Planning of Optimum Production From a Natural Gas Field

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SUMMARY

The design of an optimum development plan for a natural gas field always depends on the typical characteristics of the producing field, as well as those of the market to be served by this field. Therefore, a good knowledge of the field parameters, such as the total natural gas reserves, the well productivity, and the dependence of production rates on pipeline pressure and depletion of natural gas reserves, is required prior to designing the development scheme of the field, which in fact depends on the gas-sales contract to be concluded in order to commit the natural gas reserves to the market.

In the present paper these various technical parameters are discussed in some detail, and on this basis a theoretical/economical analysis of natural gas production is given. For this purpose a simplified economical/mathematical model for the field is proposed, from which optimum production rates at various future dates can be calculated. The results of these calculations are represented in a dimensionless diagram which may serve as an aid in designing optimum development plans for a natural gas field. The use of these graphs is illustrated in a few examples.

INTRODUCTION

The present paper gives a survey of steps to be taken in analysing a natural gas discovery, and to arrive at an optimum production and development programme for such a field.

There is a great difference between gas and oil production, not only because of the different physical characteristics of gas and oil, but also for purely economic reasons. Production from an oilfield can be according to an optimum development and depletion pattern, based on its own merits, but a gas reservoir is always directly linked to a market by means of a pipeline; therefore the physical characteristics of the reservoir cannot always determine the best depletion pattern because the market must be able to accept the gas. Thus, for a gas field a very close interrelationship exists between the production and marketing phases.

Another big difference is that an oilfield can be developed gradually. Production of oil begins at an early stage of field development and additional information about the oil reservoir is obtained during this stage. The optimum field development pattern is finally decided several years after the field begins to produce and this decision is then based on a detailed knowledge of the reservoir. In the case of a gas field, however, it is not possible to start production until the gas-sales contract has been signed and therefore the basic parameters required to determine the optimum development pattern of the field will have to be known before the field development has begun. It is, of course, not possible to obtain as detailed a knowledge of these parameters as it is in the case of oilfield development and therefore the planning of the gas field development pattern in connection with a gas-sales contract is liable to many uncertainties. In the following sections the various steps leading to the determination of optimum gas field development are necessarily over-simplified in order to avoid undue complication. The uncertainty inherent in the estimation of the various parameters required to make such a stepwise analysis must be borne in mind when judging the risks of the venture.

RESERVES

Gas in Place. Fig 1 is a simplified map of a hypothetical gas field in which the contour lines indicate places of equal depth at the top of the reservoir. The only thing definitely known about a gas field when it is first discovered is a seismic map which, as shown in Fig 1, gives the various contour lines on the top of the formation and the true depth of the formation in the first

NATURAL GAS IN PLACE

Fig 1.

discovery well. In order to build up a better picture of the structure it is necessary to drill a number of appraisal wells; the actual number needed depends on the quality of the seismic evaluation.

Next, from such a map, the amount of gas in place must be estimated and this necessitates measuring the total gross volume of rock that is gas-bearing. One has also to measure the porosity of the formation, which is that proportion of the rock containing pores filled with reservoir fluids, and the fraction of this which contains water (*i.e.* the water saturation) since the remainder will contain gas. The porosity can be measured in the formation by coring and logging in the wells which have been drilled, but as these of course represent only a very small proportion of the total rock volume, and since the data obtained from this small area are being applied to the total probable area of the field it is clear that there is considerable room for inaccuracies. The total amount of gas in place can now be calculated by multiplying the gross volume of rock by the porosity and the gas saturation (which equals one minus the water saturation). The product of these three values gives the gas volume at reservoir conditions, *i.e.* at reservoir pressure and temperature. This volume must be converted at standard conditions. This calculation is shown in the equation in Fig 1, in which the symbol "E" is the ratio between the gas volume under standard atmospheric conditions and the volume occupied by the same amount of gas in the reservoir where the pressure is much higher. This so-called "gas expansion factor" could be calculated using Boyle's law if the gas were an ideal gas, *i.e.* "E" would equal the reservoir pressure divided by the standard atmospheric pressure, multiplied by the atmospheric temperature divided by the reservoir temperature. In fact, since the gas is not an "ideal" gas, the result has to be qualified by a gas compressibility factor "Z", which is normally smaller than 1, and which is a function of the pressure which is not the same throughout the life of the reservoir. Once the gas composition is known, the dependence of "Z" on pressure and temperature can be determined or, even better, accurately measured on a sample of the gas, and hence all the basic information for determining the gas in place will then be available. This, however, does not represent the amount of gas that can be economically recovered. For this purpose the recovery factor must be known, *i.e.* that fraction of the gas in place which is recoverable under normal economic operating conditions. To assess the value of the recovery factor it is necessary to have an understanding of the producing performance of the reservoir.

Reservoir Performance. Fig 2 illustrates the factors affecting gas recovery. If the reservoir is a closed unit, *i.e.* not underlain by water, it is known as a depletion reservoir, and as gas is produced the pressure will drop, as indicated on the line marked "depletion". (If the gas were an "ideal" gas this line would be straight).

Recovery of gas is possible from such a field up to a certain abandonment pressure. This is the lowest pressure at which gas can still be produced from the wells at a rate sufficiently high to cover the operating costs. The point at which the line representing the abandonment pressure crosses the depletion line indicates the ultimate economic gas recovery, which in this case is between 80 per cent and 90 per cent of the gas in place. This is an average figure for a depletion-type reservoir.

In actual fact, the majority of gas fields are underlain by water and as the gas pressure in the field begins to drop, water will start to flow and enter the gas reservoir, and this so-called water encroachment will then maintain the reservoir pressure to a greater or lesser extent. This water encroachment mechanism is called "production by water drive". Fig 2 indicates three types of water drive: weak, moderate, and strong.

NATURAL GAS RECOVERY

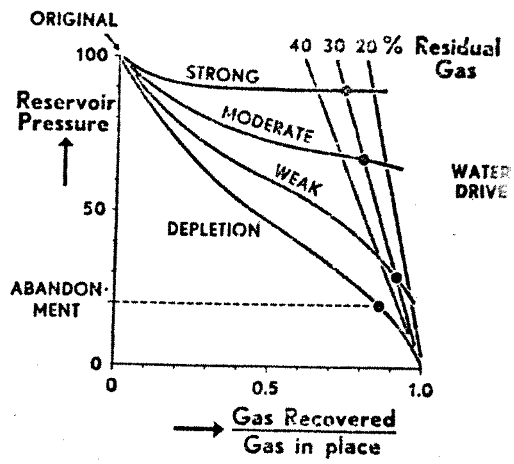

FIG 2.

It is here that an important difference between gas and oil reservoirs arises. In the case of oil reservoirs the water drive fields usually have much higher recovery factors than depletion type fields; as far as gas reservoirs are concerned, frequently the reverse is the case. The reason for this lies in the fact that the water encroaching into a gas field does not displace all the gas. An appreciable amount of the gas is trapped by capillary forces in the pores of the rock and is by-passed and left behind. This gas is known as residual gas and, expressed as a percentage of the original pore volume filled with gas, is in practice anything from 40 per cent to 20 per cent. Fig 2 indicates that for a strong water drive, where pressure is maintained at its initial value, it is impossible to recover more than 60 per cent of the gas in place if residual gas saturation is as high as 40 per cent. This compares with 80 per cent to 90 per cent ultimate recovery in the case of a depletion field. If the water drive decreases in strength, the ultimate recovery will be higher and it is in fact even possible that with a very weak water drive ultimate recovery may be slightly higher than in the case of depletion.

The strength of the water drive depends principally on three factors, First, on the permeability of the formation: this is the ability of the formation to allow flow of fluids through it. Through formations with high permeability, the flow of gas is relatively easy and occurs at low pressure drops, whilst through low permeability formations even high pressure drops will result in low flow rates of gas. The same applies for water: the lower the permeability, the less chance there is of a strong water drive occurring. Secondly, the strength of the water drive depends on the size of the reservoir. The larger the reservoir the weaker the water drive is likely to be. This is because, quite simply, the volume of water needed to maintain pressure is dependent on the area of the field, which, likened to a circle, is, in simplified terms, proportional to the radius of the field squared. The circumference of such a field through which all water fluid must pass is, however, directly proportional to the radius. Consequently, the amount of water influx in a given period of time and for a given pressure drop is roughly proportional to the radius, but the amount of water required to maintain the reservoir pressure at the given level during this period of time, expressed as a fraction of the volume of the reservoir, will be proportional to the inverse of the radius squared. Combining the two effects, one might say that the relative strength of the water drive is roughly proportional to the inverse of the radius, and consequently, for comparable conditions, is relatively weaker for larger-sized fields. Thirdly, there is the time factor. It takes time for the water to flow into the reservoir. If a high rate of production is being maintained from the field, a high amount of influx is required during a short time period, and consequently the water drive may be weak, whereas the same field with a low rate of production may have a strong water drive.

Summarizing, the reservoir engineer has to estimate the likely depletion pattern of the field; he has to know how permeable the rock is; he has to assess which other factors will be of importance, what the expected reservoir drive will be, and also the amount of gas which will be left behind if there is water encroachment. He is then in a position to make an estimate of the recovery percentage; this, multiplied by the gas in place, will give him an estimate of the gas reserves.

GAS WELL PERFORMANCE

The Problem. We now come to the problem of actually producing the gas in the formation. How many wells are needed; when should they be drilled; how much should be produced from any one well? To assess this, a number of tests have to be carried out, more specifically production tests in the first discovery well, the results of which will be supplemented by further tests in the appraisal wells at a later date. Fig 3 is a simple diagrammatic representation of actual gas flow from the reservoir. The gas stored in the reservoir must of course flow through the formation to the well bore, this process being called the inflow performance of the gas well. It then has to flow upwards through the well tubing to the surface. During this phase of the production two factors are important, first, the friction loss experienced in the well tubing and the resultant pressure drop, and secondly, the amount of suspended water present. Even in the case of a well producing hardly any water at all, an accumulation of water in the well tubing will build up in time, depending on the production rate. This will lead to an increased overall density of the flowing gas with a consequent higher hydrostatic pressure drop. This phenomenon is known as "liquid hold-up" and is particularly important at low flow rates. Finally, after leaving the well-head, the gas will have to be dehydrated and treated to pipeline quality before delivery. Under special circumstances, when reservoir pressures will have dropped to low values, compression of the gas may also be required before delivery into the pipeline.

GASFIELD - SCHEMATIC

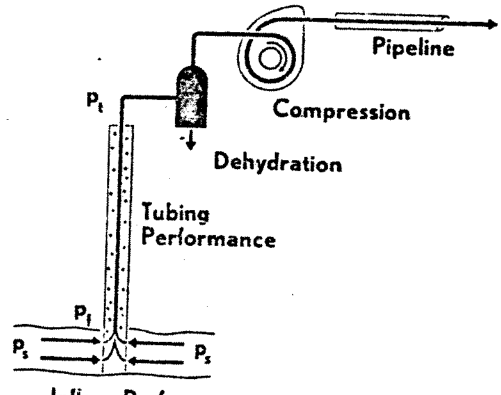

FIG 3.

Production Test. Fig 4 indicates in principle the various phases of a production test. First, the static pressure p_s of the discovery well is measured; then gas is produced from the well at a number of different rates—three are shown in this figure, and the pressure drawdown, *i.e.* the flowing pressure p_f, is measured during these tests. Finally, the well is shut in to determine whether the pressure builds up again to the original static reservoir pressure p_s. If the well bore pressure does not build up to its original value, it is clear that the reservoir is very small indeed. The shape of the curves in Fig 4 and their end points are very important for future estimations since they permit calculation of the permeability of the formation. The level of the end points indicates whether the gas can flow unhindered into the well bore or whether there is completion damage, and whether the effect of non-Darcy flow (explained later) is important or not.

PRODUCTION TEST

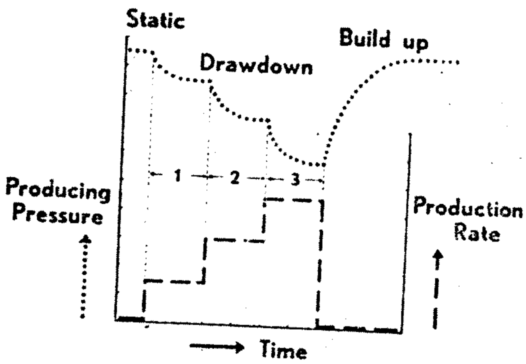

FIG 4.

Inflow Performance. The results of an analysis of such a production test are shown in Fig 5, which is an inflow performance graph. This figure indicates the "drawdown squared", which is the difference between the static reservoir pressure p_s squared and the flowing pressure p_f squared at the bottom of the well. Normally a straight line is drawn through the points measured and this is then called the inflow performance relationship. Any point on this line relates the production rate to the "drawdown squared", the maximum value of which is equal to the static reservoir pressure squared. The production rate obtained at this maximum value is called the "open flow potential". This value is frequently used in the U.S.A. in connection with gas regulation, although the open flow potential does not correspond to the actual production rate that can be achieved. In practice the maximum production rate is likely to be less than the open flow potential, as indicated by the unbroken curve in Fig 5. How is this difference explained?

INFLOW PERFORMANCE

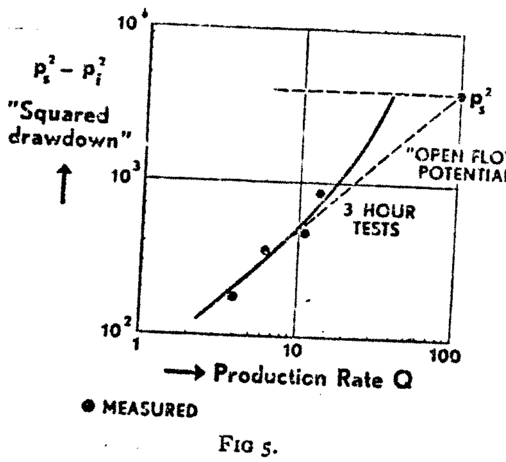

FIG 5.

In theory the flow of oil or gas in a formation obeys Darcy's law, which states that the pressure drop over a given distance is proportional to the flow rate of the fluid considered. This is true for oil but not always for gas, because Darcy's law ignores the forces of inertia. This law (strictly) is valid only in cases where the flowing liquid or gas has no mass at all and in actual practice only for low flow rates. In the case of high flow rates, however, inertia has an appreciable effect. These conditions are frequently met under normal conditions of gas production and are not often encountered under oil production conditions. This may be explained by the following considerations. According to Darcy's law, the rate of flow through a porous reservoir is directly proportional to the pressure drop over a certain distance, and inversely proportional to the viscosity of the flowing liquid or gas. The proportionality constant in this equation is the formation permeability referred to above. The viscosity of gas at reservoir conditions is about 1/50 that of light oil. Consequently, all other conditions being the same, gas flow rates may be about 50 times higher than oil flow rates. The effect of inertia is, however, not only proportional to the flow rate but also to the density of the flowing fluid, and as oil densities under reservoir conditions may be about four times higher than the gas density, the resulting mass flow rate of gas is about ten times larger in magnitude than the mass flow rate of oil. This explains why, in gas production practice, deviations from Darcy's law due to the inertia effect are more frequently encountered than in oil production practice. From this it will be apparent that the mass flow effect in gas fields is generally much more important than in oilfields, and it is in fact this mass flow effect which causes the flow performance line indicated in Fig 5 to deviate from a straight line drawn through the observed points. Analysis of the well production test will indicate the importance of this effect.

Well Spacing. Fig 5 indicates the results of a 3-hr flow test. This is the well inflow performance if the length of the production period is 3 hr, but in 3 hr the pressure drop will not have been propagated throughout the reservoir. The gas produced during the 3-hr test originates from that portion of the reservoir which is close to the well bore, whilst after producing for a longer period gas will have to be drawn from an area of the reservoir further away. Consequently, larger pressure drops will then be required in order to obtain the same rate of flow. If the well were left on production for a longer period, its performance would be less favourable. It is necessary therefore to correct the "3-hr inflow performance" to a so-called "stabilized inflow performance", which is itself also dependent on the number of wells which will have to be drilled in the reservoir and also on well spacing.

Fig 6 indicates how a gas field might be developed, in particular an offshore gas field. In the large Slochteren

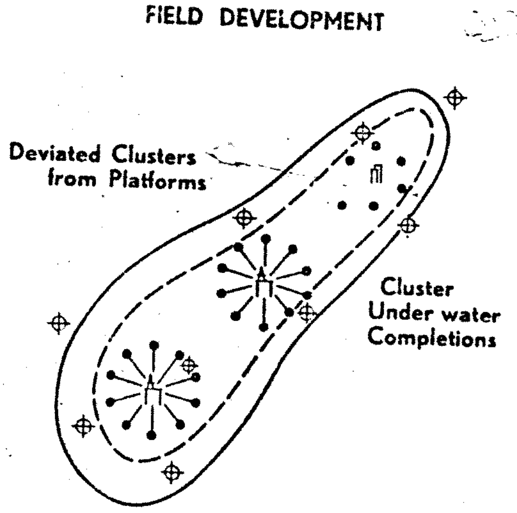

FIG 6.

gas field in the Netherlands, the wells are not evenly spaced over the reservoir but are bunched together, only some 80 yards apart, in so-called "clusters". This cluster idea can be developed further for offshore drilling by drilling a number of deviated holes from a fixed platform. Another way of developing such a gas field would be by drilling wells vertically, more or less situated on a circle, completing them on the sea-bed, and laying flow lines from the subsea well heads to a pipeline platform where the gas produced can be treated and then sent ashore; this is what is known as "underwater completion". Both these methods are forms of cluster drilling.

In modern gas field development it is increasingly becoming the practice, especially when drilling in deep water, to abandon appraisal wells because the cost of completing them and providing them with supporting structures to prevent the well head from being bent off by the action of the sea may be more than the cost of drilling new wells. Apart from this, they may not be in very favourable positions in the reservoir for future production.

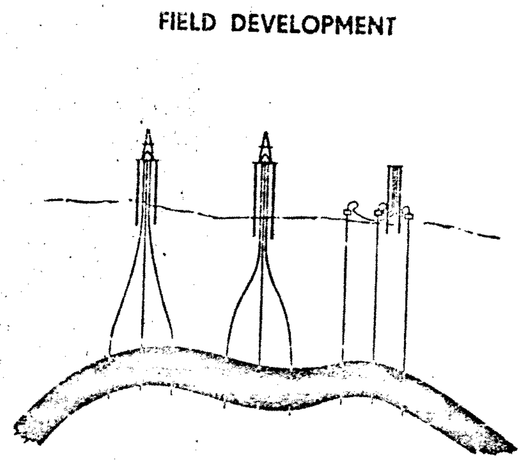

FIG 7.

Fig 7 is a diagrammatic representation of the two forms of cluster development. It is technically simple and convenient to drill wells closely together from fixed platforms, but it does have one disadvantage. The distance between the wells down-hole is much closer than if the wells had been evenly spaced over the reservoir and this can lead to mutual well interference.

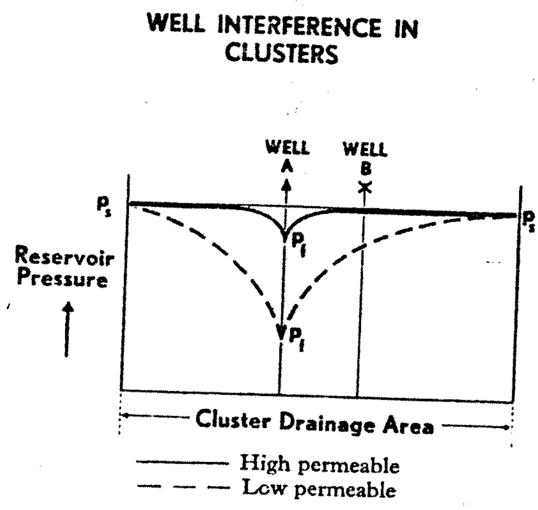

FIG 8.

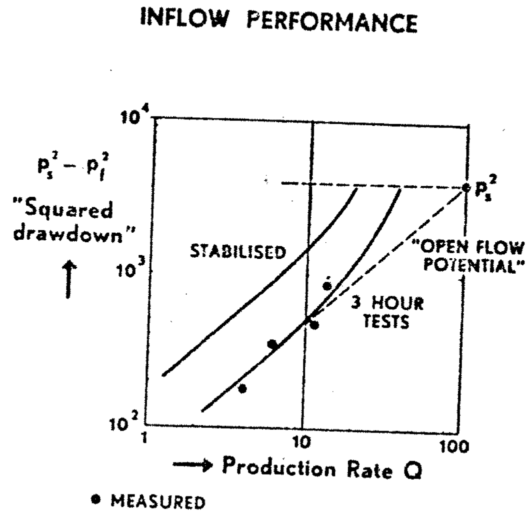

FIG 9.

Fig 8 illustrates this effect. In the case of the high permeability formations the wells can be closer together than in the case of low permeability formations. In the high permeability case the pressure drop is mainly around the well bore, and may be mainly due to the well completion effect (so-called skin effect) and the influence of inertia effects as discussed above (in gas production practice also frequently referred to as "turbulent flow effect") which are most pronounced in the vicinity of the well bore. The latter two effects can be very important in deciding how close together the wells can be. Formulae have been designed to estimate the interference between wells in a cluster from measured values of permeability, turbulence, and skin effect. From this it is possible to assess the well inflow performance for stabilized conditions as shown in Fig 9. This is basically Fig 8 with one line added to show the stabilized performance, which is the performance of the individual wells in the clusters after the development of the field is completed and the field has reached its designed maximum production rate. From the two curves drawn in Fig 9 it may be observed that the difference in squared drawdown between the two curves is relatively smaller at the high production end than the low, and this is due to the turbulence effect, which is most pronounced at high flow rates. The more there is this deviation from Darcy's law, the less serious is the effect of the mutual interference between the wells. From the example given in Fig 9 it may now be observed that the open flow potential of the well does not correspond to the maximum production that could be achieved from a gas well in the field after development has been completed. First, the influence of the turbulent flow effect will in this particular case reduce the potential to some 40 per cent of the open flow potential, and a further 20 per cent reduction is caused by well interference in the clusters at the time the field development is completed. Moreover, the flow rate obtained at well flowing pressures equal to atmospheric pressure cannot be obtained in practice.

A further reduction of well flow rate is caused by the so-called vertical flow performance or tubing performance.

Tubing Performance. The effect of vertical flow performance is demonstrated in Fig 10. There is a difference between the well flowing pressure p_f at the bottom of the well and the tubing head pressure p_t at the well head that is a function of the production rate. Normally, the value of the tubing head pressure p_t is dictated by the required inlet pressure of the pipeline. At various production rates Q values for the flowing well bore pressure p_f can be calculated from this requirement, and if the static reservoir pressure p_s is known, the difference between the squares of this static reservoir pressure and the flowing well bore pressure may be calculated. The result of these calculations is that for a given tubing head pressure p_t and a given static reservoir pressure p_s, a relationship exists between the production rate Q and the "squared drawdown", which is shown in Fig 10.

Fig 10 gives two curves, one for a 3-inch tubing and the other for a 5-inch tubing. Two points should be noted here. For low production rates, as shown on the left-hand side of the graph, there is hardly any difference between the two performance curves, because there is virtually no friction, and in fact the tubing sizes are probably too large for such low rates of flow. For high production rates at the other end of the scale it is, however, clear that beyond a certain production rate,

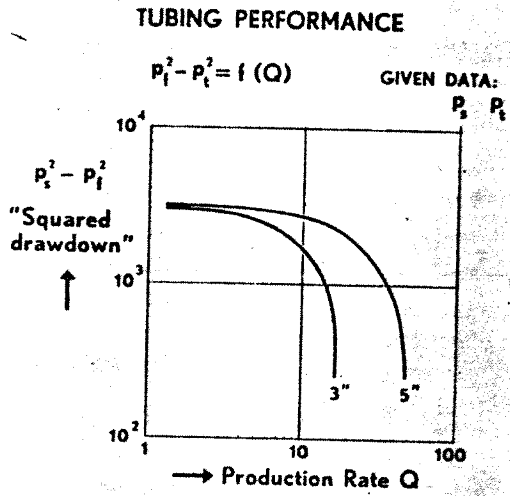

FIG 10.

dependent on the tubing size, an increase in "squared drawdown" does not improve the production rate. The tubing acts virtually as a choke nipple at these rates and has reached maximum flow conditions. The maximum possible production rate for 5-inch tubing is of course larger than for 3-inch tubing.

In order to determine the effect of the tubing performance on the performance of the well, the lines shown in Fig 10 should be superimposed on the inflow performance graphs in Fig 9.

Well Performance. The well performance obtained by superimposition of inflow performance and tubing performance as shown in Figs 9 and 10 respectively is given in Fig 11. The intersection of the inflow performance curve and the tubing performance curve determines the well production rate at the given conditions. These conditions are static reservoir pressure, type of well inflow performance, tubing size, and required tubing head pressure. It may be observed that, for the example given, the maximum production rate

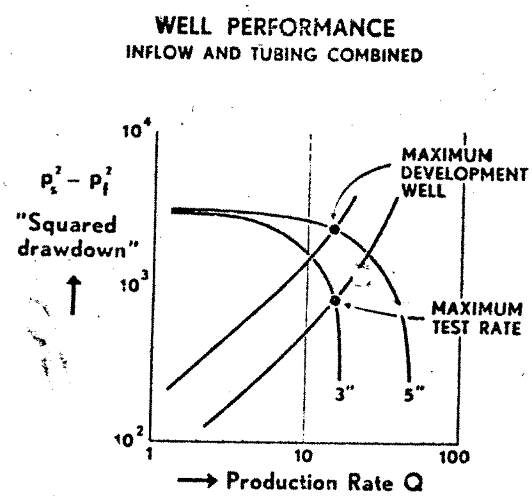

FIG 11.

obtained from the well after a 3-hr production test is practically the same as that predicted for stabilized conditions. This is of course a coincidence which is caused by the fact that the well might have been tested using a 3-inch tubing and may eventually be completed with a larger 5-inch tubing. The loss incurred by the stabilized inflow performance with respect to the 3-hr test inflow performance will then be compensated by the increase due to the larger tubing size. The flow rates indicated in Fig 11 are valid for one value of the static reservoir pressure only, and for a given tubing size and a fixed tubing head pressure p_t. It is possible to calculate a number of tubing performance curves as shown in Fig 10, each valid for a distinct value of the static reservoir pressure p_s, and to superimpose the set of curves thus obtained on the stabilized inflow performance curve shown in Fig 9. The result of this is shown in Fig 12, which indicates the reservoir deliverability throughout the future reservoir life. The representation is in fact an over-simplification because other factors, not discussed here, will also play a role. When reservoir pressure drops, gas viscosity will increase and gas condensate may collect in the formation around the well bore. The combination of these effects may cause a deterioration of the inflow performance which is not shown in Fig 12. The intersections of the well inflow performance line in Fig 12 with the various tubing

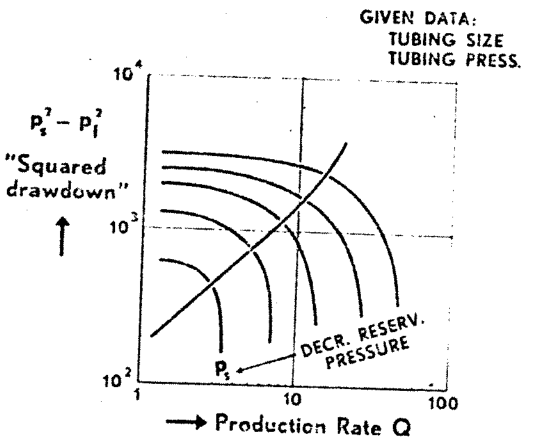

Fig 12.

performance lines yield a relationship between well production rate Q and static reservoir pressure p_s. With the help of the appropriate reservoir performance curve given in Fig 2, the latter may be related to the amount of gas produced from the reservoir. It is thus possible to eliminate the static reservoir pressure p_s from those two relationships and directly plot the well production rate Q as a function of the amount of gas produced from the reservoir (expressed as a fraction of the gas in place). This is shown in Fig 13. In this figure three well performance lines are indicated: for a high, a medium, and a low tubing head pressure respectively. Each of these lines may be obtained from plots as shown in Fig 12 but of course drawn for different values of the tubing head pressure p_t. The resulting performance relationship as shown in Fig 13 is used to select future

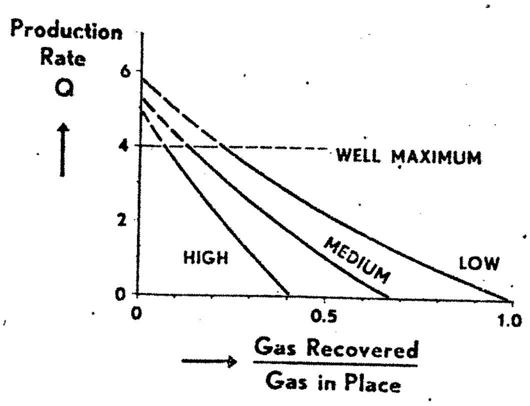

Fig 13.

field operating conditions. A high or a low tubing head pressure can be chosen. This choice is significant because it may be necessary to deliver gas a long distance through the pipeline to a buyer who requires the gas at a specific pressure. The level of the tubing head pressure with respect to the pipeline pressure moreover has an important influence on the selection of the most economical dehydration process. However, it must be appreciated that whatever the level of tubing head pressure chosen, a well can only be maintained for a limited period at its maximum rate Q_m, after which it will be necessary either to reduce the tubing head pressure and install compressors, or drill more wells to maintain a constant field deliverability. Nevertheless, in order to achieve a high gas recovery factor, a low tubing head pressure will eventually be needed and compressors will have to be installed. It is possible, of course, to postpone the time when compression is needed by accepting a lower maximum well production rate and drilling more wells. As may be observed from Fig 13, a lower value for the Q_m will allow a larger fraction of the gas in place to be produced at high tubing head pressures and permit postponement of compressor installation. This choice will also be influenced by future expectations of well inflow performance. The well tested is not necessarily situated in the best or worst part of the reservoir and it may be proved during subsequent tests in appraisal wells that the inflow performance of the discovery well was not representative for the reservoir. This is demonstrated in Fig 14, where, at a given tubing head pressure, well performances are shown for low, medium, and high productivity wells. Of course, lowering the design production rate Q_m for a high productivity well will hardly result in any signi-

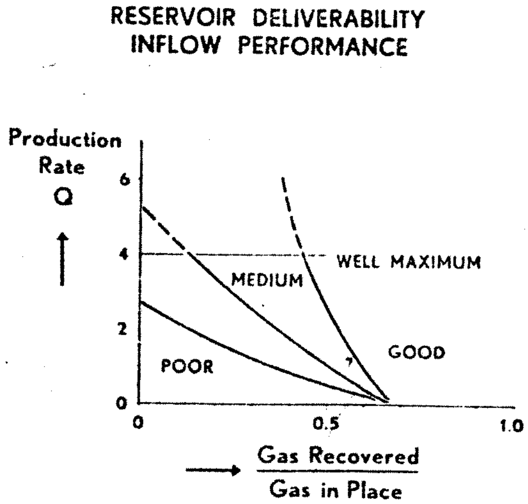

FIG 14.

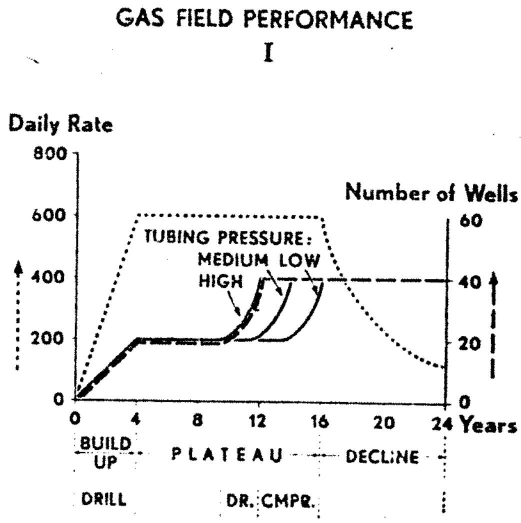

FIG 15.

ficant postponement of compressor installation, whilst for the low productivity well, lower well potentials may have a significant effect. It is the task of the reservoir engineer, who will formulate plans for the future development of the field, to choose between the various alternatives indicated, in order to arrive at an acceptable gas field performance pattern. Apart from the technical aspects, this is also an economic problem, which will be dealt with in the following sections.

GAS FIELD PERFORMANCE

Offtake Pattern. The offtake pattern of a gas field is limited by two constraints. In the first place, the offtake should be such that the market can absorb the gas produced, and this will normally lead to a restriction of the rate at which production can be built up. The maximum rate of offtake may be restricted by the length of time during which a certain field is required to serve a given market. On the other hand, the rate of build-up may be restricted by the physical capabilities of drilling wells and constructing facilities to transport the gas from well-head to pipeline. The maximum offtake rate of the field may also be limited by gas field economic considerations.

In the following section this latter economic optimum, maximum production rate will be determined regardless of market conditions. In actual practice the maximum offtake rate for the market should be less but never more than this field optimum offtake rate. In order to simplify the economic analysis, first an arbitrarily chosen offtake pattern for the field will be studied. This is shown in Fig 15, where the production pattern consists of three parts: a period of production build-up; a period of constant production; and a period of production decline. The field development associated with such a production pattern can be determined by using reservoir deliverability graphs as shown in Fig 13. At each point in time the total amount of gas produced since the beginning of production can be determined from the chosen production pattern, and at a given tubing head pressure the corresponding well production rate obtained from Fig 13. By dividing the total field production rate by the corresponding well production rate, the number of wells required at any selected point of time will be obtained. In doing this, account should be taken of the peak production rate that may be required at any time and which is normally higher than the annual average daily production rate indicated on the graph. The resulting number of wells required is indicated in Fig 15 for three values of the tubing head pressure. To obtain the selected field production pattern, the example shown in Fig 15 indicates a period of drilling during the build-up of the field, followed by a period of production at constant rate without further drilling, after which additional wells are drilled to maintain total field output at the same level whilst still producing the wells at high tubing head pressures. The drilling of additional wells cannot continue indefinitely, as the number of wells required to maintain field potential under these conditions becomes prohibitive at a given time. Thereafter, field potential can only be maintained by lowering the tubing head pressures and subsequently installing gas compressors. This so-called "compressor phase" will then continue until well-head pressures fall below that which is an efficient and economic compressor intake pressure, and this marks the beginning of the decline period of the field.

In some instances a different procedure may be followed and this is shown in Fig 16. Here wells are drilled during the build-up period, and this is followed by a period of constant rate without drilling additional wells as before. Instead of maintaining the tubing head pressure at a high value and drilling additional wells to maintain field potential, it is now preferred to first lower tubing head pressures and install gas compressors and then drill additional wells to maintain field potential till the start of the production decline period. An

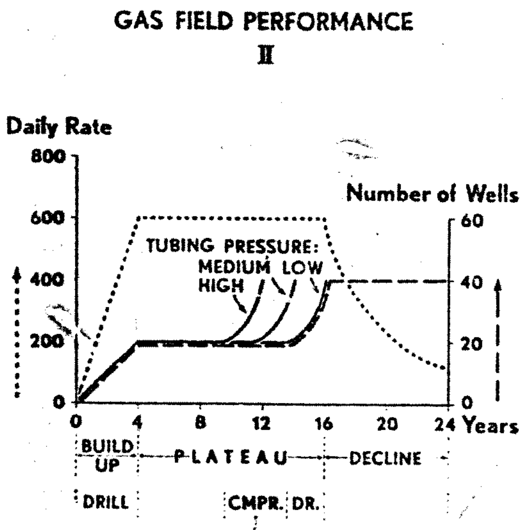

Fig 16.

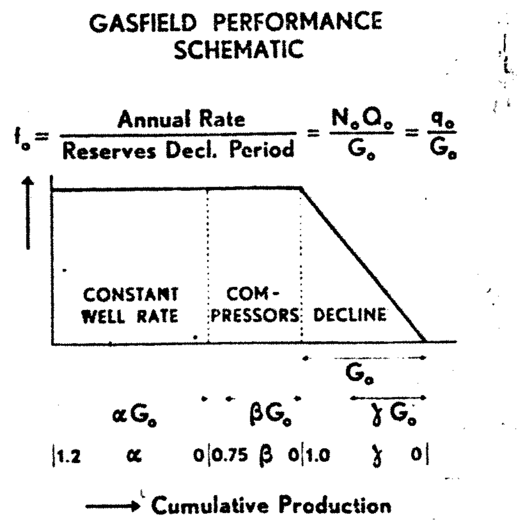

Fig 17.

important feature that may be observed from both Figs 15 and 16 is that at the time production decline starts, the cumulative amount of gas produced up to that time is the same and consequently, as may be observed from Fig 13, at the given minimum tubing head pressure individual well production rates will be the same. This leads to the same number of wells being required in both cases and consequently at the end of the constant rate period both fields will be similar. The difference between the gas field performance shown in Fig 15 and in Fig 16 is only the way in which one arrives at this end point and consequently the phasing of the various expenditure items, but not in the total amount of facilities built and wells drilled. The total amount of capital expenditure required in the given examples depends only on the maximum level of field production and not on the particular development pattern selected to maintain this level. As the phasing of capital expenditure is different in both examples given, this means that the economic results will not be the same if the interest on capital is taken into account. In the following sections it will be studied whether there is an optimum manner to arrive at this end point, and at what maximum production rate such an optimum is reached.

Basic Production Pattern. In this section, the economic result of future gas production is calculated on the basis of a production pattern for a gas field in which no further drilling will take place; in other words, at a point of time at the end of the build-up period shown in Fig 16, and excluding the drilling period which follows the compressor period in this case. From the previous sections it may be understood that in a gas field where a number of wells (N_o) have been drilled, the future production performance will consist of a period of constant rate, during which well potentials exceed the maximum permissible rate, followed by a period during which compressors are installed to maintain offtake rates at a constant level, compensating for the effect of decreasing well-head pressures, and finally by a period of declining production. A diagrammatic representation of such future performance is given in Fig 17, where a particular system of units has been used to represent the values of production rate on the vertical axis and cumulative gas production on the horizontal axis. On the horizontal scale of this figure, cumulative gas recoveries have been plotted, but they are expressed in a unit of gas production G_o which represents the total amount of gas that will be recovered from the start of the decline period until the economic limit of the field is reached, *i.e.* the total amount of gas that will be produced during the decline period when the wells are produced at minimum tubing head pressure. As a consequence it will be seen that the total amount of gas produced during the decline period expressed in this unit equals one. On the vertical axis the total field production rate is represented, and this is again expressed as a fraction, f_o, obtained by dividing the total annual rate of production q_o by the amount of gas recovered during the decline period G_o. The total annual rate of production q_o is of course obtained by multiplying the number of wells N_o and the maximum production rate per well Q_o*. Studying the graph one observes that from the present onwards, future production is characterized by a maximum production rate q_o, to be maintained until a total amount of gas equal to αG_o (in the particular example $1{\cdot}2 \times G_o$) has been produced, whereafter a gradual installation of compressors will take place until a further amount of gas equal to βG_o (in the particular example the $0{\cdot}75 \times G_o$) will have been produced, following a total amount of gas

* The maximum annual production rate Q_o of a well is here expressed as the maximum that can be produced during a year under existing load conditions and meeting the peak rates whenever required. It is therefore equivalent to the maximum rate of the well Q_m, times the load factor expressed in number of days maximum production per annum.

equal to G_o is produced during the decline period. Of course, any future points in time may be characterized in mathematical language by choosing either a value of the parameter α between zero and 1·2, a value of the parameter β between zero and 0·75, or a value of the parameter γ between zero and 1. The parameter γ is a measure of the total amount of gas that still has to be produced until the end of the field's economic life, at any point in time during the decline period.

With the help of this particular system of units, which are in fact dimensionless, the future life of any particular gas field may be represented mathematically. In order to express the future economic value of gas production, this system of units may be used as explained in the next section.

Present Value Calculation. The economic value of future gas production can be expressed in terms of money by calculating its so-called "present value". The present value of an amount of money to be received at some future date is defined by the amount of money placed at compound interest today, which will increase to that same amount of money at that future date. From this definition it is clear that the present value of an amount of money to be received in the future is always less (if the interest rate is larger than zero, of course) than the future amount to be received. One can calculate the present value of the future amount to be received by multiplying it by a so-called "deferment factor". Such deferment factors can be obtained from tabulations at various interest rates. In Fig 18, first line,

OPTIMUM RATE

PV PRODUCTION

$$Q^* = N_o Q_o \dot{u} (D_1 + D_2 + D_3)$$

PV PROFIT

$$P^* = Q^* - N_o V_w - N_o V_c D_2'$$

UNIT CASH GENERATION

$$u = S_p - C_o - R - t_x (S_p - C_o - C_d - R)$$

MAXIMUM PV PROFIT

$$\frac{dP^*}{dN_o} = 0$$

PARAMETER

$$s_o = \frac{uQ_o}{V_w}$$

FIG 18.

the "present value production" has been calculated for the basic production pattern shown in Fig 17. The total annual production rate $N_o Q_o$ has been multiplied by some monetary value u, the significance of which will be explained later, and the sum of three deferment factors D_1, D_2, and D_3, valid for the total production obtained during the constant rate period, the compressor period, and the decline period respectively. The value of u is called the cash generation per unit of gas sold.

This unit cash generation is obtained by subtracting from the unit sales price of the gas S_p, the unit operating costs C_o, the unit royalty R, and the corporate taxes. The latter are obtained, for example, by applying the tax fraction t_x to the result obtained by subtracting from the unit sales price S_p, the unit operating costs C_o, the unit capital allowance C_d, and the unit royalty R. This is demonstrated on the third line of Fig 18. The present value of future profits obtained from the gas production can now be calculated by subtracting from present value future gas production Q^*, *inter alia* the total amount of capital investment required to drill N_o wells. The capital investment per well, V_w, is defined by the costs required to drill a well and to construct the production facilities required to transport the gas produced from that individual well to the pipeline. One must also subtract from Q^* the present value of future compressor investments necessary to compress the total amount of gas produced from the field from minimum tubing-head pressure to pipeline pressure. This is obtained by multiplying the number of wells N_o by the compressor investment V_c required to compress the amount of gas produced by a single well and by the deferment factor D_2' valid during the compressor period. The result of this calculation is shown mathematically in line 2 of Fig 18.

From this formula it may be deduced that the present value profit of P^* is proportional to the number of wells N_o but this is not entirely true, since the values of the deferment factors D_1, D_2, D_2', and D_3 and the present value production Q^* also depend on the number of wells N_o. This is because a larger number of wells will lead to more rapid depletion of the reserves and this in turn will result in increased values of the deferment factors and also in an increased value of Q^*. The relationship between the present value production Q^*, the deferment factors D_1, D_2, D_2', and D_3 and N_o can be calculated and expressed mathematically, but is not given here.

OPTIMUM OFFTAKE PATTERNS

Optimization. The optimum production rate of a gas field will be reached if any further increase in the production rate by increasing the number of wells will no longer contribute to an increase in present value profit. This may be expressed mathematically by putting the differential coefficient of the present value profit P^* with respect to the number of wells N_o equal to zero. The corresponding mathematical expression is given in the third line of Fig 18.

The mathematical procedure to develop this expression and to arrive at the required result is not given here. From the mathematical treatment it appears that the optimum dimensionless production rate f_o (*cf* Fig 17) which might be reached at any given future point in time, depends on the values of the parameters α, β, and γ and further on the value of two economic parameters, s_o and I_r. The value of the parameter s_o is the most significant. It is obtained as shown in the fourth line of Fig 18 by dividing the annual cash generation of an individual well (*i.e.* the unit cash generation u multiplied by the maximum annual production Q_o) by the individual well investment V_w. The value of the other parameter I_r expresses the ratio between the compressor investment V_c and the well investment V_w.

Optimum Rate Calculations. The results of the optimum rate calculations for various values of the parameter s_o, and valid only for the particular production pattern given in Fig 17, are given in Fig 19. In

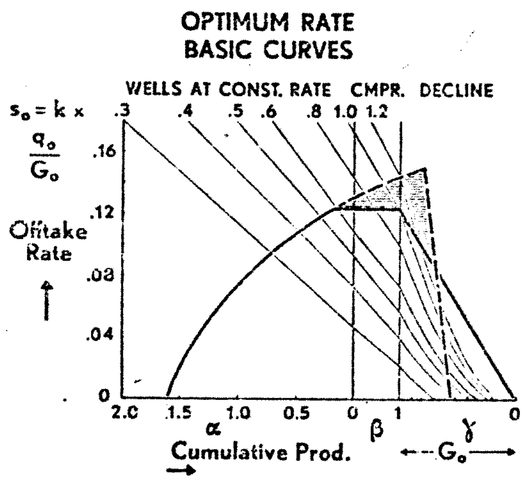

FIG 19.

this figure dimensionless annual field production rates, f_o, are plotted vertically, against the future cumulative gas production horizontally, using the particular system of units explained above in the context of Fig 17. To illustrate the use of this graph, an example is given of an uneconomic offtake pattern, represented by the broken line in Fig 19, and its modification to an economic offtake pattern represented by the continuous line. On the graph, a number of basic curves may be observed, marked by various parameter values s_o. These curves represent the mathematical relationship between optimum rate f_o and the parameters α, β, and γ for the selected s_o value. At any future point in time, characterized by the pertinent values of the parameters α, β, or γ, this line indicates the maximum economic rate of the field that may be obtained by drilling additional wells. The condition $dP^*/dN_o = 0$ is satisfied for production rates indicated by this line, and this means that any further drilling will result in a decrease of present value profit, and consequently in a return on the additional investment, which is smaller than the basic interest rate applied for the present value calculations. In the particular example given in Fig 19, the ultimate recovery of the field, or in other words the gas reserves at the start of production, equals

$$(1{\cdot}5 + 0{\cdot}4 + 1{\cdot}0)\,G_o.$$

This point in time can be characterized mathematically by the value $\alpha = 1{\cdot}5$. During the period of production build-up (*cf* Figs 15 and 16) the production rates will increase with time and consequently with cumulative production, which results in Fig 19 in decreasing values of α. The production build-up period is represented in Fig 19 by the steadily rising continuous graph starting at the point $\alpha = 1{\cdot}5$ on the horizontal axis. During its rise, this graph will cross various "s_o curves" and at a certain time will intersect with the curve corresponding to the pertinent value of s_o for the reservoir under study. In the example given in Fig 19 this value of $s_o = k \times 0{\cdot}6$. Any further increase in production rate beyond this point will result, as discussed above, in a decrease of present value profit and consequently from this point onwards production ought to remain constant until the vertical line characterized by the value $\beta =$ zero (or $\gamma = 1$, which is the same) has been reached. Thereafter production will decline according to a straight line drawn from the intersection with the line $\beta =$ zero to a point $\beta =$ zero situated on the horizontal axis. The example given above demonstrates that for any given build-up pattern, there exists a maximum optimum offtake rate of the field beyond which further investments would not yield the minimum rate of return required. A few more examples of the use of the "optimum rate basic curves" are discussed in the next section.

Examples of Optimum Offtake Patterns. In Fig 20 three hypothetical examples of offtake patterns are given for three particular gas fields. The highest graph demonstrates the optimum development pattern for a field having a large fraction of reserves which can be produced before the installation of compressors is necessary. This field corresponds, for instance, to the high productivity field shown in Fig 14.

The line representing the production build-up rises until the parameter value $s_o = k \times 0{\cdot}6$ is reached. Drilling will cease at this moment and a period of constant rate production will be obtained, until the horizontal line intersects the vertical line corresponding to the value $\alpha =$ zero (or $\beta = 0{\cdot}4$, which is the same). Thereafter, in order to maintain constant production rates, the installation of compressors will be required (compare the production pattern given in Fig 16), whereafter the field will start to decline. It should be observed that in this example, in contrast to the example given in Fig 16, no further drilling will occur after the installation of compressors. This might be understood by the following reasoning.

During the decline period, after some decline has occurred, it may be considered worthwhile to drill one or more additional wells to restore the field production capacity. This proposal could be analysed economically in a manner similar to the principles given previously, with the difference that in this particular case no further investment in compressors will be necessary, as a total amount of compressor horsepower sufficient to compress the maximum annual production rate of the field prior to the decline is already installed. This reduced investment requirement can be expressed numerically by reducing the amount of capital investment required to drill and equip an additional well and which is in fact represented by the total amount $V_w + V_c \times D_2'$ (see above). As the value of V_c can now be put equal to zero and as the value of $D_2' = 1$ in the decline period, this has the effect of decreasing the capital investment V_w to a value corresponding to $V_w^2/(V_w + V_c)$ in the formula for calculating the parameter value s_o. This in our particular example, if $V_w = V_c$, would result in a limiting s_o value of twice the value used before and this would then correspond to a parameter value of $s_o = k \times 1{\cdot}2$.

For the case of the field with high productivity, this increased s_o value is crossed by the field performance line before reaching the beginning of the decline period and consequently no further economic drilling is possible beyond this limit. This is not the case for the medium productivity field shown in Fig 20, where from the start of production comes first a period of production build-up whilst drilling, followed by a continued production build-up, with drilling and compressor installation, and then a period of constant rate is maintained by first installing compressors and secondly drilling additional wells before the field decline starts. In this particular example the line corresponding to the parameter value $s_o = k \times 1{\cdot}2$ is intersected by the field performance line after reaching a cumulative production corresponding to the value $\beta =$ zero (or $\gamma = 1$, which is the same), *i.e.* during the so-called "well decline period".

OPTIMUM OFFTAKE PATTERNS

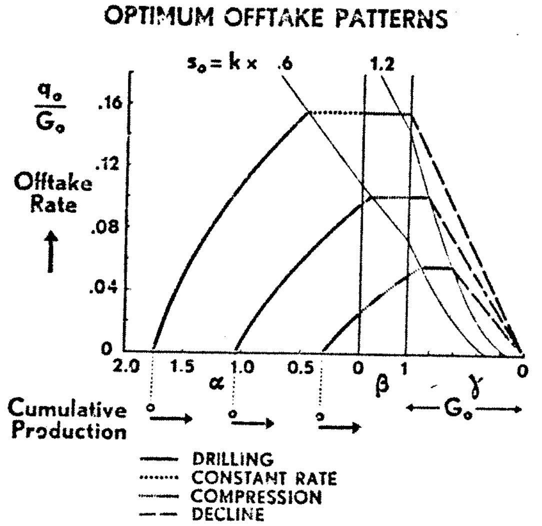

FIG 20.

Finally, a very complicated field development pattern is shown for a poor productivity field where only very limited production will occur, prior to the period during which compressors must be installed in order to deliver gas to the pipeline. Although the latter example is given mainly in order to show the use that can be made of the basic "optimum rate curves" shown in Fig 19, it could correspond to a gas field at shallow depth with low original reservoir pressure.

From Fig 20 it may be observed that a production pattern for such a field could consist of a period of production build-up whilst drilling additional wells, followed by an additional build-up, whilst concurrently drilling wells and installing compressors, followed by a further production build-up, drilling additional wells to offset individual well decline and the installing of additional compressors to increase field output, until the line marked by the parameter value $s_o = k \times 0{\cdot}1$ has been reached. Thereafter production can be kept constant by drilling additional wells (but not installing any additional compressors) until the curve corresponding to the parameter value $s_o = k \times 1{\cdot}2$ has been reached and only thereafter will field decline occur.

From the examples given above it is clear that, depending on the economic circumstances, different optimum development patterns are obtained for different fields and that for any particular given field a careful analysis of the various physical parameters will be required before such an optimum pattern can be formulated.

CONCLUSIONS

In the preceding sections a summary has been given of the step-by-step procedure required for analysing the future performance of a given gas field and formulating an optimum development pattern for such a field. The first step in this procedure is to assess as rapidly as possible the size of the field and the amount of gas in place contained in the reservoir. Secondly, a prediction must be made of the future reservoir performance to determine the amount of gas that will be recovered under normal economic circumstances of operation. Extensive well production tests will then be necessary to determine future well and field deliverabilities, from which, by means of economic calculations, likely optimum offtake patterns can be formulated. Of course, as said before, a gas field is closely linked to a particular gas market and optimum offtake patterns from the field's point of view are not always optimum offtake patterns from the market's point of view. It is the task of the gas producer to conclude a gas-sales contract to serve the market in such a manner that the resulting offtake pattern will be as close as possible to the field's optimum. It is the task of the reservoir engineer to indicate a range of optimum patterns within the limits of his uncertain knowledge of the field's parameters, shortly after discovery of the field, that will allow the gas marketers to conclude such an optimum gas-sales agreement.

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