improvement which brings into the picture the deviation of individual permeabilities from the average. I have wanted for some time to make some such refinement, so this is a welcome contribution.

There is another aspect of the averaging of varying permeabilities. When a given sand interval contains permeabilities varying over a wide range, the resultant flow capacity of the interval for a given fluid depends very greatly not only on the statistical but also on the physical distribution of the sand streaks of varying permeability. To illustrate: if in a 20-ft. sand interval, half the sand has a permeability of 10 md., the other half, 100 md., the arithmetic average permeability is 55 md., regardless of the physical distribution. One of the two extreme cases would be represented by two 10-ft. homogeneous strata in contact, one of 10 md., the other of 100 md. permeability. The other extreme would be represented by a regular alternation of thin streaks of sands of the two permeabilities. In the first case, the flow capacity would be approximately equivalent to that of a solid sand interval of uniform permeability of 55 md. In the second case the flow capacity would be equivalent to that of a 20-ft. homogeneous sand body of much higher permeability, possibly 99 md., if the sand streaks were thin enough and permeably connected. This is because the oil or gas in the tight streaks would quickly migrate obliquely into the more permeable sand, and flow into the well along these better conductors, effectively by-passing the tighter streaks.

The phenomenon of oblique flow is mathematically so complex that even Dr. Muskat has been unwilling to tackle it in generalized form, and has published so far only a simple special case. With variations in permeability of 100:1, the proper solution of an extreme case of oblique flow could change well-productivity estimates by as much as 2:1 from the estimate made from a simple arithmetic average. If there is any way in which Mr. Law's statistical analysis can help to give us a solution to the oblique-flow problem, I am sure we should welcome the result. As it stands, Mr. Law's contribution is of considerable value, and, among other things, should lead the way to a better understanding of the deviations of individual wells from the average curve relating specific productivity index to permeability.[1]

J. Law (author's reply).—In this author's opinion, because of the lack of knowledge of the incidence of interstitial heterogeneity, a solution to the problem of oblique flow is not indicated.

[1] N. Johnston and J. E. Sherborne: Permeability as Related to Productivity Index. Amer. Petr. Inst. Drill. and Prod. Practice (1943).