The Flow of Real Gases Through Porous Media

R. AL-HUSSAINY
JUNIOR MEMBER AIME
H. J. RAMEY, JR.*
P. B. CRAWFORD
MEMBERS AIME

TEXAS A&M U.
COLLEGE STATION, TEX.

ABSTRACT

The effect of variations of pressure-dependent viscosity and gas law deviation factor on the flow of real gases through porous media has been considered. A rigorous gas flow equation was developed which is a second order, non-linear partial differential equation with variable coefficients. This equation was reduced by a change of variable to a form similar to the diffusivity equation, but with potential-dependent diffusivity. The change of variable can be used as a new pseudo-pressure for gas flow which replaces pressure or pressure-squared as currently applied to gas flow.

Substitution of the real gas pseudo-pressure has a number of important consequences. First, second degree pressure gradient terms which have commonly been neglected under the assumption that the pressure gradient is small everywhere in the flow system, are rigorously handled. Omission of second degree terms leads to serious errors in estimated pressure distributions for tight formations. Second, flow equations in terms of the real gas pseudo-pressure do not contain viscosity or gas law deviation factors, and thus avoid the need for selection of an average pressure to evaluate physical properties. Third, the real gas pseudo-pressure can be determined numerically in terms of pseudo-reduced pressures and temperatures from existing physical property correlations to provide generally useful information. The real gas pseudo-pressure was determined by numerical integration and is presented in both tabular and graphical form in this paper. Finally, production of real gas can be correlated in terms of the real gas pseudo-pressure and shown to be similar to liquid flow as described by diffusivity equation solutions.

Applications of the real gas pseudo-pressure to radial flow systems under transient, steady-state or approximate pseudo-steady-state injection or production have been considered. Superposition of the linearized real gas flow solutions to generate variable rate performance was investigated and found satisfactory. This provides justification for pressure build-up testing. It is believed that the concept of the real gas pseudo-pressure will lead to improved interpretation of results of current gas well testing procedures, both steady and unsteady-state in nature, and improved forecasting of gas production.

INTRODUCTION

In recent years a considerable effort has been directed to the theory of isothermal flow of gases through porous media. The present state of knowledge is far from being fully developed. The difficulty lies in the non-linearity of partial differential equations which describe both real and ideal gas flow. Solutions which are available are approximate analytical solutions, graphical solutions, analogue solutions and numerical solutions.

The earliest attempt to solve this problem involved the method of successions of steady states proposed by Muskat.[1] Approximate analytical solutions[2] were obtained by linearizing the flow equation for ideal gas to yield a diffusivity-type equation. Such solutions, though widely used and easy to apply to engineering problems, are of limited value because of idealized assumptions and restrictions imposed upon the flow equation. The validity of linearized equations and the conditions under which their solutions apply have not been fully discussed in the literature. Approximate solutions are those of Heatherington *et al.*,[3] MacRoberts[4] and Janicek and Katz.[5] A graphical solution of the linearized equation was given by Cornell and Katz.[6] Also, by using the mean value of the time derivative in the flow equation, Rowan and Clegg[7] gave several simple approximate solutions. All the solutions were obtained assuming small pressure gradients and constant gas properties. Variation of gas properties with pressure has been neglected because of analytic difficulties, even in approximate analytic solutions.

Green and Wilts[8] used an electrical network for simulating one-dimensional flow of an ideal gas. Numerical methods using finite difference equations and digital computing techniques have been used extensively for solving both ideal and real gas equations. Aronofsky and Jenkins[9, 10] and Bruce *et al.*[11] gave numerical solutions for linear and radial gas flow. Douglas *et al.*[12] gave a solution for a square drainage area. Aronofsky[13] included the effect of slippage on ideal gas flow. The most important contribution to the theory of flow of ideal gases through porous media was the conclusion reached by Aronofsky and Jenkins[14] that solutions for the liquid flow case[15] could be used to generate approximate solutions for constant rate production of ideal gases.

An equation describing the flow of real gases has been solved for special cases by a number of investigators using numerical methods. Aronofsky and Ferris[16] considered linear flow, while Aronofsky and Porter[17] considered radial gas flow. Gas properties were permitted to vary as linear functions of pressure. Recently, Carter[18] proposed an empirical correlation by which gas well behavior can be estimated from solutions of the diffusivity equation using instantaneous values of pressure-dependent gas

Original manuscript received in Society of Petroleum Engineers office June 28, 1965. Revised manuscript of SPE 1243A received Feb. 18, 1966. Paper was presented at SPE Annual Fall Meeting held in Denver, Colo., Oct. 3-6, 1965.

*Presently on the faculty at Stanford U.

[1]References given at end of paper.

properties evaluated at an average pressure also defined empirically. Carter gave a limited number of numerical solutions as a basis, and suggested some relations which might give a better correlation. However, the proposed relations were not evaluated in the mentioned work. Solutions have been presented by Eilerts *et al.*[19, 20] for flow of gas-condensate fluids in linear and radial systems.

It has been observed that as the gas flow velocity increases, departure from Darcy's law occurs.[1, 2] Such flow is termed non-Darcy, or turbulent flow. Flow is transitional, and not truly turbulent. A gas flow equation including a quadratic velocity term to account for turbulence near the producing well has been solved by Swift and Kiel[21] and Tek *et al.*[22] for ideal gases. Eilerts *et al.*[19, 20] and Carter[15, 23] also included non-Darcy flow in their solutions for real gases. An approximate solution including non-Darcy flow has been presented by Rowan and Clegg.[24]

Two other calculational procedures appear in the works of Roberts[25] and Kidder[26] for solving the one-dimensional flow equation for an ideal gas. Roberts used a stepwise forward integration in time by joining together a sequence of solutions for linearized differential equations. Kidder, applying perturbation technique and using the well-known Boltzmann transformation in the theory of diffusion, gave an exact analytic solution for gas flow in a semi-infinite porous medium. Kidder's solution is very similar to a more general one reported by Polubarinova-Kochina[27] on the movement of ground water.

In summary, only a limited number of solutions for flow of real gases are available, and these are not of general utility. Furthermore, methods of analyzing gas reservoir performance in current use are generally based on solutions for the flow of ideal gases *under the assumption of small pressure gradients*. These methods fail to describe the behavior of low permeability and high pressure reservoirs.

FLOW OF REAL GASES

The following concerning the flow of real gases through porous media is drawn from an analogy with the theory of heat conduction in solids.[28] Variation of gas physical properties with the pressure correspond to that of temperature-dependent properties in the theory of heat conduction.

The mechanism of fluid flow through a porous medium is governed by the physical properties of the matrix, geometry of flow, PVT properties of the fluid and pressure distribution within the flow system. In deriving the flow equations and establishing the solutions, the following assumptions are made. The medium is homogeneous, the flowing gas is of constant composition and the flow is laminar and isothermal. Assumption of laminar flow can be removed, but will be used to simplify the presentation.

The principle of conservation of mass for isothermal fluid flow through a porous medium is expressed by the well-known continuity equation[1]:

$$\nabla\cdot[\rho \vec{v}] = -\phi \frac{\partial \rho}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The velocity vector in Eq. 1 is given by Darcy's law for laminar flow as:

$$\vec{v} = -\frac{k(p)}{\mu(p)} \nabla p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Substituting Eq. 2 in Eq. 1 yields:

$$\nabla\cdot\left[\rho \frac{k(p)}{\mu(p)} \nabla p\right] = \phi \frac{\partial \rho}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For real gases:

$$\rho = \frac{M}{RT}\left[\frac{p}{z(p)}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Density can be eliminated from Eq. 3 to yield:

$$\nabla\cdot\left[\frac{k(p)}{\mu(p)\, z(p)}\, p\nabla p\right] = \phi \frac{\partial}{\partial t}\left[\frac{p}{z(p)}\right] . \quad . \quad . \quad . \quad . \quad (5)$$

Eq. 5 is one form of the fundamental non-linear partial differential equation describing isothermal flow of real gases through porous media.

The pressure-dependent permeability for gas was expressed by Klinkenberg[29] as:

$$k(p) = k_l\left(1 + \frac{b}{p}\right) , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where k_l = effective permeability to liquids; and b = the slope of a linear plot of $k(p)$ vs $\frac{1}{p}$.

However, the dependency of permeability on pressure is usually negligible for pressure conditions associated with gas reservoirs, as pointed out by Aronofsky.[13]* In a subsequent paper, Aronofsky and Ferris[16] indicated that variations of gas properties with pressure are more important than variations of permeability with pressure. Therefore, liquid permeability can be used for gas flow, and the following equation is correct for all practical purposes:

$$\nabla\cdot\left[\frac{p}{\mu(p)z(p)} \nabla p\right] = \frac{\phi}{k} \frac{\partial}{\partial t}\left[\frac{p}{z(p)}\right] . \quad . \quad . \quad . \quad (7)$$

Eq. 7 can be expanded to many different forms. For example, Eq. 7 can be rearranged to point out explicitly the real gas diffusivity

$$\frac{k}{\phi\mu(p)c_g(p)} .$$

Since

$$p\nabla p = \frac{1}{2} \nabla p^2 \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Eq. 7 becomes, after some rearrangement:

$$\nabla^2 p^2 - \frac{d[\ln \mu(p)z(p)]}{dp^2}(\nabla p^2)^2 = \frac{2\phi\mu(p)z(p)}{k} \frac{\partial}{\partial t}\left[\frac{p}{z(p)}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From the definition of the isothermal compressibility of gas:

$$c_g(p) = \frac{1}{\rho}\frac{d\rho}{dp} = \frac{z(p)}{p}\frac{d}{dp}\left(\frac{p}{z(p)}\right) = \frac{1}{p} - \frac{1}{z(p)}\frac{dz(p)}{dp} \quad . \quad . \quad . \quad . \quad (10)$$

Thus:

$$\frac{\partial}{\partial t}\left[\frac{p}{z(p)}\right] = \frac{pc_g(p)}{z(p)}\frac{\partial p}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Combining Eqs. 9 and 11:

*Permeability can be considered an important function of pressure for a wet condensate gas as used by Eilerts.[19, 20] This case can be handled, as will be shown later in this paper.

$$\nabla^2 p^2 - \frac{d[\ln \mu(p)z(p)]}{dp^2}(\nabla p^2)^2 = \frac{\phi\mu(p)c_g(p)}{k}\frac{\partial p^2}{\partial t} \quad (12)$$

If it is assumed that viscosity and gas law deviation factors change slowly with pressure change, the pressure differential of $[\ln \mu(p)z(p)]$ becomes negligible. On the other hand, the assumption that pressure gradients are small will permit omission of terms of order $(\nabla p^2)^2$. In either event, Eq. 12 can be simplified to:

$$\nabla^2 p^2 = \frac{\phi\mu(p)c_g(p)}{k}\frac{\partial p^2}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Eq. 13 is similar in form to the diffusivity equation. However, the diffusivity is a function of pressure, even for a perfect gas. In this form, the close analogy with liquid flow found by Jenkins and Aronofsky[10, 14] is emphasized. *However, the assumption that pressure gradients are small everywhere in the flow system cannot be justified in many important cases.* The assumption of small pressure gradients is implicit in *all* of the pressure build-up and drawdown methods currently in use which are based upon ideal gas flow solutions or liquid flow analogies. We return, then, to the rigorous Eq. 7.

Eq. 7 can be transformed to a form similiar to that of Eq. 13 *without assuming small pressure gradients*, by making a scale change in pressure. Define a new pseudo-pressure $m(p)$ as follows:

$$m(p) = 2\int_{p_m}^{p} \frac{p}{\mu(p)z(p)}\,dp \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where p_m is a low base pressure. The variable $m(p)$ has the dimensions of pressure-squared per centipoise. Since $\mu(p)$ and $z(p)$ are functions of pressure alone for isothermal flow, this is a unique definition of $m(p)$. It follows that:

$$\frac{\partial m(p)}{\partial t} = \frac{\partial m(p)}{\partial p}\cdot\frac{\partial p}{\partial t} = \left(\frac{2p}{\mu(p)z(p)}\right)\frac{\partial p}{\partial t} \quad , \quad . \quad . \quad . \quad (15)$$

and

$$\frac{\partial m(p)}{\partial x} = \left(\frac{2p}{\mu(p)z(p)}\right)\frac{\partial p}{\partial x} \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

with similar expressions for $\frac{\partial m(p)}{\partial y}$ and $\frac{\partial m(p)}{\partial z}$.

Therefore, Eq. 7 can be rewritten in terms of the variable $m(p)$ using the definition of $c_g(p)$ given by Eq. 10 as:

$$\nabla\cdot[\nabla m(p)] = \frac{\phi\mu(p)c_g(p)}{k}\frac{\partial m(p)}{\partial t} \quad , \quad . \quad . \quad . \quad . \quad . \quad (17)$$

or

$$\nabla^2 m(p) = \frac{\phi\mu(p)c_g(p)}{k}\frac{\partial m(p)}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Comparison of Eqs. 13 and 18 shows that the form of the diffusivity equation is preserved in terms of the new variable $m(p)$. However, Eq. 18 is still non-linear because diffusivity is a function of potential. The gas law deviation factor z does not appear in the equation, but is involved in $m(p)$ and $c_g(p)$. Eq. 18 does not involve the assumptions of small pressure gradients, nor that of slow variation of $[\mu(p)z(p)]$.

The importance of Eq. 18 deserves emphasis. It is a fundamental partial differential equation which describes the flow of real gases. To the authors' knowledge, this equation has not been presented previously in connection with gas flow. Equations of this type have been called quasi-linear flow equations.[27, 36] The real importance lies in the extreme utility of this form of the equation. As will be shown, the form of the equation suggests a powerful engineering approach to the flow of real gases.

To solve Eq. 18, it is necessary to convert the usual initial and boundary conditions into terms of the new pseudo-pressure $m(p)$. Important considerations are as follows.

The gas mass flux is:

$$\vec{v}\rho = \frac{q}{A}\rho = \frac{-Mk}{RT}\,\frac{p}{\mu(p)z(p)}\,\nabla p \quad . \quad . \quad . \quad (19)$$

In terms of $m(p)$, the mass flux is:

$$\frac{q}{A}\rho = \frac{-Mk}{2RT}\,\nabla m(p) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The usual boundary conditions are either specification of pressure or the gas flux across bounding surfaces. When pressure is fixed, $m(p)$ can be determined from Eq. 14. If flux is specified, the boundary conditions can be determined from Eq. 20. If the outer boundary is impermeable, then:

$$\frac{dm(p)}{dn} = 0 \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where n is the direction normal to the boundary.

Steady-state flow occurs when pressure distribution and fluid velocity are independent of time. Eq. 18 reduces to:

$$\nabla^2 m(p) = 0 \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

which is Laplace's equation. Thus, previous solutions of the Laplace equation can be used if $m(p)$ is used as the potential.

Steady-state flow can rarely be obtained in reality because gas wells usually produce gas from a limited, finite reservoir or drainage volume. There can be no flow across the outer boundary. Thus, pressure must decline as production continues. True steady-state would require pressure to remain constant at the outer boundary, which implies flow across the outer boundary. Production of a bounded reservoir at constant production is an important problem, which will be considered later in this paper.

REAL GAS PSEUDO-PRESSURE

To obtain generally useful solutions for Eq. 18, the proper physical properties for natural gases must be specified. Fortunately, all required physical properties have been correlated as functions of pseudo-reduced pressures and temperatures for many gases met in field work. It should be emphasized that the concept of the real gas pseudo-pressure is not limited to use of specific gas property correlations. Pseudo-reduced pressure and temperature are defined, respectively, as:

$$p_{pr} = \frac{p}{p_{pc}} \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and

$$T_{pr} = \frac{T}{T_{pc}} \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where p_{pc} is the pseudo-critical pressure and T_{pc} is the pseudo-critical temperature. Real gas law deviation factors $z(p)$ have been presented by Standing and Katz.[31] Viscosities of natural gases have been correlated by Carr *et al.*[22] as the ratio of viscosity at any pressure to that at one atmosphere. Thus:

$$\frac{\mu(p)}{\mu_1} = f(p_{pr}, T_{pr}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Compressibilities of natural gases have been correlated by Trube[23] as reduced compressibilities, the product of compressibility and pseudo-critical pressure. That is:

$$c_{pr} = c_g(p) \cdot p_{pc} = f(p_{pr}, T_{pr}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Substitution of Eqs. 23 to 25 in Eq. 14 yields:

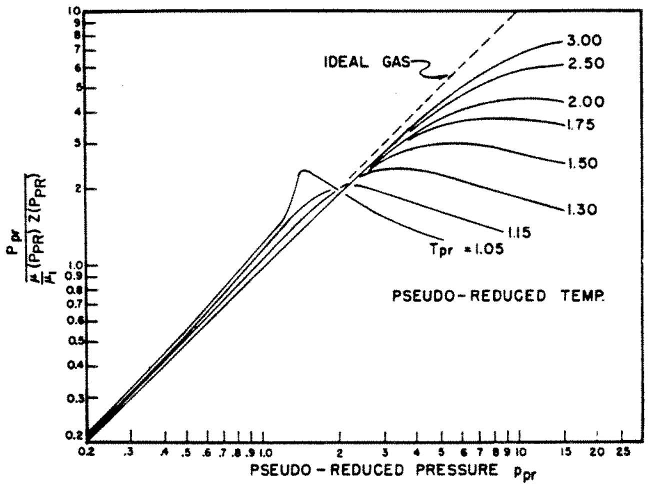

FIG. 1—RATIO OF PSEUDO-REDUCED PRESSURE TO VISCOSITY—GAS LAW DEVIATION FACTOR PRODUCT VS PSEUDO-REDUCED PRESSURE.

$$m(p) = \frac{2(p_{pc})^2}{\mu_1} \int_{(p_{pr})_m}^{p_{pr}} \frac{p_{pr}}{\frac{\mu}{\mu_1}(p_{pr}) z(p_{pr})} dp_{pr} \quad . \quad . \quad (27)$$

The integral can be evaluated generally from reduced properties correlations.

EVALUATION OF REAL GAS PSEUDO-PRESSURE

To establish the relationship between p_{pr} and $m(p)$, the integral must be evaluated numerically for various isotherms. The lower limit of the integration $(p_{pr})_m$ can be set arbitrarily. A value of 0.20 was chosen. Selected isotherms from pseudo-reduced temperatures of 1.05 to 3.0 were used.

Fig. 1 presents the argument of the integral in Eq. 27 vs pseudo-reduced pressure for various pseudo-reduced temperatures. The dashed line represents the ideal gas case with both viscosity ratio and gas law deviation factor equal to unity. The magnitude of gross variations of gas properties with pressure and temperature is apparent.

Fig. 2 presents $m(p)$ integrals as functions of pseudo-reduced pressures and temperatures. The integrals were evaluated by means of the Trapezoidal rule using an IBM 709 digital computer. Values of the integrals are also presented in Table 1. Interpolation between the curves or between the values presented in the table can be performed easily.

Use of Fig. 2 or Table 1 is limited to gases containing small amounts of contaminants for which changes in viscosity and gas law deviation factor can be handled by appropriate changes in the pseudo-critical properties, as suggested by Carr *et al.*[22] However, useful charts can be prepared for gases containing large amounts of contaminants if complete properties are known. See Robinson

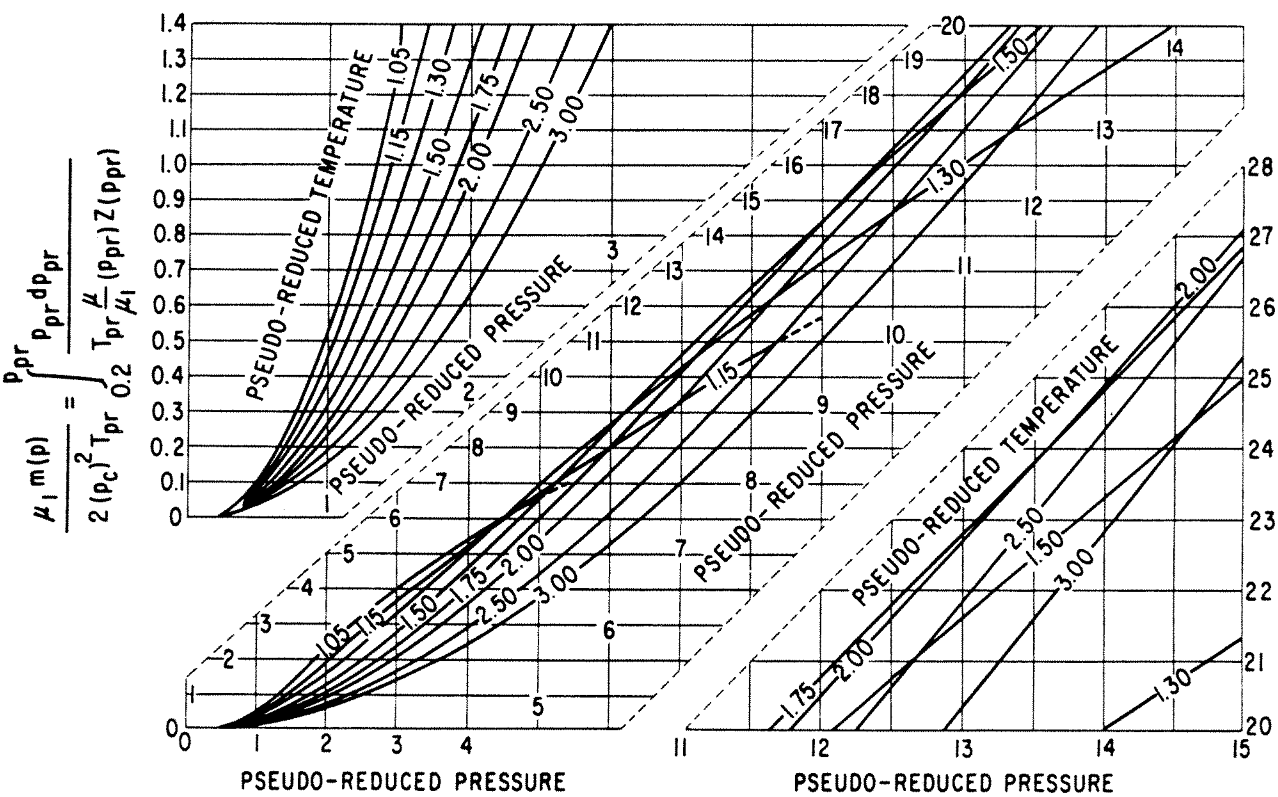

FIG. 2—REAL GAS PSEUDO-PRESSURE INTEGRALS VS PSEUDO-REDUCED PRESSURE.

et al[9] for density data for gases containing large amounts of contaminants.

In general, it is useful to prepare a chart of $m(p)$ in units of psi-squared per centipoise vs pressure in psi for any given reservoir to aid engineering use of the real gas pseudo-pressure. The $m(p)$ can be computed readily for *any* specific gas and reservoir temperature if density and viscosity are known as functions of pressure. The integration can be performed using the Trapezoidal rule or graphical integration. More sophisticated integrations are usually not required.

The $m(p)$ values in Fig. 2 and Table 1 are presented as a convenience because it is necessary to assume many gases do follow the existing correlations because of lack of specific data. It is emphasized that the concept of the real gas potential is *general* and is *not* limited to use of the $m(p)$ values presented herein. If viscosity and density data are available for a specific gas, it should be used in preference to Fig. 2 and Table 1 to prepare $m(p)$ plots for the specific gas.

TRANSIENT FLOW

CONSTANT-RATE PRODUCTION

As has been described in the introduction of this paper, Eq. 7 has been solved for specific flow cases under appropriate boundary and initial conditions by a number of authors using finite difference solutions. We seek a general solution which can be used for engineering purposes without the aid of a digital computer. Eq. 18 and the work of Aronofsky and Jenkins[14] provide a basis for an approach. For radial flow of *ideal* gas, the continuity equation leads to:

$$\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r}\frac{\partial p^2}{\partial r} = \frac{\phi \mu c_g(p)}{k}\frac{\partial p^2}{\partial t}, \quad . \; . \; . \; . \quad (28)$$

where $c_g(p)$ for an ideal gas is the reciprocal of the pressure. Several features of Eq. 28 are noteworthy. First,

TABLE 1—$\dfrac{\mu_i m(p)}{2(p_{pc})^2 T_{pr}} = \displaystyle\int_{0.2}^{p_{pr}} \frac{p_{pr}\, dp_{pr}}{T_{pr}\, \frac{\mu}{\mu_i}(p_{pr})\, Z\,(p_{pr})}$

Pseudo-Reduced Pressure	Values of Integral for Pseudo-Reduced Temperature T_{pr} of							
p_{pr}	1.05	1.15	1.30	1.50	1.75	2.00	2.50	3.00
0.30	0.0257	0.0229	0.0198	0.0170	0.0145	0.0126	0.0100	0.0083
0.40	0.0623	0.0553	0.0477	0.0409	0.0348	0.0303	0.0241	0.0200
0.50	0.1102	0.0971	0.0839	0.0716	0.0609	0.0530	0.0421	0.0349
0.60	0.1698	0.1485	0.1283	0.1091	0.0927	0.0807	0.0640	0.0532
0.70	0.2418	0.2105	0.1810	0.1532	0.1303	0.1132	0.0898	0.0747
0.80	0.3264	0.2835	0.2419	0.2037	0.1734	0.1505	0.1194	0.0993
0.90	0.4236	0.3678	0.3111	0.2608	0.2221	0.1927	0.1529	0.1271
1.00	0.5326	0.4631	0.3889	0.3246	0.2763	0.2397	0.1902	0.1580
1.10	0.6546	0.5691	0.4755	0.3954	0.3358	0.2915	0.2312	0.1920
1.20	0.7903	0.6855	0.5707	0.4734	0.4004	0.3483	0.2761	0.2292
1.30	0.9484	0.8126	0.6734	0.5579	0.4702	0.4098	0.3248	0.2695
1.40	1.1444	0.9503	0.7838	0.6484	0.5452	0.4758	0.3773	0.3129
1.50	1.3671	1.0980	0.9020	0.7449	0.6255	0.5461	0.4335	0.3594
1.60	1.5828	1.2546	1.0277	0.8473	0.7114	0.6209	0.4932	0.4090
1.70	1.7924	1.4191	1.1606	0.9558	0.8025	0.7001	0.5566	0.4616
1.80	1.9959	1.5883	1.3001	1.0703	0.8983	0.7840	0.6235	0.5173
1.90	2.1926	1.7595	1.4457	1.1906	0.9988	0.8724	0.6940	0.5760
2.00	2.3821	1.9321	1.5966	1.3164	1.1042	0.9653	0.7679	0.6378
2.10	2.5649	2.1071	1.7526	1.4474	1.2146	1.0624	0.8454	0.7025
2.20	2.7424	2.2841	1.9138	1.5838	1.3298	1.1636	0.9264	0.7701
2.30	2.9147	2.4619	2.0791	1.7253	1.4498	1.2687	1.0111	0.8407
2.40	3.0825	2.6399	2.2473	1.8712	1.5744	1.3777	1.0994	0.9143
2.50	3.2464	2.8172	2.4186	2.0214	1.7034	1.4904	1.1912	0.9907
2.60	3.4066	2.9937	2.5935	2.1758	1.8370	1.6068	1.2862	1.0700
2.70	3.5633	3.1683	2.7710	2.3341	1.9751	1.7268	1.3846	1.1522
2.80	3.7169	3.3403	2.9504	2.4957	2.1169	1.8504	1.4864	1.2373
2.90	3.8679	3.5094	3.1320	2.6612	2.2626	1.9778	1.5915	1.3252
3.00	4.0165	3.6766	3.3153	2.8308	2.4123	2.1091	1.6998	1.4159
3.25	4.3788	4.0876	3.7771	3.2685	2.8038	2.4534	1.9849	1.6550
3.50	4.7278	4.4874	4.2400	3.7223	3.2178	2.8178	2.2896	1.9112
3.75	5.0653	4.8766	4.7052	4.1897	3.6504	3.2016	2.6119	2.1841
4.00	5.3938	5.2579	5.1693	4.6678	4.0997	3.6049	2.9516	2.4731
4.25	5.7144	5.6367	5.6277	5.1539	4.5638	4.0268	3.3077	2.7782
4.50	6.0276	6.0088	6.0822	5.6459	5.0406	4.4663	3.6788	3.0994
4.75	6.3347	6.3697	6.5308	6.1412	5.5280	4.9203	4.0649	3.4357
5.00	6.6368	6.7235	6.9714	6.6377	6.0234	5.3860	4.4664	3.7865
5.25	—	7.0706	7.4044	7.1355	6.5252	5.8621	4.8825	4.1511
5.50	—	7.4124	7.8304	7.6343	7.0326	6.3472	5.3130	4.5286
5.75	—	7.7495	8.2497	8.1338	7.5449	6.8412	5.7575	4.9194
6.00	—	8.0821	8.6632	8.6336	8.0622	7.3442	6.2150	5.3241
6.25	—	8.4099	9.0711	9.1326	8.5836	7.8551	6.6844	5.7413
6.50	—	8.7330	9.4731	9.6297	9.1085	8.3739	7.1643	6.1699
6.75	—	9.0520	9.8703	10.1249	9.6364	8.8993	7.6544	6.6104
7.00	—	9.3670	10.2635	10.6185	10.1665	9.4298	8.1543	7.0633
7.25	—	9.6786	10.6531	11.1091	10.6973	9.9647	8.6633	7.5283
7.50	—	9.9876	11.0398	11.5957	11.2279	10.5034	9.1808	8.0049
7.75	—	10.2936	11.4223	12.0794	11.7587	11.0452	9.7064	8.4921
8.00	—	10.5963	11.7998	12.5615	12.2897	11.5897	10.2398	8.9884
8.25	—	10.8961	12.1731	13.0416	12.8211	12.1377	10.7812	9.4932
8.50	—	11.1935	12.5433	13.5194	13.3532	12.6897	11.3308	10.0062
8.75	—	—	12.9102	13.9939	13.8858	13.2440	11.8872	10.5281
9.00	—	—	13.2735	14.4644	14.4187	13.7993	12.4497	11.0583
9.25	—	—	13.6340	14.9322	14.9513	14.3558	13.0182	11.5962
9.50	—	—	13.9925	15.3980	15.4834	14.9128	13.5926	12.1421
9.75	—	—	14.3483	15.8609	16.0146	15.4700	14.1700	12.6952
10.00	—	—	14.7011	16.3205	16.5447	16.0274	14.7499	13.2545
10.50	—	—	15.3996	17.2313	17.6030	17.1463	15.9178	14.3923
11.00	—	—	16.0892	18.1318	18.6590	18.2662	17.0928	15.5560
11.50	—	—	16.7703	19.0212	19.7090	19.3931	18.2738	16.7372
12.00	—	—	17.4427	19.8976	20.7507	20.5120	19.4614	17.9315
12.50	—	—	18.1069	20.7640	21.7858	21.6135	20.6575	19.1388
13.00	—	—	18.7642	21.6238	22.8166	22.7156	21.8627	20.3556
13.50	—	—	19.4147	22.4762	23.8434	23.8144	23.0724	21.5838
14.00	—	—	20.0588	23.3216	24.8616	24.9057	24.2820	22.8246
14.50	—	—	20.9676	24.1596	25.8642	25.9948	25.4964	24.0719
15.00	—	—	21.3318	24.9921	26.8596	27.0862	26.7197	25.3268

the second degree pressure gradient term $(\partial p/\partial r)^2$ does not appear for an ideal gas. Second, Eq. 28 has the form of the diffusivity equation, but the diffusivity is proportional to pressure. Viscosity is a function of temperature, but not of pressure for an ideal gas. Aronofsky and Jenkins found that for constant rate production of an ideal gas from a closed radial system, the pressure at the producing well could be correlated as a function of a dimensionless time based on a compressibility evaluated at the initial pressure. The correlation was slightly sensitive to the production rate, but not sensitive enough to affect engineering accuracy.

Aronofsky and Jenkins demonstrated that production of ideal gas from a closed radial system could be approximated very closely from the solutions for transient liquid flow of van Everdingen and Hurst.[15] Matthews[35] later pointed out the application of this conclusion to pressure build-up analysis for gas wells as a liquid case analog.

For radial flow of a real gas, Eq. 18 becomes:

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{\phi\mu(p)c_g(p)}{k}\frac{\partial m(p)}{\partial t} \quad . \quad . \quad (29)$$

The close analogy between Eqs. 28 and 29 suggests that the real gas pseudo-pressure $m(p)$ should correlate as a function of a dimensionless time based on viscosity and compressibility evaluated at the initial pressure, if the variation of the viscosity-compressibility product with $m(p)$ for a real gas is similar to the variation of compressibility for an ideal gas $(1/p)$ with pressure squared. Fig. 3 shows the comparison.

In view of the close resemblance between (μc_g) vs $m(p)$ for the real gas, and p^{-1} vs p^2 for the ideal gas, it is reasonable to expect solutions for the flow of real gases to correlate as functions of a dimensionless time based on initial values of viscosity and compressibility. That is, let:

$$t_D = \frac{kt}{\phi(\mu c_g)_i r_w^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Further, define a dimensionless real gas pseudo-pressure drop $m_D(r_D, t_D)$:

$$m_D(r_D, t_D) = \frac{\pi k h T_{sc}\,[m(p_i) - m(r, t)]}{q_{sc} p_{sc} T} \quad , \quad . \quad . \quad . \quad (31)$$

where $r_D = r/r_w$. The dimensionless real gas pseudo-pressure drop is thus analogous to the van Everdingen-Hurst[15] dimensionless pressure drop $p_D(r_D, t_D)$.

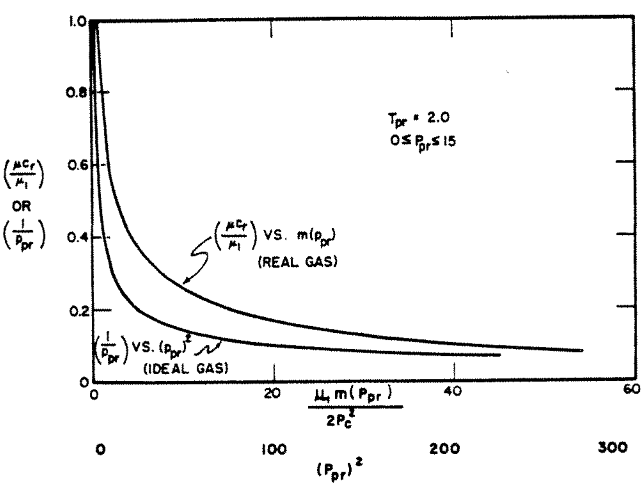

FIG. 3—PRESSURE-DEPENDENT DIFFUSIVITY TERMS VS POTENTIAL FOR IDEAL AND REAL GAS FLOW.

Fig. 4 shows the comparison between $p_D(t_D)$ for the liquid flow solutions and $m_D(t_D)$ obtained from Eilerts *et al.*[20] solutions for the radial flow of natural gases. The solid line represents the liquid case, while points shown are computed from the Eilerts *et al.* solutions. As can be seen, the comparison is excellent for the entire range of flow considered by Eilerts *et al.* for both natural gases and condensate gases. The transient flow data computed by Carter[15, 23] correlate just as well. The Eilerts *et al.* data are a severe test of the linearization of the real gas flow equation, because production included a tenfold range in production rate, and almost complete depletion over a pressure range from 10,000 to 1,000 psi. Carter's results covered a range from 4,700 to 1,180 psia, and a more restricted range of flow rate.

The $m_D(t_D)$ correlation (Fig. 4) is actually not as good as it appears. Although it is quite good at times before the boundary effect is felt, at long times there may be a considerable difference between $m_D(t_D)$ and $p_D(t_D)$ values

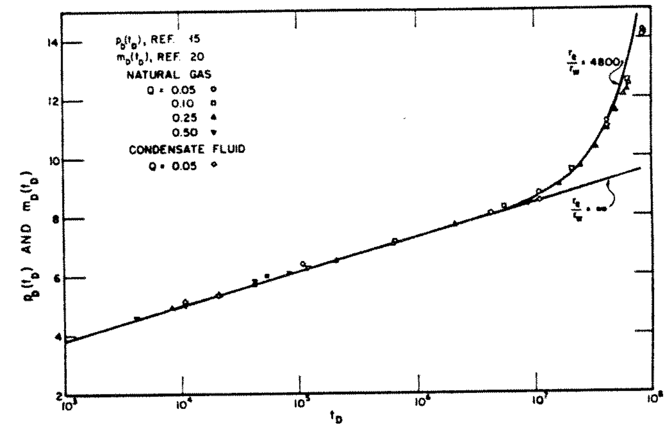

FIG. 4—$p_D(t_D)$ AND $m_D(t_D)$ VS t_D FOR A CLOSED, RADIAL RESERVOIR PRODUCED AT CONSTANT RATE.

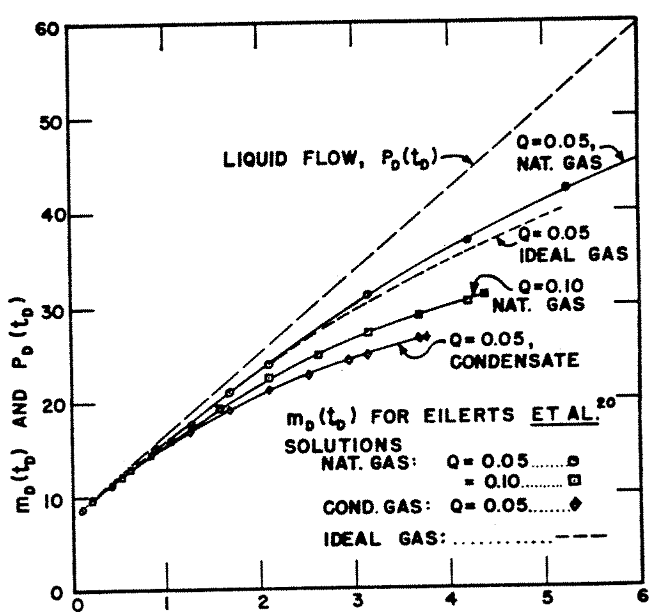

FIG 5—$m_D(t_D)$ AND $p_D(t_D)$ VS t_D FOR RADIAL FLOW OF LIQUID, IDEAL GAS AND REAL GASES.

(Fig. 5). Also shown on Fig. 5 are the Aronofsky-Jenkins[16] ideal gas flow results. It is clear that both the ideal and real gas cases lead to dimensionless pressure drops which are lower than the liquid case—and which are flow-rate dependent. Another important difference is illustrated by the case $Q = 0.05$. The ideal gas line terminates at the point where the well pressure is zero. The real gas solutions terminate at a well pressure of 10 per cent of the initial pressure. Although not shown on Fig. 5, the production times for the real gas cases to reach a limiting production pressure are about two and a half times that required for the ideal gas flow cases. Clearly, production forecasts based on the ideal gas solutions will be far too conservative.

Another important observation can be made from Fig. 5 by comparing the real gas solutions for natural gas and condensate for a flow rate Q of 0.05. Although the natural gas line is close to the liquid case, the condensate line is far below the liquid case line. The terminal producing pressure is reached earlier for the condensate line than for the natural gas line. This indicates the importance of gas property variations upon the results. That is, no single set of $m_D(t_D)$ correlations could be expected to apply to all real gases at long production times. It is also clear from Fig. 5 that the real gas results tend to approach the liquid case results as flow rate decreases, and at small production times.

Aronofsky and Jenkins introduced the concept of a transient drainage radius r_d. This term should not be confused with the dimensionless radial coordinate r_D. From the Aronofsky-Jenkins definition of the transient drainage radius, we write for real gas flow:

$$\ln\frac{r_d}{r_w} = \frac{\pi k h T_{sc}}{q_{sc} p_{sc} T}\left[m(\bar{p}) - m(p_w)\right] = m_D(p_w) - m_D(\bar{p}) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (32)$$

The Eilerts *et al.*[20] results can also be correlated as transient drainage radii vs dimensionless time. The results are presented in Fig. 6, and agree with the Aronofsky-Jenkins results and the liquid flow results almost exactly. Actually, the correlation of the real gas flow solutions in terms of the transient drainage radius (Fig. 6) is a much better correlation than the correlation in terms of $m_D(t_D)$ (Figs. 4 and 5). The drainage radius correlation is excellent for all values of production time. Thus, Eq. 32 provides the most useful engineering approach to the transient flow of real gases. As recommended by Jenkins and Aronofsky for ideal gas flow, the transient drainage radius for real gas flow can be found from:[11]

$$\ln\frac{r_d}{r_w} = p_D(t_D) - 2t_D\left(\frac{r_w}{r_e}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \quad (33)$$

and the $m(\bar{p})$ can be found from the materials balance:

$$\left(\frac{p}{z}\right)_i - \left(\frac{\bar{p}}{\bar{z}}\right) = \frac{T p_{sc} q_{sc} t}{\pi h r_e^2 \phi T_{sc}} = \frac{1}{2}\int_{m(\bar{p})}^{m(p_i)} (\mu c_g)\, dm(p) \approx \frac{(\mu c_g)_{avg}}{2}\left[m(p_i) - m(\bar{p})\right] \quad . \; . \quad (34)$$

Eqs. 32 through 34 are not strictly a solution to Eq. 18. They represent an excellent engineering approximation which applies for a wide range of conditions. The method appears to be every bit as good as the Jenkins-Aronofsky result for ideal gas flow.

Fig. 6 shows that at long production times r_d takes the constant value 0.472 r_e. This is similar to the Aronofsky-Jenkins finding for ideal gas. Substitution of long-time values for $p_D(t_D)$ in Eq. 33 also leads to this conclusion. Thus, Eq. 32 becomes similar in form to the liquid case pseudo-steady-state equation at times long enough that the outer boundary effect is controlling. The fact that r_d eventually becomes constant at 0.472 r_e does not mean the physical drainage radius stabilizes about half-way out in the reservoir. The entire reservoir volume is being drained, as can be seen by inspection of any of the Eilerts *et al.*[20] production figures.

The Eilerts *et al.* solutions have provided an excellent set of information to test the linearization of the real gas flow solutions for production. Eilerts *et al.* specified that the effective permeability was a function of pressure (assuming pressure drop would result in condensation and reduction of effective permeability). Effective permeability can thus be taken within the $m(p)$ integral. Correlations in Figs. 4 through 6 do include a pressure-dependent permeability. Thus, if an approximation of the effect of pressure drop upon liquid condensation and reduction in permeability near the wellbore can be made, the performance can be estimated from:

$$\ln\frac{r_d}{r_w} = \frac{\pi h T_{sc}}{q_{sc} p_{sc} T}\left[m'(\bar{p}) - m'(p_w)\right] \quad , \; . \; . \; . \; . \quad (35)$$

where

$$m'(p) = 2\int_{p_m}^{p} \frac{k p\, dp}{\mu z} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (36)$$

and k is a known function of pressure.

The usefulness of considering k a function of pressure to handle condensate flow might be open to question. Nevertheless, it is clearly indicated that variation of k as a function of pressure can be included in the real gas pseudo-pressure.

Correlation of the Eilerts *et al.*[20] data presented previously involves calculation of $m(p)$ and determination of relationships between the Eilerts *et al.* nomenclature and that used in this paper. (Necessary relationships are in the Appendix).

Eilerts *et al.*[20] also determined performance with a steady-state, non-Darcy flow region near the producing well. As a result, a steady-state skin effect can also be introduced to yield the following approximation for the radial flow of real gases during production:

$$\frac{\pi k h T_{sc}\left[m(\bar{p}) - m(p_w)\right]}{q_{sc} p_{sc} T} = \ln\frac{r_d}{r_w} + s + Dq_{sc} \quad , \; . \quad (37)$$

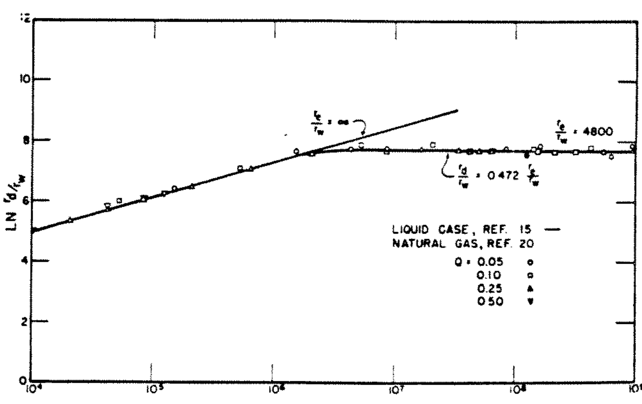

FIG. 6—JENKINS-ARONOFSKY DRAINAGE RADIUS VS t_D FOR A CLOSED, RADIAL RESERVOIR PRODUCED AT CONSTANT RATE.

where s is the skin effect and D is the non-Darcy flow coefficient.

CONSTANT RATE INJECTION

All of the preceding discussion of real and ideal gas transient flow deals with *production* only. *Injection* results, as was clearly shown by Aronofsky and Jenkins[10] for radial ideal gas flow, cannot be linearized in as simple a fashion. Aronofsky and Jenkins correlated injection well pressures for radial flow of an ideal gas as functions of a dimensionless time based on gas compressibility evaluated at the initial formation pressure before injection. The dimensionless pressure rise at a given dimensionless time was generally greater than that for a liquid case, and increased with injection rate. Aronofsky and Jenkins showed that injection case results were very close to the liquid case for low injection rates. Although injection is of practical importance in itself, the major utility of injection case correlations is in application of the principle of superposition to generate variable rate production cases, including the important pressure build-up case.

Superposition, as it has been applied in gas well testing, requires that dimensionless times for both injection and production be based on the same gas physical property evaluation. Although superposition could be based on different dimensionless times for injection and production, the added complexity of such a scheme does not appear justified. Thus, an obvious question is: will injection solutions correlate closer to the liquid case if dimensionless times are based on physical properties evaluated at a pressure above the initial, low formation pressure?

We rule out the scheme of using a point evaluation at the existing injection pressure because this would yield a result not usable for forecasting. That is, it would be necessary to know the injection pressure-time history before it could be calculated. An obvious possibility is to evaluate physical properties at the final, elevated injection pressure, or in the case of superposition applied to reservoir production or build-up, at the initial formation pressure before production was started. This idea is fundamentally the basis for all gas well pressure build-up applications currently in use.

In brief, correlations for injection based on an elevated pressure are no better (or worse) than those based on physical properties evaluated at the initial, low formation pressure. This is true for *both* the ideal and real gas flow cases. Fig. 7 presents the dimensionless real gas potential rise for the Eilerts *et al.*[20] injection case (their Fig. 8) correlated vs dimensionless times based on both the initial, low formation pressure and the final injection pressure. The dashed line presents the liquid flow solution. Two facts are apparent: the slopes of the correlations are similar, and correlations based on final injection pressure are no worse than those based on initial, low formation pressure. From the Jenkins-Aronofsky studies of ideal gas flow, we can also conclude that the difference between the injection case correlations and the liquid case become smaller as injection rate decreases; in any case, the differences aren't large.

Fig. 7 can lead to another idea. Correlation based on a dimensionless time evaluated with physical properties about half-way between the extremes might be quite good. This idea follows immediately from the *theorem of the mean.* That is, if flowing fluid physical properties vary monotonically with potential, the proper result is limited by those evaluated at the extreme values of physical properties. Friedmann[36] proved that results must lie between those evaluated at the extremes of physical properties whether physical properties are monotonic functions or not. The injection problem has been the subject of much investigation in the fields of heat transmission and ground-water movement (Friedmann,[36] Storm[37] and Polubarinova-Kochina[21]). As has been shown by these authors, it cannot always be assumed that evaluation at an average property will yield good results. Sometimes the answer will vary from one extreme to the other.

SUPERPOSITION OF LINEARIZED SOLUTIONS

Superposition is rigorously correct only for linear partial differential equations. Nevertheless, the extremely close check between the linearized real gas solutions correlated on the basis of the $m_D(t_D)$, as given by Eq. 31, and a t_D given by Eq. 30, and the liquid flow solutions of van Everdingen and Hurst, indicates the possibility that superposition might be quite good for matching an *increasing* rate production schedule. An increasing rate

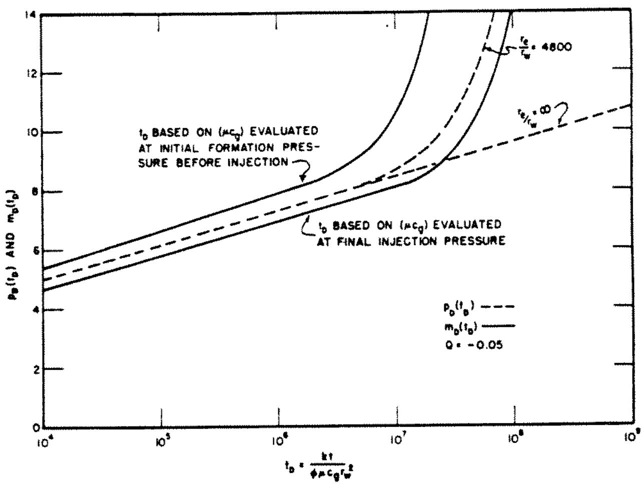

FIG. 7—$p_D(t_D)$ AND $m_D(t_D)$ VS t_D FOR INJECTION OF A REAL GAS (CORRELATION OF EILERTS *et al.*[20] DATA).

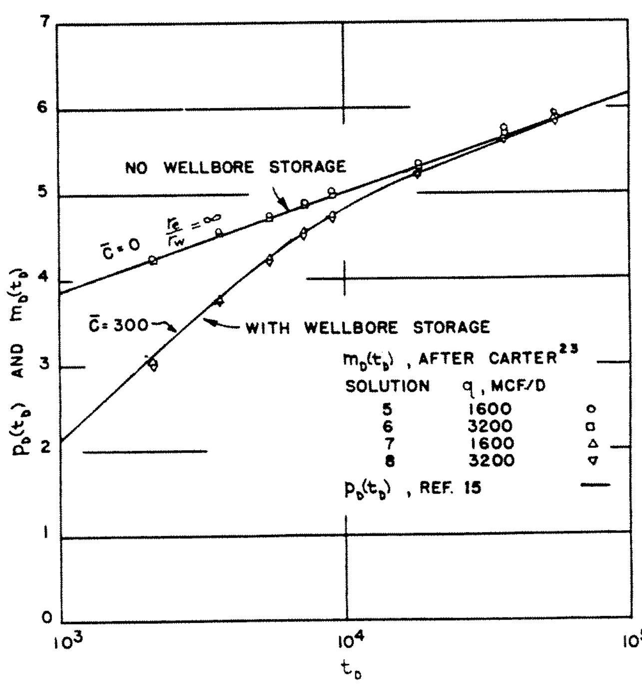

FIG. 8—$p_D(t_D)$ AND $m_D(t_D)$ VS t_D FOR CONSTANT RATE PRODUCTION WITH WELLBORE STORAGE.

schedule would require superposition of positive incremental rates. However, the real gas flow solutions do depend slightly upon production rate. Thus, the only way that the application of the principle of superposition (as an acceptable approximation) to real gas flow can be established is by comparison with finite-difference solutions of variable-rate, real gas flow problems.

Such a comparison can be made for an increasing production rate schedule from data for real gas flow published by Carter.[23] Carter studied the effect of wellbore storage on gas production. For his solutions, it was assumed that the surface flow rate was held constant, but 0.02965 Mcf was withdrawn from the wellbore per psi pressure drop in the wellbore. This resulted in the sand face flow rate increasing as a function of time toward the constant surface flow rate. This case is almost exactly analogous to the wellbore unloading case presented by van Everdingen and Hurst[15] in their Eq. VIII-II. The wellbore storage constant $\overline{C}$ for Carter's solutions can be determined from Eq. 6 presented by Ramey.[10] The value of $\overline{C}$ for Carter's solutions does vary slightly with pressure, but a value of 300 is quite good. Fig. 8 presents a comparison between the $m_D(t_D)$ obtained from Carter's solutions, both with and without wellbore storage, and the van Everdingen-Hurst $p_D(t_D)$ solutions for the liquid flow case. As can be seen, the comparison with constant rate liquid flow *without* storage is excellent. This was previously shown for the Eilerts *et al.*[20] solutions. Of more interest, the comparison between the liquid flow case *with* wellbore storage and Carter's two solutions with wellbore storage are also excellent. This establishes that superposition of the linearized real gas flow solutions for an increasing flow rate should be a very good approximation—at least before outer boundary effects are controlling.

Although superposition in an increasing production rate schedule appears quite good, it is not apparent that a *decreasing* rate schedule is susceptible to superposition. This results because the dimensionless real gas injection pressure increases do not correlate with the liquid case as well as do production data. Even for transient injection of an ideal gas,[10] the resulting dimensionless pressure rise appears to depend upon injection rate, but does approach the liquid case solution as injection rate decreases. The fact that injection results do approach the liquid case as injection rate approaches zero suggests that superposition of small *positive* incremental rates (injections) would be feasible. Again, the possibility can only be checked by comparison with finite-difference solutions.

Fortunately, both Carter[18] and Dykstra[41] have presented finite-difference solutions for decreasing flow-rate production. Dykstra's data provide an excellent set for comparison of finite-difference solutions with superposition of the linearized solutions. Fig. 9 presents a comparison of Dykstra's computed flowing pressures with those obtained by superposition of linearized real gas flow solutions. The line is Dykstra's result, while points represent results of superposition using only four or five incremental rate changes to represent a rapidly changing flow rate. The flow rate is shown by the dashed line. For the example shown, the permeability was 0.25 md, thickness was 179 ft, initial pressure 6,150 psia and flow rate declined quadratically as a function of time from 6,556 to 2,500 Mcf/D by 50 days' producing time. Superposition was accomplished using dimensionless times based on the *initial* pressure and the $m_D(t_D)$ taken equal to the liquid case $p_D(t_D)$ values. The maximum difference between Dykstra's result and those computed by superposition was 20 psi out of a drawdown of 2,150 psi—a difference of 0.9 per cent. The 50-day production period was long enough that initial rate changes were influenced by the outer boundary. Thus, we conclude that superposition can be used to reproduce variable-rate drawdown data with acceptable accuracy.

The previous remarks concerning superposition of incremental rate *increases* are, of course, directly applicable to pressure build-up testing. Although insufficient comparisons between finite-difference build-up solutions and superposition solutions for the real gas flow case have been made to completely explore this problem, it does appear that build-up theory can be used with good accuracy. An interesting test of pressure build-up can be made by comparison of Dykstra's[41] solutions with superposition solutions. Because Dykstra's cases involved a variable-rate production period, permeability was low and pressure gradients high, it is believed that a fairly extreme test results. Fig. 10 presents the build-up following the drawdown of Fig. 9. As can be seen, the

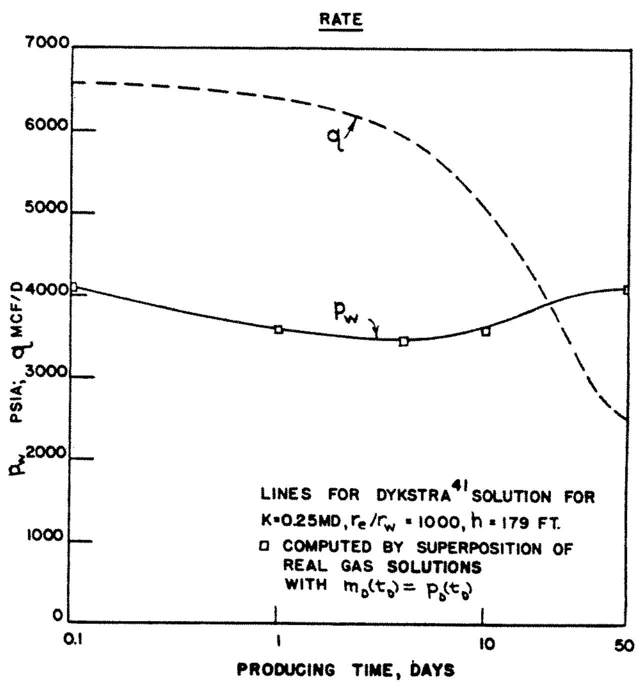

FIG. 9—COMPARISON OF FINITE DIFFERENCE AND SUPERPOSITION FLOWING PRESSURES FOR A DECREASING PRODUCTION RATE.

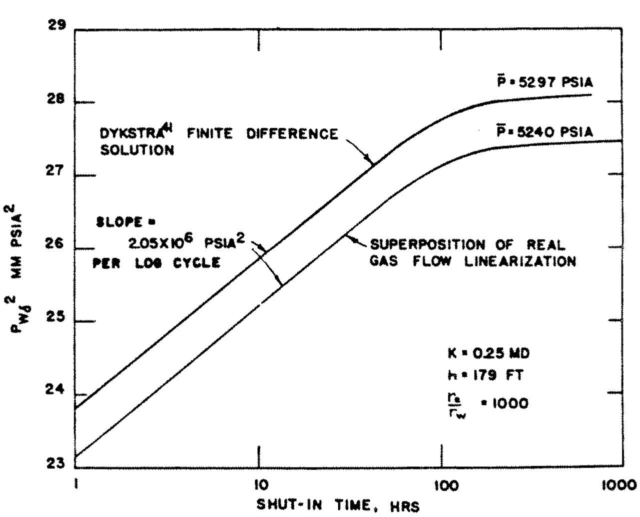

FIG. 10—COMPARISON OF FINITE DIFFERENCE AND SUPERPOSITION BUILD-UP PRESSURES FOR A REAL GAS.

superposition result yields a similar build-up curve of identical slope, but about 60 psi below Dykstra's finite-difference solutions. Again, the percentage difference is small; the final static pressure is about 1.1 per cent too low. It appears that superposition of the real gas flow linearization will always yield a pressure build-up static pressure that is too low, but as good or better than results of current methods. Furthermore, field application would be to the field measured data—the real solution—which would tend to correct for this error. We conclude that pressure-build-up analysis based on superposition can be done for real gas flow with acceptable accuracy, but that further study of pressure build-up for real gas flow is desirable.

STEADY-STATE AND PSEUDO-STEADY-STATE FLOW

Radial gas flow at constant production rate will be considered. A horizontal homogeneous porous medium of constant thickness h with impermeable upper and lower boundary, and a well of radius r_w located in the center of a radial reservoir, constitutes the flow system. The outer radius r_e represents either the real boundary or the radius of drainage. Two cases will be considered: (1) constant pressure at r_e, and (2) no flow across r_e.

CONSTANT PRESSURE AT OUTER BOUNDARY

The steady-state equation for a real gas in axisymmetrical coordinates can be written from Eq. 22 as:

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dm(p)}{dr}\right] = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The boundary conditions for two concentric cylinders of radii r_w and r_e are:

$$r = r_w : m(p) = m(p_w) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

$$r = r_e : m(p) = m(p_e) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Integrating Eq. 38 and using the boundary conditions, the steady-state pressure distribution in the system is:

$$m(p_r) - m(p_w) = \frac{q_{sc}p_{sc}T}{\pi khT_{sc}}\left(\ln\frac{r}{r_w}\right) \quad . \quad . \quad . \quad (41)$$

Eq. 41 can be evaluated for $p = p_e$ at $r = r_e$ and rearranged to provide an equation analogous to the normal radial flow equation:

$$q_{sc} = \frac{\pi khT_{sc}\,[m(p_e)-m(p_w)]}{Tp_{sc}\,\ln\frac{r_e}{r_w}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Both Eqs. 41 and 42 are in darcy, or cgs units. Thus, the $m(p)$ have the units of sq atm/cp.

NO FLOW ACROSS OUTER BOUNDARY

As was shown previously by Eq. 32 and Fig. 6 at long times, a flow equation for the closed outer boundary, constant mass rate production, radial flow case can be written:

$$\ln\frac{r_d}{r_w} = \ln\frac{0.472\,r_e}{r_w} = \frac{\pi khT_{sc}}{q_{sc}p_{sc}T}[m(\bar{p})-m(p_w)] \quad . \quad (43)$$

Since the $m(\bar{p})$ values were determined from a materials balance, the $\bar{p}$ argument represents the average pressure which would yield the proper average density, or the static pressure following a complete pressure build-up. It is *not* a volumetric average pressure. Eq. 43 coupled with the normal material balance for a bounded drainage volume provides a useful means to couple production rate and gas recovery.

In the case of liquid flow, an equation similar to Eq. 43 can be derived using the concept of pseudo-steady-state flow. That is, a condition is eventually reached for constant rate liquid production when the rate of pressure decline becomes constant everywhere in the reservoir. This condition is expressed mathematically by setting the Laplacian of the pressure equal to a constant (other than zero). Although it can be shown that the Laplacian of pressure-squared for an ideal gas, or the Laplacian of the real gas pseudo-pressure cannot be equal to a constant rigorously, a flow condition similar to pseudo-steady-state does appear to exist for both ideal and real gas flow, for all practical purposes. The existence of such a condition is suggested by Eq. 43. Fig. 11 presents an interesting inspection of the pressure behavior during the period that Eq. 43 applies for one of the Eilerts *et al.*[20] cases. Also shown is the $p_D(t_D)$ for comparison with the liquid case. As was seen previously in Fig. 5, the $m_D(t_D)$ does not change at a constant rate during this period. Although it matches the liquid case solution at early times, eventually the $m_D(t_D)$ drops below the liquid case solution. The most interesting feature of Fig. 11, however, is that the $m_D(r_D, t_D)$ for all radial locations are *essentially* parallel. Thus, the $m(p)$ profile is essentially independent of time. This condition can be described approximately by setting the Laplacian of $m(p)$ equal to a constant. As shown in Refs. 39 and 42, this leads to an equation similar to Eq. 32, but in terms of an average $\overline{m}(p)$ rather than $m(\bar{p})$. Although it can be shown that these two averages tend to be equivalent for practical ranges of conditions, it does not appear worthwhile to show the development here. In any event, Eq. 32 describes the long-time flow behavior of closed radial systems with remarkable accuracy.

Another consequence of inspection of Fig. 11 is that the $m(p)$ distribution can be obtained readily. For ex-

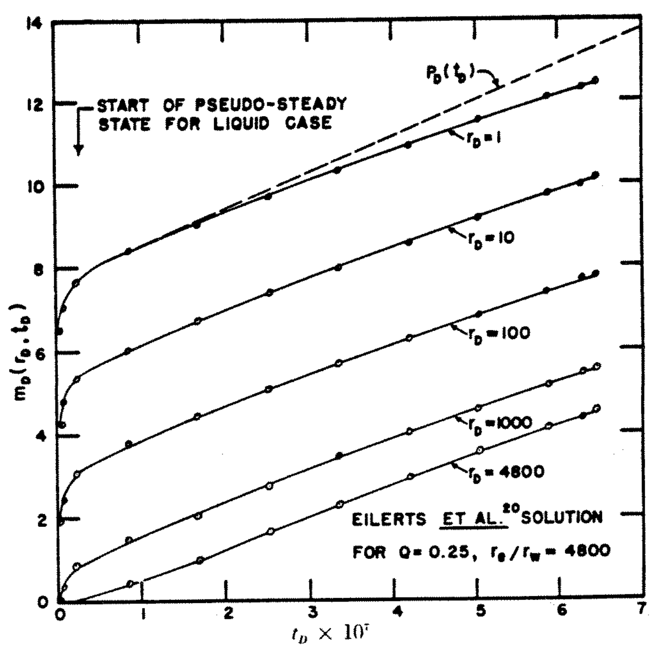

FIG. 11—$m_D(r_D, t_D)$ VS t_D FOR CONSTANT RATE PRODUCTION OF A REAL GAS FROM A CLOSED, RADIAL RESERVOIR.

ample, the following equation also describes flow reasonably well:

$$\ln\left(0.606\ \frac{r_e}{r_w}\right) = \frac{\pi k h T_{sc}}{q_{sc} p_{sc} T}\ [m(p_e) - m(p_w)] \quad . \quad (44)$$

DISCUSSION AND CONCLUSIONS

The purpose of the preceding was to describe fundamental considerations which can be used successfully to analyze the flow of real gases. The concept of the real gas pseudo-pressure promises a considerable simplification and improvement in all phases of gas well testing analysis and gas reservoir calculations. Such applications will be described in useful engineering form in a companion paper.

Several remarks concerning the real gas pseudo-pressure are in order. No claim of originality can be made for the substitution we have called the *real gas pseudo-pressure*. Carslaw and Jaeger[25] reviewed application of a similar transformation which was used in solution of heat conduction problems as early as 1894 and the early 1930's. Recently, McMordie[38] pointed out the utility of this sort of transformation in heat conduction problems. There have even been numerous mentions of the use of a transformation similar to the $m(p)$ function in connection with flow through porous media. In 1949, Muskat[43] used the same transformation in a discussion of the theory of potentiometric models. In 1953, Leibenzon[41] used the transformation, and Russian authors refer to it as the Leibenzon transformation. In 1951, Fay and Prats[30] discussed use of a similar transformation in connection with transient liquid flow. In 1955, Atkinson and Crawford[45] evaluated numerically a similar function but with constant viscosity. In 1962, Carter[18] used a gas mobility term $M(p)$, which was defined as:

$$M(p) = \frac{khp}{T\mu z} \quad .$$

Clearly, the $m(p)$ function is proportional to the pressure integral of Carter's $M(p)$. In 1963, Hurst *et. al.*[46] used a similar integral, but with constant viscosity. To our knowledge, however, this paper represents the first application of the real gas pseudo-pressure to linearization of transient real gas flow. Perhaps the most surprising fact is that the realization of the utility of this concept has been so long in coming.

In the original draft of this paper and the companion paper,[42, 47] we called the $m(p)$ function the *real gas potential*. It was stated in those papers that the $m(p)$ transformation was not a true potential. Carslaw and Jaeger[25] termed a similar substitution in heat conduction an *effective potential*, while Muskat[43] called the transformation a *potential* as a matter of convenience. We feel that the $m(p)$ transformation will be an important function in gas reservoir engineering, and it is important that the function be given a suitable name. If we were to name the transformation as Russian authors have, we would call it the Muskat transformation. In the belief that the name should be reasonably descriptive and brief, the term real gas pseudo-pressure was finally selected. This name was originally suggested to us by M. Prats, with Shell Development Co.

It appears that the following conclusions are justified. The transformation called the real gas pseudo-pressure in this paper reduces a rigorous partial differential equation for the flow of real gas in an ideal system to a form similar to the diffusivity equation, but with potential-dependent diffusivity. Because the variation of the diffusivity of real gas with pressure was similar to that of an ideal gas, it was possible to correlate finite difference solutions for the ideal radial production of real gas from a bounded system with the liquid flow solutions of van Everdingen and Hurst, and the ideal gas solutions of Aronofsky and Jenkins. This correlation avoids the assumption of small pressure gradients in the reservoir and offers generally useful solutions for the radial flow of real gas.

An investigation of the injection of real gas into a bounded radial system also gave a reasonable correlation —but not as good a correlation as production data. The correlation was as good as, or better than, the correlation of ideal gas flow results made by Aronofsky and Jenkins.

An investigation of the possibility of superposition of the linearized results indicated that superposition can be used as an acceptible engineering approximation to generate variable rate flow of real gases in a radial system. Pressure build-up for real gas flow was thus justified for the first time. (No justification for pressure build-up for the non-linear problem of ideal gas flow has yet been presented.)

Accurate and simple equations can be written to describe unsteady flow of real gases which properly consider variation of gas physical properties.

NOMENCLATURE

∇ = grad
$\nabla\cdot$ = divergence
∇^2 = Laplacian operator
A = area, sq cm
b = slope of a straight line in a plot of $k(p)$ vs $1/p$
$c_g(p)$ = real gas compressibility defined by Eq. 10
h = thickness, cm
$k(p)$ = effective permeability, darcies
M = molecular weight
$m(p)$ = real gas pseudo-pressure defined by Eq. 14
p = pressure, atm
q = flow rate, cm³/sec
r = radius, cm
R = gas constant
t = time, sec
T = temperature, °K
v = velocity, cm/sec
V = pore volume, cm³
x, y, z = direction notation
$z(p)$ = gas deviation factor, a function of pressure at constant temperature
ρ = density, gm/cm³
$\mu(p)$ = real gas viscosity, a function of pressure at constant temperature, cp
μ_i = viscosity at atmospheric pressure, cp
n = normal distance scale
ϕ = hydrocarbon porosity, fraction

SUBSCRIPTS

e = external boundary
l = liquid
pc = pseudo-critical
r = radius
sc = standard conditions
w = internal boundary, the well

ACKNOWLEDGMENTS

The authors wish to acknowledge financial support of the Texas A&M U., the Texas Engineering Experiment Station of Texas A&M and the Texas Petroleum Research Committee. This paper represents a composite of research effort conducted over a period of time by several agencies. Crawford first evaluated the $m(p)$ function during the past decade. Use of the $m(p)$ function and a more recent evaluation of the function were described by Al-Hussainy.[39] The authors also wish to acknowledge the encouragement in the course of this study by R. L. Whiting. Portions of this work were done by Al-Hussainy in partial fulfillment of graduate degree requirements in petroleum engineering at Texas A&M U. Finally, and sincerely, the authors wish to acknowledge the numerous helpful suggestions made by the reviewers of the original draft of this paper.

REFERENCES

1. Muskat, M.: *The Flow of Homogeneous Fluids*, J. E. Edwards, Inc., Ann Arbor, Mich. (1946).
2. Katz, D. L., Cornell, D., Kobayashi, R., Poettman, F., Vary, J. A., Elenbaas, J. R. and Weinaug, C. F.: *Handbook of Natural Gas Engineering*, McGraw-Hill Book Co. Inc., New York, N. Y. (1959).
3. Heatherington, C. R., MacRoberts, D .T. and Huntington, R. L.: "Unsteady Flow of Gas Through Porous Media", *Trans.*, AIME (1942) **146**, 166.
4. MacRoberts, D. T.: "Effect of Transient Conditions in Gas Reservoirs", *Trans.*, AIME (1949) **186**, 36.
5. Janicek, J. D. and Katz, D. L.: "Applications of Unsteady State Gas Flow Calculations", paper presented at Research Conference on Flow of Natural Gas Reservoirs, U. of Michigan, Ann Arbor, Mich. (June 30, 1955).
6. Cornell, D. and Katz, D. L.: "Pressure Gradients in Natural Gas Reservoirs", *Trans.*, AIME (1953) **198**, 61.
7. Rowan, G. and Clegg, M. W.: "An Approximate Method for Transient Radial Flow", *Soc. Pet. Eng. Jour.* (Sept., 1962) 225.
8. Green L. and Wilts, C. H.: "Nonsteady Flow of Gas Through a Porous Wall", *Proc.*, First U. S. National Congress Applied Mechanics, J. E. Edwards, Inc., Ann Arbor, Mich. (1962) 777.
9. Aronofsky, J. S. and Jenkins, R.: "Unsteady Flow of Gas Through Porous Media: One-dimensional Case", *Proc.*, First U. S. National Congress Applied Mechanics, J. E. Edwards, Inc., Ann Arbor, Mich. (1952) 763.
10. Jenkins, R. and Aronofsky, J. S.: "Unsteady Radial Flow of Gas Through Porous Media", *Jour. Appl. Mech.* (1953) **20**, 210.
11. Bruce, G. D., Peaceman, D. W., Rachford, H. H. and Rice, J.: "Calculations of Unsteady-State Gas Flow Through Porous Media", *Trans.*, AIME (1953) **198**, 79.
12. Douglas, J., Jr., Peaceman, D. W. and Rachford, H. H.: "Calculation of Unsteady-State Gas Flow in a Square Reservoir", *Trans.*, AIME (1955) **204**, 190.
13. Aronofsky, J. S.: "Effect of Gas Slip on Unsteady Flow of Gas Through Porous Media", *Jour. Appl. Phys.* (1954) **25**, 48.
14. Aronofsky, J. S. and Jenkins, R.: "A Simplified Analysis of Unsteady Radial Gas Flow", *Trans.*, AIME (1954) **201**, 149.
15. van Everdingen, A. F. and Hurst, W.: "The Application of the LaPlace Transformation to Flow Problems in Reservoirs", *Trans.*, AIME (1949) **186**, 305.
16. Aronofsky, J. S. and Ferris, O. D.: "Transient Flow of Nonideal Gases in Porous Media—One-dimensional Case", *Jour. Appl. Phys.* (1954) **25**, 289.
17. Aronofsky, J. S. and Porter, J. D.: "Unsteady Radial Flow of Gas Through Porous Media: Variable Viscosity and Compressibility", *Jour. Appl. Mech.* (1956) **23**, 128.
18. Carter, R. D.: "Solutions of Unsteady-State Radial Gas Flow", *Jour. Pet. Tech.* (May, 1962) 549.
19. Eilerts, C. K.: "Integration of Partial Differential Equation for Transient Linear Flow of Gas-Condensate Fluids in Porous Structures", *Soc. Pet. Eng. Jour.* (Dec., 1964) 291.
20. Eilerts, C. K., Sumner, E. F. and Potts, N. L.: "Integration of Partial Differential Equation for Transient Radial Flow of Gas-Condensate Fluids in Porous Structures", *Soc. Pet. Eng. Jour.* (June, 1965) 141.
21. Swift, G. W. and Kiel, O. G.: "The Prediction of Gas Well Performance Including the Effect of Non-Darcy Flow", *Jour. Pet. Tech.* (July, 1962) 791.
22. Tek, M. R., Coats, K. H. and Katz, D. L.: "The Effect of Turbulence on the Flow of Natural Gas Through Porous Reservoirs", *Jour. Pet. Tech.* (July, 1962) 799.
23. Carter, R. D.: Supplemental Appendix to "Determination of Stabilized Gas Well Performance from Short Flow Tests", *ADI Doc. No. 7471*, Library of Congress, Washington, D. C.
24. Rowan, G. and Clegg, M. W.: "An Approximate Method for Non-Darcy Radial Gas Flow", *Soc. Pet. Eng. Jour.* (June, 1964) 96.
25. Roberts, R. C.: "Unsteady Flow of Gas Through a Porous Medium", *Proc.*, First U. S. National Congress Applied Mechanics, J. E. Edwards, Inc., Ann Arbor, Mich. (1952) 773.
26. Kidder, R. E.: Unsteady Flow of Gas Through a Semi-Infinite Porous Medium", *Jour. Appl. Mech.* (1957) **24**, 329.
27. Polubarinova-Kochina, P. Ya.: *Theory of Ground Water Movement*, Princeton U. Press, Princeton, N. J. (1962).
28. Carslaw, H. S. and Jaeger, J. C.: *Conduction of Heat in Solids*, 2nd Ed., Oxford U. Press, London, England (1959) 11.
29. Klinkenberg, L. J.: "The Permeability of Porous Media to Liquids and Gases", *Drill. and Prod. Prac.*, API (1941) 200.
30. Fay, C. H. and Prats, M.: "The Application of Numerical Methods to Cycling and Flooding Problems", *Proc.*, Third World Petroleum Congress, Section II (1951) 555.
31. Standing, M. B. and Katz, D. L.: "Density of Natural Gases", *Trans.*, AIME (1942) **146**, 140.
32. Carr, N. L., Kobayashi, R. and Burrows, D. B.: "Viscosity of Hydrocarbons Under Pressure", *Trans.*, AIME (1954) **201**, 264.
33. Trube, A. S.: "Compressibility of Natural Gases", *Trans.*, AIME (1957) **210**, 69.
34. Robinson, D. B., Macrygeorgas, C. A. and Govier, G. W.: "The Volumetric Behavior of Natural Gases Containing Hydrogen Sulfide and Carbon Dioxide", *Trans.*, AIME (1960) **219**, 54.
35. Matthews, C. S.: "Analysis of Pressure Build-Up and Flow Test Data", *Jour. Pet. Tech.* (Sept., 1961) 862.
36. Friedmann, N. E.: "Quasilinear Heat Flow", *Trans.*, ASME (April, 1958) 635.
37. Storm, M. L.: "Heat Conduction in Simple Metals", *Jour. Appl. Phys.* (July, 1951) **22**, No. 7, 192.
38. McMordie, R. K.: "Steady-State Conduction With Variable Thermal Conductivity", *Trans.*, ASME (Feb., 1962) 92.
39. Al-Hussainy, R.: "Flow of Real Gases Through Porous Media", MS Thesis, Texas A&M U. (Jan., 1965).
40. Ramey, H. J., Jr.: "Non-Darcy Flow and Wellbore Storage Effects in Pressure Buildup and Drawdown of Gas Wells", *Jour. Pet. Tech.* (Feb., 1965) 223.
41. Dykstra, H.: "Calculated Pressure Build-up for a Low Permeability Gas-Condensate Well", *Jour. Pet. Tech.* (Nov., 1961) 1131.
42. Al-Hussainy, R., Ramey, H. J., Jr. and Crawford, P. B.: "The Theory of the Real Gas Potential", Paper No. 1243-A, presented at SPE Annual Fall Meeting, Denver, Colo. (Oct. 3-6, 1965).
43. Muskat, M.: "The Theory of Potentiometric Models", *Trans.*, AIME (1949) **179**, 216.
44. Leibenzon, L. S.: "Subsurface Hydraulics of Water, Oil and Gas", *Publ. Acad. Sci.*, collected works, U.S.S.R. (1953) **2**.
45. Atkinson, D. O. and Crawford, P. B.: "Generalized Pressure Distribution and Gas Mass in Pipe Lines", *Pet. Prod. Tech.*, AIME Conference Proc. of 1956, Miscellaneous Publication of the Texas Engineering Experiment Station (Sept., 1956) 59.
46. Hurst, W., Goodson, W. C. and Leeser, R. E.: "Aspects of Gas Deliverability", *Jour. Pet. Tech.* (June, 1963) 668.
47. Al-Hussainy, R. and Ramey, H. J., Jr.: "Application of the Real Gas Potential", Paper No. 1243-B, presented at the SPE Annual Fall Meeting, Denver, Colo. (Oct. 3-6, 1965).

APPENDIX

CORRELATION OF EILERTS *ET AL.*[20] SOLUTIONS

Eilerts *et al.*[20] solved the following equation numerically (in their nomenclature):

$$\frac{\partial}{\partial U}\left[W(P)\frac{\partial P^2}{\partial U}\right] = e^{2U}\frac{\partial}{\partial H}\left[\frac{P}{Z(P)}\right] \quad , \quad . \quad . \quad . \quad \text{(A-1)}$$

where

$p = p_iP$, $K(p) = k(p_i)K(P)$, $z(p) = z(p_i)Z(P)$, $\mu(p) = \mu(p_i)\,\mu(P)$, $w(p) = k(p)/\mu(p)z(p)$

and

$$w(p) = w(p_i)W(P) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(A-2)}$$

Dimensionless time is defined as:

$$H = \frac{p_ik(p_i)}{2\phi\mu(p_i)r_b^2}\,t \quad , \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(A-3)}$$

and the dimensionless radius:

$$U = \ln\frac{r}{r_b} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(A-4)}$$

In terms of the $m(p)$ function, and using the dimensionless variables Eq. A-2, the flow equation takes the form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,\frac{\partial m(P)}{\partial r}\right] = \frac{\phi\mu(p_i)\mu(P)c_g(P)}{p_ik(p_i)K(P)}\,\frac{\partial m(P)}{\partial t}, \quad \text{(A-5)}$$

where

$$m(P) = 2\int_0^P PW(P)dP \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(A-6)}$$

Let the coefficient on the right side of Eq. A-5 be evaluated at the initial conditions, and define:

$$r_D = \frac{r}{r_w} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(A-7)}$$

$$t_D = \frac{k(p_i)p_i}{\phi\mu(p_i)c_g(P_i)r_w^2}\,t = \frac{2H}{c_g(P_i)}\left(\frac{r_b}{r_w}\right)^2 \quad . \quad \text{(A-8)}$$

Notice that $\mu(P)$ and $K(P)$ are equal to one at the initial P. Hence, Eq. A-5 takes the form:

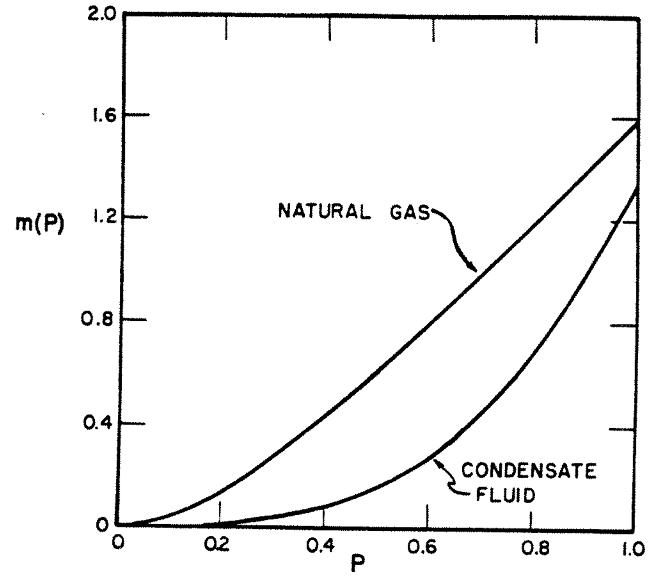

Fig. 12—$m(P)$ vs P for the Eilerts *et al.*[20] Gases.

$$\frac{1}{r_D}\frac{1}{\partial r_D}\left[r_D\frac{\partial m(P)}{\partial r_D}\right] = \frac{\partial m(P)}{\partial t_D} \quad . \; . \; . \; . \; . \quad \text{(A-9)}$$

The flow rate at the producing face as given by Eilerts *et al.*[20] is:

$$Q = 2W(P)\frac{\partial P^2}{\partial U} = 2\frac{\partial m(P)}{\partial U} \quad , \; . \; . \; . \quad \text{(A-10)}$$

and the closed boundary:

$$\frac{\partial m(P)}{\partial U} = 0 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(A-11)}$$

Thus, in terms of the dimensionless real gas pseudo-pressure drop:

$$m_D(r_D, t_D) = \frac{2}{Q}\Delta m(P) \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(A-12)}$$

The $m(P)$ for the Eilerts *et al.* natural gas and condensate fluid are shown in Fig. 12. The large difference between physical properties of the two fluids is apparent.

★★★

Application of Real Gas Flow Theory to Well Testing and Deliverability Forecasting

R. AL-HUSSAINY
JUNIOR MEMBER AIME
H. J. RAMEY, JR.*
MEMBER AIME

TEXAS A&M U.
COLLEGE STATION, TEX.

ABSTRACT

Previous gas well test analyses have been based mainly upon linearizations of ideal gas flow results, although a method for drawdown analysis based upon real gas flow results has been proposed. Linearizations based upon ideal gas flow require estimation of gas physical properties at some sort of average pressure, and implicitly involve the assumption that pressure gradients are small everywhere in the reservoir. A new real gas flow equation has been developed by means of a substitution which couples pressure, viscosity and gas law deviation factor. This substitution has been called the real gas pseudo-pressure. Use of the real gas pseudo-pressure leads to simple equations describing real gas flow which do not contain pressure-dependent gas properties, and which do not require the assumption of small pressure gradients everywhere in the flow system.

Equations required to determine flow capacity, well condition and static formation pressure from pressure drawdown and build-up tests with the real gas pseudo-pressure concept are presented. Also shown are applications of the real gas pseudo-pressure to back-pressure testing, the gas materials balance and rigorous determination of average gas properties for previous gas flow equations. Included are sample calculations for well test analyses.

INTRODUCTION

A recent study[1] of the flow of the transient flow of real gases in ideal radial systems showed that it is possible to consider gas physical property dependence upon pressure by means of a substitution called the real gas pseudo-pressure. The substitution has the advantage that it enables engineering solutions for steady and transient flow of real gases that are more accurate and general than those previously available. The solutions are particularly important for the case of gas flow in tight, high-pressure formations under large drawdowns. Even for more ideal flow conditions, the real gas pseudo-pressure concept leads to important rules for finding average gas physical properties pertinent for older well test analyses.

Ref. 1 provides a detailed study of the theory behind the real gas flow correlations to be used in this paper. The purpose of this paper is to provide necessary engineering forms for use of the real gas flow results, and to illustrate applications.

The following considers flow of real gases in an ideal radial flow system. It is assumed that: (1) formation thickness, porosity, water saturation, absolute permeability, temperature and gas composition are constant; (2) gas compressibility and density are functions of pressure as described by the gas law $pv = znRT$; (3) gas viscosity is a function of pressure; (4) effective permeability to gas may be a function of pressure to account for liquid condensation; (5) condensate is immobile; and (6) the formation has no dip.

SUMMARY OF REAL GAS FLOW EQUATIONS

Combination of the continuity equation and Darcy's law for radial flow of a real gas yields the equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{p}{\mu(p)z(p)}\frac{\partial p}{\partial r}\right] = \frac{\phi}{k}\frac{\partial}{\partial t}\left[\frac{p}{z(p)}\right] \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the left-hand side of Eq. 1 is differentiated, a term involving $(\partial p^2/\partial r)^2$ arises which does not occur in ideal gas flow equations. Thus, modification of ideal gas flow solutions to fit real gas flow involves the assumption that pressure gradients are small. An alternate procedure is to make a change of variable in Eq. 1.

We define the real gas pseudo-pressure $m(p)$ as:

$$m(p) = 2\int_{p_m}^{p}\frac{p}{\mu(p)z(p)}dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

As shown in Ref. 1, substitution of $m(p)$ in Eq. 1 leads to:

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{\phi\mu(p)c_g(p)}{k}\frac{\partial m(p)}{\partial t} \quad . \quad . \quad . \quad . \quad (3)$$

Eq. 3 looks like the diffusivity equation, but is still non-linear because the diffusivity depends upon pressure, or $m(p)$. Eq. 3 also closely resembles the equation for the flow of an ideal gas. Furthermore, the dependence of the diffusivity term upon $m(p)$ for the real gas is very similar to the dependence of the ideal gas diffusivity upon pressure squared. As is shown in Ref. 1, this leads immediately to practical solutions for Eq. 3 which are similar to the Aronofsky-Jenkins[2] ideal gas flow solutions. Engineering units will be used throughout the following. For production of a real gas in an ideal radial system at constant rate with a closed outer boundary:

Original manuscript received in Society of Petroleum Engineers office June 28, 1965. Revised manuscript of SPE 1243B received Feb. 18, 1966. Paper was presented at SPE Annual Fall Meeting held in Denver, Colo., Oct. 3-6, 1965.

*Presently on the faculty at Stanford U.

[1]References given at end of paper.

$$\frac{1.987 \times 10^{-5}\, khTsc}{qp_{sc}T}\left[m(\bar{p})-m(p_w)\right] = \ln\frac{r_d}{r_w} + s + Dq \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

where

$$\ln\frac{r_d}{r_w} = p_D(t_D) - 2\,t_D\left(\frac{r_w}{r_e}\right)^2 \quad , \; . \; . \; . \; . \; . \; . \; . \; . \; (5)$$

and

$$t_D = \frac{0.000264\, kt}{\phi(\mu C_t)_i r_w^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (6)$$

The $m(\bar{p})$ is evaluated at an average pressure determined from a materials balance:

$$\left(\frac{p}{z}\right)_i - \left(\frac{\bar{p}}{z}\right) = \frac{Tp_{sc}G_p}{\pi hr_e^2\phi S_g T_{sc}} = \int_{m(\bar{p})}^{m(p_i)} (\mu c_g)\, dm(p)$$

$$\approx (\mu c_g)_{avg}\,[m(p_i)-m(\bar{p})] \quad . \; . \; . \; . \; . \; . \; . \; (7)$$

The $p_D(t_D)$ is the van Everdinger-Hurst[3] dimensionless pressure-drop function. For large ratios of r_e/r_w:

$$p_D(t_D) = \frac{1}{2}\,[\ln t_D + 0.80907] \;, \text{ for } 100 \leqslant t_D \leqslant \frac{1}{4}\left(\frac{r_e}{r_w}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (8)$$

$$p_D(t_D) = \ln\frac{r_e}{r_w} - \frac{3}{4} + 2\,t_D\left(\frac{r_w}{r_e}\right)^2 \;, \text{ for } t_D \geqslant \frac{1}{4}\left(\frac{r_e}{r_w}\right)^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (9)$$

Eqs. 4 through 9 are not an explicit solution to Eq. 3. But they do represent an excellent engineering approximation good for a wide range of conditions.

The total system compressibility in Eq. 6 (c_t) normally can be taken equal to the gas saturation-gas compressibility product S_gC_g. The porosity used is the *total* porosity.

In view of the finding that production of real gas can be linearized as above, we can write a very useful equation for the real gas pressure at the inner boundary of an infinite reservoir produced at constant rate, including a steady-state skin effect and non-Darcy flow term for $t_D > 100$, and before outer boundary effects are important:

$$m(p_{wf}) = m(p_i) - 5.792 \times 10^4\frac{qp_{sc}T}{khT_{sc}}\left[\log_{10}t_D + 0.3513 + 0.87s + 0.87Dq\right] \quad . \; . \; . \; . \; . \; (10)$$

If standard conditions are taken to be 14.7 psia and 60F:

$$m(p_{wf}) = m(p_i) - 1{,}637\frac{qT}{kh}\left[\log_{10} t_D + 0.3513 + 0.87s + 0.87Dq\right] . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (11)$$

And for long flowing times:

$$q = 1.987 \times 10^{-5}\frac{khT_{sc}\,[m(\bar{p})-m(p_{wf})]}{p_{sc}T\left[\ln\dfrac{0.472\,r_e}{r_w} + s + Dq\right]} \; . \; (12)$$

In the preceding equations, s is the familiar skin effect, and (Dq) is an approximate non-Darcy flow term.[4] The non-Darcy flow coefficient D is inversely proportional to gas viscosity, and slightly dependent on time for *very* short transient flows. However, D can be considered constant as an adequate engineering approximation.

APPLICATION OF REAL GAS FLOW EQUATIONS

The following describes application of the real gas flow equations to common cases of gas well flowing testing (drawdown, build-up and back-pressure testing).

DRAWDOWN TESTING

Eqs. 10 or 11 provide the basis for drawdown test analysis for real gas flow. From Eq. 10, it is apparent that a plot of $m(p_{wf})$ vs the logarithm of the producing time for constant rate production should produce a straight line of slope:

$$-\,b = 5.792 \times 10^4\,\frac{qp_{sc}T}{khT_{sc}} \quad . \; . \; . \; . \; . \; . \; . \; . \; (13)$$

Or for standard conditions of 14.7 psia and 60F:

$$-\,b = 1{,}637\,\frac{qT}{kh} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (14)$$

Eq. 14 is analogous to the equation for analyzing build-up and drawdowns for gas wells presented by Tracy,[5] except neither viscosity nor z appear. The striking feature of all of the real gas flow equations presented in the previous section is that pressure-dependent viscosity and gas law deviation factor are absent.

Procedure is as follows. Drawdown data are taken as in the past: measure sandface producing pressures as a function of producing time for constant rate of production. Next, produce a working plot of $m(p)$, psi^2/cps vs pressure in psi for the reservoir. If specific viscosity and density data are available, this can be done by graphical integration or the Trapezoidal rule using Eq. 2. If viscosity and z-factor correlations are normally used, $m(p)$ can be found without integration by reading either Table 1 or Fig. 2, Ref. 1. Drawdown pressures are converted to appropriate $m(p)$ values and plotted vs the logarithm of drawdown time, and a straight line is passed through the data. Either Eqs. 13 or 14 can then be used to find the flow capacity. An example calulation is given in the Appendix.

In general, production of an $m(p)$ vs p working plot for a given reservoir requires less effort than squaring pressures as in current methods. The plot, of course, may be used for all wells producing from the same reservoir if formation temperature and gas composition does not vary widely.

Eqs. 10 or 11 can also be solved for the skin effect and non-Darcy flow coefficient. This result is:

$$s' = s + Dq = 1.151\left[\frac{m(p_i) - m(p_{1\,hr})}{-\,b} - \log_{10}\frac{k}{\phi(\mu c_t)_i r_w^2} + 3.23\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; (15)$$

It is necessary to have drawdown tests at two flow rates to determine both the skin effects and the non-Darcy flow coefficient D separately. As described by Ramey,[6] wellbore storage effects or any variable-rate test can often be used to separate the skin effect and non-Darcy flow. The real gas potential drop across the skin can be found from:

$$\Delta m(p)_{skin} = 0.87\,(-b)s \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (16)$$

Flow efficiency can be found by means of Eqs. 12 or 13:

$$FE = J_{act.}/J_{theo.} = \frac{m(\bar{p}) - m(p_{wf}) - \Delta\, m(p)_{skin}}{m(\bar{p}) - m(p_{wf})} \quad . \quad (17)$$

in analogy with the expression presented by Matthews.[7]

The preceding for drawdown analysis for real gas flow should be very accurate. In many cases, conventional plotting of pressure or pressure squared leads to plots which are not linear, although the $m(p)$ plot is linear ~ as it should be. One example given in the Appendix yields reasonable straight lines for both $m(p)$ and pressure-squared plotting (Fig. 2), although a plot of drawdown pressure vs log time has considerable curvature. In other cases, it has been observed that the pressure-log time plot and $m(p)$-log time plots are excellent straight lines, although a pressure-squared plot shows considerable curvature. In all cases tried to date, the $m(p)$-log time plot has produced reasonable straight lines.

Often, it is necessary to analyze a variable-rate drawdown—such as Russell's[8] two-rate test, or to account for wellbore storage effects.[6] This requires superposition of constant rates to generate variable-rate cases. Ref. 1 indicates that superposition can be used as a satisfactory approximation in many cases.

An important question at this point is whether the real gas flow solutions offer a worthwhile improvement over existing gas flow solutions currently used in well test analysis?

In addition to the problem of gas physical property variation, use of the real gas flow solutions offers an important improvement over existing gas flow theory that is particularly important for low permeability formations. That is, the solution in terms of real gas pseudo-pressure includes the second-degree pressure gradient term neglected by methods based on either liquid flow or ideal gas flow solutions. In the case of low-permeability gas well testing, current well test interpretation methods can lead to formation capacity or deliverability estimates that can be grossly in error. This can be illustrated by comparison of flow capacity determinations by various methods for a known capacity system. Excellent data for testing various methods are available from Carter's[9] finite difference solutions for real gas flow.

Carter proposed a method to find flow capacity which was based on a correlation with a dimensionless time wherein viscosity and compressibility were point time functions. Thus, it is possible to compare flow capacities developed from his method, the liquid case analogy and from the real gas flow analysis of this paper. Table 1 presents the comparison. As can be seen, Carter's analysis leads to flow capacities which are high, the liquid flow analogy leads to flow capacities which are generally low, while the real gas flow analysis based on $m(p)$ values leads to capacity estimates in very close agreement with the actual values. Note particularly the good agreement for the Solution 3 case for 8 md-ft capacity. The difference between Carter's analysis and the liquid case analogy is a general result. As flow approaches that of an ideal gas, the two methods and the real gas flow method yields the same answer.

TABLE 1—COMPARISON OF FLOW CAPACITIES ESTIMATED FROM DRAWDOWN DATA BY VARIOUS METHODS FOR CARTER'S GAS FLOW SOLUTIONS (REF. 9)

Solution No.	Flow Rate (Mcf/D)	Initial Pressure (psia)	Capacity (md-ft) True	Carter's	Liquid Case	$m(p)$ Analyses*
1	2,800	3,300	50	53	45.9	49.4
2	4,000	3,300	50	55.3	44.4	50.3
3	1,200	4,700	8	10.92	5.64	8.2

*The differences between the true and real gas $m(p)$ results are not believed significant. Differences are less than normal error involved in taking slopes of drawdown curves.

BUILD-UP TESTING

The pertinent equations for analysis of pressure build-up in terms of the real gas pseudo-pressure are:

$$\text{Flow capacity} = m(p_{ws}) = m(p_i) - 5.792 \times 10^4 \frac{q p_{sc} T}{k h T_{sc}} \left[\log_{10} \frac{t_p + t_s}{t_s}\right] \quad . \quad . \quad . \quad (18)$$

Or using standard conditions of 14.7 psia and 60F:

$$m(p_{ws}) = m(p_i) - 1{,}637 \frac{qT}{kh}\left[\log_{10}\frac{t_p + t_s}{t_s}\right] \quad . \quad . \quad . \quad (19)$$

Thus, a Horner-type[7] plot of $m(p_{ws})$ should yield a straight line of slope $(-b)$, and formation flow capacity can be determined from Eqs. 13 or 14. The skin effect and non-Darcy flow coefficient can be determined from Eq. 15 if the real gas potential difference is replaced by $m(p_{1\,hr}) = (p_{wp})$. The flow efficiency can be determined from Eq. 17 if $m(p_{wp})$ is substituted for $m(p_{wf})$. The pressure drop across the skin can be computed from Eq. 16 as for drawdown.

In a completely analogous manner to previous build-up theory, Eqs. 18 and 19 indicate that the plot of $m(p_{ws})$ can be extrapolated to infinite shut-in time (a time ratio of unity) to yield $m(p^*)$. The Matthews-Brons-Hazebroek[10] pressure correction charts can be used to correct the extrapolated $m(p^*)$ value to $m(\bar{p})$, the average real gas potential, if the ordinates of their pressure correction charts are changed to:

$$2.303\left[\frac{m(p^*) - m(\bar{p})}{-b}\right], \quad . \quad . \quad . \quad . \quad (20)$$

and the dimensionless producing time is defined as:

$$(t_p)_D = \frac{0.000264\, k t_p}{\phi(\mu c_t)_i A} \quad , \quad . \quad . \quad . \quad . \quad (21)$$

where A is the drainage area in square feet.

If producing time t is long enough that the drainage radius is stabilized before the well is shut in, the best approximation for pressure build-up is:

$$m(p_{ws}) = m(\bar{p}) - 5.792 \times 10^4 \frac{q p_{sc} T}{k h T_{sc}}\left[\log_{10}\left(\frac{\phi(\mu c_t)_i r_e^2}{0.00266\, k t_s}\right)\right] \quad . \quad . \quad . \quad (22)$$

The slope of a plot of $m(p_{ws})$ vs $\log_{10} t_s$ yields the flow capacity, while the $m(p)$ can be read on the straight line at a build-up time t_s of:

$$t_s \text{ hours} = \frac{\phi(\mu c_t)_i r_e^2}{0.00266 k} \quad . \quad . \quad . \quad . \quad (23)$$

This is analogous to the Dietz[11] method for liquid flow.

BACK-PRESSURE TEST ANALYSIS

Application of the preceding to back-pressure test analysis is straightforward, but several features deserve mention. First, Eq. 4 provides an excellent match to both short-time isochronal flow tests and stabilized flow tests. The skin effect and the non-Darcy flow constant can be obtained from short-time isochronal flow testing if a single drawdown or build-up is available to obtain the flow capacity kh. Once this is done, r_d can be placed equal to 0.472 r_e, and either Eqs. 4 or 12 used to generate a plot of $\log_{10} [m(\bar{p})-m(p_{wf})]$ vs $\log_{10} q$.

This is analogous to the familiar back-pressure curve. The $m(p)$ curve has many of the characteristics of a normal back-pressure curve, but there are some important differences, also. At low rates, the non-Darcy term Dq will be negligible and the slope of the curve will be unity. At high values of flow rate, non-Darcy flow may become important and the slope of the curve will approach 2 (n value of 0.5) in a fashion similar to that described by Carter *et al.*[12] However, the $m(p)$ back-pressure curve is only slightly sensitive to static pressure level, because only the constant D depends upon pressure through dependence upon gas viscosity (Swift and Kiel[1]). Thus, a single, stabilized back-pressure curve in terms of the real gas pseudo-pressure difference can be used to generate an entire family of back-pressure curves in terms of pressure squared—or used in terms of $m(p)$ with the gas materials balance to provide a simple means of forecasting gas well performance.

AVERAGE GAS PROPERTIES

Current engineering practices in the analysis of gas well tests involves evaluation of an equivalent liquid flow system (Matthews[3]), or modification of expressions for flow of an ideal gas (Tracy[5] or Carter[9]). Matthews recommended that gas properties be evaluated at an arithmetic average pressure, while Carter found empirically that evaluation of physical properties at an average pressure equal to the square root of the average of squared pressures gave reasonable agreement with real gas flow solutions. One important result of the present study is that use of the real gas $m(p)$ leads to a solution for the proper average value for gas physical properties. For example, it is not uncommon to assume that gas vsicosity and gas law deviation factor can be regarded as constant at some average value for the flow region. For a bounded system, this would lead to the flow equation:

$$q = 1.987 \times 10^{-5} \frac{khT_{sc}(\bar{p}^2-p_{wf}^2)}{p_{sc}T(\mu z)_{avg} \ln \dfrac{0.472 r_e}{r_w}}, \quad . \quad (24)$$

where we have neglected skin effect and non-Darcy flow. If Eq. 24 is compared with Eq. 12, the result is:

$$\frac{\bar{p}^2 - p_{wf}^2}{(\mu z)_{avg}} = m(\bar{p}) - m(p_{wf}) \quad . \quad . \quad . \quad . \quad (25)$$

or

$$(\mu z)_{avg} = \frac{\bar{p}^2 - p_{wf}^2}{m(\bar{p}) - m(p_{wf})} \quad . \quad . \quad . \quad . \quad (26)$$

A similar result will hold if pressures are expressed in terms of p_e rather than $\bar{p}$. Carter[9] suggested the use of an equation similar to Eq. 26 for determination of average properties. Since many gas reservoir flow tests can be analyzed by current theory, Eq. 26 may be of much use. Note however that the $(\mu z)_{avg}$ would be used only in determination of flow capacity from the usual equation—*not* in determination of dimensionless times. Dimensionless times should still be evaluated with physical properties at the initial formation pressure before production.

If Matthew's liquid case analogy is used, an average value of (μB_g) is required for flow capacity determination. A liquid flow analogy to Eq. 12 is:

$$q = 1.987 \times 10^{-5} \frac{kh(2)(\bar{p}-p_{wf})}{\mu B_g [\ln 0.472 r_e/r_w]} \quad . \quad . \quad (27)$$

Comparison of Eqs. 12 and 27 leads to:

$$\frac{2(\bar{p}-p_{wf})}{(\mu B_g)_{avg}} = \frac{T_{sc}[m(\bar{p})-m(p_{wf})]}{p_{sc}T}, \quad . \quad . \quad . \quad . \quad (28)$$

or

$$(\mu B_g)_{avg} = \frac{2p_{sc}T(\bar{p}-p_{wf})}{T_{sc}[m(\bar{p})-m(p_{wf})]} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Again, the average value of viscosity-formation volume factor product would be used in determination of flow capacity, not in determination of dimensionless times. Extension of the averages to the transient period is obvious in view of Eq. 4.

It should be emphasized that the liquid case approximation recommended by Matthews, and Tracy's method of gas well build-up analysis, will give excellent results in many cases where pressure drawdown is not large and permeability is high enough that second-degree pressure gradients are not important. Nevertheless, it is recommended that a real gas flow analysis is useful, even in this case, to determine that the current approximate methods are applicable. In any event, it is likely that the real gas flow methods outlined in this paper will be found simpler in application than older methods once a master plot of $m(p)$ vs p is prepared for any given reservoir. This results mainly because it is no longer necessary to square pressures for plotting, and gas physical properties appear only in dimensionless times. An additional benefit is that gas properties are always evaluated at known pressures in the real gas flow method.

CONCLUSIONS

1. As a result of a change of variable called the real gas pseudo-pressure, it is possible to write approximate solutions for the production of real gases from ideal radial systems, which are analogous to the Aronofsky-Jenkins ideal gas flow solutions.

2. The approximate solutions for transient flow of a real gas using the real gas pseudo-pressure leads to methods for interpreting pressure drawdown and build-up tests which are similar to current methods, except that gas physical properties either do not appear in the engineering equations or appear at known pressures. Use of the real gas flow analysis indicates there are many well tests which can be analyzed with current procedures, but there are likely to be many others where current procedures are in error.

3. The real gas flow approximation can be used to establish proper average values of physical properties for current procedures.

Finally, it is emphasized that the real gas pseudo-pressure concept is principally a computing device. Results of back-pressure, build-up and drawdown tests can be converted from $m(p)$ solutions to solutions in terms of pressure or pressure squared to provide familiar information and displays.

NOMENCLATURE

A = drainage area, sq ft

B = formation volume factor, res vol/std vol

b = slope of build-up or drawdown plot, psi^2/cp-cycle

c = compressibility psi^{-1}
D = non-Darcy flow constant, (Mscf/D)$^{-1}$
G_p = cumulative gas produced, scf
h = net formation thickness
J = productivity index, (Mscf/D)/unit potential difference
k = effective permeability
$m(p)$ = real gas pseudo-pressure (Eq. 2) psi^2/cp
p = pressure, psi
$p_D(t_D)$ = van Everdingen-Hurst dimensionless pressure drop
q = gas rate, Mscf/D
r = radial location
s = van Everdingen-Hurst skin effect, dimensionless
s' = apparent skin effect, dimensionless
S_g = fractional gas saturation
T = temperature, °R
t = time, hours
z = real gas law deviation factor, dimensionless ($pv = nzRT$)
ϕ = total porosity, fraction pore volume
μ = viscosity of gas, cp

SUBSCRIPTS AND SUPERSCRIPTS

D = dimensionless
d = drainage
e = exterior boundary
g = gas
i = initial
1 hr = one hour on straight line
m = base
– = superbar, average
sc = standard conditions of pressure and temperature
sf = sand face
t = total
w = well, inner boundary
wf = well flowing
wp = producing before shut-in
$w\delta$ = shut-in well
* = extrapolated
avg = average

ACKNOWLEDGMENTS

The authors gratefully acknowledge financial support of the Texas A&M U. and the Texas Engineering Experiment Station of Texas A&M. The encouragement of P. B. Crawford and R. L. Whiting is also appreciated. Portions of this work were done by Al-Hussainy in partial fulfillment of the requirements toward a graduate degree at Texas A&M.

REFERENCES

1. Al-Hussainy, R., Ramey, H. J., Jr. and Crawford, P. B.: "The Flow of Real Gases Through Porous Media", *Jour. Pet. Tech.* (May, 1966) 624.
2. Aronofsky, J. S. and Jenkins, R.: "Unsteady Radial Flow of Gas Through Porous Media", *Jour. Appl. Mech.* (1953) **20,** 210.
3. van Everdingen, A. F. and Hurst, W.: "The Application of the LaPlace Transformation to Flow Problems in Reservoirs", *Trans.,* AIME (1949) **186,** 305.
4. Swift, G. W. and Kiel, O. G.: "The Prediction of Gas Well Performance Including the Effect of Non-Darcy Flow", *Jour. Pet. Tech.* (July, 1962) 791.
5. Tracy, G. W.: "Why Gas Wells Have Low Productivity", *Oil & Gas Jour.* (Aug. 6, 1956) 84.
6. Ramey, H. J., Jr.: "Non-Darcy Flow and Wellbore Storage Effects in Pressure Build-up and Drawdown of Gas Wells", *Jour. Pet. Tech.* (Feb., 1965) 223.
7. Matthews, C. S.: "Analysis of Pressure Build-up and Flow Test Data", *Jour. Pet. Tech.* (Sept., 1961) 862.
8. Russell, D. G.: "Determination of Formation Characteristics from Two-Rate Flow Tests", *Jour. Pet. Tech.* (Dec., 1963) 1347.
9. Carter, R. D.: "Solutions of Unsteady-State Radial Gas Flow", *Jour. Pet. Tech.* (May, 1962) 549.
10. Matthews, C. S., Brons, F. and Hazebroek, P.: "A Method for Determination of Average Pressure in a Bounded Reservoir", *Trans.,* AIME (1954) **201,** 182.
11. Dietz, D. N.: "Determination of Average Reservoir Pressure From Build-up Surveys", *Jour. Pet. Tech.* (Aug., 1965) 955.
12. Carter, R. D., Miller, S. C. and Riley, H. G.: "Determination of Stabilized Gas Well Performance from Short Flow Tests", *Jour. Pet. Tech.* (June, 1963) 651.
13. Carter, R. D.: Supplemental Appendix to "Determination of Stabilized Gas Well Performance from Short Flow Tests", ADI Doc. No. 7471, Library of Congress, Washington, D.C.

APPENDIX

DRAWDOWN ANALYSIS EXAMPLE

An isochronal flow test is performed on a gas well at two different rates. Given the reservoir data and fluid properties below, determine the flow capacity, skin effect and non-Darcy flow coefficient for this well. The well is completed with tubing-annulus packer.

Reservoir and Gas Data

p_i = 2,300 psia
h = 10 ft
r_w = 0.5 ft
r_e = 2,980 ft (640-acre spacing)
T = 130F
ϕ = 0.1 *fr BV*
S_g = 77 per cent PV

Gas Properties

p(psia)	z	Viscosity (cp)
400	0.95	0.0117
800	0.90	0.0125
1,200	0.86	0.0132
1,600	0.81	0.0146
2,000	0.80	0.0163
2,400	0.81	0.0180

Drawdown Data

Flowing Time (hours)	Flow No. 1 (q=1,600 Mscf/D) p_{wf} (psia)	Flow No. 2 (q=3,200 Mscf/D) p_{wf} (psia)
0.232	1,855	1,105
0.4	1,836	1,020
0.6	1,814	954
0.8	1,806	906
1.0	1,797	860
2.0	1,758	700
4.0	1,723	539
6.0	1,703	387

SOLUTION

It can be assumed that wellbore storage effects are negligible, since the well is completed with a down-hole packer. The first step is to find $m(p)$ vs p for this gas.

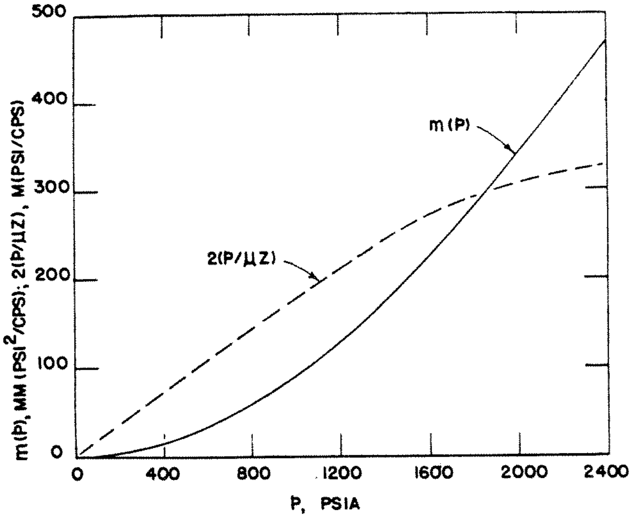

FIG. 1—$m(p)$ AND $2(p/\mu z)$ VS p FOR SAMPLE DRAWDOWN PROBLEM.

This can be done with the gas properties tabulated above and Eq. 2 in the main text. The quantity $2(p/\mu z)$ can be calculated and plotted vs pressure, as shown below and on Fig. 1. Integration can be performed in a tabular calculation by reading mid-point values of $2(p/\mu z)$ from the graph and multiplying by Δp. The computed $m(p)$, psi²/cp, is also shown on Fig. 1. This curve can be used for future tests with this well or other wells producing the same gas at the same formation temperature. Often, only gas gravity is available. In this case $m(p)$ can be found without integration from Ref. 1.

p (psia)	z	μ (cp)	$2(p/\mu z)$ (psi/cp)	Mean $2(p/\mu z)$	Δp (psi)	$2(p/\mu z)$ ($\times\Delta p$)	$m(p)$ (psi²/cp)
400	0.95	0.0117	71,975	35,988	400	14.4×10^6	14.4×10^6
800	0.90	0.0125	142,222	107,099	400	42.9×10^6	57.3×10^6
1,200	0.86	0.0132	211,416	176,819	400	70.7×10^6	128.0×10^6
1,600	0.81	0.0146	270,590	241,003	400	96.5×10^6	224.5×10^6
2,000	0.80	0.0163	306,748	288,669	400	115.5×10^6	340.0×10^6
2,400	0.81	0.0180	329,218	319,000	400	127.6×10^6	467.6×10^6

In the following, Flow 1 will be analyzed in detail. Results for Flow 2 will also be given to illustrate the importance of a second flow test at a different rate. Fig. 2 presents the drawdown data plotted in the conventional manner, and in terms of the $m(p_{wf})$.

The flow capacity for Flow 1 can be estimated from Eq. 14 in the main text:

$$kh = 1{,}637\frac{qT}{-b} = 1{,}637\frac{(1{,}600)(590)}{(32\times10^6)} = 48.4 \text{ md-ft}.$$

The total of skin effect and non-Darcy flow resistance can be estimated from Eq. 15.

$$s' = s + Dq = 1.151\left[\frac{m(p_i) - m(p_{1\,hr})}{-b} - \log_{10}\left(\frac{k}{\phi(\mu c_t)_i r_w^2}\right) + 3.23\right]$$

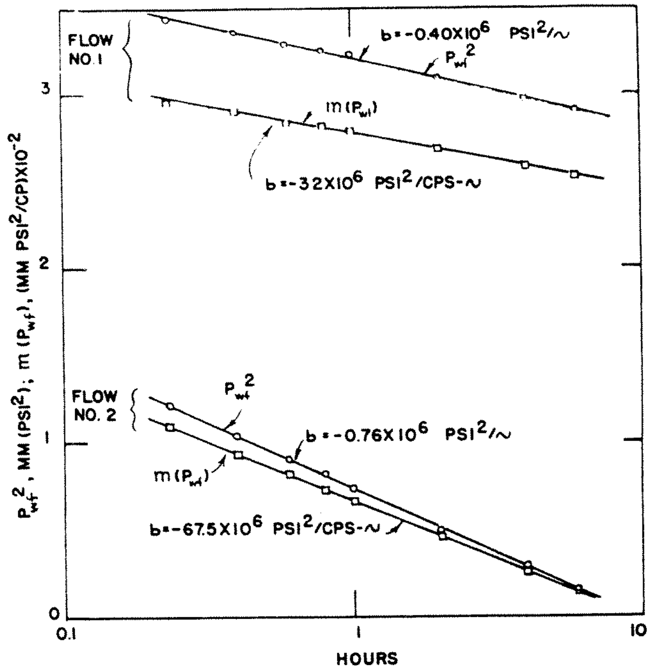

FIG. 2—$(p_{wf})^2$ AND $m(p_{wf})$ VS LOG TIME FOR SAMPLE PROBLEM.

$$s + Dq = 1.151\left[\frac{(435-279)\,10^6}{32\times10^6} - \log_{10}\left(\frac{4.84}{(0.1)(0.0176)(0.00041)(0.77)(0.5)^2}\right) + 3.23\right] = 0.657.$$

The flow capacity determined from Flow 2 is 45.8 md-ft, and the skin effect plus non-Darcy flow component is 1.36. Thus

Flow 1: $s + Dq_1 = 0.657$

Flow 2: $s + Dq_2 = 1.36$.

These two equations can be solved for $q_1 = 1{,}600$ Mscf/D, and $q_2 = 3{,}200$ Mscf/D to yield $s = -0.03$ and $D = 4.39\times10^{-4}$ (Mscf/D)$^{-1}$. Thus, the skin effect is negligible and all of the resistance near the well is caused by non-Darcy flow.

The difference in flow capacities found above for the two rates is not significant. This sample problem was taken from two computer solutions by Carter[13] because it provides a good example of an apparent skin effect caused by non-Darcy flow. The true flow capacity used by Carter in the solution was 50 md-ft. The difference between flow capacities determined above and the true value of 50 md-ft results because Carter approximated the effect of non-Darcy flow as a constant pressure-squared difference for his solutions. In real gas flow, a better approximation would be a constant difference in $m(p)$.

The next step would be to substitute the values of kh, s and D into Eq. 12 in the main text to provide a general equation for generation of stabilized deliverability curves. ★★★