Note: Mobility ratio $m=m_o/m_w$ in this paper is the ***inverse*** of the industry-accepted definition of mobility ratio M = Displacing-to-Displaced mobility ratio, $M = m_w/m_o = \lambda_w/\lambda_o$.

T.P. 2894

THE EFFECT OF PERMEABILITY STRATIFICATION IN COMPLETE WATER-DRIVE SYSTEMS

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ABSTRACT

A theory is presented for calculating the performance history of complete water-drive systems producing from idealized stratified formations. The general equations are applied to systems where the permeability stratification is either of the exponential or linear type. Calculations were carried through for different degrees of permeability stratification, but with special emphasis on the effect of the mobility ratio between the produced oil and the invading water on the resultant performance. These results are also expressed graphically as curves for the initial water breakthrough recovery, for the different degrees of stratification, as a function of the mobility ratio, and of the composition of the produced fluid stream as a function of the cumulative oil recovery. For several typical cases the latter has also been plotted as a function of the cumulative oil and water throughflow. The general result is that when the mobility of the oil is lower than that of the invading water the channelling tendency resulting from the permeability stratification becomes aggravated as the higher permeability zones become flooded out. Situations of this type would obtain when producing low gravity or highly viscous oils. Conversely, if the mobility of the oil is high compared to that of the invading water, the flooding of the high permeability zones will lead to a retarding and choking effect, and the gross bypassing phenomena will be partially suppressed. These conditions would correspond to those of flooding high gravity or low viscosity oils. A discussion is given of the various basic assumptions made in the analysis, including that of ignoring the stripping phase of the production history as implied by relative permeability concepts.

INTRODUCTION

The physical ultimate recoveries from oil reservoirs are basically determined and limited by the physical oil displacement processes associated with the reservoir producing mechanism. In practice, however, the economic ultimate recoveries are further limited by the mobility of the reservoir fluids and the uniformity and continuity of the producing formation. In fact, it is the differential depletion between the component parts of the composite reservoir which ultimately determines the total recovery at the time of field abandonment. While this observation applies to both the solution gas drive and gravity drainage mechanisms, in which use is made only of the energy contained within the original oil-bearing reservoir, it is of even more paramount importance under operations wherein the energy associated with extraneous fluids provides the ultimate oil expulsion mechanism. Whether the invading fluid is the water from an edgewater drive, water injected for pressure maintenance, gas injected for pressure maintenance, or gas returned to the formation in a cycling program, it is often the continuity and uniformity of the producing section which will control the economic efficiency of the operations.

The importance of the problem of reservoir non-uniformity does not, of course, lessen its complexity or the difficulties of its solution. In fact, these are inherently such that the concept of a "general" solution is virtually meaningless. About all that can be reasonably hoped for is the analysis of specific and well-defined types of non-uniformity which may give some degree of approximation to actual reservoir conditions. Since variations in the nature of the reservoir which depend only on the position along the streamlines will not lead directly to major differential depletion development within the reservoir, the types of non-uniformity considered thus far have involved stratification assumptions. That is, the producing section has been replaced by a multi-layer "sandwich," each uniform areally, and differing from the others only in its basic physical constants as to thickness, porosity and permeability. The fluid motion in the composite system is thus approximated by a parallel superposition of the independent fluid movements in the individual strata.

For the specific application to cycling operations the theory of the effect of permeability stratification has been developed for both discontinuous[1] and continuous types of permeability stratification. Among the latter, treatments have been given of systems in which permeability distributions are governed by exponential, linear[2] or probability[3] functions. In all these studies complete dynamical equivalence was assumed between the injected dry gas and the displaced wet gas. The overall effective permeability of each stratum was therefore considered as constant and independent of the degree of invasion of the injected fluid.

In the case of the displacement of oil by water, the assumption of dynamical equivalence between the water and oil will be strictly valid only by accident. Even if the oil viscosity should be the same as that of the water, the effective permeability to the oil in the presence of the connate water will in general be quite different from that of the water behind the water-oil interface flowing past the trapped residual oil. As a result the effective permeability for the stratum as a whole and rate of water invasion will change with time as the intrusion continues. The differential fluid motion in the individual strata will thus also vary with time. Qualitatively, it is easy to predict the resultant effects. If the permeability to viscosity ratio of the invading fluid exceeds that of the fluid displaced, the stratification and bypassing effect of the perme-

[1]References given at end of paper.

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ability variation itself will be accentuated. Conversely, if the permeability to viscosity ratio of the invading fluid is less than that of the fluid displaced, the rate of fluid advance in the more permeable strata will become increasingly retarded as the invasion continues, and the channelling tendency will be reduced. It is the quantitative aspects of these phenomena which require detailed analytical treatment and will be the subject of the following sections.

GENERAL THEORY

A little consideration will show that even for a single uniform stratum the history of the fluid motion when the invading and displaced fluids are not dynamically identical will be extremely difficult to determine. In fact, thus far only the cases of simplest geometry, namely linear, radial and spherical, have been quantitatively treated.[4] General well pattern effects are necessarily ignored in such simplified approximations, and the produced fluid is assumed to change suddenly and completely to the invading fluid when the latter first reaches the producing wells. The effect of incomplete pattern sweep efficiency has been taken into account in the study of cycling operations in formations with exponential permeability distributions.[4] It can also be formally included in a general theory of the composition history of the production from stratified formations even when the invading and displaced fluids are not assumed to be dynamically equivalent. From a practical standpoint, however, the resulting analysis is virtually intractable.

The only theoretical treatment thus far reported on the recovery of oil by invading water in undepleted stratified formations without ignoring the differences in mobility between the oil and water is that of Dykstra and Parsons.[5] A discrete layer analysis was used in this study, and a probability permeability distribution was simulated by proper choices of the individual permeabilities of a 50-layer subdivision of the composite section. In this paper will be given the general formulation of the problem for continuous permeability distributions, and this will be applied specifically to the exponential and linear distributions.

If it is assumed that the permeability-saturation characteristics of all the strata in the composite section are the same and that the residual oil saturations behind the water-oil

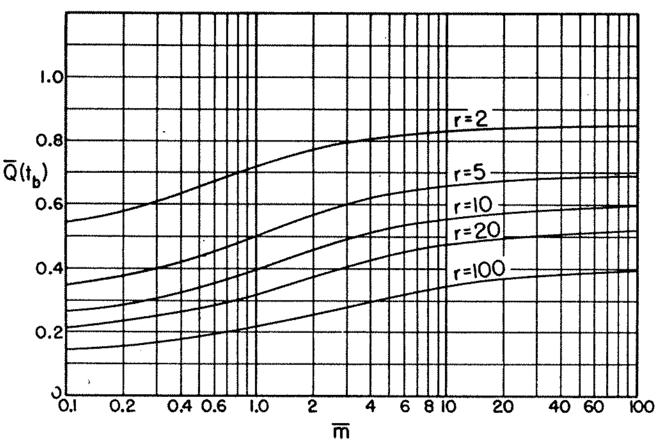

FIG. 1 — THE VARIATION OF THE INITIAL BREAKTHROUGH RECOVERY IN EXPONENTIALLY STRATIFIED SYSTEMS WITH THE MOBILITY RATIO OF THE INVADING AND DISPLACED FLUIDS. $\overline{Q}(t_b)$ = FRACTION OF TOTAL DISPLACEABLE OIL RECOVERED AT THE TIME OF INITIAL BREAKTHROUGH, t_b. $\overline{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER); r = STRATIFICATION RATIO OF ASSUMED EXPONENTIAL PERMEABILITY DISTRIBUTION.

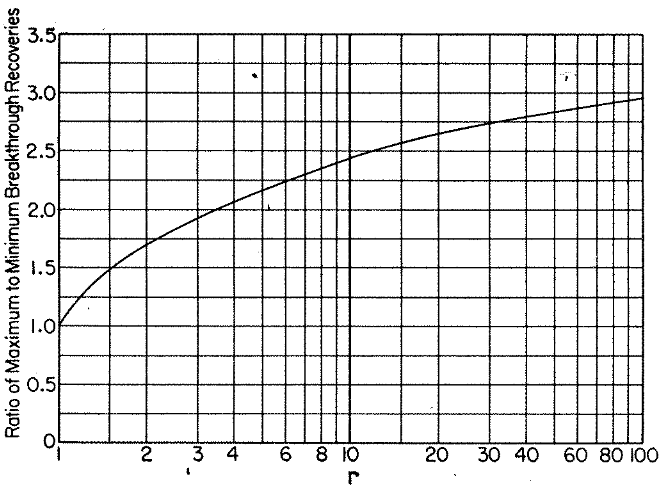

FIG. 2 — THE VARIATION OF THE RATIO OF MAXIMUM TO MINIMUM WATER BREAKTHROUGH RECOVERIES IN EXPONENTIALLY STRATIFIED MEDIA WITH THE STRATIFICATION RATIO, r, FOR THE LIMITS OF INFINITE AND VANISHING MOBILITY RATIOS.

interfaces are the same, as well as the oil saturations in the uninvaded areas,* the rate of water invasion or oil production, in reservoir volume, for any unit thickness lamina at depth z, of homogeneous permeability k, will be expressible as:

$$Q(t, z) = k\ F(kt) \qquad (1)^{**}$$

where the function $F(kt)$ expresses the effect of the displacing fluid invasion on the overall resultant permeability, and the factor k accounts for the absolute permeability of the lamina. The cumulative throughflow to the time t will then be:

$$\overline{Q}(t, z) = k \int_0^t F(kt)\ dt \qquad (2)$$

Initial water breakthrough will develop when the total throughflow equals the volume of clean recoverable oil in the lamina, i.e., when:

$$\overline{Q}(t, z) = AS\overline{f} = k \int_0^t F(kt)\ dt \qquad (3)$$

where A is the initial oil productive area, S the areal sweep efficiency, and $\overline{f}$ the net displacement porosity, i.e., the product of the porosity and the increase in local water saturation in the water invaded area or decrease in oil saturation. Since by Equation (3), $AS\overline{f}$ is evidently a function only of kt, it may be formally inverted so as to give:

$$t(breakthrough) = \frac{\Psi(AS\overline{f})}{k} \qquad (4)$$

where Ψ is the inverse of the integral of Equation (3). Equation (4) shows that the breakthrough times will in general be inversely proportional to k, as in the case of cycling operations where the differences between the invading and displaced fluids are neglected.

The cumulative recovery from the whole section of thickness

*These assumptions of uniformity, together with that for f below, are made both for reasons of simplicity and the lack of any well established alternative assumptions. Actually, however, the method of analysis given here could also be applied if these parameters were taken as variable with depth or k, although the formal analysis would become considerably more complicated.

**While the product kt is indicated in Equation (1) as the basic element of the argument F to emphasize the role played by a potentially varying permeability, it is the ratio of k to the viscosity which is actually the important dynamical variable.

H, at a time t before breakthrough, will then be, as a fraction of the recoverable oil in the reservoir,

$$\bar{Q}(t) = \int_0^H k\,dz \int_0^t F(kt)\,dt \Big/ A \int_0^H \bar{f}\,dz \quad \ldots \quad (5)$$

and that at initial water breakthrough will be:

$$\bar{Q}(t_b) = \int_0^H dz \int_0^{kt_b} F(\tau)\,d\tau \Big/ A \int_0^H \bar{f}\,dz \quad \ldots \quad (6)$$

where t_b is the breakthrough time in the highest permeability layer, as defined by Equation (4) for $k = k(z = H)$. It is assumed in Equations (5) and (6), and in all the subsequent analyses, that the various permeability strata have been rearranged in increasing sequence from $z = 0$ to $z = H$.

After the first water breakthrough in any individual stratum there will be a gradual rise in water-oil ratio as the initial water cusps expand and ultimately completely envelop the producing wells. The history of this latter phase of the oil production will evidently depend on the well pattern, field geometry, and relative producing rates of the wells. While its effect could be formally introduced into the general theory, there is little point in such a generalization, since the assumed functional form for this phase of the history would necessarily have to be quite specific and arbitrary and would not warrant the additional laborious computations required for its numerical evaluation. Accordingly, it will be assumed hereafter that the production in any layer is converted to 100 per cent water as soon as there is any breakthrough at all.* This is equivalent to setting the pattern sweep efficiency, $S = 1$ in Equation (3), etc.

Proceeding to the period after breakthrough in the most permeable layer, i.e., for $t > t_b$, the fractional oil content in the total fluid production will then be:

$$R_o(t) = \frac{\int_0^{z_o} k(z)F(kt)\,dz}{\int_0^H k(z)F(kt)\,dz} \quad \ldots \quad (7)$$

*This simplification involves the further assumption that there is no "subordinate phase" or stripping of the residual oil behind the advancing water-oil interface. This will be further discussed later.

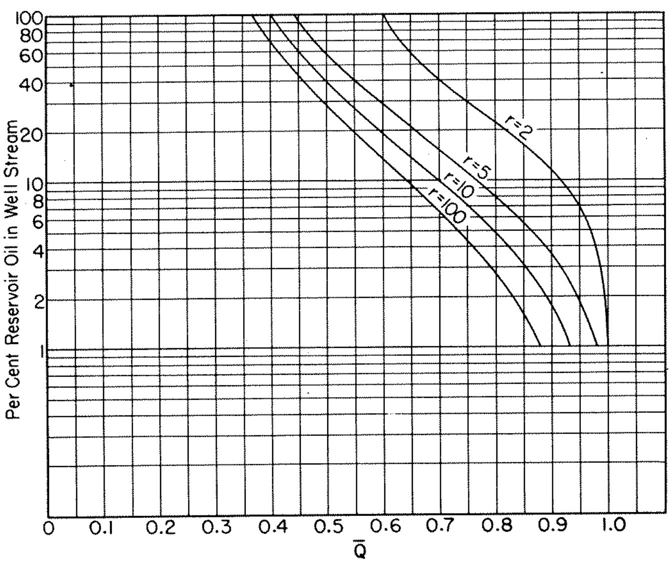

FIG. 3 — THE VARIATION OF THE OIL CONTENT IN THE PRODUCTION FROM EXPONENTIALLY STRATIFIED WATER-DRIVE RESERVOIRS WITH THE CUMULATIVE RECOVERY, FOR $\bar{m} = 0.1$; $\bar{Q}$ = CUMULATIVE RECOVERY, AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. r = STRATIFICATION RATIO; $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER).

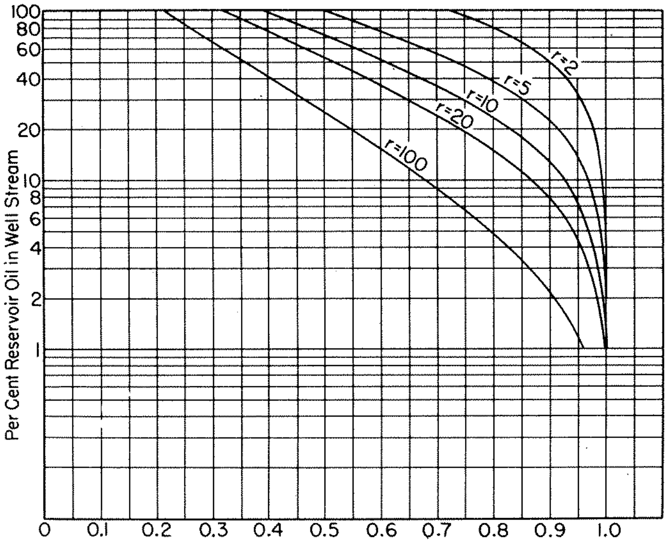

FIG. 4 — THE VARIATION OF THE OIL CONTENT IN THE PRODUCTION FROM EXPONENTIALLY STRATIFIED WATER-DRIVE RESERVOIRS WITH THE CUMULATIVE RECOVERY, FOR $\bar{m} = 1$; $\bar{Q}$ = CUMULATIVE RECOVERY, AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. r = STRATIFICATION RATIO; $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER).

where z_o is determined by the condition:

$$k(z_o) = \frac{\Psi(A\bar{f})}{t} \quad \ldots \quad (8)$$

and represents the depth to which breakthrough has already occurred.

The cumulative oil production as a fraction of the total recoverable oil in the reservoir will then be:

$$\bar{Q}(t) = \left[A \int_{z_o}^H \bar{f}\,dz + \int_0^{z_o} k(z)\,dz \int_0^t F(kt)\,dt\right] \Big/ A \int_0^H \bar{f}\,dz \quad \ldots \quad (9)$$

If the permeability included in the net productive section has a non-vanishing lower limit, the production phase governed by Equations (7) and (8) will terminate after breakthrough in the tightest layer, as determined by Equation (4). Under the assumption of 100 per cent pattern efficiency, i.e., $S = 1$, this will necessarily terminate the whole oil production history, although in practice a limiting value of R_o will generally control the state of abandonment.

The Function F for Linear Flow

As a practical matter drastic simplifications must be made in applying the general equations developed above. As a first step a functional form must be derived for the function F. For this purpose the case of an ideal linear system will be assumed here as was done by Dykstra and Parsons,[5] since anything more complex which might be of practical significance would be virtually intractable from a strictly analytical standpoint.

It may be shown that the rate of throughflow per unit cross section at any time t in a linear system in which a liquid (w) is displacing another (o), may be expressed as:

$$Q = \frac{m_w \bar{f} \alpha L/2}{\sqrt{1 - (1 - \bar{m})\alpha m_w t}}; \quad m = m_o/m_w; \quad \alpha = \frac{2\bar{m}\Delta p}{\bar{f}L^2} \quad . \quad (10)^*$$

*Equation (10) can be derived by applying the analysis given in "Flow of Homogeneous Fluids Through Porous Media," §8.3, for two-fluid linear encroachment systems.

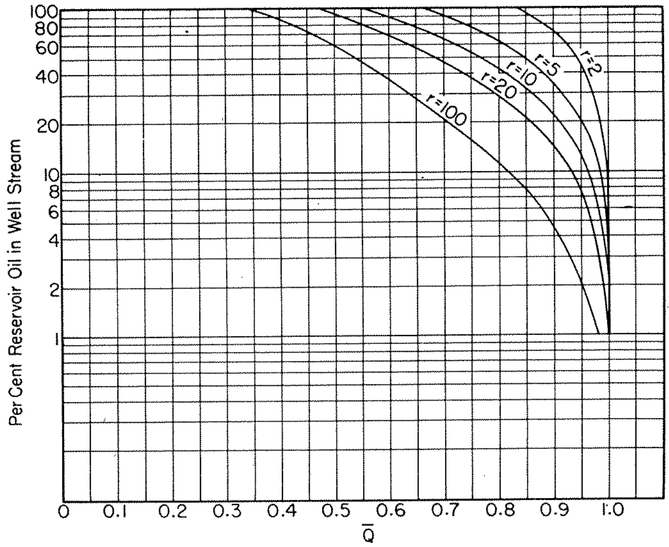

FIG. 5 — THE VARIATION OF THE OIL CONTENT IN THE PRODUCTION FROM EXPONENTIALLY STRATIFIED WATER-DRIVE RESERVOIRS WITH THE CUMULATIVE RECOVERY, FOR $\bar{m} = 10$; $\bar{Q}$ = CUMULATIVE RECOVERY. AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. r = STRATIFICATION RATIO; $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/MOBILITY OF INVADING WATER).

where m_w, m_o are the "mobilities" for the displacing and displaced fluids, L the total length of the system, Δp is the driving pressure differential, which is assumed to be constant,* and $\bar{f}$ is the net displacement porosity. The mobility is defined as the ratio of the effective permeability to the viscosity. By reference to Equation (1) it follows that the function F is given by:

$$F(kt) = \frac{\bar{f} a L/2\mu_w}{\sqrt{1-(1-\bar{m})am_w t}} \quad . \; . \; . \; . \; . \quad (11)$$

The cumulative throughflow per unit cross section to the time t will be, on integrating Equation (10):

$$\bar{Q}(t,z) = \frac{\bar{f}L}{1-\bar{m}}\left[1-\sqrt{1-(1-\bar{m})am_w t}\right] \quad . \quad (12)$$

The time for breakthrough is obtained by setting $\bar{Q}(t,z)$ equal to $\bar{f}L$, and is found to be:

$$t_b = \frac{1+\bar{m}}{am_w} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

To proceed further it is necessary to specify the nature of the permeability, k or m_w, distribution.** Here, too, one must make, as a matter of practical necessity, a further simplification in considering that the mobility ratio $\bar{m}$ is a constant, independent of the absolute permeability. As to the permeability distributions themselves, the exponential and linear distributions will be analyzed in detail.

*It is also tacitly assumed throughout this work that at all times the pressures are above the bubble point, corresponding to a complete water drive operation of an undersaturated reservoir.

**Under the assumption that the relative permeabilities both ahead and behind the water-oil interfaces are independent of the homogeneous fluid permeabilities, the distributions of the latter, as given by Equation (14) or (21), will apply also to the effective permeabilities and mobilities which enter directly in the analysis.

EXPONENTIAL PERMEABILITY DISTRIBUTION

As in the case of the previous[4] study of cycling operations ($\bar{m} = 1$), the exponential permeability distribution will be taken in the form:

$$k = k_{min}\, e^{bz/H};\; b = \log r \quad . \; . \; . \; . \; . \quad (14)$$

where k_{min} is the minimum permeability, at $z = 0$, rk_{min} is the maximum permeability at $z = H$, H is the net thickness of the section and, as already indicated, the various strata are considered to be rearranged in a monotonic sequence increasing with depth. The parameter r defines the particular exponential distribution of interest, and will be termed the stratification ratio. It will also be assumed that there is no cross flow between the strata, which will not be strictly true unless the vertical permeability is zero, and that the net displacement porosity $\bar{f}$ is independent of k, which also will represent an approximation assumption.

Under these assumptions the total throughflow from the whole section at a time t before any breakthrough has developed, as a fraction of the total recoverable oil, will then be given essentially by integrating Equation (12), as:

$$\bar{Q}(t) = \frac{1}{(1-\bar{m})H}\int_0^H \left[1-\sqrt{1-(1-\bar{m})am_w t}\,\right] dz$$

$$= \frac{1}{1-\bar{m}}\left[1-\frac{2}{b}\left\{\sqrt{1-(1-\bar{m}^2)t/t_b}-\sqrt{1-(1-\bar{m}^2)t/rt_b}\right\} - \frac{1}{b}\log\frac{\sqrt{1-(1-\bar{m}^2)t/t_b}-1}{\sqrt{1-(1-\bar{m}^2)t/t_b}+1} + \frac{1}{b}\log\frac{\sqrt{1-(1-\bar{m}^2)t/rt_b}-1}{\sqrt{1-(1-\bar{m}^2)t/rt_b}+1}\right]$$

$$. \; . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

where t_b is the time of initial breakthrough, as given by:

$$t_b = \frac{(1+\bar{m})}{ar\, m_{w\,min}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (16)$$

$m_{w\,min}$ corresponding to k_{min}.

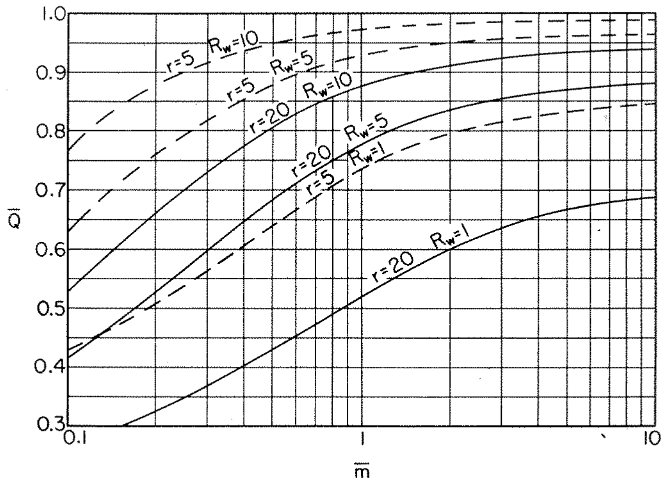

FIG. 6 — THE VARIATION OF THE CUMULATIVE RECOVERY, $\bar{Q}$, IN EXPONENTIALLY STRATIFIED WATER-DRIVE RESERVOIRS WITH THE MOBILITY RATIO $\bar{m}$, TO FIXED VALUES OF THE WATER-OIL RATIO R_w, AND FOR FIXED VALUES OF THE STRATIFICATION RATIO r. $\bar{Q}$ = CUMULATIVE RECOVERY AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER).

At this time $\bar{Q}$ will have the value:

$$\bar{Q}(t_b) = \frac{1}{1-\bar{m}}\left[\, 1-\frac{2}{b}\left\{ \bar{m}-\sqrt{1-(1-\bar{m}^2)/r}\right\} -\frac{1}{b}\log\frac{\bar{m}-1}{\bar{m}+1}+\frac{1}{b}\log\frac{\sqrt{1-(1-\bar{m}^2)/r}-1}{\sqrt{1-(1-\bar{m}^2)/r}+1}\,\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17)$$

After the initial breakthrough the cumulative fractional recovery will be, on applying Equations (10) and (14) to Equation (9):

$$\bar{Q}(t) = 1-\frac{z_o}{H}+\frac{1}{(1-\bar{m})H}\int_o^{z_o}\left[1-\sqrt{1-(1-\bar{m})\alpha m_w t}\,\right]dz$$

$$= 1+\frac{\bar{m}}{(1-\bar{m})b}\log\frac{r}{\bar{t}}-\frac{1}{(1-\bar{m})b}\left[\, 2\left\{\bar{m}-\sqrt{1-(1-\bar{m}^2)\bar{t}/r}\right\} +\log\frac{\bar{m}-1}{\bar{m}+1}-\log\frac{\sqrt{1-(1-\bar{m}^2)\bar{t}/r}-1}{\sqrt{1-(1-\bar{m}^2)\bar{t}/r}+1}\,\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

where z_o is the value of z above which there has been water breakthrough at the time t, and $\bar{t} = t/t_b$. The water-oil ratio will be:

$$R_w = \frac{Q_w}{Q_o} = \frac{\int_o^{z_o} k(z)F(kt)dz}{\int_{z_o}^{H} k(z)F(kt)dz} = \frac{(1-\bar{m}^2)(\bar{t}-1)}{2\bar{m}\left\{\sqrt{1-(1-\bar{m}^2)\bar{t}/r}-\bar{m}\right\}} \quad . \quad (19)$$

It will be readily verified that for $\bar{m} = 1$, as may be assumed for the case of cycling operations, Equations (17)-(19) reduce to:

$$\bar{Q}(t_b) = \frac{1}{rb}(r-1);\; t_b = \frac{\mu_w \bar{f} L^2}{r\Delta p k_{min}(eff)},$$

$$\bar{Q}(t>t_b) = \frac{1}{b}\left[\, 1-\frac{\bar{t}}{r}+\log\bar{t}\,\right]$$

$$R_w = \frac{\bar{t}-1}{1-\bar{t}/r} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (20)$$

which agree with those derived for this case previously.

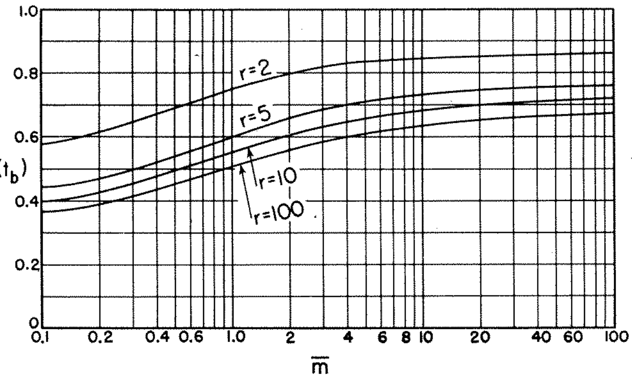

FIG. 8 — THE VARIATION OF THE INITIAL BREAKTHROUGH RECOVERY IN LINEARLY STRATIFIED SYSTEMS WITH THE MOBILITY RATIO OF THE INVADING AND DISPLACED FLUIDS. $\bar{Q}(t_b)$ = FRACTION OF TOTAL DISPLACEABLE OIL RECOVERED AT TIME OF INITIAL BREAKTHROUGH, t_b. $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER); r = STRATIFICATION RATIO OF ASSUMED LINEAR PERMEABILITY DISTRIBUTION.

On the other hand, in the limit of $\bar{m}\to\infty$, in which the invading fluid (water, m_w) has a negligible mobility as compared to that displaced (oil, m_o), Equations (17)-(19) reduce to:

$$\bar{Q}(t_b) = \frac{2}{b}\left(1-\frac{1}{\sqrt{r}}\right);\; t_b = \frac{\mu_w \bar{f} L^2}{2r\Delta p k_{min}(eff)},$$

$$\bar{Q}(t>t_c) = 1-\frac{1}{b}\log\frac{r}{\bar{t}}+\frac{2}{b}\left(1-\sqrt{\bar{t}/r}\right),$$

$$R_w = \frac{(\bar{t}-1)}{2(1-\sqrt{\bar{t}/r})} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (21)$$

And in the converse limit of $\bar{m}\to 0$, where the resistance to the invading fluid is negligible as compared to that displaced, the water-oil ratio will rise precipitously to infinite values immediately after initial breakthrough, and the total fractional recovery $\bar{Q}$ will be essentially that at t_b, as given by Equation (17) for $\bar{m} = 0$.

The values of the fractional flooding recovery at the time of first water breakthrough, $\bar{Q}(t_b)$, as given by Equation (17), are plotted vs. the mobility ratio $\bar{m}$ in Fig. 1 for fixed values of stratification ratio r. As previously anticipated, for $\bar{m}<1$, i.e., when the mobility of the displacing fluid (water) exceeds that of the fluid displaced (oil) the bypassing tendency is accentuated and the breakthrough recovery is reduced. Conversely, when the mobility of the displacing fluid is lower than that displaced, $\bar{m}>1$, the continued entry of the former in the higher permeability strata will automatically retard itself and the breakthrough recovery will be increased. These effects increase with increasing stratification ratios r. The ratio of maximum breakthrough recovery, for $\bar{m} = \infty$, to the minimum at $\bar{m} = 0$, is plotted vs. r in Fig. 2. The total maximum range of the effect of the mobility contrast is 3.26, for $r = \infty$.

Because of the rapid buildup of the water production after initial breakthrough, especially for $\bar{m}<1$, it is difficult to devise a satisfactory graphical representation of this later phase of the production history in terms of the water-oil ratio R_w. It has been found preferable to use as an index the percentage of oil in the composite fluid stream, in reservoir measure. This quantity, denoted by R_o, will be given simply by $1/(1+R_w)$, and is plotted vs. the cumulative fractional displaceable oil recovery, for fixed values of stratification ratio r and several values of mobility ratio $\bar{m}$ in Figs. 3-5.

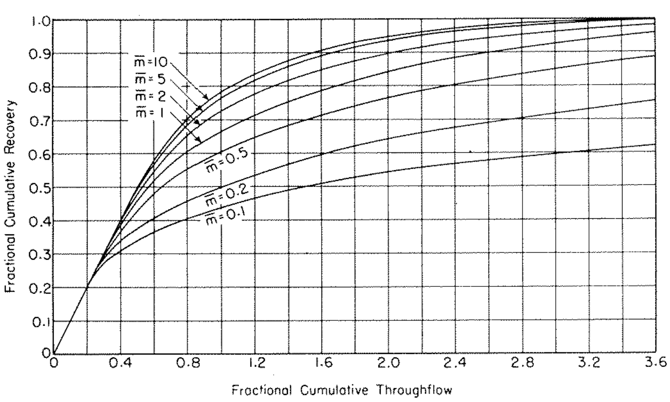

FIG. 7 — THE CUMULATIVE OIL RECOVERY HISTORY VS. THE TOTAL FRACTIONAL CUMULATIVE WATER AND OIL THROUGHFLOW IN EXPONENTIALLY STRATIFIED WATER-DRIVE RESERVOIRS FOR DIFFERENT VALUES OF THE MOBILITY RATIO, $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER). IN ALL CASES THE STRATIFICATION RATIO = 20.

The starting points of the curves of these figures represent the initial breakthrough recoveries. The general shifts of the curves, for mixed $\overline{m}$, to the right as r decreases, are of course to be expected apriori. It is of interest to note, however, that whereas for $\overline{m}<1$, Fig. 3, the systems with lowest breakthrough recoveries, greatest values of r, have the steepest initial rates of fall in oil content of the flow stream, the converse is true for $\overline{m}>1$, Fig. 5. Moreover, whereas for $\overline{m}<1$, the curves tend to be initially concave upward, they ultimately develop steep slopes, for all mobility ratios m, as the oil content of the flow stream becomes small and complete flooding develops throughout the reservoir.

Figs. 3-5 show that the differences in ultimate oil recoveries for reservoirs with different stratification and fluid characteristics will be considerably lower if the operations are continued to low oil contents than at initial water breakthrough. Thus, for $\overline{m} = 0.1$ the recovery at one per cent oil content in the well stream will be only 30 per cent greater for $r = 2$ than for $r = 100$, whereas the breakthrough recovery for $r = 2$ will be 3.7 times as great as for $r = 100$. For $\overline{m} = 10$, the recovery at one per cent oil content for $r = 2$ will exceed that for $r = 100$ by only about two per cent, as compared to a ratio of 2.4 at initial breakthrough. Similarly for r fixed at 20, the breakthrough recoveries for $\overline{m} = 10$ and 0.1 are in the ratio of 2.2, whereas the recoveries at one per cent oil content are in the ratio of only 1.06.

The variation of the recoveries to fixed water-oil ratio, R_w, with the value of $\overline{m}$, for several values of r, is plotted in Fig. 6. Here again will be seen the effect of increasing mobility ratio $\overline{m}$ in improving the resultant recoveries. It will be noted, too, that the recoveries are more sensitive to $\overline{m}$ in the range $\overline{m}<1$ than for $\overline{m}>1$. The gross comparative positions of the curves reflect the degree of reservoir stratification and the limiting water-oil ratios to which the production is continued.

The oil recovery vs. total throughflow or production history can be calculated from the relation (valid for $1 \leqslant \bar{t} \leqslant r$):

$$\overline{V}(\bar{t}) = \overline{Q}(\bar{t}) + \frac{1+\overline{m}}{2\overline{m}b}\left[\bar{t} - 1 - \log \bar{t}\right] \quad . \quad . \quad . \quad (22)$$

where $\overline{V}(\bar{t})$ is the cumulative total liquid production, in reservoir volume, expressed in units of the total displaceable reservoir liquid content. $\overline{Q}(\bar{t})$ is the cumulative fractional oil recovery given by Equation (18). A typical set of production history curves calculated by Equations (22) and (18) are those plotted in Fig. 7 for a stratification ratio $r = 20$. The

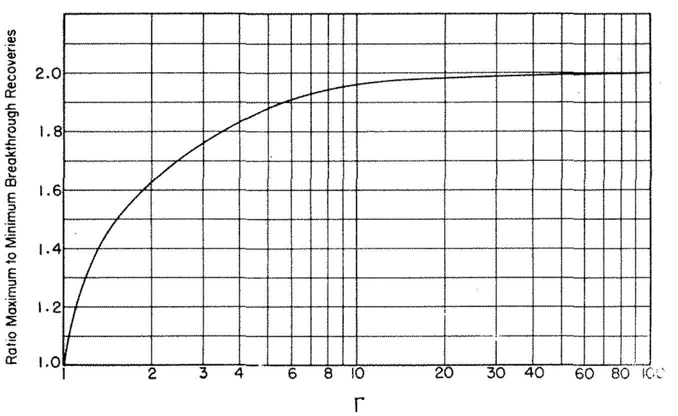

FIG. 9 — THE VARIATION OF THE RATIO OF MAXIMUM TO MINIMUM WATER BREAKTHROUGH RECOVERIES IN LINEARLY STRATIFIED MEDIA WITH THE STRATIFICATION RATIO r, FOR THE LIMITS OF INFINITE AND VANISHING MOBILITY RATIOS.

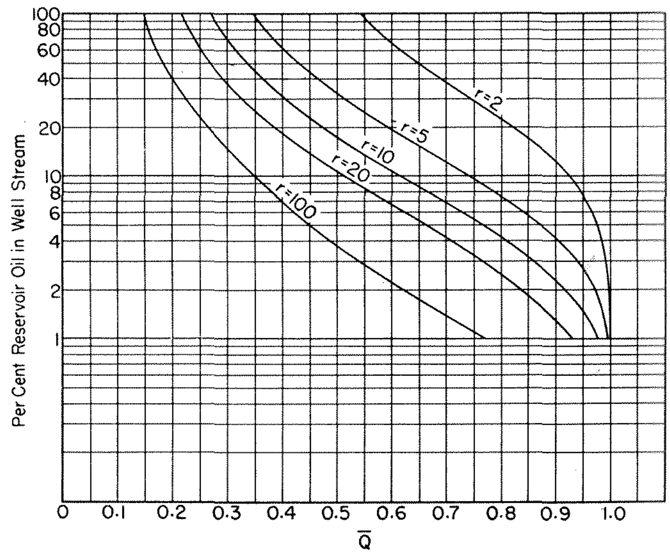

FIG. 10 — THE VARIATION OF THE OIL CONTENT IN THE PRODUCTION FROM LINEARLY STRATIFIED WATER-DRIVE RESERVOIRS WITH THE CUMULATIVE RECOVERY, FOR $\overline{m} = 0.1$; $\overline{Q}$ = CUMULATIVE RECOVERY, AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. r = STRATIFICATION RATIO; $\overline{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER).

effect of the mobility ratio $\overline{m}$ is here strikingly shown by comparative values of the cumulative throughflow, $\overline{V}$, for equal total recoveries, and especially at high values of the latter. Thus, at a total fractional recovery of 35 per cent the total fractional throughflow would be 0.35, 0.36, and 0.55 for $\overline{m} = 10$, 1, 0.1; at a recovery of 50 per cent these would be 0.51, 0.57 and 1.5 respectively; and for 80 per cent cumulative recovery the corresponding fractional throughflows or total reservoir fluid production would be 1.06, 1.68 and 8.0 for $\overline{m} = 10$, 1 and 0.1 respectively.

LINEAR PERMEABILITY DISTRIBUTION

The linear permeability distribution may be defined by:

$$k(z) = k_{min}\left[1 + (r-1)z/H\right] \quad . \quad . \quad . \quad (23)$$

where here, too, the stratification ratio r is the ratio of the maximum permeability rk_{min}, at $z = H$, to the minimum k_{min}, at $z = 0$. The total throughflow at a time t before breakthrough has developed, as a fraction of the total recoverable oil, will be, analogous to Equation (15):

$$\overline{Q}(t) = \frac{1}{(1-\overline{m})H}\int_0^H \left[1 - \sqrt{1-(1-\overline{m})am_w t}\right]dz$$

$$= \frac{1}{1-\overline{m}}\left[1 + \frac{2}{3b}\left\{(a-b)^{3/2} - a^{3/2}\right\}\right] \quad . \quad (24)$$

where:

$$a = 1 - (1-\overline{m}^2)\bar{t}/r; \quad b = (1-\overline{m}^2)(r-1)\bar{t}/r;$$

$$\bar{t} = t/t_b; \quad t_b = \frac{(1+\overline{m})\mu_w}{ar\ k_{min}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24a)$$

The fractional recovery at first water breakthrough, i.e., $\bar{t} = 1$, will then be:

$$\overline{Q}(t_b) = \frac{1}{1-\overline{m}}\left[1 + \frac{2r}{3(r-1)(1-\overline{m}^2)}\left\{\overline{m}^3 - (1-(1-\overline{m}^2)/r)^{3/2}\right\}\right.$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

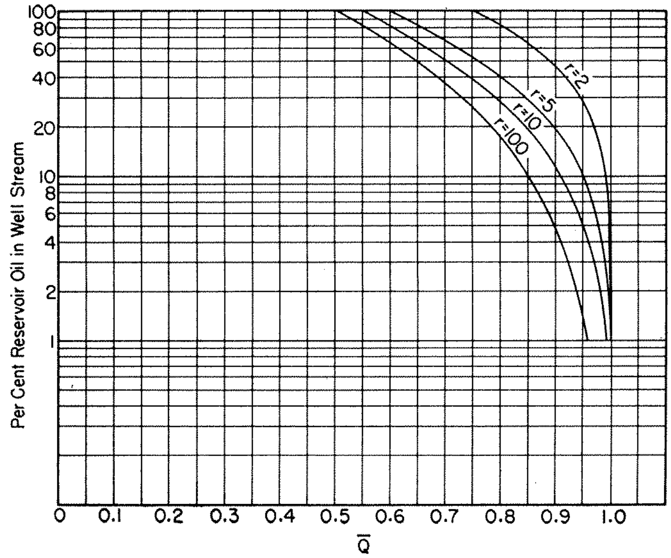

FIG. 11 — THE VARIATION OF THE OIL CONTENT IN THE PRODUCTION FROM LINEARLY STRATIFIED WATER-DRIVE RESERVOIRS WITH THE CUMULATIVE RECOVERY, FOR $\bar{m} = 1$; Q = CUMULATIVE RECOVERY, AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. r = STRATIFICATION RATIO; $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER).

After the initial water breakthrough, $\bar{t} > 1$, the cumulative recovery will be:

$$\bar{Q}(\bar{t}) = 1 - \frac{z_o}{H} + \frac{1}{(1-\bar{m})H} \int_o^{z_o} [1 - \sqrt{1-(1-\bar{m})am_w t}\,]\,dz$$
$$= 1 + \frac{\bar{m}}{1-\bar{m}}\,\frac{(r/\bar{t}-1)}{r-1} + \frac{2r/\bar{t}}{3(1-\bar{m})(1-\bar{m}^2)(r-1)}\{\bar{m}^3 - a^{3/2}\} \qquad (26)$$

where z_o is the value of z above which there has been water breakthrough. It will be noted that at $\bar{t} = 1$, Equation (26) becomes equivalent to Equation (25). And at the time of breakthrough in the tightest layer, $\bar{t} = r$, $\bar{Q}$ becomes unity, as it should under the assumed conditions.

The water-oil ratio during this period is given by:

$$R_w = \frac{Q_w}{Q_o} = \frac{3(1-\bar{m}^2)^2(\bar{t}^2-1)}{4\bar{m}} \Bigg/ \left[\left\{2+(1-\bar{m}^2)\frac{\bar{t}}{r}\right\}\sqrt{1-(1-\bar{m}^2)\bar{t}/r} - \bar{m}(3-\bar{m}^2)\right] \qquad (27)$$

In the limit of $\bar{m} = 1$, corresponding to cycling operations, R_w reduces, as it should, to:

$$R_w = \frac{\bar{t}^2 - 1}{1 - \bar{t}^2/r^2} \qquad (28)$$

It will be noted, too, that at $\bar{t} = 1$, $R_w = 0$, and that it becomes indefinitely large as $\bar{t}$ approaches r.

The total reservoir fluid throughflow, in units of the total displaceable reservoir content volume, may be shown to be given by:

$$\text{Throughflow} = \bar{Q}(\bar{t}): \quad 0 \leqslant \bar{t} \leqslant 1$$
$$= \bar{Q}(\bar{t}) - \frac{r(1+\bar{m})}{2\bar{m}(r-1)}\left[1 - \frac{1}{2}\left(\bar{t} + \frac{1}{\bar{t}}\right)\right] : \quad 1 \leqslant \bar{t} \leqslant r \quad (29)$$

The variation of the initial breakthrough recoveries with the mobility ratio $\bar{m}$, as given by Equation (25), for fixed values of r is plotted in Fig. 8. The general shapes of these curves, reflecting the effect of the mobility ratio on the breakthrough recovery, is similar to that shown in Fig. 1 for the exponential permeability distribution. As a whole the curves of Fig. 8 lie higher than those of Fig. 1, showing that for the same stratification constant and mobility parameter the linear permeability distribution represents a type of reservoir non-uniformity which is not as serious as the exponential distribution. This is further indicated by the closer grouping of the curves of Fig. 8 than the corresponding ones for Fig. 1, which again shows that the effects of the permeability variation will be much more limited when distributed linearly than when it varies exponentially. In fact, whereas the breakthrough and subsequent recoveries become vanishingly small as r approaches infinity for the exponential distribution, the breakthrough recovery approaches the non-vanishing limit $(2\bar{m}+1)/3(1+\bar{m})$ as r becomes infinitely large in the linear permeability distribution.

The gross overall effect of the mobility ratio in the linearly stratified formations, as expressed by the ratio of the maximum to minimum water breakthrough recoveries, is plotted vs. the stratification ratio r in Fig. 9. These represent the ratios $\bar{Q}(t_b)$ for $\bar{m} = \infty$ to that for $\bar{m} = 0$. The curve of Fig. 9 is of the same general shape as that of Fig. 2 for the exponentially stratified system, except that the maximum asymptotic limit has here the value 2.0, as compared to 3.26 in Fig. 2. This once more reflects the smaller total effect of permeability non-uniformity when distributed according to a linear stratification as compared to that when it follows an exponential distribution.

The resultant production histories after first water breakthrough in the linearly stratified systems are plotted in Figs. 10-12. Here, as in the case of the exponential distributions, the ordinates represent the per cent reservoir oil in the composite well stream, and the abscissas are the cumulative fractional oil recoveries. The uppermost points of the various curves give the breakthrough recoveries. The general characteristics of this set of curves are similar to those for the exponential distributions plotted in Figs. 3 to 5. Once again the

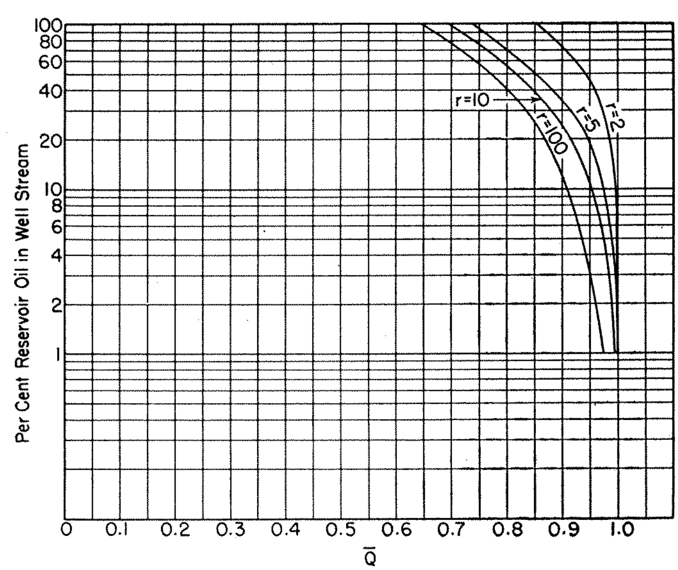

FIG. 12 — THE VARIATION OF THE OIL CONTENT IN THE PRODUCTION FROM LINEARLY STRATIFIED WATER-DRIVE RESERVOIRS WITH THE CUMULATIVE RECOVERY, FOR $\bar{m} = 10$; $\bar{Q}$ = CUMULATIVE RECOVERY, AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. r = STRATIFICATION RATIO; $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER).

more limited effects of the differential mobility in the linearly stratified system are exhibited by the closer grouping of the curves for different stratification constants and their general shift toward the region of higher recoveries. It is for the former reason that in Figs. 10-12 the curves for $r = 20$ have been omitted, because of the small separation between the curves for $r = 10$ and $r = 100$. It will be noted, too, on comparison of the corresponding figures for the linearly and exponentially stratified systems, that while as a group the curves for the former are shifted toward the right, those for the same values of r for the two types of stratification ultimately cross, so as to result in somewhat lower cumulative oil recoveries at very low oil contents in the linearly stratified formations. The general effect of the mobility ratio $\overline{m}$ is again the same for the linear case as it is for the exponential case. That is, the curves shift toward increasing recoveries as $\overline{m}$ is increased, and they become grouped more closely for the different values of the stratification constant. This implies that the effect of the permeability variation will be less serious the higher the value of the mobility ratio $\overline{m}$.

The variation of the recoveries to fixed water-oil ratios R_w with the mobility ratio m, for the two values of r, is plotted in Fig. 13. The similarities and differences between the curves of Fig. 13 and the corresponding ones in Fig. 6 for the exponential distribution are quite apparent, and of the same character as discussed with respect to the composition history curves of Figs. 10-12.

A set of curves, similar to those of Fig. 7, illustrating the cumulative oil recovery vs. the total reservoir fluid production histories is given in Fig. 14 for $r = 20$ in the linear permeability distribution. The implications of these curves are similar to those noted with respect to Fig. 7, and need no further specific discussion.

For the sake of completeness the results analogous to those of Figs. 6 and 13 for the probability distribution have been replotted in Fig. 15 from graphs of Dykstra and Parsons.[5] The parameter characterizing the probability distribution is here the "variation," V, which is related to the standard deviation σ of the probability distribution by:

$$V = 1 - 2^{-\sigma} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

V ranges between 0 and 1 as the corresponding formation changes from one of complete uniformity to infinite variability. It will be seen that the general features of the curves in Fig. 15 are similar to those of Figs. 6 and 13, although the quantitative effect of the mobility ratio $\overline{m}$ seems to be somewhat less in the case of the probability distribution than in either of the other distributions.

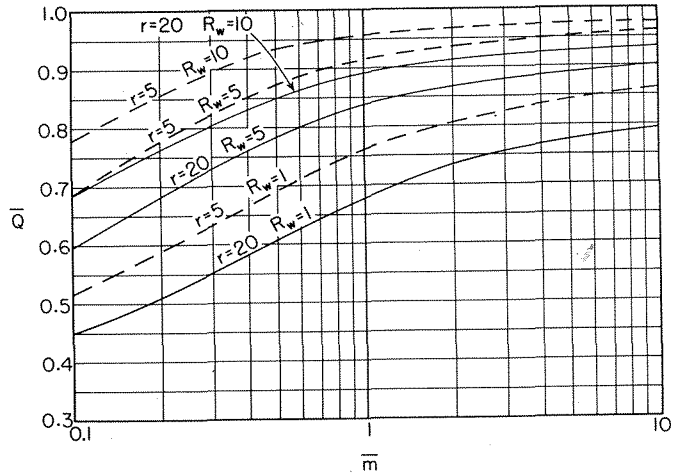

FIG. 13 — THE VARIATION OF THE CUMULATIVE RECOVERY, $\overline{Q}$, IN LINEARLY STRATIFIED WATER-DRIVE RESERVOIRS WITH THE MOBILITY RATIO $\overline{m}$, TO FIXED VALUES OF THE WATER-OIL RATIO R_w, AND FOR FIXED VALUES OF THE STRATIFICATION RATIO r. $\overline{Q}$ = CUMULATIVE RECOVERY AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. $\overline{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER).

DISCUSSION

While the general features of the implications of the theory of the performance of stratified water drive reservoirs have been discussed above, it must be understood that the basic theory involves three fundamental assumptions.

These are: (1) idealized stratification, (2) 100 per cent areal or pattern sweep efficiency, and (3) the absence of stripping action behind the water-oil interface. It is virtually necessary to make each of these assumptions in order to develop a tractable formulation of the theory. However, it is well to understand what their physical implications are and the possible effect they may have on the final numerical results which are calculated by the simplified theoretical analysis.

The assumption of idealized stratification will certainly never be satisfied in practice. Its significance lies only in the concept that it may provide a statistical equivalence to the actual non-uniformity in permeability which does obtain in the producing formation. Such equivalence may be valid, at least in an approximate degree, under conditions where the invading and displaced fluids have the same mobilities, as is generally assumed in the case of cycling operations and which is represented by the special case of $\overline{m} = 1$ in the generalized analytical theory. The steady state pressure and flow distributions under such conditions may then be represented by a steady state type of streamline distribution, which automatically provides a type of stratification which is at least analogous to that assumed in the analytical treatment. The assumption of vanishing cross-flow, which is an integral part of the idealized stratification concept, is then also automatically satisfied in the actual steady state streamline system, although microscopically the resultant streamline stratification will in no sense be coincident with the strictly parallel stratification assumed in the theory.

However, when the mobilities of the displaced and invading fluids are different, the instantaneous steady state representations will continually change as the water invasion proceeds, and with such changes will be associated new streamline distributions. The assumption of vanishing cross-flow will then no longer be valid unless by accident the cross-bedding permeability happens to be strictly zero. It is clear, therefore, that the concept of idealized stratification can at best represent only an extreme simplification. On the other hand, aside from the realistic necessity of making such an assumption in order to arrive at a tractable analysis, it does have the further justification that it probably represents the least favorable type of permeability non-uniformity, as compared to those which actually occur, from the point of view of the initial breakthrough recovery. That is, the simplified theory should at least give minimum values of the breakthrough recoveries. It also seems probable that the recovery history after breakthrough will also be inherently less favorable under idealized stratification and vanishing cross-flow than under actual conditions where there are both two and three dimensional variations of permeability. From this point of view the theory has value in providing at least lower limits for the recovery possibilities in non-uniform water-drive systems, thus giving the least favorable expectations for the performance of such reservoirs. Since it is not feasible to treat arbitrary types of reservoir non-uniformity, it is felt that means for fixing at least the poorest recovery potentialities of the actual reservoir systems does represent a constructive contribution to their analysis.

The assumption of perfect areal or pattern sweep efficiency will also never be valid in practice. It may be recalled, however, that in the earlier study of cycling operations in exponentially stratified reservoirs it was found that the effect of imperfect areal sweep efficiency was rather minor compared to the direct effect of the stratification, except in reservoirs of a very high degree of uniformity. It seems likely, therefore, that in actual producing formations the simplification made in assuming perfect areal pattern sweep efficiency will not in itself cast serious doubt on the significance of the results.

The final major assumption made in the theory developed here lies in assuming that there is no oil stripping in the flooded part of the formation and behind the advancing water-oil interface. This may be expressed by the assumption that the permeability to the oil behind the water-oil interface* is 0. A theory was developed some time ago[6] in which the advance of water in an oil bearing system was analyzed under the more general assumption that the permeability to the oil remained non-vanishing even after passage of the water front. This led to a separation of the composite well producing history into a primary phase corresponding to the displacement of the oil bank ahead of the water-oil interface and a subsequent subordinate phase giving the history of the continued stripping of the oil beind the water-oil front. This latter phase is determined primarily by the relative permeabilities and viscosities to the water and oil behind the water front.

This theory is based on a simple and straightforward application of relative permeability concepts. For the linear** water-flooding system it leads to an equation for the saturation distribution at any time t which may be expressed as:

$$x = \frac{Qt}{f}\frac{dF_w}{d\rho_w} + \Phi(\rho_w) \qquad (31)$$

*As previously noted, it has been further assumed that the amount of this residual oil is independent of the absolute permeability, since satisfactory data are not available on the relationship, if any, between these parameters.

**It may be readily shown that in the similar case of perfect radial systems, Equation (31) will still apply provided x is replaced by πr^2.

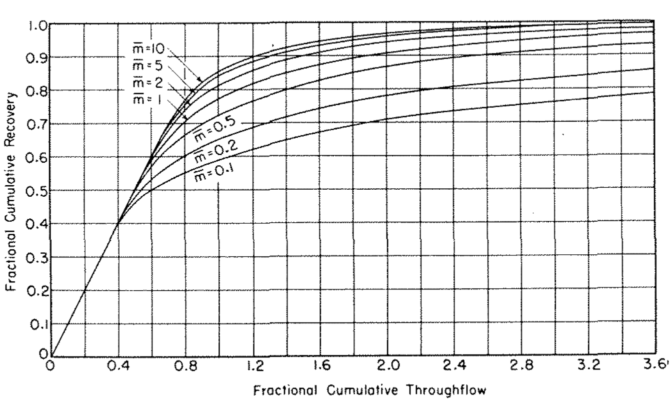

FIG. 14 — THE CUMULATIVE OIL RECOVERY HISTORY VS. THE TOTAL FRACTIONAL CUMULATIVE WATER AND OIL THROUGHFLOW IN LINEARLY STRATIFIED WATER-DRIVE RESERVOIRS FOR DIFFERENT VALUES OF THE MOBILITY RATIO, $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER). IN ALL CASES THE STRATIFICATION RATIO = 20.

where Q is the constant rate of throughflow, f the porosity, F_w the water fraction of the local flow-stream, and $\Phi(\rho_w)$ defines the initial saturation distribution. F_w is given explicitly by:

$$F_w = \frac{m_w}{m_o + m_w} \qquad (32)$$

where m_w, m_o are the mobilities of the water and oil respectively. While Equation (31) apparently involves no basic physical approximations except for the neglect of capillary pressure effects, it leads to the curious and physically meaningless result that the saturation distribution ahead of the initial interface will be multiple-valued.* As noted by Buckley and Leverett,[6,7]** this difficulty may be circumvented when applying the theory by arbitrarily imposing a sharp single valued front at the advanced interface, on requiring that the total increase in water content of the flooded area equal the total volume of invading water. From a physical standpoint, however, it is far from satisfying to have to resort to such an artifice, and the fundamental significance of the whole procedure still remains obscure. Moreover, question may be raised if the relative permeability concepts, as derived from multiphase systems in which two or more phases are continually supplied for flow across each internal cross section, are inherently applicable to displacement processes occurring at the advancing front of an identifiable wetting phase displacing liquid. Intuitively, at least, it is conceivable that under the latter conditions the non-wetting phase would be stripped locally to a discontinuous distribution of vanishing permeability as soon as it has been invaded and passed by the wetting phase "front."

Finally, it may be noted that because of the approximate symmetry of F_w and F_w' for unconsolidated sands about the saturation for maximum slope, when the oil and water viscosities are equal, Equation (31) when applied† to "oil flooding" of a water sand would give essentially the same type of invasion history as for water flooding. This, however, is hardly to be expected in view of the differential wettability between oil and water, and even though the relative permeabilities themselves presumably reflect these wettability differences.

In spite of these theoretical questions regarding the validity of Equation (31), it has apparently served to give a correlation of laboratory water-flooding data.[8] The effect of the water-oil viscosity ratio on the breakthrough recoveries appears to be subject to at least semi-quantitative description by Equation (31). Especially in the range of low invading fluid mobilities, as compared to that of the displaced fluid, the breakthrough recoveries are particularly low, and the corresponding subordinate phase recoveries become of comparable magnitude. While the permeability stratification theory would qualitatively give similar effects, it is admittedly doubtful if in these particular experiments permeability non-uniformity was the controlling factor. If these experimental results and the other implications of Equation (31) should be extended and confirmed for other systems, it will, of course, be neces-

*This ambiguity does not arise when F_w is a linear function of ρ_w or increases monotonically with ρ_w. In the former case, which implies linear k_w and k_o curves, the original water-oil boundary will advance without distortion, and in the latter initial distributions in which ρ_w decreases with x will continually become more stretched out.

**Although the inclusion of the capillary pressure term may formally resolve this ambiguity, the quantitative features of the distributions would then also be modified.

†For oil flooding, Equation (31) retains its general form with ρ_w replaced by ρ_o and F_w by $F_o = 1 - F_w$. When F_w and F_o are not symmetrical for equal viscosities, it may be possible to make them symmetrical by choosing a suitable viscosity ratio.

sary to accept its basic validity even though it has not been fully clarified in all respects.

While these considerations cast doubt on the predominating importance of the permeability non-uniformity in controlling actual water-drive recoveries, the stratification theory presented here should give the implications of at least one major factor in the recovery processes. If the validity of Equation (31) and its underlying concepts should become definitely established, it will be appropriate to consider the composite effect of these two factors. In principle, it should be possible to develop a superposition treatment, analogous to the manner in which the incompleteness of the pattern efficiency was incorporated in the theory of cycling in exponentially stratified media.[9] While it would be hazardous to predict the resultant production histories without making quantitative calculations, it seems reasonable to anticipate that the composite effect would correspond to surprisingly low breakthrough and subsequent recoveries under conditions which could occur in practice. Thus for $r = 20$ in the exponentially stratified distribution, the breakthrough recovery would be only 23 per cent for $\bar{m} = 0.2$ even for sharp water-oil interfaces. And in a uniform stratum the breakthrough recovery for a water-oil viscosity ratio of 0.2 would probably be about 50 per cent or less according to Equation (31). Although the superimposed resultant will not be the simple product of these two factors, the net theoretical effect will undoubtedly indicate discouragingly low breakthrough and overall recoveries. It is evident that this whole subject warrants a great deal of further experimental and theoretical study.

ACKNOWLEDGMENTS

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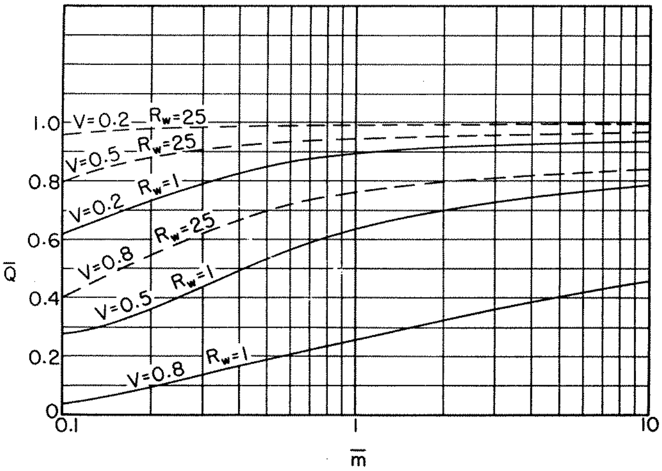

FIG. 15 — THE VARIATION OF THE CUMULATIVE RECOVERY, $\bar{Q}$, IN STRATIFIED WATER-DRIVE RESERVOIRS WITH PROBABILITY PERMEABILITY DISTRIBUTIONS, WITH THE MOBILITY RATIO $\bar{m}$, TO FIXED VALUES OF THE WATER-OIL RATIO R_w, AND FOR FIXED VALUES OF THE "VARIATION" PARAMETER V. $\bar{Q}$ = CUMULATIVE RECOVERY AS A FRACTION OF THE TOTAL RESERVOIR DISPLACEABLE RECOVERY. $\bar{m}$ = (MOBILITY OF DISPLACED OIL)/(MOBILITY OF INVADING WATER).

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