Notes on Mobility Ratios Used in Fluid Displacement Calculations.

All calculations that involve one fluid displacing a second fluid involve the ratio of the mobilities of the two fluids.

The mobility of a fluid is defined as the ratio of effective permeability to viscosity.

$$\lambda_o = k_o/\mu_o \quad \text{md/centipoise} \qquad (1)$$

$$\lambda_g = k_g/\mu_g \quad \text{"} \qquad (2)$$

$$\lambda_w = k_w/\mu_w \quad \text{"} \qquad (3)$$

Note that effective permeability in the above equations depend on saturation, saturation history (drainage or imbibition process) as well as the character of the porous rock.

In systems in which one fluid is displacing a second, it is common practice to call the fluid that is increasing in saturation the "displacing fluid" or "displacing phase". The fluid that is decreasing in saturation is called the "displaced fluid" or "displaced phase". The mobility ratio is the ratio of the mobility of the displacing fluid to that of the displaced fluid.

$$M = \lambda_{displacing} / \lambda_{displaced} \qquad (4)$$

For example, when water is displacing oil, as when aquifer water moves into an oil reservoir, the mobility ratio of the operation is written

$$M_{wo} = \lambda_w/\lambda_o = \frac{k_w}{k_o} \cdot \frac{\mu_o}{\mu_w} \qquad (5)$$

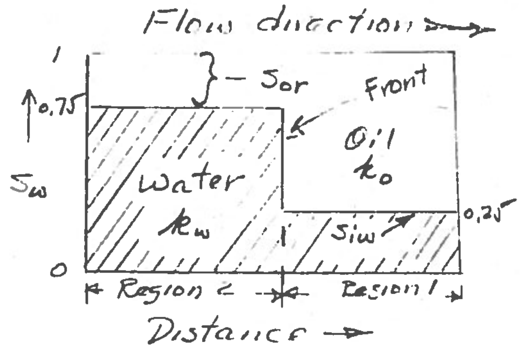


Figure 1.

Figure 1 illustrates partial displacement of oil by water. Only water is flowing in Region 2 behind the water/oil front, while only oil is flowing in Region 1 ahead of the front. In specifying the mobility ratio of this system, the effective water permeability in Region 2 is evaluated at the residual oil saturation, Sor. The effective oil permeability in Region 1 is evaluated at the irreducible water saturation Siw. The mobility ratio is,

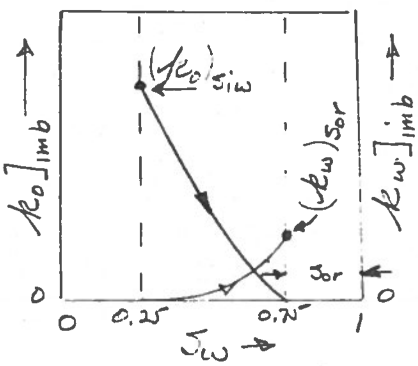


Figure 2

$$M = \frac{(k_w)_{Sor}}{(k_o)_{Siw}} \cdot \frac{\mu_o}{\mu_w} \qquad (6)$$

The situation of constant fluid saturation behind the front with some residual displaced phase is often refered to as a "leaky piston" displacement.

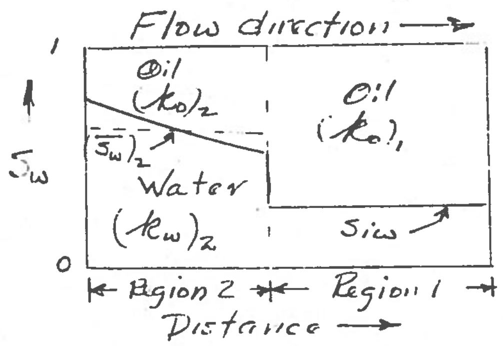


Figure 3

Figure 3 illustrates what is termed a "Buckley-Leverett" type displacement. With this type of displacement both oil and water are flowing in Region 2 (as the front advances). Only oil is flowing in Region 1. In specifying the mobility ratio of this system, the mobility of Region 2 is evaluated at the average saturation condition by the relationship.

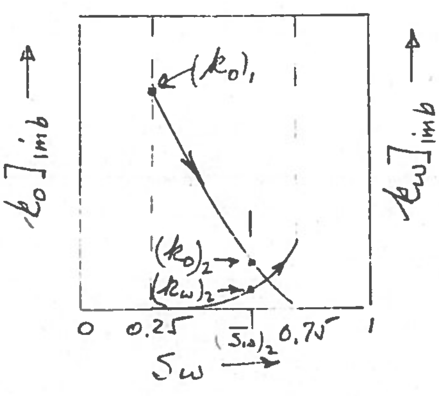


Figure 4.

$$\lambda_2 = \left(\frac{(k_w)_2}{\mu_w} + \frac{(k_o)_2}{\mu_o} \right) \qquad (7)$$

The mobility ratio becomes.

$$M_{wo} = \frac{\lambda_2}{\lambda_1} = \left[\frac{(k_w)_2}{\mu_w} + \frac{(k_o)_2}{\mu_o} \right] \Big/ \frac{(k_o)_1}{\mu_o} \qquad (8)$$

The above effective permeabilities are illustrated in Figure 4.

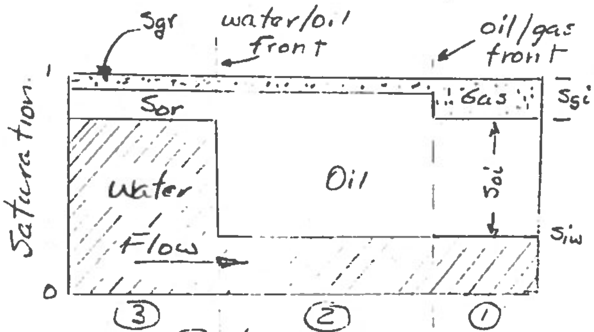


Figure 5

Figure 5 illustrates two "leaky piston" displacement fronts often used in calculating a process of water displacing oil and gas. In Region 1 only gas phase is flowing. In Region 2 only oil phase is flowing. And in Region 3 only water is flowing. The two mobility ratios would be written as.

$$M_{og} = \frac{\lambda_2}{\lambda_1} = \frac{(k_o)_{siw, Sgr}}{(k_g)_{Sgi}} \cdot \frac{\mu_g}{\mu_o} \qquad (9)$$

and.

$$M_{wo} = \frac{\lambda_3}{\lambda_2} = \frac{(k_w)_{Sor+Sgr}}{(k_o)_{siw, Sgr}} \cdot \frac{\mu_o}{\mu_w} \qquad (10)$$

Mobility ratios for most reservoir displacements lie between about 0,1 and 10. Mobility ratios less than 1 are generally considered as "favorable" in that the displacement front has fairly regular (smooth) features. This is illustrated by the front appearance at three successive times in Figure 6. Mobility ratios greater than 1 give more ragged displacement fronts as illustrated by Figure 7. The term "fingering" is often used to designate the erratic front behavior of high mobility ratio displacements.

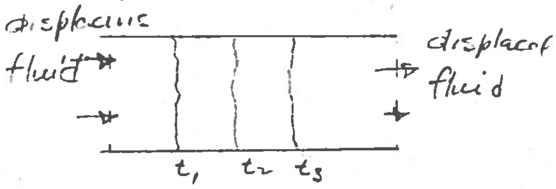


Figure 6

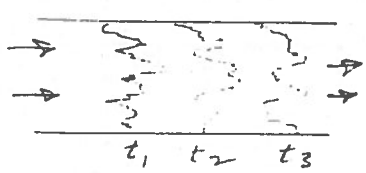
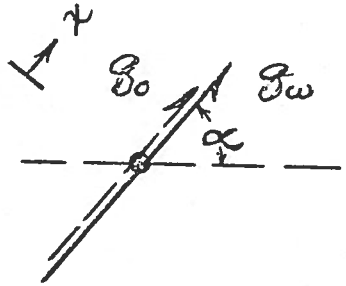


Figure 7

FRACTIONAL FLOW EQUATION, I



$$q_o = -\frac{k_o}{\mu_o}\left[\frac{\partial p_o}{\partial x} + \rho_o g \sin\alpha\right]$$

$$q_w = -\frac{k_w}{\mu_w}\left[\frac{\partial p_w}{dx} + \rho_w g \sin\alpha\right]$$

TOTAL FLOW, $q = q_o + q_w$

CAPILLARY PRESSURE, $p_c = p_o - p_w$

$$\frac{\partial p_c}{\partial x} = \frac{\partial p_o}{\partial x} - \frac{\partial p_w}{\partial x}$$

$$\frac{\partial p_c}{\partial x} = -\frac{(q-q_w)\mu_o}{k_o} - \rho_o g \sin\alpha + \frac{q_w \mu_w}{k_w} + \rho_w g \sin\alpha$$

LET $\Delta\rho = \rho_w - \rho_o$

$$q_w\left[\frac{\mu_o}{k_o} + \frac{\mu_w}{k_w}\right] = \frac{q\mu_o}{k_o} + \frac{\partial p_c}{\partial x} - \Delta\rho g \sin\alpha$$

FRACTIONAL FLOW EQUATION, II

DIVIDING BY $\frac{q\mu_o}{k_o}$;

$$\frac{q_w}{q}\left[1+\frac{k_o}{\mu_o}\cdot\frac{\mu_w}{k_w}\right]=1+\frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right]$$

$$\frac{q_w}{q}=f_w=\frac{1+\frac{k_o}{q\mu_o}\left[\frac{\partial P_c}{\partial x}-\Delta\rho g\sin\alpha\right]}{1+\frac{k_o}{k_w}\cdot\frac{\mu_w}{\mu_o}}$$

AS $k_o = k\cdot k_{ro}$ AND IGNORING $\frac{\partial P_c}{\partial x}$ BECAUSE SMALL

$$f_w=\frac{1-ak_{ro}}{1+\frac{k_{ro}}{k_{rw}}\cdot\frac{\mu_w}{\mu_o}}$$ WHERE

$$a=\frac{0.488\,k\,\Delta\rho\sin\alpha}{q\mu_o}$$ IF

$\Delta\rho$ = gm/cc
k = DARCY
q = BBL/DAY/FT^2
μ_o = cp

FRACTIONAL FLOW EQUATION, III

$$a = \frac{7.84(10^{-6})\,k\,\Delta\rho\,\sin\alpha}{q\,\mu_o}$$

IF $\Delta\rho = \frac{\text{lbs}}{\text{ft}^3}$

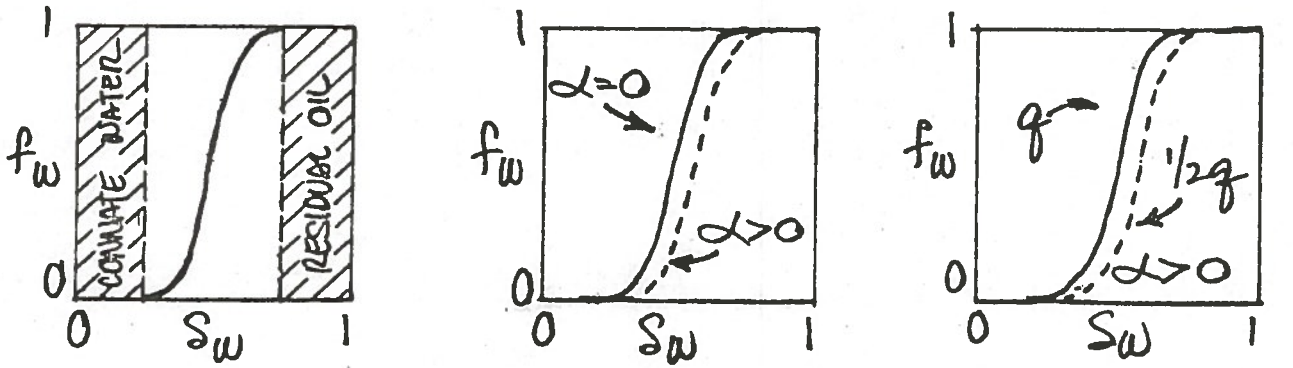
k = md.

q = BBL/DAY/FT^2

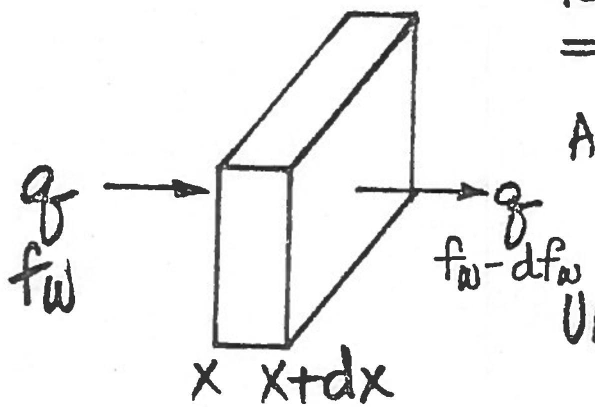
μ_o = cp.

WHEN $\alpha = 0$

$$f_w = \frac{1}{1 + \frac{k_{ro}}{k_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$



FRONTAL ADVANCE EQUATION, I



RATE OF WATER ACCUMULATION = WATER IN MINUS WATER OUT

$$\text{ACCUM. RATE} = q f_w - q(f_w - df_w)$$
$$= q\, df_w$$

$$\text{UNIT PORE VOL.} = \frac{A \cdot dx \cdot \phi}{5.615}$$

RATE OF WATER SATURATION CHANGE, $\frac{dS_w}{dt}$, IS RATE OF WATER ACCUMULATION DIVIDED BY PORE VOLUME.

$$\frac{dS_w}{dt} = \frac{q\, df_w}{A\, dx\, \phi} \cdot 5.615$$

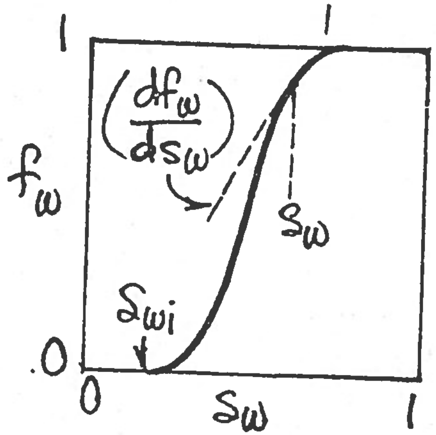
$$dx = \frac{5.615\, q}{A\phi} \left(\frac{df_w}{dS_w}\right) dt$$

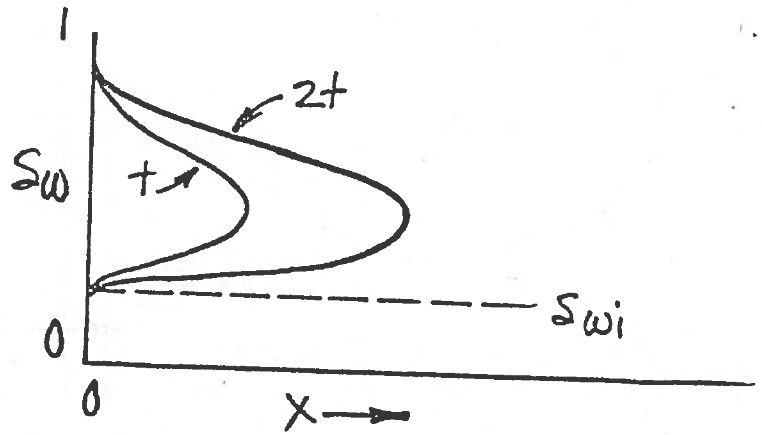
$$\int_{x_o}^{x_{S_w}} dx = 5.615 \cdot \frac{q}{A\phi} \left(\frac{df_w}{dS_w}\right) \int_{t=0}^{t} dt$$

FRONTAL ADVANCE EQUATION, II

$$(x)_{S_w} - X_o = \frac{5.615}{A\phi} \, qt \left(\frac{df_w}{dS_w}\right)$$

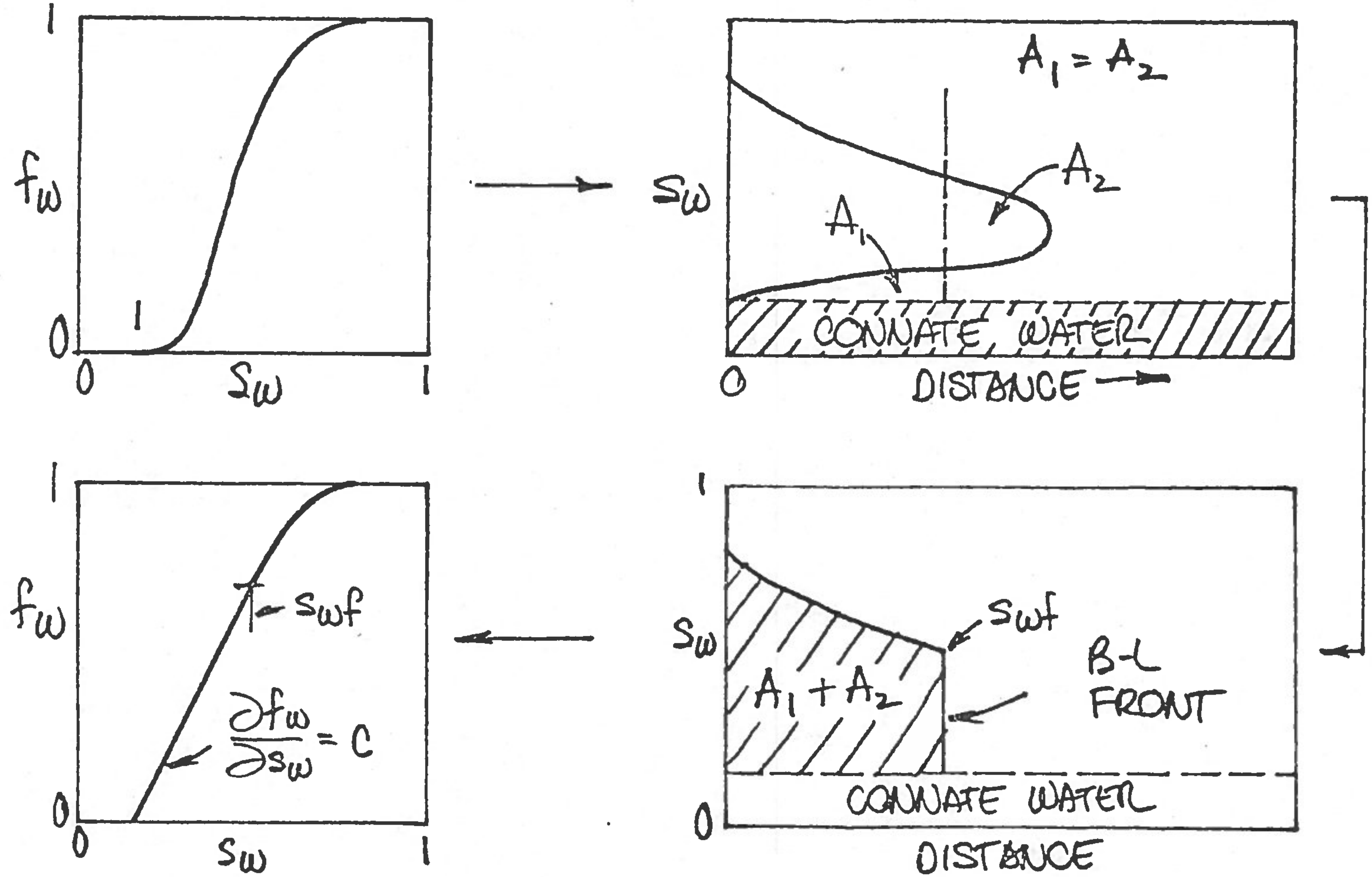
THIS IS THE BUCKLEY-LEVERETT FRONTAL ADVANCE EQUATION. IT GIVES THE DISTANCE, $[(x)_{S_w} - X_o]$, THAT A GIVEN SATURATION, S_w, MOVES IN TIME t.





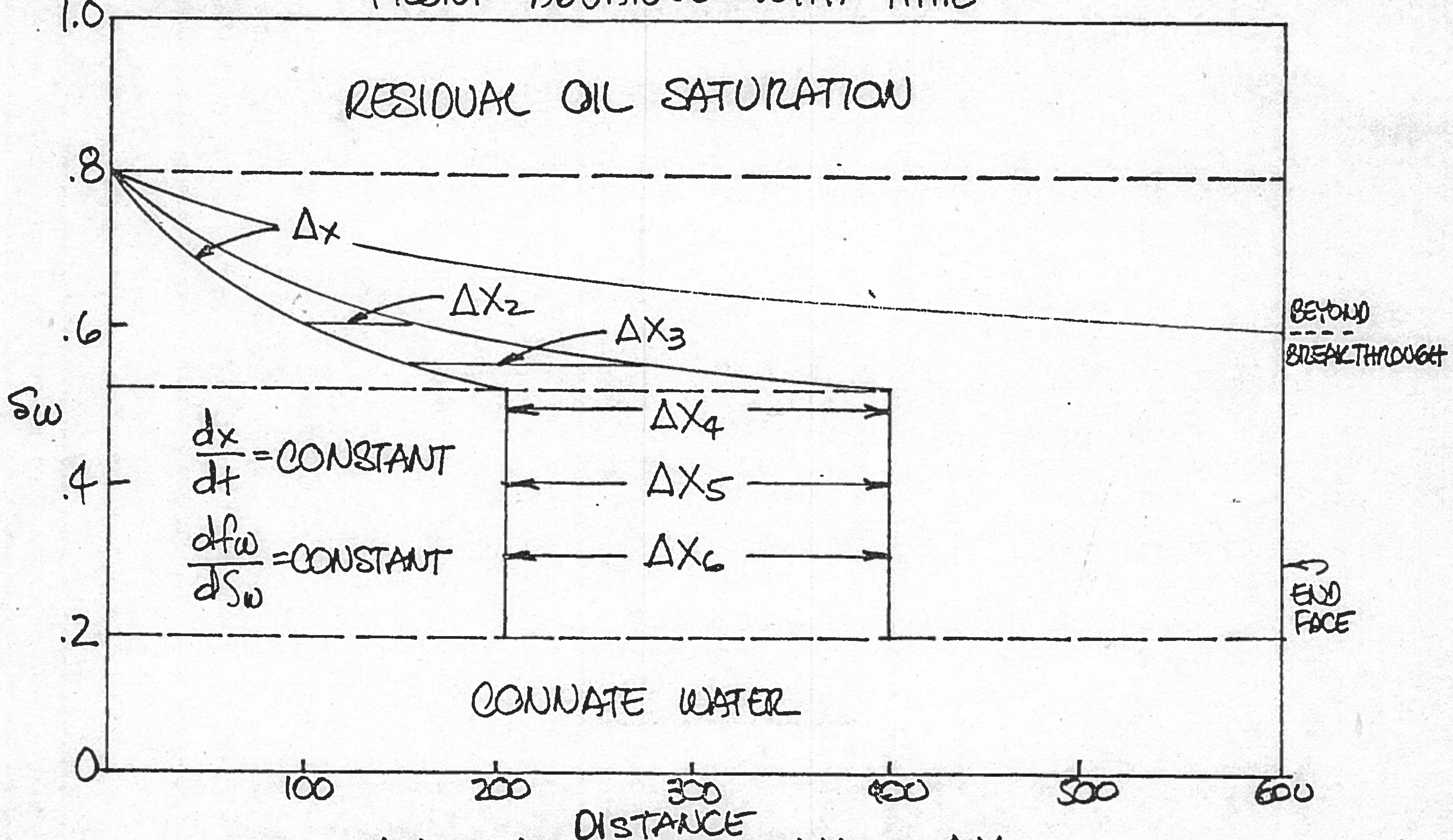
FRONTAL ADVANCE EQUATION, III

MODIFICATION OF B-L RELATIONSHIP TO AVOID TRIPLE SATURATION VALUES.



FRONTAL ADVANCE EQUATION, IV

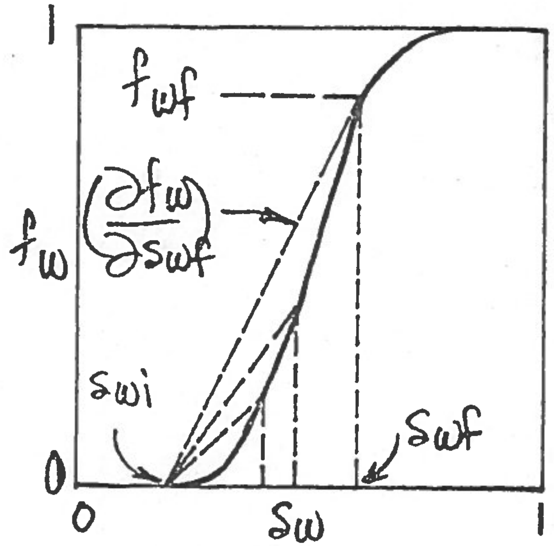
FRONT ADVANCE WITH TIME



$$\Delta X_1 < \Delta X_2 < \Delta X_3 < \Delta X_4 = \Delta X_5 = \Delta X_6$$

FRONTAL ADVANCE EQUATION VI

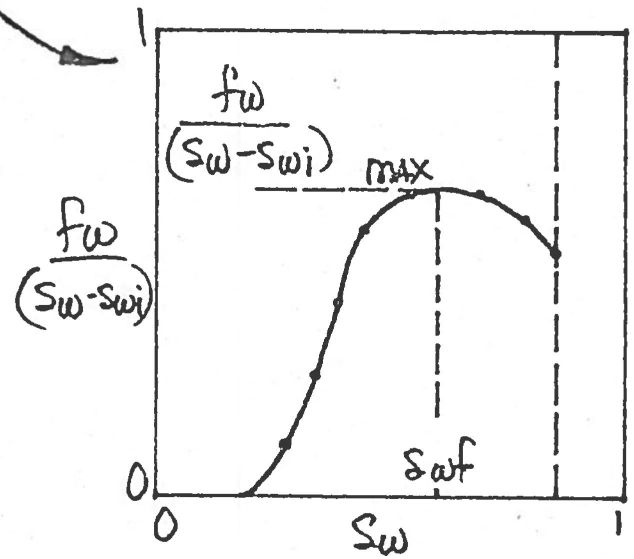
DETERMINATION OF SATURATION AND FRACTIONAL FLOW AT THE FRONT.



ALSO CAN BE OBTAINED GRAPHICALLY

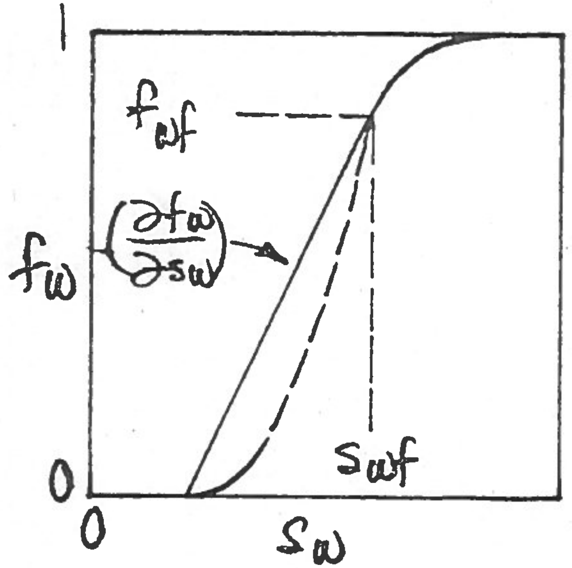
CALCULATE $\frac{f_w}{(S_w - S_{wi})}$ AND PLOT AGAINST S_w.

FRONT CONDITIONS ARE WHERE THIS HAS THE MAXIMUM VALUE.

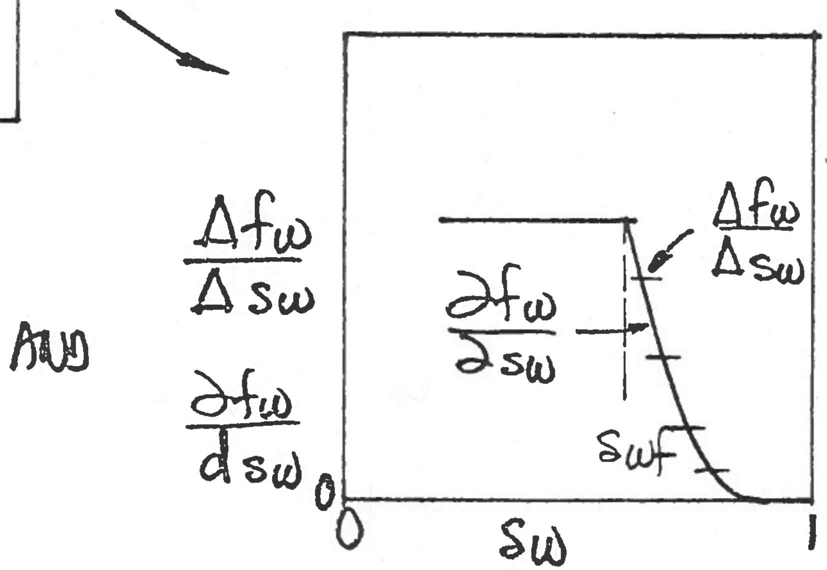


FRONTAL ADVANCE EQUATION VI

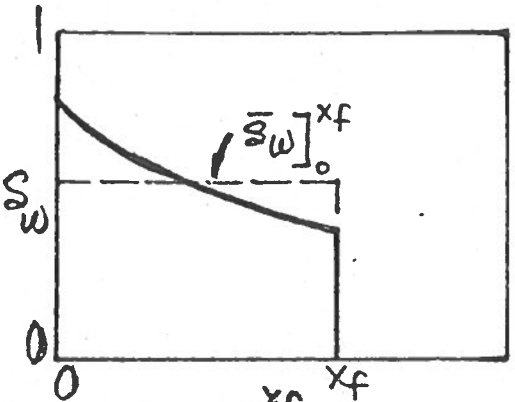
DETERMINATION OF $\frac{\partial f_w}{\partial s_w}$

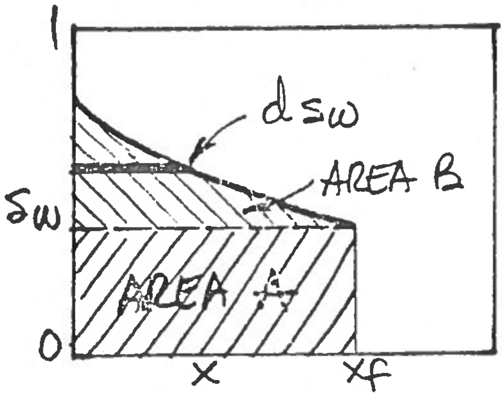


PERFORM GRAPHICAL DIFFERENTIATION OF $f_w - s_w$ DATA AT $S_w > S_{wf}$ · to OBTAIN VALUES OF $\frac{\partial f_w}{\partial s_w}$ VS S_w



AVERAGE SATURATION BEHIND FRONT - I





$$\bar{S}_w\Big]_0^{x_f} = \frac{\int_0^{x_f} S_w\,dx}{x_f} = \frac{\text{AREA A} + \text{AREA B}}{x_f}$$

FROM BUCKLEY-LEVERETT

$$x_f = \frac{5.615}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_f \int_0^t q\,dt$$

$$= \frac{5.615\ Q}{A\phi}\cdot\left(\frac{\partial f_w}{\partial S_w}\right)_f$$

$$\text{AREA A} = S_{wf}\cdot x_f$$

$$\text{AREA B} = \int_{S_{wf}}^{1} x\,dS_w$$

$$\text{AS}\quad x = \frac{5.615Q}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_x$$

$$\text{AREA B} = \frac{5.615\ Q}{A\phi}\int_{S_{wf}}^{1}\frac{df_w}{\cancel{dS_w}}\cdot\cancel{dS_w}$$

AVERAGE SATURATION BEHIND FRONT, II

$$\text{AREA } B = \frac{5.615\,Q}{A\phi} \int_{f_w @ s_{wf}}^{f_w @ s_w = 1} df_w$$

$$= \frac{5.615\,Q}{A\phi} \left[(f_w @ s_w = 1) - (f_w @ s_{wf}) \right]$$

BUT: $(f_w @ s_w = 1) = 1$; $(f_w @ s_{wf}) = f_{wf}$

THEREFORE

$$\text{AREA } B = \frac{5.615\,Q}{A\phi} \left[1 - f_{wf} \right]$$

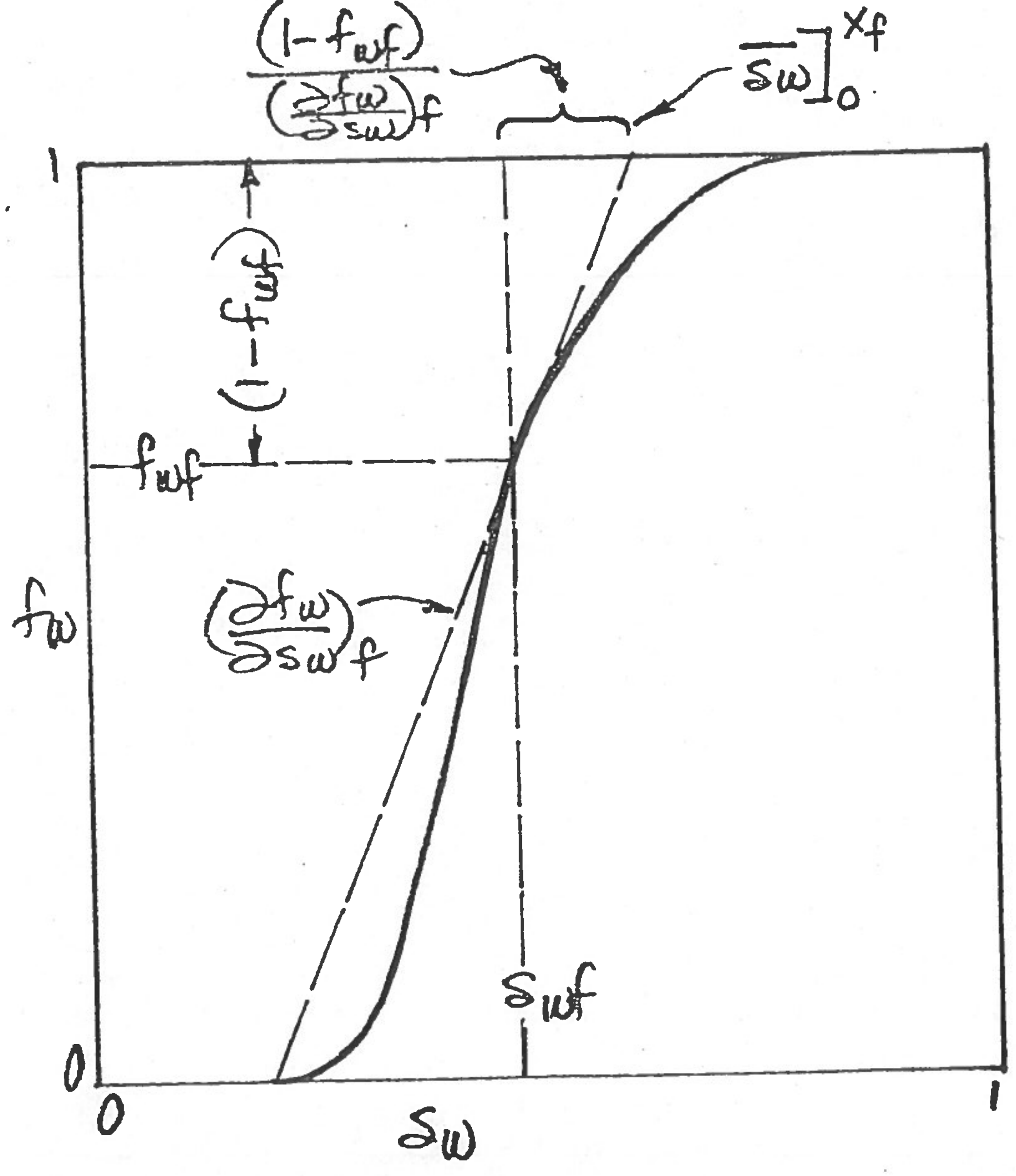
$$\text{AS } \bar{S}_w \Big]_0^{x_f} = \frac{\text{AREA A} + \text{AREA B}}{x_f}$$

$$\bar{S}_w \Big]_0^{x_f} = \frac{\frac{5.615\,Q}{A\phi} \left[S_{wf} \left(\frac{\partial f_w}{\partial s_w} \right)_f + (1 - f_{wf}) \right]}{\frac{5.615\,Q}{A\phi} \left(\frac{\partial f_w}{\partial s_w} \right)_f}$$

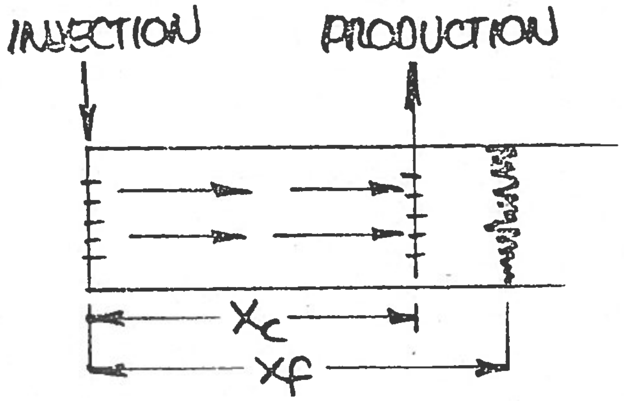
$$\bar{S}_w \Big]_0^{x_f} = S_{wf} + \frac{(1 - f_{wf})}{\left(\frac{\partial f_w}{\partial s_w} \right)_f}$$

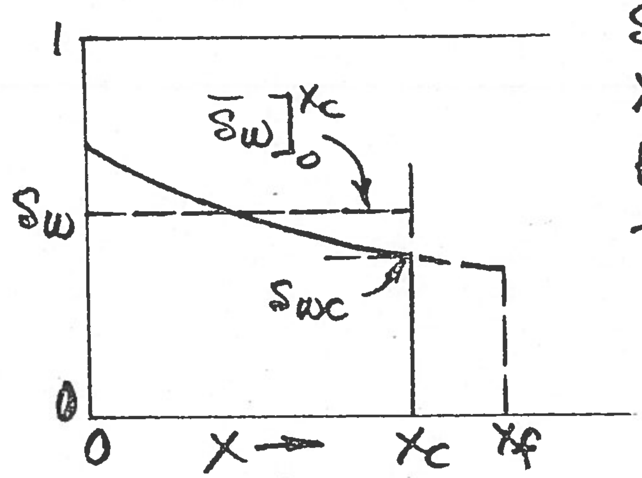
AVERAGE SATURATION BEHIND FRONT III

$$\bar{S}_w\Big]_0^{x_f} = S_{wf} + \frac{(1 - f_{wf})}{\left(\frac{\partial f_w}{\partial S_w}\right)_f}$$



AVERAGE SATURATION BEHIND FRONT - III



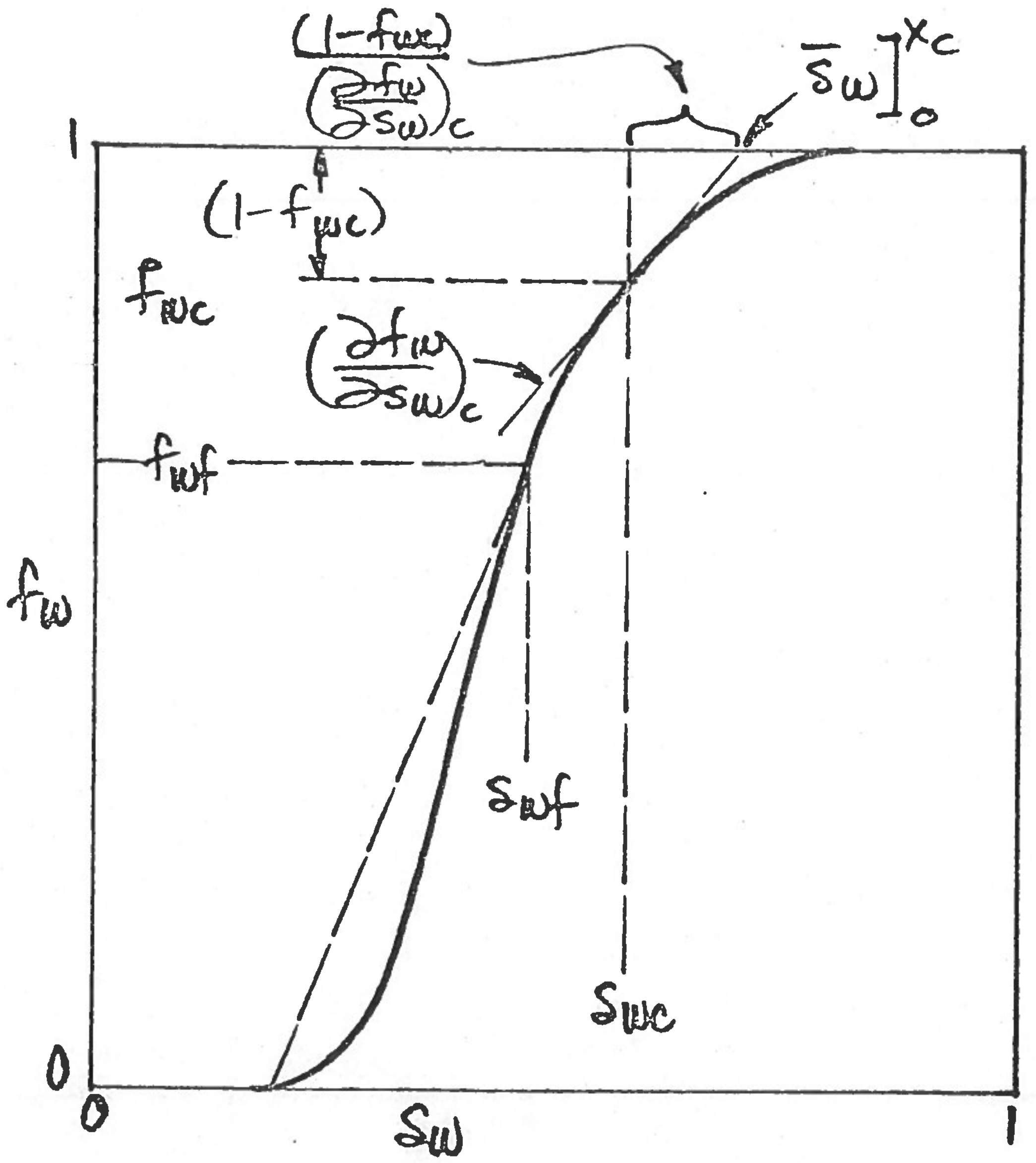


WHEN THE FRONT LINE HAS PASSED POSITION x_c, THE AVERAGE WATER SATURATION BEHIND x_c CAN BE DEVELOPED BY A SIMILAR APPROACH. THE FINAL EQUATION IS

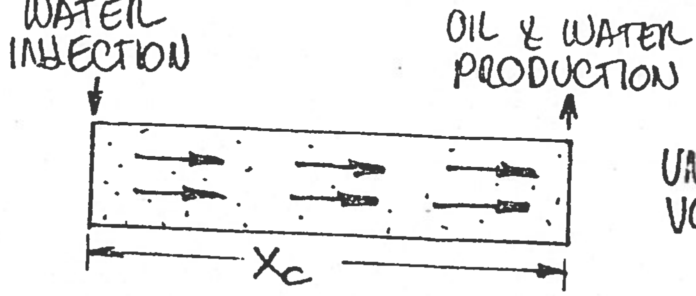
$$\bar{S}_w\Big]_0^{x_c} = S_{wc} + \frac{(1-f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$

AVERAGE SATURATION BEHIND FRONT - II

$$\bar{S}_w\Big]_0^{x_c} = S_{wc} + \frac{(1-f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$



INJECTION VOLUME VS. RECOVERY RELATIONS, I



UNIT PORE VOLUME, $V_p = \dfrac{A \phi x_c}{5.615}$

DIMENSIONLESS PORE VOLUMES INJECTED

$$V_{iD} = \frac{W_i}{V_p} = \frac{W_i}{\frac{A \phi x_c}{5.615}} = \frac{5.615\, W_i}{A \phi x_c}$$

FROM BUCKLEY - LEVERETT

$$x_c = \frac{5.615\, W_i}{A \phi} \left(\frac{\partial f_w}{\partial s_w} \right)_c$$

OR

$$\frac{5.615\, W_i}{A \phi x_c} = 1 \Big/ \left(\frac{\partial f_w}{\partial s_w} \right)_c$$

THEREFORE, THE PORE VOLUMES INJECTED TO REACH A GIVEN WATER SATURATION AT THE OUTFLOW FACE, s_{wc} IS,

$$V_{iD} = 1 \Big/ \left(\frac{\partial f_w}{\partial s_w} \right)_c$$

INJECTION VOLUME VS. RECOVERY RELATIONS II

RESERVOIR OIL DISPLACED $= V_p (\Delta \bar{S}_w)_o^{x_c}$

$$= \frac{A \phi x_c}{5.615} \left[(\bar{S}_w)_o^{x_c} - S_{wi} \right]$$

STOCK TANK OIL PRODUCED $= \dfrac{\text{RES. OIL DISP.}}{B_o}$

$$N_p = \frac{A \phi x_c}{5.615\, B_o} \left[(\bar{S}_w)_o^{x_c} - S_{wi} \right]$$

FLOWING WATER-OIL RATIO AT OUTFLOW FACE;

$$WOR = \frac{f_{wc}}{1 - f_{wc}}$$

SURFACE PRODUCING WATER-OIL RATIO,

$$F_{wo} = \frac{f_{wc} \cdot B_o}{(1 - f_{wc})\, B_w}$$

SURFACE WATER CUT $= \dfrac{\dfrac{f_{wc}}{B_w}}{\dfrac{f_{wc}}{B_w} + \dfrac{(1 - f_{wc})}{B_o}} = \dfrac{1}{1 + F_{wo}}$

TIME, $t = \dfrac{W_i}{i_w}$

$$= \frac{V_{iD} \cdot A \phi x_c}{5.615\, i_w} \text{ (DAYS)}$$

WHERE i_w IS BBL PER DAY

NOTES ON FLUID DISPLACEMENT IN POROUS ROCKS

1. Introduction

With few exceptions, production of petroleum reservoirs involves one or more aspects of fluid displacement - that is, displacement of one fluid by a second. Natural fluid displacement occurs, for example, when water influxes into the reservoir and displaces oil, or gas, that lay originally near the hydrocarbon-water contact. A second example is the "drive" or displacement of oil by gas as the gas comes out of solution in the oil phase. The material balance methods presented in Section E allowed one to evaluate, in the case of water influx, the amount and rate at which the water enters the petroleum reservoir. This becomes the amount and rate at which hydrocarbons are displaced in the invaded reservoir volume. To calculate the invaded reservoir volume, one must have an appropriate value for the volumetric displacement efficiency. This is shown by Equation 1.1.

$$W_e = 7758\ \phi\ V_i \cdot E \qquad (1.1)$$

where

V_i = reservoir volume invaded, acre-ft.
E = volumetric displacement efficiency, fraction
ϕ = porosity, fraction
W_e = influx volume, bbl.

Material presented in this section concerns the evaluation of E in the above equation. This is best done by considering that the volumetric displacement efficiency to be composed of three other efficiency numbers with the relationship:

$$E = E_D \cdot E_A \cdot E_V \qquad (1.2)$$

The internal displacement efficiency, E_D, in Equation 1.2 represents hydrocarbon saturation change within portions of the reservoir where fluid displacement has been in effect, per unit of initial hydrocarbon saturation. That is:

$$E_D = \frac{\Delta S_h}{S_{hi}} \qquad (1.3)$$

where

ΔS_h = hydrocarbon saturation change
S_{hi} = hydrocarbon saturation at start of displacement

The areal displacement efficiency, E_A, accounts for the fraction of the reservoir that is contacted by displacing fluid, in an areal sense while the vertical displacement efficiency, E_V, represents the fraction of reservoir that is contacted by displacing fluid, in a vertical sense. The product, $E_A \cdot E_V$, represents the fraction of the reservoir in which fluid displacement has occured.

Basics of calculating internal displacement efficiency, E_D, will be covered in sub-sections 2 and 3. Areal and vertical efficiency concepts will be introduced in the fourth sub-section. Sub-section five will cover the Dykstra-Parsons method of handling vertical efficiency. The last section will list computer abstracts that pertain to fluid displacement calculations.

2. Fractional Flow

We can say that a unit weight of any fluid at a given point has a potential Φ. Insofar as subsurface flow is concerned, we usually are interested only in the fluid pressure at the location and its position. Thus, we can say,

$$\Phi = p + \rho g h \tag{2.1}$$

where h is the distance above some arbitrary plane.

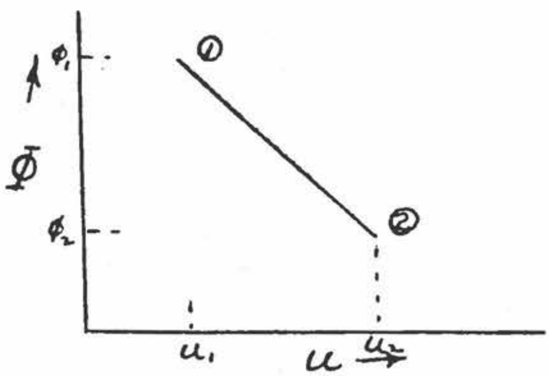
We know from basic fluid flow mechanics that:

$$\vec{q} = -\frac{k}{\mu}\ \text{grad}\ \Phi \tag{2.2}$$

If we consider flow in the u direction, then:

$$\Big[q\Big]_u = -\frac{k}{\mu}\left(\frac{d\Phi}{du}\right) \tag{2.3}$$

It is very important to remember that $\frac{d\Phi}{du}$ is a negative value; that is, flow goes from high Φ to a low Φ.



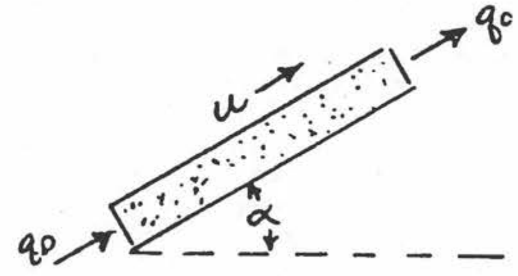
$$\frac{d\Phi}{du} = \lim_{\Delta u \to o} \frac{\Delta\Phi}{\Delta u} = \lim_{\Delta u \to o} \frac{\Phi_2 - \Phi_1}{u_2 - u_1}$$

The fact that $\frac{d\Phi}{du}$ is negative is why the - sign is placed before the k/μ.

If we consider a linear section of porous media inclined at angle α from the horizontal and in which flow is going in the upward (u) direction, we can write:

$$\Big[q_D\Big]_u = -\frac{k_D}{\mu_D}\left(\frac{d\,\Phi_D}{du}\right) = -\frac{k_D}{\mu_D}\left[\frac{d\,p_D}{du} + \rho_D g\,\frac{dh}{du}\right] \tag{2.4}$$

$$\Big[q_o\Big]_u = -\frac{k_o}{\mu_o}\left(\frac{d\,\Phi_o}{du}\right) = -\frac{k_o}{\mu_o}\left[\frac{d\,p_o}{du} + \rho_o g\,\frac{dh}{du}\right] \tag{2.5}$$



where

q	=	flow rate per unit area
k	=	effective permeability
μ	=	viscosity
p	=	pressure
u	=	distance in the direction of flow
ρ	=	density of the fluid
g	=	gravitational constant
sub D	=	displacing fluid
sub o	=	oil (displaced) fluid

It might be well to inspect Equations 2.4 and 2.5 to see if they are correct. Let's first consider the situation for a horizontal bed. Here α is equal to zero. $\frac{dh}{du}$ must also be equal to zero as $\frac{dh}{du} = \sin\alpha$. q_D then has a positive value (flow towards the right because $\frac{dp_o}{du}$ is negative, and we have two negatives. So we are alright with regard to the pressure gradient. Let's now let $\frac{dp_D}{du} = o$ and consider only the effect of the bed dip. For the situation illustrated $\frac{dh}{du}$, or $\sin\alpha$, is a positive value. q_D, however, is negative (because of the negative sign before the brackets) - which means that flow would be down hill. This is what we would expect.

Since two phases are present and the interface between phases is curved, we must include any effect of capillarity. We will arbitrarily define the capillary pressure, P_c, as

$$P_c = p_D - p_o \tag{2.6}$$

It follows directly that

$$\frac{\partial P_c}{\partial u} = \frac{\partial p_D}{\partial u} - \frac{\partial p_o}{\partial u} \tag{2.7}$$

Solving Equations 2.4 and 2.5 for the pressure gradients and putting them into Equation 2.7 gives:

$$\frac{\partial P_c}{\partial u} = -\frac{q_D\mu_D}{k_D} - \rho_D g \sin\alpha + \frac{q_o\mu_o}{k_o} + \rho_o g \sin\alpha \, . \tag{2.8}$$

If we arbitrarily let

$$\Delta\rho = \left(\rho_D - \rho_o\right) \tag{2.9}$$

and consider the two fluids incompressible, flowing at constant total rate, q,

$$q = q_o + q_D \tag{2.10}$$

Equation 2.8 develops into:

$$\frac{\partial P_c}{du} + \Delta\rho g \sin\alpha - \frac{q\,\mu_o}{k_o} = q_D\left(\frac{k_o}{\mu_o} + \frac{k_D}{\mu_D}\right) \tag{2.11}$$

Dividing through by $\frac{q\,\mu_o}{k_o}$ and changing signs:

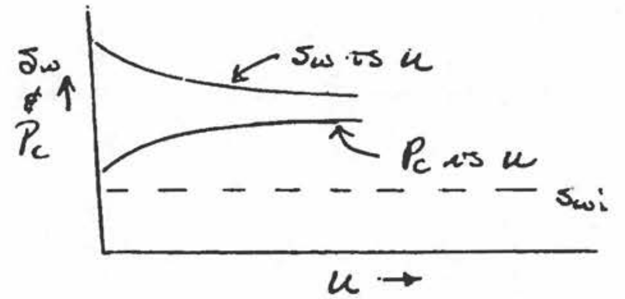
$$\frac{q_D}{q}\left[1 + \frac{k_o}{\mu_o}\cdot\frac{\mu_D}{k_D}\right] = 1 - \frac{k_o}{q\,\mu_o}\left[\frac{\partial P_c}{\partial u} + \Delta\rho g \sin\alpha\right] \tag{2.12}$$

As $\frac{q_D}{q} = f_D =$ fraction of displacing phase flowing,

$$f_D = \frac{1 - \frac{k_o}{q\,\mu_o}\left[\frac{\partial P_c}{\partial u} + \Delta\rho g \sin\alpha\right]}{\left[1 + \frac{k_o}{k_D}\cdot\frac{\mu_D}{\mu_o}\right]} \tag{2.13}$$

Equation 2.13 is completely general. If flow is down-dip, sin α is a negative value. If gas is displacing oil, Δρ is negative because of the manner of defining it in Equation 2.9. The equation as it now stands considers oil as the displaced phase, but any displaced fluid could be substituted.

In the "normal" water-wet sand, the capillary pressure decreases as one moves back from the flood front. (See sketch). Therefore, the sign of $\frac{\partial P_c}{\partial u}$ in Equation 2.13 is positive as is the $\Delta\rho g \sin\alpha$ term. In other words, the usual situation is that capillarity assists



displacement in water-wet sands and reduces displacement in oil-wet sands. However, because ΔP_c across reservoir distances of hundreds of feet is apt to be only a few psi, the values of $\frac{\partial P_c}{\partial u}$ in most instances, are so small as to be negligable. Therefore, for $\partial P_c/\partial u$, essentially zero:

$$f_D = \frac{1 - \frac{k_o}{\mu_o q} \Delta\rho g \sin\alpha}{1 + \frac{k_o}{k_D} \cdot \frac{\mu_D}{\mu_o}} \tag{2.14}$$

As $k_o = k \cdot k_{ro}$ and $\frac{k_o}{k_D} = \frac{k_{ro}}{k_{rD}}$, Equation 2.14 can be written:

$$f_D = \frac{1 - \frac{kk_{ro}}{\mu_o q} \Delta\rho g \sin\alpha}{1 + \frac{k_{ro}}{k_{rD}} \cdot \frac{\mu_D}{\mu_o}} \tag{2.15}$$

If k = millidarcys
$\Delta\rho$ = lbs/ft^3
μ_o, μ_D = centipoise
q = $B/D/ft^2$ cross section,

then

$$f_D = \frac{1 - \left[\frac{7.84(10^{-6})\ k\ k_{ro}\ \Delta\rho \sin\alpha}{\mu_o q}\right]}{1 + \frac{k_{ro}}{k_{rD}} \cdot \frac{\mu_D}{\mu_o}} \tag{2.16}$$

However, the more usual units of k = darcys
$\Delta\rho$ = Δ specific gravity with respect to water = $(\Delta\gamma)_w$

yield

$$f_D = \frac{1 - \left[\frac{0.488\ k\ k_{ro}\ (\Delta\gamma)_w \sin\alpha}{\mu_o q}\right]}{1 + \frac{k_{ro}}{k_{rD}} \cdot \frac{\mu_D}{\mu_o}} \tag{2.17}$$

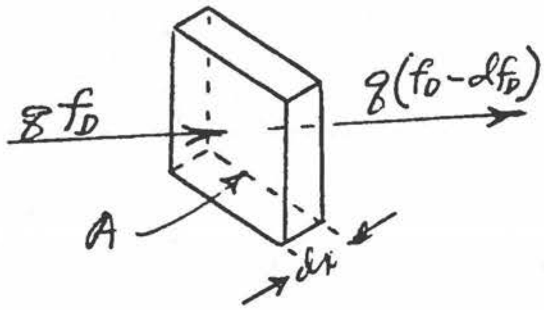
Notice that both Equation 2.16 and 2.17 can, for a particular system, be simplified to:

$$f_D = \frac{1 - a\, k_{ro}}{1 + \frac{k_{ro}}{k_{rD}} \cdot \frac{\mu_D}{\mu_o}} \tag{2.18}$$

Note that the values of a can be either positive or negative.

3. Frontal Advance (Buckley-Leverett Equation)

We consider a linear flow system as shown of cross sectional area A, porosity ϕ, and thickness dx, and flow q barrels a day through it. f_D is the fraction of q that is displacing fluid and $(f_D - df_D)$ is the fraction leaving. Rate (barrels a day) of displacing fluid accumulation in the element =



displacing fluid in - displacing fluid out.

$$= q\, f_D - q\, (f_D - df_D) \tag{3.1}$$

$$= q\, df_D \tag{3.2}$$

As the pore volume = $\frac{A \cdot dx \cdot \phi}{5.615}$ barrels, the accumulation causes a saturation change rate of:

$$\frac{dS_D}{dt} = \frac{q df_D}{A\ dx\ \phi/5.615} \tag{3.3}$$

or

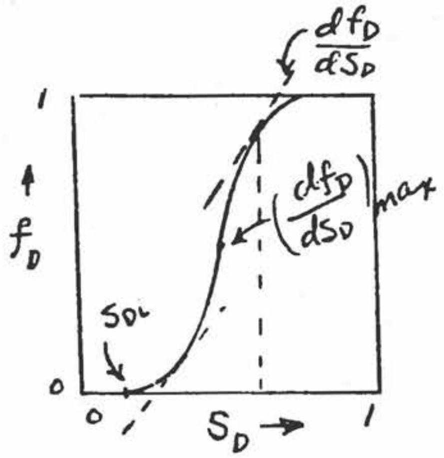
$$dx = \frac{5.615\ q}{A\phi} \left(\frac{df_D}{dS_D}\right)_{S_D} dt \tag{3.4}$$

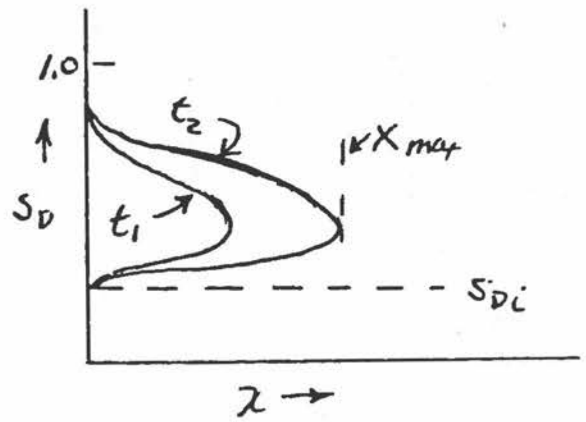
Integrating, this yields:

$$\int_{x_D}^{x_{S_D}} dx = \frac{5.615\ q}{A\phi} \left(\frac{df_D}{dS_D}\right) \int_{t=o}^{t} dt \tag{3.5}$$

$$x_{S_D} = x_o + \frac{5.615}{A\phi} \cdot qt \left(\frac{d\ f_D}{dS_D}\right)_{S_D} \tag{3.6}$$

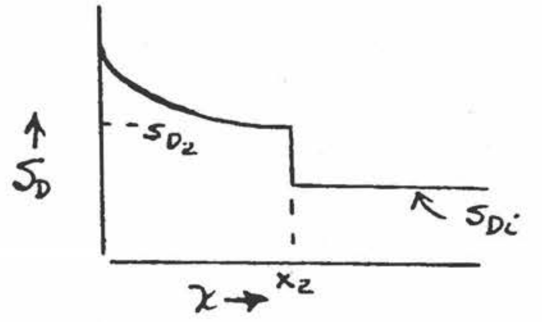
Equation 3.6 is very important. It says that the distance, x_{S_D}, to which a plane of saturation, S_D, moves to, having started from location x_o, is equal to a constant $5.615/\phi$ times the throughput per unit area, $\frac{qt}{A}$, and the slope of the f_D vs. S_D curve at the value of S_D.





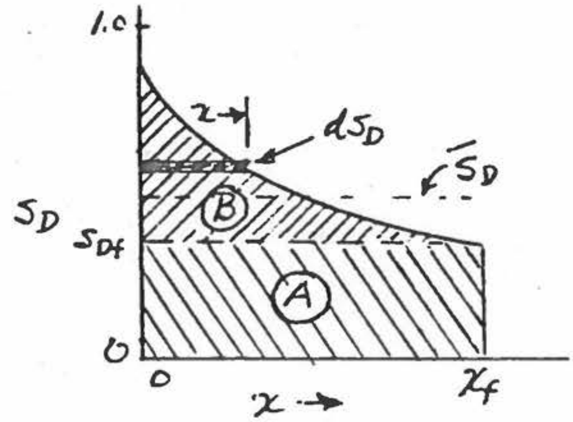
From the left-hand sketch we can see that the maximum value of $\left(\frac{df_D}{dS_D}\right)$ occurs at about $S_D = 0.4$. This saturation plane advances most rapidly per Equation 3.6.

Mathematical analyses indicate that the shape of the S vs. x curve can be modified at the nose to give the following shape. Location x_2 is the displacement front location. The displacing phase saturation at the front is S_{D2}.



Calculation of Average Saturation Behind the Front

Previous theory allows the computation of the S_D - x curve. If we consider the case where the B - L front has advanced to x_f, we can say that the the average saturation of displacing phase, $\bar{S}_D$, behind the front is equal to the sum of areas A and B divided by x_f.



$$\bar{S}_D = \frac{\text{Area A} + \text{Area B}}{x_f} \tag{3.7}$$

But

$$\text{Area A} = S_{Df} \cdot x_f \tag{3.8}$$

$$\text{Area B} = \int_{S_{Df}}^{1.0} x \, dS_D \tag{3.9}$$

As

$$x = \frac{5.615\, q}{A\phi}\left(\frac{df_D}{dS_D}\right) \tag{3.10}$$

$$\text{Area B} = \frac{5.615\, q}{A\phi} \int_{S_{Df}}^{1.0} \frac{df_D}{\cancel{dS_D}} \cdot \cancel{dS_D} \tag{3.11}$$

$$= \frac{5.615\, q}{A\phi} \int_{f_D @ S_{Df}}^{f_D @ S_D = 1} df_D \tag{3.12}$$

$$\text{Area B} = \frac{5.615q}{A\phi}\left[(f_D \ @\ S_D = 1) - (f_D \ @\ S_{Df})\right] \tag{3.13}$$

As $(f_D \ @\ S_D = 1) = 1$ and $(f_D \ @\ S_{Df}) = f_{Df}$ (3.14)

$$\text{Area B} = \frac{5.615\ q}{A\phi}(1 - f_{Df}) \tag{3.15}$$

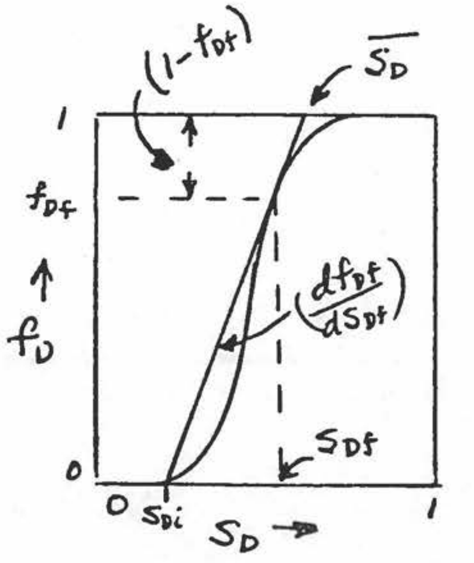
Going back to Equations 3.7 and 3.8 and replacing x_f by its equivalent

$$x_f = \frac{5.615\ q}{A\phi}\left(\frac{df_D}{dS_D}\right)_f \tag{3.16}$$

we obtain

$$\bar{S}_D = \frac{S_{Df}\cdot\cancel{\frac{5.615\ q}{A\phi}}\left(\frac{df_D}{dS_D}\right)_f + \cancel{\frac{5.615\ q}{A\phi}}(1 - f_{Df})}{\cancel{\frac{5.615\ q}{A\phi}}\left(\frac{df_D}{dS_D}\right)_f} \tag{3.17}$$

$$\bar{S}_D = S_{Df} + \frac{(1 - f_{Df})}{\left(\frac{df_D}{dS_D}\right)_f} \tag{3.18}$$



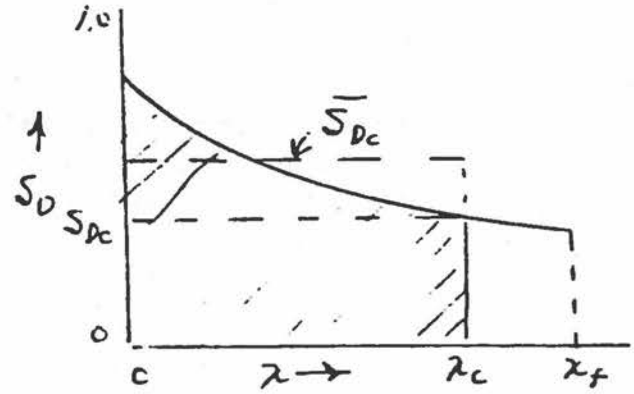
Significance of Equation 3.18 is shown by the sketch. The average saturation $\overline{S_D}$ can be obtained by extending the operating line (tangent at S_{Df}) to $f_D = 1$ and reading the value.

As the saturation change of the displacing fluid must be equal to the hydrocarbon saturation change, the hydrocarbon displaced at <u>breakthrough</u> will be:

$$(\bar{S}_D - S_{Di}) = S_{Df} - S_{Di} + \frac{(1 - f_{Df})}{\left(\frac{df_D}{dS_D}\right)_f} \tag{3.19}$$

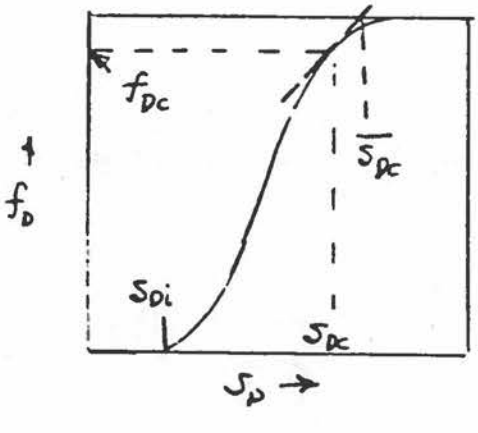
$$(\bar{S}_D - S_{Di}) = \frac{1}{\left(\frac{df_D}{dS_D}\right)_f} \tag{3.20}$$

Let's now consider how to obtain the hydrocarbon recovery after breakthrough of the front. This will be the recovery during the subordinate phase of production. To do this we picture the situation of the front having progressed beyond the outflow face (or has continued past the producing well). Referring to the figure, the outflow face is at distance x_c from the start, while the front has progressed to distance x_f. It is immediately apparent that we have the same situation as when the front was located at distance x_c. Therefore, without going through the development, we can say:



After breakthrough

$$\bar{S}_{Dc} = S_{Dc} + \frac{(1 - f_{Dc})}{\left(\frac{df_D}{dS_D}\right)_c} \tag{3.21}$$

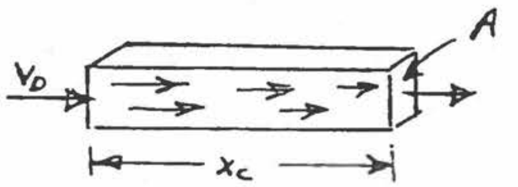


As previously, the hydrocarbon recovery will be:

$$\bar{S}_{Dc} - S_{Di} = S_{Dc} - S_{Di} + \frac{(1 - f_{Dc})}{\left(\frac{df_D}{dS_D}\right)_c} \tag{3.22}$$

Calculation of Injection Volume Required to Reach Average Saturation Conditions

The fractional flow equation and the Buckley-Leverett relationships permit one to calculate average displacing phase saturation behind an advancing displacement front from measured or correlatable reservoir parameters. This in turn leads to the evaluation of the hydrocarbon phase recovery and the internal displacement efficiency number, E_D. The relationships to be developed in this section relate to the quantity of displacing fluid required to achieve a given hydrocarbon recovery and to the producing volume ratio of displacing phase/displaced phase at breakthrough of displacing phase at the outflow face of the system.



Consider a segment of reservoir as indicated in the sketch. Let the cross sectional area perpendicular to flow be A square feet, and length between inlet and outlet faces be x_c feet. The unit pore volume, in barrels is:

$$V_p = \frac{Ax_c\,\phi}{5.615} \tag{3.23}$$

Consider that at a particular time, V_D barrels of displacing fluid has entered the unit volume. Assuming constant pressure prevails during displacement an equal volume of fluids will be displaced from the unit. The dimensionless pore volumes of displacing fluid will be:

$$V_{iD} = \frac{V_D}{V_p} = \frac{V_D}{A\ x_c\ \phi/5.615} \tag{3.24}$$

Equation 3.6 can be modified to fit the present situation by writing it as:

$$(x_{S_D} - x_o) = x_c = \frac{5.615}{A\phi} \cdot V_D \left(\frac{df_D}{dS_D}\right)_c \tag{3.25}$$

Solving Equations 3.24 and 3.25 yields:

$$V_{iD} = 1/\left(\frac{df_D}{dS_D}\right)_c \tag{3.26}$$

This relationship is important. It says that the reciprical of the slope of the fracional flow curve at saturation conditions existing at the outflow face is the dimensionless pore volumes of displacing fluid required to achieve this saturation.

The relationship of average saturation between inflow and outflow faces of the unit reservoir volume and injection volume comes from Equations 3.21 and 3.26.

That is,

$$\bar{S}_{Dc} = S_{Dc} + (1 - f_{Dc})\, V_{iD} \tag{3.27}$$

Note that one requires the saturation and fraction of displacing phase flowing at the outflow face to use Equation 3.27.

Because calculations of water displacing oil are so frequently encountered in reservoir and production engineering calculations, certain of the above equations will be rewritten for the water→oil displacements.

$$\text{Reservoir oil displaced} = V_p\, (\Delta S_o)\Big|_o^{x_c} \tag{3.28}$$

$$= \frac{A\, x_c\, \phi}{5.615} \left[(\overline{S_w})\Big|_o^{x_c} - S_{wi} \right] \tag{3.29}$$

Stock tank oil displaced/produced:

$$N_p = \frac{A\, x_c\, \phi}{5.615\, B_o} \left[(\overline{S_w})\Big|_o^{x_c} - S_{wi} \right] \tag{3.30}$$

Flowing water-oil ratio at outflow face,

$$WOR = \frac{f_{wc}}{f_{oc}} = \frac{f_{wc}}{1 - f_{wc}} \tag{3.31}$$

Surface producing water-oil ratio,

$$F_{wo} = \frac{f_{wc}}{(1 - f_{wc})} \cdot \frac{B_o}{B_w} \tag{3.32}$$

Surface water cut,

$$Cut = \frac{f_{wc}/B_w}{f_{wc}/B_w + (1 - f_{wc})/B_o} = \frac{F_{wo}}{1 + F_{wo}} \tag{3.33}$$

Injection time, $$t = \frac{W_i}{i_w} = \frac{V_{iD} \cdot A\, x_c\, \phi}{5.615\, i_w}$$

4. Mobility Ratio, Sweep Efficiency, Stratification

These three parameters are very important to the recovery of hydrocarbons by displacement processes. Mobility ratio is fixed by the viscosities and saturations on each side of a displacement front. Sweep efficiency depends on mobility ratio and the geometrical relationship of injection/producing wells (in injection projects). Stratification effects are caused by permeability differences (usually in a vertical sense) in the reservoir sections in which fluids are moving.

Mobility Ratio

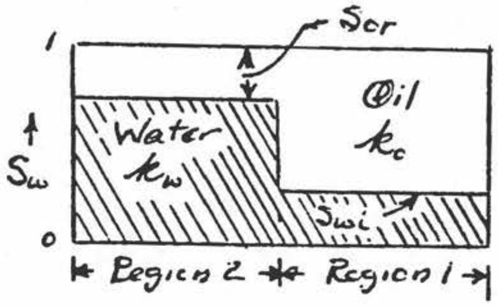
The mobility of a fluid is defined as the ratio of effective permeability to viscosity.

$$\lambda_o = \frac{k_o}{\mu_o} \tag{4.1}$$

Note that effective permeability depends both on saturation and saturation history, i.e., imbibition or drainage process.

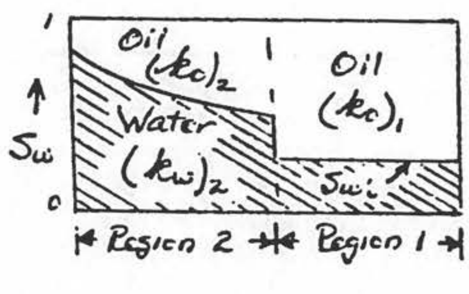
Mobility ratio expresses something of the ability of a displacing fluid to do an effective displacement. It is defined as the ratio of fluid mobility behind the front to fluid mobility ahead of the front. For example, the mobility ratio of water displacing oil would be defined by the relationship:

$$M_{wo} = \frac{\lambda_w}{\lambda_o} = \frac{k_w}{\mu_w} \cdot \frac{\mu_o}{k_o} \tag{4.2}$$



The sketch illustrates what is called "piston displacement." Only water is flowing behind the front, and only oil is flowing ahead of the front. The effective permeabilities for this situation would be water permeability at residual oil saturation and oil permeability at interstitial water saturation.

If more than one fluid is moving behind the front, as illustrated in this sketch, it is preferable to use an effective mobility for the two phase flow in region 2. In this instance:



$$\lambda_2 = \left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o}\right)_2 \tag{4.3}$$

where the effective permeabilities are evaluated at the average saturation of region 2.

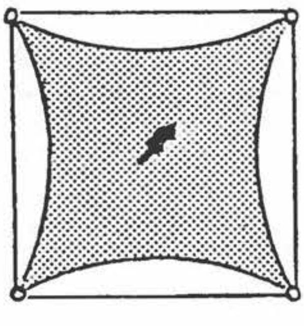
The mobility then becomes:

$$M_{wo} = \frac{\lambda_2}{\lambda_1} = \left(\frac{k_w}{\mu_o} + \frac{k_o}{\mu_o}\right)_2 \Bigg/ \left(\frac{k_o}{\mu_o}\right)_1 \qquad (4.4)$$

Mobility ratios for most reservoir displacements range between 0.1 and several hundred. Displacement efficiency decreases as mobility ratio increases.

Sweep Efficiency

The term "sweep efficiency" usually is used in connection with pattern displacements, although one can very well consider sweep efficiency of, say, water displacing oil updip as a result of water influx. Other terms are often used in place of sweep efficiency. Amoung these are areal displacement efficiency (used in previous discussions), areal sweep, areal coverage, coverage, area swept, and pattern efficiency. Regardless of the term used, the purpose is to express the fraction of the basic area that has been swept or processed by the displacing fluid at any particular time.



An example of areal sweep efficiency is shown in the sketch. The shaded portion represents the area within a repeated five spot pattern that has been contacted by displacing fluid injected at the center well at breakthrough into the four producing wells. As sketched, the areal sweep efficiency is about 0.7

Areal sweep efficiency is a function mobility ratio, pore volumes of fluid injected, and the geometrical relationship of injection and producing wells. The chart immediately below (Figure 4.1) shows the areal sweep efficiency at breakthrough as a function of mobility ratio for repeated five spot geometry.

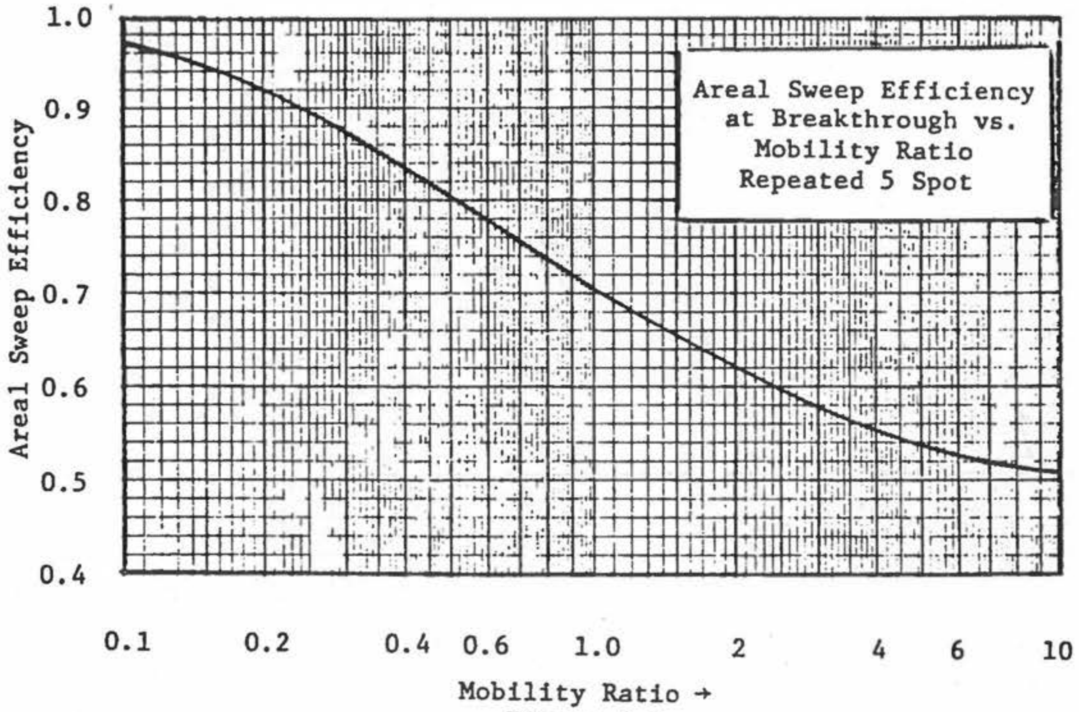


Figure 4.1

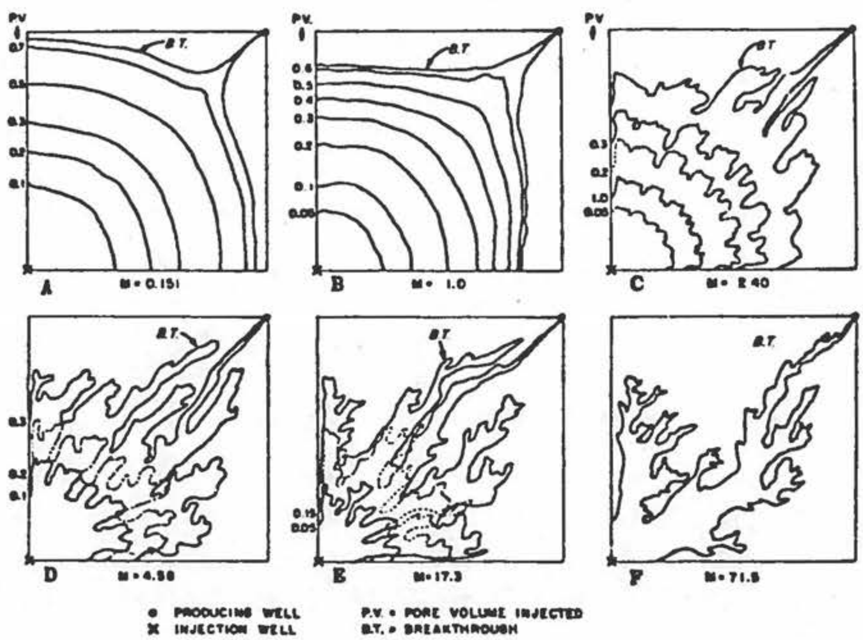


Figure 4.2

Figure 4.2 illustrates shapes of displacement fronts and areal sweep efficiencies at breakthrough found by Haberman (Trans AIME 219 (1960) 264). Note the "dendritic" type of displacement encountered at high mobility ratios.

Areal sweep efficiency continues to increase after breakthrough of displacing fluid to the producing well, although not as fast as before breakthrough. Figure 4.3 shows areal sweep efficiency values as a function of displaceable pore volumes injected and mobility ratio. The "ticks" on the 45° line indicate breakthrough sweep efficiencies.

A displaceable pore volume represents a volume consistant with maximum saturation change of the displacing fluid. Thus, for water as the displacing fluid, one displaceable pore volume is represented by:

$$V_D = V_R \, \phi \, (\Delta S_w)_{max} \qquad (4.5)$$

where

V_R = bulk reservoir volume, bbl.

ϕ = porosity, fraction

$(\Delta S_w)_{mx}$ = $(1 - S_{or} - S_{wi})$ if no gas phase is present.

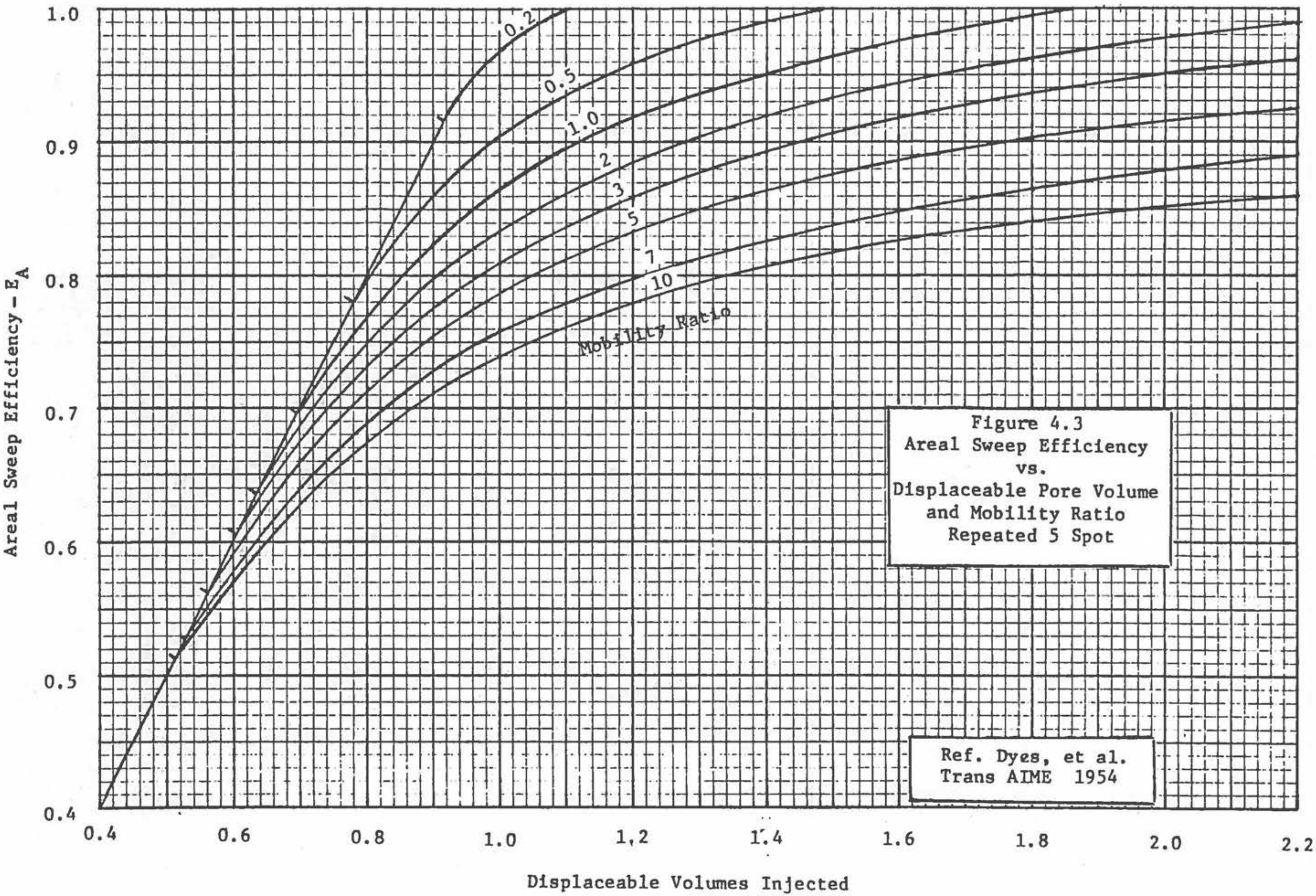
= $(1 - S_{gr} - S_{or} - S_{wi})$ if gas phase is present and displaced.

S_{gr}, S_{or} = residual hydrocarbon saturations at infinite water throughput.

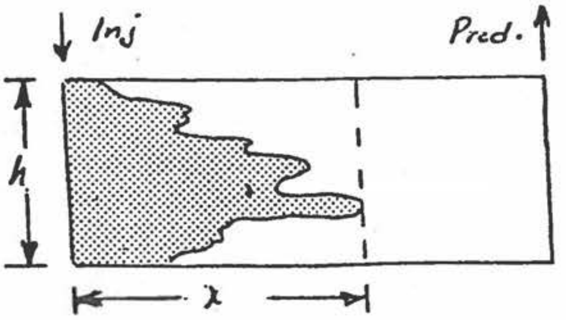
Stratification

Variability of permeability in a vertical sense results in lowering recovery of displaced phase for a given amount injected phase. The reason for this is that the displacement process moves much faster through high permeability portions of the reservoir than it does through the remaining bulk of the reservoir. As a consequence, breakthrough occurs earlier (lower value of displaceable pore volumes injected) and, if the mobility ratio is adverse, the high permeability "streak" continues to transport a large proportion of the injected fluid to the producing wells and effectively slows displacement in the remainder of the reservoir.

Effects of stratification on displacement processes are difficult to predict mainly because we can make observations only at wellbores. We have no direct knowledge of the permeable paths, or stratification into layers, in the inter-well distances. However, experience has shown that sands that show a large degree of variability of permeability in well cores will give poorer flooding results than will sands that are more homogeneous.



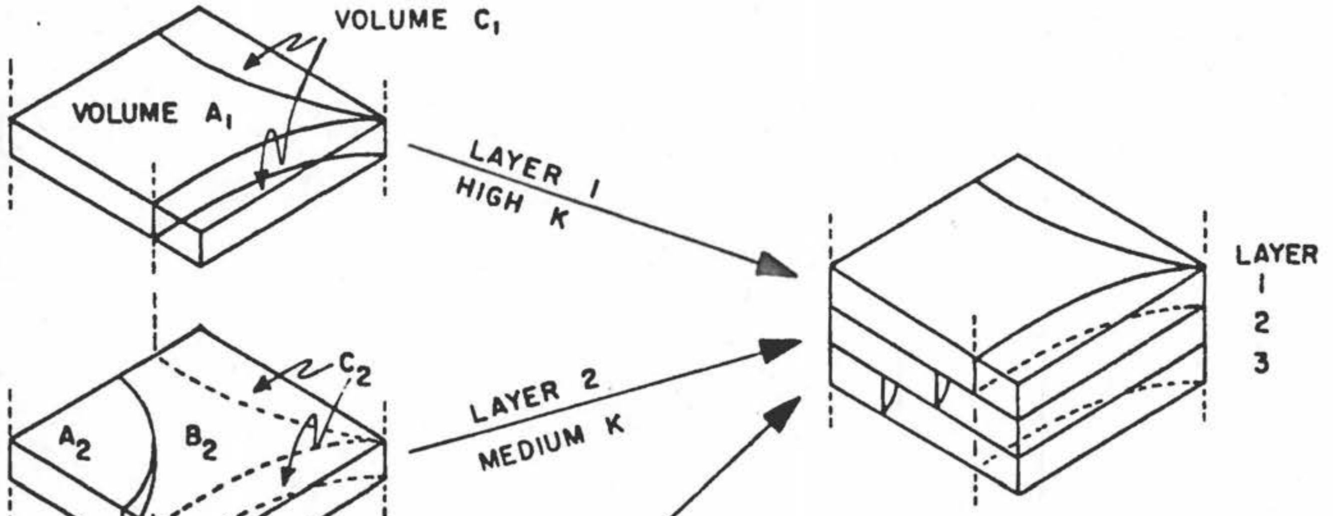
0.2
0.5
1.0
2
3
5
7
10
Mobility Ratio
Areal Sweep Efficiency - E_A
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
Displaceable Volumes Injected
Figure 4.3
Areal Sweep Efficiency
vs.
Displaceable Pore Volume
and Mobility Ratio
Repeated 5 Spot
Ref. Dyes, et al.
Trans AIME 1954

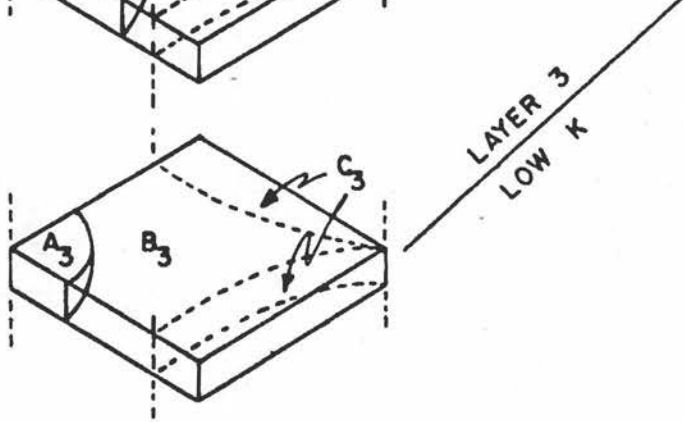


The adjacent sketch illustrates the elementary concept of displacement in stratified reservoirs and the manner of defining vertical displacement efficiency, E_v. Depicted is a vertical cross section through an injection well and producing well. Injection fluid has advanced irregularly as a result of permeability variation in the section as indicated by the shaded area. Vertical displacement efficiency is defined in terms of the distance of furthest advance, x , and in this instance would be the shaded area divided by the area h • x .

A method of evaluating E_v, for given mobility ratios and permeability variation was developed by H. Dykstra and R. L. Parson in 1948 and will be looked at in detail in the next section.

SCHEMATIC OF EFFICIENCY FACTORS





$$E_D = \frac{S_{HC_i} \text{ in Volume A} - S_{HC} \text{ in Volume A}}{S_{HC_i} \text{ in Volume A}}$$

$$E_V = \frac{\sum \text{A Volumes}}{\text{A Volumes} + \text{B Volumes}}$$

$$E_A = \frac{\sum \text{A Volumes} + \sum \text{B Volumes}}{\text{Total Pattern Volume}}$$

$$E_R = E_D \cdot E_V \cdot E_A$$

RESERVOARTEKNIKK III

DYKSTRA - PARSONS METHOD OF CALCULATING LINEAR DISPLACEMENTS IN LAYERED SYSTEMS

In this method an oil reservoir is characterized as a layered system and recovery is calculated as a function of the permeability variation of this layered system and the mobility ratio.

Assumptions:

1. The reservoir consists of isolated layers of uniform permeability with no cross flow between layers.
2. Piston-like displacement; that is only one phase is flowing in any given volume element.
3. Flow is linear.
4. The fluids are incompressible; that is there are no transient pressure effects.
5. The pressure drop across every layer is the same.
6. Mobility ratio, porosity, and fluid saturation are the same in each layer. (This is not a necessary assumption for the method but was made in the interest of simplifying the calculations of coverage charts.)

Theory

It will be assumed that the reservoir may be thought of as a series of layers piled one on top of the other. It may be true that two adjacent layers have the same permeability. In general, however, the absolute permeability will vary from one layer to the next. In any given layer the permeability is taken to be constant. In each layer it will be assumed that there is piston-like displacement, i.e., only oil is flowing ahead of the front and only water behind the front. This means that in any layer all the oil is produced at breakthrough that will be produced.

Consider first the determination of the velocity of the front in any layer. By Darcy's law:

$$q_o = -\frac{k_o}{\mu_o}\frac{dP}{dx} \tag{1a}$$

$$q_w = -\frac{k_w}{\mu_w}\frac{dP}{dx} \tag{1b}$$

Suppose the flood front is located at x_1, and let ΔP_1 be the difference in pressure between the point x_1 and the influx end of the layer. Then

$$q_w = -\frac{k_w}{\mu_w}\frac{\Delta P_1}{x_1} \tag{2}$$

The difference in pressure between the efflux end of the layer and x_1 is

$$\Delta P - \Delta P_1 \tag{3}$$

where P is the difference in pressure between the efflux end of the layer and the influx end. Hence

$$q_o = -\frac{k_o}{\mu_o}\frac{(\Delta P - \Delta P_1)}{L-x_1} \tag{4}$$

When equations (2) and (3) are added:

$$\frac{\mu_w q_w x_1}{k_w} + \frac{\mu_o q_o (L-x_1)}{k_o} = -\Delta P \tag{5}$$

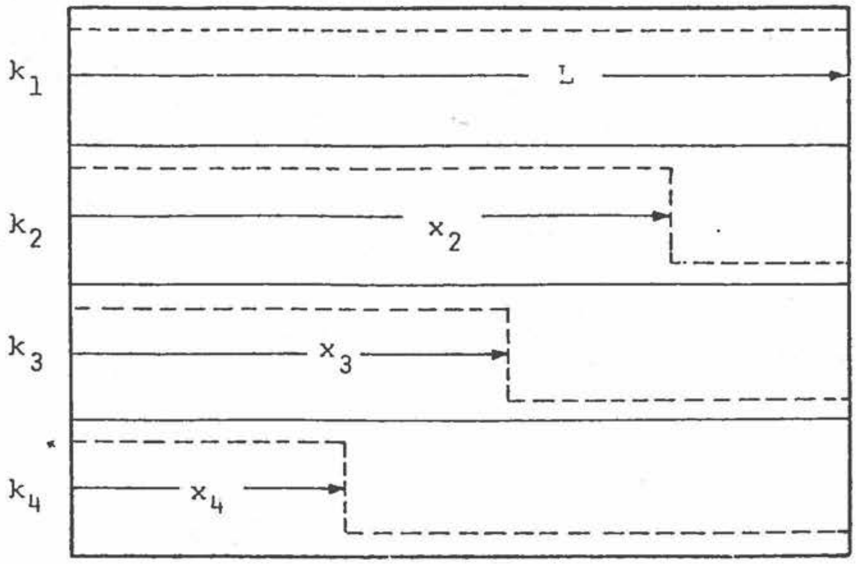
However, inasmuch as the fluids are incompressible and only all oil or all water is flowing,

$$q_w = q_o = q \tag{6}$$

It follows that:

$$q_o = q = \frac{-k\ \Delta P}{\frac{\mu_w}{k_{rw}}x_1 + \frac{\mu_o}{k_{ro}}(L-x_1)} \tag{7}$$

It is now desired to find the ratio of the distance of advance in one layer where $x_1 = L$, i.e., where breakthrough has just occurred, to the interface position x_1 in any other layer with a smaller permeability (see sketch). The pressure drop will be



assumed to be the same across all the layers. The ratio of rates is given by

$$\frac{q_i}{q_1} = + \frac{k_i}{k_1} \frac{\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}} (L-x_1)}{\frac{\mu_w}{k_{rw}} x_i + \frac{\mu_o}{k_{ro}} (L-x_i)} \tag{8}$$

where k_{rw} and k_{ro} are taken to be the same in each layer. But

$$q_i = \frac{dx_i}{dt} \; ; \quad q_1 = \frac{dx_1}{dt} \tag{9}$$

Hence

$$\frac{dx_i}{dx_1} = + \frac{k_i}{k_1} \frac{\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}} (L-x_1)}{\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}} (L-x_i)} \tag{10}$$

To find x_i it is necessary to integrate the above equation:

$$\int_o^L \left[\frac{\mu_w}{k_{rw}} x_1 + \frac{\mu_o}{k_{ro}}(L-x_1)\right] dx_1 = + \frac{k_1}{k_i} \int_o^{x_i} \left[\frac{\mu_w}{k_{rw}} x_i + \frac{\mu_o}{k_{ro}}(L-x_i)\right] dx_i \tag{11}$$

The limits are chosen so that both interfaces start off at the inlet at the same time. It is desired to find the ratio x_i/x_1 when $x_1 = L$.

Integration gives:

$$(1 + M)L^2 = + \frac{k_1}{k_i} [x_i^2 + 2M(Lx_i) - Mx_i^2] \tag{12}$$

where

$$M = \frac{k_{rw}}{k_{ro}} \frac{\mu_o}{\mu_w}$$

On rearrangement:

$$(1-M)\left[\frac{x_i}{L}\right]^2 + 2M\left[\frac{x_i}{L}\right] - \frac{k_i}{k_1}(1+M) = 0 \tag{13}$$

Solving this quadratic equation gives

$$\frac{x_i}{L} = \frac{M \pm \sqrt{M^2 + \frac{k_i}{k_1}(1 - M^2)}}{(M - 1)} \tag{14}$$

When x_i also refers to the first layer $k_i = k_1$ and $x_i = L$, or

$$1 = \frac{M \pm 1}{M - 1} \tag{15}$$

Hence the minus sign must be chosen in equation (14). Thus when the jth layer has broken through,

$$\frac{x_i}{x_j} = \frac{x_i}{L} = \frac{M - \sqrt{M^2 + \frac{k_i}{k_j}(1 - M^2)}}{(M - 1)} \tag{16}$$

which gives the distance of advance of the front in layer i having a permeability less than j when layer j has broken through.

If in equation (13) the mobility ratio M is set equal to one, then:

$$\frac{x_i}{x_1} = \frac{k_i}{k_1} \tag{17}$$

This is just an expression for the basic assumption of the Stiles method, that the ratio of the distances of advance in the various layers is the same as the corresponding permeability ratio. Thus the Stiles method will give the same answers as the Dykstra-Parsons method when the mobility ratio is unity.

It is now of interest to find an expression for the coverage when the nth layer has broken through. The coverage is defined as the fraction of the reservoir which has been invaded by water. Let N be the total number of layers in the system. Number the layers in order of decreasing permeability.

When the nth layer has broken through, all the layers with permeability greater than that of the nth layer will also have broken through. Hence the fraction of the reservoir for which the layers have been completely flooded out is n/N. The remaining layers, which have permeabilities less than the nth layer, will be only partially swept out. The distances the flood has advanced in the jth layer (j>n) when the nth layer has just broken through is

$$\frac{x_j}{x_n} = \frac{M - \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{M - 1} \tag{18}$$

$$\frac{x_j}{x_n} = \frac{x_j}{L} \tag{19}$$

is just the fraction of the jth layer which has been swept out. The complete coverage is then just:

$$\text{COVERAGE} = \frac{n + \sum_{j>n} \left[\frac{x_j}{x_n}\right]}{N} \tag{20}$$

or

$$\text{COVERAGE} = \frac{n + \sum_{j>n} \left[\frac{M - \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{(M - 1)}\right]}{N} \tag{21}$$

but

$$\sum_{j>n} M = (N - n)\, M. \tag{22}$$

Hence

$$\text{COVERAGE} = \frac{n + \frac{(N-n)M}{M-1} - \frac{1}{M-1} \sum_{j>n} \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{N} \tag{23}$$

The above formula makes it possible to calculate the coverage, or fraction of the reservoir which has been invaded by water, when the nth layer has broken through.

An expression for computing the water-oil ratio when the nth layer has broken through will now be derived. When the nth layer has broken through, only water is flowing in the layers with permeability greater than that of the nth layer. The total flow rate of water per unit breadth is:

$$Q_{w_n} = \sum_{j<n} H \Delta Z q_{w_j} = \Delta Z \sum_{j<n} H \left[\frac{- k_j k_{rw}}{\mu_w} \frac{\Delta P}{L} \right] \tag{24}$$

Only oil is flowing out in the layers with permeability less than that in the nth layer. The total flow rate of oil per unit breadth is:

$$Q_{o_n} = \Delta Z \sum_{j>n} H q_{o_j} \tag{25}$$

Equation (7) must be used for q_{oi}, since there is a front moving along in the layers where oil is flowing out. Thus

$$Q_{o_n} = \Delta Z \sum_{j>n} \left[- \frac{H k_j \Delta P}{\frac{\mu_w}{k_{rw}} x_j + \frac{\mu_o}{k_{ro}} (L - x_j)} \right] \tag{26}$$

$$Q_{o_n} = \Delta Z \sum_{j>n} \left[- \frac{H k_j \frac{\Delta P}{L}}{\frac{\mu_w}{k_{rw}} \frac{x_j}{L} + \frac{\mu_o}{k_{ro}} (1 - \frac{x_j}{L})} \right] \tag{27}$$

But equation (18) may be used for x_j/L so that:

$$Q_{o_n} = \Delta Z \sum \left[- \frac{H k_j \frac{\Delta P}{L}}{\frac{\mu_w}{k_{rw}} \left\{ \frac{M - \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{M - 1} \right\} + \frac{\mu_o}{k_{ro}} \left\{ \frac{-1 + \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{M - 1} \right\}} \right] \tag{28}$$

$$Q_{o_n} = \Delta Z \sum \left[- \frac{H k_j \frac{k_{rw}}{\mu_w} \frac{\Delta P}{L}}{\frac{M - \sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{M - 1} + \frac{-M + M\sqrt{M^2 + \frac{k_j}{k_n}(1 - M^2)}}{M - 1}} \right] \tag{29}$$

$$Q_{o_n} = \Delta Z \sum_{j>n} \left[- \frac{H k_j \frac{k_{rw}}{\mu_w} \frac{\Delta P}{L}}{\sqrt{M^2 + \frac{k_j}{k_n} (1 - M^2)}} \right] \tag{30}$$

The water-oil ratio when the nth layer has broken through and for layers of equal thickness is then:

$$WOR_n = \frac{Q_{w_n}}{Q_{o_n}} = \frac{\sum\limits_{j<n} k_j}{\sum\limits_{j>n} \dfrac{k_j}{\sqrt{M^2 + \dfrac{k_j}{k_n}(1 - M^2)}}} \tag{31}$$

This expression can be used to calculate the water-oil ratio when the nth layer has just broken through. The producing water-oil ratio F_{wo} is then given by B_o times the value given by equation (31), or

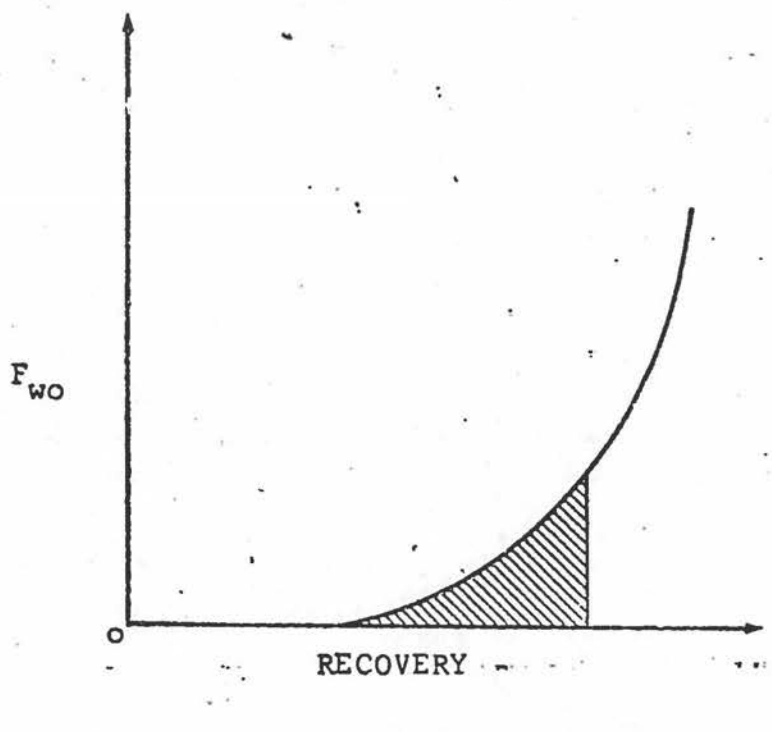
$$F_{wo} = B_o \times WOR$$

The cumulative oil recovery, Np, is calculated from the following equation:

$$Np = \frac{7758\ Ah\phi C\ (S_{oi} - S_{or})\ Ea}{B_o} \tag{32}$$

where Ah is area thickness product in acre-feet
ϕ is fractional porosity
C is coverage from equation (23), or from charts
S_{oi} and S_{or} are initial and residual oil saturations
Ea is areal sweep efficiency
B_o is oil formation volume factor.

The calculation of the recovery as a function of time will now be considered. If the F_{wo} is plotted against the recovery on rectangular coordinates, a curve would be obtained which looks something like that in the following sketch.



Now the water-oil ratio, F_{wo}, is given by

$$F_{wo} = \frac{q_w}{q_o} = \frac{\frac{dW_p}{dt}}{\frac{dN_p}{dt}} = \frac{dW_p}{dN_p} \qquad (33)$$

where W_p is the cumulative water produced. The question can then be asked, what does the area under the F_{wo} vs recovery curve represent? The area is just:

$$\text{AREA} = \int_0^{N_p} (F_{wo})dN_p = \int_0^{N_p} \frac{dW_p}{dN_p} = W_p \qquad (34)$$

Thus the area under the curve is just the water produced up to the given recovery N_p. The water injected W_i when the recovery is N_p is just:

$$W_i = W_p + B_oN_p + W_F \qquad (35)$$

W_F being the volume of water required for fill-up and is equal to 7758 $Ah\phi(S_{gi}-S_{gr})$. The time required to reach a given recovery is just:

$$\text{TIME, } t = \frac{W_i}{i_w} \qquad (36)$$

where i_w is the water injection rate (assumed to be constant). Thus by finding the area under the F_{wo}-N_p curve up to a given N_p, it is possible to obtain curves for the cumulative water injected as a function of F_{wo} and the cumulative production as a function of time. To find the production rate it is only necessary to divide the differences in recovery by the corresponding differences in time.

Coverage Charts

The above equations hold for an arbitrary permeability distribution. Dykstra and Parsons wanted to obtain generalized curves in which the permeability distribution could be characterized by a single number, so that engineers would not need to perform the rather long calculations necessary to compute the F_{wo}-N_p curve. To obtain such generalized curves, they assumed that if the percentage of the permeabilities greater than a given value was plotted against that permeability on log probability paper, a straight line results. They characterized this straight line by the permeability variation which was defined to be the median permeability minus the permeability at 84.1 cumulative percent, this difference divided by the median permeability. The permeability variation essentially measures the slope of the straight line. Only this permeability variation is necessary to characterize the distribution as far as the calculations are concerned. The reason for this is that the magnitudes of the permeability are not important, inasmuch as only ratios of permeabilities appear in the calculations. Thus it was possible to calculate curves giving the coverage as a function of the permeability variation and mobility ratio for any given water-oil ratio. These calculations were made and put in the form of two charts in the original paper as Figures 9 and 10 for water-oil ratios of 1 and 25. Since the time, computers have become available and made it possible to make calculations covering a wide range of water-oil ratios. New coverage charts have been prepared recently and are now available covering a WOR range of 0.1 to 100. A set of 10 charts are included with this write-up.

Correlations of Recovery with Coverage

In the original paper, a correlation was presented (Figure 11) of a "recovery modulus" and coverage, C. The recovery modulus is defined as $R\left[1 - S_w\ (WOR)^{-0.2}\right]$, where R is the fractional recovery of the original oil. Coverage, C, is a function of WOR and also of permeability variation and mobility ratio. The correlation was based on results of laboratory core floods and gave best results for initial oil saturations lying between 45 and 60 per cent. The correlation was particularly useful in estimating a recovery factor when oil saturation were assumed to lie close to the range mentioned above.

C. E. Johnson (Trans AIME 207 (1956) 345) was able to simplify use of the correlation by constructing charts for WOR's of 1, 2, 5, and 100. On these charts, lines of constant recovery modulus values are plotted as functions of mobility ratio, M, and permeability variation, V. These charts are shown on pages 100 and 101. The procedure is to find the value of recovery modulus from the appropriate Johnson chart and, knowing the water saturation, S_w, calculate the fractional recovery, R. Stocktank oil recovery can then be calculated from the relationship:

$$N_p = \frac{7758\ Ah\phi\ S_{oi}\ E_A \cdot R}{B_o}$$

Proceaure for Using the Dykstra-Parsons Method

The steps for calculating recovery with the aid of the coverage charts is as follows:

1. Assemble permeability data in descending order. Calculate "percentage equal to or greater than" for each entry.

2. Plot percentage against log permeability on probability paper. Calculate permeability variation from

$$V = \frac{k_{50} - k_{84.1}}{K_{50}}$$

3. Calculate mobility ratio,

$$M = \frac{k_w \mu_o}{\mu_w k_o}$$

4. From charts get coverage, C.

5. Calculate recovery from the following equation:

$$N_p = \frac{7758 \text{ AhOC } (S_{oi} - S_{or}) E_a}{B_o}$$

6. Plot N_p vs F_{wo}.

7. Integrate N_p - F_{wo} curve graphically to get W_p.

8. Calculate $W_i = W_F + N_p B_o + W_p$.

9. Time in years is given by

$$\frac{W_t}{i_w \times 365}$$

10. Calculate oil and water rates from

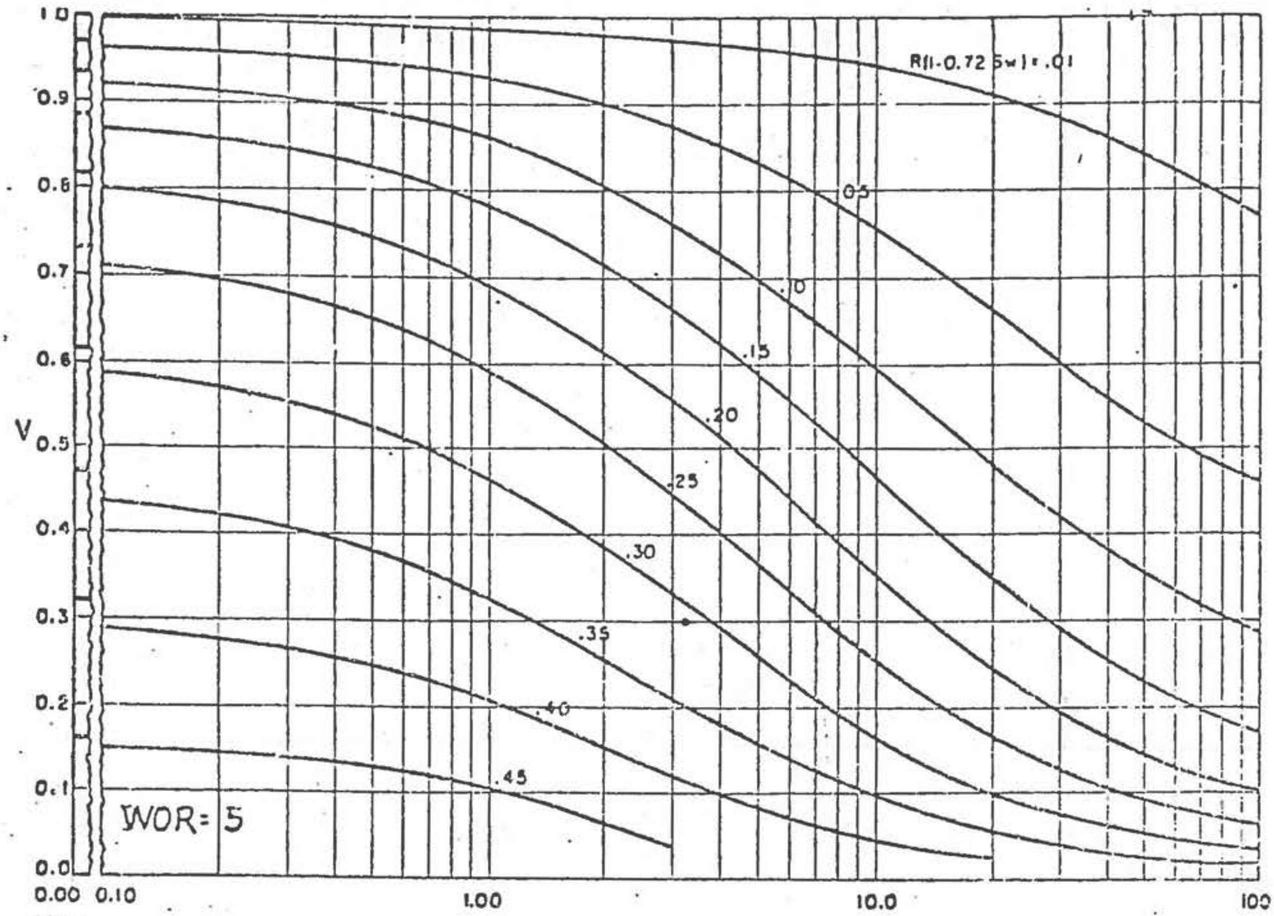
$$q_o = \frac{i_w}{B_o + F_{wo}}$$

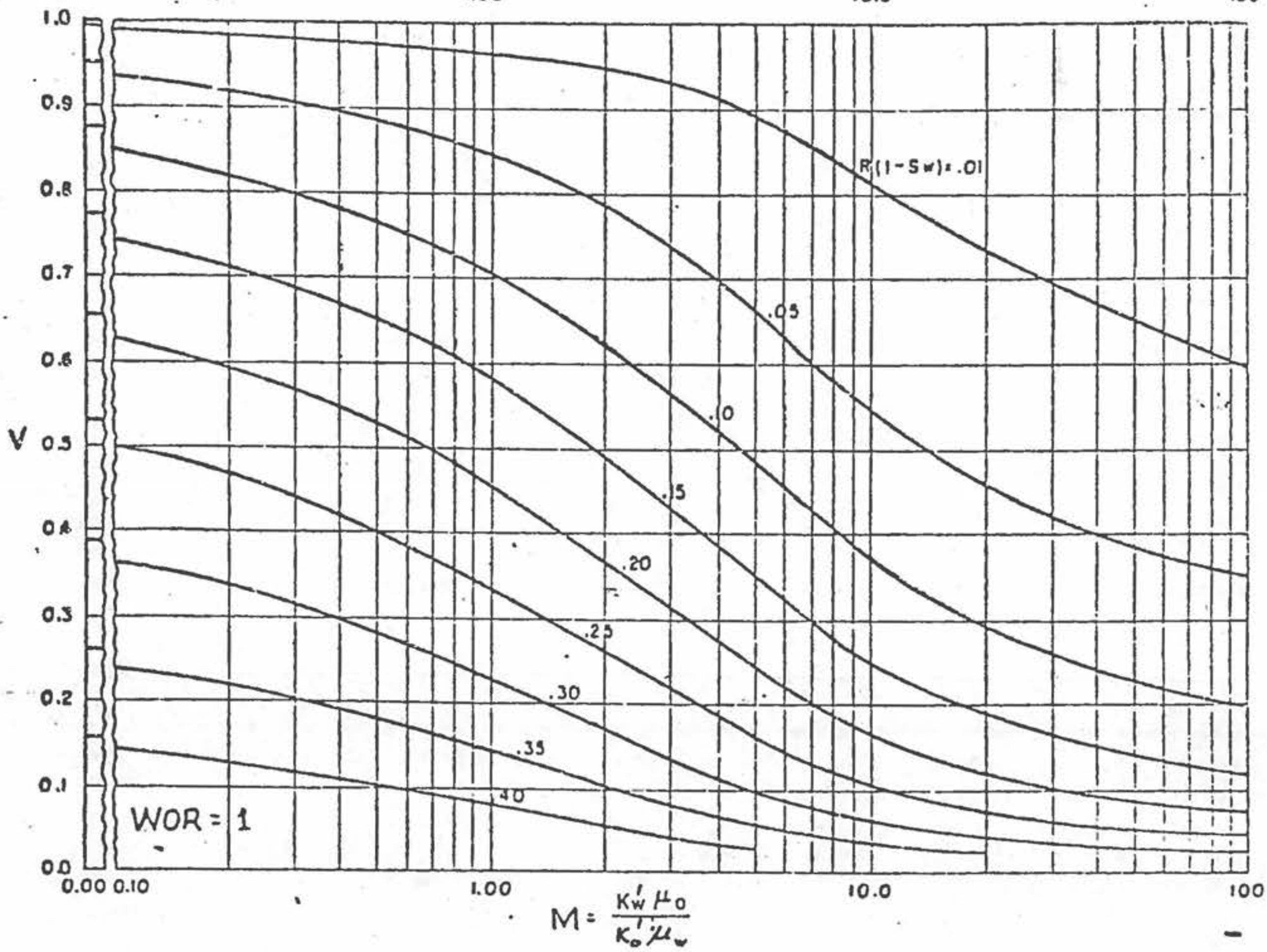
$$q_w = q_o F_{wo} \quad \text{or} \quad q_w = i_w - B_o q_o$$

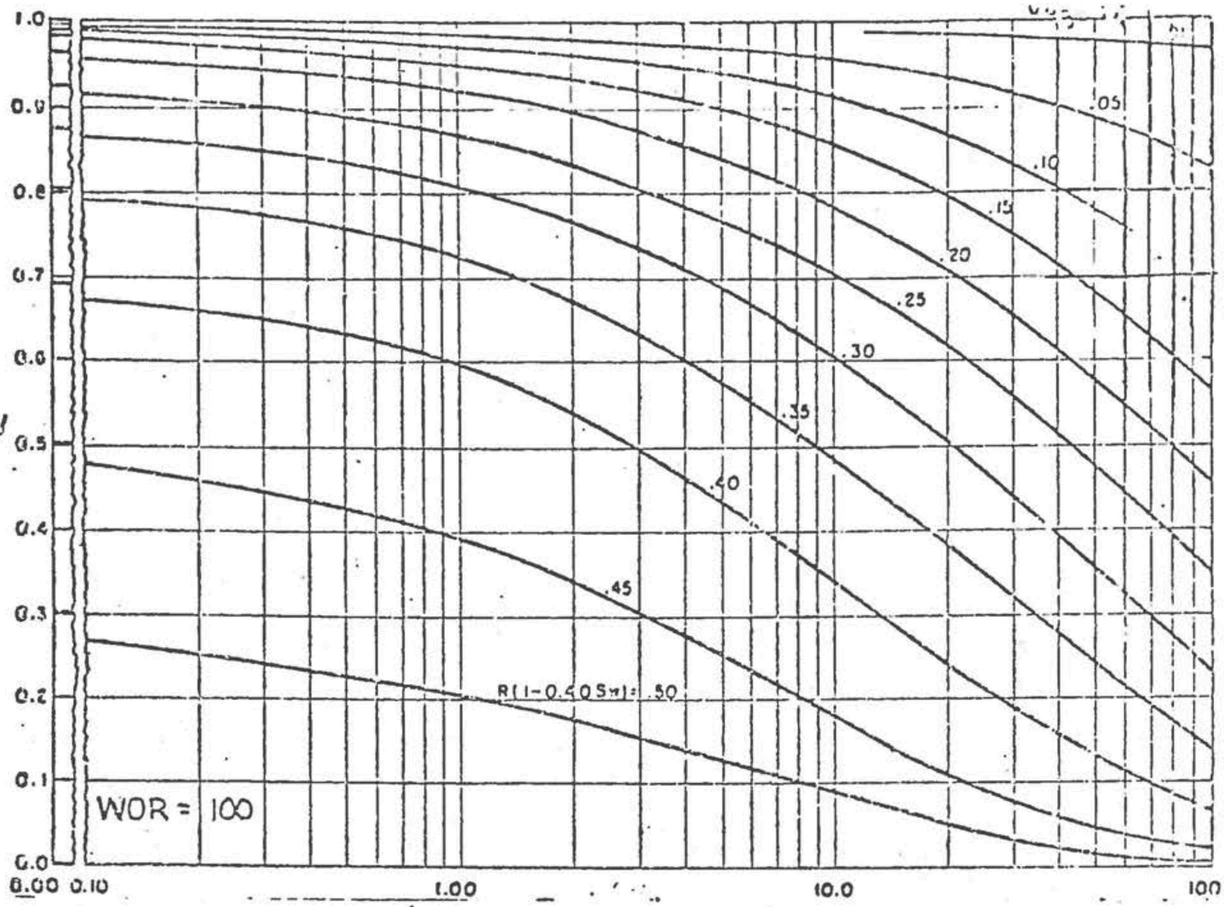
REFERENCES

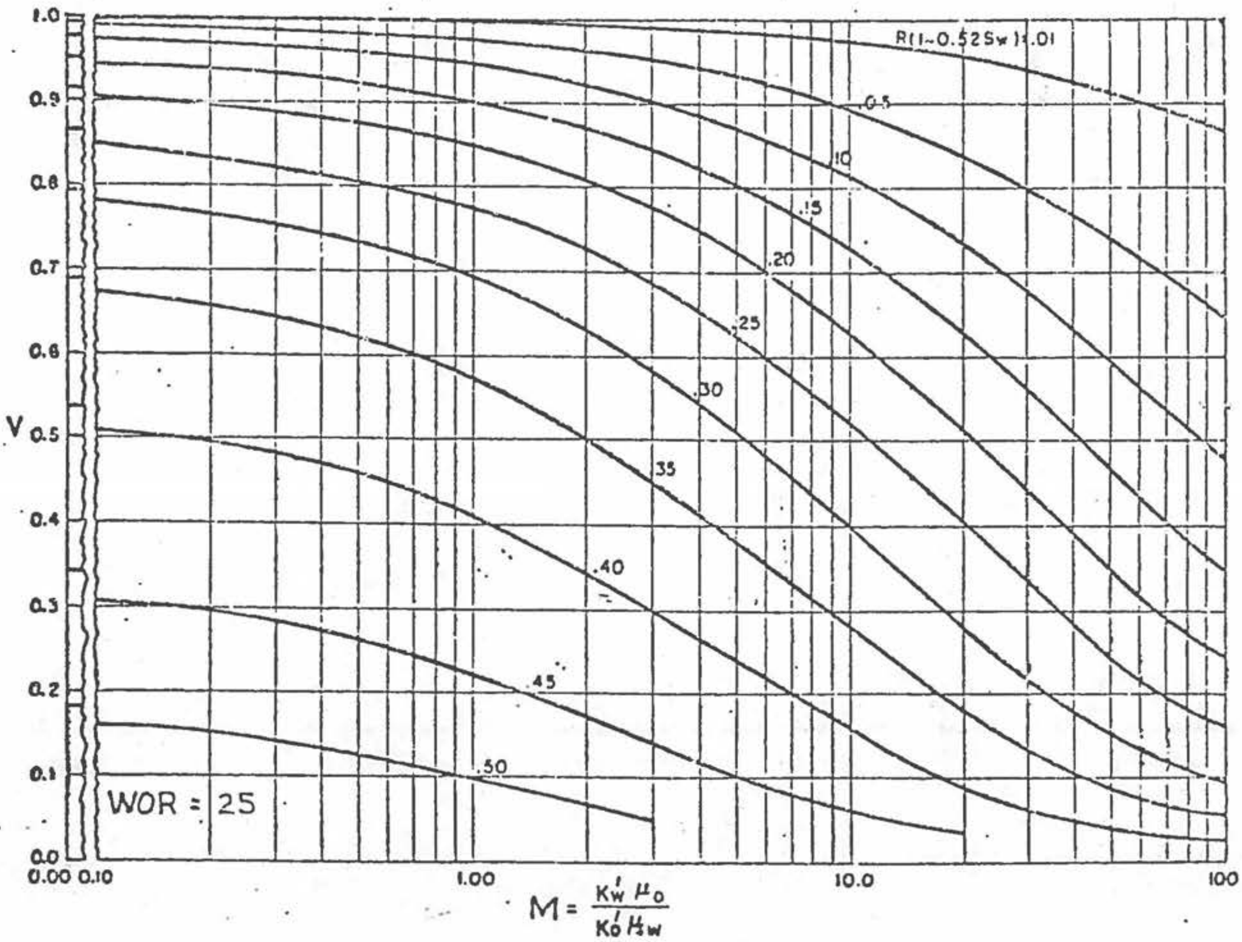
1. Dykstra and Parsons, API Sec. Rec. of Oil in the U.S., p.160 2nd Ed, 1950.

2. Johnson, Trans AIME 207, 345 (1956).


R(1-0.72 Sw) = .01
.05
.10
.15
.20
.25
.30
.35
.40
.45
WOR = 5
V
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.00 0.10
1.00
10.0
100


R(1-Sw) = .01
.05
.10
.15
.20
.25
.30
.35
.40
WOR = 1
V
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.00 0.10
1.00
10.0
100
M = $\frac{K_w' \mu_o}{K_o' \mu_w}$


V
WOR = 100
R(1-0.40Sw)= .50
.05
.10
.15
.20
.25
.30
.35
.40
.45
0.00 0.10
1.00
10.0
100


V
WOR = 25
R(1-0.52Sw)=.01
.05
.10
.15
.20
.25
.30
.35
.40
.45
.50
0.00 0.10
1.00
10.0
100
$M = \frac{K_w' \mu_o}{K_o' \mu_w}$

Notes on Predicting Pattern Performance During Water Injection Operations

Reservoir engineers are oftem called upon to estimate the profitability of water flooding a given property or reservoir. There are a number of methods described in the literature that can be used. Each method involves several assumptions about the sub-surface reservoir behavior and each has a different degree of complexability in carrying out the calculations. The Higgins-Leighton method, for example, is so detailed that it can only be used in conjunction with a large computer. It problbly offers the better solution, however, all things being equal. Simpler methods, capable of being carried out with hand-held calculators, often offer solutions (predictions) sufficiently accurate for preliminary evaluations of the flood and may, in certain instances, be sufficiently accurate for use as the final prediction method.

The method described in these notes falls in the second catagory. It is relative simple and straight forward and can be handled by desk calculator. It models a multi-layer pattern flood with provision for the displacement of gas ahead of the oil bank as well as displacement or oil by the injected water. The primary assumptions in the method are these:

1. The reservoir volume to be flooded (pattern) is handled as a group of layers with no cross flow between layers. Thus, it is similar to an assumption in the Dykstra-Parsons method.
2. Within a given layer there are no saturation gradients behind the two fronts. This means that the displacement is piston like (or leaky-piston) in nature. This also is similar to an assumption of the Dykstra-Parsons method. However, one can develop a similar method that uses a Buckley-Leverett type of displacement.
3. Water intake into each layer is proportional to the layer injectivity at the time (which changes in each layer as the flood progresses). However, to simplify these notes and concentrate on describing the displacement process model involved, it will be assumed that the layer injectivities are constant in time and proportional to the kh product of the layer.

The first section of the notes will describe the behavior of a single layer of the model. Later sections will describe the process of combining the behavior of all layers to yield the desired pattern behavior. Field behavior, which invols superposition in time of a number of pattern behaviors will not be covered in these notes.

Single Layer Behavior

The geometry of the segment of reservoir to be considered is that of a developed 5-spot pattern consisting of one production well at the center and four injection wells located at the corners. This is illustrated in Fig. 1. To simplify description behavior in only ¼ of the pattern (cross-hatched area) will be considered.

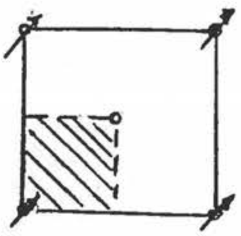


Figure 1

Figure 2 illustrates layering within the unit volume. As indicated previously, no cross flow of fluids occurs between layers. This means that water entering a given layer at the injection wellbore remains in and displaces hydrocarbons from that layer.

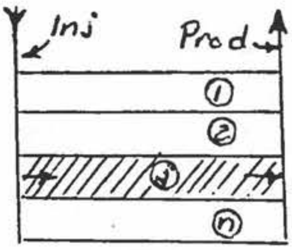


Figure 2

Figure 3 illustrates location of two displacement fronts in a given (j) layer prior to fillup. At this point only free gas has been produced for the layer. The area processed by the injected water is labled Region 3 and has an areal coverage value of E_{aw}. Region 2 is an area which contains oil that has been displaced from Region 3 plus some residual gas remaining from the displacement at the oil-gas front. The areal coverage factor of the oil-gas front is E_{ao}. Region 1 contains initial saturation conditions.

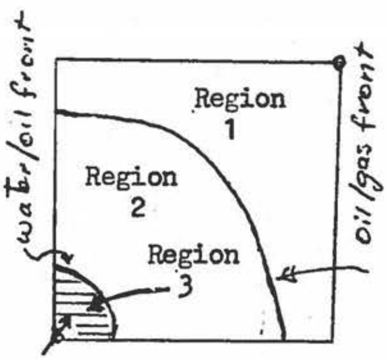


Figure 3

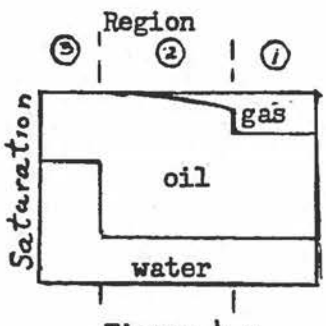


Figure 4 a

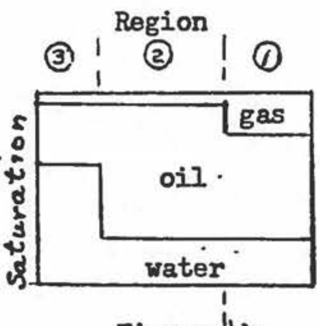


Figure 4b

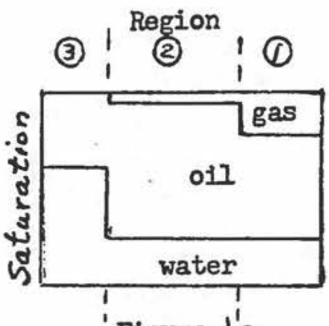


Figure 4c

Figure 4, above, illustrate several saturation-distance situation along the diagonal connecting the injection and producing well is Fig 3. In the field, pressure increases from the producing well to the injection well. Gas that is left behind the oil-gas displacement front is consequently compressed and taken into solution in the oil in Region 2. Thus there is the possibility that the gas saturation - distance profile should look somewhat like that in Fig 4a. On the other hand, not knowing how far back in Region 2 gas saturation exists it is easier to assume saturation profiles like those of Fig 4b or 4c. Figure 4b assumes residual gas saturation exists all the way through Regions 2 and 3, while Fig 4c assumes that residual gas remains only in Region 2. Some small diferences in pattern performance result from how the residual gas phase is assumed to lie. Figure 4b is often used and results in the easiest analysis of the pattern behavior. Figure 4c seems a little more in keeping with postulated field behavior and will be the situation used in these notes.

Considering the situation pictured in Fig 3 (prior to fillup) the oil and gas displaced from Region 3 must be in Region 2 or have been produced. Gas originally in Region 3, or part of it, could have been produced but oil from Region 3 must still be in Region 2. In terms of the areal coverage factors E_{aw} and E_{ao} and a volume balance of the displaced oil phase we can compute the relative areas processed by the two fronts as follows:

Given:

E_{aw} = area included in Region 3

$E_{ao} - E_{aw}$ = area included in Region 2

$(\Delta S_o)_3$ = oil saturation change, Region 3

$(\Delta S_g)_2$ = gas saturation change, Region 2

$$E_{aw}(\Delta S_o)_3 = (E_{ao} - E_{aw})(\Delta S_g)_2 \qquad (1)$$

This leads to.

$$\frac{E_{ao}}{E_{aw}} = F = 1 + \frac{(\Delta S_o)_3}{(\Delta S_g)_2} \qquad (2)$$

Equation 2 is completely general. Nothing has been said about the shapes of the two displacement fronts or of the mobility ratios that pertain to the fronts.

In general, the mobility ratio at the oil-gas front is apt to be quite low - probably less than 0.1, while the water-oil mobility ratio at the water-oil front is apt to be greater than one. Each mobility ratio can be calculated from the saturation conditions considered and the phase viscosity ratio.

Areal coverage factors, E_a, for a number of standard patterns (5-spot, 9-spot, direct line drive, etc.) are available in various forms in the literature (see, for example, SPE Monograph #3 on waterflooding). The attached chart shows 5-spot values determined by Caudle and Witte, (Trans AIME 216,(1959) 446). While not shown on this chart, E_a values for mobility ratios less than 0.15 are, for practical purposes, unity.

The term, displaceable pore volume, used in waterflood calculations is defined as the unit pore volume multiplied by the maximum achievable difference of water saturation in the area processed by the injected

4

water. Thus, for a single layer of the 5-spot pattern illustrated in Fig 1, the displaceable pore volume, V_d, amounts to

$$V_d(\text{bbl}) = 7758\ A\ h\ \phi\ \Delta S_w \tag{3}$$

where

A = pattern (layer) area, acres
h = layer thickness, feet
ϕ = layer porosity, fraction
$\Delta S_w = (S_{w\,max} - S_{wi})$ in Region 3
$= (1 - S_{or} - S_{gr} - S_{wi})$ in Region 3

The term displaceable pore volume injected, V_{di}, is, of course, the volume of injected fluid divided by the displaceable pore volume.

$$V_{di} = W_i / V_d \tag{4}$$

when the injected fluid is water.

To illustrate some of the calculation procedures involved in predicting pattern behavior it is helpful to use specified values for variables in the calculations. In the following calculations the pattern is assumed to be a 5-spot and the following factors.

Pattern area, A = 10 acres
Pattern thickness (total) = 16 feet
Number of layers = 4 (equal thickness of 4 feet)
Porosity, ϕ = 0.25
Mobility ratios =
M_{wo} = 2 (water-oil front)
M_{og} = 0.1 (oil-gas front)
Layer permeabilies, k_j = 700,500 400, and 200 md.
Saturations, S

Region	S_w	S_o	S_g
1	0.30	0.58	0.12
2	0.30	0.65	0.05
3	0.68	0.32	0.00

Injection rate, i_w = 450 barrels per day

The above values result in the following values for Vp, V_d, and F.

$V_p(\text{pattern}) = 7758\ A h \phi = 7758 \cdot 10 \cdot 16 \cdot 0.25 = 310320$ bbl.

$V_{p_j}(\text{layer}) = 310320/4 = 77580$ bbl.

$V_d(\text{pattern}) = V_p \cdot \Delta S_w = 310320\,(0.68 - 0.30) = 117922$ bbl

$V_{d_j} = V_d / 4 = 29480$ bbl

$F = \dfrac{E_{ao}}{E_{aw}} = 1 + \dfrac{(\Delta S_o)_3}{(\Delta S_g)_2} = 1 + \dfrac{(0.58 - 0.32)}{(0.12 - 0.05)} = 4.71$

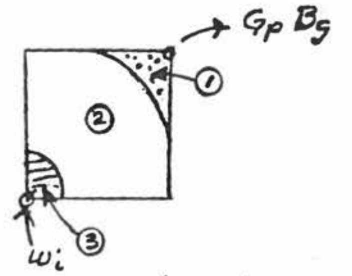
$V_{di} = W_i / V_d = \dfrac{450 \cdot 365}{117922} = 1.39$ per year

At this point it is well to develop equations for the cumulative products produced from the layer and their rate of production at various times. To do so we will consider three situations. these are:

1. Prior to fillup. Only gas is being produced
2. After fillup but before water breakthrough. Both gas and oil phases are being produced.
3. After water breakthrough. Both oil and water phases are being produced.

The first set of equations are for the cumulative productions removed from the layer. Separate equations for rates will be developed later.

1. Prior to Fillup - Only Gas Production



The basic approach is a volume balance on the gas phase; i.e., the produced volume is equal to the original volume minus what is still there. Refering to Fig 4c, it seen that original gas saturation is in Region 1, residual gas saturation is in Region 2, and zero gas is in Region 3. Consequently, the gas phase volume balance is:

$$(G_p B_g)_j = V_{p_j}\left[S_{gi} - (1-E_{ao})S_{g1} - (E_{ao}-E_{aw})S_{g2}\right] \qquad (5)$$

where

S_{gi}, S_{g1} = original gas saturation value
S_{g2} = gas saturation in Region 2

Because pattern efficiency charts only provide E_{aw} values Eq 5 can be modified by recalling that $E_{ao} = F\ E_{aw}$ (eq 2,pg 3). Also, noting that Region 1 contains the original gas saturation so that S_{gi} and S_{g1} are the same, Eq 5 developes into:

$$(G_p B_g)_j = V_{p_j}\ E_{aw_j}\left[F S_{gi} - S_{g2}(F-1)\right] \qquad (6)$$

Equations for oil and water phase productions during the "prior to fillup" regime are, of course:

$$(N_p B_o)_j = 0 \qquad (7)$$
$$(W_p B_w)_j = 0 \qquad (8)$$

At this point let's consider an example calculation of the gas produced at fillup from one of the layers described on page 4. Under these conditions:

$E_{ao} = 1$: $E_{aw} = E_{ao}/F = 1/F = 1/4.71 = 0.212$

From the 5-spot chart for $E_{aw} = 0.212$ and $M_{wo} = 2$. $V_{d_{ij}} = 0.212$

Therefore, $W_{ij} = 0.212\ V_{dj} = 0.212 \cdot 29480 = 6250$ barrels.

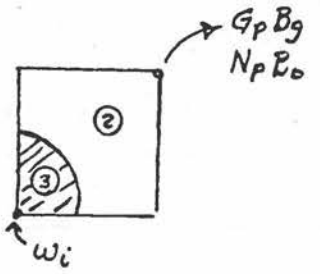
$$(G_p B_g)_j = 77580 \cdot 0.212\left(4.71\cdot 0.12 - 3.71\cdot 0.05\right) \qquad (Eq\ 6)$$

$$= 6245 \text{ bbl.}$$

$$\text{Producing time, } t = \frac{W_{ij}}{i_j} = \frac{6250}{450/4} = 55 \text{ days.}$$

Note: The slight difference of 5 barrels between the water injected and the produced gas comes from round-off errors. They should be the same, of course.

2. After Fillup but Before Water Breakthrough



During this production phase gas is still being produced because of expansion of Region 3 and the fact a gas saturation change occurs at the water-oil front. Oil is also being produced because of the expansion of Region 3.

The gas volume balance can be obtained directly from Eq 5 by noting that $E_{ao} = 1$. Or, looking at the sketch at the left, it can be seen that:

$$(G_p B_g)_j = V_{p_j}\left[S_{gi} - (1 - E_{aw}) S_{g2}\right] \qquad (9)$$

The cumulative oil production comes from an oil volume balance. Refering to the sketch above, it can be seen that:

$$(N_p B_o)_j = V_{p_j}\left[S_{oi} - E_{aw} S_{o3} - (1 - E_{aw}) S_{o2}\right] \qquad (10)$$

As we have specified that water breakthrough has not yet taken place the water production equation is still

$$(W_p B_w)_j = 0 \qquad (11)$$

An illustrative use of Eqs 9 and 10 is as follows: what will be the reservoir gas and oil cumulative productions when the layer has been produced for 100 days?

$$W_{i_j} = 450/4 \cdot 100 = 11250 \text{ barrels.}$$

$$V_{di_j} = \frac{W_{ij}}{V_{dj}} = \frac{11250}{29480} = 0.382$$

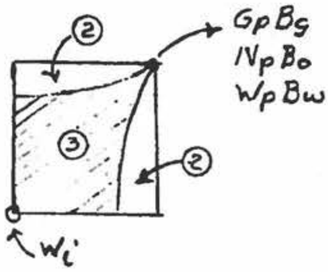
From the 5-spot chart for $V_{di} = 0.382$, $E_{aw} = 0.382$

$$\therefore (G_p B_g)_j = 77580\left[0.12 - (1 - 0.382)\cdot 0.05\right] = 6912 \text{ barrels.}$$

$$(N_p B_o)_j = 77580\left[0.58 - 0.382\cdot 0.32 - (1 - 0.382)\, 0.65\right]$$

$$= 4349 \text{ barrels.}$$

3. After Water Breakthrough



After water breakthrough there is still gas production because Region 3 is increasing in size and the gas saturation change at the water-oil front. Oil phase is being produced because of the diminishing size of Region 2.

As can be seem from the sketch at the left, there is really no difference insofar as the gas and oil phases are concerned, than the situation depicted on the previous page. Only the relative sizes of Regions 2 and 3 are different. Therefore, Eqs 9 and 10 are valid for this period.

The volume balance on the water that has entered the layer and that remaining is simple. The produced water volume is:

$$W_{p_j} = W_i - V_{p_j} \cdot E_{aw}(S_{w3} - S_{wi}) \qquad (12)$$

There is some question as to whether the water quantities in Eq 12 should not contain the water formation volume factor, B_w. It is more correct to include it but as it is usually very close to unity it often is omitted.

An alternate equation that can be used to calculate produced water comes from a volume balance of all fluids in the pattern, that is:

$$W_{pj} = W_{ij} - (N_pB_o)_j - (G_pB_g)_j \qquad (13)$$

To illustrate the after breakthrough production equations let's calculate the layer production after one year of injection.

$$W_{ij} = 450/4 \cdot 365 = 41062 \text{ barrels}$$

$$V_{dij} = \frac{41062}{29480} = 1{,}393$$

From 5-spot chart for $V_{di} = 1{,}39$; $M_{wo} = 2$,

$E_{aw} = 0{,}879.$

$$\therefore (G_pB_g)_j = 77580[0{,}12 - (1-0{,}879)0{,}05]$$
$$= 8840 \text{ barrels.} \qquad (Eq. 9)$$

$$(N_pB_o)_j = 77580[0{,}58 - 0{,}879 \cdot 0{,}32 - (1-0{,}879)0{,}65]$$
$$= 17073 \text{ barrels.} \qquad (Eq\ 10)$$

$$W_{p_j} = 41063 - 77580 \cdot 0.879(0.68 - 0{,}30)$$
$$= 15150 \text{ barrels}$$

Layer Producing Rate Equations. Up to this point the equations have been for the cumulative volumes of reservoir gas, oil, and water produced from the layer as functions of time or cumulative water injected. Ordinarilly one is equally interested in the rates of production of oil and water (not so much for gas) during the flood as these values are used to size equipment and calculate income to the project. As rate is the time derivative of cumulative production, e.g. $q_o = dN_p/dt$, one can always resort to plotting cumulative production values against time and determining tangent slopes at various time. But the method to be used in the following portion of the note is to differentiate with respect to time eqs 6,9,10 and 12.

Gas Rate Prior to Fillup. If one considers the sketch on page 5 that represents the "prior to fillup" case, it is instantly apparent that gas is being removed from the layer at the same rate that water is entering the pattern. So, for this case

$$(q_g B_g)_j = i_{wj} \tag{14}$$

However, it is instructive to see if this relationship can be developed from Eq 6 (or Eq 5) as the method of doing so will apply to developing the other fluid rate equations. Starting with Eq 6 then:

$$(G_p B_g)_j = V_{p_j} \cdot [F S_{gi} - S_{g2}(F-1)] E_{aw} \tag{6}$$

As V_{p_j}, F, S_{gi} and S_{g2} are constants

$$(q_c B_g)_j = \frac{d}{dt}(G_p B_g)_j = V_{p_j}[F S_{gi} - S_{g2}(F-1)] \cdot \frac{d E_{aw}}{dt} \tag{15}$$

$$\text{But } \frac{d E_{awj}}{dt} = \frac{d E_{awj}}{d V_{dij}} \cdot \frac{d V_{dij}}{dt} \tag{16}$$

$$\text{and } \frac{d V_{dij}}{dt} = \frac{d}{dt} \frac{W_{ij}}{V_j \Delta S_w} = \frac{1}{V_j \Delta S_w} \cdot i_{wj}$$

Therefore.

$$(q_g B_g)_j = \frac{\cancel{V_p}[F S_{gi} - S_{g2}(F-1)] \cdot i_w}{\cancel{V_p} \Delta S_w} \frac{d E_{aw}}{d V_{di}} \tag{17}$$

As $\Delta S_w = [F S_{gi} - S_{g2}(F-1)]$ and at small values of E_{aw} $\frac{d E_{aw}}{d V_{di}} = 1$, we get

$$(q_g B_g)_j = i_{wj}$$

At th instant that fillup is achieved there will be a major decrease in the gas production rate. This is because the gas was being produced in accordance with the volumetric rate of advance of the oil-gas front and, after fillup, the oil and gas being produced is a function of the water-oil front advance rate. This will be handled in the next section.

Gas Rate After Fillup. Equation 9 on page 6 gives the cumulative gas produced after fillup. This can be written as:

$$(G_p B_g)_j = V_{p_j}(S_{gi} - S_{g2}) + V_{p_j} S_{g2} E_{aw}. \qquad (18)$$

By following the exact same procedure as on the previous page we get:

$$(q_g B_g)_j = \frac{S_{g2}}{\Delta S_w} i_{wj} \left(\frac{dE_{aw}}{dV_{di}}\right)_j \qquad (19)$$

where $\Delta S_w = S_{w3} - S_{wi}$

$\left(\frac{dE_{aw}}{dV_{di}}\right)$ = slope of areal efficiency curve.

Equation 19 applies any time after fillup of the layer. Note that a sharp change in rate will occur at water breakthrough as this is the condition at which the slope of the areal efficiency curve changes unity to some smaller value.

Oil Rate After Fillup. As pointed out previously, the equation for the cumulative oil production is the same before and after water breakthrough. Of course, before fillup the produced oil volume is zero. Rewritting Eq 10 in slightly different form, we have:

$$(N_p B_o)_j = V_{p_j}(S_{oi} - S_{o2}) + V_{p_j}(S_{o2} - S_{o3}) E_{aw} \qquad (20)$$

Again applying the procedures shown on page 8 we obtain:

$$(q_o B_o)_j = \frac{(S_{o2} - S_{o3})}{\Delta S_w} \cdot i_{wj} \left(\frac{dE_{aw}}{dV_{di}}\right)_j \qquad (21)$$

Water Rate After Breakthrough. It is quite easy to see from Eq. 12 that the water rate equation after breakthrough is:

$$(q_w)_j = i_{wj}\left(1 - \frac{dE_{aw}}{dV_{di}}\right) \qquad (22)$$

In fact, it can be seen that before breakthrough this equation predicts that $q_w = 0$.

This concludes the development of the rate equations. It is apparent that the slope of the areal efficiency curve is an important parameter, as are certain key saturation values and the injection rate.

If many calculations are to be made (10 or more layers) it is probably best to fit simple equations to the efficiency curve and develop the slope by taking the derivative of the curve fit equation. When only a few calculations are to be made it is sufficient to construct tangents to the efficiency curve and determine the slope by that means. The tangent slopes are then plotted against V_{di} for easy reference.

Multilayer Behavior

The basic method of calculating the behavior of a multilayer system is to apply the relationships developed for a single layer and add the results. Because the permeability of the layers are different, the fronts advance at different rates in the layers. In essence, the calculation boils down to keeping track of the water that has entered each layer and computing the layer outputs.

The following illustrative calculations are based on four layers. It would be better to use more layers, say 10 or 20, but it is not done as it involves too much calculation. In the calculations all layers are of the same thickness, and have the same porosity. This is not necessary to the method as layers may have different thickness and porosity. They may also have different saturation values postulated in the different regions. But these are complicating factors and generally are not worth carrying out in a preliminary evaluation of a field prospect.

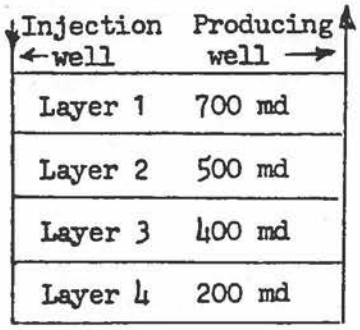


Figure 5

Figure 5 shows the permeability of the four layer system to be calculated. The calculation scheme is facilitated by placing the highest permeability at the top and working downward. This, of course, may bear no resemblance to the real situation in the field.

A tabular calculation format seems to work best. This will be illustrated on the following pages. The first calculation will be for the condition of fillup in the first layer. The second calculation will be for 3 months of injection and the third for one year of injection. In each calculation it is assumed that the water entering a layer is proportional to the permeability , k_j , of the layer.

1. Condition - Fillup in Layer 1

$N_p = 0$; $W_p = 0$; $F = 4.71$; $E_{ao} = 1.0$; $\Delta S_w = 0.38$; $S_{gi} = 0.12$
$S_{g2} = 0.05$; $S_{o1} = 0.58$; $S_{o2} = 0.65$; $S_{o3} = 0.32$; $V_{pj} = 77580$ bbl ;
$V_{dj} = 29480$ bbl. Pattern injection rate, $i_w = 450$ bbl/day

①	②	③	④	⑤	⑥	⑦
k_j	$k_j/\Sigma k_j$	E_{aoj}	E_{awj}	V_{dij}	$(G_p B_g)_j$	$(S_g B_g)_j$
700	0.389	[1.0]	0.212	0.212	6253	175
500	0.278	0.714	0.152	0.152	4467	125
400	0.222	0.571	0.121	0.121	3570	100
200	0.111	0.286	0.061	0.061	1790	50
Σ 1800	1.00			0.546	16080	450

Water injected, $W_i = V_{dj} \, \Sigma V_{dij} = 0.546 \cdot 29480$
$= 16096$ barrels

$$Time = \frac{W_i}{i_w} = \frac{16096}{450} = 36 \text{ days.}$$

Calculation notes:

④ $E_{awj} = E_{aoj}/F$

⑥ From Eq 5

⑦ From Eq 14

[ ] = starting value.

2. Condition - 3 Months of Injection

$W_i = 450 \cdot 91.25 = 41063$ barrels. ; $V_{pj} = 77580$ bbl ; $V_{dj} = 29480$ bbl
$\Delta S_w = 0.38$; $S_{o1} = 0.58$; $S_{o2} = 0.65$; $S_{o3} = 0.32$; $S_{gi} = 0.12$; $S_{g2} = 0.05$

①	②	③	④	⑤	⑥
$\frac{k_j}{\Sigma k_j}$	i_{wj}	V_{dij}	E_{awj}	E_{aoj}	$\left(\frac{dE_{aw}}{dV_{di}}\right)_j$
0.389	175	0.542	0.542	1	1
0.278	125	0.387	0.387	1	1
0.222	100	0.309	0.309	1	1
0.111	50	0.155	0.155	0.728	1
1.000	450	1.393			

Layer	⑦ $(G_pB_g)_j$	⑧ $(q_gB_g)_j'$	⑨ $(N_pB_o)_j$	⑩ $(q_oB_o)_j'$	⑪ $(W_p)_j$
1	7533	23	8445	152	0
2	6932	17	4477	109	
3	6629	13	2480	87	
4	4566*	50*	0	0	
	25660	103	15402	248	0

Calculation notes:

⑤ $E_{aoj} = F \cdot E_{awj}$ Note that layer 4 has not filled up.

⑥ E_{awbt} for $M_{wo} = 0.585$ Breakthrough has not occured in any layer.

⑦⑧ Eq 9 & 17 except for last layer. Last layer Eq 6, 14.

⑨⑩ Eqs 10 & 20

3. Condition - One Year of Injection

W_i = 450 • 365 = 164250 barrels. See previous calculations for other variables.

① i_{wj}	② V_{dij}	③ E_{awj}	④ $\left(\frac{dE_{aw}}{dV_{di}}\right)_j$	⑤ $(G_pB_g)_j$	⑥ $(q_gB_g)_j'$
175	2,168	0.952	0,06	9123	1.4
125	1,548	0,897	0,12	8910	2.0
100	1,236	0.851	0,20	8732	2.6
50	0.620	0,615	0.75	7816	4.9
450				34581	10.9

⑦ $(N_pB_o)_j$	⑧ $(q_oB_o)_j'$	⑨ $(W_p)_j$	⑩ q_{wj}
18742	9.1	35810	164.5
17734	13.0	19181	110.0
16356	17.4	11412	80.0
10314	32.6	119	12.5
63146	72.0	66522	367.0

Calculation Notes:

④ tangent method

⑤ ⑥ Eqs 9 & 17

⑦ ⑧ Eqs 10 & 21

⑨ ⑩ Eqs 12 & 22

The one year of injection results are quite interesting. Note that about 94% of the water entering the first layer is being produced. Oil production from the first layer amounts to only about 13% of the total. The initial quantity of oil in the pattern amounted to 161,366 reservoir barrels. At the end of this first year of injection a total of 63,146 reservoir barrels have been recovered. This is a a pattern recovery factor of 39%. The other thing to note is the current surface water-oil ratio. Assuming an oil formation volume factor of 1.1, the surface stock tank oil rate is 65.5 STB/day. The water producing rate is 367 barrels a day. This means that the expense involved in treating 450 barrels per day of injection water and lifting and disposing of 367 barrels a day of produced water must be met by income generated from the 65.5 stock tank oil barrels.

One can see from the form of the equations developed in this note that a relative simple computer program can be written to handle many layers and many time step conditions. The assumption of layer injection rates proportional to their permeability is not the best that could be made. A separate set of calculations could have been made to handle the change in layer conductivity as the two fronts progress through the layer (Deppe's method) but the general result would not have been much different.

Summary of Equations

1. Prior to Fillup

$$(G_p B_g)_j = V_{pj} E_{awj} \left[F S_{gi} - (F-1) S_{g2} \right] \quad (6)$$

$$(q_g B_g)_j = i_{wj} \quad (14)$$

2. After Fillup But Before Water Breakthrough

$$(G_p B_g)_j = V_{pj} \cdot \left[S_{gi} - (1 - E_{aw}) S_{g2} \right] \quad (9)$$

$$(q_g B_g)_j = \frac{S_{g2}}{(S_{w3} - S_{wi})} \, i_{wj} \left(\frac{d E_{aw}}{d V_{di}} \right)_j \quad (19)$$

$$(N_p B_o)_j = V_{pj} \cdot \left[S_{oi} - E_{aw} S_{o3} - (1 - E_{aw}) S_{o2} \right] \quad (10)$$

$$(q_o B_o)_j = \frac{(S_{o2} - S_{o3})}{(S_{w3} - S_{wi})} \cdot i_w \cdot \left(\frac{d E_{aw}}{d V_{di}} \right)_j \quad (21)$$

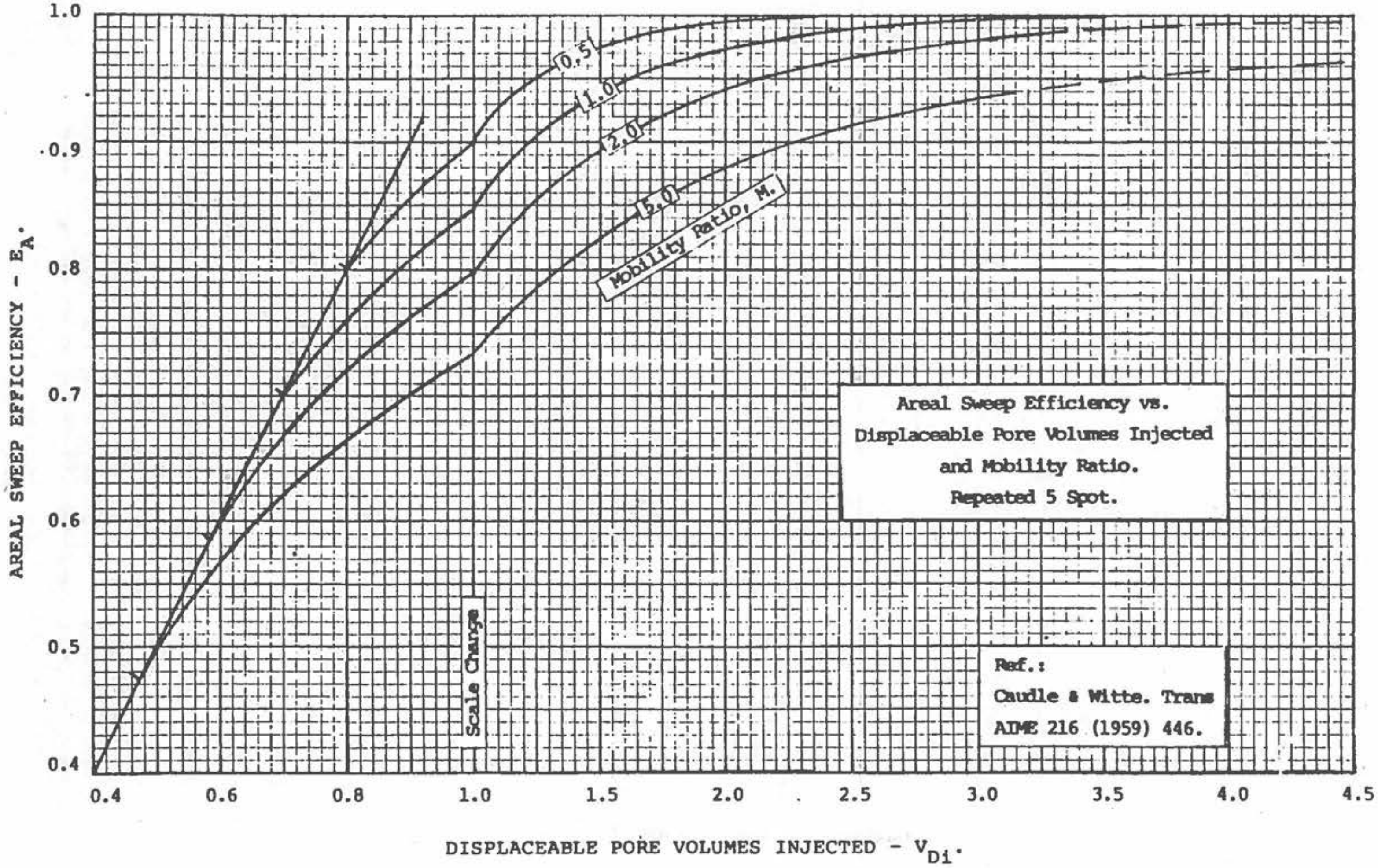
3. After Water Breakthrough

Oil and gas equations same as in 2, above.

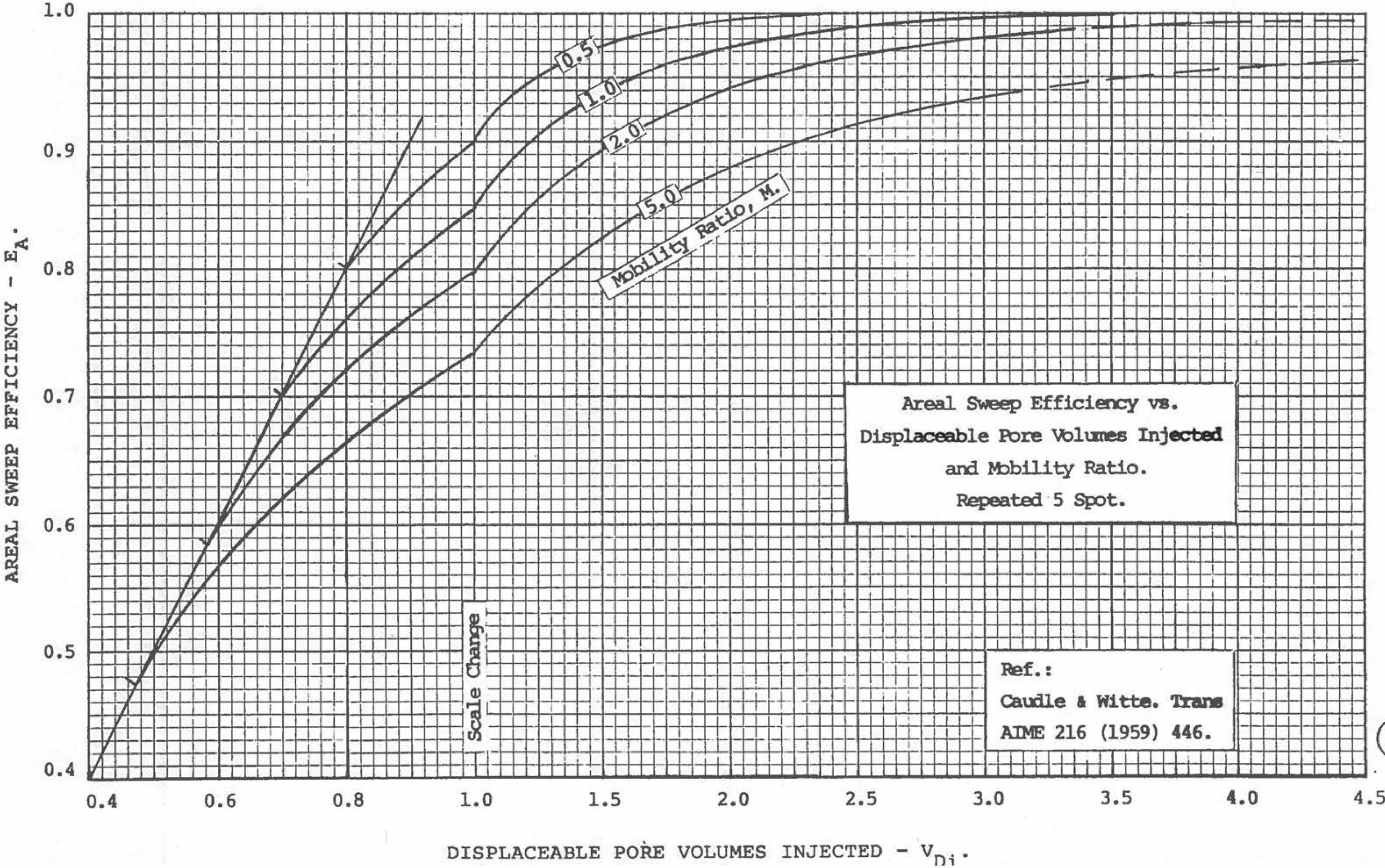
$$W_{pj} = W_{ij} - V_{pj} \cdot E_{aw} (S_{w3} - S_{wi}) \quad (12)$$

$$q_{wj} = i_{wj} \left(1 - \left(\frac{d E_{aw}}{d V_{di}} \right)_j \right) \quad (22)$$

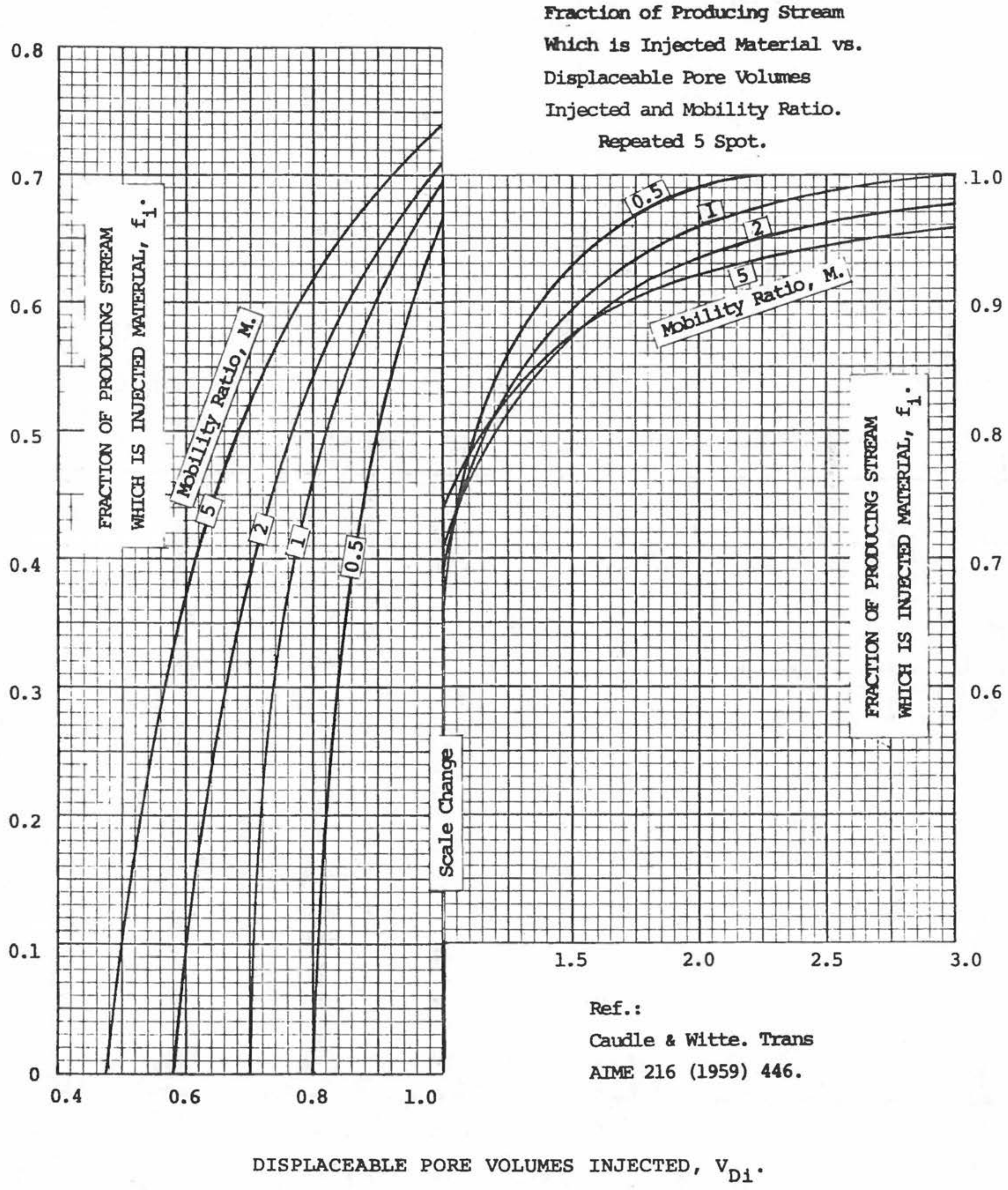
M.B. Standing
February 28,1981



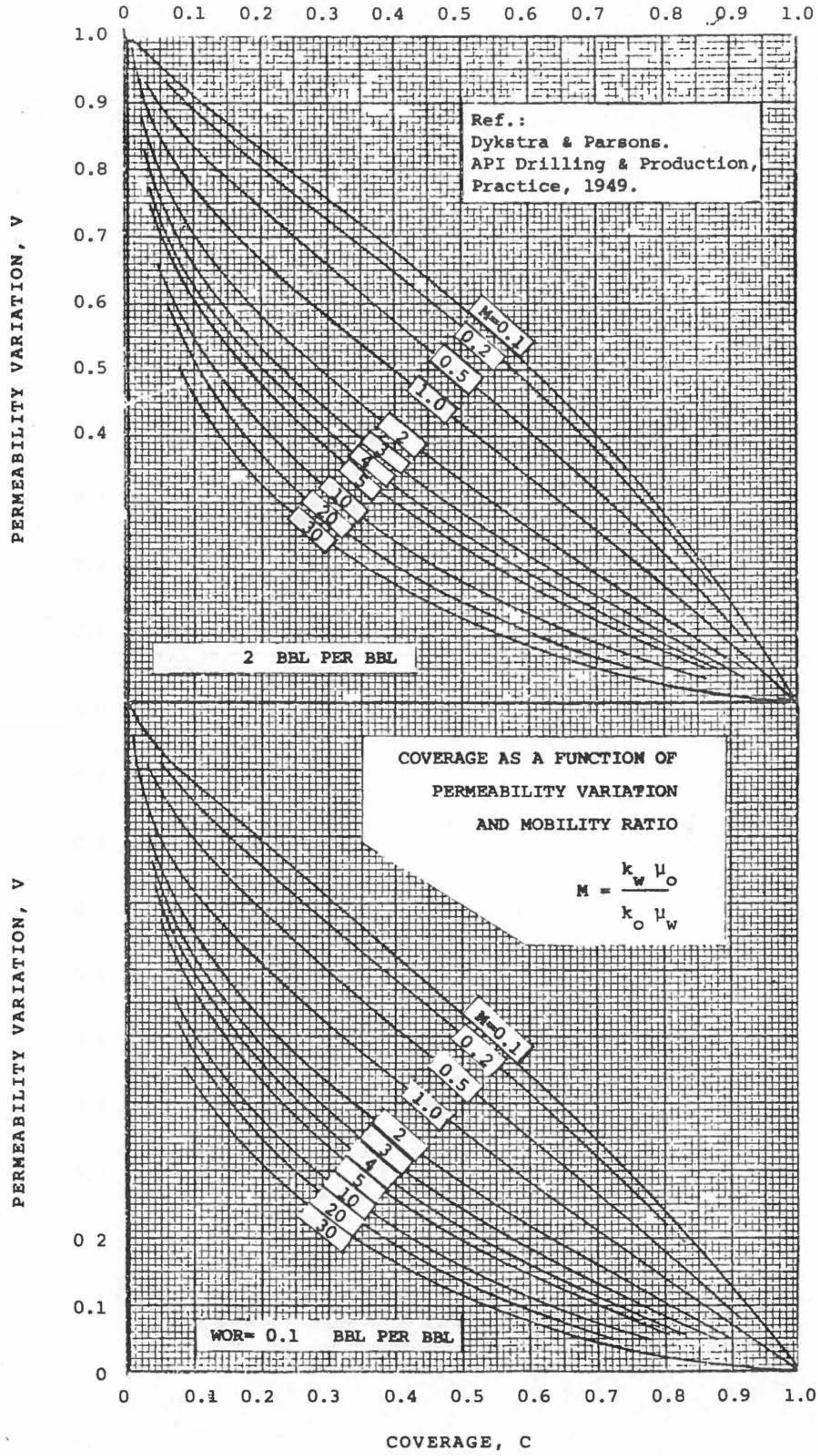
1.0
0.9
0.8
0.7
0.6
0.5
0.4
AREAL SWEEP EFFICIENCY - E_A.
0.4
0.6
0.8
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
DISPLACEABLE PORE VOLUMES INJECTED - V_{Di}.
0.5
1.0
2.0
5.0
Mobility Ratio, M.
Scale Change
Areal Sweep Efficiency vs.
Displaceable Pore Volumes Injected
and Mobility Ratio.
Repeated 5 Spot.
Ref.:
Caudle & Witte. Trans
AIME 216 (1959) 446.

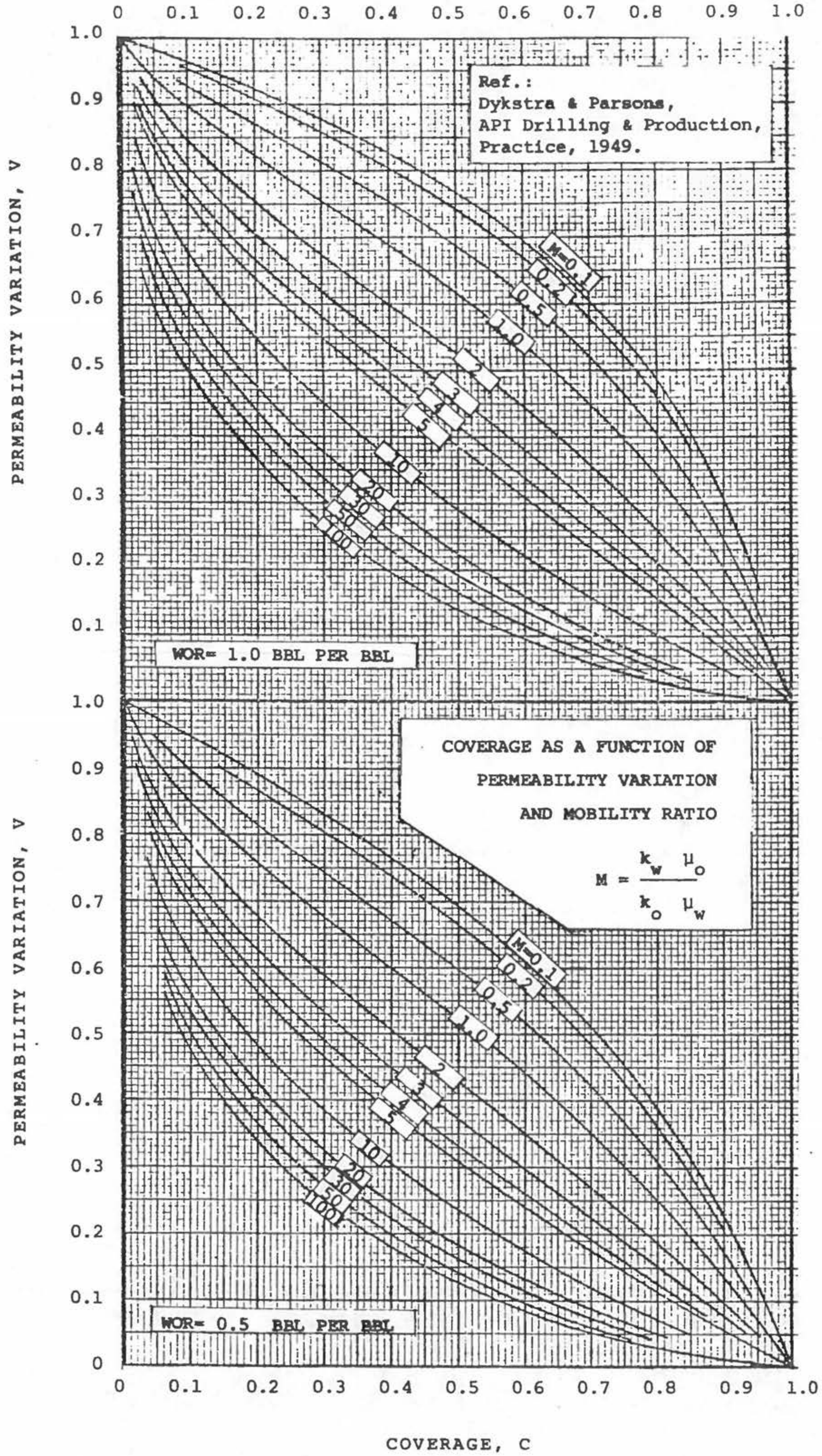

AREAL SWEEP EFFICIENCY - E_A.
DISPLACEABLE PORE VOLUMES INJECTED - V_{Di}.
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.4
0.6
0.8
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
0.5
1.0
2.0
5.0
Mobility Ratio, M.
Scale Change
Areal Sweep Efficiency vs.
Displaceable Pore Volumes Injected
and Mobility Ratio.
Repeated 5 Spot.
Ref.:
Caudle & Witte. Trans
AIME 216 (1959) 446.

74


Fraction of Producing Stream
Which is Injected Material vs.
Displaceable Pore Volumes
Injected and Mobility Ratio.
Repeated 5 Spot.
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
FRACTION OF PRODUCING STREAM
WHICH IS INJECTED MATERIAL, f_i.
Mobility Ratio, M.
5
2
1
0.5
Scale Change
0.5
1
2
5
Mobility Ratio, M.
FRACTION OF PRODUCING STREAM
WHICH IS INJECTED MATERIAL, f_i.
1.0
0.9
0.8
0.7
0.6
0.4
0.6
0.8
1.0
1.5
2.0
2.5
3.0
Ref.:
Caudle & Witte. Trans
AIME 216 (1959) 446.
DISPLACEABLE PORE VOLUMES INJECTED, V_Di.

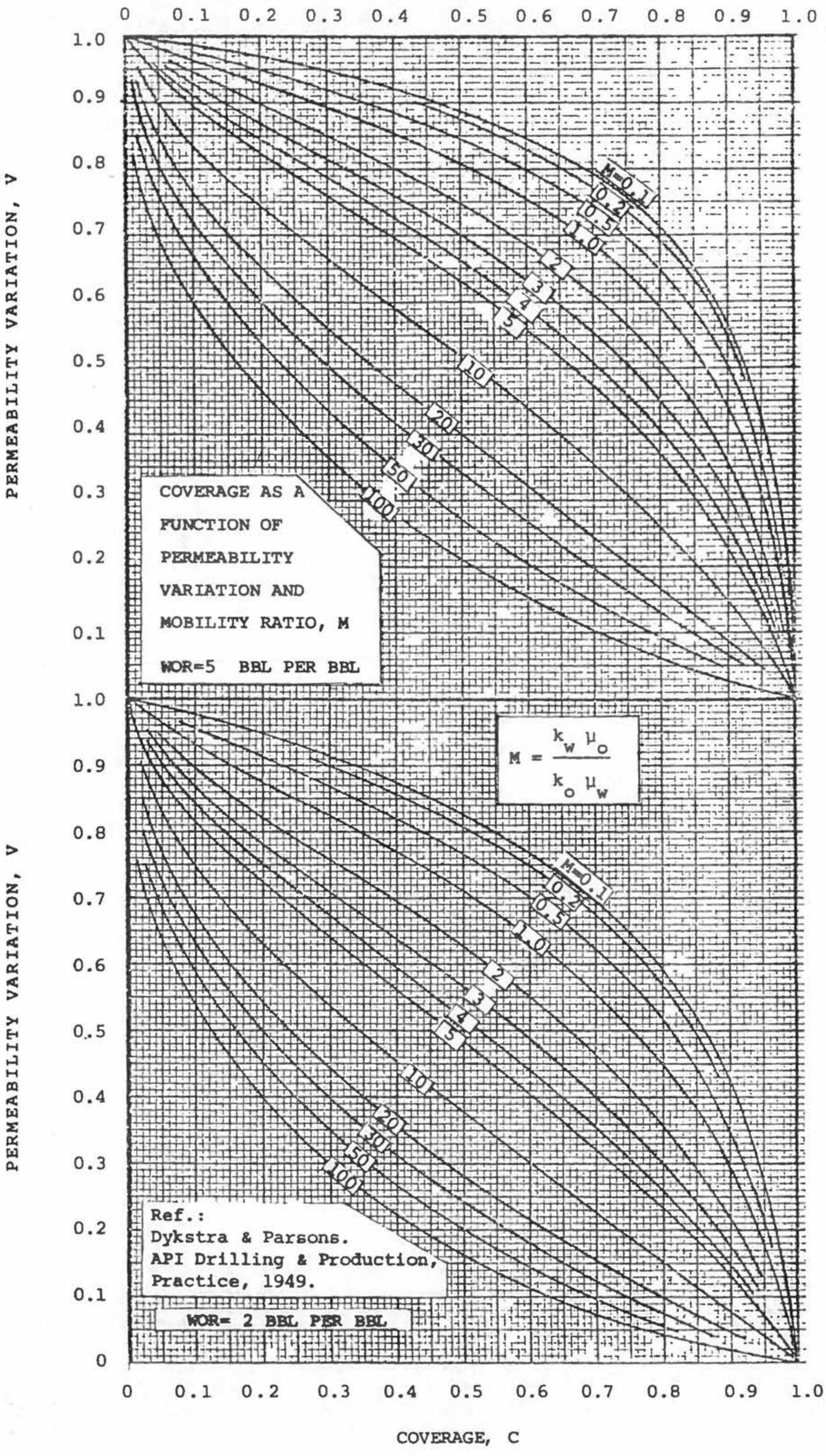
76


Ref.:
Dykstra & Parsons.
API Drilling & Production,
Practice, 1949.
2 BBL PER BBL
COVERAGE AS A FUNCTION OF
PERMEABILITY VARIATION
AND MOBILITY RATIO
$M = \frac{k_w \mu_o}{k_o \mu_w}$
M=0.1
0.2
0.5
1.0
2
3
4
5
10
20
30
WOR= 0.1 BBL PER BBL
PERMEABILITY VARIATION, V
COVERAGE, C
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

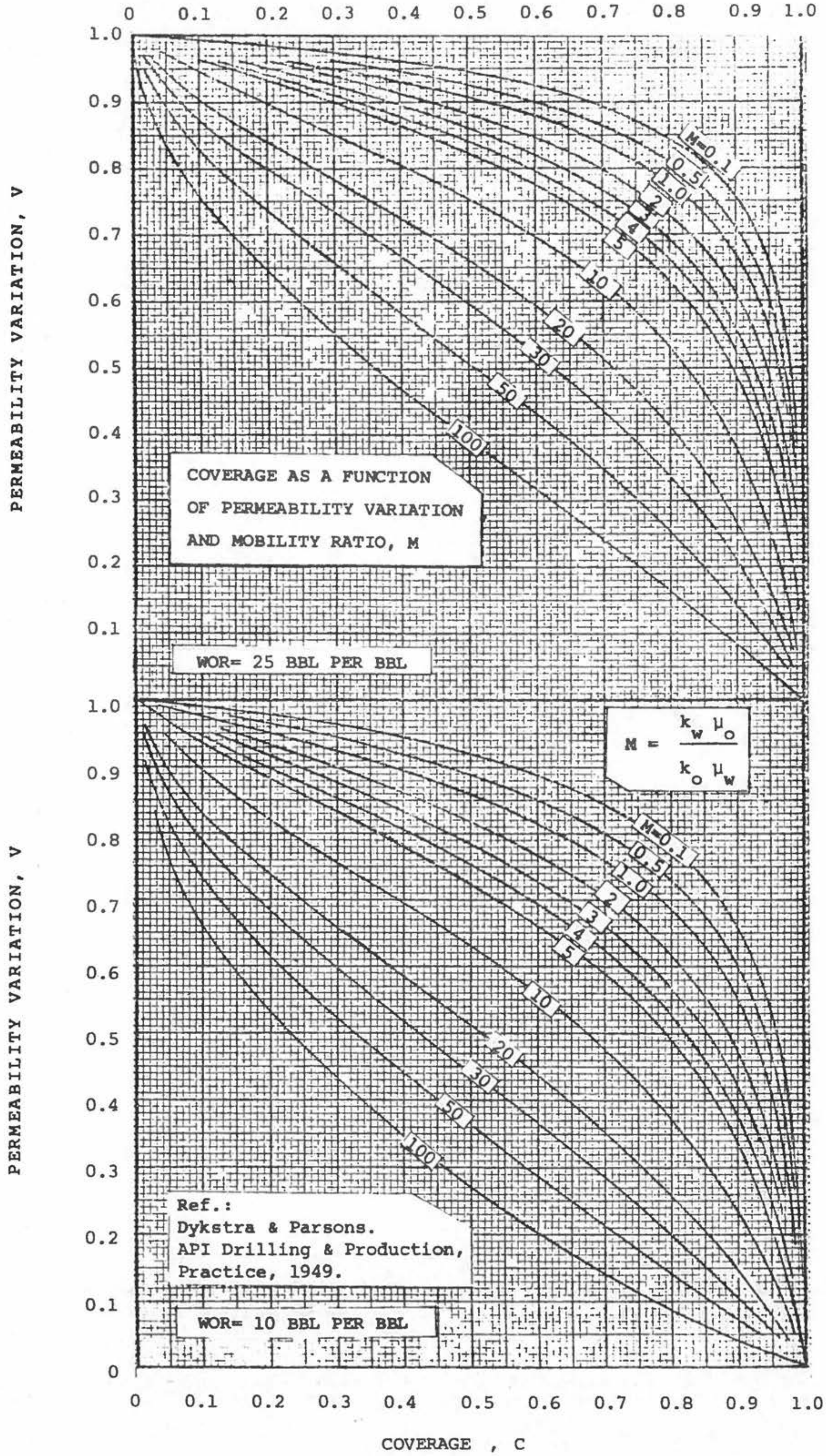


0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
PERMEABILITY VARIATION, V
Ref.:
Dykstra & Parsons,
API Drilling & Production,
Practice, 1949.
M=0.1
0.2
0.5
1.0
2
3
4
5
10
20
30
50
100
WOR= 1.0 BBL PER BBL
COVERAGE AS A FUNCTION OF
PERMEABILITY VARIATION
AND MOBILITY RATIO
$M = \frac{k_w \; \mu_o}{k_o \; \mu_w}$
WOR= 0.5 BBL PER BBL
COVERAGE, C

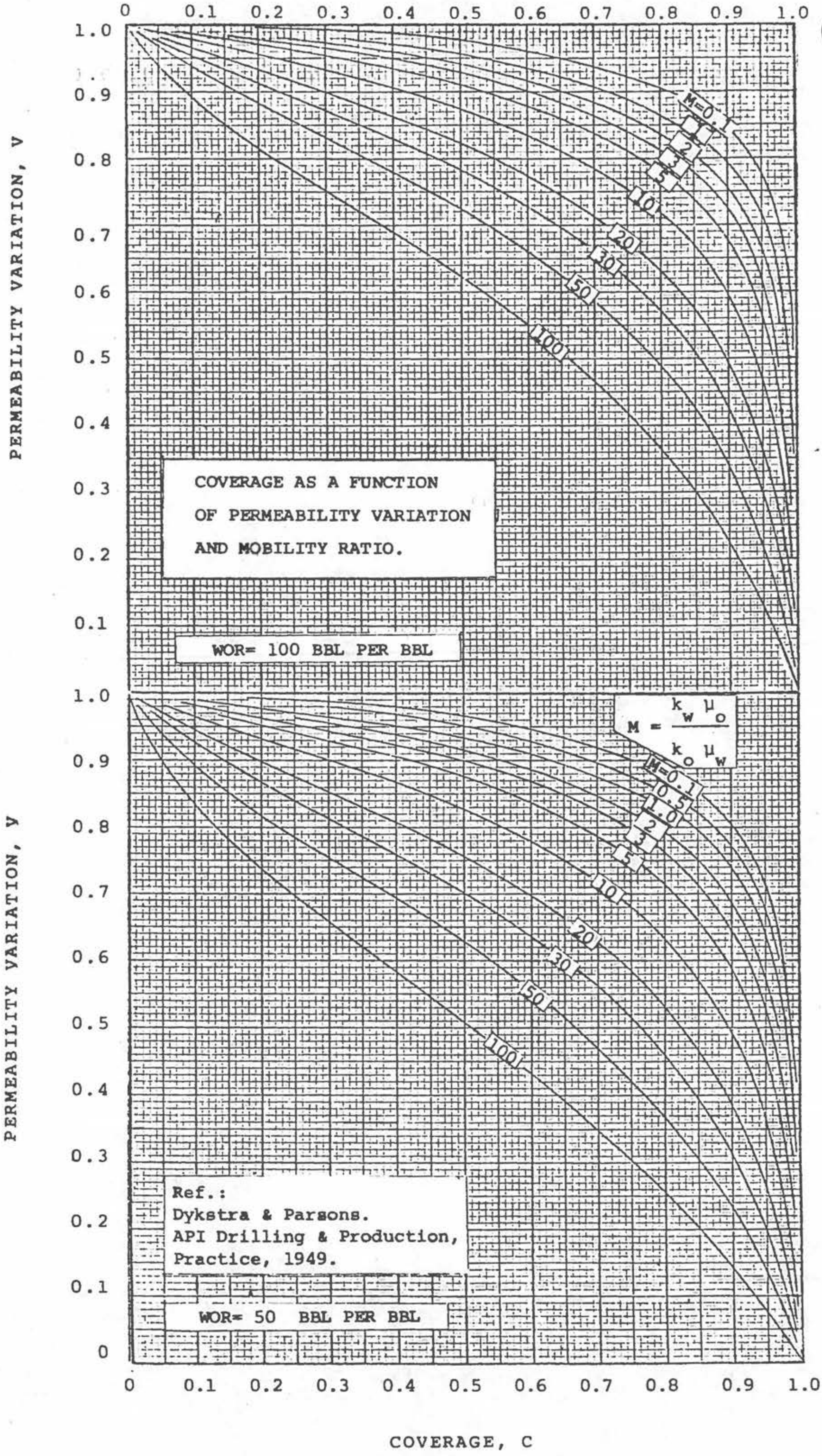
(78)


COVERAGE AS A FUNCTION OF PERMEABILITY VARIATION AND MOBILITY RATIO, M
WOR=5 BBL PER BBL
M=0.1
0.2
0.5
1.0
2
3
4
5
10
20
30
50
100
PERMEABILITY VARIATION, V
$M = \frac{k_w \mu_o}{k_o \mu_w}$
Ref.: Dykstra & Parsons. API Drilling & Production, Practice, 1949.
WOR= 2 BBL PER BBL
COVERAGE, C
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

79


PERMEABILITY VARIATION, V
COVERAGE AS A FUNCTION OF PERMEABILITY VARIATION AND MOBILITY RATIO, M
WOR= 25 BBL PER BBL
M=0.1
0.5
1.0
2
3
4
5
10
20
30
50
100
$M = \frac{k_w \mu_o}{k_o \mu_w}$
Ref.: Dykstra & Parsons. API Drilling & Production, Practice, 1949.
WOR= 10 BBL PER BBL
COVERAGE , C

80


COVERAGE AS A FUNCTION
OF PERMEABILITY VARIATION
AND MOBILITY RATIO.
WOR= 100 BBL PER BBL
$M = \frac{k_w \mu_o}{k_o \mu_w}$
M=0.1
0.5
1.0
2
3
5
10
20
30
50
100
Ref.:
Dykstra & Parsons.
API Drilling & Production,
Practice, 1949.
WOR= 50 BBL PER BBL
PERMEABILITY VARIATION, V
COVERAGE, C

Procedure for Buckley-Leverett Calculation:

The general procedure is to calculate values of f_w at various values of S_w, (or f_g as function of S_g) and to plot f_w vs S_w. Values from the $f_w - S_w$ curve and slopes of tangent lines drawn to the curve are used to calculate recovery and injection volumes.

1. Calculate viscosity ratio, μ_w/μ_o or μ_g/μ_o.
2. Calculate "a". $a = \dfrac{7.84(10^{-6}) k (\rho_w - \rho_o)\sin\alpha}{q_t \mu_o}$.

If gravity effects are not important, $a = 0$

3. Calculate f_w from frontal advance equation

$$f_w = \frac{1 - a k_{ro}}{1 + \dfrac{\mu_w/\mu_o}{k_{rw}/k_{ro}}}$$

Set up a calculation sheet like this:

①	②	③	④	⑤	⑥	⑦
S_w	k_{ro}	k_{rw}/k_{ro}	$1 - a k_{ro}$	$1 + \dfrac{\mu_w/\mu_o}{k_{rw}/k_{ro}}$	f_w	$\dfrac{f_w}{S_w - S_{iw}}$

4. Plot on fairly large scale values of f_w and $f_w/(S_w - S_{iw})$ against S_w, starting at S_{iw}. Draw in <u>smooth</u> curves.

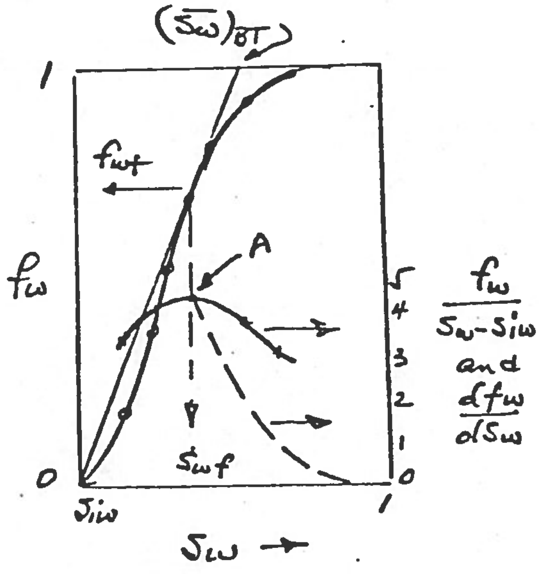


Figure 1.

5. Determine values for the saturation at the front, S_{wf}, the fraction of water flowing at the front, f_{wf}, and the average saturation back of the front, $(\bar{S}_w)_{BT}$ (also called average saturation at breakthrough.) This is done by:

a) Drawing a tangent to the $f_w - S_w$ curve that starts at S_{iw}. Reading values as indicated in Figure 1, or. b) determine maximum value of $\dfrac{f_w}{S_w - S_{iw}}$ curve (Point A in Fig 1) and draw a vertical line through it to obtain S_{wf} and f_{wf}. Calculate $(\bar{S}_w)_{BT}$ from equation

$$(\bar{S}_w)_{BT} = S_{wf} + \frac{1-f_{wf}}{\left(\frac{f_w}{S_w - S_{iw}}\right)_{max}}$$

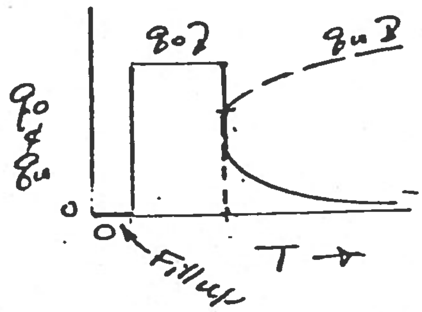
6. To determine parameters for calculations after breakthrough, set up a second calculation sheet with these headings.

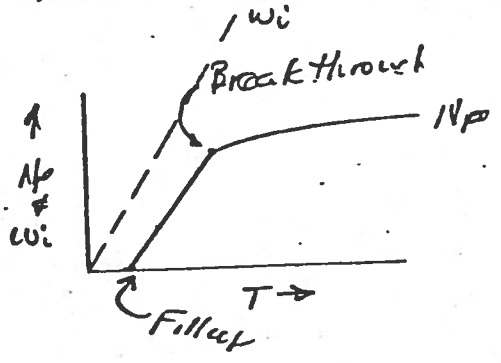
⑧ $S_w]_c$	⑨ $f_w]_c$	⑩ $\left(\frac{df_w}{dS_w}\right)_c$	⑪ $V_{ID}]_c$	⑫ $\bar{S}_w]_c^{\circ}$	---	⑮ W_F	⑯ W_i	⑰ T

Other columns for q_o, q_w, N_p, W_p, F_{wo}, etc may be added depending on the problem.

7. Select about 5 values of $S_w]_c$ between breakthrough, (S_{wf}), and where the f_w curve becomes one. Include S_{wf} as one of the values. Read values of $f_w]_c$ and $(df_w/dS_w)_c$ from smoothed curves on Figure 1. Calculate values of remaining variables from equations given in previous notes.

8. Plot variables as time functions to yield performance. Examples are:





Note that q_o and q_w have two values at breakthrough, because

$$q_o = \frac{i_w}{B_o}(1 - f_{wi}) \ ; \ q_w = i_w \cdot f_w$$

and f_w changes from zero to f_{wf} at breakthrough.

MB Standing

CSD Petroleum Industry Course

Displacement Problem 1

(This problem pertains to the displacement of oil up-structure by an influxing aquifer. The problem is to be solved using the Buckley-Leverett method).

The sketches below illustrate a section of reservoir in which water is advancing up-structure as a result of pressure reduction in the oil band section. To simplify this problem two assumptions will be used : (1) the initial water in the oil zone amounts to 32% saturation and is constant with height. To say it differently, we will neglect any effects of the initial transition zone saturation. (2) water breakthrough into the 1st line well occurs when the front reaches the elevation of the well. In other words, we will neglect effects of "cusping" of water into the well.

1980

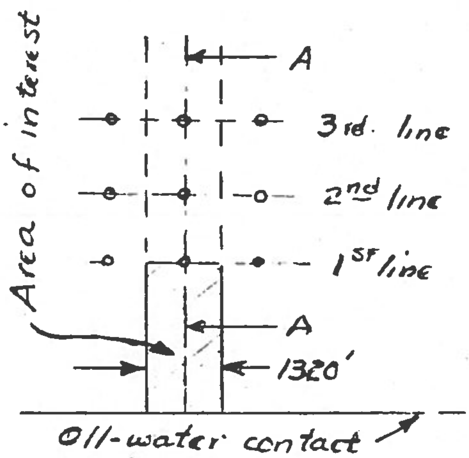


Figure 1
Plan View of "Window"

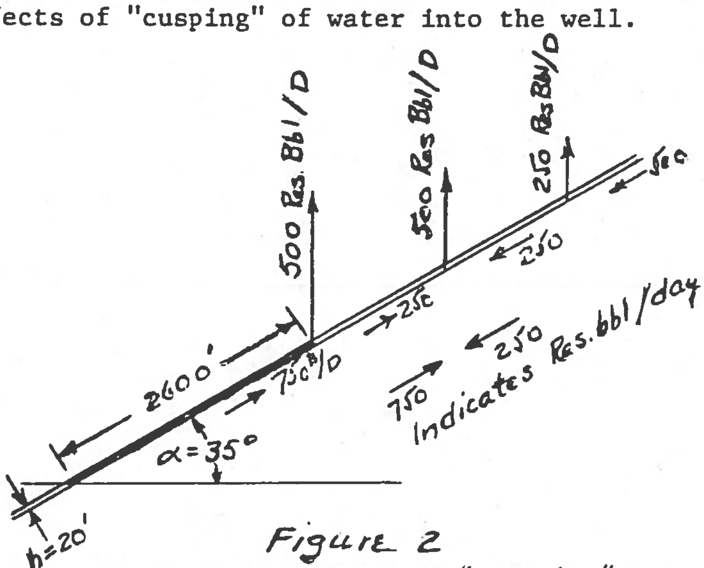


Figure 2
Section A-A in "Window"

"Window" Data

Window length = 2600'
Window width = 1320'
Formation thickness = 20'
Formation dip = +35°
Porosity = 0.235
Permeability (abs) = 100 md
Up-dip flow rate at oil-water contact = 750 Res.bbl/day

Fluid Saturations

S_{iw} = 0.30; $\overline{S}_{wi}$ = 0.32
$\overline{S}_{oi}$ = 0.68 S_{gi} = 0.00

Fluid Properties

B_o = 1.27 B_w = 1.02
μ_o = 1.16 cp μ_w = 0.38 cp
ρ_o = 45 lb/ft^3 ρ_w = 65 lb/ft^3

Relative Permeability Data

S_w	k_{ro}:imb	k_{rw}/k_{ro}] imb
0.30	0.725	0.0000 (S_{iw})
0.32	0.615	0.0195
0.35	0.470	0.072
0.40	0.315	0.280
0.45	0.210	0.790
0.50	0.133	2.000
0.55	0.077	4.750
0.60	0.036	11.85
0.65	0.012	33.50
0.67	0.007	55.53

Displacement Problem 1 Cont.

The oil that is displaced up-structure by the influxing aquifer water will presumably be captured by the oil wells. When the water-oil front reaches the elevation of any well there will be an instantaneous jump in water-oil ratio (this is because we are considering only one layer - the jump would be more gradual if many layers were considered). We are, of course, interested in the amount of oil that can be recovered from the invaded volume (cross-hatched area in Fig 1) as we continue to produce the wells. The items to be calculated are:

(1) How many barrels of stocktank oil will be displaced from the invaded volume (and recovered) when the front first breaks through into the first line well ? What fraction of initial oil in this volume does this amount to ?

(2) What will be the surface producing water-oil ratio immediately after breakthrough ? Assuming that the well continues to produce at the same <u>total</u> fluid rate, what will the stocktank oil rate be ?

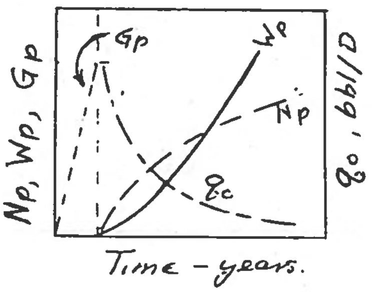
(3) When the first line well's cut reaches 95 %,what will be the amount of oil recovered from the invaded volume ? How long (years) will it take to reach this cut ? (Consider that the the aquifer influx rate into the window remains constant at 750 reservoir barrels per day.)

CSD Petroleum Industry Course

Displacement Problem 2

(The purpose of this problem is to illustrate the application of the Dykstra-Parsons method to evaluating the water flood potential of a depleted reservoir).

Calculate the expected production (cumulative) of oil, water and gas as a function of time using the Dykstra-Parsons coverage charts. Also calculate the oil producing rate, bbl/day , as a function of time. Prepare a plot of the produced quantities and the oil production rate as illustrated by the sketch at the left. Base your calculations on a reservoir pore volume of one million barrels and a constant injection rate of 100,000 barrels a year. Assume that all gas will be displaced (fill-up) from the unit before any oil production is achieved. Carry out your calculations to a producing water-oil ratio, F_{wo} , of 25.



Data for the calculation are these:

Permeability of 14 core samples are as follows:

Core #	Air Perm. md.
1	31
2	10
3	82
4	170
5	105
6	47
7	19
8	70
9	20
10	22
11	38
12	63
13	42
14	135

Initial gas saturation, S_{gi}	0.15
Initial oil saturation, S_{oi}	0.55
Initial water saturation, S_{wi}	0.30
Residual gas saturation, S_{gr}	0.05
Residual oil Saturation, S_{or}	0.28
Oil viscosity, μ_o	2.0 cp
Water viscosity, μ_w	0.5 cp
Oil form. vol. fact., B_o	1.21
Water form. vol. fact., B_w	1.01
Relative water permeability, k_{rw} at $(S_{or} + S_{gr})$	0.225
Relative oil permeability, k_{ro} at $(S_{gi}$ & $S_{wi})$	0.450

Bbl pore volume, $V_p = 7758\ Ah\phi = 1(10^6)$

Water injection rate, $i_w = 1(10^5)$ bbl/Yr

CSD Petroleum Industry Course

Displacement Problem 3

(This problem illustrates the calculation of fluid production volumes and rates from a four-layer 5-spot pattern.)

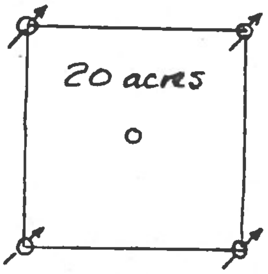


Figure 1

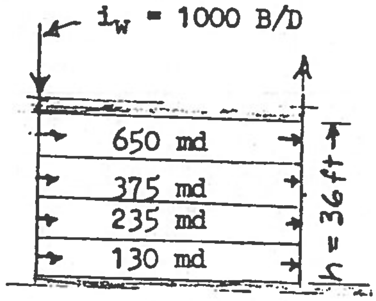


Figure 2

Saturations	Regions 1	2	3
Water, S_w	0.28	0.28	0.85
Oil, S_o	0.62	0.68	0.15
Gas, S_g	0.10	0.04	0.00
	1.00	1.00	1.00

k_{rw}(at S_{or}, Region 3) = 0.655

k_{ro}(at S_{wi} + S_{gr}, Region 2) = 0.498

A = 20 acres $\quad$ u_o = 2.66 cp
h = 36 feet $\quad$ u_w = 0.70 cp
$\emptyset$ = 0.23 $\quad$ B_o = 1.085

Pattern injection rate, i_w = 1000 B/D

Assume layer intakes proportional to $(kh)_j$

Use the above data to calculate the following parameters:

(1) The injection time (days) required to obtain fillup in second (375 md) layer.

(2) The injection time (days) at water breakthrough in the first (650 md) layer, and the cumulative production(s) at this time

(3) The following quantities at 5 years of injection.
  a. Cumulative water injected, W_i
  b. Cumulative stock tank oil produced, N_p
  c. The oil production rate, q_o
  d. The water production rate, q_w

Guide Answers

(1) 77 days
(2) 186 days; N_p = 91212 STB
(3b) N_p = 468971 STB

CSD Petroleum Industry Course

Solution Displacement Problem 1

A. Calculation of fractional flow curve

$$f_w = \frac{1 - a\,k_{ro}}{1 + \frac{k_{ro}}{k_{rw}} \cdot \frac{\mu_w}{\mu_o}} \qquad a = \frac{7.84(10^{-6})\,k_{abs}\,(\rho_w - \rho_o)\sin\alpha}{q/A\ \mu_o}$$

$\alpha = 35°$

$$a = \frac{7.84(10^{-6}) \cdot 100\ (65-45)\ 0.5736}{750/_{20 \cdot 1320} \cdot 1.16} = 0.2729$$

$S_{iw} = 0.30$
$S_{wi} = 0.32$

$$\mu_w/\mu_o = \frac{0.38}{1.16} = 0.3276$$

$$\therefore\ f_w = \frac{1 - 0.2729\ k_{ro}}{1 + \frac{0.3276}{k_{rw}/k_{ro}}}$$

S_w	k_{ro}	k_{rw}/k_{ro}	f_w	$\frac{f_w - f_{wi}}{S_w - S_{wi}}$	$\frac{\Delta f_w}{\Delta S_w}$
0.30 ← S_{iw}	0.725	0.0000	0	—	
0.32 ← S_{wi}	0.615	0.0195	0.0467 ← f_{wi}	—	
0.35	0.470	0.072	0.1571	3.680	
0.40	0.315	0.280	0.4212	4.681	
0.45	0.210	0.790	0.6664	4.767	4.904
0.50	0.133	2.000	0.8281	4.341	3.234
0.55	0.077	4.750	0.9158	3.779	1.754
0.60	0.036	11.85	0.9635	3.274	0.954
0.65	0.012	33.50	0.9871	2.850	0.472
0.67	0.007	55.53	0.9922	2.701	0.255

From plot of $\frac{f_w - f_{wi}}{S_w - S_{wi}}$ vs S_w the maximum occurs at $S_w = 0.430$. Maximum value is 4.80, which is slope of tangent line.

1. At break through at the first line well, the average saturation behind the front is

$$\bar{S}_w\big]_{BT} = S_{wf} + \frac{1 - f_{wf}}{(f_w - f_{wi})/(S_w - S_{wi})_{max}} = 0.430 + \frac{(1 - 0.575)}{4.80} = 0.5185$$

STB recovered @ BT

$$N_p = \frac{V_p[\bar{S}_w)_{BT} - \bar{S}_{wi})}{B_o} = \frac{(2600 \cdot 1320 \cdot 20) \cdot 0.235}{1.27 \cdot 5.615}[0.5185 - 0.320]$$

$N_p = \underline{449.0\ (10^3)}$ STB. ans

$$\text{Rec. Fraction} = \frac{(0.5185 - 0.320)}{(1 - 0.320)} = \underline{\underline{0.292}}\ \text{ans}$$

2. What will be surface water-oil ratio at B.T.

$$\text{Subsurface } w/o = \frac{f_w}{f_o} = \frac{0.5185}{(1-0.5185)} = 1.077 = \text{WOR}$$

$$\text{Surface water-oil ratio} = 1.077 \cdot \frac{B_o}{B_w} = 1.077 \cdot \frac{1.27}{1.02} = \underline{\underline{1.34}}\ \text{ans}$$

$$q_o = \frac{q_t \cdot f_o}{B_o} = \frac{500\,(1-0.5185)}{1.27} = \underline{189.6\ \text{STB/D}}\ \text{ans}$$

<u>3</u> Cut @ first line well = 95%

$$\text{Cut} = 0.95 = \frac{q_w}{q_w + q_o} \quad \therefore F_{wo} = \frac{q_w}{q_o} = \frac{95}{5} = 19$$

$$\text{WOR (subsurface)} = F_{wo} \cdot \frac{B_w}{B_o} = 19 \cdot \frac{1.02}{1.27} = 15.26 = \frac{q_w B_w}{q_o B_o}$$

$$\therefore f_w = \frac{15.26}{15.26+1} = \frac{q_w B_w}{q_w B_w + q_o B_o} = 0.9385; \quad S_{wc} = 0.569$$

$$\frac{\partial f_w}{\partial S_w} = 0.95$$

$$\therefore \bar{S}_w\Big]_o^c = S_{wc} + \frac{(1-f_{wc})}{\partial f_w/dS_w} = 0.569 + \frac{(1-0.9385)}{0.95} = 0.6337$$

$$\therefore \text{Recovery @ 95\% cut} = \frac{(2600 \cdot 1320 \cdot 20)\,0.235}{5.615 \cdot 1.27}(0.6337 - 0.320)$$

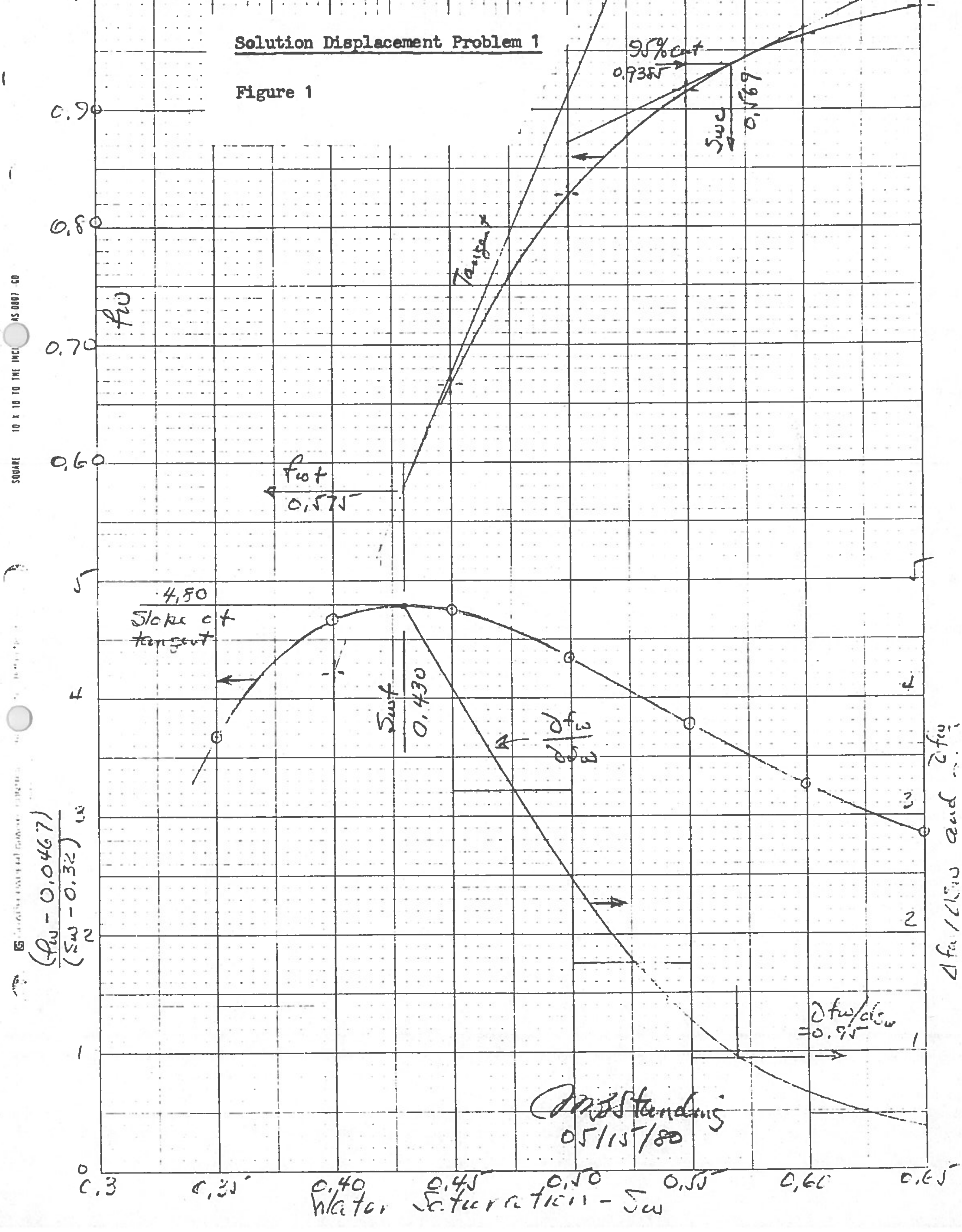
$$= \underline{\underline{709.588\ (10^3)\ \text{STB}}}\ \text{ans}$$

$$\text{Pore Vol inj} = V_{iD} = 1/\partial f_w/dS_w = \frac{1}{0.95} = 1.053$$

$\therefore$ Time to reach 95% cut

$$t = \frac{W_i}{q_i} = \frac{(2600 \cdot 1320 \cdot 20) \cdot 0.235 \cdot 1.053}{5.615 \cdot 750 \cdot 365} = \underline{\underline{11.05\ \text{years}}}\ \text{ans}$$

JMH
05/15/93


Solution Displacement Problem 1
Figure 1
Sw]BT = 0.5185
Sw]c = 0.6337
95% cut
0.9385
Swc
0.569
Tangent
fw
fwf
0.575
4.80
Slope at tangent
Swf
0.430
∂fw/∂Sw
(fw - 0.0467)/(Sw - 0.32)
∂fw/dSw = 0.95
dfw/dSw and ∂fw
1.0
0.90
0.80
0.70
0.60
5
4
3
2
1
0
0.3
0.35
0.40
0.45
0.50
0.55
0.60
0.65
Water Saturation - Sw
05/15/80

CSD Petroleum Industry Course

Solution Displacement Problem 2

A. Calculation of mobility ratio, M_{wo}

$$M_{wo} = \frac{\lambda_w}{\lambda_o} = \frac{k_{rw}/\mu_w}{k_{ro}/\mu_o} = \frac{0.225/0.5}{0.450/2.0} = 2.0$$

B. Calculation of permeability variation, V.

14 cores + 1 = 15

k	% ≥ k
170	6.7
135	13.3
105	20.0
82	26.7
70	33.3
63	40.0
47	46.7
42	53.3
38	60.0
31	66.7
22	73.3
20	80.0
19	86.7
10	93.3

From log k probability plot (Figure 1)
Dykstra-Parsons permeability variation = 0.61

C. Calculation of produced oil & F_{wo}

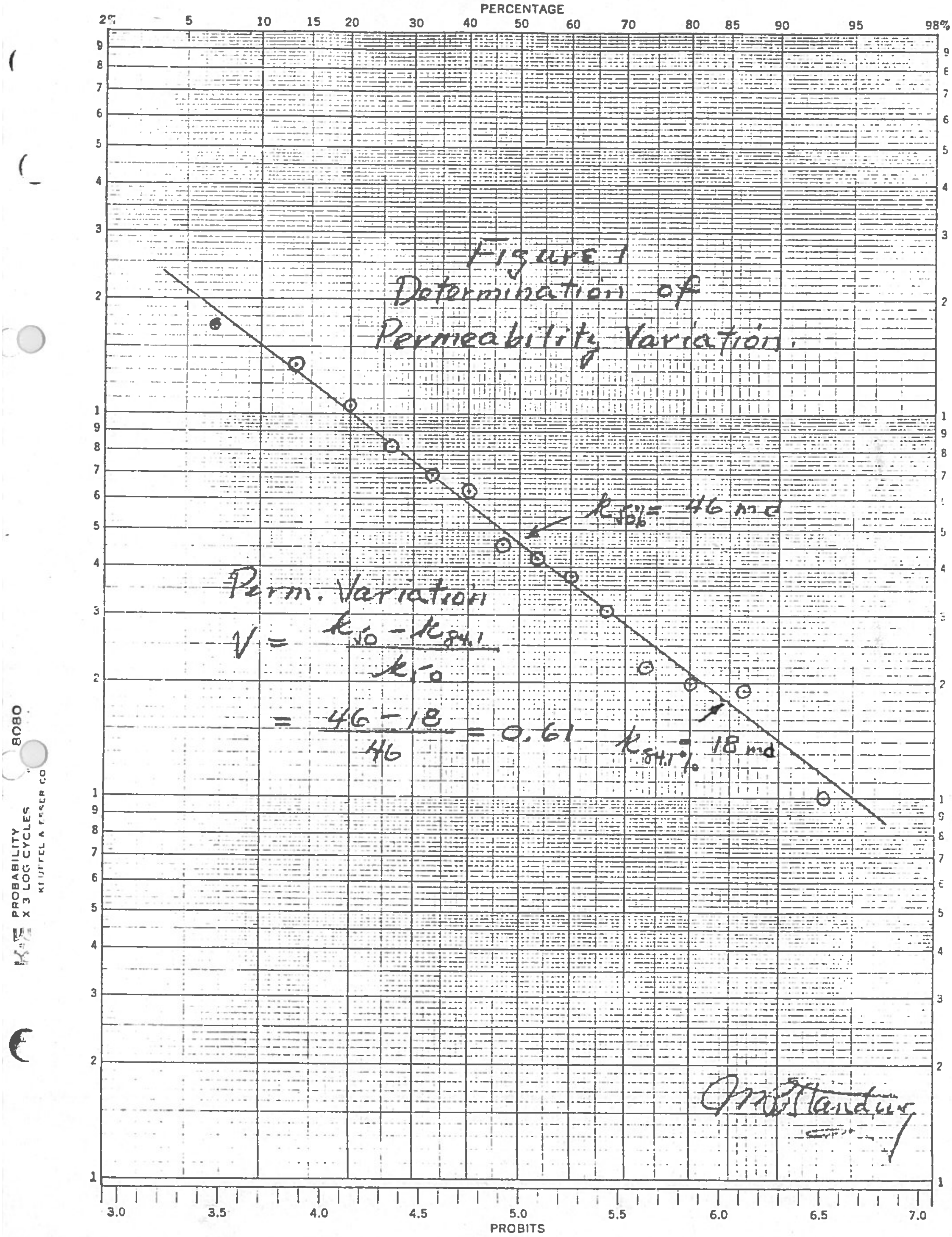
$$N_p = \frac{V_p \cdot \Delta S_o \cdot C}{B_o} = \frac{1(10^6)(S_{oi} - S_{or}) \cdot C}{B_o}$$

$$= \frac{1(10^6)(0.55 - 0.28)\,C}{1.21}$$

$$= 2.23\,(10^5)\,C.$$

$$F_{wo} = WOR \cdot \frac{B_o}{B_w} = 1.20\ WOR$$

Solution Displacement Problem 2



Solution Displacement Problem 2 Cont

WOR	F_{wo}	C	Np
0.1	0.120	0.130	0.290 (10^5)
0.2	0.240	0.175	0.390
0.5	0.600	0.273	0.609
1	1.20	0.385	0.859
2	2.40	0.520	1.160
5	6.00	0.700	1.561
10	12.0	0.805	1.795
25	30.0	0.900	2.007

D. Calculation of produced Water.

$$F_{wo} = \frac{q_w}{q_o} = \frac{dW_p/dt}{dN_p/dt} = \frac{dW_p}{dN_p}$$

$$\therefore W_p = \int_0^N F_{wo}\, dN_p.$$

Water production is calculated by determining the area under the F_{wo} vs N_p smooth curves shown on Figure 2. Increments of 0.2 (10^5) barrels were used and the areas determined by Simpson rule integration.

Increment of oil prod. 10^5 bbl	Increment Water prod. 10^5 bbl	Cum. Water Production 10^5 bbl
0 - 0.2	0.0047	0.0047
0.2 - 0.4	0.0276	0.0323
0.4 - 0.6	0.0813	0.1136
0.6 - 0.8	0.1513	0.2649
0.8 - 1.0	0.2657	0.5306
1.0 - 1.2	0.4067	0.9373
1.2 - 1.4	0.6633	1.6006
1.4 - 1.6	1.0533	2.6539
1.6 - 1.8	1.7833	4.4372
1.8 - 2.0	3.733	8.170

$(W_p)_\Delta = \int_{N-1}^{N_p} F_{wo}\, dN_p$ $\qquad$ $W_p = \int_0^{N_p} F_{wo}\, dN_p$

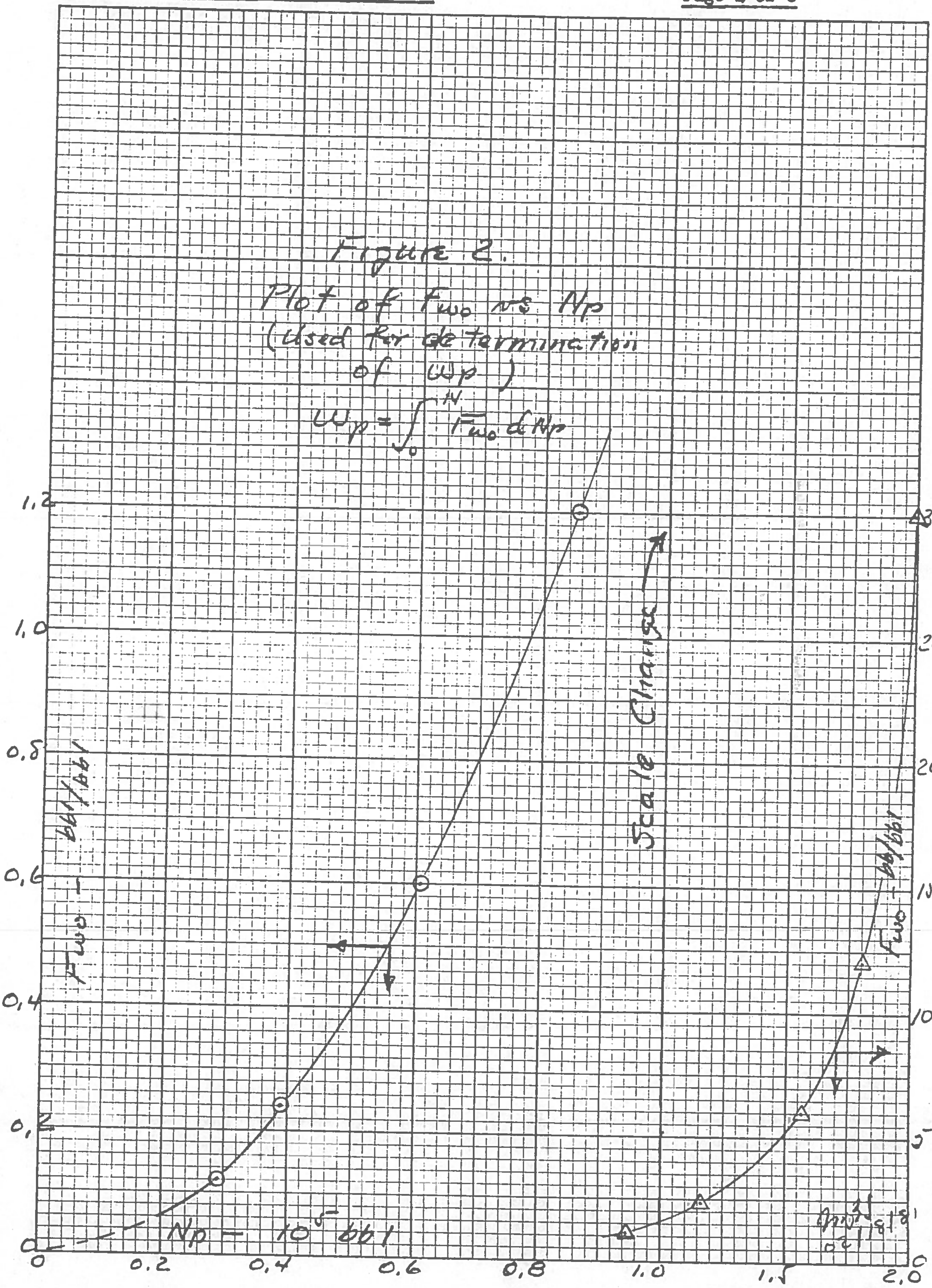


Figure 2.
Plot of Fwo vs Np
(Used for determination of Wp)
$W_p = \int_0^N F_{wo}\, dN_p$
Scale Change
Fwo - bbl/bbl
Fwo - bbl/bbl
Np - 10⁵ bbl
0
0.2
0.4
0.6
0.8
1.0
1.2
0
0.2
0.4
0.6
0.8
1.0
1.5
2.0
0
5
10
15
20
25
30

Solution Displacement Problem 2 Cont.

E. Calculation of system performance.

$i_w = 100\,000$ bbl/year. $= 274$ bbl/day.

Fill-up vol. $V_F = V_p \cdot \Delta S_g = 1(10^6)(S_{gi} - S_{gr})$

$= 1(10^6)(0.15 - 0.05) = 1(10^5)$ bbl

Fill-up time $= \dfrac{1(10^5)}{100000} = 1$ year.

$W_i = W_F + W_p + N_p B_o \qquad t = \dfrac{W_i}{100000}$

$q_o = \dfrac{i_w}{(B_o + F_{wo})}$

① N_p 10^5 bbl	② W_p 10^5 bbl	③ W_F 10^5 bbl	④ $N_p B_o$ 10^5 bbl	⑤ W_i 10^5 bbl	⑥ t years	⑦ F_{wo}	⑧ q_o bbl/D
—	—	—	—	0	0		
0.0	0.0	1	—	1	1	0.00	226
0.2	0.0047		0.242	1.2467	1.25	0.06	216
0.4	0.0323		0.484	1.5163	1.52	0.25	188
0.6	0.1136		0.726	1.8396	1.84	0.59	152
0.8	0.2649		0.968	2.233	2.23	1.06	121
1.0	0.5306		1.210	2.740	2.74	1.60	98
1.2	0.9373		1.452	3.389	3.39	2.60	72
1.4	1.6006		1.694	4.295	4.30	4.1	52
1.6	2.6537		1.936	5.590	5.59	6.7	35
1.8	4.4372		2.178	7.615	7.62	12.0	21
2.0	8.170	↓	2.42	11.59	11.6	30.0	9

Figure 3 shows performance curves for the system. Data in Columns 1, 2, & 8 are plotted against Column 6 data.

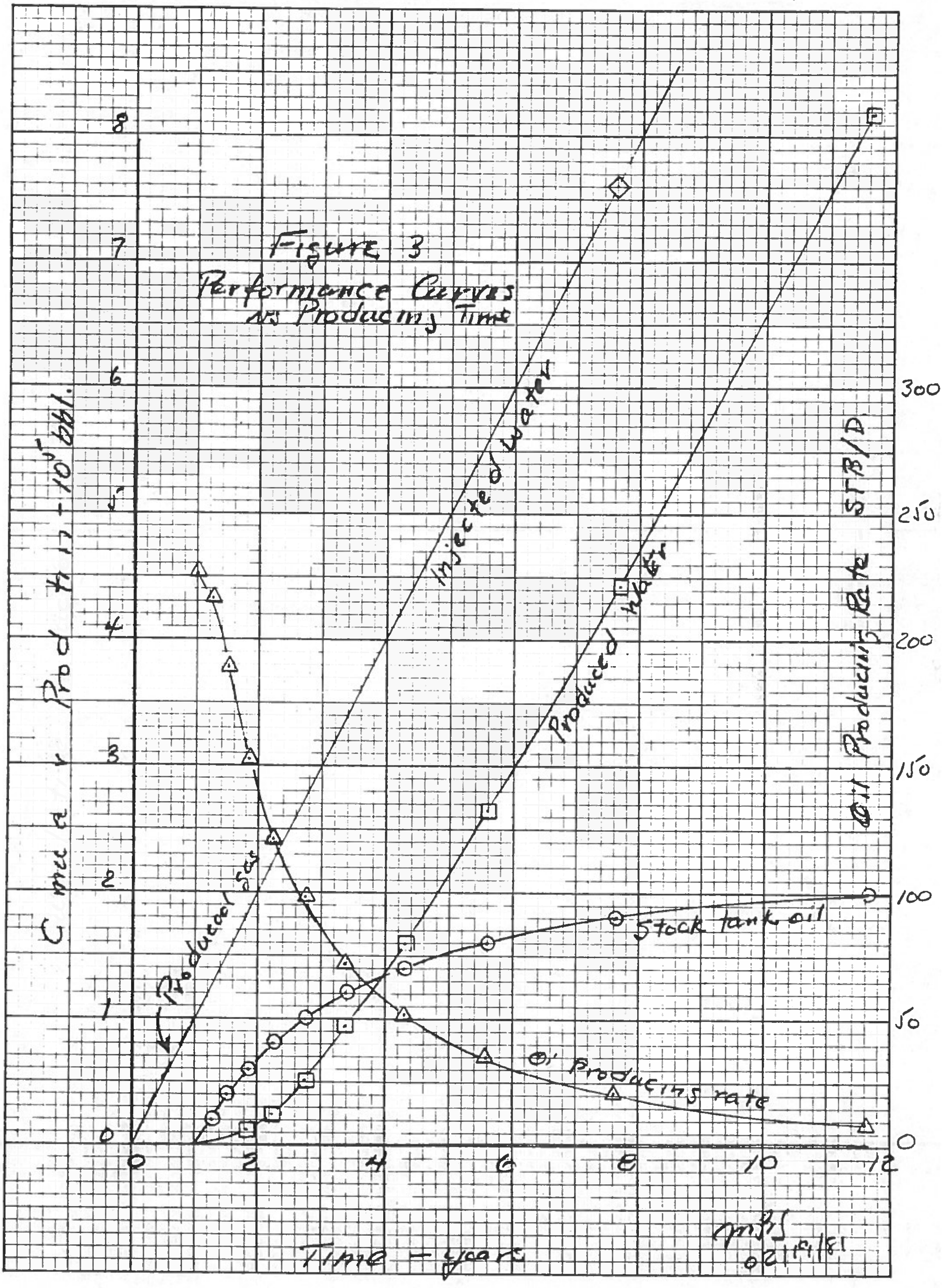


Figure 3
Performance Curves
vs Producing Time
Cumulative Prod $\times 10^{5}$ - bbl.
Oil Producing Rate STB/D
Time - years
Injected Water
Produced Water
Produced Gas
Stock tank oil
Oil Producing rate
0
1
2
3
4
5
6
7
8
0
2
4
6
8
10
12
0
50
100
150
200
250
300
02/19/81

CSD Petroleum Industry Course

Solution Displacement Problem 3

Mobility ratio, $M_{wo} = \frac{k_{rw3}}{k_{ro2}} \cdot \frac{\mu_o}{\mu_w} = \frac{0.655}{0.498} \cdot \frac{2.66}{0.70} = 5$

$F = 1 + \frac{\Delta S_{o3}}{\Delta S_{g2}} = 1 + \frac{(0.62-0.15)}{(0.10-0.04)} = 8.83$

$\Delta S_w = S_{w3} - S_{wi} = (0.85 - 0.28) = 0.57$

Pattern pore vol. $= 7758\, Ah\phi = 7758 \cdot 20 \cdot 36 \cdot 0.23$
$= 1,284725$ bbl

Pattern displaceable vol. $= V_p \cdot \Delta S_w = 732,293$ bbl

Layer pore vol, $V_{pj} = 321181$ bbl.

Layer displaceable pore vol, $V_d = 183073$ bbl.

1. Condition of fillup in 375 md layer.

k_j	$\frac{k_j}{\Sigma k_j}$	E_{aoj}	E_{awj}	V_{dij}
650	0.468	1.733	0.196	→
375	0.270	$\boxed{1.000}$	0.113	→
235	0.169	0.626	0.071	→
130	0.093	0.344	0.039	→
1390			0.419	0.419

Note: E_{awbt} for $M_{wo} = 5$ is 0.475

$W_i = 0.419 \cdot V_{dj} = 0.419 \cdot 183073 = 76708$ bbl

$\therefore$ Time to fillup $= \frac{76708}{1000} = 77$ days.
ans

2. Condition of water break through in 650 md layer. Time and cum. production.

$E_{awbt} = 0.475$

$\frac{k_j}{\Sigma k_j}$	E_{awj}	E_{aoj}	V_{dij}	$(G_p B_g)_j$	$(N_p B_o)_j$	W_{pj}
0.468	$\boxed{0.475}$	4.19	0.475	25373	61586	0
0.270	0.274	2.42	0.274	22791	27371	↓
0.169	0.172	1.52	0.172	21480	10008	
0.093	0.094	0.83	0.094	17203	0	
			1.015	86847	98965	0

fillup

ans!!

Solution Displacement Problem 3 Cont.

$W_i = 1.015 \cdot V_{di_j} = 1.015 \cdot 183073 = 185819$ bbl

∴ Time to break through = 186 days.

Cum Oil Prod = $\frac{98965}{1.085}$ = 91,212 bbl. } ans

Cum gas Prod = 86847 reservoir bbl.

3. Condition at 5 years injection

$W_i = 5 \cdot 365 \cdot 1000 = 1,825,000$ bbl. ← ans a.

$\frac{k_j}{\Sigma k_j}$	w_{ij}	i_w	V_{di_j}	E_{aw_j}	$\left(\frac{dE_{aw}}{dV_{di}}\right)_j$
0.468	554100	468	4.67	0.965	0.015
0.270	492750	270	2.69	0.922	0.032
0.169	308425	169	1.68	0.845	0.115
0.093	169725	93	0.93	0.710	0.350
1.000	1825000	1000	9.97		

$\frac{k}{\Sigma k_j}$	$(N_p B_o)_j$	$(q_o B_o)_j$	W_{p_j}	q_{w_j}
0.468	144997	6.5	661937	461
0.270	137677	8.0	309150	261
0.169	124570	18.0	140158	149
0.093	101590	30.0	28341	60
	508834	62.5	1139586	931

$\therefore N_p = \frac{508834}{1.085} = 468971$ Stock tank bbl. ← b.

$q_o = 62.5/1.085 = 57.6$ STB/D. ← c.

$W_p = 1,139,586$ bbl.

$q_w = 931$ bbl/day. ← d.

ans

Oil Recovery Factor $= \frac{508834}{1,284,725 \cdot 0.62} = 64\%$

Present water cut $= \frac{q_w}{q_w + q_o} = 93\%$

[signature] Standing,
04/04/81