

GENERALIZED PSEUDOPRESSURE WELL TREATMENT IN RESERVOIR SIMULATION

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ABSTRACT

This paper presents a generalized treatment of wells in a reservoir simulator which accounts for localized near-wellbore multiphase flow behavior using a pseudopressure approach. The method has its most important application to gas condensate wells where condensate blockage can have a significant impact on well deliverability. The formulation is general, however, in that it handles all reservoir fluid systems from single-phase gas to saturated volatile oils to undersaturated black-oil oils.

INTRODUCTION

The procedures presented are designed to compute the relation between molar or volumetric rate (q) from a well grid block and the grid-blocks's flowing bottomhole pressure (BHFP). The pseudopressure method is accurate for the treatment of varying (and sometimes highly non-linear) total gas+oil mobilities within a well-grid cell, independent of the well completion geometry.

The method presented is based on the work on gas condensate wells presented in Ref. 1 by Fevang and Whitson. The well treatment given here is generalized for any fluid type, including single-phase gas, single-phase oil, saturated oil, saturated gas condensate, and two-phase gas/oil systems.

The general idea behind the pseudopressure method is as follows. First, at a given well-grid cell at a given time step, the flowing composition produced from (or injected into) a cell is assumed known. The proposed well calculation relates rate to BHFP and average grid-cell pressure using:

1. produced or injected composition,
2. appropriate relative permeabilities
3. PVT calculations at a few pressures from the grid-cell average pressure to the limiting (minimum or maximum) flowing BHFP.

The PVT calculations constitute exactly a constant-composition expansion (CCE) of the produced or injected composition, starting at the average grid-cell pressure and extending to the limiting (minimum or maximum) BHFP. Only a few select pressures are needed in the CCE calculation. The CCE calculations may include single-phase property evaluations, a stability test at the limiting BHFP, and determination of the upper- and lower-saturation pressures.

In a black-oil model, the pseudopressure well treatment can be pre-calculated before the simulation starts, with pseudopressure tables generated as a function of rate, BHFP, grid-cell pressure, and producing GOR (R_p). A similar pre-calculated table can also be used in a compositional simulator, but an on-the-fly calculation may still be computationally efficient.

DEFINITIONS

For black-oil ("BO") and compositional equation-of-state ("EOS") formulations:

Pseudopressure Function:

$$m(p) = m_t(p) = \int_{p_{\min}}^p \lambda_t(p) dp \quad \dots\dots\dots (1)$$

Gas Rate:

$$q_g = \beta_s C [m_t(p_G) - m_t(p_{wf})] \quad \dots\dots\dots (2)$$

Oil Rate:

$$q_o = (1 - \beta_s) C [m_t(p_G) - m_t(p_{wf})] \quad \dots\dots\dots (3)$$

Steady-State Well Constant

$$C = \frac{2\pi a_1 kh}{\ln(r_o/r_w) + s} \quad \dots\dots\dots (4)$$

where a_1 = units constant, r_o = Peaceman² radius or some properly-chosen equivalent drainage radius, and skin s accounts for physical skin effects and well geometry (vertical, vertically fractured, horizontal, etc).

Surface Phase Separation:

$$\begin{aligned} \beta_s &= \text{volume fraction separator gas (BO)} = 1/(1+R_p) \quad \dots\dots\dots (5) \\ \beta_s &= \text{mole fraction separator gas (EOS)} \end{aligned}$$

Total Mobility:

$$\lambda_t = \lambda_g + \lambda_o \quad \dots\dots\dots (6)$$

Gas Mobility:

$$\lambda_g = \frac{k_{rg}}{\mu_g B_g} + \frac{k_{ro}}{\mu_o B_o} R_s \quad \text{(BO)} \quad \text{or} \quad \lambda_g = \frac{k_{rg}}{\mu_g} \frac{\rho_g}{M_g} \quad \text{(EOS)} \quad \dots\dots\dots (7)$$

Oil Mobility:

$$\lambda_o = \frac{k_{ro}}{\mu_o B_o} + \frac{k_{rg}}{\mu_g B_g} R_v \quad \text{(BO)} \quad \text{or} \quad \lambda_o = \frac{k_{ro}}{\mu_o} \frac{\rho_o}{M_o} \quad \text{(EOS)} \quad \dots\dots\dots (8)$$

General Behavior of $\lambda_t(p)$

Figs. 1 and 2 show the behavior of $\lambda_t(p)$ for different producing compositions from a dry gas to a dead oil. Also shown are the pressure-temperature phase envelopes of the producing compositions and the vertical line representing CCE calculations at reservoir temperature. Fig. 1 represents a single-phase well grid cell and Fig. 2 represents a two-phase gas/oil well grid cell. Fig. 3 illustrates the graphical integration of the pseudopressure function.

GENERAL PROCEDURE

The general procedure for computing the pseudopressure function $m(p)$ involves creating a piecewise-linear table of $m(p)$ versus pressure from the grid cell pressure p_G to the limiting (minimum or maximum) BHFP.

Once the $m(p)$ piecewise-linear table is created, any iteration on rate or BHFP during a time step is straight-forward. Without undue loss in accuracy, the produced or injected composition from the previous timestep can probably be used.

To calculate the $m(p)$ table, the produced or injected wellstream composition is used to make the CCE calculations with the following procedures.

Single-Phase Grid Cell (Production)

1. Calculate the upper saturation pressure which defines the pressure boundary p between Region 1¹ and upper Region 3.

If a saturation pressure can not be found, the mixture is assumed single-phase at all pressures from p_G to p_{\min} - e.g. a dry gas.

2. Make a stability test at p_{\min} , which by default is the minimum BHFP (or wellhead) constraint for the well.

a. If p_{\min} is unstable (two phase), a lower single-phase Region 3 does not exist.

b. If p_{\min} is stable (single phase), compute the lower dewpoint pressure which defines a lower limit of Region 1, $p > p_{\min}$. This situation (a single-phase lower Region 3) will be extremely seldom.

3. The boundaries and existence of Region 1 and Region(s) 3 are now defined by (p_{\min} , p , and p_G).

Single-phase λ_t values at any of these pressures are used in constructing the piecewise-linear $m(p)$ function in upper and lower Regions 3.

¹ As defined in Ref. 1 and shown in Fig. 4, Region 1 is the steady-state flow region near the wellbore within which gas and oil flow simultaneously and the flowing composition (i.e. producing GOR of the flowing mixture) is constant. Region 2, if it exists, lies outside Region 1 and is a region of condensate accumulation where the oil phase has no (or very small) mobility. *For the application of pseudopressure proposed in this paper, it is a fundamental assumption that Region 2 does not exist within the well grid cell but that the numerical grids surrounding the well grid cell treat Region 2 sufficiently accurate.* Region 1 is single phase flow.

4. If a two-phase Region 1 exists, at least two Region 1 "interior" λ_t values should be calculated at equidistant pressures between the upper and lower bounds of Region 1. A user-defined value (N_1) for the number of interior λ_t values can be specified, but usually two values should be adequate.

Linear extrapolation of the two right-most interior λ_t values should be used to determine λ_t at the upper bound of Region 1. Likewise, if a lower Region 3 exists then linear extrapolation of the two left-most interior λ_t values should be used to determine λ_t at the lower bound of Region 1.

Two-Phase Gas/Oil Grid Cell (Production)

No single-phase upper Region 3 exists, so $p=p_G$.

1. Make a stability test at p_{min} , which by default is the minimum BHFP (or wellhead) constraint for the well.
 - a. If p_{min} is unstable (two phase), a lower single-phase Region 3 does not exist.
 - b. If p_{min} is stable (single phase), compute the lower dewpoint pressure which defines a lower limit of Region 1, $p > p_{min}$. This situation (a single-phase lower Region 3) will be extremely seldom.
2. The boundaries and existence of Region 1 and Region 3 are now defined by (p_{min} , p , $p=p_G$).

Single-phase λ_t values at any of these pressures are used in constructing the piecewise-linear $m(p)$ function in lower Region 3 (if it exists). Additional single-phase Region 3 λ_t values can also be calculated by user specification.

3. At least two Region 1 "interior" λ_t values should be calculated at equidistant pressures between the upper and lower bounds of Region 1. A user-defined value for the number of interior λ_t values can be specified, but two values should usually be adequate.

Linear extrapolation of the two right-most interior λ_t values should be used to determine λ_t at the upper bound of Region 1 (do not use the "average" λ_t evaluated at p_G , which is already known for grid-to-grid calculations). Likewise, if a lower Region 3 exists then linear extrapolation of the two left-most interior λ_t values should be used to determine λ_t at the lower bound of Region 1.

Injection Into a Grid Cell

The procedure for calculating $m(p)$ for a grid cell with injection is exactly the same as outlined above for production from a grid cell. The only difference is that the lower pressure bound is p_G and the upper pressure bound is some higher pressure.

Two situations will cause injection into a grid cell: (1) injection from a well with specified injection composition, or (2) backflow within the wellbore from one or more "other" well-cell grids (connected through the same wellbore).

For injection by a well, the injection composition at the current time step could be used. However, for the sake of generality (and particularly the case of backflow where the "injected" composition can range from dead oil to dry gas), the flowing (produced or injected) composition at the previous time step should probably be used (as for production from the grid cell). Ideally, using an implicitly-determined injected composition would give the most accurate (and stable?) results, but computationally this may be prohibitive.

Calculating λ_t in Two-Phase Region 1

At "interior" Region 1 pressures where the produced/injected composition exists as two phases (gas and oil), λ_t is calculated according to the steady state theory of Evinger and Muskat³, as described in Ref. 1. Based on a flash calculation at an "interior" two-phase pressure, the oil relative volume $V_{ro}=V_o/(V_o+V_g)$ is calculated. Knowing V_{ro} and the phase viscosities (also from the flash calculation of produced/injected composition), k_{rg}/k_{ro} is computed by the relation,

$$k_{rg} / k_{ro} = \left(\frac{1}{V_{ro}} - 1 \right) \left(\frac{\mu_g}{\mu_o} \right) \quad (9)$$

From relative permeability tables, k_{rg} and k_{ro} values are determined at this k_{rg}/k_{ro} value. With k_{rg} and k_{ro} known at this "interior" pressure in Region 1, total mobility is calculated.

CAPILLARY NUMBER DEPENDENCE

We are currently testing procedures for including velocity and gas/oil interfacial tension (IFT) effects directly in the pseudopressure function. The capillary number N_c is used to correlate simultaneously for the dependence of relative permeabilities on velocity and IFT. A heuristic approach is proposed, based on radial or linear flow and the single-phase flow analogy. Extension to more complicated well geometries is possible, but flow velocities in non-radial geometries will generally have smaller pressure gradients near the wellbore and, consequently, less capillary-number effect than for the radial-flow approximation. The linear-flow model yields a "minimum" capillary-number effect (as will be discussed below).

To include capillary number in the pseudopressure function $m(p)$ it is first necessary to assume a model that describes the variation in relative permeabilities as a function of N_c . We are aware of only one model in the open literature⁴, while several proprietary models are currently used in non-commercial reservoir simulators. Our approach does not depend on a specific model, but we recommend using the N_c -dependence proposed in Ref. 1 (discussed below).

Next it is necessary to find an approximation that gives capillary number (i.e. velocity) as a function of pressure. Our proposal is outlined here for radial flow. For single-phase Darcy flow the velocity in a radial (cylindrical) geometry is given by

$$v(r) = \frac{qB}{2\pi hr} \quad (10)$$

with the steady-state rate equation for pressure p at an arbitrary radius r ,

$$q = \frac{2\pi kh}{\mu B \ln r / r_w} (p - p_{wf}) \quad (11)$$

$$p(r) = p_{wf} + \frac{q\mu B}{2\pi kh} \ln r / r_w \quad (12)$$

$$r(p) = r_w \exp \left[\frac{2\pi kh}{q\mu B} (p - p_w) \right] \quad (13)$$

where p_w is the wellbore pressure at radius r_w . Radius r in the velocity equation can now be expressed in terms of pressure only, yielding

$$v(p) = \frac{qB}{2\pi h r_w} \exp \left[-\frac{2\pi kh}{q\mu B} (p - p_w) \right] \quad (14)$$

Knowing velocity v as a function of pressure, capillary number is also given in terms of pressure, and the pseudopressure function can be evaluated including capillary number dependence of relative permeabilities.

This extension of Eqs. 10-14 is easily extended to linear flow. Eq. 12 can be refined using the pseudopressure itself (following single-phase flow analogy as originally proposed for dry gas by Al Hussainy et al.),

$$m(p_{(r)}) = m(p_w) + \frac{q}{2\pi kh} \ln r / r_w \quad (15)$$

Unfortunately this refinement introduces a trial-and-error solution to the pseudopressure function (which is probably overkill in the context of full-field reservoir simulation).

This approach must be verified by testing against fine-grid well models using a numerical simulator that includes capillary number dependence on relative permeabilities.²

Our original procedure for modeling the dependence of relative permeability on capillary number¹ uses a correction factor f to k_{rg} and k_{ro} at a fixed k_{rg}/k_{ro} value. The correction f is correlated with capillary number. This model is an alternative to the more traditional model (proposed by Coats) for describing a semi-continuous change from immiscible to miscible (straight-line) relative permeabilities as a function of IFT only. In the Coats model, immiscible (rock) curves are used for all IFT greater than a threshold value, and only below this threshold IFT will the rock curves be altered towards straight lines.

Based on the limited relative permeability data which are currently available, where velocity/IFT effects have been measured under controlled conditions, a simple expression has been found to fit the measurements,

² Implementation of capillary number dependence on relative permeability in a numerical simulator at the finite-difference level is highly susceptible to erroneous use. The reason is that capillary number dependence will be almost exclusively limited to the near-wellbore region (Region 1) where velocities are "high". Using coarse grids where flowing velocities are low will effectively "ignore" the velocity/IFT effect. Fine-gridding will be necessary to correctly use capillary number dependence on relative permeabilities. This problem is avoided with our formulation of the $m(p)$ function including capillary number dependence.

$$k_{rg} = f k_{rgI} + (1-f) k_{rgM} \quad (16)$$

$$f = 1 - [(\alpha N_c)^n + 1]^{-1} \quad (17)$$

where $n \approx 0.75$ seems to fit data of Hendersen et al. and our own measurements conducted at ResLab. The scaling parameter α is used to fit measured data. Immiscible (or "rock") curves are those measured in the laboratory and usually input to the simulator. Miscible (or "straight-line") relative permeabilities (for zero critical and residual saturations) are given by,

$$k_{rgM} = \frac{1 - S_{wi}}{1 + (k_{rgM} / k_{roM})^{-1}} = \frac{S_g}{1 - S_{wi}} \quad (18)$$

$$k_{roM} = \frac{k_{rgM}}{(k_{rgM} / k_{roM})} = \frac{1 - S_{wi} - S_g}{1 - S_{wi}} \quad (19)$$

CONCLUSIONS

1. A pseudopressure method is proposed for improving the estimation of near-well pressure losses in coarse-grid simulators. This gives more accurate estimation of the bottomhole flowing pressure for a well on rate constraint, and more accurate estimation of rate for a well on pressure constraint.
2. The proposed method is general for all reservoir fluid types, for black-oil and EOS compositional PVT formulations, for production and injection wells, and for any well geometry.
3. A proposal is given for including the capillary-number dependence on relative permeabilities within the pseudopressure function. Capillary number defines the combined effect of velocity and gas/oil interfacial tension (IFT) on relative permeability.
4. The proposed methods need further testing in coarse-grid reservoir simulation models, but results from work already presented in Ref. 1 show the pseudopressure method has great promise for improving well treatment.

REFERENCES

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NOMENCLATURE

B	formation volume factor
B_g	gas formation volume factor
B_o	oil formation volume factor
C	well constant
f	miscibility factor
h	producing layer thickness
k	absolute permeability used as base to define relative permeabilities
k_{rg}	gas relative permeability
k_{ro}	oil relative permeability
k_{rgI}	immiscible ("rock") gas relative permeability
k_{roI}	immiscible ("rock") oil relative permeability
k_{rgM}	miscible ("straight-line") gas relative permeability
k_{roM}	miscible ("straight-line") oil relative permeability
$m(p)$	total gas+oil pseudopressure function, $m_t(p)$
M_g	reservoir gas molecular weight
M_o	reservoir oil molecular weight
N_c	capillary number = v_x/σ
n	exponent in capillary number correlation
p	pressure
p_{wf}	bottomhole (wellbore) flowing pressure
p_b	bubblepoint pressure
p_d	dewpoint pressure
p_G	well-grid cell average pressure
q	surface volumetric rate
q_g	gas surface volumetric rate
q_o	oil surface volumetric rate
r	radius
r_o	Peaceman or equivalent well radius
r_w	wellbore radius
R_s	solution gas-oil ratio of reservoir oil
R_p	production (or injection) gas-oil ratio
R_v	solution oil-gas ratio of reservoir gas
s	skin factor
S_{wi}	irreducible water saturation
v	Darcy velocity
v_x	velocity = $v/[\phi(1-S_{wi})]$
V_{ro}	oil volume relative to total gas+oil volume
α	capillary number scaling factor
β_s	volumetric or molar separator gas fraction of wellstream
λ_g	gas mobility
λ_o	oil mobility
λ_t	total gas+oil mobility
μ	viscosity
μ_g	gas viscosity
μ_o	oil viscosity
ϕ	porosity
ρ_g	reservoir gas density
ρ_o	reservoir oil density

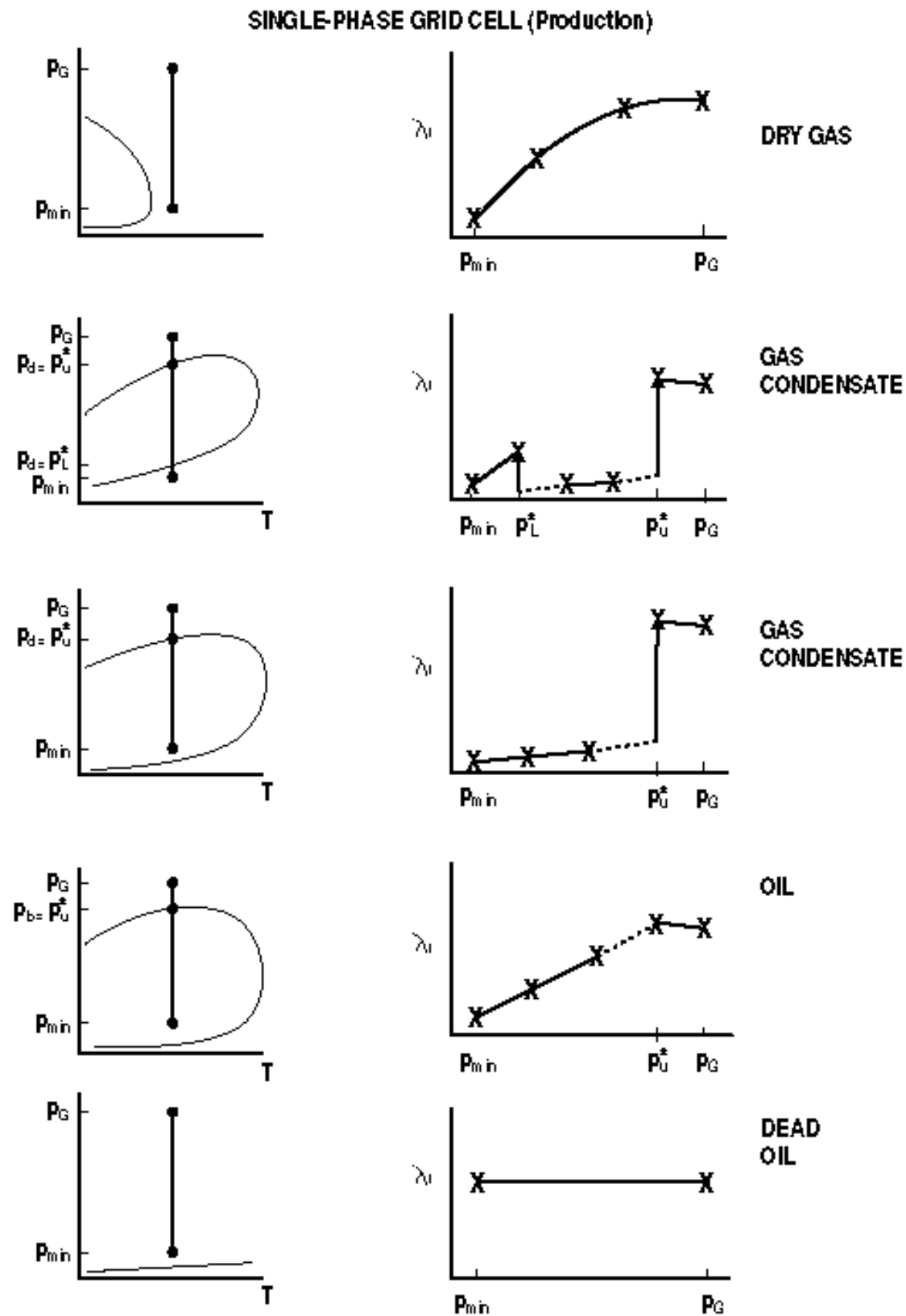


Fig. 1

**TWO-PHASE GAS/OIL GRID CELL
 (Production)**

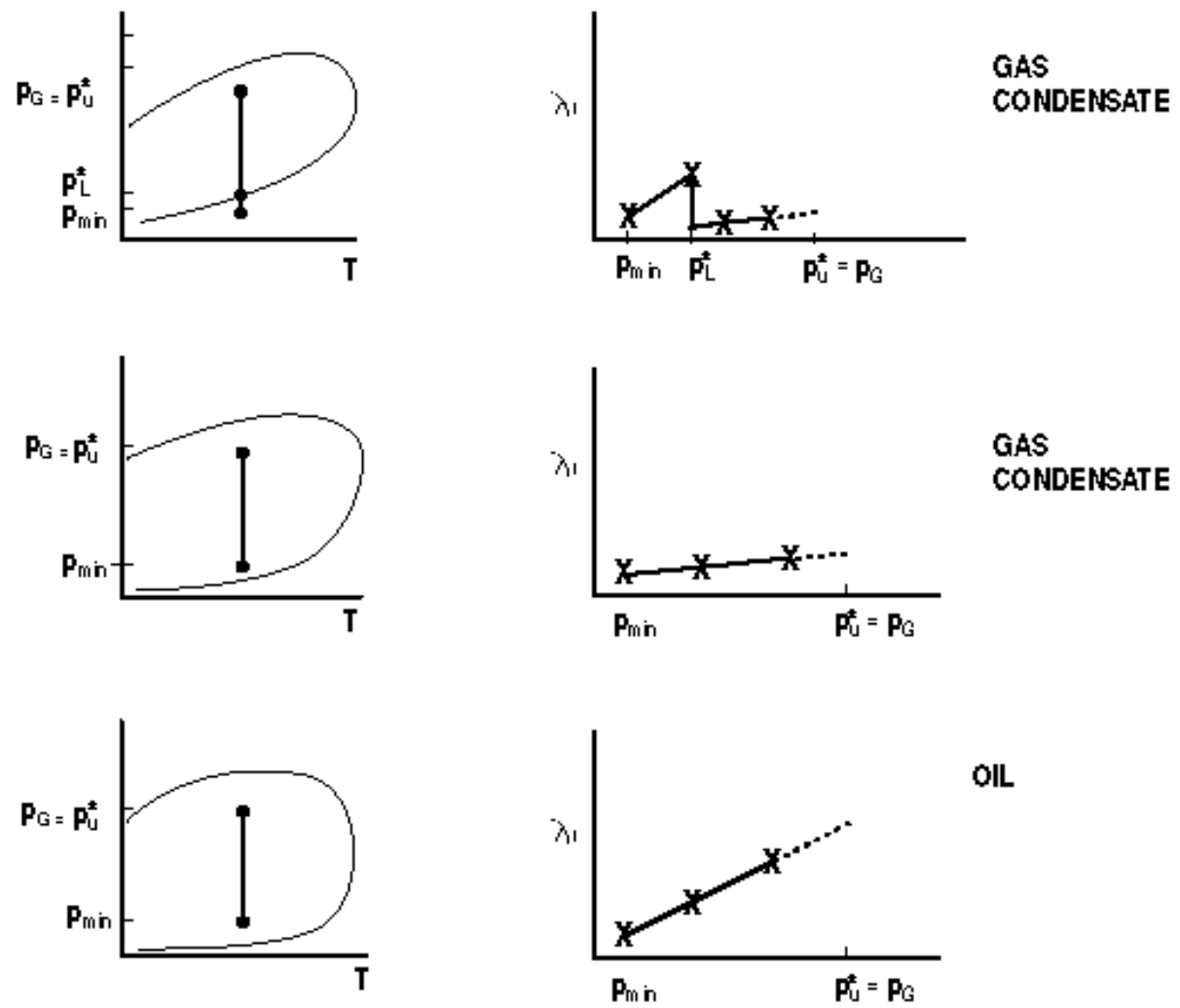


Fig. 2

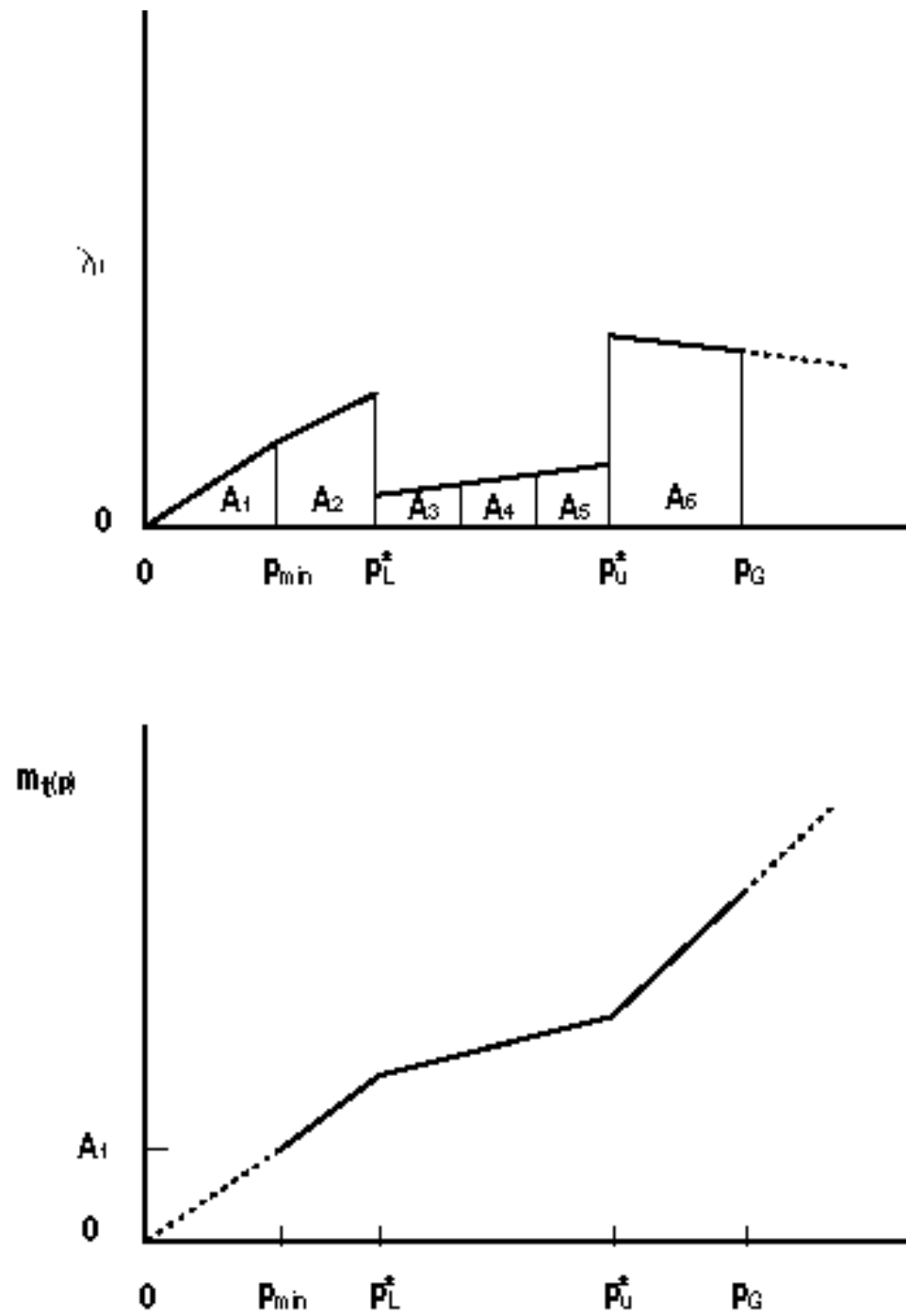
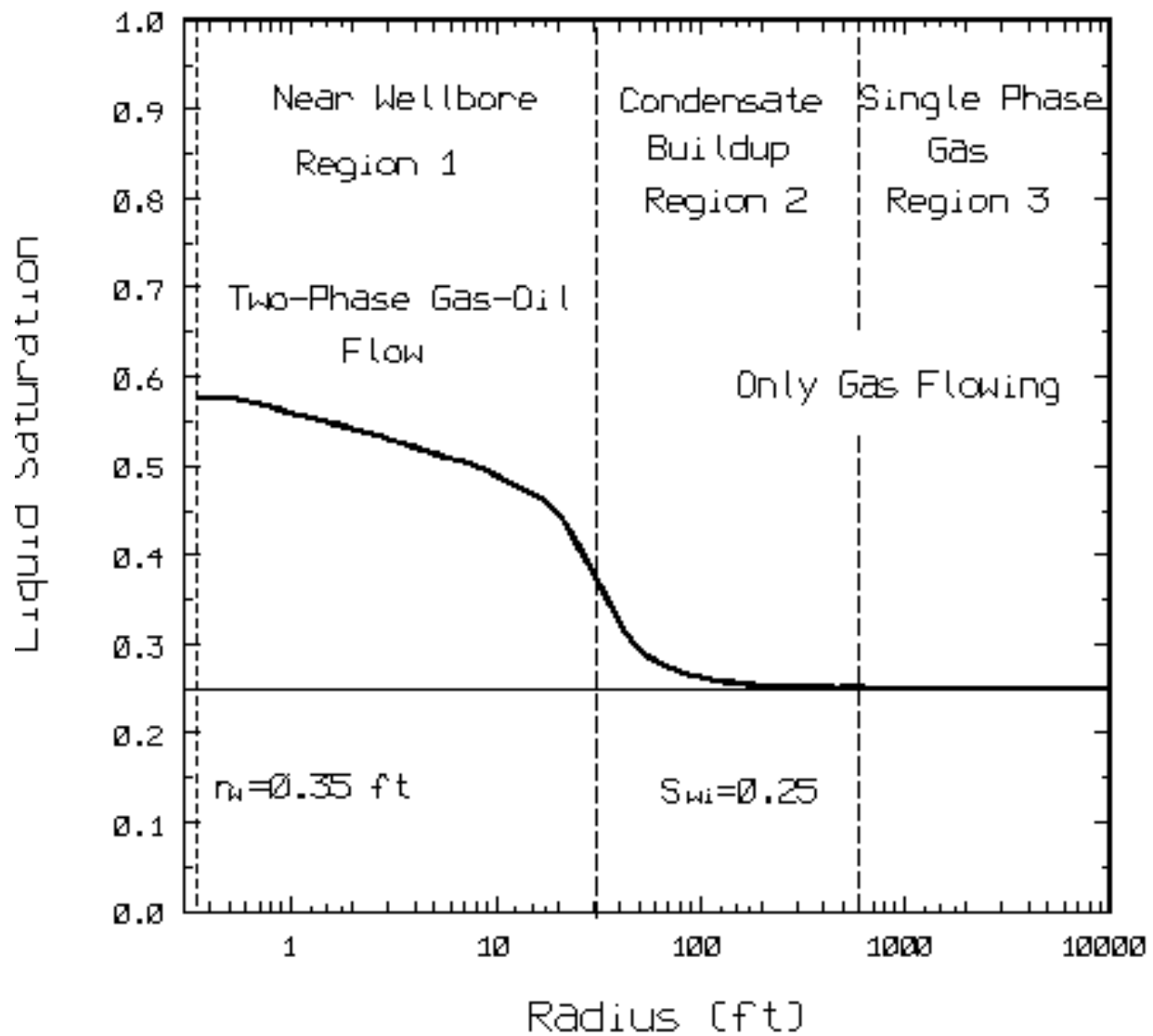


Fig. 3



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Fig. 4