NOTES ON FLUID DISPLACEMENT IN POROUS ROCKS

1. Introduction

With few exceptions, production of petroleum reservoirs involves one or more aspects of fluid displacement - that is, displacement of one fluid by a second. Natural fluid displacement occurs, for example, when water influxes into the reservoir and displaces oil, or gas, that lay originally near the hydrocarbon-water contact. A second example is the "drive" or displacement of oil by gas as the gas comes out of solution in the oil phase. The material balance methods presented in Section E allowed one to evaluate, in the case of water influx, the amount and rate at which the water enters the petroleum reservoir. This becomes the amount and rate at which hydrocarbons are displaced in the invaded reservoir volume. To calculate the invaded reservoir volume, one must have an appropriate value for the volumetric displacement efficiency. This is shown by Equation 1.1.

$$W_e = 7758\ \phi\ V_i \cdot E \tag{1.1}$$

where

V_i = reservoir volume invaded, acre-ft.
E = volumetric displacement efficiency, fraction
ϕ = porosity, fraction
W_e = influx volume, bbl.

Material presented in this section concerns the evaluation of E in the above equation. This is best done by considering that the volumetric displacement efficiency to be composed of three other efficiency numbers with the relationship:

$$E = E_D \cdot E_A \cdot E_V \tag{1.2}$$

The internal displacement efficiency, E_D, in Equation 1.2 represents hydrocarbon saturation change within portions of the reservoir where fluid displacement has been in effect, per unit of initial hydrocarbon saturation. That is:

$$E_D = \frac{\Delta S_h}{S_{hi}} \tag{1.3}$$

where

ΔS_h = hydrocarbon saturation change
S_{hi} = hydrocarbon saturation at start of displacement

The areal displacement efficiency, E_A, accounts for the fraction of the reservoir that is contacted by displacing fluid, in an areal sense while the vertical displacement efficiency, E_V, represents the fraction of reservoir that is contacted by displacing fluid, in a vertical sense. The product, $E_A \cdot E_V$, represents the fraction of the reservoir in which fluid displacement has occured.

Basics of calculating internal displacement efficiency, E_D, will be covered in sub-sections 2 and 3. Areal and vertical efficiency concepts will be introduced in the fourth sub-section. Sub-section five will cover the Dykstra-Parsons method of handling vertical efficiency. The last section will list computer abstracts that pertain to fluid displacement calculations.

2. Fractional Flow

We can say that a unit weight of any fluid at a given point has a potential Φ. Insofar as subsurface flow is concerned, we usually are interested only in the fluid pressure at the location and its position. Thus, we can say,

$$\Phi = p + \rho g h \tag{2.1}$$

where h is the distance above some arbitrary plane.

We know from basic fluid flow mechanics that:

$$\vec{q} = - \frac{k}{\mu} \text{ grad } \Phi \tag{2.2}$$

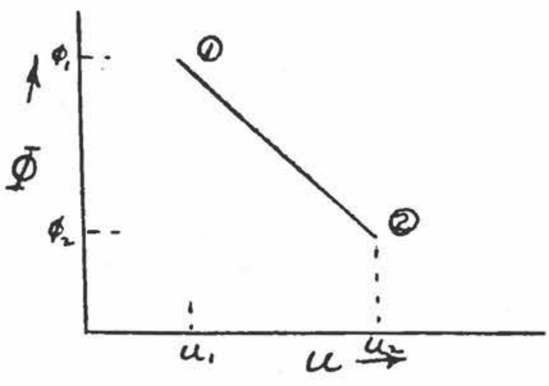
If we consider flow in the u direction, then:

$$\Big[q \Big]_u = - \frac{k}{\mu} \left(\frac{d\Phi}{du} \right) \tag{2.3}$$

It is very important to remember that $\frac{d\Phi}{du}$ is a negative value; that is, flow goes from high Φ to a low Φ.

$$\frac{d\Phi}{du} = \lim_{\Delta u \to o} \frac{\Delta \Phi}{\Delta u} = \lim_{\Delta u \to o} \frac{\Phi_2 - \Phi_1}{u_2 - u_1}$$

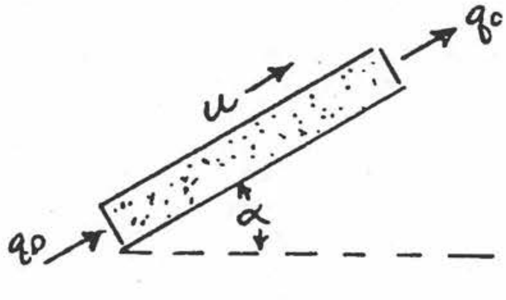
The fact that $\frac{d\Phi}{du}$ is negative is why the - sign is placed before the k/μ.



If we consider a linear section of porous media inclined at angle α from the horizontal and in which flow is going in the upward (u) direction, we can write:

$$\Big[q_D \Big]_u = - \frac{k_D}{\mu_D} \left(\frac{d\, \Phi_D}{du} \right) = - \frac{k_D}{\mu_D} \left[\frac{d\, p_D}{du} + \rho_D g \frac{dh}{du} \right] \tag{2.4}$$

$$\Big[q_o \Big]_u = - \frac{k_o}{\mu_o} \left(\frac{d\, \Phi_o}{du} \right) = - \frac{k_o}{\mu_o} \left[\frac{d\, p_o}{du} + \rho_o g \frac{dh}{du} \right] \tag{2.5}$$



where

q = flow rate per unit area
k = effective permeability
μ = viscosity
p = pressure
u = distance in the direction of flow
ρ = density of the fluid
g = gravitational constant
sub D = displacing fluid
sub o = oil (displaced) fluid

It might be well to inspect Equations 2.4 and 2.5 to see if they are correct. Let's first consider the situation for a horizontal bed. Here α is equal to zero. $\frac{dh}{du}$ must also be equal to zero as $\frac{dh}{du} = \sin\alpha$. q_D then has a positive value (flow towards the right because $\frac{dp_o}{du}$ is negative, and we have two negatives. So we are alright with regard to the pressure gradient. Let's now let $\frac{dp_D}{du} = o$ and consider only the effect of the bed dip. For the situation illustrated $\frac{dh}{du}$, or $\sin\alpha$, is a positive value. q_D, however, is negative (because of the negative sign before the brackets) - which means that flow would be down hill. This is what we would expect.

Since two phases are present and the interface between phases is curved, we must include any effect of capillarity. We will arbitrarily define the capillary pressure, P_c, as

$$P_c = p_D - p_o \tag{2.6}$$

It follows directly that

$$\frac{\partial P_c}{\partial u} = \frac{\partial p_D}{\partial u} - \frac{\partial p_o}{\partial u} \tag{2.7}$$

Solving Equations 2.4 and 2.5 for the pressure gradients and putting them into Equation 2.7 gives:

$$\frac{\partial P_c}{\partial u} = - \frac{q_D \mu_D}{k_D} - \rho_D g \sin\alpha + \frac{q_o \mu_o}{k_o} + \rho_o g \sin\alpha \, . \tag{2.8}$$

If we arbitrarily let

$$\Delta\rho = \left(\rho_D - \rho_o\right) \qquad (2.9)$$

and consider the two fluids incompressible, flowing at constant total rate, q,

$$q = q_o + q_D \qquad (2.10)$$

Equation 2.8 develops into:

$$\frac{\partial P_c}{du} + \Delta\rho g \sin\alpha - \frac{q\,\mu_o}{k_o} = q_D\left(\frac{k_o}{\mu_o} + \frac{k_D}{\mu_D}\right) \qquad (2.11)$$

Dividing through by $\frac{q\,\mu_o}{k_o}$ and changing signs:

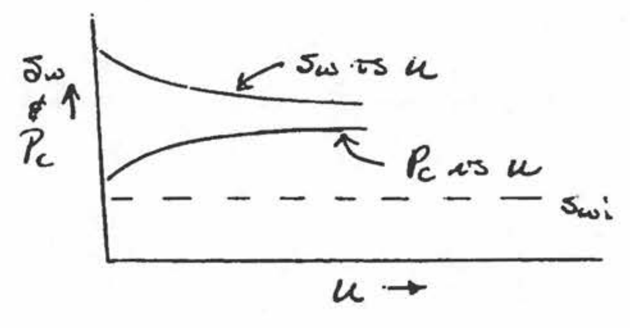
$$\frac{q_D}{q}\left[1 + \frac{k_o}{\mu_o}\cdot\frac{\mu_D}{k_D}\right] = 1 - \frac{k_o}{q\,\mu_o}\left[\frac{\partial P_c}{\partial u} + \Delta\rho g \sin\alpha\right] \qquad (2.12)$$

As $\frac{q_D}{q} = f_D =$ fraction of displacing phase flowing,

$$f_D = \frac{1 - \frac{k_o}{q\,\mu_o}\left[\frac{\partial P_c}{\partial u} + \Delta\rho g \sin\alpha\right]}{\left[1 + \frac{k_o}{k_D}\cdot\frac{\mu_D}{\mu_o}\right]} \qquad (2.13)$$

Equation 2.13 is completely general. If flow is down-dip, sin α is a negative value. If gas is displacing oil, Δρ is negative because of the manner of defining it in Equation 2.9. The equation as it now stands considers oil as the displaced phase, but any displaced fluid could be substituted.

In the "normal" water-wet sand, the capillary pressure decreases as one moves back from the flood front. (See sketch). Therefore, the sign of $\frac{\partial P_c}{\partial u}$ in Equation 2.13 is positive as is the $\Delta\rho g \sin\alpha$ term. In other words, the usual situation is that capillarity assists



displacement in water-wet sands and reduces displacement in oil-wet sands. However, because ΔP_c across reservoir distances of hundreds of feet is apt to be only a few psi, the values of $\frac{\partial P_c}{\partial u}$ in most instances, are so small as to be negligable. Therefore, for $\partial P_c/\partial u$, essentially zero:

$$f_D = \frac{1 - \frac{k_o}{\mu_o q} \Delta\rho g \sin\alpha}{1 + \frac{k_o}{k_D} \cdot \frac{\mu_D}{\mu_o}} \qquad (2.14)$$

As $k_o = k \cdot k_{ro}$ and $\frac{k_o}{k_D} = \frac{k_{ro}}{k_{rD}}$, Equation 2.14 can be written:

$$f_D = \frac{1 - \frac{kk_{ro}}{\mu_o q} \Delta\rho g \sin\alpha}{1 + \frac{k_{ro}}{k_{rD}} \cdot \frac{\mu_D}{\mu_o}} \qquad (2.15)$$

If k = millidarcys
$\Delta\rho$ = lbs/ft^3
μ_o, μ_D = centipoise
q = $B/D/ft^2$ cross section,

then

$$f_D = \frac{1 - \left[\frac{7.84(10^{-6})\, k\, k_{ro}\, \Delta\rho \sin\alpha}{\mu_o q}\right]}{1 + \frac{k_{ro}}{k_{rD}} \cdot \frac{\mu_D}{\mu_o}} \qquad (2.16)$$

However, the more usual units of k = darcys
$\Delta\rho$ = Δ specific gravity with respect to water = $(\Delta\gamma)_w$

yield

$$f_D = \frac{1 - \left[\frac{0.488\, k\, k_{ro}\, (\Delta\gamma)_w \sin\alpha}{\mu_o q}\right]}{1 + \frac{k_{ro}}{k_{rD}} \cdot \frac{\mu_D}{\mu_o}} \qquad (2.17)$$

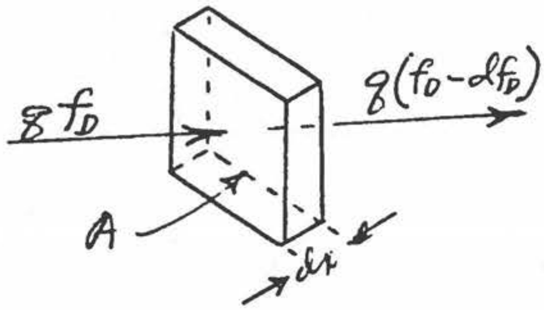
Notice that both Equation 2.16 and 2.17 can, for a particular system, be simplified to:

$$f_D = \frac{1 - a\,k_{ro}}{1 + \frac{k_{ro}}{k_{rD}} \cdot \frac{\mu_D}{\mu_o}} \qquad (2.18)$$

Note that the values of a can be either positive or negative.

3. Frontal Advance (Buckley-Leverett Equation)

We consider a linear flow system as shown of cross sectional area A, porosity ϕ, and thickness dx, and flow q barrels a day through it. f_D is the fraction of q that is displacing fluid and $(f_D - df_D)$ is the fraction leaving. Rate (barrels a day) of displacing fluid accumulation in the element =



displacing fluid in - displacing fluid out.

$$= q\,f_D - q\,(f_D - df_D) \qquad (3.1)$$

$$= q\,df_D \qquad (3.2)$$

As the pore volume = $\frac{A \cdot dx \cdot \phi}{5.615}$ barrels, the accumulation causes a saturation change rate of:

$$\frac{dS_D}{dt} = \frac{q df_D}{A\ dx\ \phi/5.615} \qquad (3.3)$$

or

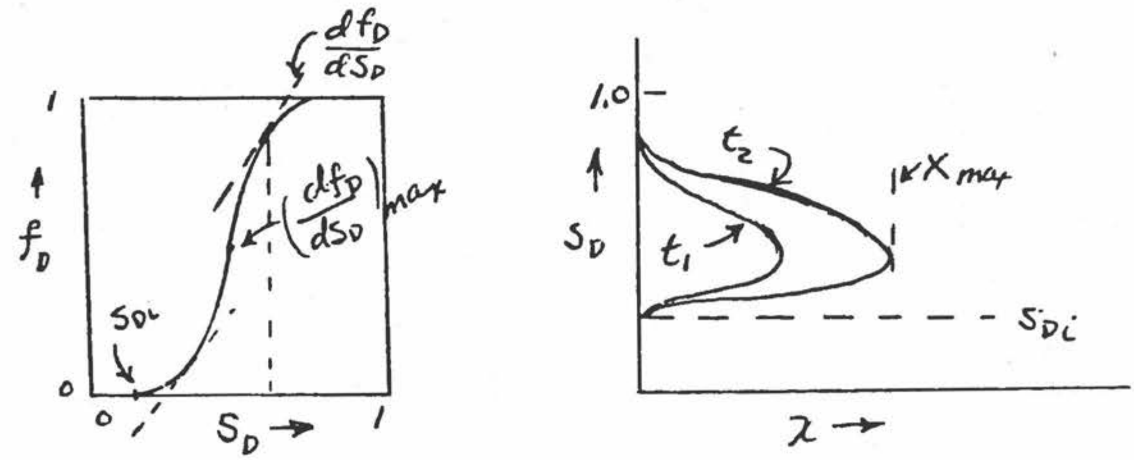
$$dx = \frac{5.615\ q}{A\phi}\left(\frac{df_D}{dS_D}\right)_{S_D} dt \qquad (3.4)$$

Integrating, this yields:

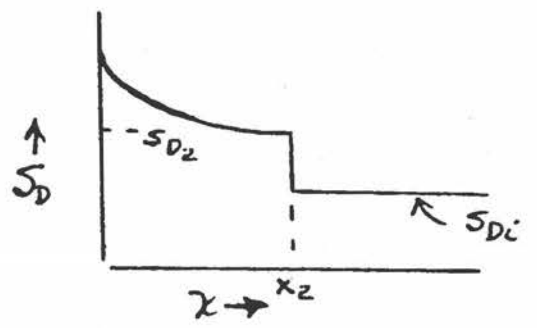
$$\int_{x_D}^{x_{S_D}} dx = \frac{5.615\ q}{A\phi}\left(\frac{df_D}{dS_D}\right) \int_{t=o}^{t} dt \qquad (3.5)$$

$$x_{S_D} = x_o + \frac{5.615}{A\phi} \cdot qt\left(\frac{d\ f_D}{dS_D}\right)_{S_D} \qquad (3.6)$$

Equation 3.6 is very important. It says that the distance, x_{S_D}, to which a plane of saturation, S_D, moves to, having started from location x_o, is equal to a constant $5.615/\phi$ times the throughput per unit area, $\frac{qt}{A}$, and the <u>slope of the</u> f_D <u>vs.</u> S_D <u>curve</u> at the value of S_D.



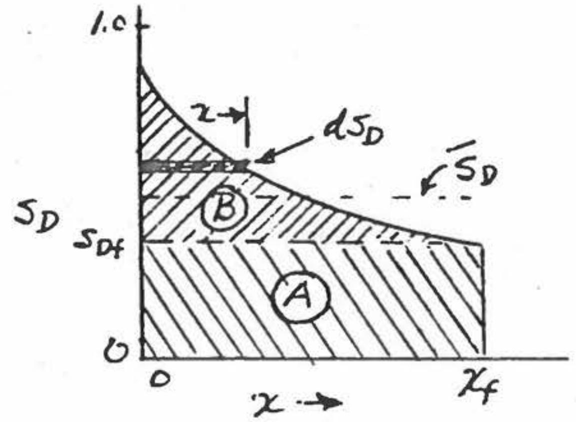
From the left-hand sketch we can see that the maximum value of $\left(\frac{df_D}{dS_D}\right)$ occurs at about $S_D = 0.4$. This saturation plane advances most rapidly per Equation 3.6.



Mathematical analyses indicate that the shape of the S vs. x curve can be modified at the nose to give the following shape. Location x_2 is the displacement front location. The displacing phase saturation at the front is S_{D2}.

Calculation of Average Saturation Behind the Front

Previous theory allows the computation of the S_D - x curve. If we consider the case where the B - L front has advanced to x_f, we can say that the the average saturation of displacing phase, $\bar{S}_D$, behind the front is equal to the sum of areas A and B divided by x_f.



$$\bar{S}_D = \frac{\text{Area A + Area B}}{x_f} \tag{3.7}$$

But

$$\text{Area A} = S_{Df} \cdot x_f \tag{3.8}$$

$$\text{Area B} = \int_{S_{Df}}^{1.0} x \, dS_D \tag{3.9}$$

As

$$x = \frac{5.615\,q}{A\phi}\left(\frac{df_D}{dS_D}\right) \tag{3.10}$$

$$\text{Area B} = \frac{5.615\,q}{A\phi} \int_{S_{Df}}^{1.0} \frac{df_D}{\cancel{dS_D}} \cdot \cancel{dS_D} \tag{3.11}$$

$$= \frac{5.615\,q}{A\phi} \int_{f_D @ S_{Df}}^{f_D @ S_D = 1} df_D \tag{3.12}$$

$$\text{Area B} \quad = \quad \frac{5.615q}{A\phi}\left[(f_D \; @ \; S_D = 1) - (f_D \; @ \; S_{Df})\right] \qquad (3.13)$$

As $(f_D \; @ \; S_D = 1) = 1$ and $(f_D \; @ \; S_{Df}) = f_{Df}$ (3.14)

$$\text{Area B} \quad = \quad \frac{5.615\ q}{A\phi}\ (1 - f_{Df}) \qquad (3.15)$$

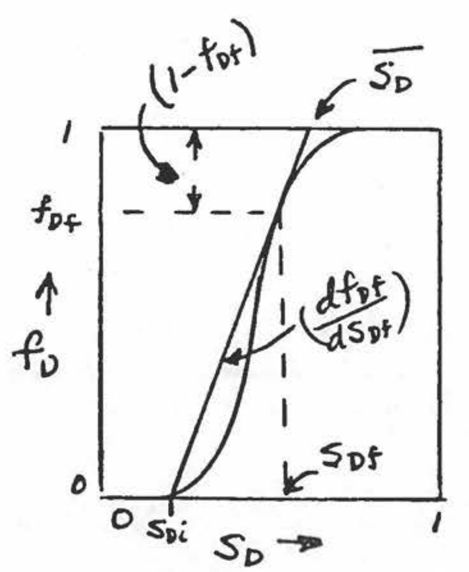
Going back to Equations 3.7 and 3.8 and replacing x_f by its equivalent

$$x_f = \frac{5.615\ q}{A\phi}\left(\frac{df_D}{dS_D}\right)_f \qquad (3.16)$$

we obtain

$$\bar{S}_D = \frac{S_{Df} \cdot \cancel{\frac{5.615\ q}{A\phi}}\left(\frac{df_D}{dS_D}\right)_f + \cancel{\frac{5.615\ q}{A\phi}}\ (1 - f_{Df})}{\cancel{\frac{5.615\ q}{A\phi}}\left(\frac{df_D}{dS_D}\right)_f} \qquad (3.17)$$

$$\bar{S}_D = S_{Df} + \frac{(1 - f_{Df})}{\left(\frac{df_D}{dS_D}\right)_f} \qquad (3.18)$$



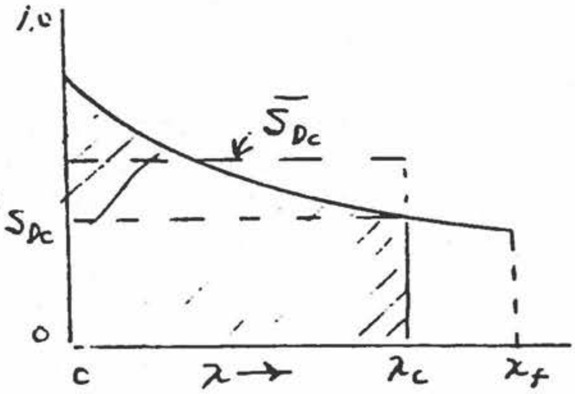
Significance of Equation 3.18 is shown by the sketch. The average saturation $\overline{S_D}$ can be obtained by extending the operating line (tangent at S_{Df}) to $f_D = 1$ and reading the value.

As the saturation change of the displacing fluid must be equal to the hydrocarbon saturation change, the hydrocarbon displaced at <u>breakthrough</u> will be:

$$(\bar{S}_D - S_{Di}) = S_{Df} - S_{Di} + \frac{(1 - f_{Df})}{\left(\frac{df_D}{dS_D}\right)_f} \tag{3.19}$$

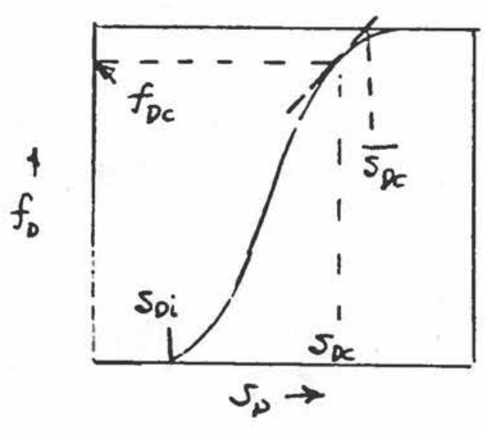
$$(\bar{S}_D - S_{Di}) = \frac{1}{\left(\frac{df_D}{dS_D}\right)_f} \tag{3.20}$$

Let's now consider how to obtain the hydrocarbon recovery after breakthrough of the front. This will be the recovery during the subordinate phase of production. To do this we picture the situation of the front having progressed beyond the outflow face (or has continued past the producing well). Referring to the figure, the outflow face is at distance x_c from the start, while the front has progressed to distance x_f. It is immediately apparent that we have the same situation as when the front was located at distance x_c. Therefore, without going through the development, we can say:



After breakthrough

$$\bar{S}_{Dc} = S_{Dc} + \frac{(1 - f_{Dc})}{\left(\frac{df_D}{dS_D}\right)_c} \tag{3.21}$$

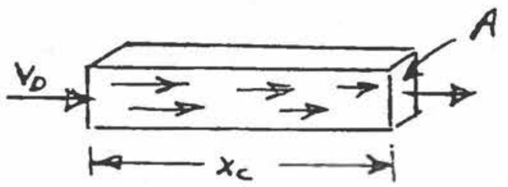


As previously, the hydrocarbon recovery will be:

$$\bar{S}_{Dc} - S_{Di} = S_{Dc} - S_{Di} + \frac{(1 - f_{Dc})}{\left(\frac{df_D}{dS_D}\right)_c} \tag{3.22}$$

Calculation of Injection Volume Required to Reach Average Saturation Conditions

The fractional flow equation and the Buckley-Leverett relationships permit one to calculate average displacing phase saturation behind an advancing displacement front from measured or correlatable reservoir parameters. This in turn leads to the evaluation of the hydrocarbon phase recovery and the internal displacement efficiency number, E_D. The relationships to be developed in this section relate to the quantity of displacing fluid required to achieve a given hydrocarbon recovery and to the producing volume ratio of displacing phase/displaced phase at breakthrough of displacing phase at the outflow face of the system.



Consider a segment of reservoir as indicated in the sketch. Let the cross sectional area perpendicular to flow be A square feet, and length between inlet and outlet faces be x_c feet. The unit pore volume, in barrels is:

$$V_p = \frac{A x_c \phi}{5.615} \tag{3.23}$$

Consider that at a particular time, V_D barrels of displacing fluid has entered the unit volume. Assuming constant pressure prevails during displacement an equal volume of fluids will be displaced from the unit. The dimensionless pore volumes of displacing fluid will be:

$$V_{iD} = \frac{V_D}{V_p} = \frac{V_D}{A\ x_c\ \phi/5.615} \tag{3.24}$$

Equation 3.6 can be modified to fit the present situation by writing it as:

$$(x_{S_D} - x_o) = x_c = \frac{5.615}{A\phi} \cdot V_D \left(\frac{df_D}{dS_D}\right)_c \tag{3.25}$$

Solving Equations 3.24 and 3.25 yields:

$$V_{iD} = 1/\left(\frac{df_D}{dS_D}\right)_c \tag{3.26}$$

This relationship is important. It says that the reciprical of the slope of the fracional flow curve at saturation conditions existing at the outflow face is the dimensionless pore volumes of displacing fluid required to achieve this saturation.

The relationship of average saturation between inflow and outflow faces of the unit reservoir volume and injection volume comes from Equations 3.21 and 3.26.

That is,

$$\bar{S}_{Dc} = S_{Dc} + (1 - f_{Dc})\, V_{iD} \tag{3.27}$$

Note that one requires the saturation and fraction of displacing phase flowing at the outflow face to use Equation 3.27.

Because calculations of water displacing oil are so frequently encountered in reservoir and production engineering calculations, certain of the above equations will be rewritten for the water→oil displacements.

$$\text{Reservoir oil displaced} = V_p \, (\Delta S_o)_o^{x_c} \tag{3.28}$$

$$= \frac{A\, x_c\, \phi}{5.615} \left[(\bar{S}_w)_o^{x_c} - S_{wi} \right] \tag{3.29}$$

Stock tank oil displaced/produced:

$$N_p = \frac{A\, x_c\, \phi}{5.615\, B_o} \left[(\bar{S}_w)_o^{x_c} - S_{wi} \right] \tag{3.30}$$

Flowing water-oil ratio at outflow face,

$$WOR = \frac{f_{wc}}{f_{oc}} = \frac{f_{wc}}{1 - f_{wc}} \tag{3.31}$$

Surface producing water-oil ratio,

$$F_{wo} = \frac{f_{wc}}{(1 - f_{wc})} \cdot \frac{B_o}{B_w} \tag{3.32}$$

Surface water cut,

$$\text{Cut} = \frac{f_{wc}/B_w}{f_{wc}/B_w + (1 - f_{wc})/B_o} = \frac{F_{wo}}{1 + F_{wo}} \tag{3.33}$$

Injection time, $$t = \frac{W_i}{i_w} = \frac{V_{iD} \cdot A\, x_c\, \phi}{5.615\, i_w}$$

4. Mobility Ratio, Sweep Efficiency, Stratification

These three parameters are very important to the recovery of hydrocarbons by displacement processes. Mobility ratio is fixed by the viscosities and saturations on each side of a displacement front. Sweep efficiency depends on mobility ratio and the geometrical relationship of injection/producing wells (in injection projects). Stratification effects are caused by permeability differences (usually in a vertical sense) in the reservoir sections in which fluids are moving.

Mobility Ratio

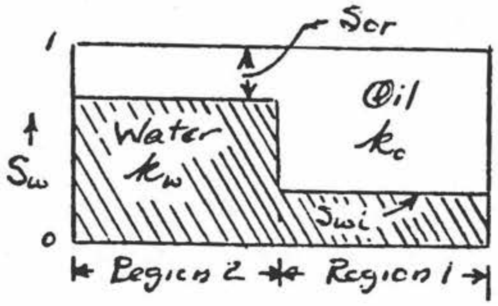
The mobility of a fluid is defined as the ratio of effective permeability to viscosity.

$$\lambda_o = \frac{k_o}{\mu_o} \tag{4.1}$$

Note that effective permeability depends both on saturation and saturation history, i.e., imbibition or drainage process.

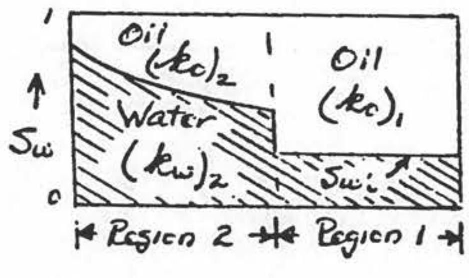
Mobility ratio expresses something of the ability of a displacing fluid to do an effective displacement. It is defined as the ratio of fluid mobility behind the front to fluid mobility ahead of the front. For example, the mobility ratio of water displacing oil would be defined by the relationship:

$$M_{wo} = \frac{\lambda_w}{\lambda_o} = \frac{k_w}{\mu_w} \cdot \frac{\mu_o}{k_o} \tag{4.2}$$



The sketch illustrates what is called "piston displacement." Only water is flowing behind the front, and only oil is flowing ahead of the front. The effective permeabilities for this situation would be water permeability at residual oil saturation and oil permeability at interstitial water saturation.

If more than one fluid is moving behind the front, as illustrated in this sketch, it is preferable to use an effective mobility for the two phase flow in region 2. In this instance:



$$\lambda_2 = \left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o}\right)_2 \tag{4.3}$$

where the effective permeabilities are evaluated at the average saturation of region 2.

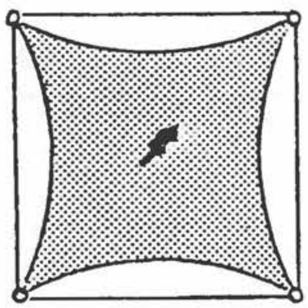
The mobility then becomes:

$$M_{wo} = \frac{\lambda_2}{\lambda_1} = \left(\frac{k_w}{\mu_o} + \frac{k_o}{\mu_o}\right)_2 \Big/ \left(\frac{k_o}{\mu_o}\right)_1 \quad (4.4)$$

Mobility ratios for most reservoir displacements range between 0.1 and several hundred. Displacement efficiency decreases as mobility ratio increases.

Sweep Efficiency

The term "sweep efficiency" usually is used in connection with pattern displacements, although one can very well consider sweep efficiency of, say, water displacing oil updip as a result of water influx. Other terms are often used in place of sweep efficiency. Amoung these are areal displacement efficiency (used in previous discussions), areal sweep, areal coverage, coverage, area swept, and pattern efficiency. Regardless of the term used, the purpose is to express the fraction of the basic area that has been swept or processed by the displacing fluid at any particular time.



An example of areal sweep efficiency is shown in the sketch. The shaded portion represents the area within a repeated five spot pattern that has been contacted by displacing fluid injected at the center well at breakthrough into the four producing wells. As sketched, the areal sweep efficiency is about 0.7

Areal sweep efficiency is a function mobility ratio, pore volumes of fluid injected, and the geometrical relationship of injection and producing wells. The chart immediately below (Figure 4.1) shows the areal sweep efficiency at breakthrough as a function of mobility ratio for repeated five spot geometry.

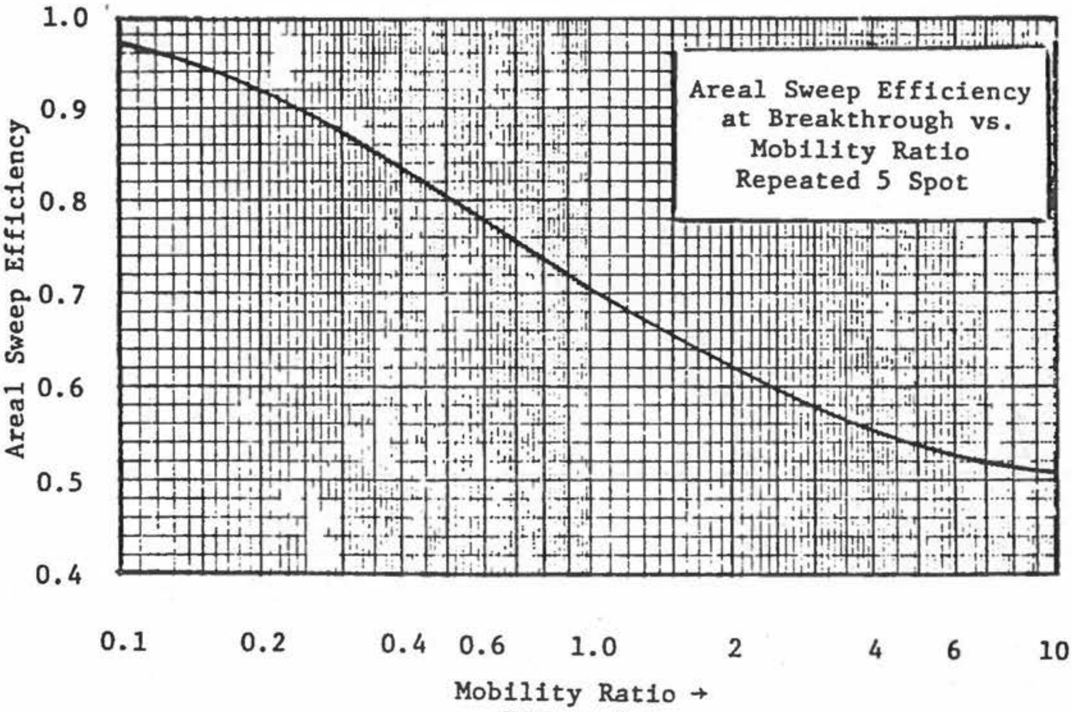


Figure 4.1

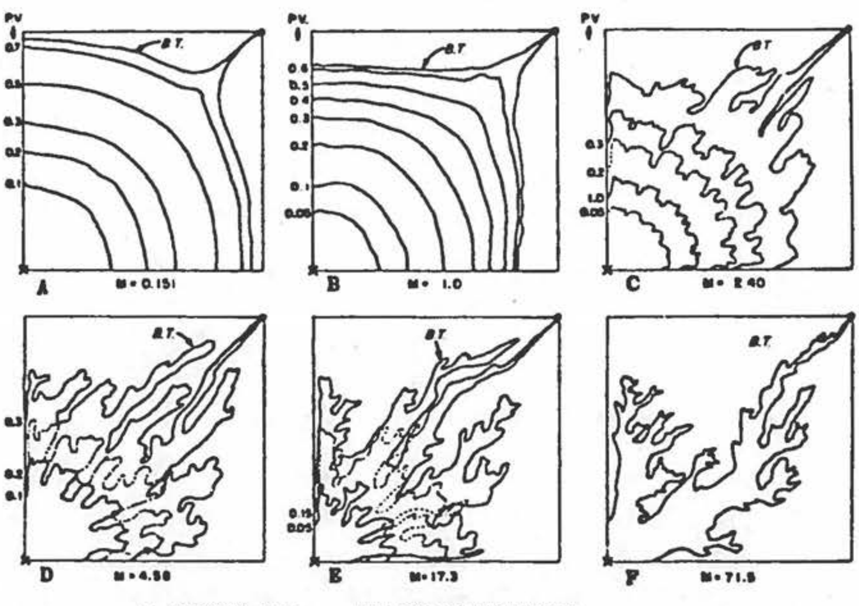


Figure 4.2

Figure 4.2 illustrates shapes of displacement fronts and areal sweep efficiencies at breakthrough found by Haberman (Trans AIME 219 (1960) 264). Note the "dendritic" type of displacement encountered at high mobility ratios.

Areal sweep efficiency continues to increase after breakthrough of displacing fluid to the producing well, although not as fast as before breakthrough. Figure 4.3 shows areal sweep efficiency values as a function of displaceable pore volumes injected and mobility ratio. The "ticks" on the 45^{o} line indicate breakthrough sweep efficiencies.

A displaceable pore volume represents a volume consistant with maximum saturation change of the displacing fluid. Thus, for water as the displacing fluid, one displaceable pore volume is represented by:

$$V_D = V_R \phi (\Delta S_w)_{max} \qquad (4.5)$$

where

V_R = bulk reservoir volume, bbl.

ϕ = porosity, fraction

$(\Delta S_w)_{mx}$ = $(1 - S_{or} - S_{wi})$ if no gas phase is present.

= $(1 - S_{gr} - S_{or} - S_{wi})$ if gas phase is present and displaced.

S_{gr}, S_{or} = residual hydrocarbon saturations at infinite water throughput.

Stratification

Variability of permeability in a vertical sense results in lowering recovery of displaced phase for a given amount injected phase. The reason for this is that the displacement process moves much faster through high permeability portions of the reservoir than it does through the remaining bulk of the reservoir. As a consequence, breakthrough occurs earlier (lower value of displaceable pore volumes injected) and, if the mobility ratio is adverse, the high permeability "streak" continues to transport a large proportion of the injected fluid to the producing wells and effectively slows displacement in the remainder of the reservoir.

Effects of stratification on displacement processes are difficult to predict mainly because we can make observations only at wellbores. We have no direct knowledge of the permeable paths, or stratification into layers, in the inter-well distances. However, experience has shown that sands that show a large degree of variability of permeability in well cores will give poorer flooding results than will sands that are more homogeneous.

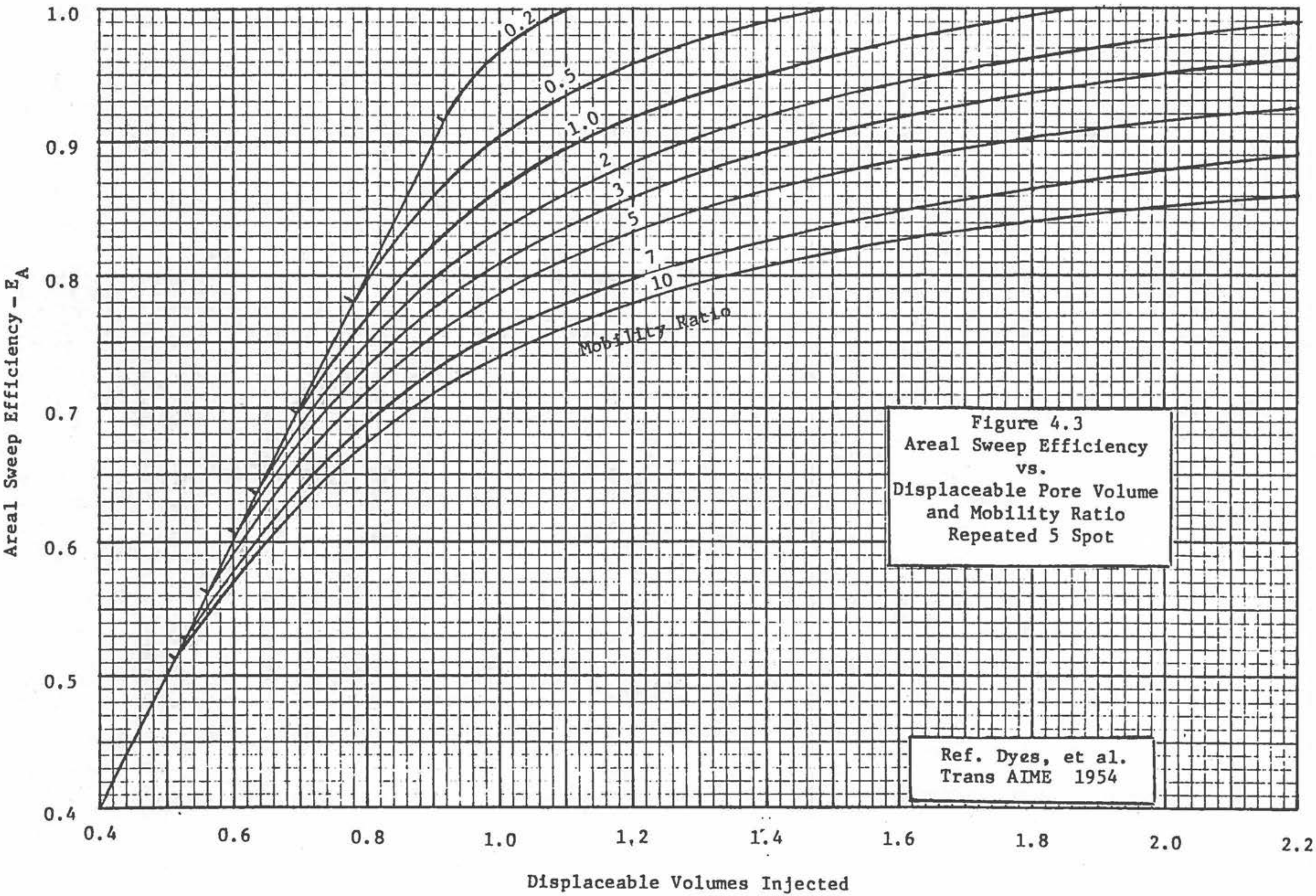
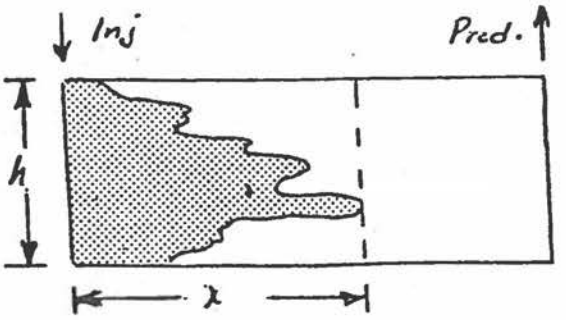


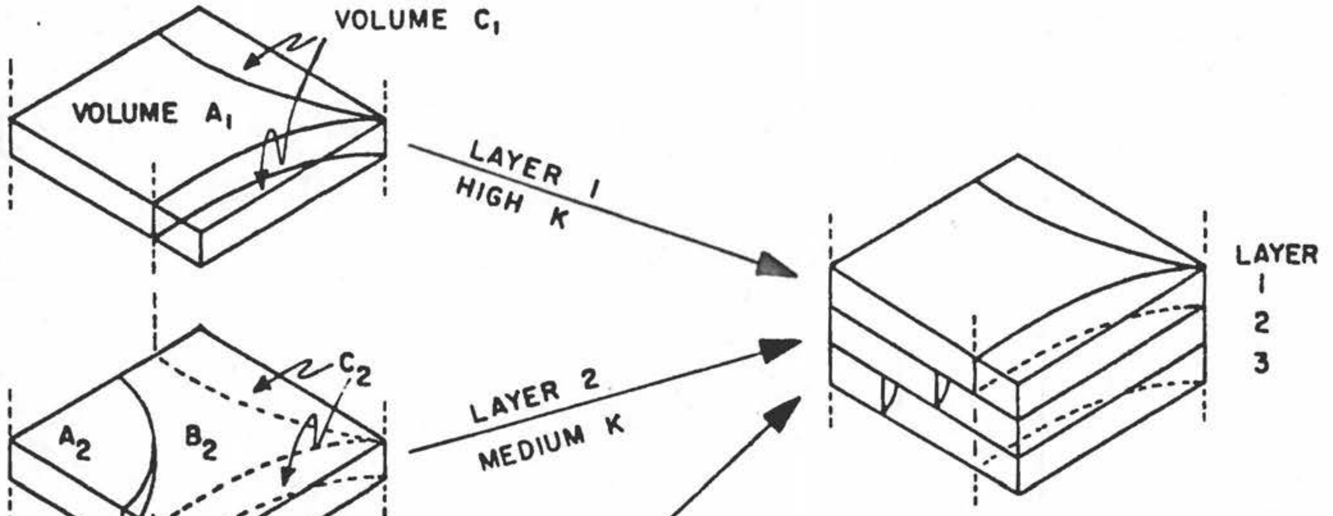
Figure 4.3
Areal Sweep Efficiency
vs.
Displaceable Pore Volume
and Mobility Ratio
Repeated 5 Spot
Ref. Dyes, et al.
Trans AIME 1954
Areal Sweep Efficiency - E_A
Displaceable Volumes Injected
Mobility Ratio
0.2
0.5
1.0
2
3
5
7
10
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2

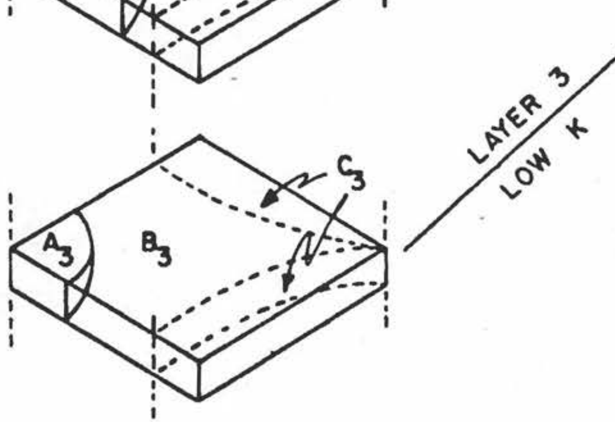


The adjacent sketch illustrates the elementary concept of displacement in stratified reservoirs and the manner of defining vertical displacement efficiency, E_v. Depicted is a vertical cross section through an injection well and producing well. Injection fluid has advanced irregularly as a result of permeability variation in the section as indicated by the shaded area. Vertical displacement efficiency is defined in terms of the distance of furthest advance, x , and in this instance would be the shaded area divided by the area $h \cdot x$.

A method of evaluating E_v, for given mobility ratios and permeability variation was developed by H. Dykstra and R. L. Parson in 1948 and will be looked at in detail in the next section.

SCHEMATIC OF EFFICIENCY FACTORS





$$E_D = \frac{S_{HC_i} \text{ in Volume A} - S_{HC} \text{ in Volume A}}{S_{HC_i} \text{ in Volume A}}$$

$$E_V = \frac{\sum \text{A Volumes}}{\text{A Volumes} + \text{B Volumes}}$$

$$E_A = \frac{\sum \text{A Volumes} + \sum \text{B Volumes}}{\text{Total Pattern Volume}}$$

$$E_R = E_D \cdot E_V \cdot E_A$$