Analysis and Prediction of Minimum Flow Rate for the Continuous Removal of Liquids from Gas Wells

R. G. Turner, SPE-AIME, Baker Oil Tools, Inc.
M. G. Hubbard, SPE-AIME, U. of Houston
A. E. Dukler, U. of Houston

Introduction

Gas phase hydrocarbons produced from underground reservoirs will, in many instances, have liquid phase material associated with them, the presence of which can affect the flowing characteristics of the well. Liquids can come from condensation of hydrocarbon gas (condensate) or from interstitial water in the reservoir matrix. In either case, the higher density liquid phase, being essentially discontinuous, must be transported to the surface by the gas. In the event the gas phase does not provide sufficient transport energy to lift the liquids out of the well, the liquids will accumulate in the wellbore. The accumulation of the liquid will impose an additional back pressure on the formation that can significantly affect the production capacity of the well. In low pressure wells the liquid may completely kill the well; and in the higher pressure wells there can occur a variable degree of slugging or churning of the liquids, which can affect calculations used in routine well tests. Specifically, the calculated bottom-hole pressures used in multirate backpressure tests will be erroneous if the well is not removing liquids on a continuous basis, and gas:liquid ratios observed during such a test may not be correct.

Several authors[1, 3, 8, 14] have suggested methods to determine if the flow rate of a well is sufficient to remove liquid phase material. Vitter[14] and Duggan[1] proposed that wellhead velocities observed in the field would be adequate for keeping wells unloaded. Jones[8] and Dukler[3] presented analytical treatments resulting in equations for calculating, from physical properties, the minimum necessary flow rate. An analysis of these studies indicates the existence of two proposed physical models for the removal of gas well liquids: (1) liquid film movement along the walls of the pipe and (2) liquid droplets entrained in the high velocity gas core. Although there probably is a continuous exchange of liquid between the gas core and the film, they will be treated separately for the purposes of this study. The development and comparison of these separate models with experimental data will permit the determination of which, if either, is the controlling mechanism for the removal of liquids from gas wells.

The Continuous Film Model

Liquid phase accumulation on the walls of a conduit during two-phase gas/liquid flow is inevitable due to the impingement of entrained liquid drops and the condensation of vapors. The movement of the liquid on the wall is therefore of interest in the analysis of liquid removal from gas wells. If the annular liquid film must be moved upward along the walls in order to keep a gas well from loading up, then the minimum gas flow rate necessary to accomplish this is of primary interest. The analysis technique used follows Dukler[2] and Hewitt[5] and involves describing the profile of the velocity of a liquid film moving upward on the inside of a tube. The minimum rate of gas flow required to move the film upward is then calculated.

From an analysis of two models — in one, the movement observed is of a liquid film on the wall of a tubular conduit where the liquid is moved upward by interfacial shear, and in the other it is of the entrained liquid drops in a vertically upward flowing gas stream — it is evident that the minimum condition required to unload a gas well is that which will move the largest liquid drops that can exist in a gas stream.

This page of *Petroleum Transactions, AIME* follows pages 1360. The intervening non-Transactions pages appeared in *Journal of Petroleum Technology*.

The results of this analysis are presented in Fig. 3 and Table 1, and the mathematical film flow model is developed in the Appendix.

Entrained Drop Movement

The existence of liquid drops in the gas stream presents a different problem in fluid mechanics, namely, that of determining the minimum rate of gas flow that will lift the drops out of the well. Since the drop is a particle moving relative to a fluid in the gravitational field, particle mechanics may be employed to determine this minimum gas flow rate.

A freely falling particle in a fluid medium (Fig. 1) will reach a terminal velocity, which is the maximum velocity it can attain under the influence of gravity alone, i.e., when the drag forces equal the accelerating (gravitational) forces. This terminal velocity is therefore a function of the size, shape and density of the particle and of the density and viscosity of the fluid medium.

By a transformation of coordinates, a drop of liquid being transported by a moving gas stream becomes a free falling particle and the same general equations apply. If the gas were moving at a velocity sufficient to hold a drop in suspension (i.e., motionless relative to the conduit), then the gas velocity (the relative velocity between the gas and the drop) would be equal to the free fall terminal velocity of the drop. Since any further increase in the gas velocity would make the drop move upward, the limiting gas flow velocity for upward drop movement is the terminal free settling velocity of the drop.

$$v_t = \sqrt{\frac{2\, g\, m_p\, (\rho_p - \rho)}{\rho_p\, \rho\, A_p\, C_d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The general free settling velocity equation (Eq. 1) shows dependence on the densities of the phases and on the mass and projected area of the particle. Since the surface tension of the liquid phase acts to draw the drop into a spheroidal shape, Eq. 1 can be rewritten in terms of the drop "diameter" (Eq. 2).

$$v_t = 6.55 \sqrt{\frac{d\, (\rho_L - \rho_g)}{\rho_g\, C_d}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Eq. 2 shows that the larger the drop, the higher the terminal velocity, all other things equal. Hence, the larger the drop, the higher the gas flow rate necessary to remove it. The problem, therefore, requires determining the diameter of the largest drop that can exist in a given flow field, and then calculating the terminal velocity of this largest drop. This will insure the upward movement of all drops in the gas stream.

Hinze[6] showed that liquid drops moving relative to a gas are subjected to forces that try to shatter the drop, while the surface tension of the liquid acts to hold the drop together. He determined that it is the antagonism of two pressures, the velocity pressure, $v^2\, \rho_g/g_c$, and the surface tension pressure, σ/d, that determines the maximum size a drop may attain. The ratio of these two pressures is the Weber number $N_{We} = v^2\, \rho_g\, d/\sigma g_c$. Hinze showed that if the Weber number exceeded a critical value, a liquid drop would shatter. For free falling drops, the value of the critical Weber number was found to be on the order of 20 to 30. If the larger of the observed values is used, a relationship between the maximum drop diameter and the velocity of a liquid drop is obtained.

$$d_m = \frac{30\, \sigma\, g_c}{\rho_g\, v_t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Substituting the maximum diameter expression into Eq. 2, the terminal velocity equation becomes

$$v_t = \frac{1.3\, \sigma^{1/4}\, (\rho_L - \rho_g)^{1/4}}{C_d^{1/4}\, \rho_g^{1/2}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The solution of Eq. 4 requires a knowledge of the interfacial tension and the drag coefficient. The interfacial tension can be obtained with sufficient accuracy from handbooks, since it appears to the fourth root. The drag coefficient is influenced by the drop shape and the Drop Reynolds number, $N_{Re} = d\, \rho_g\, v/\mu_g$. A correlation of C_d vs N_{Re} for spheres[11] shows that for a N_{Re} range from 1,000 to 200,000 the drag coefficient is approximately constant (the Newton's law region). For typical field conditions,[1,12] the particle Reynolds number ranges from 10^4 to 10^5, based on the drop size prediction of Eq. 3. This is the range where the drag coefficient is relatively constant at a value of 0.44. If this value is used, and the coefficient is corrected to allow the use of the values of surface tension in dynes per centimeter, Eq. 4 reduces to

$$v_t = 17.6\, \frac{\sigma^{1/4}\, (\rho_L - \rho_g)^{1/4}}{\rho_g^{1/2}} \quad . \quad . \quad . \quad . \quad (5)$$

Eq. 5 may be used to calculate the minimum gas flow velocity necessary to remove liquid drops.

Comparisons with Field Data

The film and drop models have been tested independently with field data obtained from gas wells producing liquids. A small portion of the data was the result of tests performed specifically to determine the minimum lift flow rate. Because of the limited range of conditions involved, these data were insufficient; therefore, previously published data[1,12] and conventional well test data were combined with them to form the current test data matrix.

Included in the data matrix are the two most common flow geometries, standard production tubing in API sizes, and annular completions where the gas is flowed between the casing and the tubing (as in single-tubing-string dual completions).

The conduit sizes included in the data range from 1.750 in. ID (2⅙ in. OD) for tubing to 8 in. for casing. Several annular areas are included, with both 5½-in. and 7-in. OD casings being represented.

Liquid phase material included salt water and condensate, ranging in API gravity from 43° to 70°.

Some of the data were incomplete for the purpose of this investigation, and it was necessary to estimate the values of some properties.

Interfacial tension is not usually determined in routine analysis and it was therefore not obtainable for the individual well fluids. The surface tension of the hydrocarbon liquids was estimated from a correlation based on molecular weight.[10]

Virtually all of the data were incomplete in that the *bottom-hole temperatures* were not reported. In these cases, estimates were made from area geothermal gradient charts, since the location and depth of the wells were known.

The *density of the liquid phase and gravity of the gas* are very important to the developments and, unfortunately, were not available for most of the data. However, the data that were insufficient in this respect did contain the liquid:gas ratio. It is generally true that in wells that produce a small quantity of liquid, the liquid will be clear, very light (high API gravity), and volatile and there will be a correspondingly light (low gravity) gas. And a rich well with a high liquid:gas ratio will generally have more dense liquid and gas phases. Based on these principles and on a knowledge of the ranges of these quantities normally encountered in the field, approximations were made. In the case of water, the specific gravity was taken to be 1.08.

The use of data collected primarily for purposes other than to determine the minimum lift velocity requires a special technique. The conditions (pressure, temperature, tubing size, etc.) of a datum point are used to calculate minimum flow rates by each of the models. The calculated rates are then compared with the observed rate. If the observed rate is known to be adequate, then it should be higher than a properly calculated minimum. If the observed rate is not adequate, then it should be lower than the calculated minimum. Sufficient data should provide statistical validation or invalidation of the mathematical models. An IBM 7094 computer was programmed to test the data in both the film and drop models. Eq. 5 was used to calculate gas velocities in developing the drop model, and integration of Eq. A-3 in the Appendix was performed for the film model calculations. The results are shown graphically in Figs. 2 and 3 and are listed on Table 1.

The figures are constructed in such a way that if a well's actual test flow rate equals its minimum calculated flow rate for liquid removal, the datum point will plot on the diagonal. If the method for calculating the minimum flow rate is accurate, then all wells that are tested at conditions near load-up (shown as circles on the graphs) should plot near this diagonal. Wells that unload easily during a test (shown as squares) should plot above the diagonal and those that do not unload (shown as triangles) should plot below the line. The ability of a given analytical model to achieve this data separation is a measure of its validity.

The drop model (Fig. 2) shows a good separation of the adequate and inadequate flow rates; however, the calculated minima are, in most cases, too low. This can be attributed to the use of drag coefficients for solid spheres rather than for oscillating liquid drops in the development of Eq. 5, and to the fact that the mathematical development predicts stagnation velocity, which must be exceeded by some finite quantity to guarantee removal of the largest drops. Another contributing factor could be the Critical Weber number, which was established for drops falling in air experimentally and not for conditions that

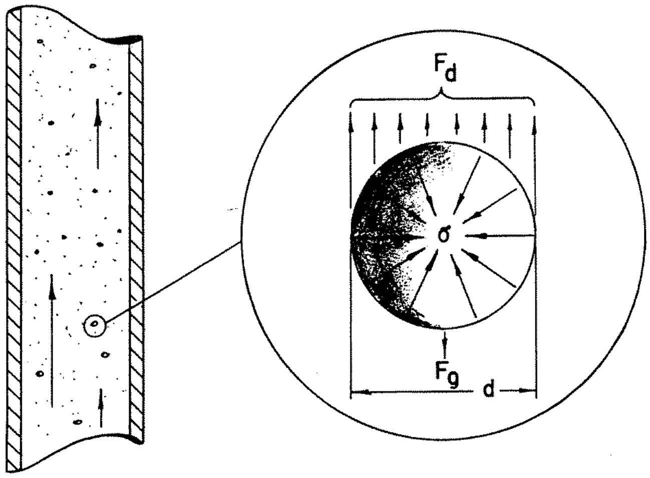

Fig. 1—Entrained drop movement.

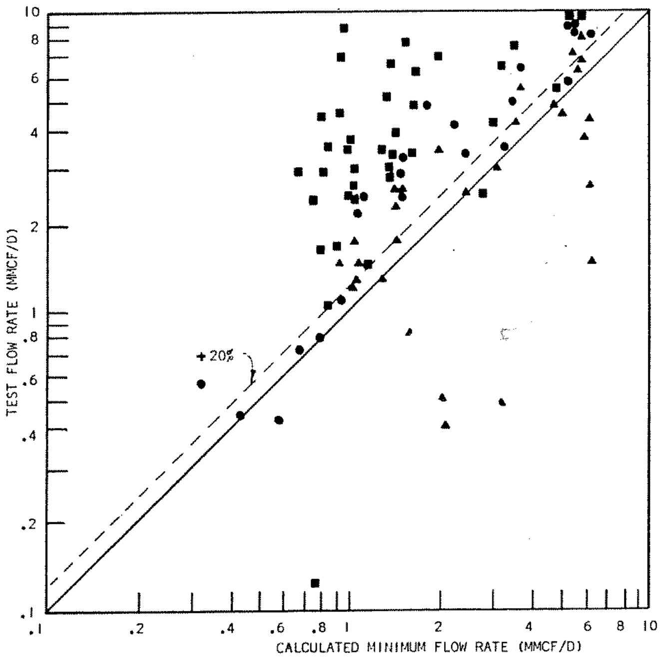

Fig. 2—The drop removal model.

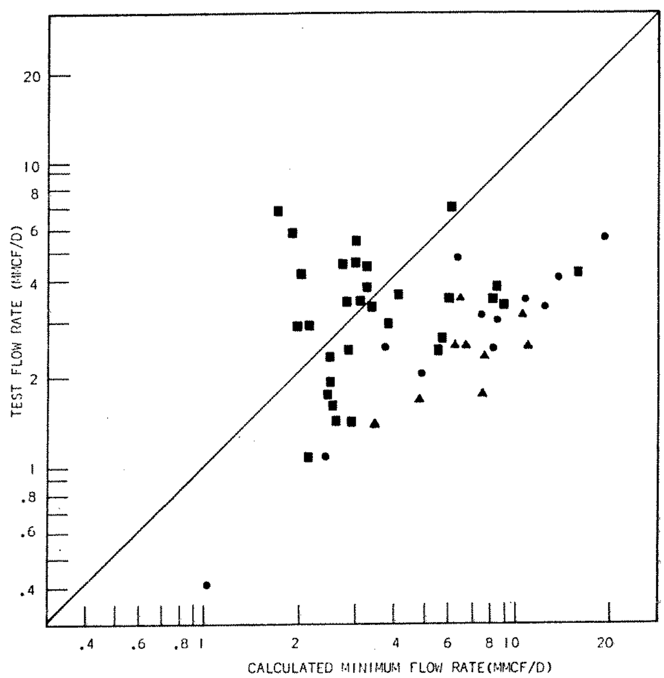

Fig. 3—The film movement model.

TABLE 1—DATA AND PREDICTIONS OF MINIMUM GAS FLOW RATE FOR UNLOADING GAS WELLS

Producing Depth (ft)	Wellhead Pressure (psi)	Condensate Gravity (°API)	Condensate Make (bbl/MM)	Water Make (bbl/MM)	Tubing ID (inches)	Tubing OD (inches)	Casing ID (inches)	Test Flow (Mcf/D)	Drop Model (Mcf/D)	Film Model (Mcf/D)	Status During Test
6404	725	63.8	6.0	0.	2.441			775	779		Near L.U.
6739	400		0.	18.0	1.995			417	583	1098	Near L.U.
6529	108	64.3	9.6	12.4	2.041			568	306		Near L.U.
6700	540	70.8	10.5	10.5	1.995			712	661		Near L.U.
6770	450	61.0	11.3	0.	1.995			442	419		Near L.U.
11200	3607	61.0	37.4	0.	1.995			1525	1156	3453	Loaded Up
11200	3434	61.0	37.4	0.	1.995			2926	1150	3866	Unloaded
11340	3773	58.0	36.8	0.	1.995			2494	1158	3811	Questionable
11340	3660	58.0	36.8	0.	1.995			3726	1142	4235	Unloaded
11416	3340	56.4	130.8	0.	2.992			2611	2412	13028	Loaded Up
11416	3295	56.4	130.8	0.	2.992			3264	2401	14199	Questionable
11416	3280	56.4	130.8	0.	2.992			4095	2395	15511	Questionable
11417	3540	56.4	113.5	0.	2.441			1814	1635	7247	Loaded Up
11417	3330	56.4	113.5	0.	2.441			2915	1600	8551	Questionable
11426	3525	55.0	106.9	0.	1.995			1792	1108	4780	Loaded Up
11426	3472	55.0	106.9	0.	1.995			2572	1085	5410	Unloaded
11355	3338	55.0	117.6	0.	2.441			2261	1623	7952	Loaded Up
11355	3245	55.0	117.6	0.	2.441			2503	1610	8212	Questionable
11355	3092	55.0	117.6	0.	2.441			3351	1574	8992	Unloaded
11390	3556	55.0	104.3	0.	1.995			2069	1091	4916	Questionable
11390	3455	55.0	104.3	0.	1.995			2769	1082	5505	Unloaded
8690	3665	60.0	68.3	0.	2.441			2542	1660	6867	Loaded Up
8690	3644	60.0	68.3	0.	2.441			3182	1654	7439	Questionable
8690	3615	60.0	68.3	0.	2.441			3890	1648	8040	Unloaded
8840	3212	60.0	54.8	0.	2.441			2547	1604	6057	Loaded Up
8840	3025	60.0	54.8	0.	2.441			3517	1569	6580	Unloaded
11850	8215	67.5	10.8	0.	2.441			3472	1956	6495	Loaded Up
11850	7950	67.5	10.8	0.	2.441			4896	1941	6524	Questionable
11850	7405	67.5	10.8	0.	2.441			6946	1930	6676	Unloaded
6995	2335	65.0	17.9	0.	1.995			1116	936	2563	Questionable
6995	2226	65.0	17.9	0.	1.995			1959	910	2504	Unloaded
5725	2182	70.0	2.5	0.		4.500	6.184	5501	3767		Loaded Up
5725	2175	70.0	2.5	0.		4.500	6.184	6405	3757		Questionable
5725	2169	70.0	2.5	0.		4.500	6.184	7504	3747		Unloaded
5515	1590	65.0	13.1	0.	3.958			3009	3281	10983	Loaded Up
5515	1550	65.0	13.1	0.	3.958			3551	3233	10820	Questionable
5515	1520	65.0	13.1	0.	3.958			4150	3195	10711	Unloaded
6180	1245	67.0	10.3	0.		2.875	6.184	4441	4920		Loaded Up
6180	1184	67.0	10.3	0.		2.875	6.184	4843	4793		Loaded Up
6180	1117	67.0	10.3	0.		2.875	6.184	5513	4649		Unloaded
6031	1958	62.5	24.8	0.		2.875	6.184	8185	5931		Loaded Up
6031	1938	62.5	24.8	0.		2.875	6.184	9039	5902		Questionable
6031	1913	62.5	24.8	0.		2.875	6.184	9897	5857		Unloaded
5962	2040	65.0	31.8	0.		2.875	6.184	6702	6082		Loaded Up
5962	1993	65.0	31.8	0.		2.875	6.184	8210	6015		Questionable
5962	1953	65.0	31.8	0.		2.875	6.184	9289	5957		Unloaded
5906	2284	67.5	15.1	0.		3.500	6.184	7109	5580		Loaded Up
5906	2271	67.5	15.1	0.		3.500	6.184	8406	5559		Questionable
5906	2256	67.5	15.1	0.		3.500	6.184	9747	5535		Unloaded
5934	2352	70.0	3.7	0.		3.500	6.184	6361	5641		Loaded Up
5934	2338	70.0	3.7	0.		3.500	6.184	8057	5671		Questionable
5934	2223	70.0	3.7	0.		3.500	6.184	9860	5485		Unloaded
5934	2003	70.0	3.7	0.		3,500	6.184	11767	5212		Unloaded
6850	2042	65.0	26.7	0.		4.500	6.184	4124	3613		Loaded Up
6850	1818	65.0	26.7	0.		4.500	6.184	4998	3412		Questionable
6850	1600	65.0	26.7	0.		4.500	6.184	6423	3199		Unloaded
7346	1835	52.7	27.8	0.4	1.995			8672	1239		Unloaded
7346	2421	52.7	27.8	0.4	1.995			6654	1407		Unloaded
7346	2705	52.7	27.8	0.4	1.995			5136	1467		Unloaded
7346	2884	52.7	27.8	0.4	1.995			3917	1502		Unloaded
8963	5056	43.9	7.5	1.4	1.995			3376	1770		Unloaded
8963	4931	43.9	7.5	1.4	1.995			4830	1732		Unloaded
8963	4786	43.9	7.5	1.4	1.995			6221	1705		Unloaded
8963	4575	43.9	7.5	1.4	1.995			7792	1659		Unloaded
5294	1902	71.0	30.9	0.	1.995			1138	851	2276	Unloaded

exist in gas wells. Analysis of the data reveals that the total contribution of these factors requires an upward adjustment of approximately 20 percent. Instead of being distributed as individual contributions among the pertinent parameters in the development, this value is lumped in the constant of Eq. 5 to produce Eq. 6.

$$v_t = 20.4 \frac{\sigma^{1/4} (\rho_L - \rho_g)^{1/4}}{\rho_g^{1/2}} \quad . \quad . \quad . \quad . \quad (6)$$

Since the contributing factors are individually obtained from experimental correlations, their adjustment, in this case, to fit the specific data does not alter or affect the rigor of the development.

The predictions of the film model (Fig. 3) do not provide as clear a definition between the adequate and inadequate rates as do those of the drop model. Additionally, the theoretical development for the film model indicates that the minimum lift velocity depends upon the gas:liquid ratio. Analysis of the available field data shows no such dependence in the range of liquid production associated with most gas wells (1 to 100 bbl/MMcf). The drop model, on the other hand, is independent of a liquid rate. This indicates that the film model does not represent the controlling liquid transport mechanism.

The data were tested for the minimum flow rate that would be required at the top and the bottom of the conduit. The results indicated that the wellhead conditions were, in most instances, controlling (i.e., required the higher flow rate). This is fortunate, since it allows the use of the more easily obtained surface data.

Since in some of the field observations the wells were known to be unloading, but the film model predicted the gas rates to be inadequate, it appears that the liquids can be continuously removed by liquid drop movement alone. It is of interest, therefore, to know what happens to a film that is not moving upward with the gas. If the liquid film moves downward,

TABLE 1 (Contd.)—DATA AND PREDICTIONS OF MINIMUM GAS FLOW RATE FOR UNLOADING GAS WELLS

Producing Depth (ft)	Wellhead Pressure (psi)	Condensate Gravity (°API)	Condensate Make (bbl/MM)	Water Make (bbl/MM)	Tubing ID (inches)	Tubing OD (inches)	Casing ID (inches)	Test Flow (Mcf/D)	Drop Model (Mcf/D)	Film Model (Mcf/D)	Status During Test
5294	1737	71.0	0.9	0.	1.995			1712	814		Unloaded
5294	1480	71.0	0.9	0.	1.995			2473	750		Unloaded
5294	1246	71.0	0.9	0.	1.995			2965	686		Unloaded
5234	1895	71.7	54.1	0.	1.995			1797	875	2652	Unloaded
5234	1861	71.7	54.1	0.	1.995			2502	859	2863	Unloaded
5234	1784	71.7	54.1	0.	1.995			3460	832	3108	Unloaded
5234	1680	71.7	54.1	0.	1.995			4439	803	3309	Unloaded
7639	2814	53.5	3.3	1.0	1.750			1596	1216		Unloaded
7639	2582	53.5	3.3	1.0	1.750			2423	1176		Unloaded
7639	2104	53.5	3.3	1.0	1.750			3598	1070		Unloaded
7639	1575	53.5	3.3	1.0	1.750			4410	918		Unloaded
7475	2783	52.4	3.4	0.	1.750			2939	834	2155	Unloaded
7475	2655	52.4	3.4	0.	1.750			4140	817	2097	Unloaded
7475	2406	52.4	3.4	0.	1.750			5820	770	1953	Unloaded
7475	2205	52.4	3.4	0.	1.750			6871	746	1884	Unloaded
7546	2574	52.2	4.1	0.6	1.750			1943	899		Unloaded
7546	2224	52.2	4.1	0.6	1.750			2910	833		Unloaded
7546	1839	52.2	4.1	0.6	1.750			3742	755		Unloaded
7546	1509	52.2	4.1	0.6	1.750			4485	683		Unloaded
7753	2611	52.6	5.5	0.	1.995			3436	1082	2954	Unloaded
7753	2527	52.6	5.5	0.	1.995			4471	1058	2881	Unloaded
8162	2556	56.7	7.7	0.	1.995			1550	1026	2801	Unloaded
8162	2415	56.7	7.7	0.	1.995			1804	996	2697	Unloaded
8162	2149	56.7	7.7	0.	1.995			2385	941	2512	Unloaded
8162	1765	56.7	7.7	0.	1.995			2949	856	2246	Unloaded
7810	2862	52.2	5.0	0.		2.375	4.974	3024	5098		Unloaded
7810	2823	52.2	5.0	0.		2.375	4.974	3863	5045		Loaded Up
7531	760	54.9	46.1	45.1	2.441			1247	1148		Loaded Up
7531	704	54.9	31.6	40.8	2.441			1313	1099		Loaded Up
7531	822	54.9	26.7	26.3	2.441			1356	1197		Loaded Up
7531	1102	54.9	26.1	23.8	2.441			1365	1419		Loaded Up
7531	552	54.9	25.1	22.3	2.441			1607	958		Near L.U.
3278	315	50.0	10.0	0.	7.386			5740	5093	19974	Loaded Up
3278	422	50.0	10.0	0.	7.386			3890	5923		Loaded Up
3278	459	50.0	10.0	0.	7.386			2780	6186		Loaded Up
3278	484	50.0	10.0	0.	7.386			1638	6359		Loaded Up
5080	500	50.0	14.0	0.		2.375	4.974	400	2184		Loaded Up
7200	500	0.	0.	5.0		2.375	4.052	800	1726		Loaded Up
6776	660	0.	0.	3.5		2.375	6.276	4300	6367		Loaded Up
3077	280	0.	0.	28.0		2.375	4.974	500	2083		Loaded Up
2250	210	0.	0.	24.0		2.375	6.276	470	3248		Loaded Up

it is then moving countercurrent to the gas and "flooding" occurs. This is a condition in which the film thickens and bridges the tube, causing film break-up and slugging, which leads to the production of drops and to increased entrainment. The flooding of the film, along with the activity of the liquid at discontinuities such as the coupling recess, provides an ample source of liquid drops for transport by the drop mechanism.

Application to Field Design

For field application it is highly desirable to have a simple method of determining the minimum flow rate necessary to insure continuous liquid removal. Although the equations required to calculate this rate are not particularly complex, a slide rule or logarithm tables are necessary. It is worthwhile, therefore, to investigate methods of simplifying the equations.

Since drop removal is the limiting liquid removal mechanism, Eq. 6 for terminal drop velocity will be used for the field application. The grouping of parameters is such that we can simplify the equation to a relationship suitable for graphical solution.

Since the fourth root of the surface tension of low molecular weight hydrocarbons varies only slightly with changes in molecular weight and temperature, a consolidation of the $\sigma^{1/4}$ term into a constant for condensates is indicated. For water, another constant may be used. (Values of 20 dynes/cm for condensate and 60 dynes/cm for water were chosen.) The liquid phase density for condensates will vary between 51.5 lb mass/cu ft (40° API) and 43.8 lb mass/cu ft (70° API). Therefore, the liquid phase density for condensates (the fourth root of which is also used) may be treated as a constant (45 lb mass/cu ft). Water will also have a relatively constant density (67 lb mass/cu ft).

This leaves two equations (one each for water and condensate) in which the terminal velocity is a function of the gas phase density. Gas density is a function of the pressure, temperature, and gas gravity. An investigation of the relative impact of variations of these parameters in ranges normally encountered in gas wells shows that gas gravity and absolute temperature have less effect than do variations in pressure. Further simplification is possible by using an average value of gas gravity (0.6) and gas temperature (120°F). This yields Eqs. 7 and 8, which are the gas velocity equations for water and condensate, respectively.

$$v_g\,(\text{water}) = \frac{5.62\,(67 - 0.0031p)^{1/4}}{(0.0031p)^{1/2}} \quad . \quad (7)$$

$$v_g\,(\text{condensate}) = \frac{4.02\,(45 - 0.0031p)^{1/4}}{(0.0031p)^{1/2}} \quad . \quad (8)$$

The interdependence of flow rate and pressure, due to reservoir deliverability, precludes having a direct minimum flow rate calculation for a particular well. However, a minimum flow rate for a particular set of conditions (pressure and conduit geometry) can be calculated using Eqs. 7 and 8 and Eq. 9.

$$q_g\,(\text{MMcf/D}) = \frac{3.06\,p\,v_g\,A}{T\,z} \quad . \; . \; . \; . \; . \quad (9)$$

Eqs. 7 through 9 allow the construction of a nomograph for direct solution of these equations (Fig. 4). Fig. 4 allows consideration of all values in the foregoing equations except the gas deviation factor z.

The nomograph is used by starting at the pressure of interest, going vertical to the proper line, then horizontal to the edge of the grid. This is the minimum gas velocity. From this point a line is drawn through the p/T line to the intermediate line, and from this line through the flow area line to the qz line.

For accurate flow rates, the deviation factor for the existing conditions should be divided into the qz term. The sample problem shown in Fig. 4 was for a hypothetical well with a wellhead pressure of 1,150 psia, producing through a 5½-in., 15.5-lb × 2⅜-in., 4.5-lb casing-tubing annulus (0.11 sq ft) and a wellhead temperature of 140°F, producing salt water along with the gas. The grid portion of the nomograph shows a required minimum gas velocity of 8.2 ft/sec, and subsequent progression through the nomograph shows a qz product of 5.4 MMcf/D. For these conditions a deviation factor of approximately 0.88 would exist, and the resulting minimum required flow rate would be 6.15 MMcf/D.

Conclusion

The minimum flow conditions necessary to remove liquids from gas wells are those that will provide a gas velocity sufficient to remove the largest drops that can exist. This velocity can be calculated using particle and drop break-up mechanics. The equation derived must be adjusted upward by approximately 20 percent to insure removal of all drops. The gas flow rate required to produce this velocity may be calculated and compared with existing conditions to determine the adequacy or inadequacy of the particular flow test. The derived equations are not limited to tubing, but can be used in annular and other flow geometries also. The gas:liquid ratio does not influence the minimum lift velocity in the observed ranges of liquid production up to 130 bbl/MMcf, and the liquid may be water and/or condensate. If both liquids are present, the properties of the denser of the two should be used in the equation, since the higher density material will be the controlling factor.

Nomenclature

A = flow area of conduit, sq ft
A_p = projected area, sq ft
C_d = drag coefficient
d = diameter of conduit, ft
d_p = diameter of liquid drop, ft
d_m = maximum diameter of liquid drop, ft
g_c = gravitational constant = 32.17 lb mass ft/lb force sec²
g = local acceleration of gravity, ft/sec²
k = constant = 0.36
h = film thickness, ft
m_p = mass of falling particle, lb mass
N_{Re} = Reynolds number = $\rho\, d\, v/\mu$
N_{We} = Weber number = $\rho\, v^2\, d/\sigma\, g_c$
p = pressure, lb force/sq in
q_g = gas flow rate, MMcf/D
T = temperature, °R

v = velocity, ft/sec
v_t = terminal velocity of free falling particle, ft/sec
w_L = liquid phase flow rate, lb mass/sec
y_m^+ = dimensionless distance parameter evaluated at center of conduit
z = gas deviation factor
$\left(\frac{\Delta p}{\Delta x}\right)_{TP}$ = two-phase pressure drop, lb force/sq ft/ft
μ_g = gas phase viscosity, lb mass/ft sec
μ_L = liquid phase viscosity, lb mass/ft sec
ρ_g = gas phase density, lb mass/cu ft
ρ_L = liquid phase density, lb mass/cu ft
ρ_p = density of particle, lb mass/cu ft
σ = interfacial tension, dynes/cm
τ = shear stress, lb force/sq ft
τ_0 = shear stress at the wall, lb force/sq ft
τ_i = shear stress at the gas/liquid interface, lb force/sq ft
$\phi = (y_m^+ - 60)/22$

References

1. Duggan, J. O.: "Estimating Flow Rate Required to Keep Gas Wells Unloaded", *J. Pet. Tech.* (Dec., 1961) 1173-1176.
2. Dukler, A. E.: "Fluid Mechanics and Heat Transfer in Vertical Falling Film Systems", *Chem. Eng. Prog.* (1960) **56,** No. 30.
3. Dukler, A. E.: "Design of Multi-Phase Flow Systems", paper presented at Southwestern District Meeting, API, Albuquerque, N. M., March, 1961.
4. Gill, W. N. and Scher, M.: "A Modification of the Momentum Transport Hypothesis", *AIChE Jour.* (March, 1961) **7,** No. 1, 61.
5. Hewitt, G. F.: "Analysis of Annular Two-Phase Flow: Application of the Dukler Analysis to a Vertical Upward Flow in a Tube", R-4033 Atomic Energy Royal Establishment, Harwell, England (1961).
6. Hinze, J. O.: "Fundamentals of the Hydrodynamic Mechanism of Splitting in Dispersion Processes", *AIChE Jour.* (Sept., 1955) **1,** No. 3, 289.
7. Hinze, J. O.: "Critical Speeds and Sizes of Liquid Globules", *Applied Scientific Research* (1949) **A-1,** No. 4, 273.
8. Jones, Park J.: *Petroleum Production,* Reinhold Publishing Corp., New York (1947) **3,** 100.
9. Lockhart, R. W. and Martinelli, R. C.: "Proposed Correlation of Data for Isothermal Two-Phase, Two-Component Flow in Pipes", *Chem. Eng. Prog.* (Jan., 1949) **45,** No. 1, 39-48.
10. *Natural Gasoline Supply Men's Association Engineering Data Book* (1951) 144.
11. Perry, J. H.: *Chemical Engineers Handbook,* 4th Ed., McGraw-Hill Book Co., Inc., New York (1963) 5-60.
12. Rawlins, E. L. and Schellhardt, M. A.: "Back Pressure Data on Natural Gas Wells and Their Application to Production Practices", *Monograph No. 7,* USBM (1935) 34.
13. Turner, R. G.: "An Analysis of the Continuous Removal of Liquids from Gas Wells", MS thesis, U. of Houston, (1967).
14. Vitter, A. L.: "Back Pressure Tests on Gas-Condensate Wells", *Drill. and Prod. Prac.,* API (1942) 79.

APPENDIX

Film Model Development

The co-current vertical upward flow of gas core-liquid film systems has been studied in several laboratory investigations, and its theoretical understanding has advanced to a point where mathematical modeling is possible. The approach presented here is after Hewitt[5] and his treatment of the Dukler[2] analysis.

In an annular liquid film (thickness h) on the walls of a vertical tube, the transport in the upward direction is a result of the interfacial shear (τ_i) of the moving gas on the surface of the liquid (Fig. 5). This motion is resisted by the action of gravity and wall friction. At any point y distance from the wall, there

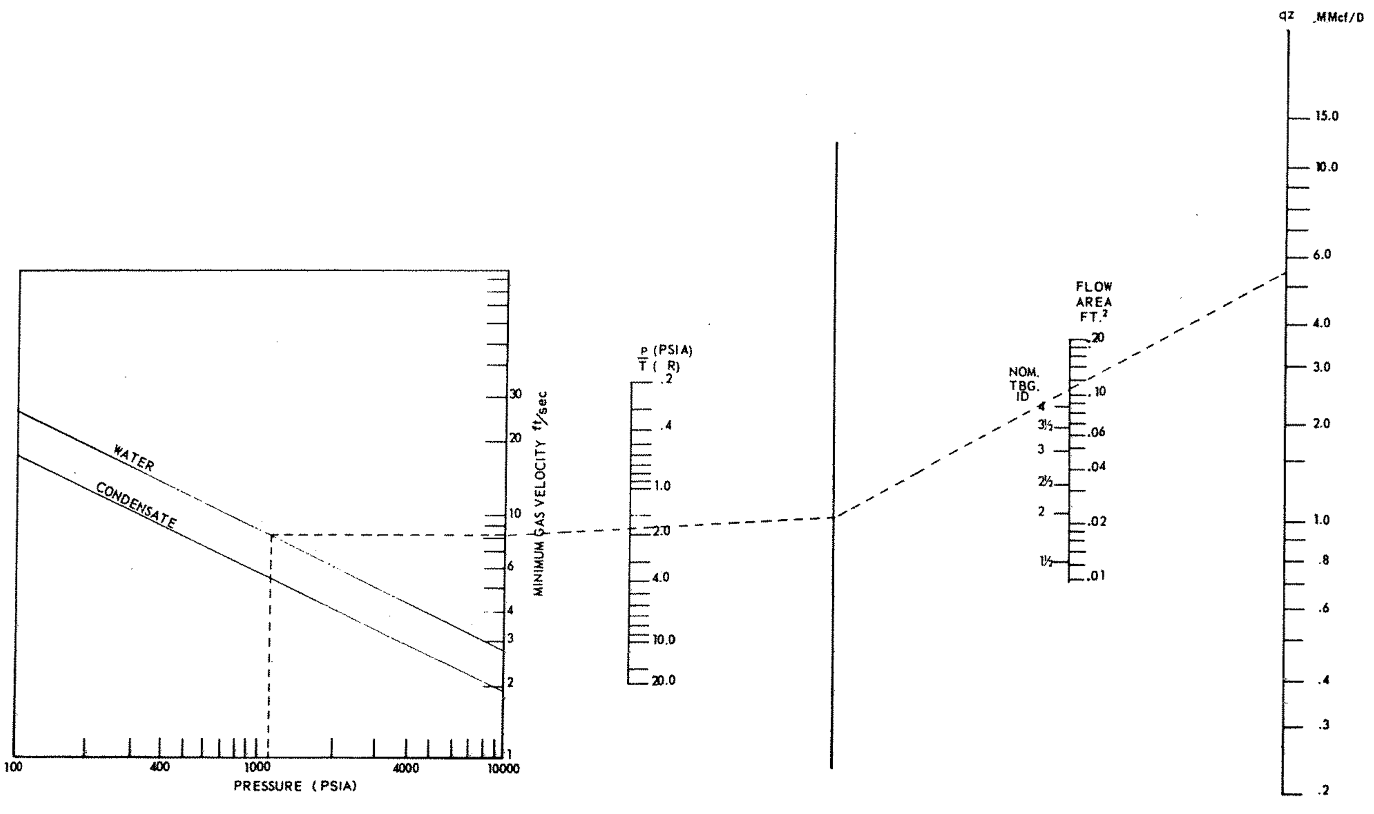

Fig. 4—Minimum flow rate nomograph.

exists a velocity v and a shear stress τ. The resisting shear stress at the wall is τ_0. A steady-state force balance shows that at any point y,

$$\frac{\tau}{\tau_0} = 1 + \frac{y\,\rho_L\,g}{\tau_0\,g_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A\text{-}1)$$

In dimensionless form, Eq. A-1 becomes

$$\frac{\tau}{\tau_0} = 1 + y^+\frac{\sigma^3}{\eta} \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad (A\text{-}2)$$

where

$$\sigma^3 = \frac{h^3\,\rho_L^2\,g}{\eta^2\,\mu_L^2}$$

$$y^+ = \frac{v^*\,y\,\rho_L}{\mu_L} \text{ (dimensionless distance parameter)}$$

$$v^* = \sqrt{\frac{\tau_0\,g_c}{\rho_L}} \text{ ("friction" velocity)}$$

$$v^+ = \frac{v}{v^*} \text{ (dimensionless velocity parameter)}$$

$$\eta = \frac{h\,v^*\,\rho_L}{\mu_L} \text{ (dimensionless film thickness)}$$

Eq. A-2 is the shear stress distribution as a function of the distance from the wall of the tube. By using the Gill and Scher[4] momentum transport hypothesis (eddy viscosity equation) and Eq. A-2, the dimensionless velocity distribution in the flow stream is obtained.

$$v^+ = \int_0^{y^+} \cdot \frac{2\left(1 + y\frac{\sigma^3}{\eta}\right)}{1 + \sqrt{1 + 4k^2y^{+2}\left(1 - e^{\frac{-\phi y^+}{y^{+m}}}\right)^2\left(1 + y^+\frac{\sigma^3}{\eta}\right)}}\,dy^+$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A\text{-}3)$$

The velocity distribution in the liquid film can then be integrated to find the liquid-phase flow rate:

$$w_L = \pi\,d\,\mu_L \int_0^{\eta} v^+\,dy^+ \quad . \quad . \quad . \quad . \quad . \quad (A\text{-}4)$$

Eqs. A-3 and A-4 may be used to evaluate the minimum gas flow rate required to move the film steadily upward. For this application it is necessary to establish the relationship between the shear stresses and the gravitational forces in the film at the minimum condition of upward flow. Since the interfacial shear (τ_i) provides the motivating force for moving the film upward, and the gravitational shear stress, $h\,\rho_L\,g/g_c$, and the shear stress at the wall (τ_0) are resisting movement, the minimum flow condition for film movement will be when the interfacial shear (τ_i) approaches the

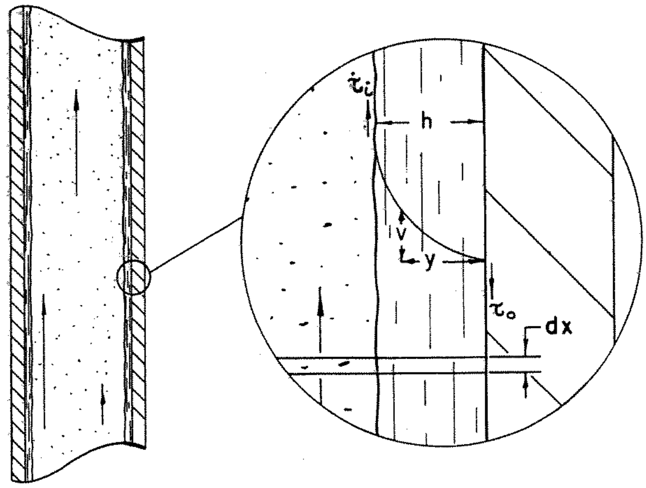

Fig. 5—Liquid film movement.

value of the gravitational "shear" and the shear stress at the wall (τ_0) approaches zero.

The ratio $\frac{h\,\rho_L\,g/g_c}{\tau_i} = X$ approaches 1.0 (i.e., the gravitational shear stress approaches the interfacial shear stress) at the limiting condition. For the purpose of analysis, X must be slightly less than 1 (i.e., the interfacial shear must be slightly larger than the gravitational shear stress, and τ_0 must be greater than zero).

If it is assumed that $X = 0.99$ at the minimum gas flow rate condition, it is possible to evaluate the necessary parameters to integrate Eqs. A-3 and A-4. The relationships utilized are

$$\sigma^3 = \frac{X}{1 - X}; \quad \frac{\beta}{\eta^{2/3}} = \frac{1}{X^{2/3}\,(1 - X)^{2/3}}$$

where

$$\beta = \frac{F\,d\,\rho_L^{2/3}\,g^{1/3}}{4\mu_L^{2/3}}; \quad F = \frac{\frac{\Delta p}{\Delta x} - \rho_g\frac{g}{g_c}}{\rho_L\frac{g}{g_c}}.$$

$\Delta p/\Delta x - \rho_g(g/g_c)$ = the two-phase pressure drop = $(\Delta p/\Delta x)_{TP}$. A modification of the Martinelli[9] two-phase pressure drop correlation is employed to evaluate the $(\Delta p/\Delta x)_{TP}$.

The calculation procedure to test the development against field data requires numerical integration and iteration. A computer program was written to perform these calculations and the results are shown in Table 1 and Fig. 3. **JPT**

Original manuscript received July 8, 1968. Revised manuscript received Aug. 1, 1969. Paper (SPE 2198) was presented at SPE Gas Technology Symposium held in Omaha, Nebr., Sept. 12-13, 1968; and at SPE 43rd Annual Fall Meeting held in Houston, Tex., Sept. 29-Oct. 2, 1968.