Notes on Predicting Pattern Performance During Water Injection Operations

Reservoir engineers are oftem called upon to estimate the profitability of water flooding a given property or reservoir. There are a number of methods described in the literature that can be used. Each method involves several assumptions about the sub-surface reservoir behavior and each has a different degree of complexability in carrying out the calculations. The Higgins-Leighton method, for example, is so detailed that it can only be used in conjunction with a large computer. It problbly offers the better solution, however, all things being equal. Simpler methods, capable of being carried out with hand-held calculators, often offer solutions (predictions) sufficiently accurate for preliminary evaluations of the flood and may, in certain instances, be sufficiently accurate for use as the final prediction method.

The method described in these notes falls in the second catagory. It is relative simple and straight forward and can be handled by desk calculator. It models a multi-layer pattern flood with provision for the displacement of gas ahead of the oil bank as well as displacement or oil by the injected water. The primary assumptions in the method are these:

1. The reservoir volume to be flooded (pattern) is handled as a group of layers with no cross flow between layers. Thus, it is similar to an assumption in the Dykstra-Parsons method.
2. Within a given layer there are no saturation gradients behind the two fronts. This means that the displacement is piston like (or leaky-piston) in nature. This also is similar to an assumption of the Dykstra-Parsons method. However, one can develop a similar method that uses a Buckley-Leverett type of displacement.
3. Water intake into each layer is proportional to the layer injectivity at the time (which changes in each layer as the flood progresses). However, to simplify these notes and concentrate on describing the displacement process model involved, it will be assumed that the layer injectivities are constant in time and proportional to the kh product of the layer.

The first section of the notes will describe the behavior of a single layer of the model. Later sections will describe the process of combining the behavior of all layers to yield the desired pattern behavior. Field behavior, which invols superposition in time of a number of pattern behaviors will not be covered in these notes.

Single Layer Behavior

The geometry of the segment of reservoir to be considered is that of a developed 5-spot pattern consisting of one production well at the center and four injection wells located at the corners. This is illustrated in Fig. 1. To simplify description behavior in only ¼ of the pattern (cross-hatched area) will be considered.

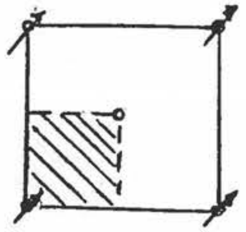


Figure 1

Figure 2 illustrates layering within the unit volume. As indicated previously, no cross flow of fluids occurs between layers. This means that water entering a given layer at the injection wellbore remains in and displaces hydrocarbons from that layer.

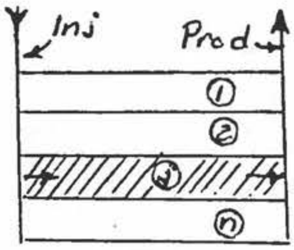


Figure 2

Figure 3 illustrates location of two displacement fronts in a given (j) layer prior to fillup. At this point only free gas has been produced for the layer. The area processed by the injected water is labled Region 3 and has an areal coverage value of E_{aw}. Region 2 is an area which contains oil that has been displaced from Region 3 plus some residual gas remaining from the displacement at the oil-gas front. The areal coverage factor of the oil-gas front is E_{ao}. Region 1 contains initial saturation conditions.

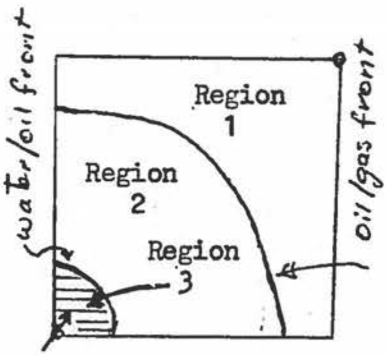


Figure 3

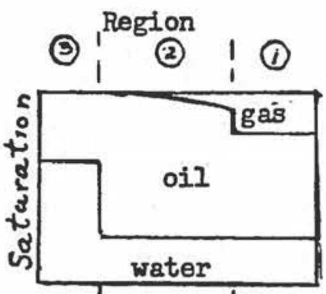


Figure 4 a

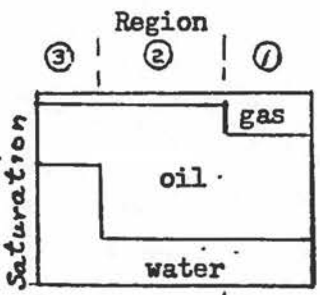


Figure 4b

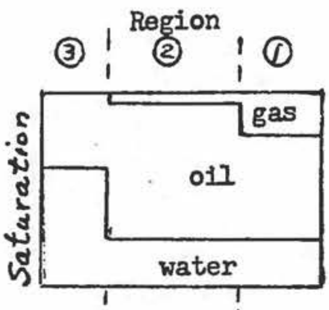


Figure 4c

Figure 4, above, illustrate several saturation-distance situation along the diagonal connecting the injection and producing well is Fig 3. In the field, pressure increases from the producing well to the injection well. Gas that is left behind the oil-gas displacement front is consequently compressed and taken into solution in the oil in Region 2. Thus there is the possibility that the gas saturation - distance profile should look somewhat like that in Fig 4a. On the other hand, not knowing how far back in Region 2 gas saturation exists it is easier to assume saturation profiles like those of Fig 4b or 4c. Figure 4b assumes residual gas saturation exists all the way through Regions 2 and 3, while Fig 4c assumes that residual gas remains only in Region 2. Some small diferences in pattern performance result from how the residual gas phase is assumed to lie. Figure 4b is often used and results in the easiest analysis of the pattern behavior. Figure 4c seems a little more in keeping with postulated field behavior and will be the situation used in these notes.

Considering the situation pictured in Fig 3 (prior to fillup) the oil and gas displaced from Region 3 must be in Region 2 or have been produced. Gas originally in Region 3, or part of it, could have been produced but oil from Region 3 must still be in Region 2. In terms of the areal coverage factors E_{aw} and E_{ao} and a volume balance of the displaced oil phase we can compute the relative areas processed by the two fronts as follows:

Given:

E_{aw} = area included in Region 3

$E_{ao} - E_{aw}$ = area included in Region 2

$(\Delta S_o)_3$ = oil saturation change, Region 3

$(\Delta S_g)_2$ = gas saturation change, Region 2

$$E_{aw}(\Delta S_o)_3 = (E_{ao} - E_{aw})(\Delta S_g)_2 \tag{1}$$

This leads to.

$$\frac{E_{ao}}{E_{aw}} = F = 1 + \frac{(\Delta S_o)_3}{(\Delta S_g)_2} \tag{2}$$

Equation 2 is completely general. Nothing has been said about the shapes of the two displacement fronts or of the mobility ratios that pertain to the fronts.

In general, the mobility ratio at the oil-gas front is apt to be quite low - probably less than 0.1, while the water-oil mobility ratio at the water-oil front is apt to be greater than one. Each mobility ratio can be calculated from the saturation conditions considered and the phase viscosity ratio.

Areal coverage factors, E_a, for a number of standard patterns (5-spot, 9-spot, direct line drive, etc.) are available in various forms in the literature (see, for example, SPE Monograph #3 on waterflooding). The attached chart shows 5-spot values determined by Caudle and Witte, (Trans AIME 216,(1959) 446). While not shown on this chart, E_a values for mobility ratios less than 0.15 are, for practical purposes, unity.

The term, displaceable pore volume, used in waterflood calculations is defined as the unit pore volume multiplied by the maximum achievable difference of water saturation in the area processed by the injected water. Thus, for a single layer of the 5-spot pattern illustrated in Fig 1, the displaceable pore volume, V_d, amounts to

$$V_d(\text{bbl}) = 7758\ A\ h\ \phi\ \Delta S_w \tag{3}$$

where

A = pattern (layer) area, acres
h = layer thickness, feet
ϕ = layer porosity, fraction
ΔS_w = $(S_{w\ max} - S_{wi})$ in Region 3
= $(1-S_{or}-S_{gr}-S_{wi})$ in Region 3

The term displaceable pore volume injected, V_{di}, is, of course, the volume of injected fluid divided by the displaceable pore volume.

$$V_{di} = W_i/V_d \tag{4}$$

when the injected fluid is water.

To illustrate some of the calculation procedures involved in predicting pattern behavior it is helpful to use specified values for variables in the calculations. In the following calculations the pattern is assumed to be a 5-spot and the following factors.

Pattern area, A = 10 acres
Pattern thickness (total) = 16 feet
Number of layers = 4 (equal thickness of 4 feet)
Porosity, ϕ = 0.25
Mobility ratios =
M_{wo} = 2 (water-oil front)
M_{og} = 0.1 (oil-gas front)
Layer permeabilies, k_j = 700, 500 400, and 200 md.
Saturations, S

Region	S_w	S_o	S_g
1	0.30	0.58	0.12
2	0.30	0.65	0.05
3	0.68	0.32	0.00

Injection rate, i_w = 450 barrels per day

The above values result in the following values for Vp, V_d, and F.

$V_p(\text{pattern}) = 7758\ A h \phi = 7758 \cdot 10 \cdot 16 \cdot 0.25 = 310320$ bbl.

$V_{p_j}(\text{layer}) = 310320/4 = 77580$ bbl.

$V_d(\text{pattern}) = V_p \cdot \Delta S_w = 310320\,(0.68-0.30) = 117922$ bbl

$V_{d_j} = V_d/4 = 29480$ bbl

$$F = \frac{E_{ao}}{E_{aw}} = 1 + \frac{(\Delta S_o)_3}{(\Delta S_g)_2} = 1 + \frac{(0.58-0.32)}{(0.12-0.05)} = 4.71$$

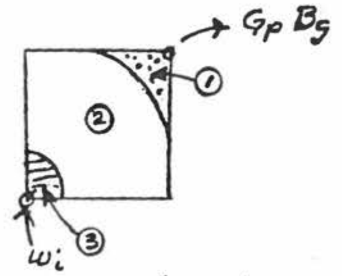
$$V_{di} = W_i/V_d = \frac{450 \cdot 365}{117922} = 1.39 \text{ per year}$$

At this point it is well to develop equations for the cumulative products produced from the layer and their rate of production at various times. To do so we will consider three situations. these are:

1. Prior to fillup. Only gas is being produced
2. After fillup but before water breakthrough. Both gas and oil phases are being produced.
3. After water breakthrough. Both oil and water phases are being produced.

The first set of equations are for the cumulative productions removed from the layer. Separate equations for rates will be developed later.

1. Prior to Fillup - Only Gas Production



The basic approach is a volume balance on the gas phase; i.e., the produced volume is equal to the original volume minus what is still there. Refering to Fig 4c, it seen that original gas saturation is in Region 1, residual gas saturation is in Region 2, and zero gas is in Region 3. Consequently, the gas phase volume balance is:

$$(G_p B_g)_j = V_{p_j}\left[S_{gi} - (1-E_{ao})S_{g1} - (E_{ao}-E_{aw})S_{g2}\right] \tag{5}$$

where

S_{gi}, S_{g1} = original gas saturation value
S_{g2} = gas saturation in Region 2

Because pattern efficiency charts only provide E_{aw} values Eq 5 can be modified by recalling that $E_{ao} = F\,E_{aw}$ (eq 2,pg 3). Also, noting that Region 1 contains the original gas saturation so that S_{gi} and S_{g1} are the same, Eq 5 developes into:

$$(G_p B_g)_j = V_{p_j}\, E_{aw_j}\left[F S_{gi} - S_{g2}(F-1)\right] \tag{6}$$

Equations for oil and water phase productions during the "prior to fillup" regime are, of course:

$$(N_p B_o)_j = 0 \tag{7}$$

$$(W_p B_w)_j = 0 \tag{8}$$

At this point let's consider an example calculation of the gas produced at fillup from one of the layers described on page 4. Under these conditions:

$E_{ao} = 1$: $E_{aw} = E_{ao}/F = 1/F = 1/4.71 = 0.212$

From the 5-spot chart for $E_{aw} = 0.212$ and $M_{wo} = 2$. $V_{d_{ij}} = 0.212$

Therefore, $W_{ij} = 0.212\ V_{dj} = 0.212 \cdot 29480 = 6250$ barrels.

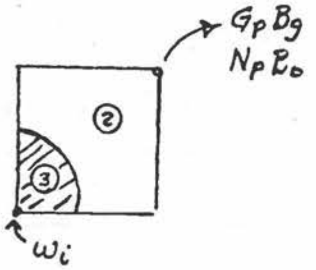
$$(G_p B_g)_j = 77580 \cdot 0.212 \left(4.71 \cdot 0.12 - 3.71 \cdot 0.05\right) \qquad (\text{Eq } 6)$$

$$= 6245 \text{ bbl.}$$

$$\text{Producing time, } t = \frac{W_{ij}}{i_j} = \frac{6250}{450/4} = 55 \text{ days.}$$

Note: The slight difference of 5 barrels between the water injected and the produced gas comes from round-off errors. They should be the same, of course.

2. After Fillup but Before Water Breakthrough



During this production phase gas is still being produced because of expansion of Region 3 and the fact a gas saturation change occurs at the water-oil front. Oil is also being produced because of the expansion of Region 3.

The gas volume balance can be obtained directly from Eq 5 by noting that $E_{ao} = 1$. Or, looking at the sketch at the left, it can be seen that:

$$(G_p B_g)_j = V_{p_j} \left[S_{gi} - (1 - E_{aw}) S_{g2} \right] \qquad (9)$$

The cumulative oil production comes from an oil volume balance. Refering to the sketch above, it can be seen that:

$$(N_p B_o)_j = V_{p_j} \left[S_{oi} - E_{aw} S_{o3} - (1 - E_{aw}) S_{o2} \right] \qquad (10)$$

As we have specified that water breakthrough has not yet taken place the water production equation is still

$$(W_p B_w)_j = 0 \qquad (11)$$

An illustrative use of Eqs 9 and 10 is as follows: what will be the reservoir gas and oil cumulative productions when the layer has been produced for 100 days?

$$W_{i_j} = 450/4 \cdot 100 = 11250 \text{ barrels.}$$

$$V_{di_j} = \frac{W_{ij}}{V_{dj}} = \frac{11250}{29480} = 0.382$$

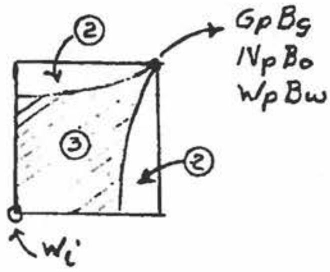
From the 5-spot chart for $V_{di} = 0.382$, $E_{aw} = 0.382$

$$\therefore (G_p B_g)_j = 77580 \left[0.12 - (1 - 0.382) \cdot 0.05\right] = 6912 \text{ barrels.}$$

$$(N_p B_o)_j = 77580 \left[0.58 - 0.382 \cdot 0.32 - (1 - 0.382)\, 0.65\right]$$

$$= 4349 \text{ barrels.}$$

3. After Water Breakthrough



After water breakthrough there is still gas production because Region 3 is increasing in size and the gas saturation change at the water-oil front. Oil phase is being produced because of the diminishing size of Region 2.

As can be seem from the sketch at the left, there is really no difference insofar as the gas and oil phases are concerned, than the situation depicted on the previous page. Only the relative sizes of Regions 2 and 3 are different. Therefore, Eqs 9 and 10 are valid for this period.

The volume balance on the water that has entered the layer and that remaining is simple. The produced water volume is:

$$W_{p_j} = W_i - V_{p_j} \cdot E_{aw}(S_{w3} - S_{wi}) \qquad (12)$$

There is some question as to whether the water quantities in Eq 12 should not contain the water formation volume factor, B_w. It is more correct to include it but as it is usually very close to unity it often is omitted.

An alternate equation that can be used to calculate produced water comes from a volume balance of all fluids in the pattern, that is:

$$W_{pj} = W_{ij} - (N_pB_o)_j - (G_pB_g)_j \qquad (13)$$

To illustrate the after breakthrough production equations let's calculate the layer production after one year of injection.

$$W_{ij} = 450/4 \cdot 365 = 41062 \text{ barrels}$$

$$V_{dij} = \frac{41062}{29480} = 1{,}393$$

From 5-spot chart for $V_{di} = 1{,}39$; $M_{wo} = 2$,

$$E_{aw} = 0{,}879.$$

$$\therefore (G_pB_g)_j = 77580[0{,}12 - (1 - 0{,}879)0{,}05]$$
$$= 8840 \text{ barrels.} \qquad (Eq. 9)$$

$$(N_pB_o)_j = 77580[0{,}58 - 0{,}879 \cdot 0{,}32 - (1 - 0{,}879)0{,}65]$$
$$= 17073 \text{ barrels.} \qquad (Eq\ 10)$$

$$W_{p_j} = 41063 - 77580 \cdot 0.879(0.68 - 0{,}30)$$
$$= 15150 \text{ barrels}$$

Layer Producing Rate Equations. Up to this point the equations have been for the cumulative volumes of reservoir gas, oil, and water produced from the layer as functions of time or cumulative water injected. Ordinarilly one is equally interested in the rates of production of oil and water (not so much for gas) during the flood as these values are used to size equipment and calculate income to the project. As rate is the time derivative of cumulative production, e.g. $q_o = dN_p/dt$, one can always resort to plotting cumulative production values against time and determining tangent slopes at various time. But the method to be used in the following portion of the note is to differentiate with respect to time eqs 6,9,10 and 12.

Gas Rate Prior to Fillup. If one considers the sketch on page 5 that represents the "prior to fillup" case, it is instantly apparent that gas is being removed from the layer at the same rate that water is entering the pattern. So, for this case

$$(q_g B_g)_j = i_{wj} \tag{14}$$

However, it is instructive to see if this relationship can be developed from Eq 6 (or Eq 5) as the method of doing so will apply to developing the other fluid rate equations. Starting with Eq 6 then:

$$(G_p B_g)_j = V_{p_j} \cdot [F S_{gi} - S_{g2}(F-1)] E_{aw} \tag{6}$$

As V_{p_j}, F, S_{gi} and S_{g2} are constants

$$(q_c B_g)_j = \frac{d}{dt}(G_p B_g)_j = V_{P_j}[F S_{gi} - S_{g2}(F-1)] \cdot \frac{d E_{aw}}{dt} \tag{15}$$

$$\text{But } \frac{d E_{awj}}{dt} = \frac{d E_{awj}}{d V_{dij}} \cdot \frac{d V_{dij}}{dt} \tag{16}$$

$$\text{and } \frac{d V_{dij}}{dt} = \frac{d}{dt} \frac{W_{ij}}{V_j \Delta S_w} = \frac{1}{V_j \Delta S_w} \cdot i_{wj}$$

Therefore.

$$(q_g B_g)_j = \frac{\cancel{V_p}[F S_{gi} - S_{g2}(F-1)] \cdot i_w}{\cancel{V_p} \Delta S_w} \frac{d E_{aw}}{d V_{di}} \tag{17}$$

As $\Delta S_w = [F S_{gi} - S_{g2}(F-1)]$ and at small values of E_{aw} $\frac{d E_{aw}}{d V_{di}} = 1$, we get

$$(q_g B_g)_j = i_{wj}$$

At th instant that fillup is achieved there will be a major decrease in the gas production rate. This is because the gas was being produced in accordance with the volumetric rate of advance of the oil-gas front and, after fillup, the oil and gas being produced is a function of the water-oil front advance rate. This will be handled in the next section.

Gas Rate After Fillup. Equation 9 on page 6 gives the cumulative gas produced after fillup. This can be written as:

$$(G_p B_g)_j = V_{p_j} (S_{gi} - S_{g2}) + V_{p_j} S_{g2} E_{aw} \tag{18}$$

By following the exact same procedure as on the previous page we get:

$$(q_g B_g)_j = \frac{S_{g2}}{\Delta S_w} i_{wj} \left(\frac{dE_{aw}}{dV_{di}}\right)_j \tag{19}$$

where $\Delta S_w = S_{w3} - S_{wi}$

$\left(\frac{dE_{aw}}{dV_{di}}\right)$ = slope of areal efficiency curve.

Equation 19 applies any time after fillup of the layer. Note that a sharp change in rate will occur at water breakthrough as this is the condition at which the slope of the areal efficiency curve changes unity to some smaller value.

Oil Rate After Fillup. As pointed out previously, the equation for the cumulative oil production is the same before and after water breakthrough. Of course, before fillup the produced oil volume is zero. Rewritting Eq 10 in slightly different form, we have:

$$(N_p B_o)_j = V_{p_j} (S_{oi} - S_{o2}) + V_{p_j} (S_{o2} - S_{o3}) E_{aw} \tag{20}$$

Again applying the procedures shown on page 8 we obtain:

$$(q_o B_o)_j = \frac{(S_{o2} - S_{o3})}{\Delta S_w} \cdot i_{wj} \left(\frac{dE_{aw}}{dV_{di}}\right)_j \tag{21}$$

Water Rate After Breakthrough. It is quite easy to see from Eq. 12 that the water rate equation after breakthrough is:

$$(q_w)_j = i_{wj} \left(1 - \frac{dE_{aw}}{dV_{di}}\right) \tag{22}$$

In fact, it can be seen that before breakthrough this equation predicts that $q_w = 0$.

This concludes the development of the rate equations. It is apparent that the slope of the areal efficiency curve is an important parameter, as are certain key saturation values and the injection rate.

If many calculations are to be made (10 or more layers) it is probably best to fit simple equations to the efficiency curve and develop the slope by taking the derivative of the curve fit equation. When only a few calculations are to be made it is sufficient to construct tangents to the efficiency curve and determine the slope by that means. The tangent slopes are then plotted against V_{di} for easy reference.

Multilayer Behavior

The basic method of calculating the behavior of a multilayer system is to apply the relationships developed for a single layer and add the results. Because the permeability of the layers are different, the fronts advance at different rates in the layers. In essence, the calculation boils down to keeping track of the water that has entered each layer and computing the layer outputs.

The following illustrative calculations are based on four layers. It would be better to use more layers, say 10 or 20, but it is not done as it involves too much calculation. In the calculations all layers are of the same thickness, and have the same porosity. This is not necessary to the method as layers may have different thickness and porosity. They may also have different saturation values postulated in the different regions. But these are complicating factors and generally are not worth carrying out in a preliminary evaluation of a field prospect.

← Injection well	Producing well →
Layer 1	700 md
Layer 2	500 md
Layer 3	400 md
Layer 4	200 md

Figure 5

Figure 5 shows the permeability of the four layer system to be calculated. The calculation scheme is facilitated by placing the highest permeability at the top and working downward. This, of course, may bear no resemblance to the real situation in the field.

A tabular calculation format seems to work best. This will be illustrated on the following pages. The first calculation will be for the condition of fillup in the first layer. The second calculation will be for 3 months of injection and the third for one year of injection. In each calculation it is assumed that the water entering a layer is proportional to the permeability , k_j , of the layer.

1. Condition - Fillup in Layer 1

$N_p = 0$; $W_p = 0$; $F = 4.71$; $E_{ao} = 1.0$; $\Delta S_w = 0.38$; $S_{gi} = 0.12$
$S_{g2} = 0.05$; $S_{oi} = 0.58$; $S_{o2} = 0.65$; $S_{o3} = 0.32$; $V_{pj} = 77580$ bbl ;
$V_{dj} = 29480$ bbl. Pattern injection rate, $i_w = 450$ bbl/day

	①	②	③	④	⑤	⑥	⑦
	k_j	$k_j/\Sigma k_j$	E_{aoj}	E_{awj}	V_{dij}	$(G_p B_g)_j$	$(g_g B_g)_j$
	700	0.389	[1.0]	0.212	0.212	6253	175
	500	0.278	0.714	0.152	0.152	4467	125
	400	0.222	0.571	0.121	0.121	3570	100
	200	0.111	0.286	0.061	0.061	1790	50
Σ	1800	1.00			0.546	16080	450

Water injected, $W_i = V_{dj} \Sigma V_{dij} = 0.546 \cdot 29480$
$= 16096$ barrels

$$\text{Time} = \frac{W_i}{i_w} = \frac{16096}{450} = 36 \text{ days.}$$

Calculation notes:

④ $E_{awj} = E_{aoj}/F$

⑥ From Eq 5

⑦ From Eq 14

[ ] = starting value.

2. Condition - 3 Months of Injection

$W_i = 450 \cdot 91.25 = 41063$ barrels. ; $V_{pj} = 77580$ bbl ; $V_{dj} = 29480$ bbl
$\Delta S_w = 0.38$; $S_{oi} = 0.58$; $S_{o2} = 0.65$; $S_{o3} = 0.32$; $S_{gi} = 0.12$; $S_{g2} = 0.05$

①	②	③	④	⑤	⑥
$\frac{k_j}{\Sigma k_j}$	i_{wj}	V_{dij}	E_{awj}	E_{aoj}	$\left(\frac{d E_{aw}}{d V_{di}}\right)_j$
0.389	175	0.542	0.542	1	1
0.278	125	0.387	0.387	1	1
0.222	100	0.309	0.309	1	1
0.111	50	0.155	0.155	0.728	1
1.000	450	1.393			

Layer	⑦ $(G_pB_g)_j$	⑧ $(q_gB_g)_j'$	⑨ $(N_pB_o)_j$	⑩ $(q_oB_o)_j'$	⑪ $(W_p)_j$
1	7533	23	8445	152	0
2	6932	17	4477	109	
3	6629	13	2480	87	
4	4566*	50*	0	0	
	25660	103	15402	248	0

Calculation notes:

⑤ $E_{aoj} = F \cdot E_{awj}$ Note that layer 4 has not filled up.

⑥ E_{awbt} for $M_{wo} = 0.585$ Breakthrough has not occured in any layer.

⑦ ⑧ Eq 9 & 17 except for last layer. Last layer Eq 6, 14.

⑨ ⑩ Eqs 10 & 20

3. Condition - One Year of Injection

W_i = 450 • 365 = 164250 barrels. See previous calculations for other variables.

① i_{wj}	② V_{dij}	③ E_{awj}	④ $\left(\frac{dE_{aw}}{dV_{di}}\right)_j$	⑤ $(G_pB_g)_j$	⑥ $(q_gB_g)_j'$
175	2.168	0.952	0.06	9123	1.4
125	1.548	0.897	0.12	8910	2.0
100	1.236	0.851	0.20	8732	2.6
50	0.620	0.615	0.75	7816	4.9
450				34581	10.9

⑦ $(N_pB_o)_j$	⑧ $(q_oB_o)_j'$	⑨ $(W_p)_j$	⑩ q_{wj}
18742	9.1	35810	164.5
17734	13.0	19181	110.0
16356	17.4	11412	80.0
10314	32.6	119	12.5
63146	72.0	66522	367.0

Calculation Notes:

④ tangent method

⑤ ⑥ Eqs 9 & 17

⑦ ⑧ Eqs 10 & 21

⑨ ⑩ Eqs 12 & 22

The one year of injection results are quite interesting. Note that about 94% of the water entering the first layer is being produced. Oil production from the first layer amounts to only about 13% of the total. The initial quantity of oil in the pattern amounted to 161,366 reservoir barrels. At the end of this first year of injection a total of 63,146 reservoir barrels have been recovered. This is a a pattern recovery factor of 39%. The other thing to note is the current surface water-oil ratio. Assuming an oil formation volume factor of 1.1, the surface stock tank oil rate is 65.5 STB/day. The water producing rate is 367 barrels a day. This means that the expense involved in treating 450 barrels per day of injection water and lifting and disposing of 367 barrels a day of produced water must be met by income generated from the 65.5 stock tank oil barrels.

One can see from the form of the equations developed in this note that a relative simple computer program can be written to handle many layers and many time step conditions. The assumption of layer injection rates proportional to their permeability is not the best that could be made. A separate set of calculations could have been made to handle the change in layer conductivity as the two fronts progress through the layer (Deppe's method) but the general result would not have been much different.

Summary of Equations

1. Prior to Fillup

$$(G_p B_g)_j = V_{pj} E_{awj} \left[F S_{gi} - (F-1) S_{g2} \right] \tag{6}$$

$$(q_g B_g)_j' = i_{wj}' \tag{14}$$

2. After Fillup But Before Water Breakthrough

$$(G_p B_g)_j = V_{pj} \cdot \left[S_{gi} - (1 - E_{aw}) S_{g2} \right] \tag{9}$$

$$(q_g B_g)_j' = \frac{S_{g2}}{(S_{w3} - S_{wi})} \, i_{wj}' \left(\frac{d E_{aw}}{d V_{di}} \right)_j' \tag{19}$$

$$(N_p B_o)_j' = V_{pj} \cdot \left[S_{oi} - E_{aw} S_{o3} - (1 - E_{aw}) S_{o2} \right] \tag{10}$$

$$(q_o B_o)_j' = \frac{(S_{o2} - S_{o3})}{(S_{w3} - S_{wi})} \cdot i_w \cdot \left(\frac{d E_{aw}}{d V_{di}} \right)_j \tag{21}$$

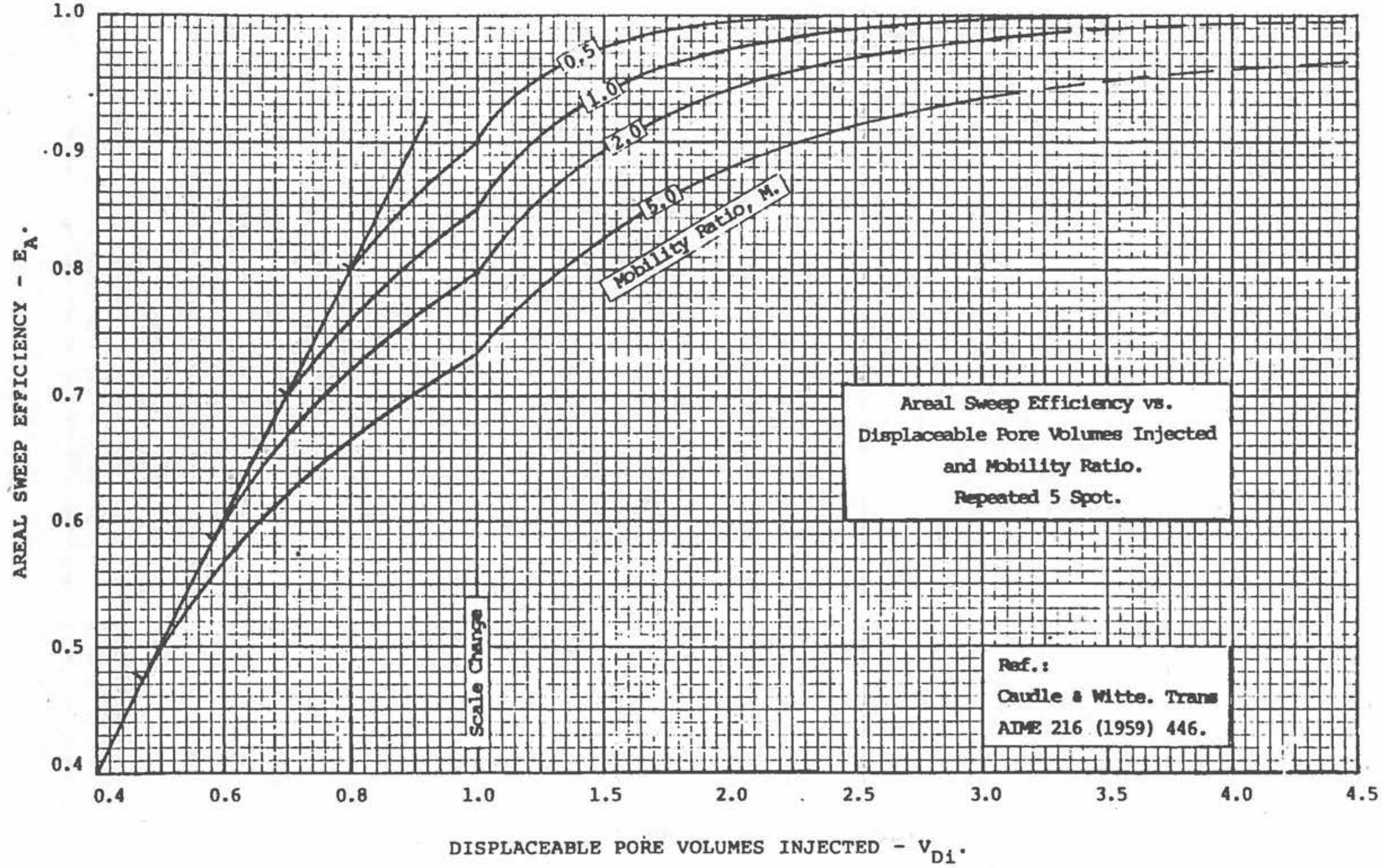
3. After Water Breakthrough

Oil and gas equations same as in 2, above.

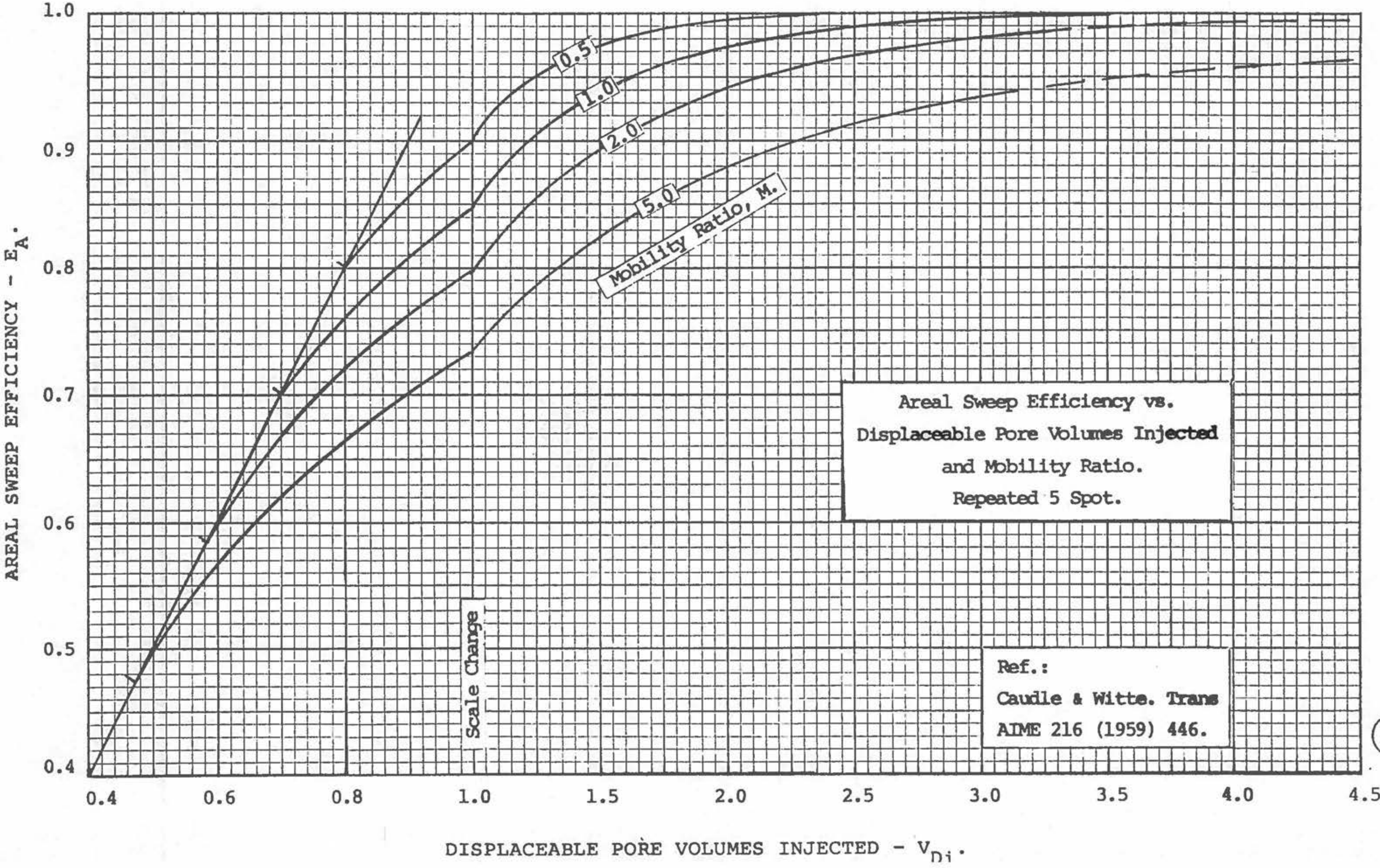
$$W_{pj} = W_{ij}' - V_{pj} \cdot E_{aw} (S_{w3} - S_{wi}) \tag{12}$$

$$q_{wj}' = i_{wj} \left(1 - \left(\frac{d E_{aw}}{d V_{di}} \right)_j \right) \tag{22}$$

M.B. Standing
February 28,1981



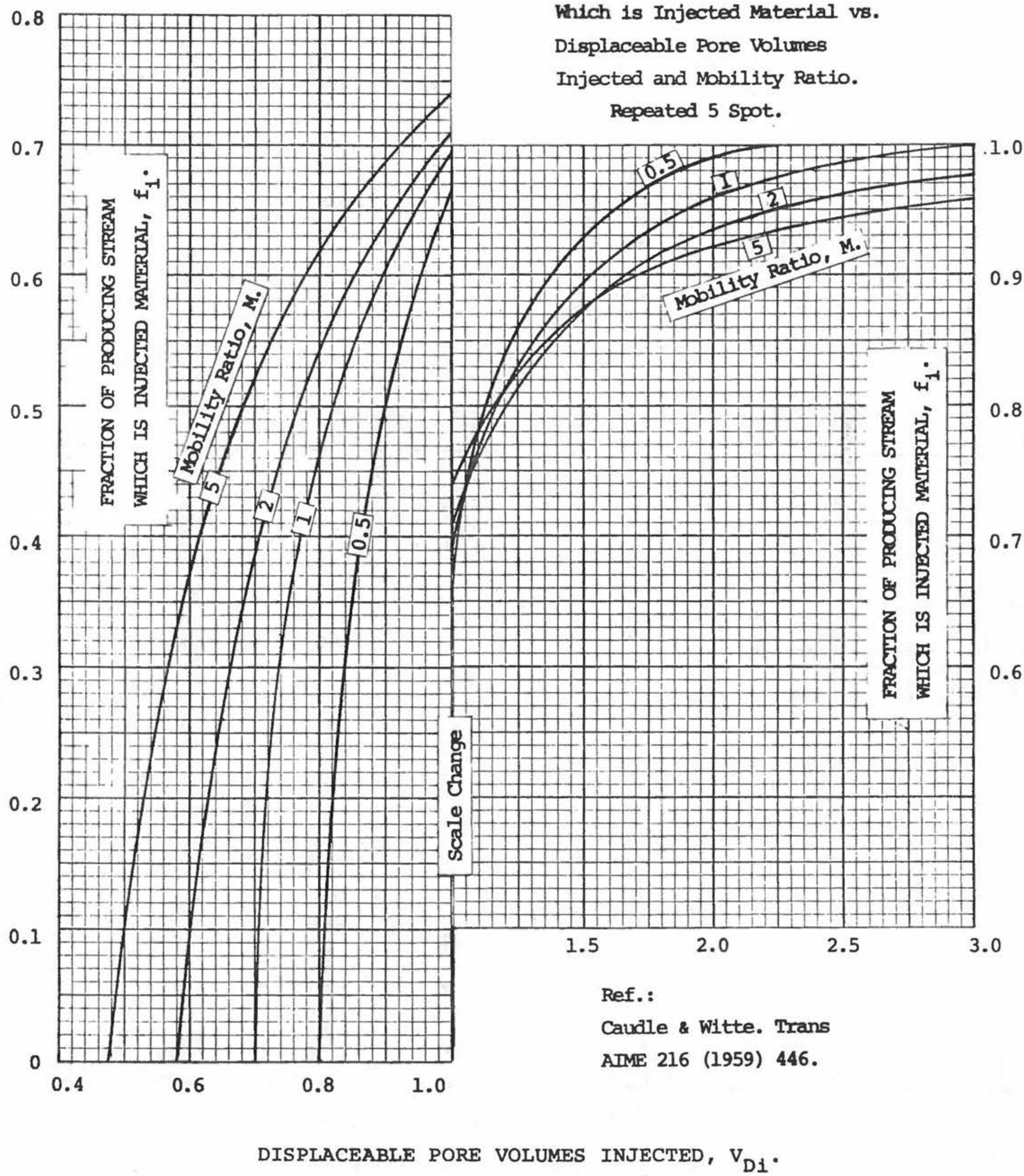
AREAL SWEEP EFFICIENCY - E_A.
DISPLACEABLE PORE VOLUMES INJECTED - V_{Di}.
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.4
0.6
0.8
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0.5
1.0
2.0
5.0
Mobility Ratio, M.
Scale Change
Areal Sweep Efficiency vs.
Displaceable Pore Volumes Injected
and Mobility Ratio.
Repeated 5 Spot.
Ref.:
Caudle & Witte. Trans
AIME 216 (1959) 446.


Areal Sweep Efficiency vs.
Displaceable Pore Volumes Injected
and Mobility Ratio.
Repeated 5 Spot.
Ref.:
Caudle & Witte. Trans
AIME 216 (1959) 446.
AREAL SWEEP EFFICIENCY - E_A.
DISPLACEABLE PORE VOLUMES INJECTED - V_{Di}.
Mobility Ratio, M.
0.5
1.0
2.0
5.0
Scale Change
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.4
0.6
0.8
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5

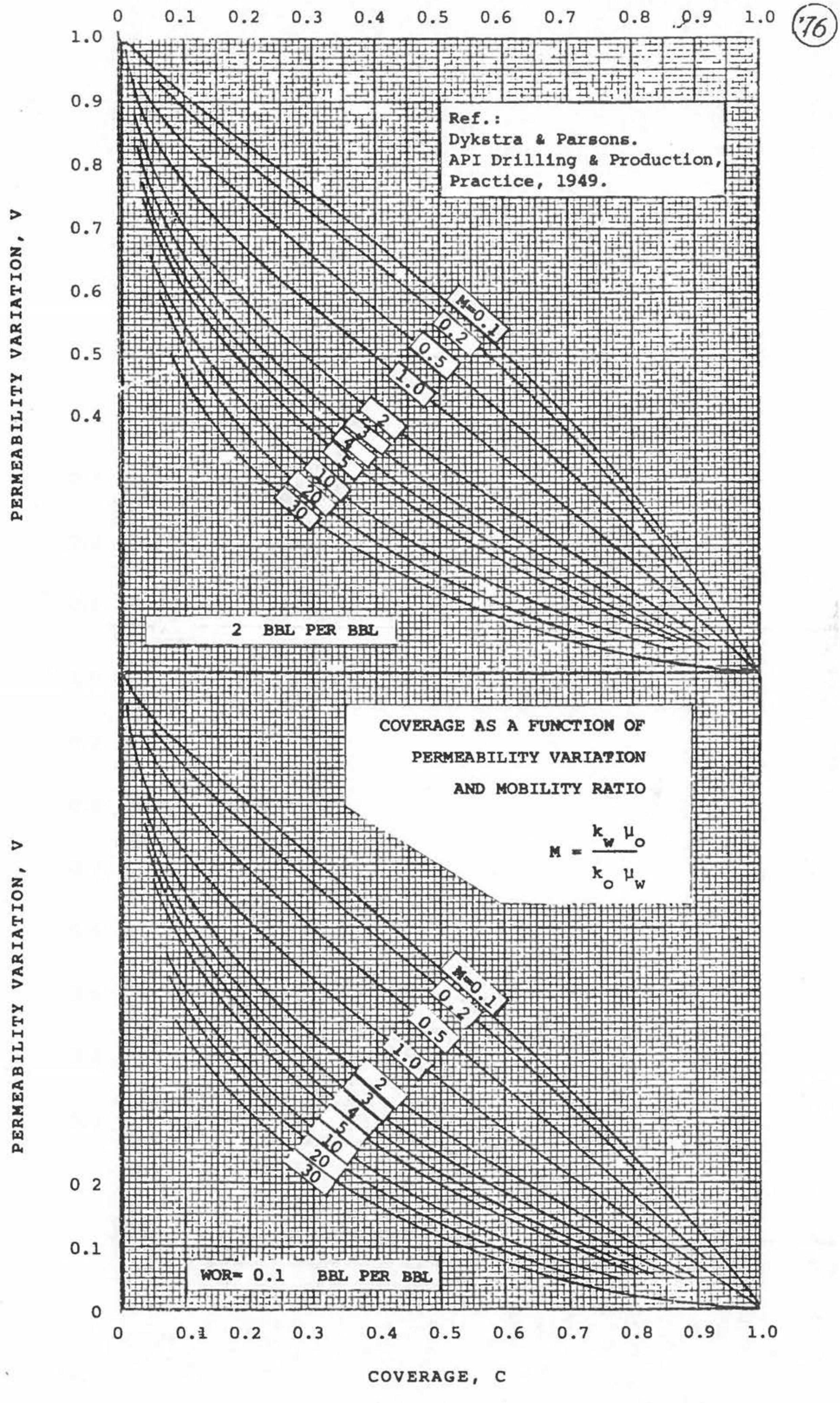
73

74

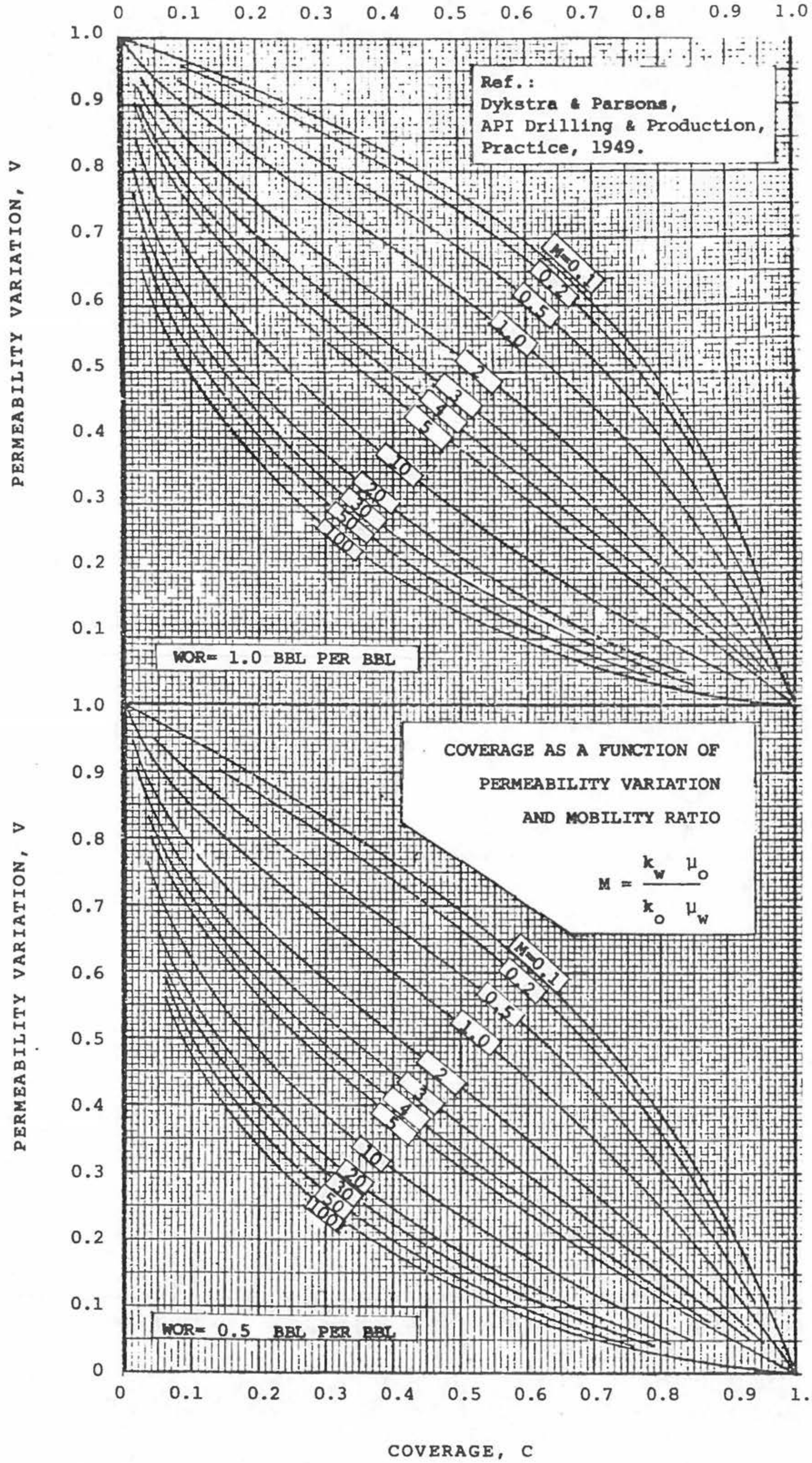
Fraction of Producing Stream Which is Injected Material vs. Displaceable Pore Volumes Injected and Mobility Ratio. Repeated 5 Spot.



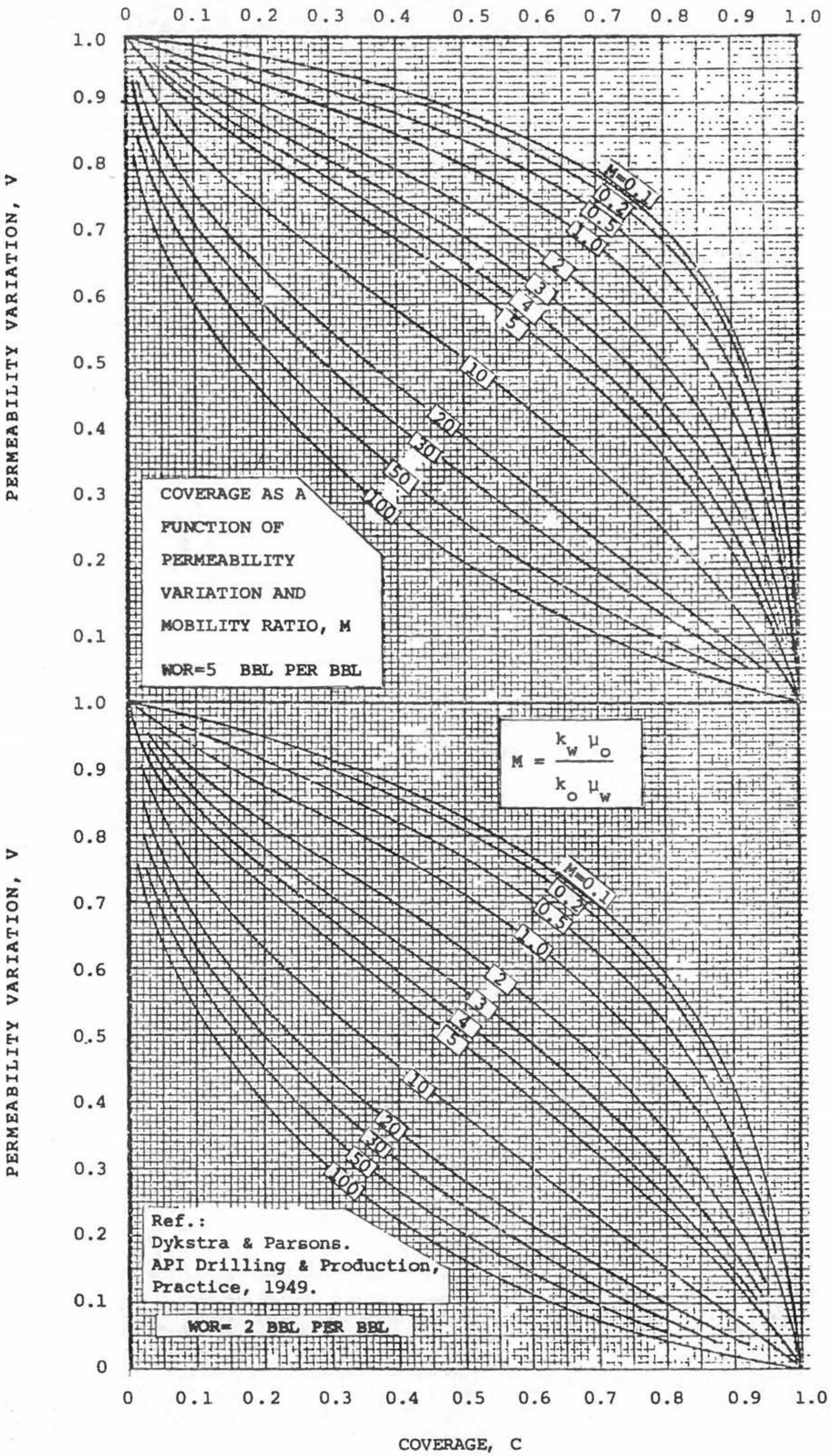
Ref.:
Caudle & Witte. Trans AIME 216 (1959) 446.


(76)
Ref.:
Dykstra & Parsons.
API Drilling & Production,
Practice, 1949.
2 BBL PER BBL
COVERAGE AS A FUNCTION OF
PERMEABILITY VARIATION
AND MOBILITY RATIO
$M = \frac{k_w \mu_o}{k_o \mu_w}$
WOR= 0.1 BBL PER BBL
M=0.1
0.2
0.5
1.0
2
3
4
5
10
20
30
PERMEABILITY VARIATION, V
COVERAGE, C
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

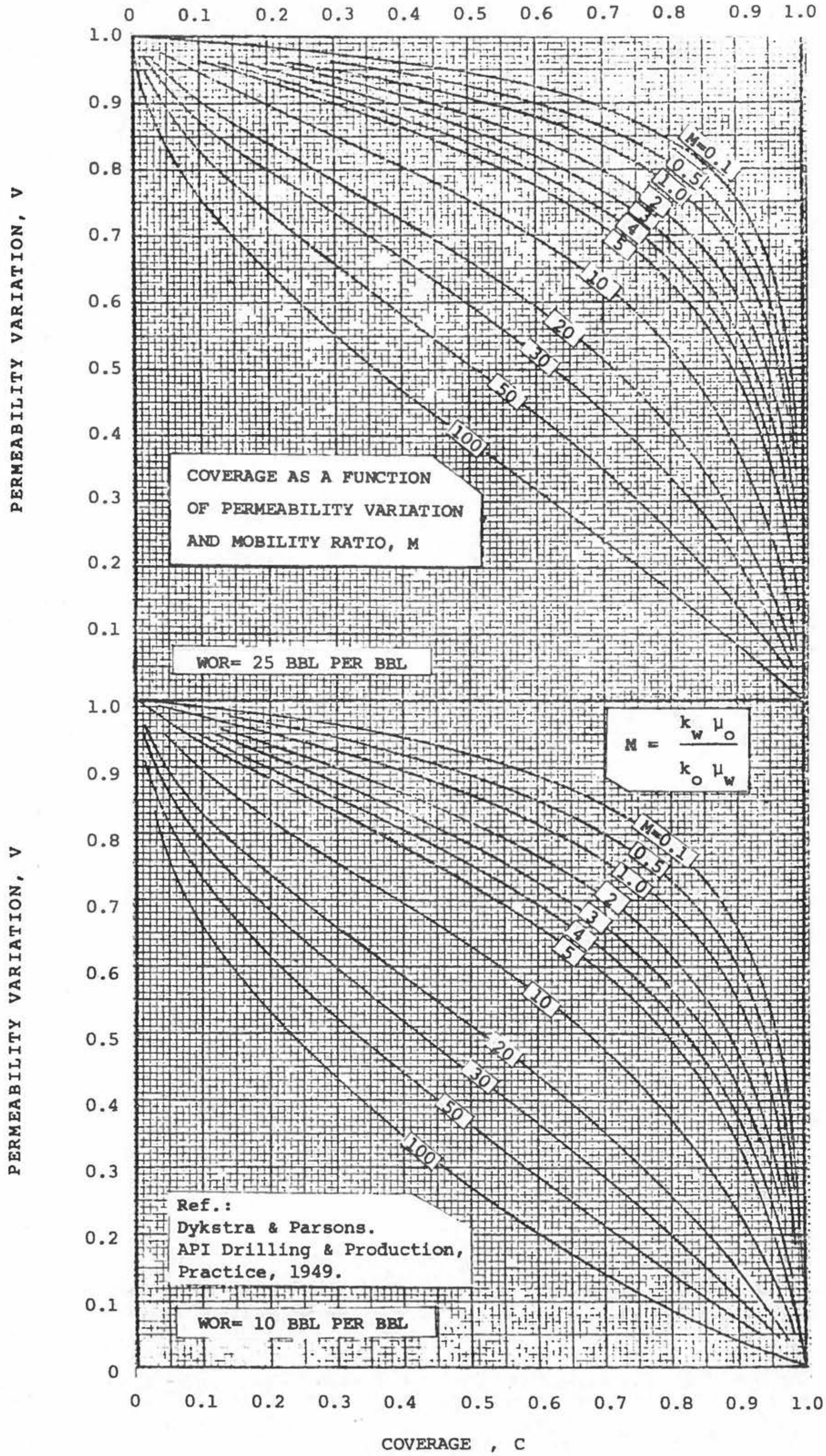
77


Ref.:
Dykstra & Parsons,
API Drilling & Production,
Practice, 1949.
PERMEABILITY VARIATION, V
M=0.1
0.2
0.5
1.0
2
3
4
5
10
20
30
50
100
WOR= 1.0 BBL PER BBL
COVERAGE AS A FUNCTION OF
PERMEABILITY VARIATION
AND MOBILITY RATIO
$M = \frac{k_w \; \mu_o}{k_o \; \mu_w}$
WOR= 0.5 BBL PER BBL
COVERAGE, C
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

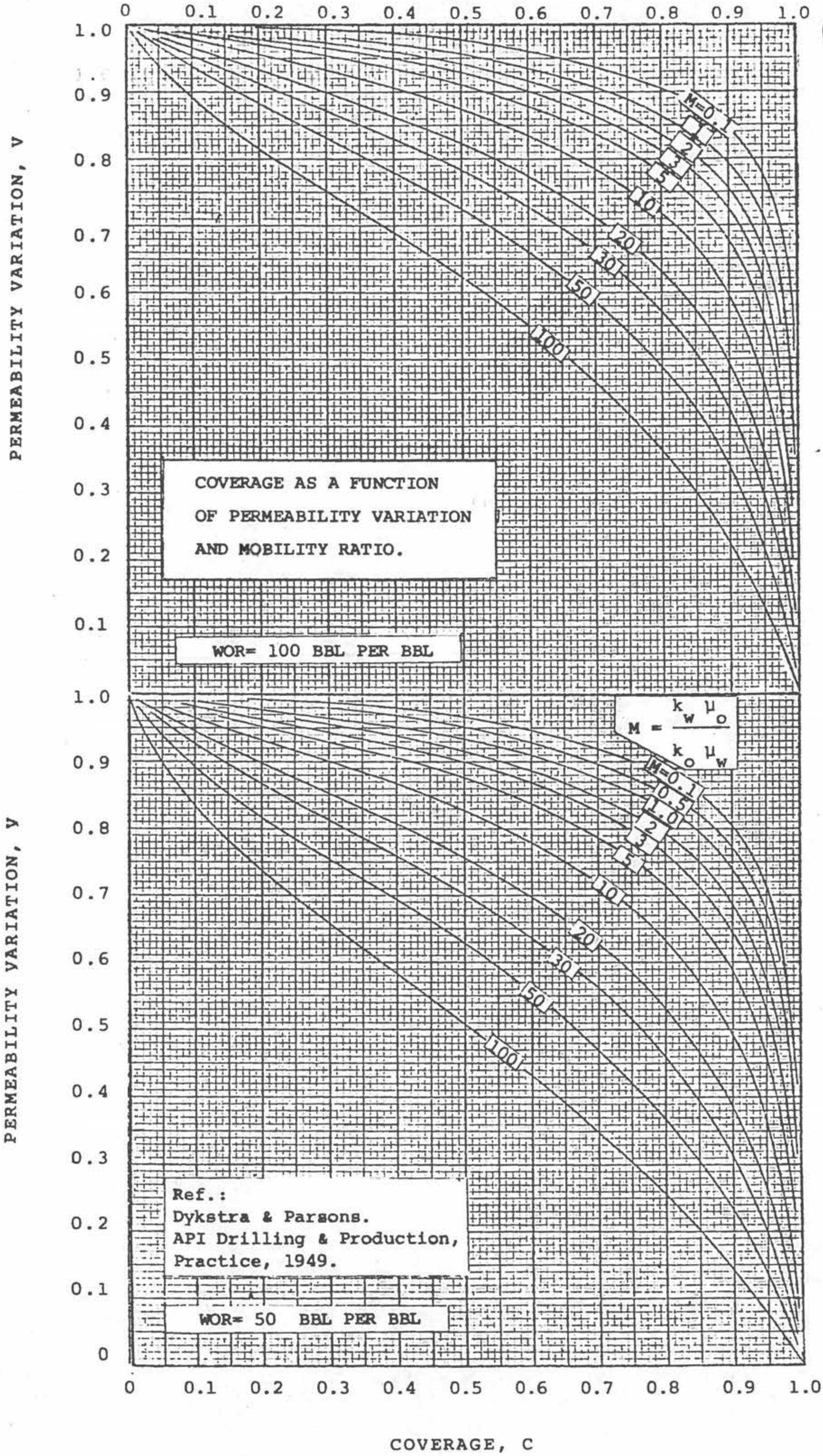
(78)


0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
PERMEABILITY VARIATION, V
COVERAGE AS A FUNCTION OF PERMEABILITY VARIATION AND MOBILITY RATIO, M
WOR=5 BBL PER BBL
M=0.1
0.2
0.5
1.0
2
3
4
5
10
20
30
50
100
M = k_w μ_o / k_o μ_w
Ref.:
Dykstra & Parsons.
API Drilling & Production, Practice, 1949.
WOR= 2 BBL PER BBL
COVERAGE, C

79


0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
PERMEABILITY VARIATION, V
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1
M=0.1
0.5
1.0
2
3
4
5
10
20
30
50
100
COVERAGE AS A FUNCTION OF PERMEABILITY VARIATION AND MOBILITY RATIO, M
WOR= 25 BBL PER BBL
$M = \frac{k_w \mu_o}{k_o \mu_w}$
PERMEABILITY VARIATION, V
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
M=0.1
0.5
1.0
2
3
4
5
10
20
30
50
100
Ref.:
Dykstra & Parsons.
API Drilling & Production,
Practice, 1949.
WOR= 10 BBL PER BBL
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
COVERAGE , C

(80)


COVERAGE AS A FUNCTION
OF PERMEABILITY VARIATION
AND MOBILITY RATIO.
WOR= 100 BBL PER BBL
WOR= 50 BBL PER BBL
$M = \frac{k_w \mu_o}{k_o \mu_w}$
M=0.1
0.5
1.0
2
3
5
10
20
30
50
100
PERMEABILITY VARIATION, V
COVERAGE, C
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Ref.:
Dykstra & Parsons.
API Drilling & Production,
Practice, 1949.