Procedure for Buckley-Leverett Calculation:

The general procedure is to calculate values of f_w at various values of S_w, (or f_g as function of S_g) and to plot f_w vs S_w. Values from the $f_w - S_w$ curve and slopes of tangent lines drawn to the curve are used to calculate recovery and injection volumes.

1. Calculate viscosity ratio, μ_w/μ_o or μ_g/μ_o.
2. Calculate "a". $a = \dfrac{7.84(10^{6})\,k\,(\rho_w - \rho_o)\sin\alpha}{q_t\,\mu_o}$.

If gravity effects are not important, $a = 0$

3. Calculate f_w from frontal advance equation

$$f_w = \frac{1 - a\,k_{ro}}{1 + \dfrac{\mu_w/\mu_o}{k_{rw}/k_{ro}}}$$

Set up a calculation sheet like this:

①	②	③	④	⑤	⑥	⑦
S_w	k_{ro}	k_{rw}/k_{ro}	$1 - a k_{ro}$	$1 + \frac{\mu_w/\mu_o}{k_{rw}/k_{ro}}$	f_w	$\frac{f_w}{S_w - S_{iw}}$

4. Plot on fairly large scale values of f_w and $f_w/(S_w - S_{iw})$ against S_w, starting at S_{iw}. Draw in smooth curves.

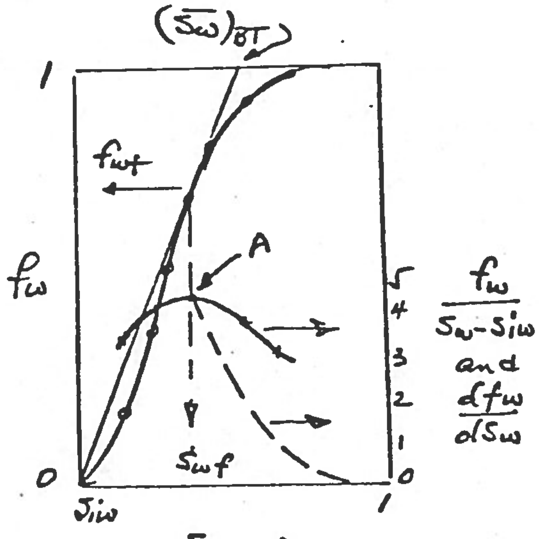


Figure 1.

5. Determine values for the saturation at the front, S_{wf}, the fraction of water flowing at the front, f_{wf}, and the average saturation back of the front, $(\bar{S}_w)_{BT}$ (also called average saturation at breakthrough.) This is done by:

a) Drawing a tangent to the $f_w - S_w$ curve that starts at S_{iw}. Reading values as indicated in Figure 1, or. b) determine maximum value of $\frac{f_w}{S_w - S_{iw}}$ curve (Point A in Fig 1) and draw a

vertical line through it to obtain S_{wf} and f_{wf}.
Calculate $(\bar{S}_w)_{BT}$ from equation

$$(\bar{S}_w)_{BT} = S_{wf} + \frac{1 - f_{wf}}{\left(\frac{f_w}{S_w - S_{iw}}\right)_{max}}$$

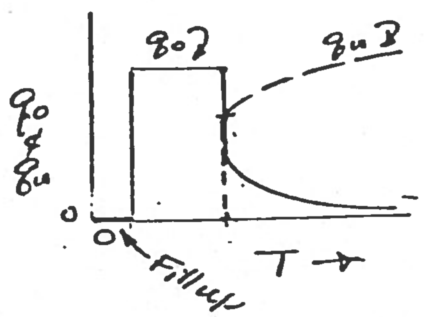
6. To determine parameters for calculations after breakthrough, set up a second calculation sheet with these headings.

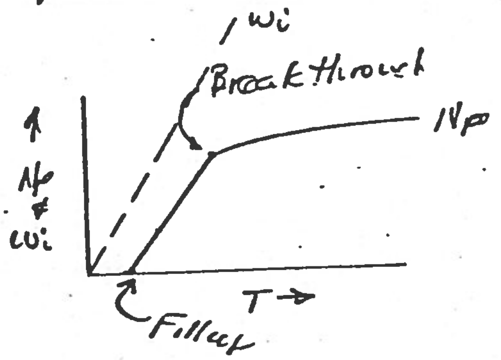
⑧ $S_w]_c$	⑨ $f_w]_c$	⑩ $\left(\frac{df_w}{dS_w}\right)_c$	⑪ $V_{ID}]_c$	⑫ $\bar{S}_w]_c^{\circ}$	---	⑮ W_F	⑯ W_i	⑰ T

Other columns for q_o, q_w, N_p, W_p, F_{wo}, etc may be added depending on the problem.

7. Select about 5 values of $S_w]_c$ between breakthrough, (S_{wf}), and where the f_w curve becomes one. Include S_{wf} as one of the values. Read values of $f_w]_c$ and $(df_w/dS_w)_c$ from smoothed curves on Figure 1. Calculate values of remaining variables from equations given in previous notes.

8. Plot variables as time functions to yield performance. Examples are:





Note that q_o and q_w have two values at breakthrough, because

$$q_o = \frac{i_w}{B_o}(1 - f_{wi}) \; ; \; q_w = i_w \cdot f_w$$

and f_w changes from zero to f_{wf} at breakthrough.

M.B. Standing

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Displacement Problem 1

(This problem pertains to the displacement of oil up-structure by an influxing aquifer. The problem is to be solved using the Buckley-Leverett method).

The sketches below illustrate a section of reservoir in which water is advancing up-structure as a result of pressure reduction in the oil band section. To simplify this problem two assumptions will be used : (1) the initial water in the oil zone amounts to 32% saturation and is constant with height. To say it differently, we will neglect any effects of the initial transition zone saturation. (2) water breakthrough into the 1st line well occurs when the front reaches the elevation of the well. In other words, we will neglect effects of "cusping" of water into the well.

1980

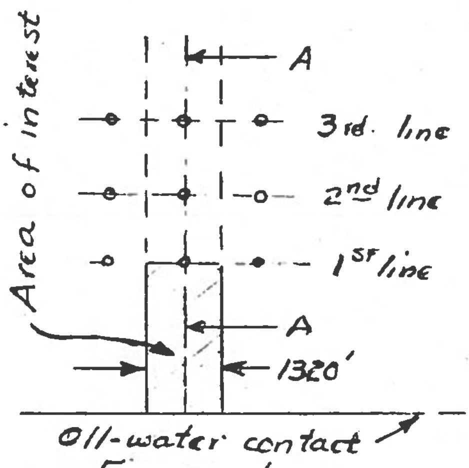

Figure 1
Plan View of "Window"

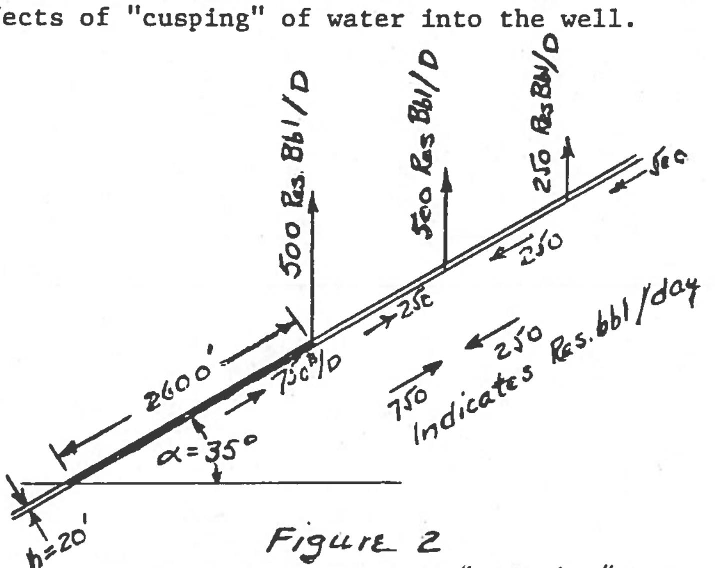

Figure 2
Section A-A in "Window"

"Window" Data

Window length = 2600'
Window width = 1320'
Formation thickness = 20'
Formation dip = +35°
Porosity = 0.235
Permeability (abs) = 100 md
Up-dip flow rate at oil-water contact = 750 Res.bbl/day

Fluid Saturations

S_{iw} = 0.30; $\bar{S}_{wi}$ = 0.32
$\bar{S}_{oi}$ = 0.68 S_{gi} = 0.00

Fluid Properties

B_o = 1.27 B_w = 1.02
μ_o = 1.16 cp μ_w = 0.38 cp
ρ_o = 45 lb/ft^3 ρ_w = 65 lb/ft^3

Relative Permeability Data

S_w	k_{ro}:imb	k_{rw}/k_{ro}] imb	
0.30	0.725	0.0000	(S_{iw})
0.32	0.615	0.0195	
0.35	0.470	0.072	
0.40	0.315	0.280	
0.45	0.210	0.790	
0.50	0.133	2.000	
0.55	0.077	4.750	
0.60	0.036	11.85	
0.65	0.012	33.50	
0.67	0.007	55.53	

Displacement Problem 1 Cont.

The oil that is displaced up-structure by the influxing aquifer water will presumably be captured by the oil wells. When the water-oil front reaches the elevation of any well there will be an instantaneous jump in water-oil ratio (this is because we are considering only one layer - the jump would be more gradual if many layers were considered). We are, of course, interested in the amount of oil that can be recovered from the invaded volume (cross-hatched area in Fig 1) as we continue to produce the wells. The items to be calculated are:

(1) How many barrels of stocktank oil will be displaced from the invaded volume (and recovered) when the front first breaks through into the first line well ? What fraction of initial oil in this volume does this amount to ?

(2) What will be the surface producing water-oil ratio immediately after breakthrough ? Assuming that the well continues to produce at the same <u>total</u> fluid rate, what will the stocktank oil rate be ?

1980

(3) When the first line well's cut reaches 95 %,what will be the amount of oil recovered from the invaded volume ? How long (years) will it take to reach this cut ? (Consider that the the aquifer influx rate into the window remains constant at 750 reservoir barrels per day.)

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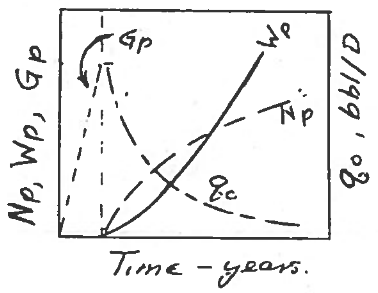
Displacement Problem 2

(The purpose of this problem is to illustrate the application of the Dykstra-Parsons method to evaluating the water flood potential of a depleted reservoir).

Calculate the expected production (cumulative) of oil, water and gas as a function of time using the Dykstra-Parsons coverage charts. Also calculate the oil producing rate, bbl/day , as a function of time. Prepare a plot of the produced quantities and the oil production rate as illustrated by the sketch at the left. Base your calculations on a reservoir pore volume of one million barrels and a constant injection rate of 100,000 barrels a year. Assume that all gas will be displaced (fill-up) from the unit before any oil production is achieved. Carry out your calculations to a producing water-oil ratio, F_{wo} , of 25.



Data for the calculation are these:

Permeability of 14 core samples are as follows:

Core #	Air Perm. md.
1	31
2	10
3	82
4	170
5	105
6	47
7	19
8	70
9	20
10	22
11	38
12	63
13	42
14	135

Initial gas saturation, S_{gi}	0.15
Initial oil saturation, S_{oi}	0.55
Initial water saturation, S_{wi}	0.30
Residual gas saturation, S_{gr}	0.05
Residual oil Saturation, S_{or}	0.28
Oil viscosity, μ_o	2.0 cp
Water viscosity, μ_w	0.5 cp
Oil form. vol. fact., B_o	1.21
Water form. vol. fact., B_w	1.01
Relative water permeability, k_{rw} at $(S_{or} + S_{gr})$	0.225
Relative oil permeability, k_{ro} at $(S_{gi}$ & $S_{wi})$	0.450

Bbl pore volume, $V_p = 7758\ Ah\phi = 1(10^6)$

Water injection rate, $i_w = 1(10^5)$ bbl/Yr

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Displacement Problem 3

(This problem illustrates the calculation of fluid production volumes and rates from a four-layer 5-spot pattern.)

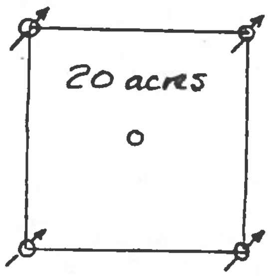

Figure 1

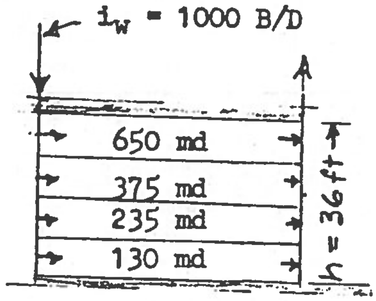

Figure 2

Saturations	Regions 1	2	3
Water, S_w	0.28	0.28	0.85
Oil, S_o	0.62	0.68	0.15
Gas, S_g	0.10	0.04	0.00
	1.00	1.00	1.00

k_{rw}(at S_{or}, Region 3) = 0.655

k_{ro}(at $S_{wi} + S_{gr}$, Region 2) = 0.498

A = 20 acres u_o = 2.66 cp
h = 36 feet u_w = 0.70 cp
ϕ = 0.23 B_o = 1.085

Pattern injection rate, i_w = 1000 B/D

Assume layer intakes proportional to $(kh)_j$

Use the above data to calculate the following parameters:

(1) The injection time (days) required to obtain fillup in second (375 md) layer.

(2) The injection time (days) at water breakthrough in the first (650 md) layer, and the cumulative production(s) at this time

(3) The following quantities at 5 years of injection.
   a. Cumulative water injected, W_i
   b. Cumulative stock tank oil produced, N_p
   c. The oil production rate, q_o
   d. The water production rate, q_w

Guide Answers

(1) 77 days
(2) 186 days; N_p = 91212 STB
(3b) N_p = 468971 STB

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Solution Displacement Problem 1

A. Calculation of fractional flow curve

$$f_w = \frac{1 - a\,k_{ro}}{1 + \frac{k_{ro}}{k_{rw}} \cdot \frac{\mu_w}{\mu_o}} \qquad a = \frac{7.84(10^{-6})\, k_{abs}\,(\rho_w - \rho_o)\,\sin\alpha}{q/A\;\mu_o}$$

$$\alpha = 35^\circ$$

$$a = \frac{7.84(10^{-6}) \cdot 100\,(65-45)\,0.5736}{750/20 \cdot 1320 \cdot 1.16} = 0.2729$$

$S_{iw} = 0.30$

$S_{wi} = 0.32$

$$\mu_w/\mu_o = \frac{0.38}{1.16} = 0.3276$$

$$\therefore\; f_w = \frac{1 - 0.2729\,k_{ro}}{1 + \frac{0.3276}{k_{rw}/k_{ro}}}$$

S_w	k_{ro}	k_{rw}/k_{ro}	f_w	$\frac{f_w - f_{wi}}{S_w - S_{wi}}$	$\frac{\Delta f_w}{\Delta S_w}$
0.30 (S_{iw})	0.725	0.0000	0	—	
0.32 (S_{wi})	0.615	0.0195	0.0467 (f_{wi})	—	
0.35	0.470	0.072	0.1571	3.680	
0.40	0.315	0.280	0.4212	4.681	
0.45	0.210	0.790	0.6664	4.767	4.904
0.50	0.133	2.000	0.8281	4.341	3.234
0.55	0.077	4.750	0.9158	3.779	1.754
0.60	0.036	11.85	0.9635	3.274	0.954
0.65	0.012	33.50	0.9871	2.850	0.472
0.67	0.007	55.53	0.9922	2.701	0.255

From plot of $\frac{f_w - f_{wi}}{S_w - S_{wi}}$ vs S_w the maximum occurs at $S_w = 0.430$. Maximum value is 4.80, which is slope of tangent line.

1. At breakthrough at the first line well, the average saturation behind the front is

$$\bar{S}_{w\,BT} = S_{wf} + \frac{1 - f_{wf}}{(f_w - f_{wi})/(S_w - S_{wi})_{max}} = 0.430 + \frac{(1 - 0.575)}{4.80} = 0.5185$$

STB recovered @ BT

$$N_p = \frac{V_p[\bar{S}_w)_{BT} - \bar{S}_{wi})}{B_o} = \frac{(2600 \cdot 1320 \cdot 20) \cdot 0.235}{1.27 \cdot 5.615}[0.5185 - 0.320]$$

$N_p = \underline{449.0\ (10^3)}$ STB. ← ans

$$\text{Rec. Fraction} = \frac{(0.5185 - 0.320)}{(1 - 0.320)} = \underline{\underline{0.292}} \leftarrow \text{ans}$$

2. What will be surface water-oil ratio at B.T.

$$\text{Subsurface } w/o = \frac{f_w}{f_o} = \frac{0.5185}{(1-0.5185)} = 1.077 = WOR$$

$$\text{Surface water-oil ratio} = 1.077 \cdot \frac{B_o}{B_w} = 1.077 \cdot \frac{1.27}{1.02} = \underline{\underline{1.34}} \leftarrow \text{ans}$$

$$q_o = \frac{q_t \cdot f_o}{B_o} = \frac{500(1-0.5185)}{1.27} = \underline{189.6 \text{ STB/D}} \leftarrow \text{ans}$$

<u>3</u> Cut @ first line well = 95%

$$\text{Cut} = 0.95 = \frac{q_w}{q_w + q_o} \quad \therefore F_{wo} = \frac{q_w}{q_o} = \frac{95}{5} = 19$$

$$WOR\ (\text{subsurface}) = F_{wo} \cdot \frac{B_w}{B_o} = 19 \cdot \frac{1.02}{1.27} = 15.26 = \frac{q_w B_w}{q_o B_o}$$

$$\therefore f_w = \frac{15.26}{15.26 + 1} = \frac{q_w B_w}{q_w B_w + q_o B_o} = 0.9385; \quad S_{wc} = 0.569$$

$$\frac{\partial f_w}{\partial S_w} = 0.95$$

$$\therefore \bar{S}_w\Big]_o^c = S_{wc} + \frac{(1 - f_{wc})}{\partial f_w / dS_w} = 0.569 + \frac{(1 - 0.9385)}{0.95} = 0.6337$$

$$\therefore \text{Recovery @ 95\% cut} = \frac{(2600 \cdot 1320 \cdot 20)\, 0.235}{5.615 \cdot 1.27}(0.6337 - 0.320)$$

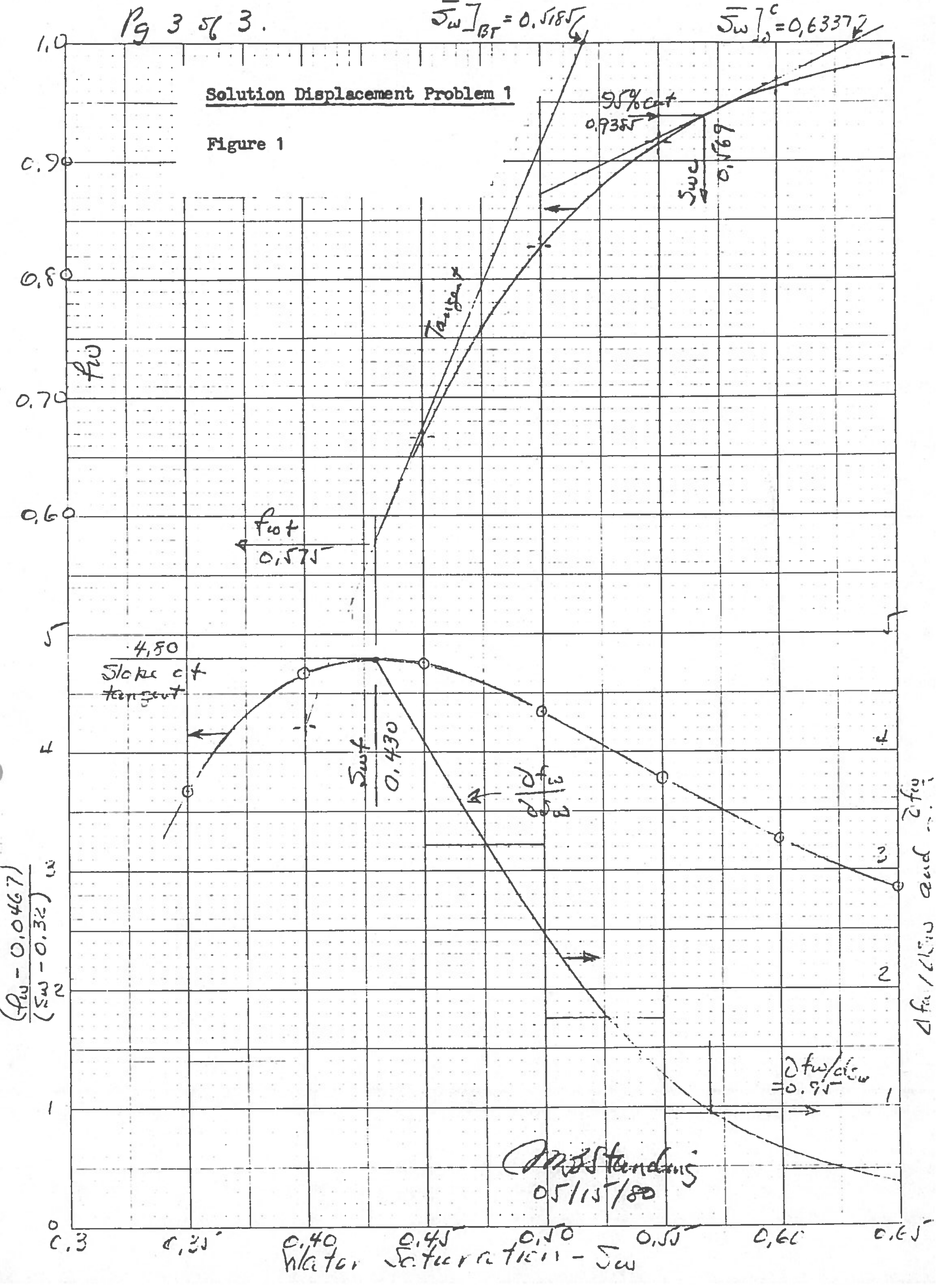
$$= \underline{\underline{709.588\ (10^3)\ \text{STB}}} \leftarrow \text{ans}$$

$$\text{Pore Vol inj} = V_{iD} = 1/\partial f_w/dS_w = \frac{1}{0.95} = 1.053$$

∴ Time to reach 95% cut

$$t = \frac{W_i}{q_i} = \frac{(2600 \cdot 1320 \cdot 20) \cdot 0.235 \cdot 1.053}{5.615 \cdot 750 \cdot 365} = \underline{\underline{11.05 \text{ years}}} \leftarrow \text{ans}$$

JRS
05/15/93


Solution Displacement Problem 1
Figure 1
$\bar{S}_w]_{BT} = 0.5185$
$\bar{S}_w]_c = 0.6337$
95% cut
0.9385
Swc
0.569
Tangent
f_w
fwf
0.575
4.80
Slope at tangent
Swf
0.430
$\partial f_w / \partial S_w$
$\partial f_w / \partial S_w$ = 0.95
$(f_w - 0.0467)/(S_w - 0.32)$
Water Saturation - Sw
05/15/80

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Solution Displacement Problem 2

A. Calculation of mobility ratio, M_{wo}

$$M_{wo} = \frac{\lambda_w}{\lambda_o} = \frac{k_{rw}/\mu_w}{k_{ro}/\mu_o} = \frac{0.225/0.5}{0.450/2.0} = 2.0$$

B. Calculation of permeability variation, V.

14 cores + 1 = 15

k	% ≥ k	k	% ≥ k
170	6.7	42	53.3
135	13.3	38	60.0
105	20.0	31	66.7
82	26.7	22	73.3
70	33.3	20	80.0
63	40.0	19	86.7
47	46.7	10	93.3

From log k probability plot (Figure 1)
Dykstra-Parsons permeability variation = 0.61

C. Calculation of produced oil & F_{wo}

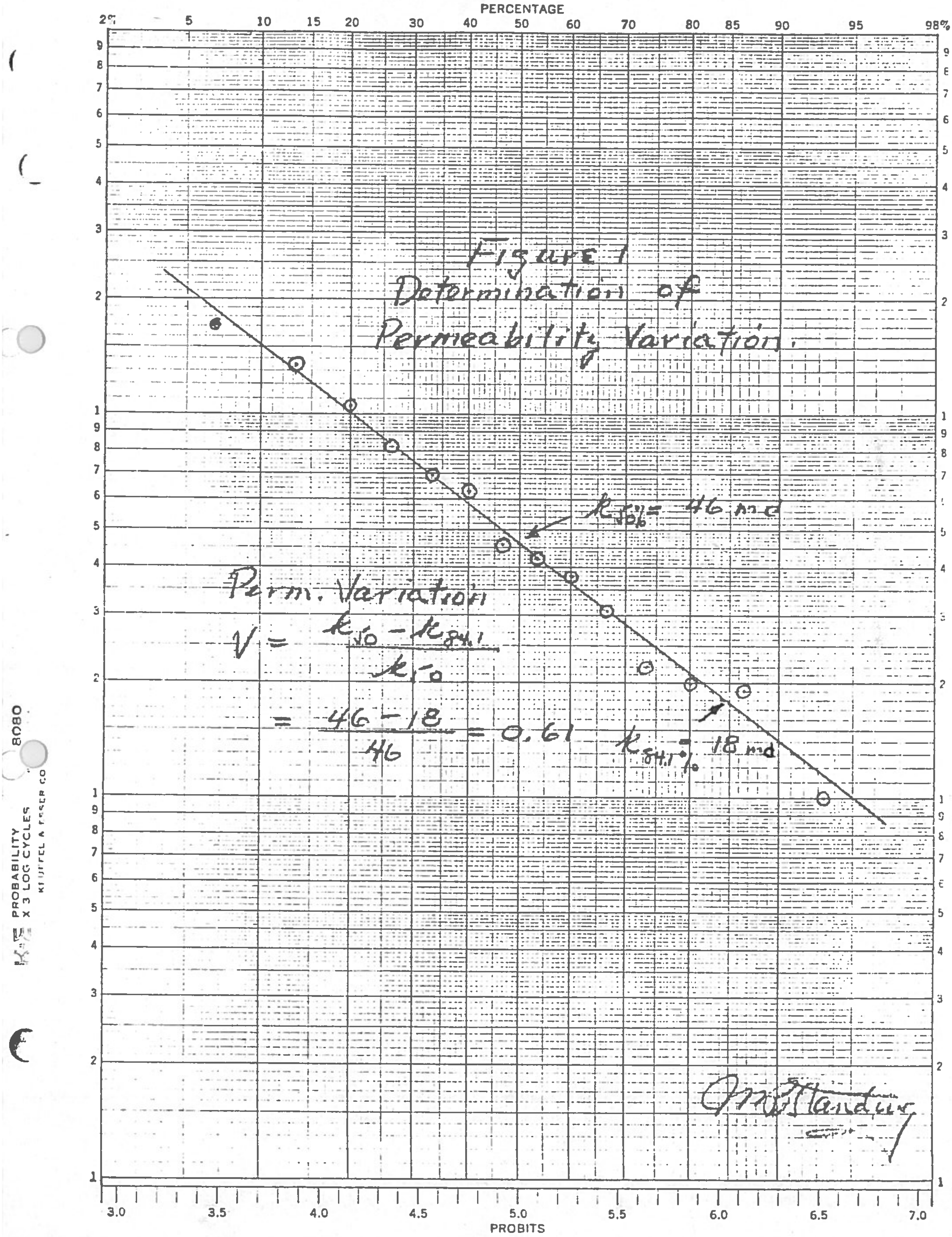
$$N_p = \frac{V_p \cdot \Delta S_o \cdot C}{B_o} = \frac{1(10^6)(S_{oi} - S_{or}) \cdot C}{B_o}$$

$$= \frac{1(10^6)(0.55 - 0.28)\, C}{1.21}$$

$$= 2.23\,(10^5)\, C.$$

$$F_{wo} = WOR \cdot \frac{B_o}{B_w} = 1.20\, WOR$$

Solution Displacement Problem 2



Solution Displacement Problem 2 Cont

WOR	F_{wo}	C	Np
0.1	0.120	0.130	0.290 (10^5)
0.2	0.240	0.175	0.390
0.5	0.600	0.273	0.609
1	1.20	0.385	0.859
2	2.40	0.520	1.160
5	6.00	0.700	1.561
10	12.0	0.805	1.795
25	30.0	0.900	2.007

D. Calculation of produced Water.

$$F_{wo} = \frac{q_w}{q_o} = \frac{dW_p/dt}{dN_p/dt} = \frac{dW_p}{dN_p}$$

$$\therefore W_p = \int_0^N F_{wo}\, dN_p.$$

Water production is calculated by determining the area under the F_{wo} vs N_p smooth curves shown on Figure 2. Increments of 0.2 (10^5) barrels were used and the areas determined by Simpson rule integration.

Increment of oil prod. 10^5 bbl	Increment Water prod. 10^5 bbl	Cum. Water Production 10^5 bbl
0 - 0.2	0.0047	0.0047
0.2 - 0.4	0.0276	0.0323
0.4 - 0.6	0.0813	0.1136
0.6 - 0.8	0.1513	0.2649
0.8 - 1.0	0.2657	0.5306
1.0 - 1.2	0.4067	0.9373
1.2 - 1.4	0.6633	1.6006
1.4 - 1.6	1.0533	2.6539
1.6 - 1.8	1.7833	4.4372
1.8 - 2.0	3.733	8.170

$$(W_p)_\Delta = \int_{N-1}^{N_p} F_{wo}\, dN_p \qquad W_p = \int_0^{N_p} F_{wo}\, dN_p$$

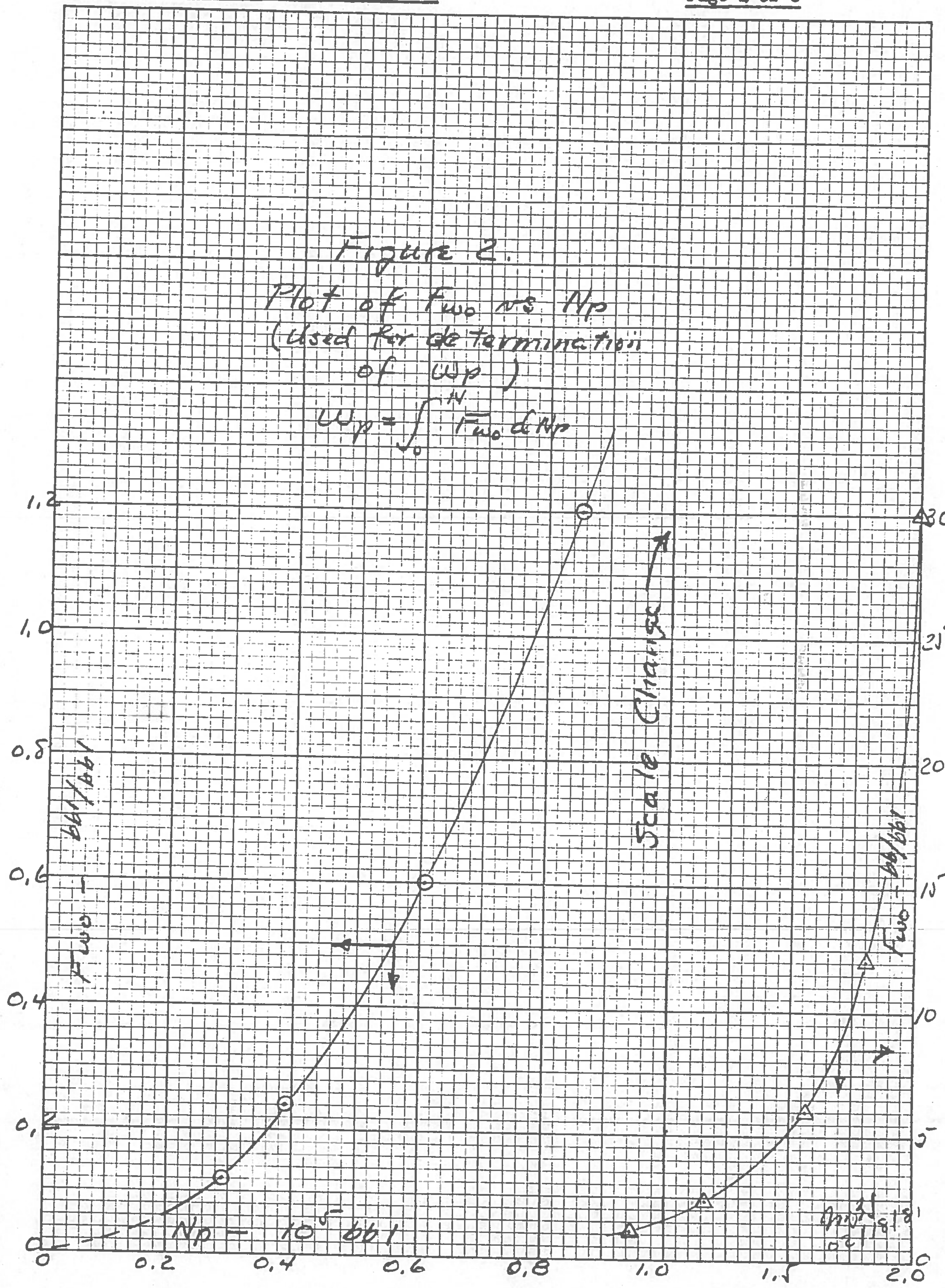


Figure 2.
Plot of Fwo vs Np
(Used for determination of Wp)
$W_p = \int_0^N F_{wo}\, dN_p$
Scale Change
Fwo - bbl/bbl
Fwo - bbl/bbl
Np - 10⁵ bbl
0
0.2
0.4
0.6
0.8
1.0
1.2
0
0.2
0.4
0.6
0.8
1.0
1.5
2.0
5
10
15
20
25
30

Solution Displacement Problem 2 Cont.

E. Calculation of system performance.

$i_w = 100\,000$ bbl/year. $= 274$ bbl/day.

Fill-up vol. $V_F = V_p \cdot \Delta S_g = 1(10^6)(S_{gi} - S_{gr})$

$= 1(10^6)(0.15 - 0.05) = 1(10^5)$ bbl

Fill-up time $= \frac{1(10^5)}{100000} = 1$ year.

$W_i = W_F + W_p + N_p B_o \qquad t = \frac{W_i}{100000}$

$q_o = \frac{i_w}{(B_o + F_{wo})}$

① N_p 10^5 bbl	② W_p 10^5 bbl	③ W_F 10^5 bbl	④ $N_p B_o$ 10^5 bbl	⑤ W_i 10^5 bbl	⑥ t years	⑦ F_{wo}	⑧ q_o bbl/D
—	—	—	—	0	0		
0.0	0.0	1	—	1	1	0.00	226
0.2	0.0047	↓	0.242	1.2467	1.25	0.06	216
0.4	0.0323		0.484	1.5163	1.52	0.25	188
0.6	0.1136		0.726	1.8396	1.84	0.59	152
0.8	0.2649		0.968	2.233	2.23	1.06	121
1.0	0.5306		1.210	2.740	2.74	1.60	98
1.2	0.9373		1.452	3.389	3.39	2.60	72
1.4	1.6006		1.694	4.295	4.30	4.1	52
1.6	2.6537		1.936	5.590	5.59	6.7	35
1.8	4.4372		2.178	7.615	7.62	12.0	21
2.0	8.170		2.42	11.59	11.6	30.0	9

Figure 3 shows performance curves for the system. Data in Columns 1, 2, & 8 are plotted against Column 6 data.

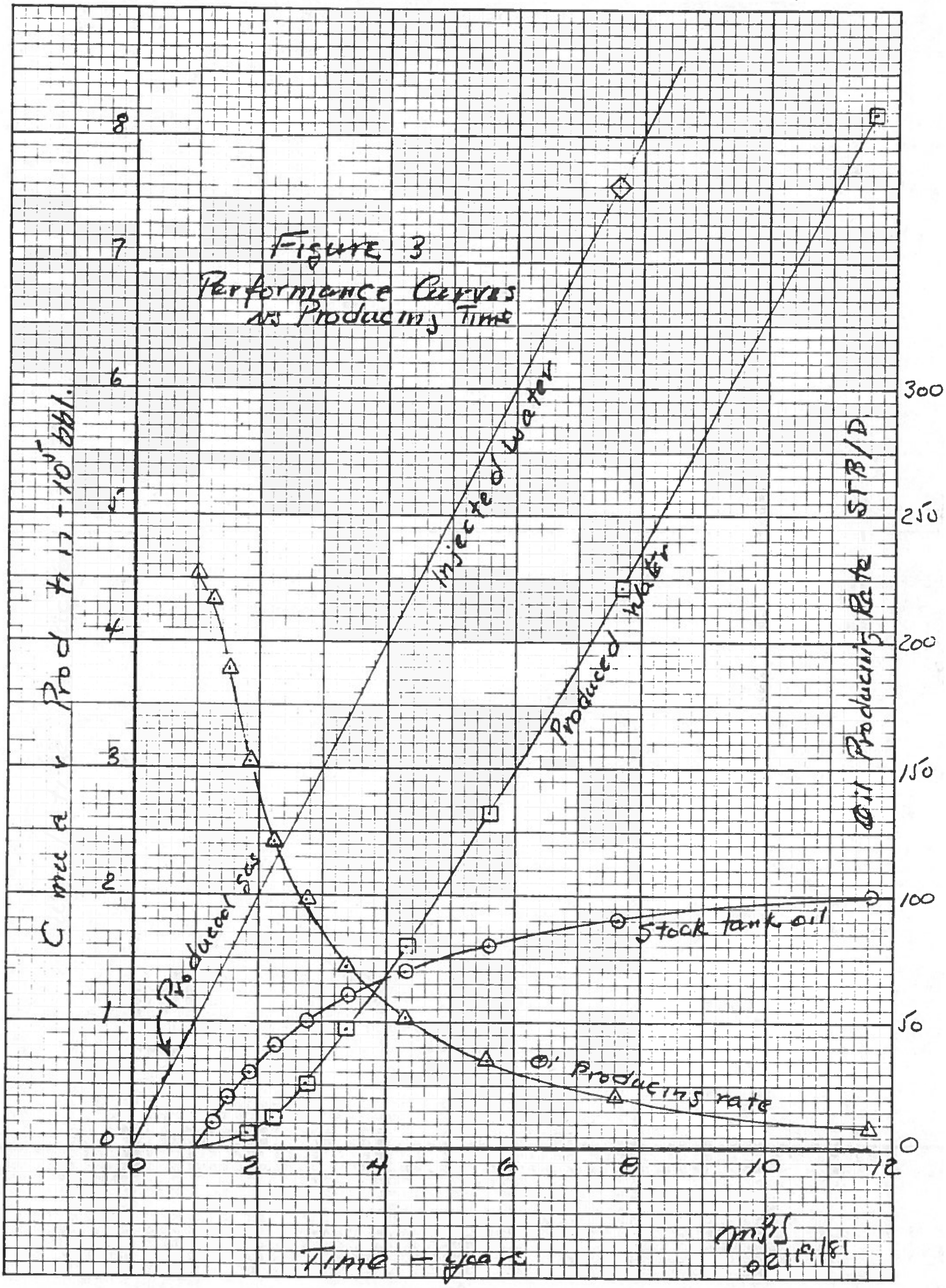


Figure 3
Performance Curves
vs Producing Time
Injected Water
Produced Water
Produced gas
Stock tank oil
Oil Producing rate
Time - years
Oil Producing Rate STB/D
0
2
4
6
8
10
12
1
3
5
7
50
100
150
200
250
300
02/19/81

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Solution Displacement Problem 3

Mobility ratio, $M_{wo} = \frac{k_{rw3}}{k_{ro2}} \cdot \frac{\mu_o}{\mu_w} = \frac{0.655}{0.498} \cdot \frac{2.66}{0.70} = 5$

$F = 1 + \frac{\Delta S_{o3}}{\Delta S_{g2}} = 1 + \frac{(0.62-0.15)}{(0.10-0.04)} = 8.83$

$\Delta S_w = S_{w3} - S_{wi} = (0.85 - 0.28) = 0.57$

Pattern pore vol. $= 7758\,Ah\phi = 7758 \cdot 20 \cdot 36 \cdot 0.23$
$= 1,284725$ bbl

Pattern displaceable vol. $= V_p \cdot \Delta S_w = 732,293$ bbl

Layer pore vol, $V_{pj} = 321181$ bbl.

Layer displaceable pore vol, $V_d = 183073$ bbl.

1. Condition of fillup in 375 md layer.

k_j	$\frac{k_j}{\Sigma k_j}$	E_{aoj}	E_{awj}	V_{dij}
650	0.468	1.733	0.196	→
375	0.270	1.000 (fillup)	0.113	→
235	0.169	0.626	0.071	→
130	0.093	0.344	0.039	→
1390			0.419	0.419

Note: E_{awbt} for $M_{wo} = 5$ is 0.475

$W_i = 0.419 \cdot V_{dj} = 0.419 \cdot 183073 = 76708$ bbl

∴ Time to fillup $= \frac{76708}{1000} = 77$ days.

2. Condition of water breakthrough in 650 md layer. Time and cum. production.

$E_{awbt} = 0.475$

$\frac{k_j}{\Sigma k_j}$	E_{awj}	E_{aoj}	V_{dij}	$(G_pB_g)_j$	$(N_pB_o)_j$	W_{pj}
0.468	0.475	4.19	0.475	25373	61586	0
0.270	0.274	2.42	0.274	22791	27371	↓
0.169	0.172	1.52	0.172	21480	10008	↓
0.093	0.094	0.83	0.094	17203	0	↓
			1.015	86847	98965	0

(fillup)

om!!

Solution Displacement Problem 3 Cont.

$W_i = 1.015 \cdot V_{dij} = 1.015 \cdot 183073 = 185819$ bbl

∴ Time to break through = 186 days.

Cum Oil Prod = $\frac{98965}{1.085}$ = 91,212 bbl.

Cum gas Prod = 86847 reservoir bbl.

} ans

3. Condition at 5 years injection

$W_i = 5 \cdot 365 \cdot 1000 = 1,825,000$ bbl. ← ans a.

$\frac{k_j}{\Sigma k_j}$	w_{ij}	i_w	V_{dij}	E_{awj}	$(\frac{dE_{aw}}{dV_{di}})_j$
0.468	854100	468	4.67	0.965	0.015
0.270	492750	270	2.69	0.922	0.032
0.169	308425	169	1.68	0.845	0.115
0.093	169725	93	0.93	0.710	0.350
1.000	1825000	1000	9.97		

$\frac{k}{\Sigma k_j}$	$(N_p B_o)_j$	$(q_o B_o)_j$	W_{pj}	q_{wj}
0.468	144997	6.5	661937	461
0.270	137677	8.0	309150	261
0.169	124570	18.0	140158	149
0.093	101590	30.0	28341	60
	508834	62.5	1139586	931

∴ $N_p = \frac{508834}{1.085}$ = 468971 Stock tank bbl. ← b.

q_o = 62.5/1.085 = 57.6 STB/D. ← c.

W_p = 1,139,586 bbl.

q_w = 931 bbl/day. ← d.

} ans

Oil Recovery Factor = $\frac{508834}{1,284,725 \cdot 0.62}$ = 64%

Present water cut = $\frac{q_w}{q_w + q_o}$ = 93%

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