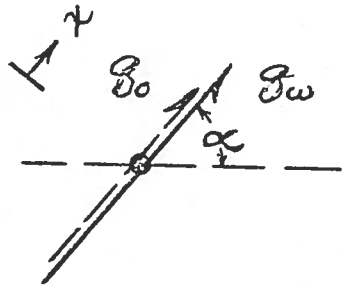


FRACTIONAL FLOW EQUATION, I



$$q_o = -\frac{k_o}{\mu_o} \left[\frac{\partial p_o}{\partial x} + \rho_o g \sin \alpha \right]$$

$$q_w = -\frac{k_w}{\mu_w} \left[\frac{\partial p_w}{\partial x} + \rho_w g \sin \alpha \right]$$

TOTAL FLOW, $q = q_o + q_w$

CAPILLARY PRESSURE, $p_c = p_o - p_w$

$$\frac{\partial p_c}{\partial x} = \frac{\partial p_o}{\partial x} - \frac{\partial p_w}{\partial x}$$

$$\frac{\partial p_c}{\partial x} = -\frac{(q - q_w)\mu_o}{k_o} - \rho_o g \sin \alpha + \frac{q_w \mu_w}{k_w} + \rho_w g \sin \alpha$$

LET $\Delta p = p_w - p_o$

$$q_w \left[\frac{\mu_o}{k_o} + \frac{\mu_w}{k_w} \right] = \frac{q \mu_o}{k_o} + \frac{\partial p_c}{\partial x} - \Delta p g \sin \alpha$$

FRACTIONAL FLOW EQUATION, II

DIVIDING BY $\frac{q \mu_o}{k_o}$;

$$\frac{q_w}{q} \left[1 + \frac{k_o}{\mu_o} \cdot \frac{\mu_w}{k_w} \right] = 1 + \frac{k_o}{q \mu_o} \left[\frac{\partial P_c}{\partial x} - \Delta \rho g \sin \alpha \right]$$

$$\frac{q_w}{q} = f_w = \frac{1 + \frac{k_o}{q \mu_o} \left[\frac{\partial P_c}{\partial x} - \Delta \rho g \sin \alpha \right]}{1 + \frac{k_o}{k_w} \cdot \frac{\mu_w}{\mu_o}}$$

AS $k_o = k \cdot k_{ro}$ AND IGNORING $\frac{\partial P_c}{\partial x}$ BECAUSE SMALL

$$f_w = \frac{1 - a k_{ro}}{1 + \frac{k_{ro} \cdot \mu_w}{k_w \mu_o}} \quad \text{WHERE}$$

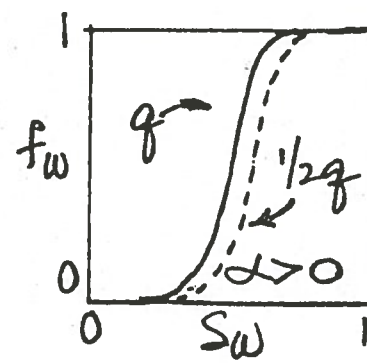
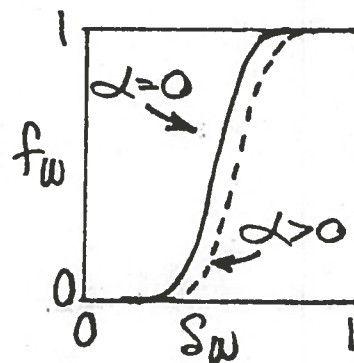
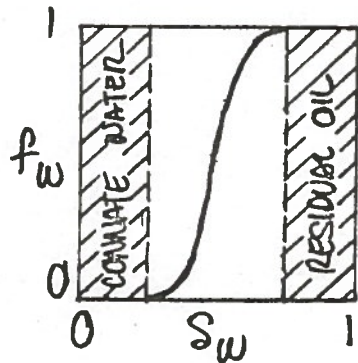
$$a = \frac{0.488 k \Delta \rho \sin \alpha}{q \mu_o} \quad \text{IF} \quad \begin{array}{l} \Delta \rho = \text{gm/cc} \\ k = \text{DARCY} \\ q = \text{BBL/DAY/FT}^2 \\ \mu_o = \text{cp} \end{array}$$

FRACTIONAL FLOW EQUATION, III

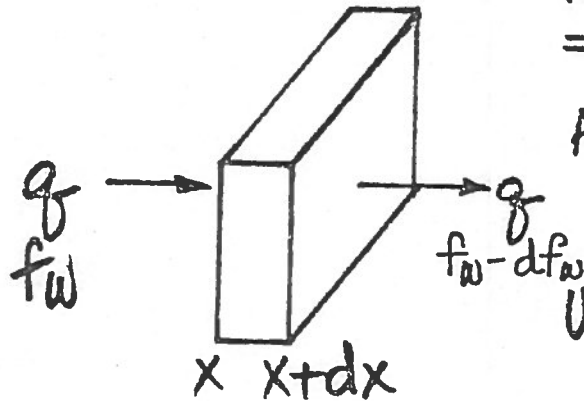
$$a = \frac{7.84(10^{-6})k \Delta \rho \sin \alpha}{q \mu_o} \quad \text{IF} \quad \begin{aligned} \Delta \rho &= \frac{\text{lbs}}{\text{ft}^3} \\ k &= \text{md.} \\ q &= \text{BBL/DAY/FT}^2 \\ \mu_o &= \text{cp.} \end{aligned}$$

WHEN $\alpha = 0$

$$f_w = \frac{1}{1 + \frac{k_{ro}}{k_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$



FRONTAL ADVANCE EQUATION, I



RATE OF WATER ACCUMULATION
= WATER IN MINUS WATER OUT

$$\begin{aligned} \text{ACCUM. RATE} &= q f_w - q (f_w - df_w) \\ &= q df_w \end{aligned}$$

$$\text{UNIT PORE VOL.} = \frac{A \cdot dx \cdot \phi}{5.615}$$

RATE OF WATER SATURATION CHANGE, $\frac{dS_w}{dt}$, IS RATE OF WATER ACCUMULATION DIVIDED BY PORE VOLUME.

$$\frac{dS_w}{dt} = \frac{q df_w}{A dx \phi} \cdot 5.615$$

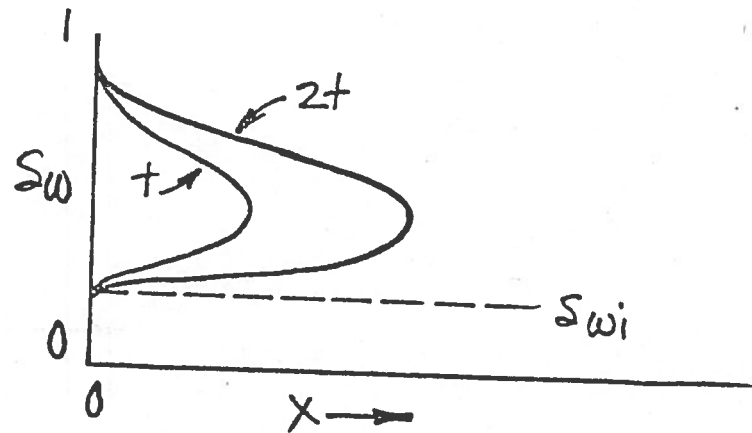
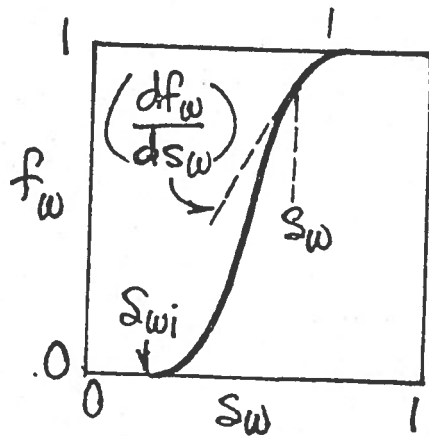
$$dx = \frac{5.615 q}{A \phi} \left(\frac{df_w}{dS_w} \right) dt$$

$$\int_{x_0}^{x_{sw}} dx = 5.615 \cdot \frac{q}{A \phi} \left(\frac{df_w}{dS_w} \right) \int_{t=0}^t dt$$

FRONTAL ADVANCE EQUATION, II

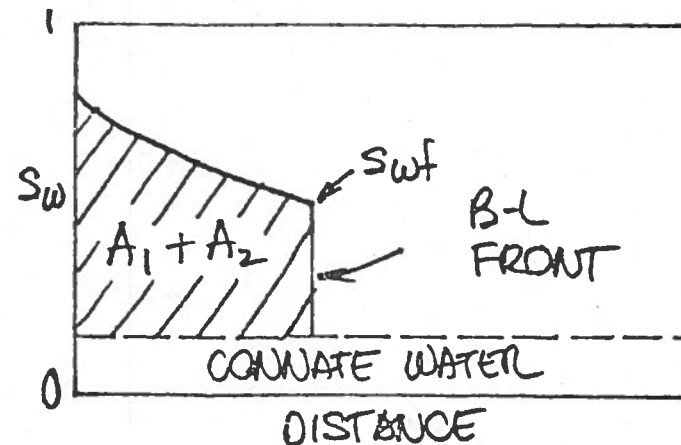
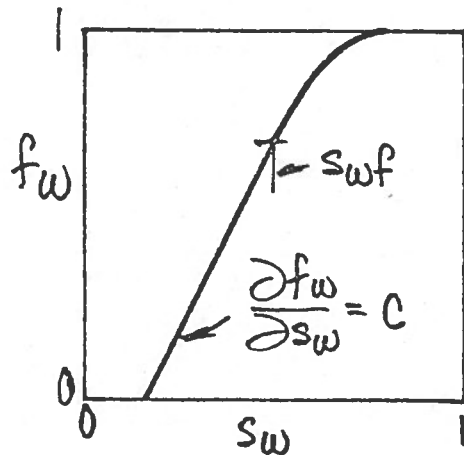
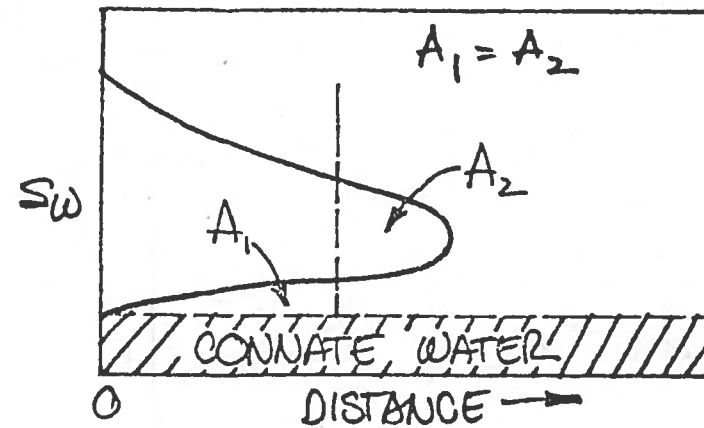
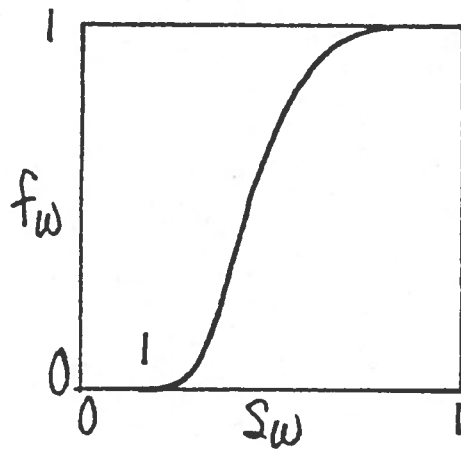
$$(x)_{s_w} - x_0 = \frac{5.615}{A \phi} q t \left(\frac{df_w}{ds_w} \right)$$

THIS IS THE BUCKLEY-LEVERETT FRONTAL ADVANCE EQUATION. IT GIVES THE DISTANCE, $[(x)_{s_w} - x_0]$, THAT A GIVEN SATURATION, s_w , MOVES IN TIME t .

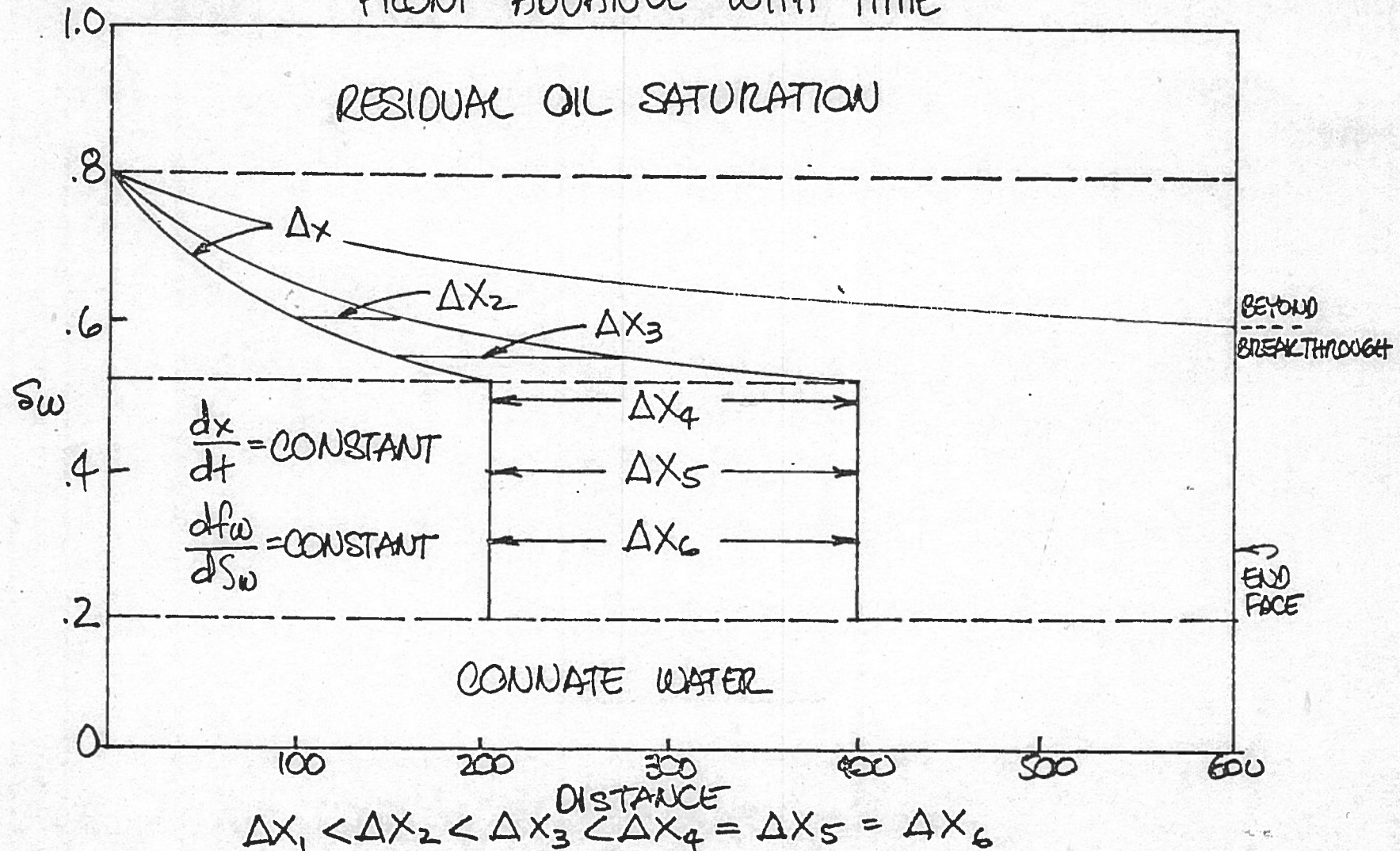


FRONTAL ADVANCE EQUATION, III

MODIFICATION OF B-L RELATIONSHIP TO AVOID TRIPLE SATURATION VALUES,

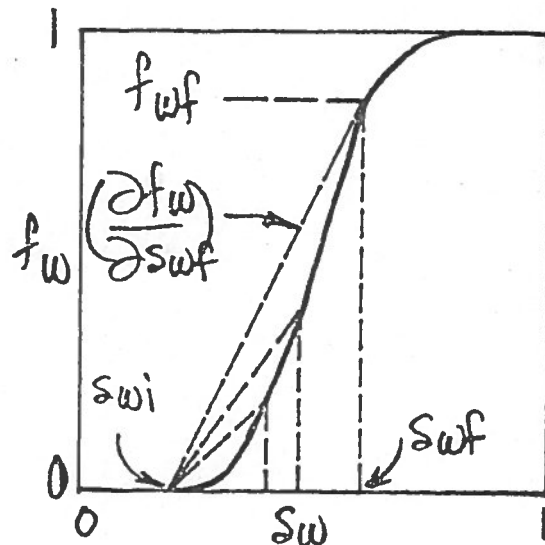


FRONTAL ADVANCE EQUATION, IV FRONT ADVANCE WITH TIME



FRONTAL ADVANCE EQUATION VI

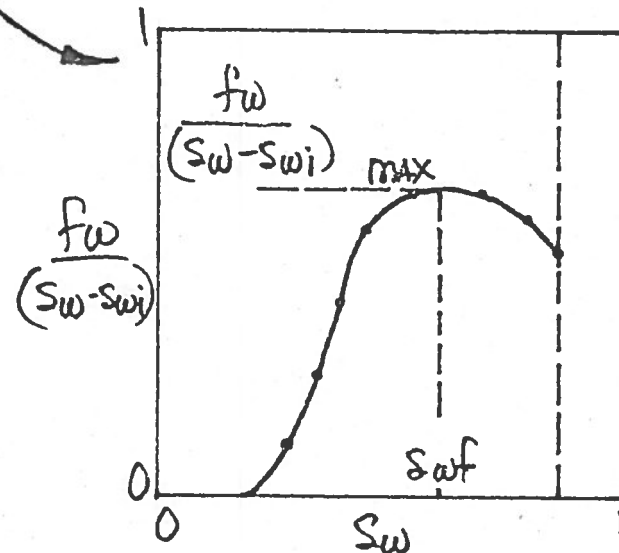
DETERMINATION OF SATURATION AND FRACTIONAL FLOW AT THE FRONT.



ALSO CAN BE
OBTAINED
GRAPHICALLY

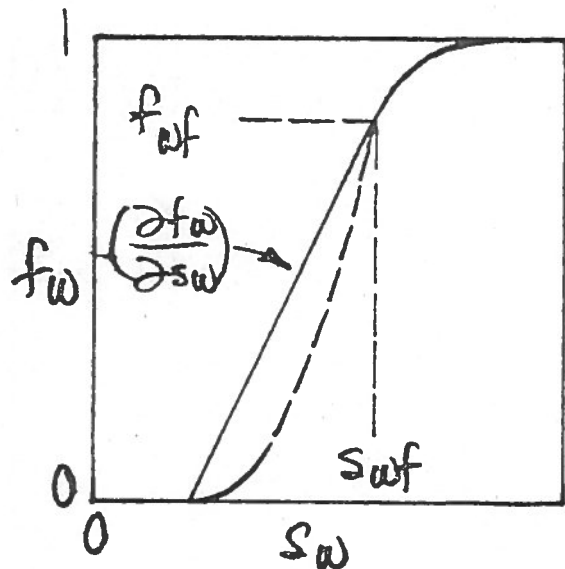
CALCULATE $\frac{f_w}{(S_w - S_{wi})}$ AND
PLOT AGAINST S_w .

FRONT CONDITIONS ARE
WHERE THIS HAS THE
MAXIMUM VALUE.



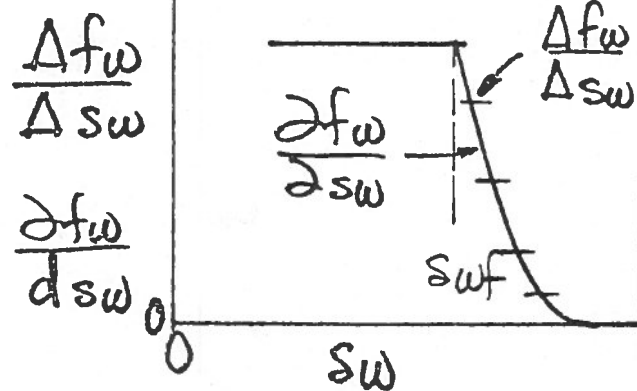
FRONTAL ADVANCE EQUATION VI

DETERMINATION OF $\frac{\partial f_w}{\partial s_w}$

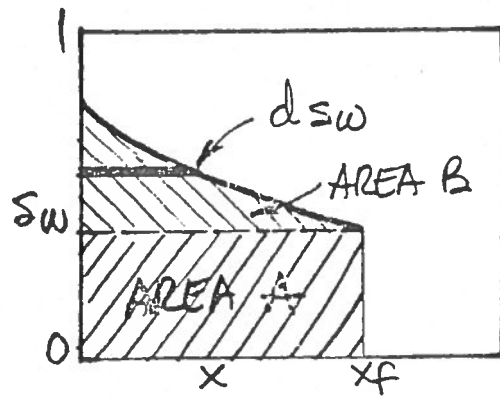
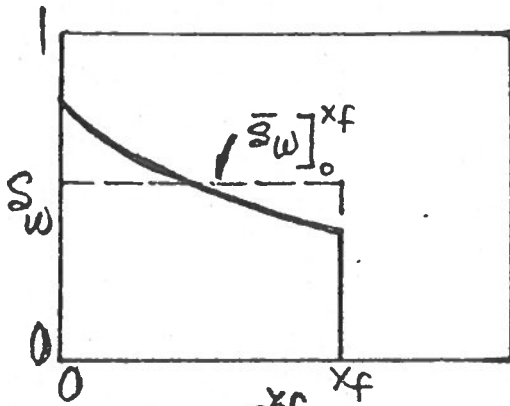


PERFORM GRAPHICAL
DIFFERENTIATION OF $f_w - s_w$
DATA AT $s_w > s_{wf}$ TO
OBTAIN VALUES OF $\frac{\partial f_w}{\partial s_w}$ VS s_w

AND



AVERAGE SATURATION BEHIND FRONT - I



$$[\bar{s}_w]_0^{x_f} = \frac{\int_0^{x_f} S_w dx}{x_f} = \frac{\text{AREA A} + \text{AREA B}}{x_f}$$

FROM BUCKLEY-LEVERETT

$$x_f = \frac{5.615}{A\phi} \left(\frac{\partial f_w}{\partial S_w} \right)_f \int_0^t q dt$$

$$= \frac{5.615 Q}{A\phi} \left(\frac{\partial f_w}{\partial S_w} \right)_f$$

$$\text{AREA A} = S_{wf} \cdot x_f$$

$$\text{AREA B} = \int_{S_{wf}}^1 x dS_w$$

$$\text{AS } x = \frac{5.615 Q}{A\phi} \left(\frac{\partial f_w}{\partial S_w} \right)_x$$

$$\text{AREA B} = \frac{5.615 Q}{A\phi} \int_{S_{wf}}^1 \frac{df_w}{dS_w} \cdot dS_w$$

AVERAGE SATURATION BEHIND FRONT, II

$$\begin{aligned} \text{AREA B} &= \frac{5.615 Q}{A \phi} \int_{f_w @ s_{wf}}^{f_w @ s_w=1} df_w \\ &= \frac{5.615 Q}{A \phi} [(f_w @ s_w=1) - (f_w @ s_{wf})] \end{aligned}$$

BUT: $(f_w @ s_w=1) = 1$; $(f_w @ s_{wf}) = f_{wf}$

THEREFORE

$$\text{AREA B} = \frac{5.615 Q}{A \phi} [1 - f_{wf}]$$

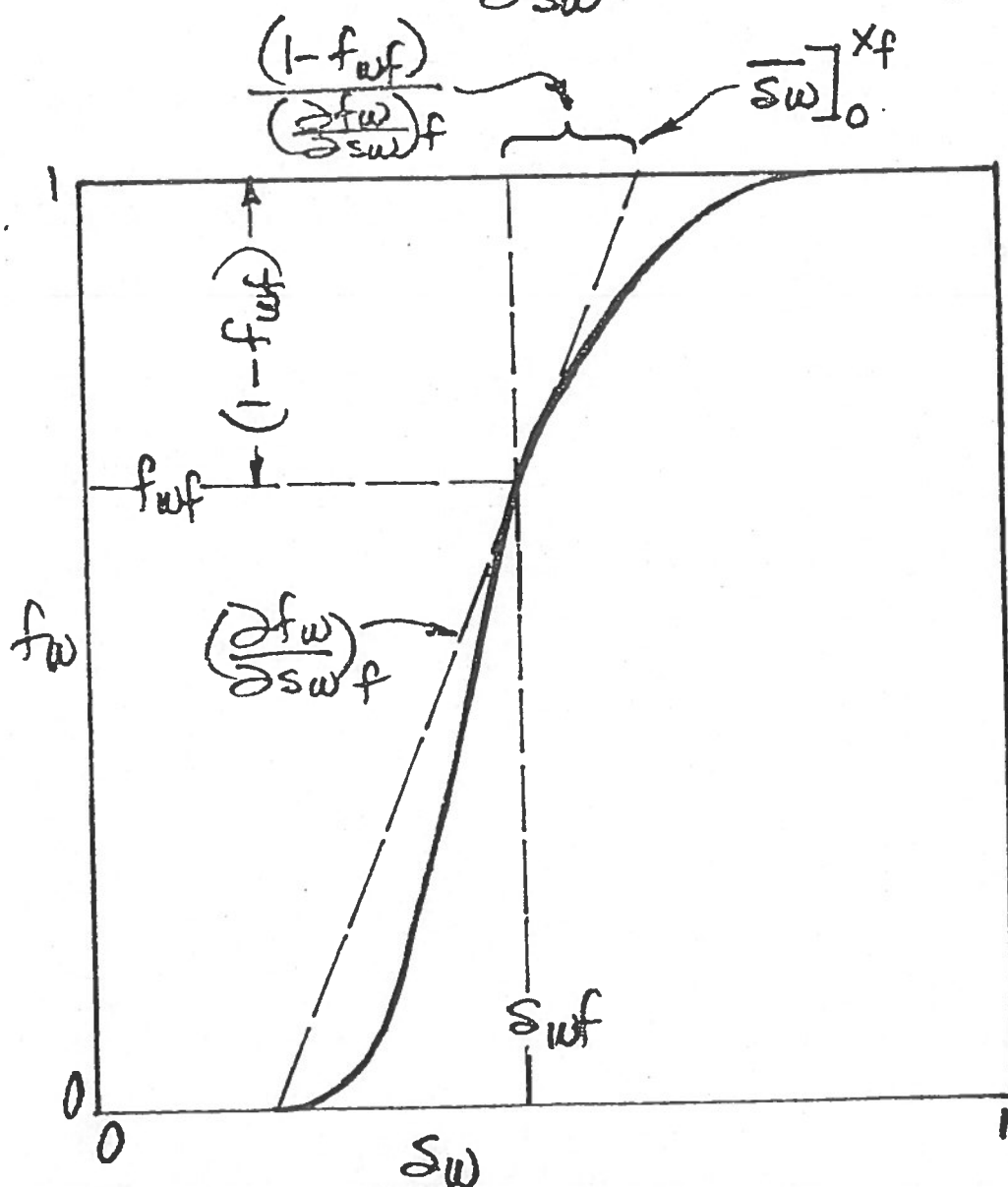
$$\text{AS } \bar{s}_w \Big|_0^{x_f} = \frac{\text{AREA A} + \text{AREA B}}{x_f}$$

$$\bar{s}_w \Big|_0^{x_f} = \frac{\frac{5.615 Q}{A \phi} [s_{wf} \left(\frac{\partial f_w}{\partial s_w} \right)_f + (1 - f_{wf})]}{\frac{5.615 Q}{A \phi} \left(\frac{\partial f_w}{\partial s_w} \right)_f}$$

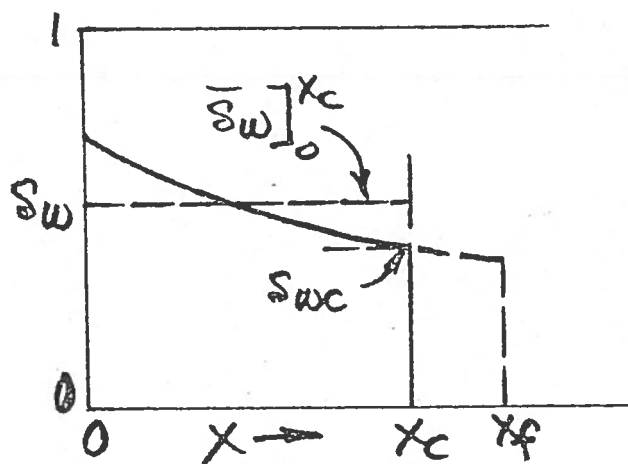
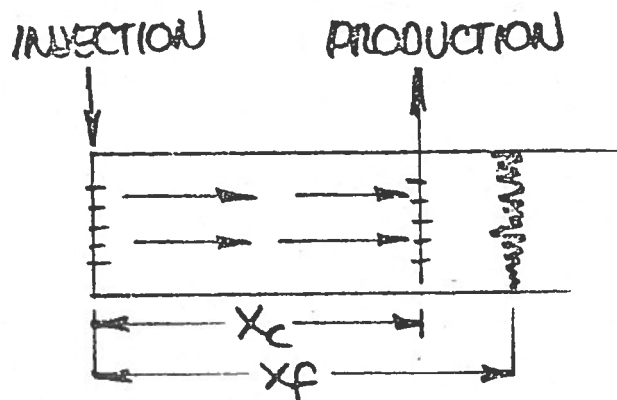
$$\bar{s}_w \Big|_0^{x_f} = s_{wf} + \frac{(1 - f_{wf})}{\left(\frac{\partial f_w}{\partial s_w} \right)_f}$$

AVERAGE SATURATION BEHIND FRONT III

$$\bar{s}_w]_0^{x_f} = s_{wf} + \frac{(1 - f_{wf})}{\left(\frac{\partial f_w}{\partial s_w}\right)_f}$$



AVERAGE SATURATION BEHIND FRONT-IV

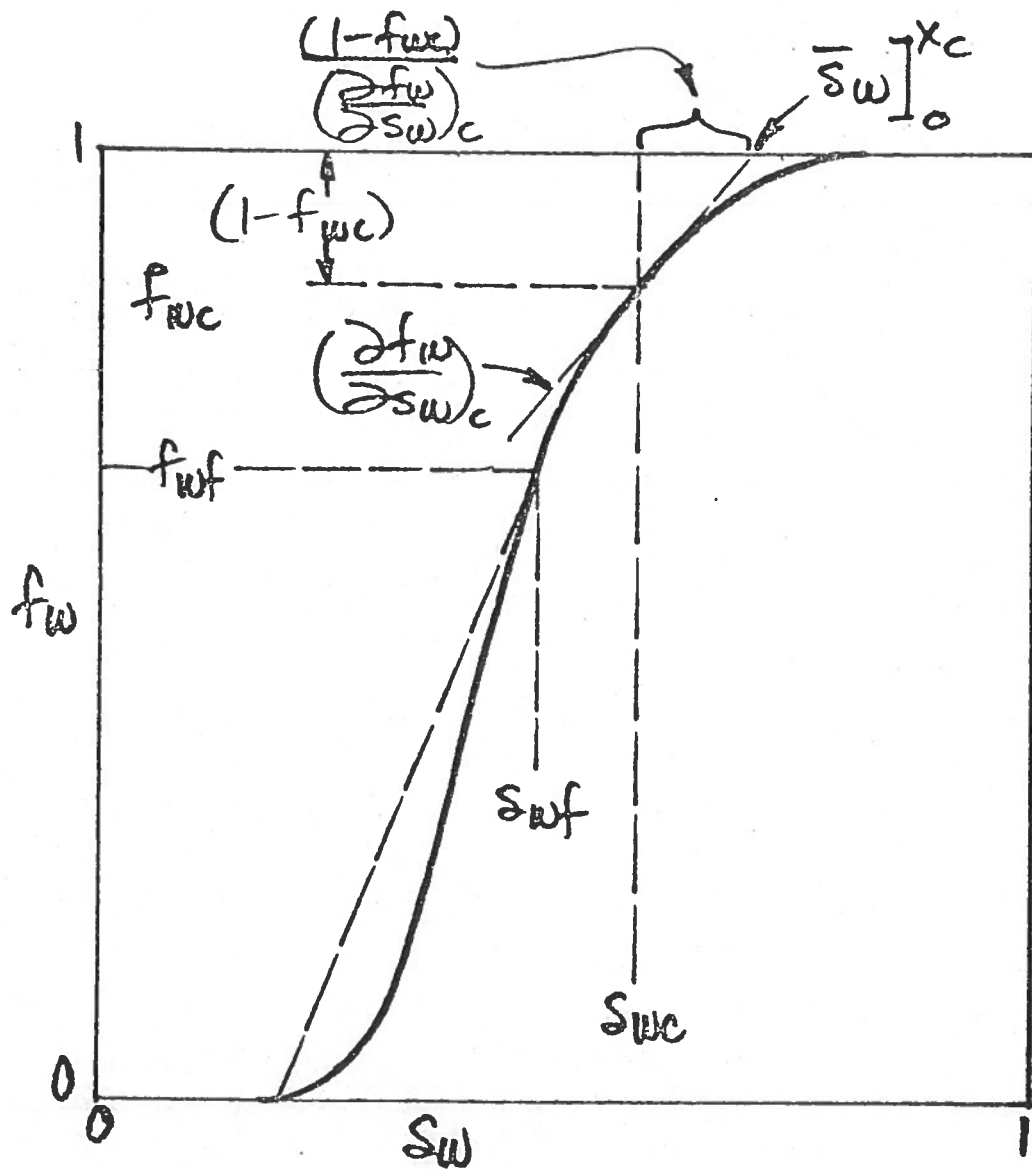


WHEN THE FRONT LINE HAS PASSED POSITION x_c , THE AVERAGE WATER SATURATION BEHIND x_c CAN BE DEVELOPED BY A SIMILAR APPROACH. THE FINAL EQUATION IS

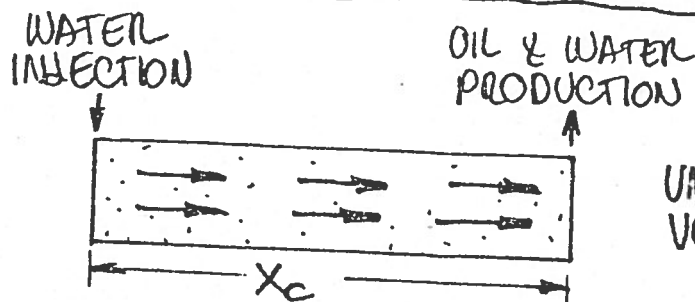
$$\bar{S}_w]_0^{x_c} = S_{wc} + \frac{(1 - f_{wc})}{\left(\frac{\partial f_w}{\partial S_w}\right)_c}$$

AVERAGE SATURATION BEHIND FRONT - II

$$\bar{s}_w \Big|_0^{x_c} = s_{wc} + \frac{(1-f_{wc})}{\left(\frac{\partial f_w}{\partial s_w}\right)_c}$$



INJECTION VOLUME VS. RECOVERY RELATIONS, I



UNIT PORE VOLUME, $V_p = \frac{A\phi x_c}{5.615}$

DIMENSIONLESS PORE VOLUMES INJECTED

$$V_{id} = \frac{W_i}{V_p} = \frac{W_i}{\frac{A\phi x_c}{5.615}} = \frac{5.615 W_i}{A\phi x_c}$$

FROM BUCKLEY - LEVERETT

$$x_c = \frac{5.615 W_i}{A\phi} \left(\frac{\partial f_w}{\partial s_w} \right)_c$$

OR

$$\frac{5.615 W_i}{A\phi x_c} = 1 / \left(\frac{\partial f_w}{\partial s_w} \right)_c$$

THEREFORE, THE PORE VOLUMES INJECTED TO REACH A GIVEN WATER SATURATION AT THE OUTFLOW FACE, s_{wc} IS,

$$V_{id} = 1 / \left(\frac{\partial f_w}{\partial s_w} \right)_c$$

INJECTION VOLUME VS. RECOVERY RELATIONS II

$$\begin{aligned}\text{RESERVOIR OIL DISPLACED} &= V_p (\Delta \bar{s}_w)_o^{x_c} \\ &= \frac{A \phi x_c}{5.615} \left[(\bar{s}_w)_o^{x_c} - s_{wi} \right]\end{aligned}$$

$$\begin{aligned}\text{STOCK TANK OIL PRODUCED} &= \frac{\text{RES. OIL DISP.}}{B_o} \\ N_p &= \frac{A \phi x_c}{5.615 B_o} \left[(\bar{s}_w)_o^{x_c} - s_{wi} \right]\end{aligned}$$

FLOWING WATER-OIL RATIO AT OUTFLOW FACE;

$$WOR = \frac{f_{wc}}{1 - f_{wc}}$$

SURFACE PRODUCING WATER-OIL RATIO,

$$F_{wo} = \frac{f_{wc} \cdot B_o}{(1 - f_{wc}) B_w}$$

SURFACE WATER CUT =

$$\frac{\frac{f_{wc} B_w}{B_w} + \frac{(1 - f_{wc})}{B_o}}{\frac{f_{wc} B_w}{B_w} + \frac{(1 - f_{wc})}{B_o}} = \frac{1}{1 + F_{wo}}$$

$$\begin{aligned}\text{TIME}_1 + &= \frac{W_i}{i_w} \\ &= \frac{V_{io} \cdot A \phi x_c}{5.615 i_w} \text{ (DAYS)}\end{aligned}$$

WHERE i_w IS BBL PER DAY